New Ideas about the Phase Structure in QCD

Larry McLerran, INT, Seattle

Yuki Fujimoto, Kenji Fukushima and Yoshimasa Hidaka;
Gyozo Kovacs, Michal Marczenko, Krzysztof Redlich

Krakow
June 2026

Scientific Questions:

Is there a phase of matter between the Quark Gluon Plasma and the Hadron Gas?
Can one quantitatively describe thermodynamics below 160 MeV from first principles, no free parameters, computation? Above 160 MeV?

Work in part motivated by some observations of Lenya Glozman, Tom Cohen, and colleagues

What is the evidence from the lattice about the existence of quarkyonic matter?

Motivation from lattice computations:

Chiral symmetry is restored about 160 MeV, but confinement as measured by Polyakov loop appears to disappear at about 300~Mev. Percolation for strings on the lattice with fermions gives same. This is the temperature of deconfinement in the pure glue theory

Everyone is entitled to his own opinion, but not his own facts.
Danial Patick Moynihan

Old Problem:

How to characterize confinement:

When dynamical quarks are present confinement at finite temperature or density is only approximately characterized by the Polyakov line because strings always break and the free energy of an isolated quark is always finite.

If we go to large N_c, or in pure gauge theory, there is truly confinement. In pure gauge theory, the deconfinement temperature is about 300 MeV, which is much different from the chiral transition in a theory with dynamical quarks.

Perhaps the region between 160 MeV and 300 MeV is distinct from that at higher or lower T.
Some evidence from lattice data showing that stringy vortices percolate at around 270 MeV. In this vortex picture, this should be the temperature of deconfinement.

How to characterize these separate regions?

Bosonic String Theory:

Originally Proposed to Describe All of Hadronic Physics

In large N_c Lmit, Free String theory has a Hagedorn Spectrum and mesons are narrow width.

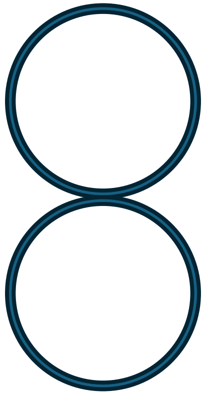
Theory formulated in d dimensions, but only in d =26 space time dimension is the theory free of tachyons or ghosts in the low mass spectrum

We will assume that strings describe the spectrum of QCD mesons in the large N_c limit except for the lowest mass states, which we take to be (almost) Goldstone bosons. We treat these states distinctly. We will take 3 spatial dimensions.

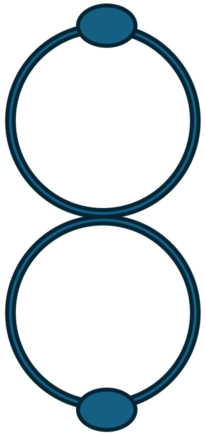
The theory is defined with no undetermined parameters, and when the physical string tension is used, the Hagedorn, or limiting temperature, is about 300 MeV

We find a quantitatively precise computation of the spectrum of mesons and glueballs, and the QCD thermodynamics for T < 160 MeV.
Chiral symmetry restored at 160 MeV and well described
For 160 MeV < T < 300 MeV, we argue gas of quarks/interacting-mesons, plus very low density of glueballs
Above 300 MeV, glueballs melt into a quark gluon plasma

The strength of interactions:
Contribution to the pressure are densities times an amplitude:



$=$



$$= \int dp dp' \; n(p) A(p, p') n(p')$$

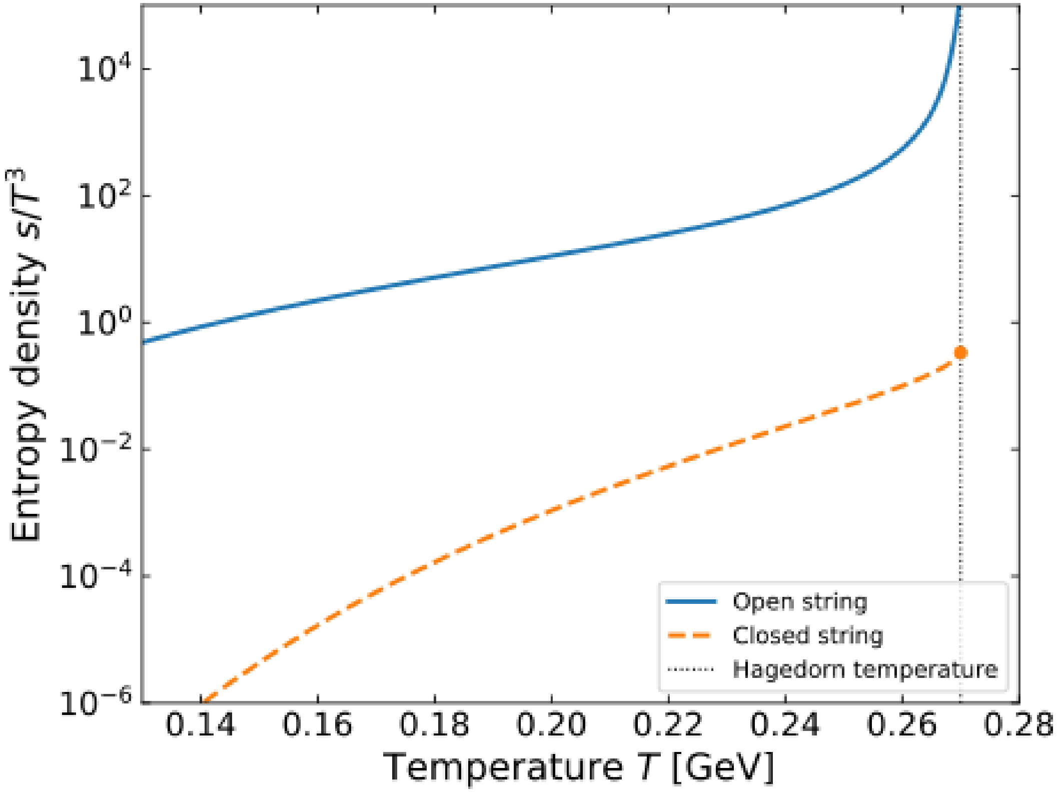
Kinetic energy of hadrons ~ 1, quarks ~N_c, gluons ~ N_c^2

Amplitude for meson-meson ~ $1/N_c$, glueball-glueball ~ $1/N_c^2$

For pressure of hadrons to be of order N_c the
hadron density should be of order N_c
Pressure for glueballs to be of order N_c^2 the
density of glueballs should be of order N_c^2

Mesons and glueballs, density ~1
Quarks and glueballs. density ~ N_c
Quarks and gluons, density ~ N_c^2

Simple model:



Contribution of glueballs is always very very small.
The closed strong, glueball contribution, is non-divergent at the Hagedorn temperature

Closed strings describe well the spectrum of glueballs and thermodynamics in pure gauge theory
H. Meyer

Mesons: open strings
Glueballs: closed strings
Quarks: Free quarks with mass between constituent mass and current quarks mass
Gluons: Mass between massless and ½ of glueball mass

Take closed and open string spectrum from string theory. Glueball and mesons must have the same Hagedorn temperature which we take to be

