

Entanglement Entropy for Elastic and Inelastic Scattering

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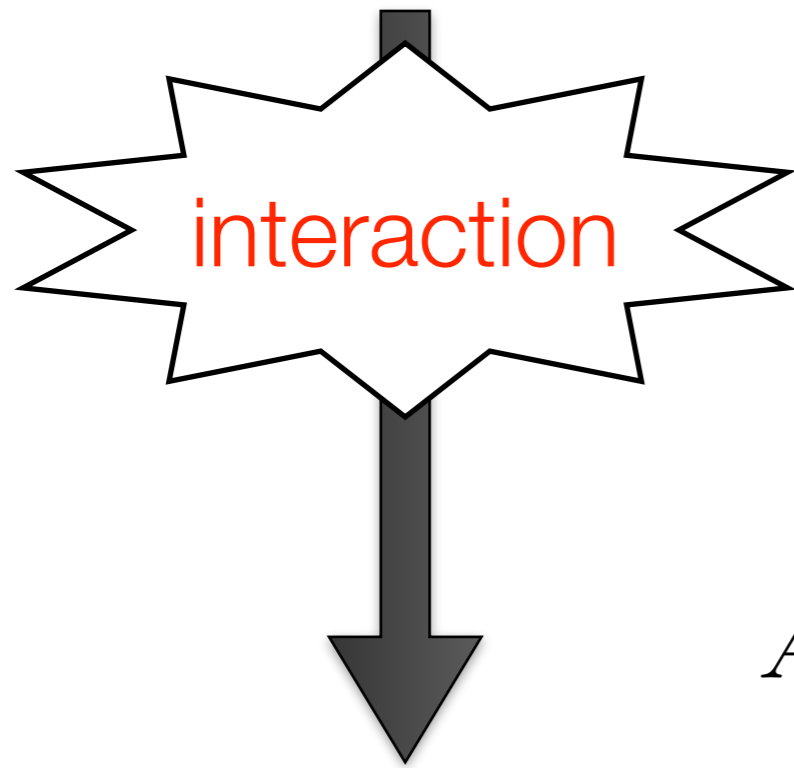
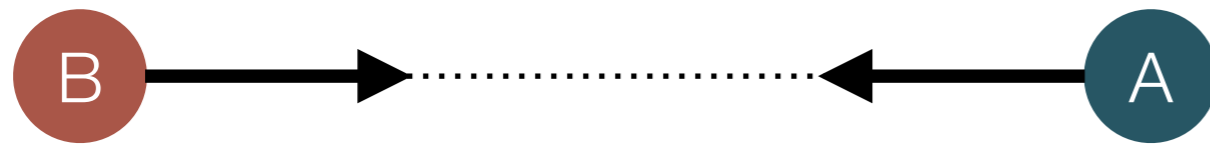
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This talk is based on the works with Robi Peschanski (IPhT, CEA-Saclay):

- R. Peschanski and S. Seki, “Entanglement in Elastic and Inelastic Two-particle Scatterings at High Energy,”
arXiv:5601.22502, accepted for publication in Phys. Rev. D.
- R. Peschanski and S. Seki, “Evaluation of Entanglement Entropy in High Energy Elastic Scattering,” Phys. Rev. D 100 (2019) no.7, 076012.
- R. Peschanski and S. Seki, “Entanglement Entropy of Scattering Particles,” Phys. Lett. B 758 (2016), 89.

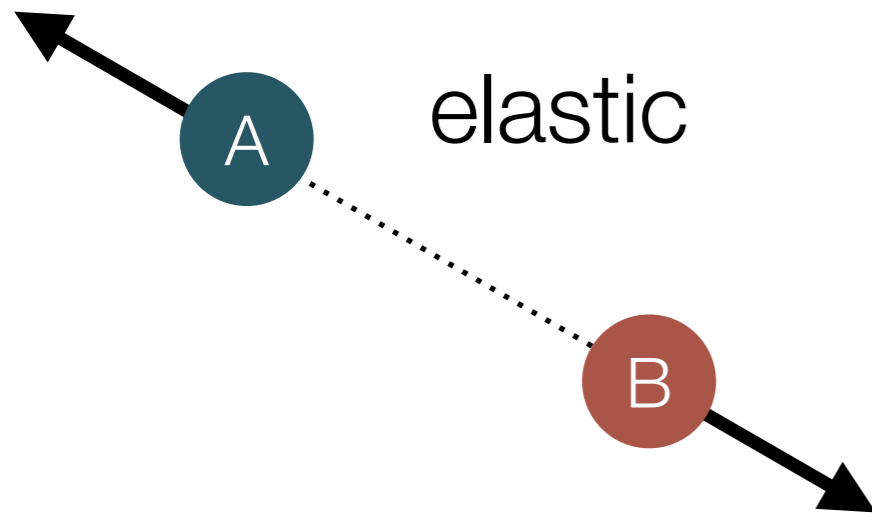
Question: What is the entanglement entropy of the final state in scattering?

an **unentangled** pair of incoming particles

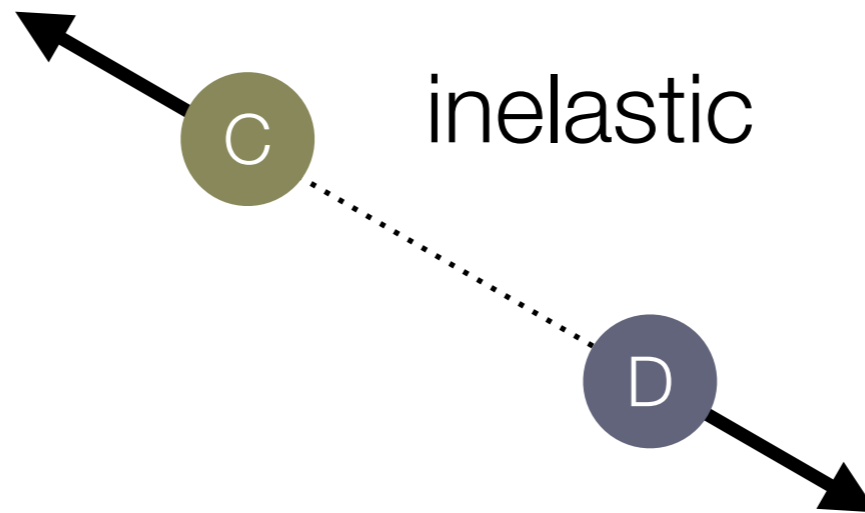


The interaction produces entanglement.

$$AB \rightarrow \begin{cases} AB & \text{(elastic)} \\ CD & \text{(inelastic)} \\ X & \text{(more than 2 particles)} \end{cases}$$



or



or

multi-particle final states

an **entangled** pair of two outgoing particles

Formulation of entanglement entropy for 2-particle final state

Let us consider

$$A_1 B_1 \rightarrow \begin{cases} A_1 B_1 & \text{(elastic channel)} \\ A_2 B_2 & \text{(two-particle inelastic channel)} \\ X & \text{(more than two-particle channels)} \end{cases}$$

