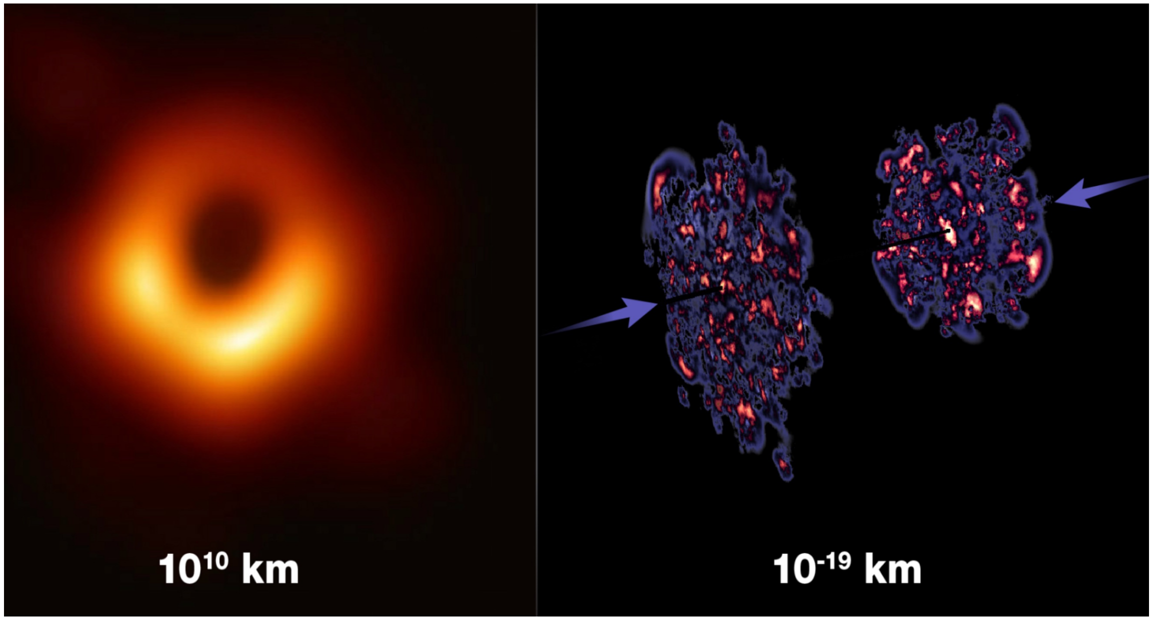
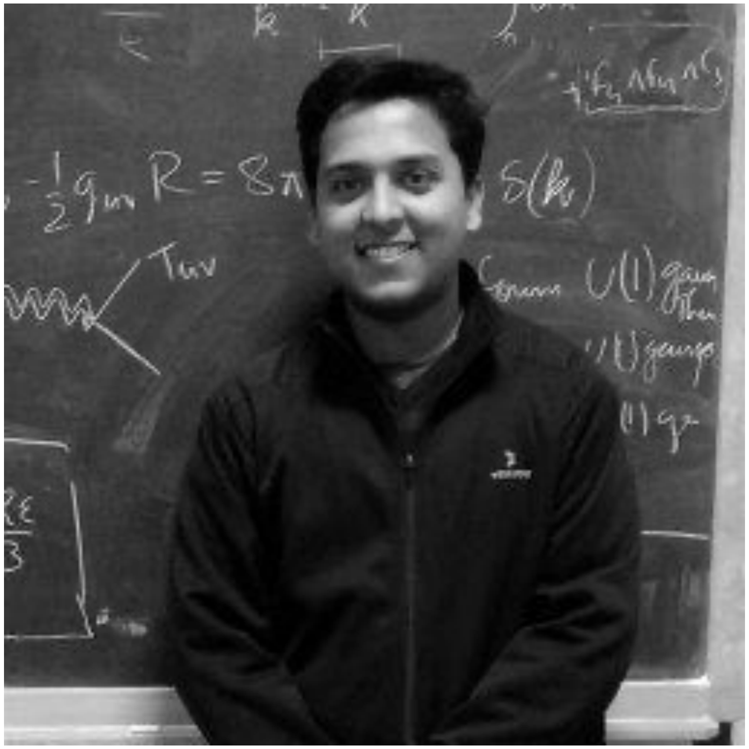
From amplitudes to shockwaves in QCD and General Relativity



Raju Venugopalan

Brookhaven National Lab, CFNS, Stony Brook, and Higgs Center, Edinburgh

66 Cracow School of Theoretical Physics, June 15, 2026

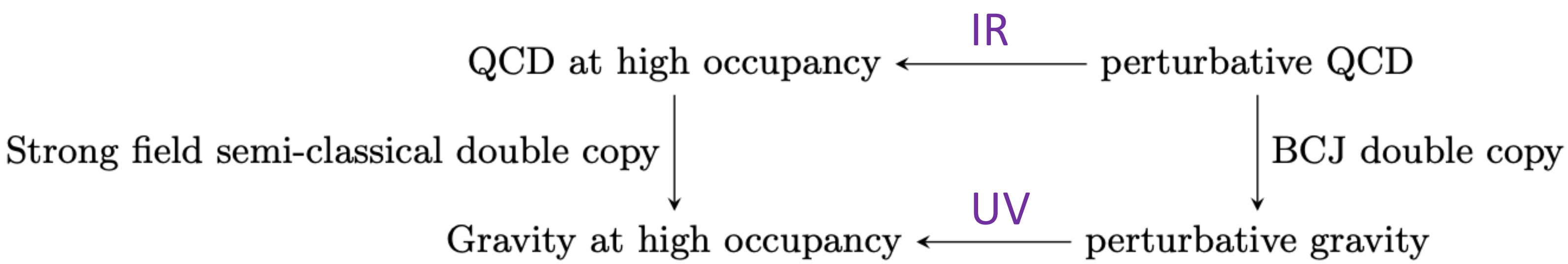


Based in part on recent work

2311.03463, 2312.03507, 2312. 11652, 2406.10483, review **2507.21252,**
2605.03038 (also with Anna Stasto), 2606.XXXXX (also with Isabelle Fite)

with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

Double Copy: gluon → gravitational radiation in shockwave collisions



Bern, Carrasco, Johannson,
arXiv: 1004.0476

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

The BCJ double copy has been exploited to perform highly precise computations of BH inspiral dynamics leading to an explosion of interest in the topic

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Lipatov, 1982-1991

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

Leonard Susskind

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13} cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys.* 36 (1995) 6377; > 4 K cites !**

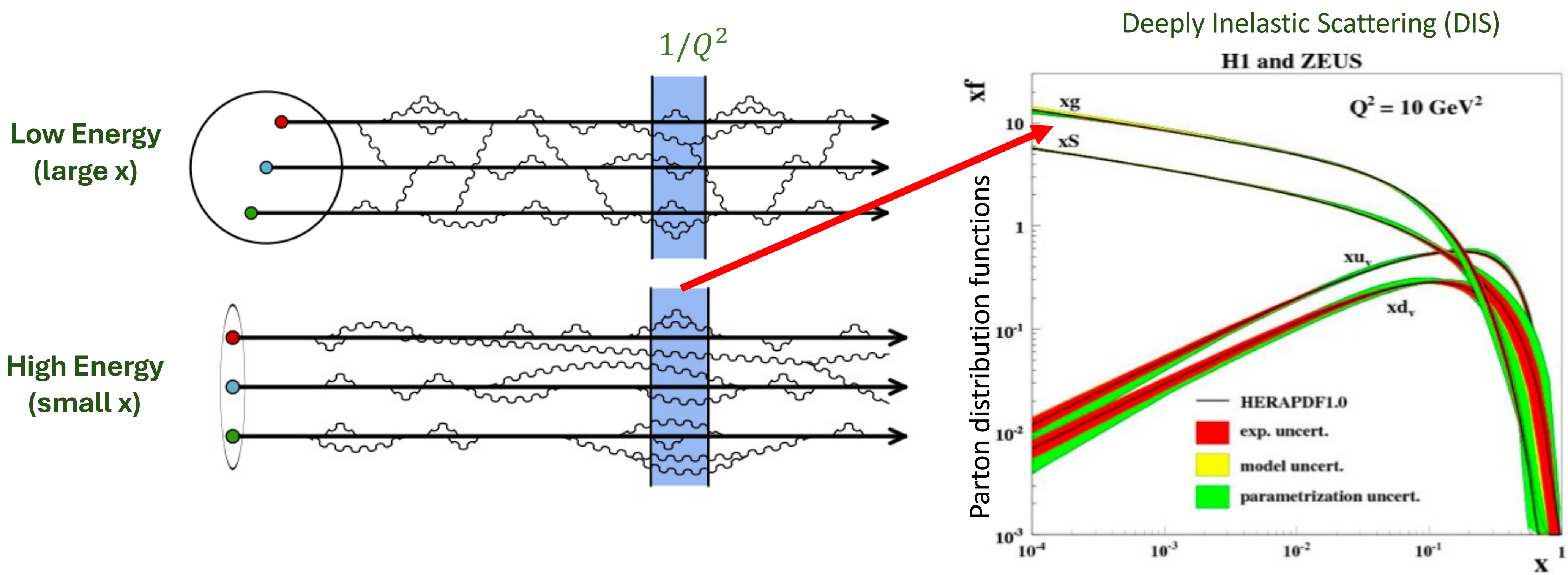
In Acknowledgements:

Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

30+ years of work by ACV et al. exploring gravitational shockwave collisions in this 2-D EFT

De Vecchia,Heissenberg,Russo,Veneziano, Phys. Rept. 1083 (2024)

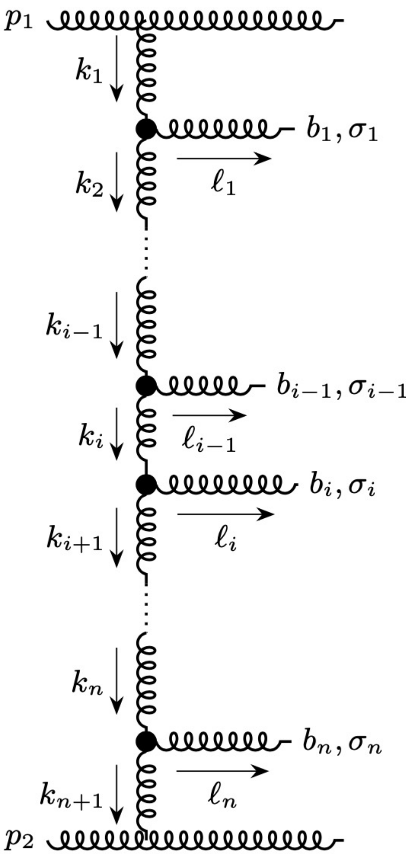
Spacetime picture of wee partons in a hadron



As the proton is boosted, “wee parton” fluctuations live longer -- released as Bremsstrahlung

Suppression in coupling compensated by large phase space for soft glue: $\alpha_S \, Ln\left(\frac{1}{x}\right) \sim 1$

Multiparticle production in QCD: BFKL paradigm for 2→ N amplitudes



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \quad \text{with} \quad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

$y_i = \mathrm{Ln}(x_i/x_{i+1})$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(``slow") but hard in transverse momentum – weak coupling Regge regime of QCD

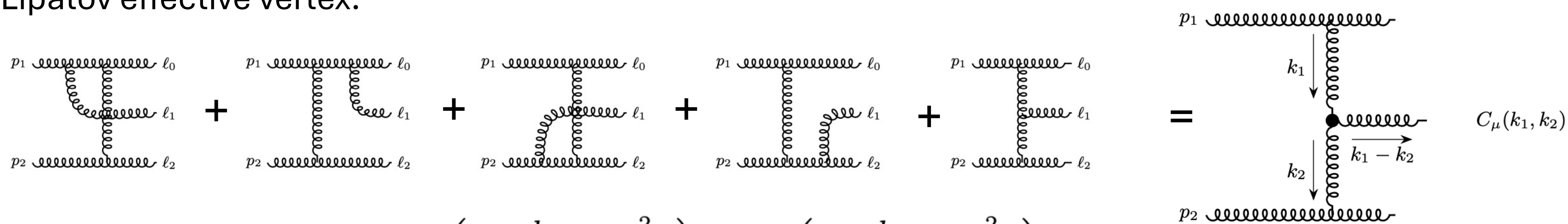
RG description rapidity of evolution given by the *BFKL Hamiltonian*
Very rapid growth of the amplitude with energy

A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

BFKL: Balitsky-Fadin-Kuraev-Lipatov (1976-1978)

BFKL: Building blocks

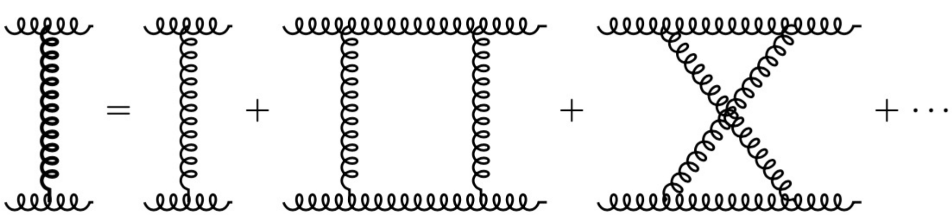
Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu C^\mu$ =0

Reggeized gluon:

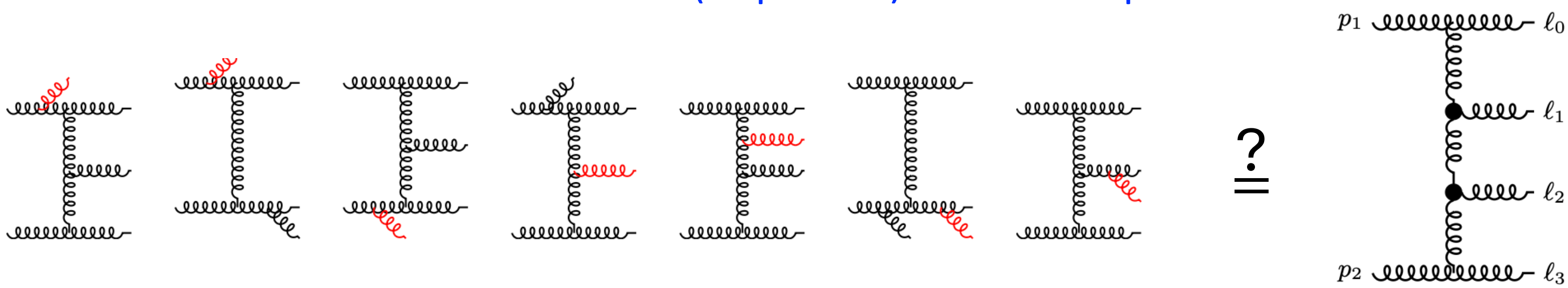


$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c \, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q} - \boldsymbol{k})^2} \,, \qquad t = -\boldsymbol{q}^2$$

At NLLx, include sub-leading correctons in Lipatov vertex and Regge trajectory – latter valid to 2-loop accuracy

2→ 4 (6 point) tree amplitude



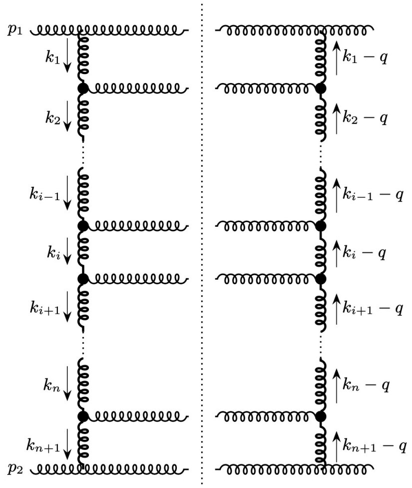
Subset of 40 graphs in MRK kinematics

Powerful use of dispersive techniques to demonstrate this equality and generalize to arbitrary N-point amplitudes. Only input is the Born diagram…

For detailed discussion, see our review arXiv:2507.21252

$2 \to \mathrm{N}+2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



The imaginary part of this $2 \to \mathrm{N}+2$ amplitude simplifies greatly in Mellin space

$$\mathcal{M}_\ell(\boldsymbol{q}^2) \equiv \int_1^\infty d\left(\frac{s}{\boldsymbol{k}^2}\right) \frac{\mathrm{Im}\,\mathcal{A}_{2\to 2}^{\mu\mu'\nu\nu'}(s,t)}{\mathcal{A}_0^{\mu\mu'\nu\nu'}(s,t)} \left(\frac{s}{\boldsymbol{k}^2}\right)^{-\ell-1}$$

with $\quad \mathcal{M}_\ell(\boldsymbol{q}^2) = 2\pi \boldsymbol{q}^2 \alpha_s N_c^2 (N_c^2 - 1) \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$