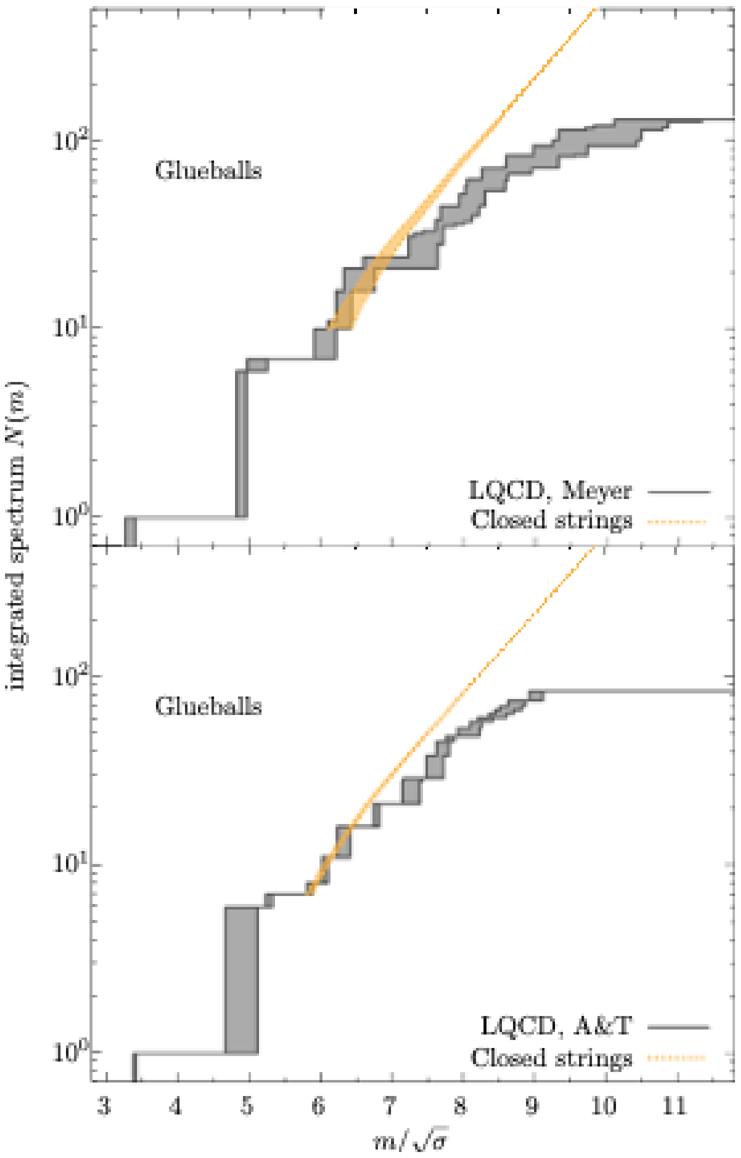
T_H \sim 300 MeV

We handle the contribution of lowest mass glueball states, and Goldstone boson meson states separately, and then treat higher mass states by a continuum integral. The meson integration begins at .6 GeV, and the glueball at about 3 GeV