We focus on a two-particle state in the [momentum Hilbert space](#).

$$\mathcal{H}_1 = \mathcal{H}_{A_1} \otimes \mathcal{H}_{B_1}, \quad \mathcal{H}_2 = \mathcal{H}_{A_2} \otimes \mathcal{H}_{B_2}$$

$$|\mathbf{p}, \mathbf{q}\rangle_1 \equiv |\mathbf{p}\rangle_{A_1} \otimes |\mathbf{q}\rangle_{B_1} \in \mathcal{H}_1, \quad |\mathbf{p}, \mathbf{q}\rangle_2 \equiv |\mathbf{p}\rangle_{A_2} \otimes |\mathbf{q}\rangle_{B_2} \in \mathcal{H}_2$$

$${}_I\langle \mathbf{p} | \mathbf{q} \rangle_J = \delta_{IJ} \sqrt{2E_{I\mathbf{p}} 2E_{J\mathbf{q}}} \delta^{(3)}(\mathbf{p} - \mathbf{q})$$

$$E_{I\mathbf{p}} = \sqrt{m_I^2 + p^2} \quad (p \equiv |\mathbf{p}|)$$

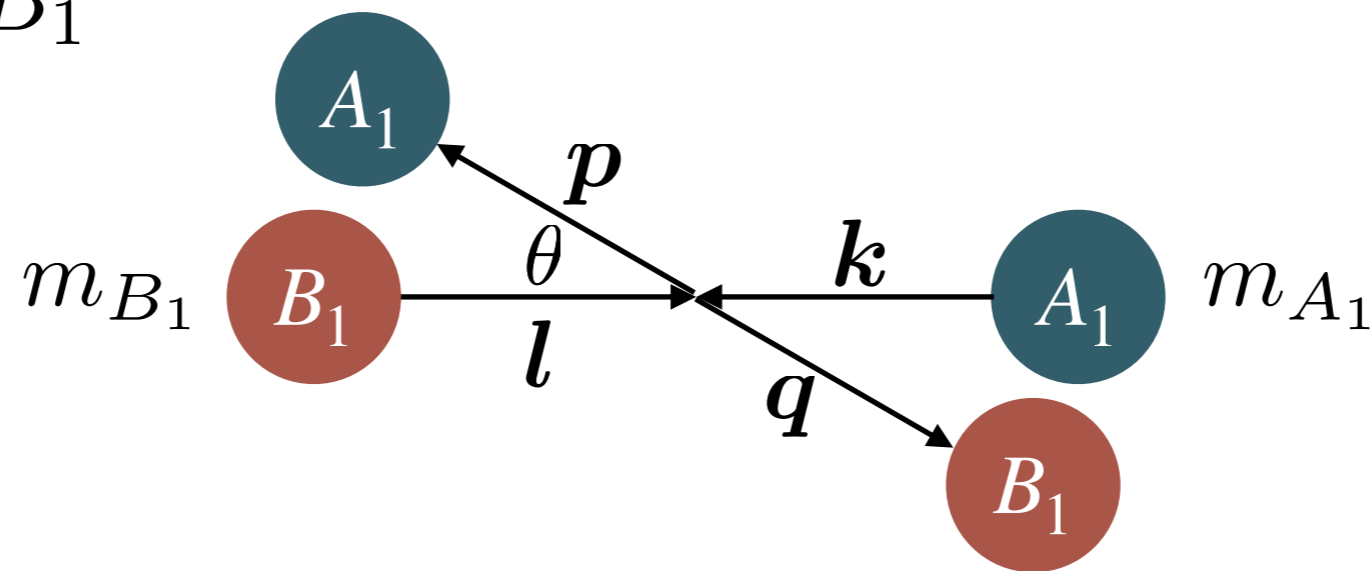
The complete set given by orthogonal basis satisfies

$$\mathbf{1} = \int \frac{d^3 \mathbf{p} d^3 \mathbf{q}}{2E_{A_1 \mathbf{p}} 2E_{B_1 \mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_1 \langle \mathbf{p}, \mathbf{q}|_1$$
$$+ \int \frac{d^3 \mathbf{p} d^3 \mathbf{q}}{2E_{A_2 \mathbf{p}} 2E_{B_2 \mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_2 \langle \mathbf{p}, \mathbf{q}|_2 + \int dX |X\rangle \langle X|$$

Elastic channel

[Peschanski and Seki, Phys. Lett. B758 (2016) 89]

$$A_1 B_1 \rightarrow A_1 B_1$$



6 steps to formulate the entanglement entropy

Step 1: Fix the initial state

$$|\text{ini}\rangle = |\mathbf{k}, \mathbf{l}\rangle_1$$

Step 2: Two-particle final state

Once we fix the initial state, the S-matrix, \mathcal{S} , gives the final state; $\mathcal{S}|ini\rangle$. However it includes not only two-particle states.

Since we are interested in the final state of two particles, we project out the states except for the two-particle ones by using the projection operator;

$$\begin{aligned} |\text{fin}\rangle_1 &= \left(\int \frac{d^3\mathbf{p}}{2E_{A_1\mathbf{p}}} \frac{d^3\mathbf{q}}{2E_{B_1\mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_1 {}_1\langle\mathbf{p}, \mathbf{q}| \right) \mathcal{S}|\mathbf{k}, \mathbf{l}\rangle_1 \\ &= \int \frac{d^3\mathbf{p}}{2E_{A_1\mathbf{p}}} \frac{d^3\mathbf{q}}{2E_{B_1\mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_1 {}_1\langle\mathbf{p}, \mathbf{q}| (1 + 2i\mathcal{T}) |\mathbf{k}, \mathbf{l}\rangle_1 \end{aligned}$$