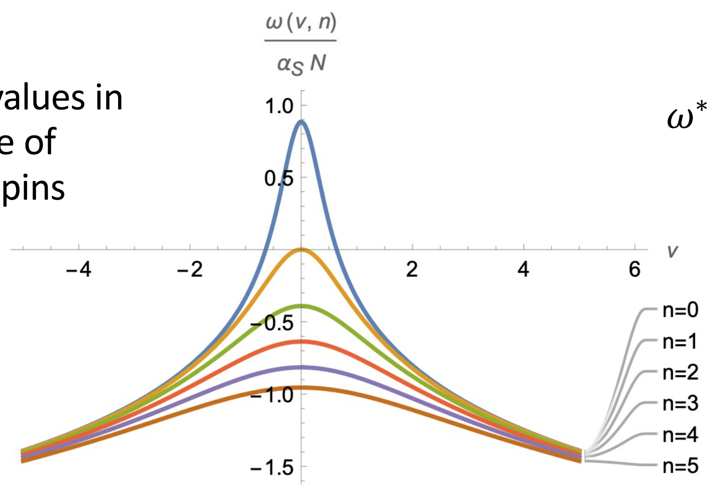
where $f_l(k,q)$ satisfies the remarkably simple BFKL integral equation

$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 - 2\alpha_s N_c \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2} \left(\boldsymbol{q}^2 - \frac{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k}')^2 + \boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k})^2}{(\boldsymbol{k}-\boldsymbol{k}')^2}\right)$$

Can be reexpressed as an RG evolution eqution for parton dists. in rapidity, governed by a BFKL Hamiltonian

$2 \to N + 2$ amplitude in the Regge limit: the BFKL equation

BFKL eigenvalues in Mellin space of conformal spins



$\omega^* = 4\, \alpha_S\, N_C \ln(2)/\pi$
~ 0.5 for $\alpha_S = 0.2$ ➡ $\sigma(s) \sim s^{0.5}$

This BFKL LLx (leading log in x) result has been extended to NLLx accuracy.

After much sophisticated analysis, (NLLx BFKL+small x resummation), gives

$\sigma(s) \sim s^{0.3}$

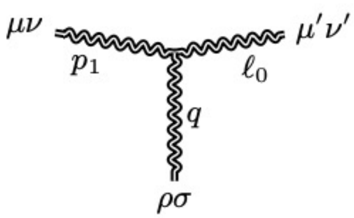
- in reasonable agreement DIS HERA data

State-of-the art BFKL review:
Del Duca, Dixon, arXiv:2203.13026

What about BFKL in GR?

On the surface, much more complicated…

3-point graviton vertex



$$
\begin{aligned}
V^{\rho\sigma}_{\mu\nu\mu'\nu'}(p_1,q) = \frac{i\kappa}{2}\Big\{ & P_{\mu\nu\mu'\nu'}\left[p_1^\rho p_1^\sigma + (p_1-q)^\rho (p_1-q)^\sigma + q^\rho q^\sigma - \frac{3}{2}\eta^{\rho\sigma} q^2\right] \\
& + 2q_\lambda q_\sigma \left[I^{\lambda\sigma}{}_{\mu\nu} I^{\rho\sigma}{}_{\mu'\nu'} + I^{\lambda\sigma}{}_{\mu'\nu'} I^{\rho\sigma}{}_{\mu\nu} - I^{\lambda\rho}{}_{\mu\nu} I^{\sigma\sigma}{}_{\mu'\nu'} - I^{\sigma\sigma}{}_{\mu\nu} I^{\lambda\sigma}{}_{\mu'\nu'}\right] \\
& + \Big[q_\lambda q^\rho \left(\eta_{\mu\nu} I^{\lambda\sigma}{}_{\mu'\nu'} + \eta_{\mu'\nu'} I^{\lambda\sigma}{}_{\mu\nu}\right) + q_\lambda q^\sigma \left(\eta_{\mu\nu} I^{\lambda\rho}{}_{\mu'\nu'} + \eta_{\mu'\nu'} I^{\lambda\rho}{}_{\mu\nu}\right) \\
& - q^2 \left(\eta_{\mu\nu} I^{\rho\sigma}{}_{\mu'\nu'} + \eta_{\mu'\nu'} I^{\rho\sigma}{}_{\mu\nu}\right) - \eta^{\rho\sigma} q^\lambda q^\sigma \left(\eta_{\mu\nu} I_{\mu'\nu',\lambda\sigma} + \eta_{\mu'\nu'} I_{\mu\nu,\lambda\sigma}\right)\Big] \\
& + \Big[2q^\lambda \big(I^{\sigma\sigma}{}_{\mu\nu} I_{\mu'\nu',\lambda\sigma}(p_1-q)^\rho + I^{\sigma\rho}{}_{\mu\nu} I_{\mu'\nu',\lambda\sigma}(p_1-q)^\sigma \\
& - I^{\sigma\sigma}{}_{\mu'\nu'} I_{\mu\nu,\lambda\sigma} p_1^\rho - I^{\sigma\rho}{}_{\mu'\nu'} I_{\mu\nu,\lambda\sigma} p_1^\sigma\big) \\
& + q^2 \left(I^{\sigma\rho}{}_{\mu\nu} I_{\mu'\nu',\sigma}{}^{\sigma} + I_{\mu\nu,\sigma}{}^{\sigma} I^{\sigma\rho}{}_{\mu\nu'}\right) + \eta^{\rho\sigma} q^\lambda q_\sigma \left(I^{\rho\sigma}{}_{\mu'\nu'} I_{\mu\nu,\lambda\rho} + I^{\rho\sigma}{}_{\mu\nu} I_{\mu'\nu',\lambda\rho}\right)\Big] \\
& + \Big[\left(p_1^2 + (p_1-q)^2\right)\left(I^{\sigma\rho}{}_{\mu\nu} I_{\mu'\nu',\sigma}{}^{\sigma} + I^{\sigma\sigma}{}_{\mu\nu} I_{\mu'\nu',\sigma}{}^{\rho} - \frac{1}{2}\eta^{\rho\sigma} P_{\mu\nu,\mu'\nu'}\right) \\
& - p_1^2 \eta_{\mu'\nu'} I^{\rho\sigma}{}_{\mu\nu} - (p_1-q)^2 \eta_{\mu\nu} I^{\rho\sigma}{}_{\mu'\nu'}\Big]\Big\}
\end{aligned}
$$

$\kappa^2 = 8\pi G$

with $P_{\mu\nu\mu'\nu'} = \frac{1}{2}\left(\eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\nu\mu'} - \eta_{\mu\nu}\eta_{\mu'\nu'}\right)$ and $I_{\mu\nu\mu'\nu'} = \frac{1}{2}\left(\eta_{\mu\mu'}\eta_{\nu\nu'} + \eta_{\mu\nu'}\eta_{\nu\mu'}\right)$

Identity operators in the space of symmetric traceless and symmetric matrices respectively

What about BFKL in GR?

In the Eikonal limit, 3-point vertex simplifies greatly! $V^{\rho\sigma}_{\mu\nu\mu'\nu'}(p_1, q) \approx i\kappa P_{\mu\nu,\mu'\nu'} p_1^{\rho} p_1^{\sigma}$

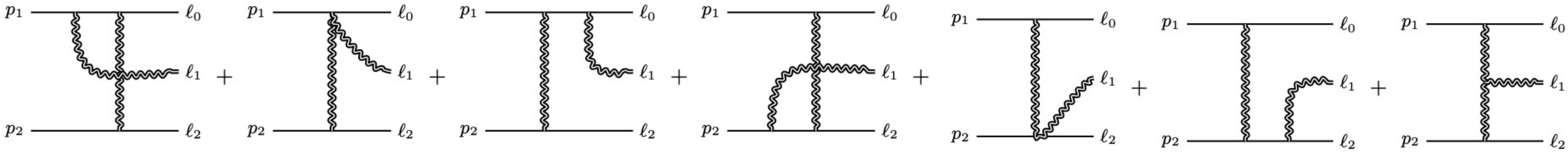
with the Grav. Born amplitude

$$\mathcal{M}^{\text{Born}}_{\mu\nu\mu'\nu'\alpha\beta\alpha'\beta'}(s,t) = \left(i\kappa P_{\mu\nu,\mu'\nu'} p_1^{\rho} p_1^{\sigma}\right) \frac{P^{\rho\sigma\rho'\sigma'}}{q^2} \left(i\kappa P_{\alpha\beta\alpha'\beta'} p_2^{\rho'} p_2^{\sigma'}\right) = \frac{1}{4}\frac{\kappa^2 s^2}{t} P_{\mu\nu\mu'\nu'} P_{\alpha\beta\alpha'\beta'}$$

Upper vertex × Propagator in deDonder gauge × Lower vertex

What about BFKL in GR?