$$T_H^{string} = \sqrt{\frac{3\sigma}{2\pi}} = 304MeV$$

for

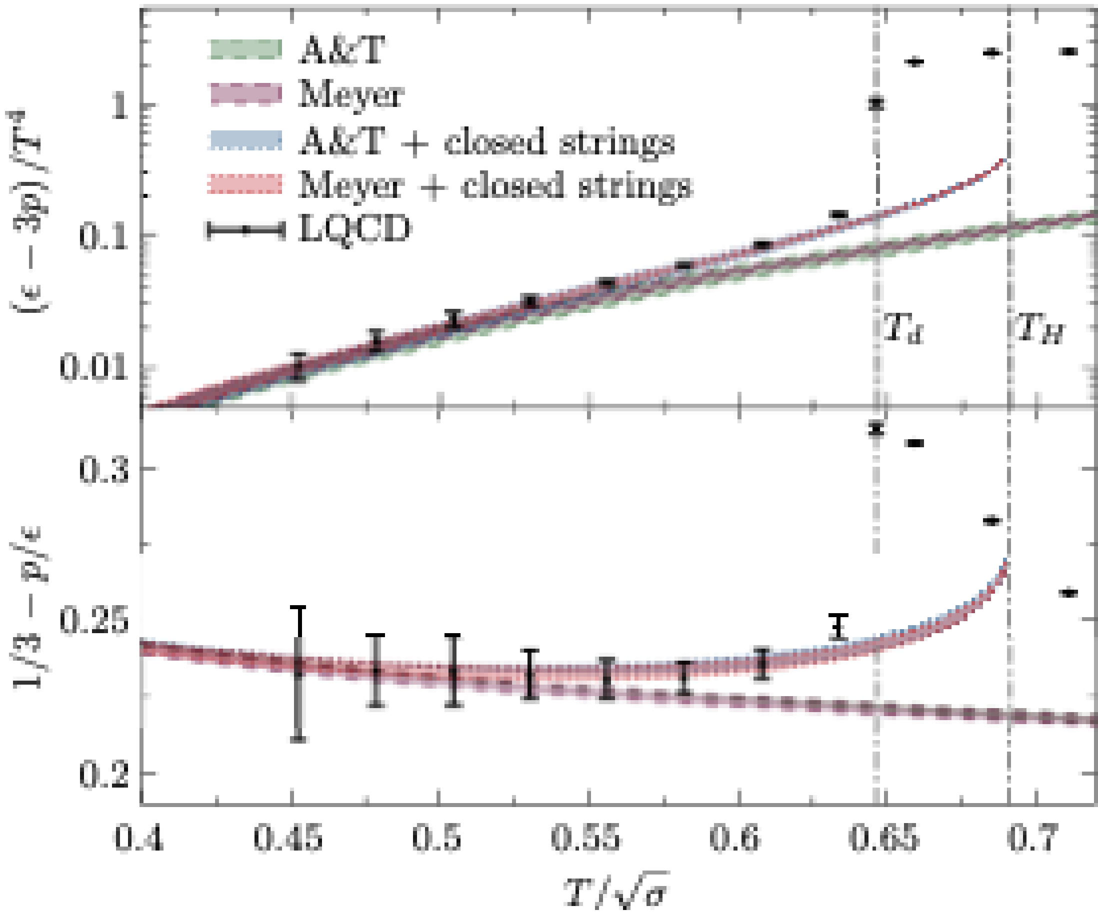
$$\sqrt{\sigma} = 440MeV$$



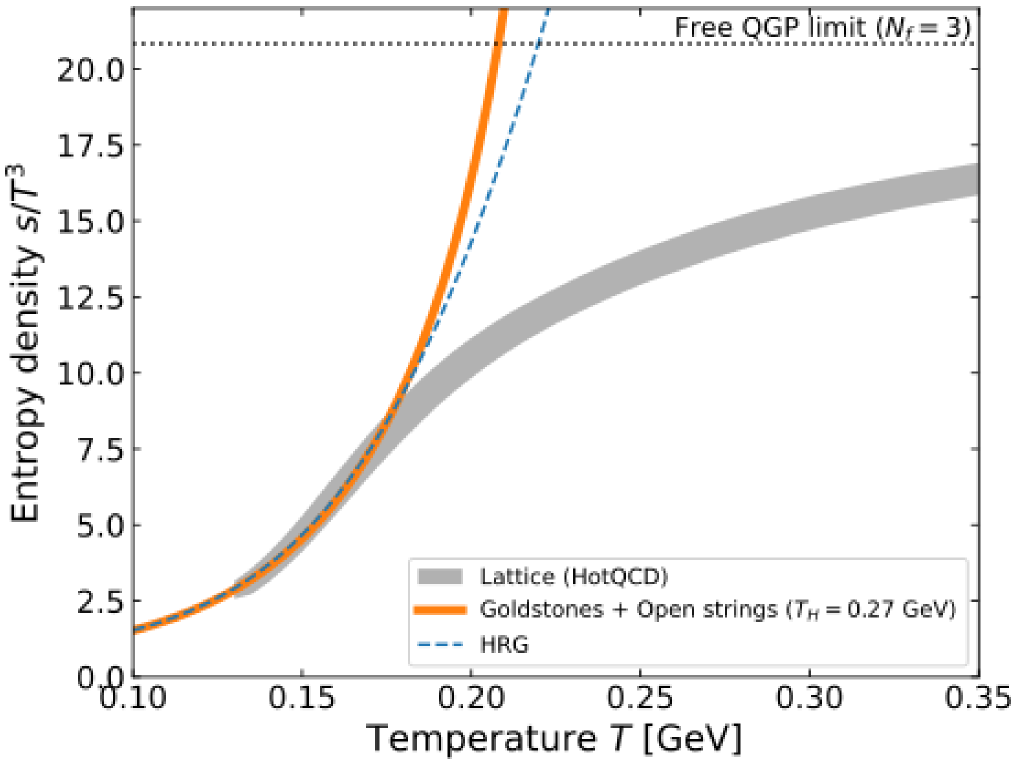
As shown by Meyer, the string provides a good description of glueball spectrum and thermodynamics for pure glue QCD (N_c = 3)