Step 3: Total density matrix

The total density matrix for the final state is defined by

$$\rho_1 = \frac{1}{\mathcal{N}_1} |\text{fin}\rangle_1 \langle \text{fin}|$$

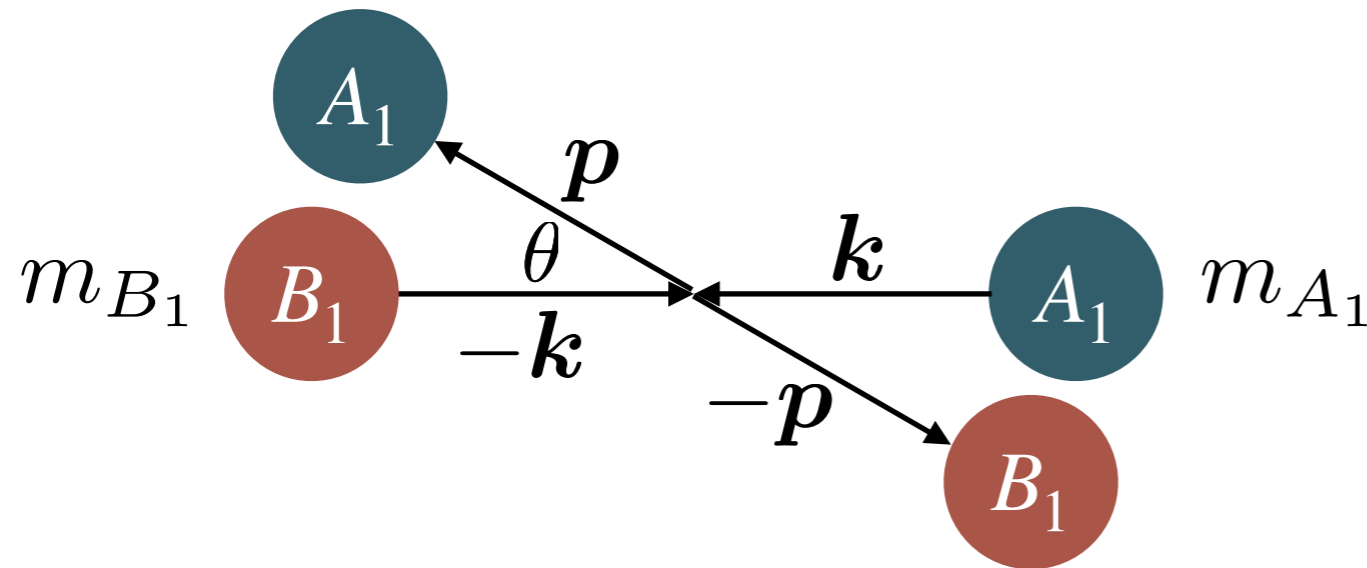
\mathcal{N}_1 is a normalization factor. Later it will be determined by $\text{tr}_{A_1} \text{tr}_{B_1} \rho_1 = 1$.

Step 4: Reduced density matrix

Tracing out the total density matrix with respect to \mathcal{H}_{B_1} , we obtain the reduced density matrix;

$$\rho_{A_1} = \text{tr}_{B_1} \rho_1 = \int \frac{d^3 \mathbf{r}}{2E_{B_1 \mathbf{r}}} {}_{B_1} \langle \mathbf{r} | \rho_1 | \mathbf{r} \rangle_{B_1}$$

Now let us move into the **center-of-mass frame** for convenience.



$$|\mathbf{p}\rangle\rangle_1 \equiv |\mathbf{p}, -\mathbf{p}\rangle_1$$

The reduced density matrix is rewritten as

$$\rho_{A_1} = \frac{1}{\mathcal{N}_1} \int \frac{d^3 \mathbf{p}}{2E_{A_1 \mathbf{p}}} \frac{\delta(p - k) \delta(0)}{4p\sqrt{s}} |{}_1\langle\langle \mathbf{p} | \mathbf{s} | \mathbf{k} \rangle\rangle_1|^2 |\mathbf{p}\rangle_{A_1} \langle \mathbf{p}|_{A_1}$$

The normalization factor determined by $1 = \text{tr}_{A_1} \text{tr}_{B_1} \rho_1$ is

$$\mathcal{N}_1 = \delta(0) \delta^{(3)}(0) \int d^3 \mathbf{p} \frac{\delta(p - k)}{4p\sqrt{s}} |{}_1\langle\langle \mathbf{p} | \mathbf{s} | \mathbf{k} \rangle\rangle_1|^2$$

Step 5: Compute $\text{tr}_{A_1} (\rho_{A_1})^n$

$$\text{tr}_{A_1} (\rho_{A_1})^n = \int d^3 \mathbf{p} \delta^{(3)}(0) \left(\frac{\delta(p - k) \delta(0)}{\mathcal{N}_1 \cdot 4p\sqrt{s}} |{}_1\langle\langle \mathbf{p} | \mathbf{s} | \mathbf{k} \rangle\rangle_1|^2 \right)^n$$

We shall rewrite it in terms of partial wave expansion.

Partial wave expansion

[Van Hove, Nuovo Cim. 28 (1963) 2344]

[Białas and Van Hove, Nuovo Cim. 38 (1965) 1385]

The partial wave expansion is often useful for analyzing scattering processes.

$$A_j B_j \rightarrow A_i B_i$$

The T-matrix element ($\mathcal{S} = \mathbf{1} + 2i\mathcal{T}$) is expanded as

$$i\langle\langle \mathbf{p} | \mathbf{t} | \mathbf{k} \rangle\rangle_j = \frac{\sqrt{s}}{\pi \sqrt{pk}} \sum_{\ell=1}^{\infty} (2\ell + 1) \tau_{ij}^{\ell} P_{\ell}(\cos \theta)$$

P_{ℓ} are Legendre polynomials.

The S-matrix element

$${}_i\langle\langle\mathbf{p}|\mathbf{s}|\mathbf{k}\rangle\rangle_j = \frac{\sqrt{s}}{\pi\sqrt{pk}} \cdot 2 \left(\delta(1 - \cos\theta)\delta_{ij} + \frac{i}{16\pi}\mathcal{A}_{ij}(s, t) \right)$$

gives the two-body scattering amplitude,

$$\mathcal{A}_{ij} = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1)\tau_{ij}^{\ell} P_{\ell}(\cos\theta)$$

We know the summation formula of Legendre polynomial,

$$\delta(1 - \cos\theta) = \frac{1}{2} \sum_{\ell=0}^{\infty} (2\ell + 1)P_{\ell}(\cos\theta)$$

Differential cross section

$$\frac{d\sigma_{ij}}{dt} = \frac{\pi}{k^4} \left| \sum_{\ell=0}^{\infty} (2\ell + 1) \tau_{ij}^{\ell} P_{\ell}(\cos \theta) \right|^2 = \frac{|\mathcal{A}_{ij}|^2}{256\pi k^4}$$

Cross section

$$\sigma_{ij} = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) |\tau_{ij}^{\ell}|^2$$

$$\sigma_{11} = \sigma_{\text{el}}, \quad \sigma_{21} = \sigma_{\text{inel}}$$

Total cross section

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) \text{Im} \tau_{11}^{\ell}$$