Gravitational Lipatov vertex from leading 2→ 3 MRK contributions:



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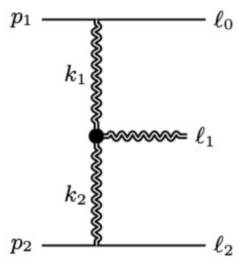

Lipatov discovered (1982) this is a double copy!

$$C^{\mu\nu}(k_1,k_2) = \frac{1}{2}C^{\mu}(k_1,k_2)C^{\nu}(k_1,k_2) - \frac{1}{2}N^{\mu}(k_1,k_2)N^{\nu}(k_1,k_2)$$

Bi-linear of QCD Lipatov vertex

∝ Bi-linear of QED bremsstrahlung vertex

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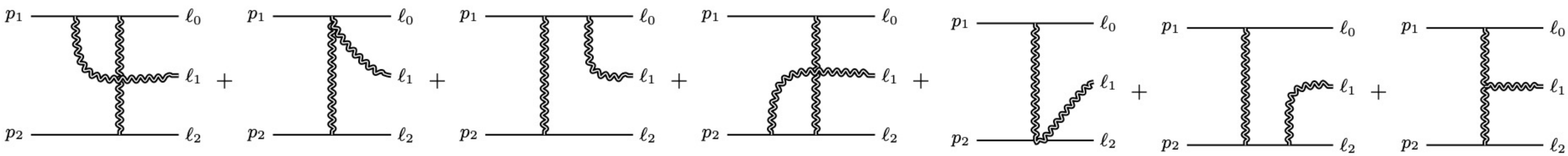

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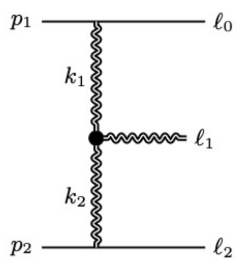

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Bi-linear of QCD Lipatov vertex   ∝ Bi-linear of QED bremsstrahlung vertex

Reggeization of the graviton propagator proceeds similarly to QCD where $\alpha(t_i)$ is the graviton Regge trajectory

$$\frac{1}{t_i} \to \frac{1}{t_i}e^{\alpha(t_i)(y_{i-1}-y_i)}$$

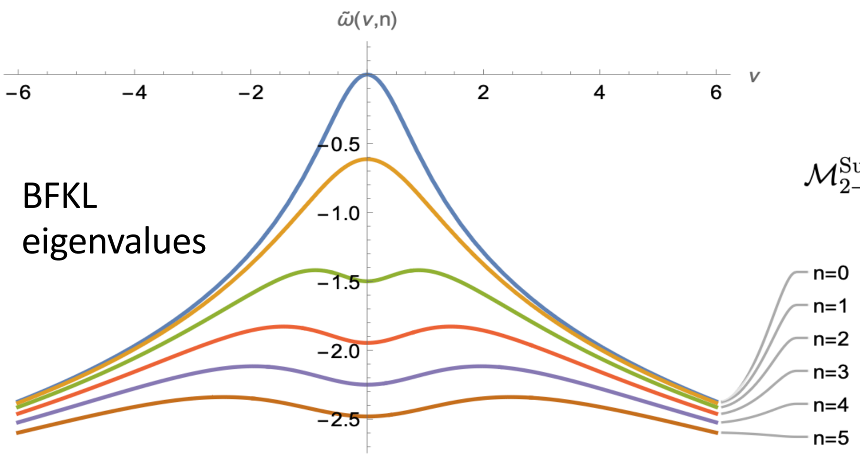
Indeed, the entire 2→ $N+2$ GR BFKL construction *follows exactly analogously as in QCD* with the GR Lipatov vertex and the graviton Regge trajectory as the building blocks

BFKL equation in Einstein Gravity

Integral equation derived by Lipatov for the Mellin amplitude: $\mathcal{M}_\ell(t) = \frac{t}{16}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$

$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 + \frac{\kappa^2}{4\pi}\int \frac{d^2\boldsymbol{k}'}{(2\pi)^2}\frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2}\mathcal{K}_G(\boldsymbol{k},\boldsymbol{k}')$$

$$C^{\mu_i\nu_i}(k_i, k_{i+1}) C_{\mu_i\nu_i}(q-k_i, q-k_{i+1})$$



$$\mathcal{M}_{2\to2}^{\text{Sudakov}+L^2} \sim \mathcal{M}^{\text{Born}}\left(-\frac{s}{t}\right)^{\kappa^2 t\log(-t/\Lambda_{\text{IR}})/8\pi^2}\frac{1}{3}\left[1+\left(-\frac{s}{t}\right)^{\sqrt{-3\kappa^2 t/8\pi^2}}+\left(-\frac{s}{t}\right)^{-\sqrt{-3\kappa^2 t/8\pi^2}}\right]$$

However, unlike QCD, growth with energy is slower than the Born amplitude $\propto s^2$

Bartels,Lipatov,Sabio-Vera, arXiv 1208.3423

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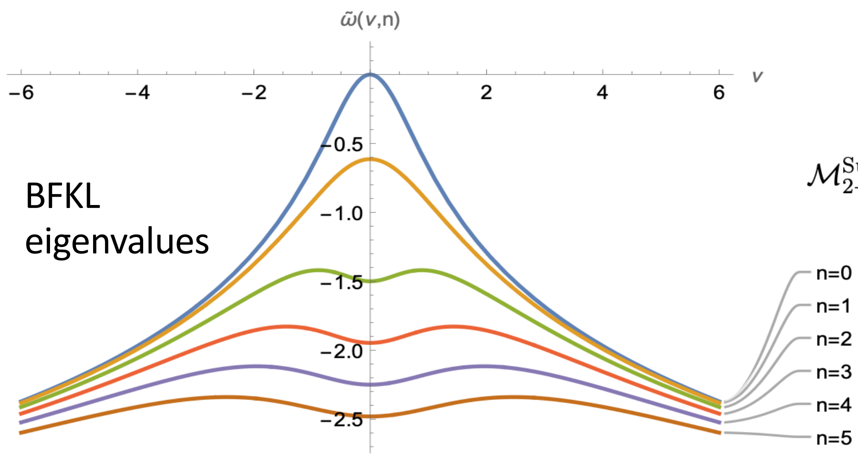
$$C^{\mu_i\nu_i}(k_i,k_{i+1})C_{\mu_i\nu_i}(q-k_i,q-k_{i+1})$$

BFKL eigenvalues



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However, unlike QCD, growth with energy is slower than the Born amplitude $\propto s^2$

Bartels,Lipatov,Sabio-Vera, arXiv 1208.3423

Interestingly, the soft limit of the Lipatov vertex smoothly goes over to the ultrarelativistic limit of Weinberg's famous radiative amplitude for soft graviton emission

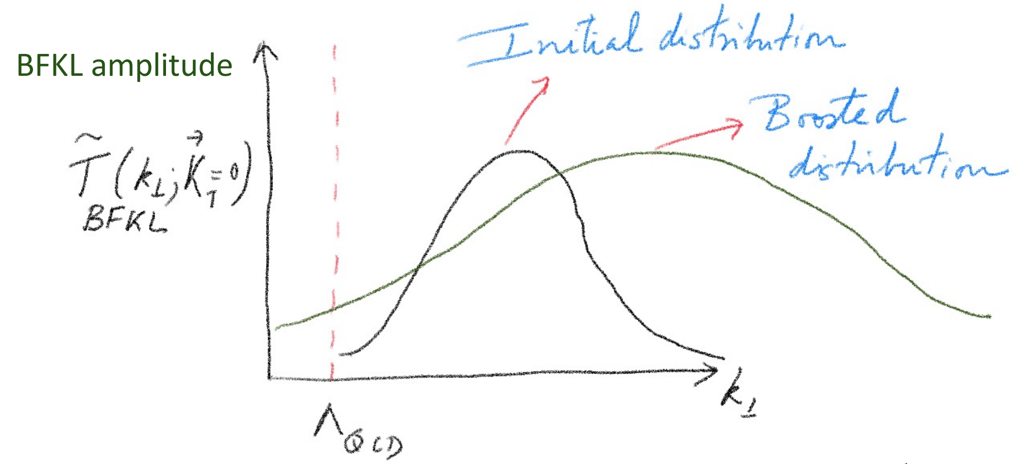
H. Raj, RV, arXiv:2507.21252

From amplitudes to shockwaves in QCD

In perturbative QCD at high energies,

BFKL breaks down -- infrared diffusion of solution

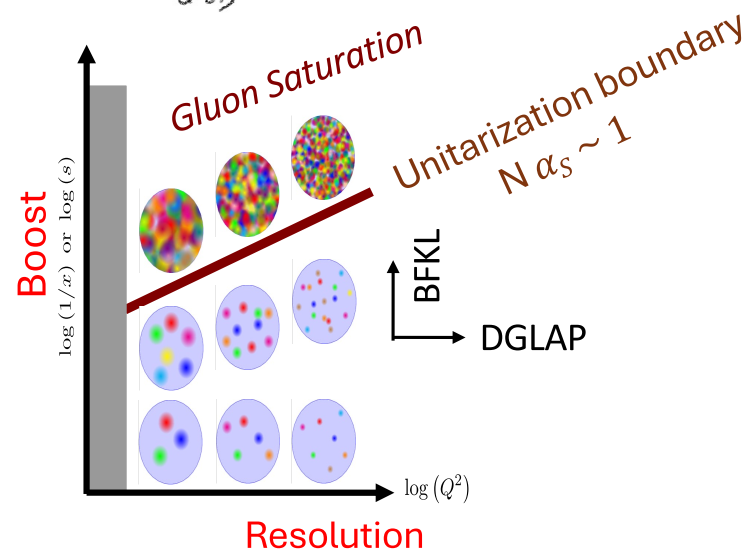
- not cured by higher order corrections…



For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251



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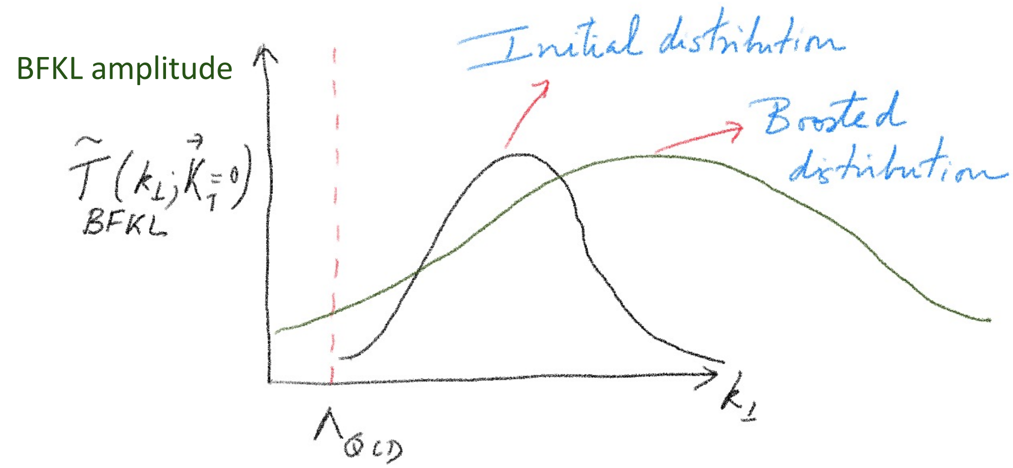
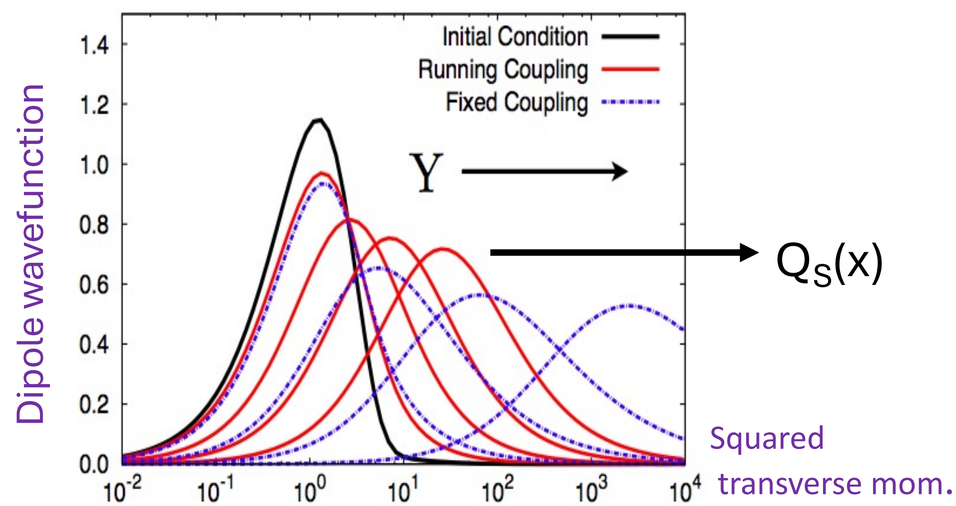


Gluon saturation cures infrared diffusion…

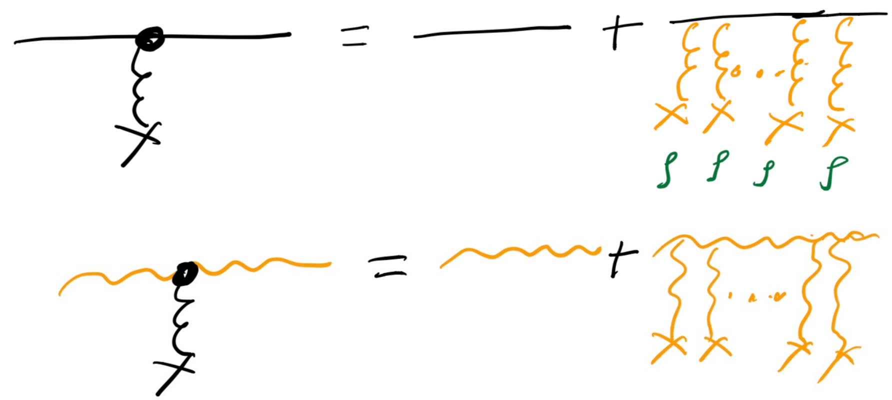
Explicitly demonstrated in solutions
of Balitsky-Kovchegov equation incorporating
physics of strong color fields in the CGC EFT

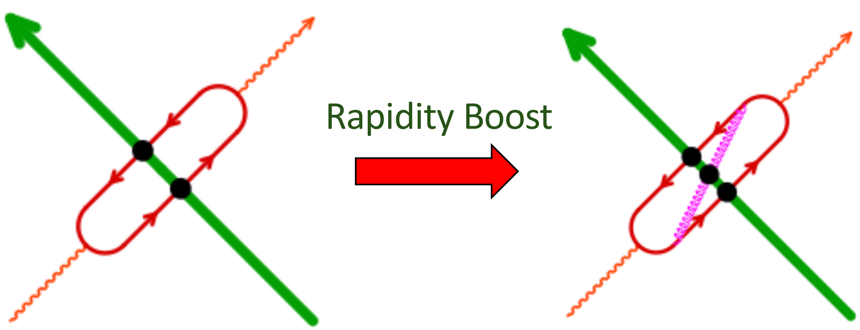


From amplitudes to shockwaves in QCD

Leading order CGC EFT power counting: *multiple scattering* (represented by lightlike Wilson lines)

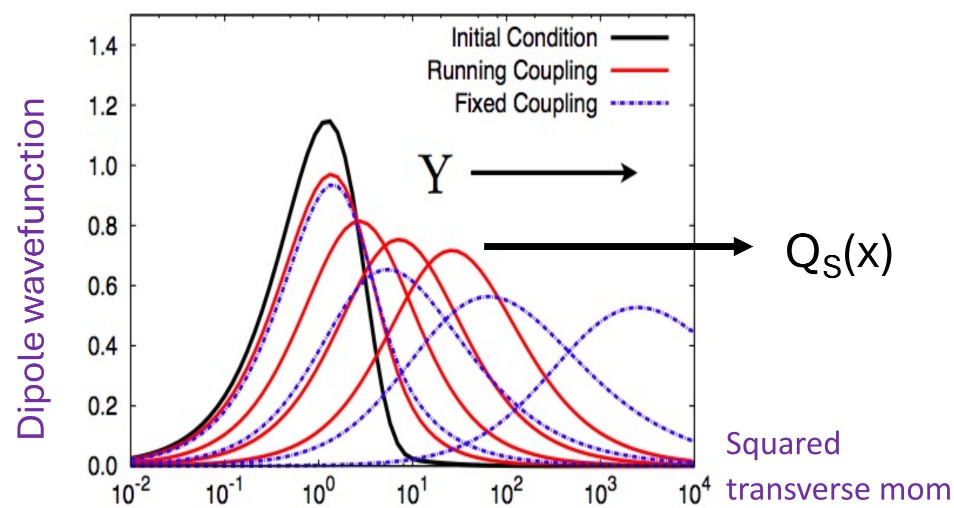
-radiative corrections computed in "shockwave" background