We obtain the following expression:

$$\text{tr}_{A_1} (\rho_{A_1})^n = \int_{-1}^1 d\cos \theta \left(\frac{\delta(0)}{2\pi k^2 \delta^{(3)}(0)} \right)^{n-1} (\mathcal{P}_{11}(\cos \theta))^n,$$

$$\mathcal{P}_{11}(\cos \theta) \equiv \frac{|\sum_{\ell=0}^{\infty} (2\ell + 1) s_{11}^{\ell} P_{\ell}(\cos \theta)|^2}{2 \sum_{\ell=0}^{\infty} (2\ell + 1) |s_{11}^{\ell}|^2}$$

$$(s_{11}^{\ell} = 1 + 2i\tau_{11}^{\ell})$$

$$\frac{4\pi k^2 \delta^{(3)}(0)}{\delta(0)} = \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(1) = \sum_{\ell=0}^{\infty} (2\ell + 1)$$

V is the divergent full phase-space “volume”:

$$V \equiv \sum_{\ell=0}^{\infty} (2\ell + 1) = 2\delta(0)$$

In terms of the cross sections, one can rewrite $\text{tr}_{A_1}(\rho_{A_1})^n$ as

$$\text{tr}_{A_1}(\rho_{A_1})^n = \left(\frac{2}{V}\right)^{n-1} \int_{-1}^1 d\cos\theta (\mathcal{P}_{11}(\cos\theta))^n$$

$$\mathcal{P}_{11}(\cos\theta) = \delta(1 - \cos\theta) \left\{ 1 - \frac{\sigma_{11}}{\frac{\pi}{k^2}V - (\sigma_{\text{tot}} - \sigma_{11})} \right\}$$

$$+ \frac{2k^2}{\frac{\pi}{k^2}V - (\sigma_{\text{tot}} - \sigma_{11})} \frac{d\sigma_{11}}{dt}$$

One can recognize \mathcal{P}_{11} as a kind of probability “density”.

$$\int_{-1}^1 d\cos\theta \mathcal{P}_{11}(\cos\theta) = 1$$

Step 6: Entanglement entropy

$$\begin{aligned} S_{\text{el}} &= - \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \text{tr}_{A_1} (\rho_{A_1})^n \\ &= \ln \frac{V}{2} - \int_{-1}^1 d\cos \theta \mathcal{P}_{11}(\cos \theta) \ln \mathcal{P}_{11}(\cos \theta) \end{aligned}$$

There is a problem, *i.e.*, this formula depends on the infinite volume V , so that this is physically meaningless. Therefore some regularization is necessary.

Volume regularization

[Peschanski and Seki, Phys. Rev. D 100 (2019) 7, 076012]

$$\mathcal{P}_{11}(\cos \theta) = \delta(1 - \cos \theta) \left\{ 1 - \frac{\sigma_{11}}{\frac{\pi}{k^2} V - (\sigma_{\text{tot}} - \sigma_{11})} \right\} + \frac{2k^2}{\frac{\pi}{k^2} V - (\sigma_{\text{tot}} - \sigma_{11})} \frac{d\sigma_{11}}{dt}$$

The first term comes from part of the two-body Hilbert space of the final states which **does not correspond to the interacting states** at the given energy. Because this term has support only at $\theta = 0$.

cf.

$${}_i\langle\langle \mathbf{p} | \mathbf{s} | \mathbf{k} \rangle\rangle_j = \frac{\sqrt{s}}{\pi \sqrt{pk}} \cdot 2 \left(\delta(1 - \cos \theta) \delta_{ij} + \frac{i}{16\pi} \mathcal{A}_{ij}(s, t) \right)$$

In order to avoid the non-interacting modes, we regularize the volume V to \tilde{V} ,

$$1 - \frac{\sigma_{11}}{\frac{\pi}{k^2} \tilde{V} - (\sigma_{\text{tot}} - \sigma_{11})} \Leftrightarrow \tilde{V} = \frac{k^2}{\pi} \sigma_{\text{tot}}$$

(Currently we do not know a concrete way to realize this regularization.)

\tilde{V} is determined only by the conditions of incoming particles.

Finally we obtain the volume-regularized entanglement entropy,

$$\tilde{S}_{\text{el}} = \ln \frac{k^2 \sigma_{\text{tot}}}{2\pi} - \int_{-1}^1 d\cos \theta \tilde{\mathcal{P}}_{11} \ln \tilde{\mathcal{P}}_{11}, \quad \tilde{\mathcal{P}}_{11} = \frac{1}{\sigma_{11}} \frac{d\sigma_{11}}{d\cos \theta}$$

By using Mandelstam variables:

$$\sqrt{s} = \sqrt{m_A^2 + k^2} + \sqrt{m_B^2 + k^2}, \quad t = 2k^2(\cos\theta - 1)$$

(At high energy, $s \approx 4k^2$.)

one can rewrite the entanglement entropy as

$$\tilde{S}_{\text{el}} = - \int_{-4k^2}^0 dt P_{\text{el}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{el}}(t) \right)$$

where

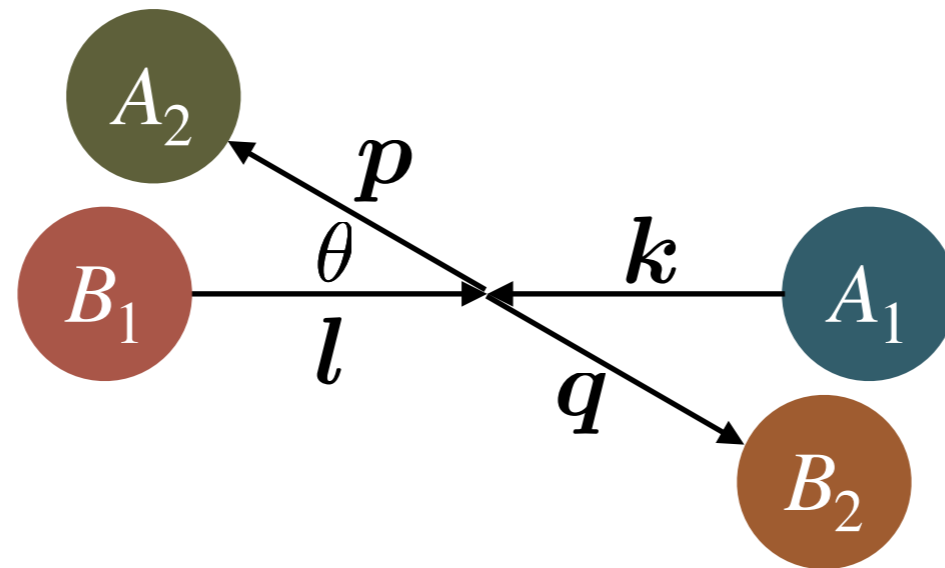
$$P_{\text{el}}(t) \equiv \frac{1}{2k^2} \tilde{\mathcal{P}}_{11}(\cos\theta) = \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{dt}$$

$$\int_{-4k^2}^0 dt P_{\text{el}}(t) = 1$$

Inelastic channel

[Peschanski and Seki, arXiv:2601.22502]

$$A_1 B_1 \rightarrow A_2 B_2$$



In the same way as for the elastic channel, we can formulate the entanglement entropy for the inelastic channel.