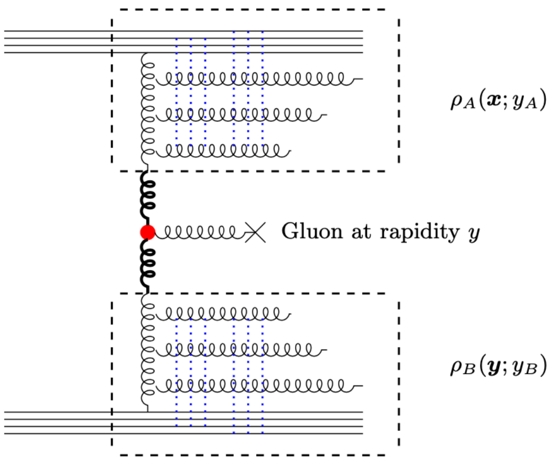


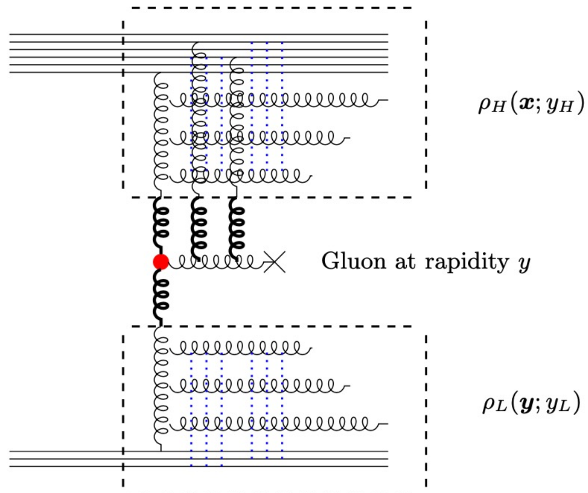
LO dipole cross-section in shockwave background

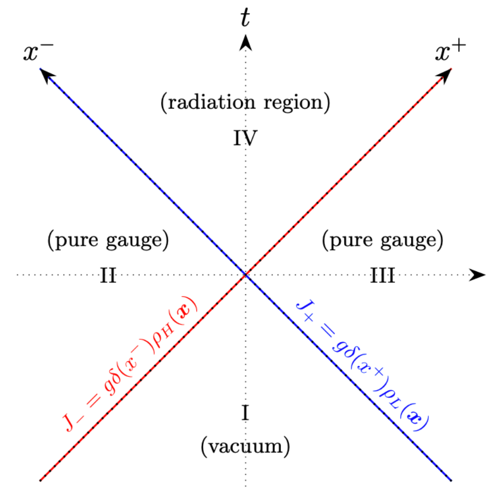
NLO dipole cross-section in shockwave background



Gluon shockwave collisions: Lipatov vertex and reggeization







Weizs$\ddot{a}$cker-Williams gluon radiation field from classical Yang-Mills shockwave solutions

$$a_i(k) = -\frac{2ig}{k^2 + i\epsilon} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \left(q_{2i} - k_i \frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2} \right) \frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \left(U(\boldsymbol{k} + \boldsymbol{q}_2) - (2\pi)^2 \delta^2(\boldsymbol{k} + \boldsymbol{q}_2) \right)$$

Lipatov vertex in $A^- = 0$ gauge

reggeized gluons from semi-classical source dists.

$$U(x^-, \boldsymbol{x})\delta\left(x^+\right) = \exp\left(ig \int_{-\infty}^{x^-} dz^- \bar{A}_-(z^-, \boldsymbol{x}) \cdot T \right)$$

$$\bar{A}_\mu(x^-, \boldsymbol{x}) = -g\delta_{\mu -}\delta\left(x^-\right) \frac{\rho_H\left(\boldsymbol{x}\right)}{\Box_\perp}$$

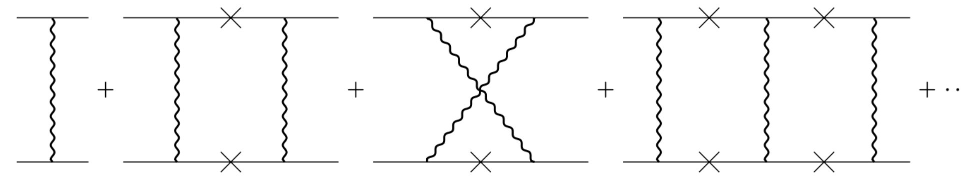
Ln (U) → *reggeized gluon*

Dumitru,McLerran (2002)
Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

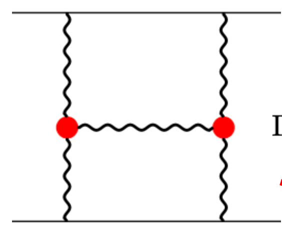
Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Power counting in gravitational scattering

In gravity, dominant contribution at large impact parameters is Eikonal multiple scattering (analogously to the CGC EFT)



Gravitational Lipatov radiation is sub-leading but increasingly important as b → R_S
So-called "H-diagram" of ACV



$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2) - \frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

Amati,Ciafaloni,Veneziano (1987, 1990)

Gravitational S-matrix:

$$S = e^{2i(\delta_0+\delta_1+\delta_2+\cdots)} \qquad \delta_0 = Gs\log\left(\frac{L}{b}\right), \qquad \delta_1 = \frac{6G^2 s}{\pi b^2}\log s\,, \qquad \delta_2 = \frac{2G^3s^2}{b^2}\left[1+\frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2}+2\right)\right]$$

Leading Eikonal term (real)

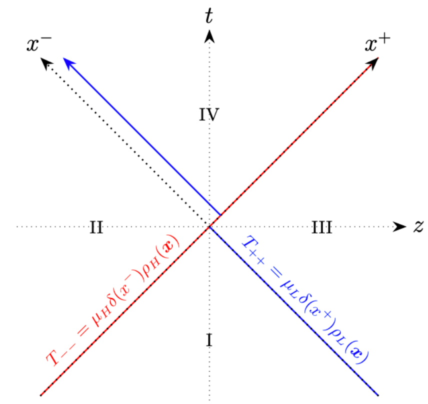
Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$
"absorptive" – radiative contribution

$\delta_2 \gg \delta_1$ for $R_S \gg l_P$

Lipatov vertex from shockwave collisions in GR

We will now sketch how the Lipatov vertex is recovered in shockwave collisions



Aichelburg-Sexl shockwave metric

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\,(dx^-)^2$$

with $\quad f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi} \mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\; \ln \Lambda |\boldsymbol{x} - \boldsymbol{y}| \rho_H(\boldsymbol{y})$

Soln of Einstein's eqns sourced by the EM tensor $\quad T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H \delta(x^-) \rho_H(\boldsymbol{x})$

$\mu_H = m_H\, \gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\, G$

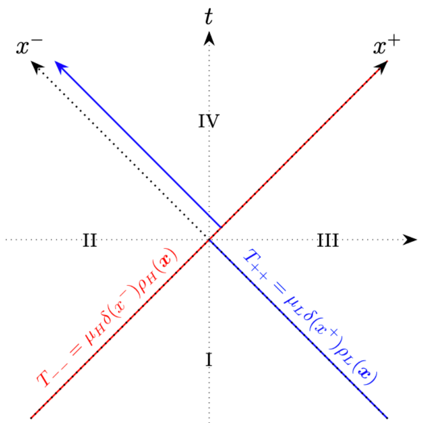
Shockwave collisions: single shock background

Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

fixing light cone gauge $h_{\mu +}$=0, find

$$h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\frac{1}{2} \int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\, \partial_+ \right)$



μ_H= $m_H\, \gamma$ = fixed for $\gamma \to \infty$

κ^2=8 π G

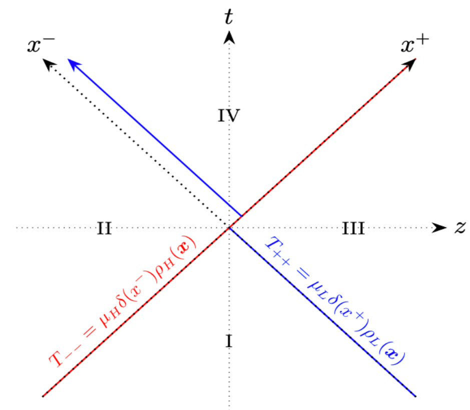
Exactly analogous to the QCD case with $A_- \to\ g_{--}$ and $T^a \to \partial_+$: Shapiro time delay

Dray and ’t Hooft (1986)

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

Saotome and Akhoury, arXiv:1210.8111

Shockwave collisions: “dilute-dense” approximation



Now consider the interaction of the “dilute” source ρ_L with the dense ρ_H shockwave:

$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H\delta\left(x^-\right)\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L\delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H\delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

We decompose the perturbation $h_{\mu\nu}$ into a term linear in ρ_L and one bi-linear in $\rho_L\rho_H$ (dilute-dilute limit)

Linearized Einstein’s equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2\tilde{h}_{ij} - \Box\tilde{h}_{ij} = \kappa^2\left[\left(2\partial_i\partial_j - \Box_\perp\delta_{ij}\right)\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}\left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right)\right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij}h \text{ where } h = \delta_{ij}h_{ij}$$

Raj, RV, arXiv:2311.03463, arXiv:2406.10483, arXiv:2507.21252

Shockwave collisions in general relativity: geodesics

Unlike QCD case, sub-eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

Not uniquely fixed by energy-momentum conservation:
Dynamics of the sources is needed. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-g}} \int_{-\infty}^{\infty} d\lambda \; \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

Solution of the corresponding null geodesic equations: $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \; , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

in shockwave background given by $X^- = \lambda \; , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

These geodesic solutions allow us to reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains, to O($\rho_{\mathrm{L}}\rho_{\mathrm{H}}$)

Gravitational radiational field

$$\tilde{h}_{ij}^{(2)}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2 \boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

$$\mathcal{M}^{(\lambda)} = k^2 \tilde{h}_{ij}^{(2)}(k) \varepsilon_{ij}^{(\lambda)}$$

$$\frac{dE^{\mathrm{GW}}}{d\omega d\Omega} = \frac{1}{2\pi^2} \omega^2 \sum_\lambda |\mathcal{M}^{(\lambda)}|^2$$

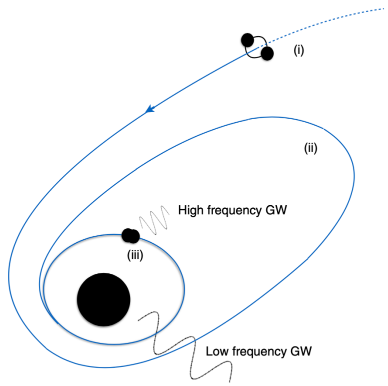
Gravitational Lipatov vertex

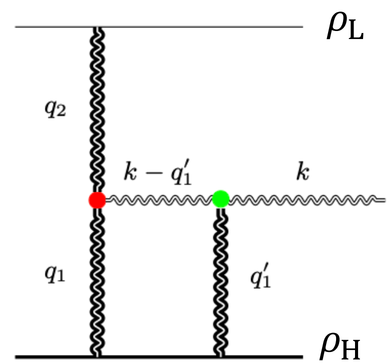
likewise for other components, recovering $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

Compare to gauge theory radiation field

$$a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2 \boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

Shockwave collisions: first dilute-dense "tidal" correction





M. Fite, H. Raj, RV, in preparation

The shockwave computation can be extended straightforwardly to O($\rho_\mathrm{L}\, \rho_\mathrm{H}^2$)

$$\tilde{h}_{ij}^{(1,2)}(k) = \frac{2\mu_L \mu_H^2 \kappa^6(-ik^-)}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_1' d^2\boldsymbol{q}_2}{(2\pi)^4} \delta^{(2)}(\boldsymbol{k} - \boldsymbol{q}_1 - \boldsymbol{q}_1' - \boldsymbol{q}_2) \frac{\tilde{\rho}_H(\boldsymbol{q}_1)}{\boldsymbol{q}_1^2} \frac{\tilde{\rho}_H(\boldsymbol{q}_1')}{\boldsymbol{q}_1'^2} \frac{\tilde{\rho}_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} V_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_1', \boldsymbol{q}_2)$$

$$\frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2} \tilde{S}_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_1', \boldsymbol{q}_2) - \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2)$$

$$\alpha\left(-2k_i k_j + \boldsymbol{k}^2 \delta_{ij}\right) + \left(q_{1i}\, q_{1j}' + q_{1i}'\, q_{1j} - \delta_{ij}\, \boldsymbol{q}_1 \cdot \boldsymbol{q}_1'\right) + \beta\left(k_i\, s_j + k_j\, s_i - \delta_{ij}\, \boldsymbol{k} \cdot \boldsymbol{s}\right)$$

$$\alpha = \frac{1}{2} + \frac{\boldsymbol{s} \cdot \boldsymbol{q}_2 - \boldsymbol{q}_1 \cdot \boldsymbol{q}_1'}{\boldsymbol{k}^2} + \frac{4(\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)(\boldsymbol{q}_1' \cdot \boldsymbol{q}_2)}{\boldsymbol{k}^4}$$

Landau-Pomeranchuk-Migdal effect in GR when coherence time for radiation exceeds the mean free path

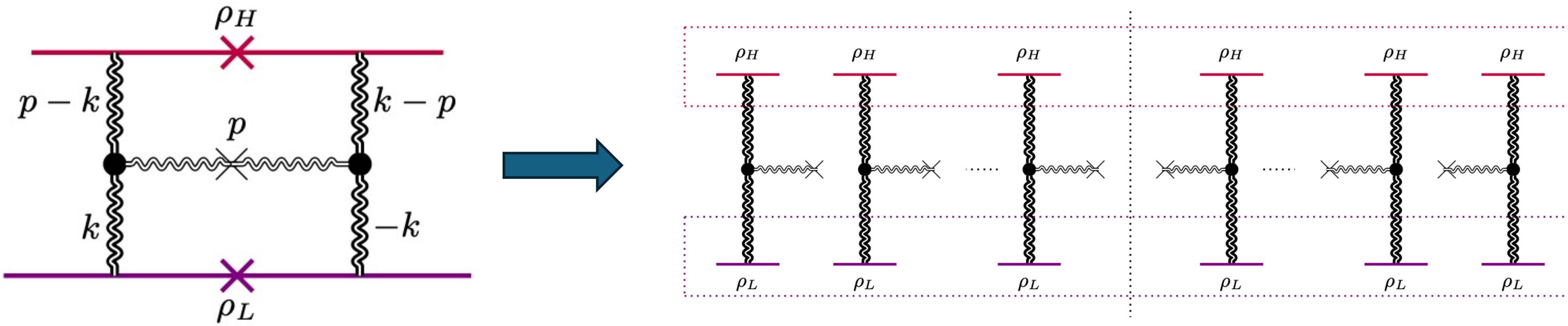
$$\beta = 1 + \frac{2\, \boldsymbol{s} \cdot \boldsymbol{q}_2}{\boldsymbol{k}^2} \qquad \boldsymbol{s} \equiv \boldsymbol{q}_1 + \boldsymbol{q}_1' = \boldsymbol{k} - \boldsymbol{q}_2$$

Multi-graviton Lipatov radiation a la CGC EFT in GR?