Step 1: The same initial state $|\text{ini}\rangle = |\mathbf{k}, \mathbf{l}\rangle_1$

Step 2: Two-particle final state

$$\begin{aligned} |\text{fin}\rangle_2 &= \left(\int \frac{d^3\mathbf{p}}{2E_{A_2\mathbf{p}}} \frac{d^3\mathbf{q}}{2E_{B_2\mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_2 {}_2\langle\mathbf{p}, \mathbf{q}| \right) \mathcal{S} |\mathbf{k}, \mathbf{l}\rangle_1 \\ &= \int \frac{d^3\mathbf{p}}{2E_{A_2\mathbf{p}}} \frac{d^3\mathbf{q}}{2E_{B_2\mathbf{q}}} |\mathbf{p}, \mathbf{q}\rangle_2 {}_2\langle\mathbf{p}, \mathbf{q}| 2i\mathcal{T} |\mathbf{k}, \mathbf{l}\rangle_1 \end{aligned}$$

Note that the unit matrix in $\mathcal{S} = \mathbf{1} + 2i\mathcal{T}$ does not contribute to the final state in the inelastic channel because ${}_2\langle\mathbf{p}, \mathbf{q}|\mathbf{1}|\mathbf{k}, \mathbf{l}\rangle_1 = 0$.

Step 3: Total density matrix

$$\rho_2 = \frac{1}{\mathcal{N}_2} |\text{fin}\rangle_2 {}_2\langle\text{fin}|$$

Step 4: Reduced density matrix

$$\rho_{A_2} = \text{tr}_{B_2} \rho_2 = \int \frac{d^3 \mathbf{r}}{2E_{B_2 \mathbf{r}}} B_2 \langle \mathbf{r} | \rho_2 | \mathbf{r} \rangle_{B_2}$$

Step 5: Compute $\text{tr}_{A_2} (\rho_{A_2})^n$

$$\begin{aligned} \rho_{A_2} &= \text{tr}_{B_2} \rho_2 = \int \frac{d^3 \mathbf{r}}{2E_{B_2 \mathbf{r}}} B_2 \langle \mathbf{r} | \rho_2 | \mathbf{r} \rangle_{B_2} \\ &= \frac{1}{\mathcal{N}_2} \int \frac{d^3 \mathbf{p}}{2E_{A_2 \mathbf{p}}} \frac{\delta(p - k') \delta(0)}{p\sqrt{s}} | {}_2 \langle \langle \mathbf{p} | \mathbf{t} | \mathbf{k} \rangle \rangle_1 |^2 | \mathbf{p} \rangle_{A_2} \langle \mathbf{p} | \end{aligned}$$

$$k' = \sqrt{\frac{(m_{A_2}^2 + m_{B_2}^2 - s)^2 - 4m_{A_2}^2 m_{B_2}^2}{4s}}$$

$$\mathcal{N}_2 = \delta(0) \delta^{(3)}(0) \int d^3 \mathbf{p} \frac{\delta(p - k')}{p\sqrt{s}} | {}_2 \langle \langle \mathbf{p} | \mathbf{t} | \mathbf{k} \rangle \rangle_1 |^2$$

In terms of partial wave modes, we can rewrite $\text{tr}_{A_2}(\rho_{A_2})^n$ as

$$\text{tr}_{A_2}(\rho_{A_2})^n = \left(\frac{2}{V}\right)^{n-1} \int_{-1}^1 d\cos\theta (\mathcal{P}_{21}(\cos\theta))^n$$

$$\mathcal{P}_{21}(\cos\theta) = \frac{1}{\sigma_{21}} \frac{d\sigma_{21}}{d\cos\theta}$$

The infinite volume V **does not appear** in \mathcal{P}_{21} , but the factor $(2/V)^{n-1}$ still remains.

Step 6: Entanglement entropy

$$S_{\text{inel}} = - \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \text{tr}_{A_2}(\rho_{A_2})^n$$

$$= \ln \frac{V}{2} - \int_{-1}^1 d\cos\theta \mathcal{P}_{21}(\cos\theta) \ln \mathcal{P}_{21}(\cos\theta)$$

Using the regularized volume, we obtain

$$\tilde{S}_{\text{inel}} = \ln \frac{k^2 \sigma_{\text{tot}}}{2\pi} - \int_{-1}^1 d\cos \theta \mathcal{P}_{21} \ln \mathcal{P}_{21}, \quad \mathcal{P}_{21} = \frac{1}{\sigma_{21}} \frac{d\sigma_{21}}{d\cos \theta}$$

or, equivalently,

$$\tilde{S}_{\text{inel}} = - \int_{-4k^2}^0 dt P_{\text{inel}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{inel}}(t) \right)$$

$$P_{\text{inel}}(t) \equiv \frac{1}{2k^2} \tilde{\mathcal{P}}_{21}(\cos \theta) = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma_{\text{inel}}}{dt}$$

$$\int_{-4k^2}^0 dt P_{\text{inel}}(t) = 1$$

Evaluation of entanglement entropies

Entanglement entropy

Elastic channel:

$$\tilde{S}_{\text{el}} = - \int_{-4k^2}^0 dt P_{\text{el}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{el}}(t) \right),$$

$$P_{\text{el}}(t) = \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{dt}$$

Inelastic channel:

$$\tilde{S}_{\text{inel}} = - \int_{-4k^2}^0 dt P_{\text{inel}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{inel}}(t) \right),$$

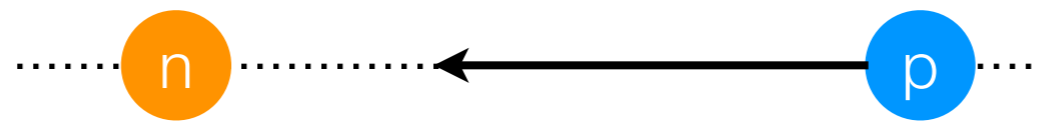
$$P_{\text{inel}}(t) = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma_{\text{inel}}}{dt}$$

In order to compute the entanglement entropy concretely, we need to know the differential cross sections as functions of t .

proton-neutron scattering

[M. N. Kreisler et al., Nucl. Phys. B 84 (1975) 3]