In QCD, multi-gluon radiation in shockwave scattering (to LLx accuracy) is given by

$$\left\langle \frac{d^n N_n}{d^3\boldsymbol{p}_1 \cdots d^3\boldsymbol{p}_n} \right\rangle_{\text{LLog}} = \int [D\rho_1]\,[D\rho_2]\; W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\;\; \frac{dN}{d^3\boldsymbol{p}_1}\bigg|_{\text{LO}} \cdots \frac{dN}{d^3\boldsymbol{p}_n}\bigg|_{\text{LO}}$$

Gelis, Lappi, RV, arXiv 0807.1306



Corresponding "n-particle" distribution: negative binomial distribution (NBD)

$$P_{n;r} = \frac{\Gamma(n+r)}{\Gamma(r)\Gamma(n+1)} \frac{\bar{n}^n r^r}{(\bar{n}+r)^{n+r}}$$

with $\quad r = \zeta \frac{(N_c^2-1) S_\perp Q_S^2}{2\pi}$

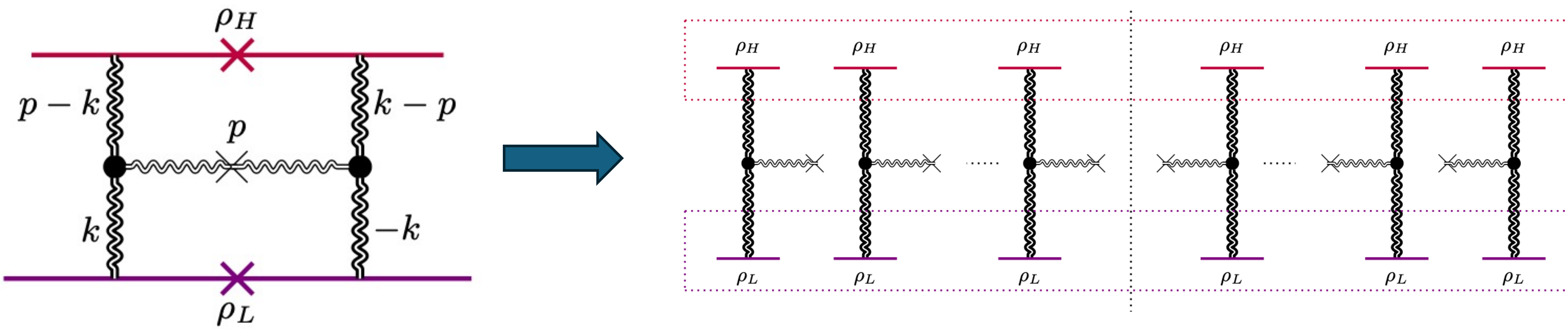
Gelis,Lappi,McLerran, arXiv: 0905.3234

Can a similar t-channel fractionation occur in GR in the strong field regime, as b→ R_S ?

Multi-graviton Lipatov radiation a la CGC EFT in GR?

In QCD, multi-gluon radiation in shockwave scattering (to LLx accuracy) is given by

$$\left\langle \frac{d^n N_n}{d^3\boldsymbol{p}_1 \cdots d^3\boldsymbol{p}_n} \right\rangle_{\rm LLog} = \int [D\rho_1]\,[D\rho_2]\; W_{Y_1}[\rho_1]\; W_{Y_2}[\rho_2]\; \left.\frac{dN}{d^3\boldsymbol{p}_1}\right|_{\rm LO} \cdots \left.\frac{dN}{d^3\boldsymbol{p}_n}\right|_{\rm LO}$$



As we will argue, such radiation can be understood as a particular squeezed coherent state

This is interesting because it has been argued that such states can enhance quantum effects that are naturally only accessible at Planck scale resolution.

T. Guerrero, Class. Quant. Grav. 37 (2020)
Parikh, Wilczek, Zaharaide, PRL 127 (2021)

Multi-graviton radiation: generalized Susskind-Glogower squeezed state

If GR radiation is a NBD, this distribution corresponds to a squeezed state

$$|z;r\rangle = (1-|z|^2)^{r/2} e^{z a^\dagger \sqrt{\hat{N}+r}} |0\rangle$$ where $|z^2| = p = \frac{\bar{n}}{\bar{n}+r}$ and $\hat{N} = a^\dagger a$

Eigenstate of the (generalized) Susskind-Glogower operator (gSG) $\hat{A} = a\sqrt{\frac{1}{\hat{N}+r-1}}$

$$\hat{A}|z;r\rangle = z\,|z;r\rangle$$

For NBD parameter r=1, this is the Susskind-Glogower-Barnett-Pegg phase operator

$$\hat{A}_{r=1}|\phi\rangle = e^{i\phi}|\phi\rangle \ \text{with}\ |\phi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} e^{in\phi}|n\rangle .$$

Stasto, Raj, RV: arXiv 2605.03038
Contribution to special APP volume

Multi-graviton radiation: generalized Susskind-Glogower squeezed state

For a pure single-mode squeezed state, $\Delta X^2\,\Delta P^2 = \frac{\hbar^2}{4}$

In gSG case of interest, $(\Delta X)^2 = \frac{1}{2} + \frac{rp}{1-p} + (1-p)^r p\, A_2(p) - 2(1-p)^{2r} p\, A_1(p)^2$;

$$(\Delta P)^2 = \frac{1}{2} + \frac{rp}{1-p} - (1-p)^r p\, A_2(p).$$

A_1 and A_2 are infinite sums in powers of p

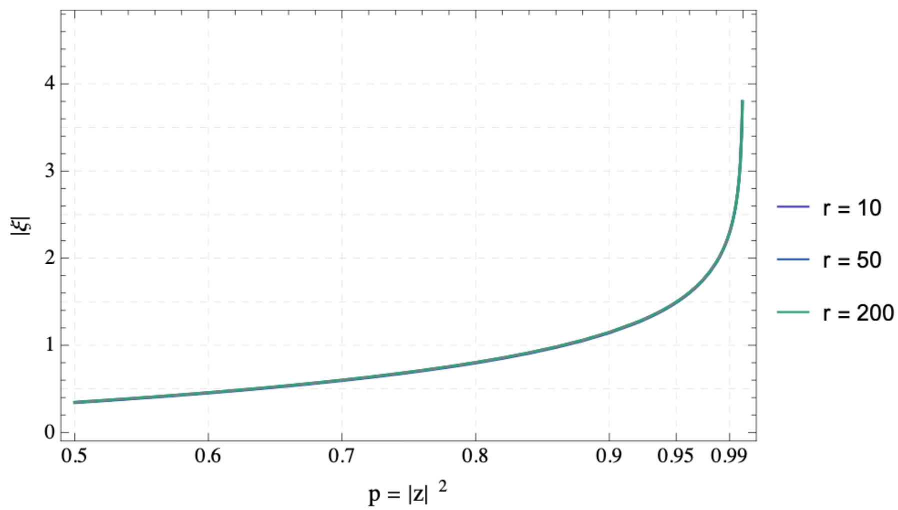
Obtained by writing $|z;r\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$ with $c_n = (1-|z|^2)^{r/2}\sqrt{\binom{n+r-1}{n}}\, z^n$ and computing expectation values of creation/annihilation operators (and squares) in this state

Writing $(\Delta X)^2 = \left(\delta + \frac{1}{2}\right) e^{-2\xi}$; and $(\Delta P)^2 = \left(\delta + \frac{1}{2}\right) e^{+2\xi}$

$\sqrt{(\Delta X)^2 (\Delta P)^2} = \frac{1}{2} + \delta$. and $\xi = \frac{1}{4} \ln \frac{(\Delta P)^2}{(\Delta X)^2}$

Can explore the parameter space of minimal values of δ sensitive to O($\hbar$) effects
and large squeezing parameters ξ

Multi-graviton radiation: generalized Susskind-Glogower squeezed state



Wide parameter space for NBD parameter r > 1 where δ is small (close to minimum uncertainty) but squeezing parameter $\xi \gg 1$

For r large, $\xi = Ln(\bar{n})$: For gravitational wave measured by LIGO, $\bar{n} \approx 4.10^{36}$, so very large squeezing values are possible…

Specifically, quantum fluctuations on the Planck scale (10^{-35} m) can be enhanced to detectable levels at current and future gravitational wave observatories

Caveat: NBD statistics are “super-Poisson” which means that quantum effects can also be mimicked by classical sources

D. Carney, arXiv:2408.00094

Summary

In QCD at very high energies (small x), nonperturbatively large phase space occupancies lead to a semi-classical ”shockwave” picture of multi-particle production

A striking double copy is seen between gluon radiation and gravitational radiation in shockwave collisions

Many-body strong field techniques in the QCD context can be ported to gravitational radiation in the strong field merger regime – a particularly valuable application are to Extreme Mass Ratio Inspirals (EMRIs)

Intriguing possibility that gravitational radiation as b→ $\mathrm{R_S}$ is in the form of a gSG squeezed state

- large squeezing parameters are feasible in such states potentially enhancing signals for quantum effects

Much work remains to solidify the computational framework and to isolate the distinctive features of classical and semi-classical effects in the strong field regime