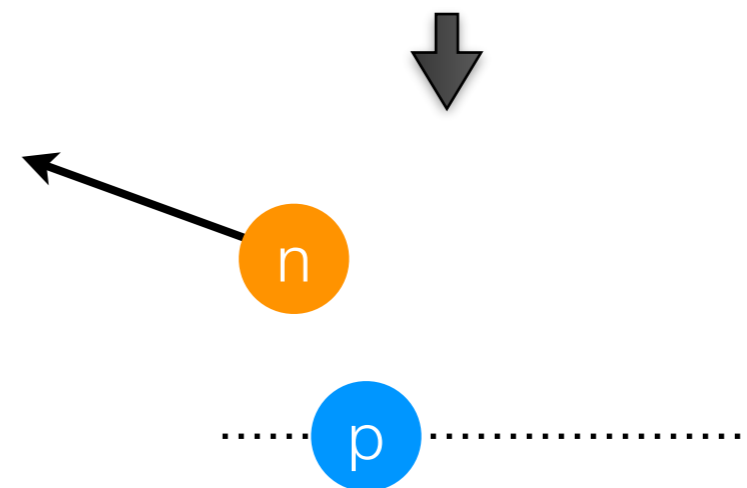
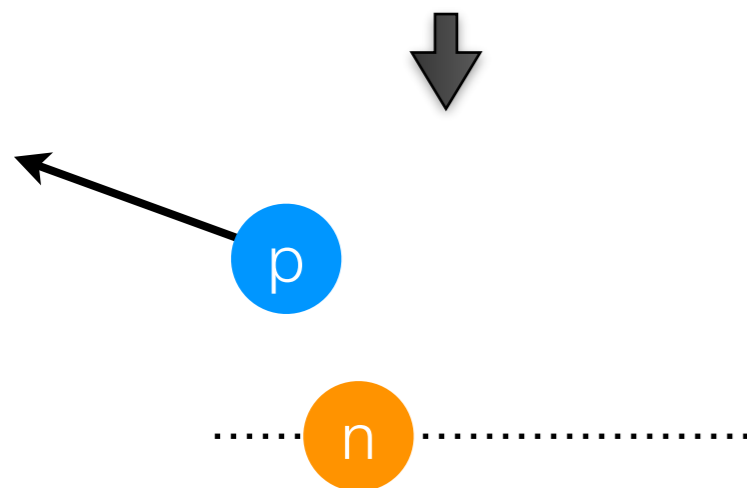
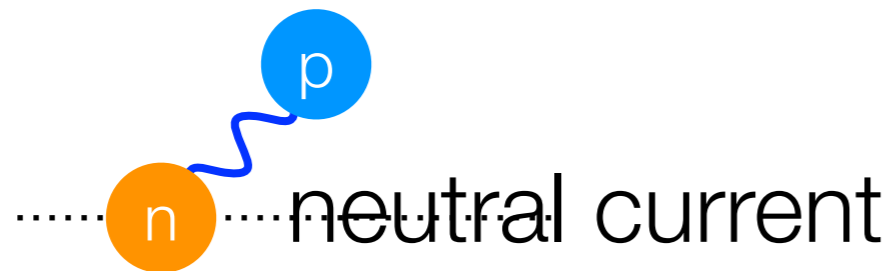
neutron target proton beam

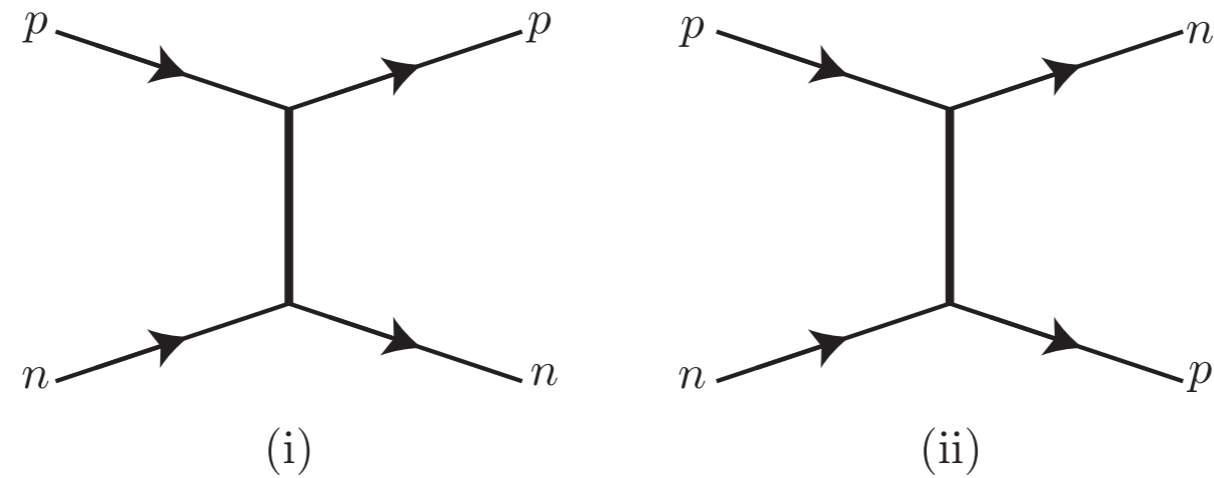


near forward scattering

$pn \rightarrow pn$ (elastic)

$pn \rightarrow np$ (inelastic)





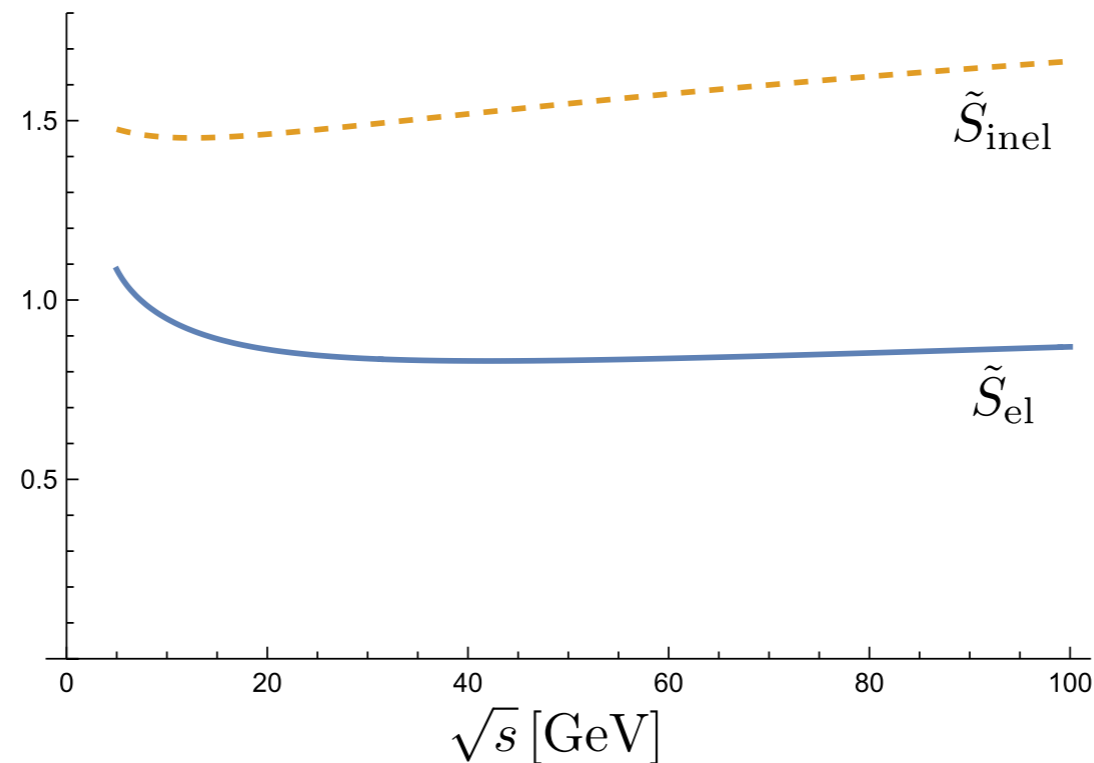
We can read the differential cross sections from the fitting functions of experimental data in CERN ISR and Fermilab.

[M. N. Kreisler et al., Nucl. Phys. B 84 (1975) 3]

[R. J. N. Phillips and V. D. Barger, Phys. Lett. B 46 (1973) 412]

[M.M. Block and R. Cahn, Phys. Lett. B 120B (1983) 229]

The elastic and inelastic entanglement entropies:

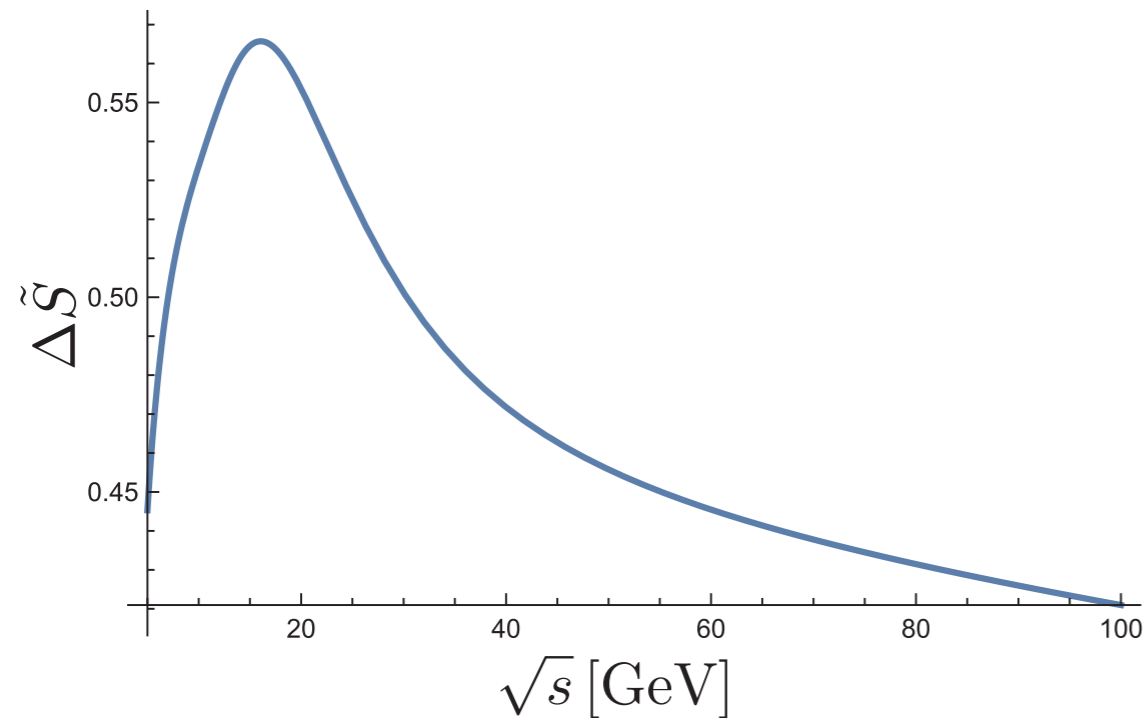


At higher energy, the entanglement entropies increase as the center-of-mass energy.

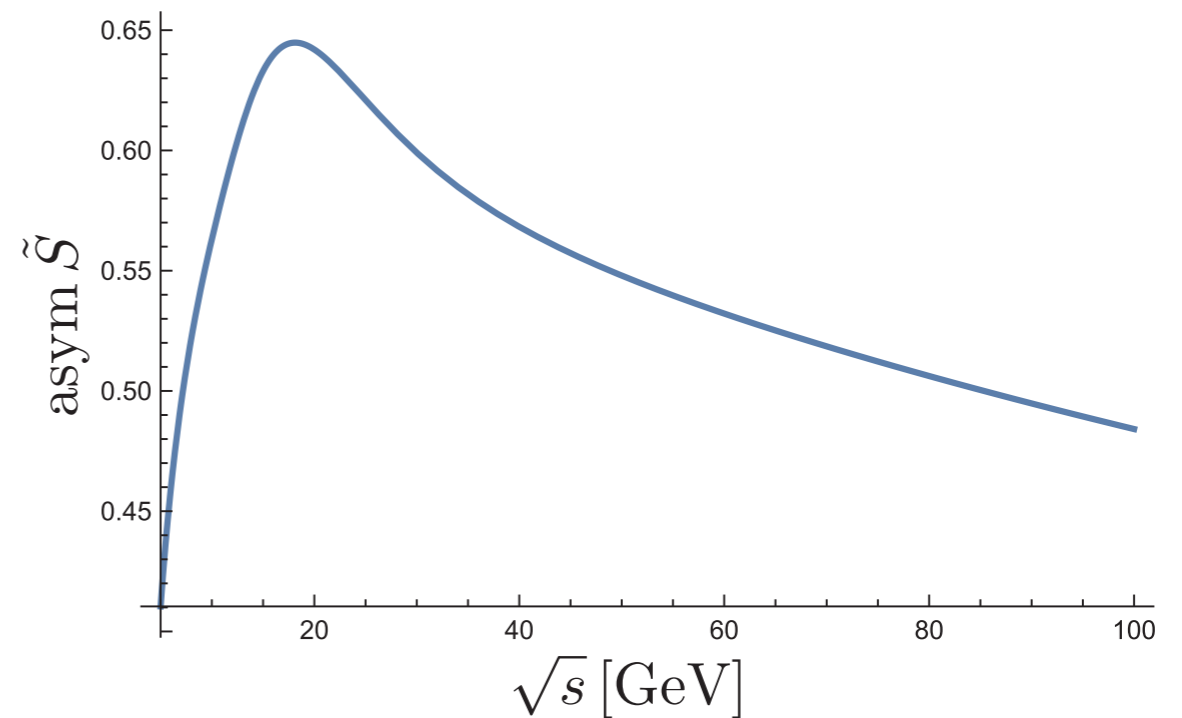
The inelastic entanglement entropy is larger than the elastic one.

$$\tilde{S}_{\text{inel}} > \tilde{S}_{\text{el}}$$

$$\Delta\tilde{S} = \tilde{S}_{\text{inel}} - \tilde{S}_{\text{el}}, \quad \text{asym } \tilde{S} = \frac{\tilde{S}_{\text{inel}} - \tilde{S}_{\text{el}}}{\tilde{S}_{\text{el}}}$$



(i)



(ii)

At lower energy the elastic entanglement entropy decreases faster than the inelastic one, while at higher energy it is contrary.

Why $\tilde{S}_{\text{inel}} > \tilde{S}_{\text{el}}$?

Naively we assume a simple exponential approximation of the dominant cross sections (the diffraction peak model)

$$\frac{d\sigma_{\text{el}}}{dt} \sim \sigma_{\text{el}}(s) e^{R_{\text{el}}^2 t}, \quad \frac{d\sigma_{\text{inel}}}{dt} \sim \sigma_{\text{inel}}(s) e^{R_{\text{inel}}^2 t}$$

(R^2 : slope)

Then the difference between the elastic and inelastic entanglement entropies is

$$\tilde{S}_{\text{el}} = \ln \left(\frac{\sigma_{\text{tot}}}{4\pi R_{\text{el}}^2} \right) + 1, \quad \tilde{S}_{\text{inel}} = \ln \left(\frac{\sigma_{\text{tot}}}{4\pi R_{\text{inel}}^2} \right) + 1,$$

$$\Delta\tilde{S} = \tilde{S}_{\text{inel}} - \tilde{S}_{\text{el}} = \ln \left(\frac{R_{\text{el}}^2}{R_{\text{inel}}^2} \right)$$

Since $R_{\text{inel}}^2 < R_{\text{el}}^2$, $S_{\text{inel}} > S_{\text{el}}$.

Conclusion and outlook

Conclusion

- We formulated the entanglement entropy of two-particle final state for the elastic and inelastic scattering.

$$\tilde{S}_{\text{el}} = - \int_{-4k^2}^0 dt P_{\text{el}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{el}}(t) \right), \quad P_{\text{el}} = \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{el}}}{dt}$$

$$\tilde{S}_{\text{inel}} = - \int_{-4k^2}^0 dt P_{\text{inel}}(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P_{\text{inel}}(t) \right), \quad P_{\text{inel}} = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma_{\text{inel}}}{dt}$$

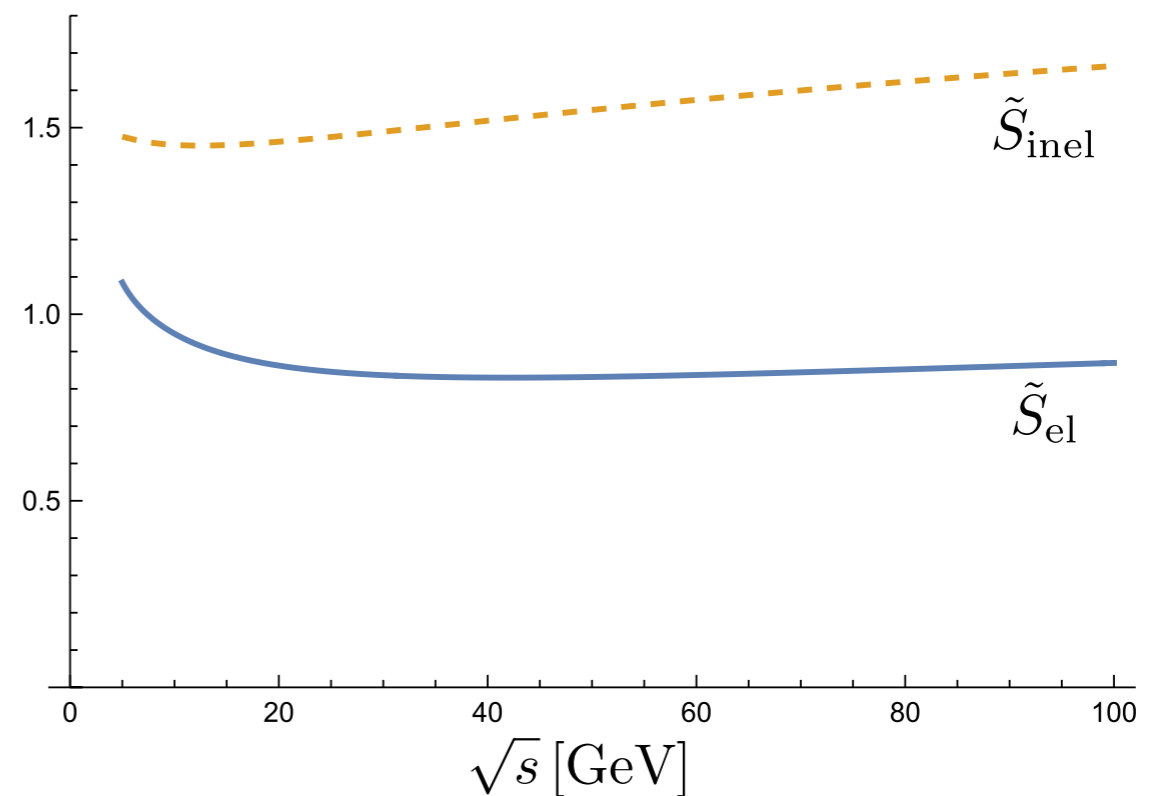
The key quantity is the normalized differential cross section;

$$P(t) = \frac{1}{\sigma} \frac{d\sigma}{dt}, \quad \int_{-4k^2}^0 dt P(t) = 1$$

- Concrete evaluation of entanglement entropies for the elastic and inelastic proton-neutron scatterings.

$$pn \rightarrow \begin{cases} pn & \text{(elastic channel)} \\ np & \text{(inelastic channel)} \end{cases}$$

$$\tilde{S}_{\text{inel}} < \tilde{S}_{\text{el}}$$



Outlook

- To evaluate the elastic-inelastic channel relation for any other cases where data exist.
Is the inelastic entanglement entropy always larger than the elastic one?
- More study about the entanglement entropy density as a function of transverse momentum;

$$\tilde{S} = \int_{-4k^2}^0 dt D(t), \quad D(t) = -P(t) \ln \left(\frac{4\pi}{\sigma_{\text{tot}}} P(t) \right)$$

Are there any universal properties for different initial particles?

Appendix

The 3-dim. Dirac delta function in spherical coordinates with azimuthal symmetry, we obtain

$$\delta^{(3)}(\mathbf{p} - \mathbf{k}) = \frac{\delta(p - k)}{4\pi k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta)$$

$$\mathbf{p} \rightarrow \mathbf{k}$$

$$\frac{4\pi k^2 \delta^{(3)}(0)}{\delta(0)} = \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(1) = \sum_{\ell=0}^{\infty} (2\ell + 1)$$