Number of spatial dimensions = 3

Resonance spectrum has missing states above twice the mass of the lowest mass glueball, because unstable states are difficult to extract



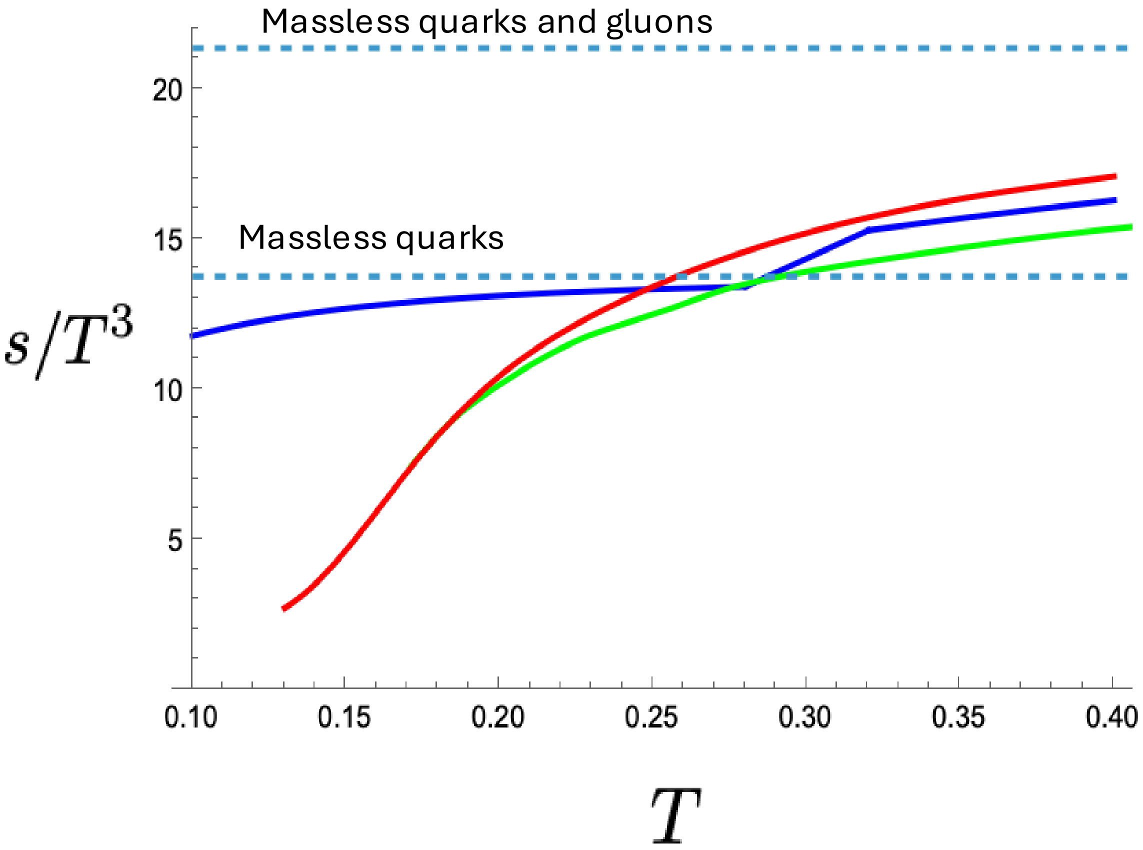
Thermodynamics for N_c = 3 is determined by the trace anomaly and is well determined below the deconfinement temperature, which is slightly below the Hagedorn temperature



Matching on to quarks plus glueballs at 160 MeV, and to quarks plus gluons at the Hagedorn temperature, the entropy is reasonably well described

There is a small correction to this from subtracting off the baryon contributions from lattice data, which makes a good fit at 310 MeV: Redlich, Marczenko, Kovacs LM

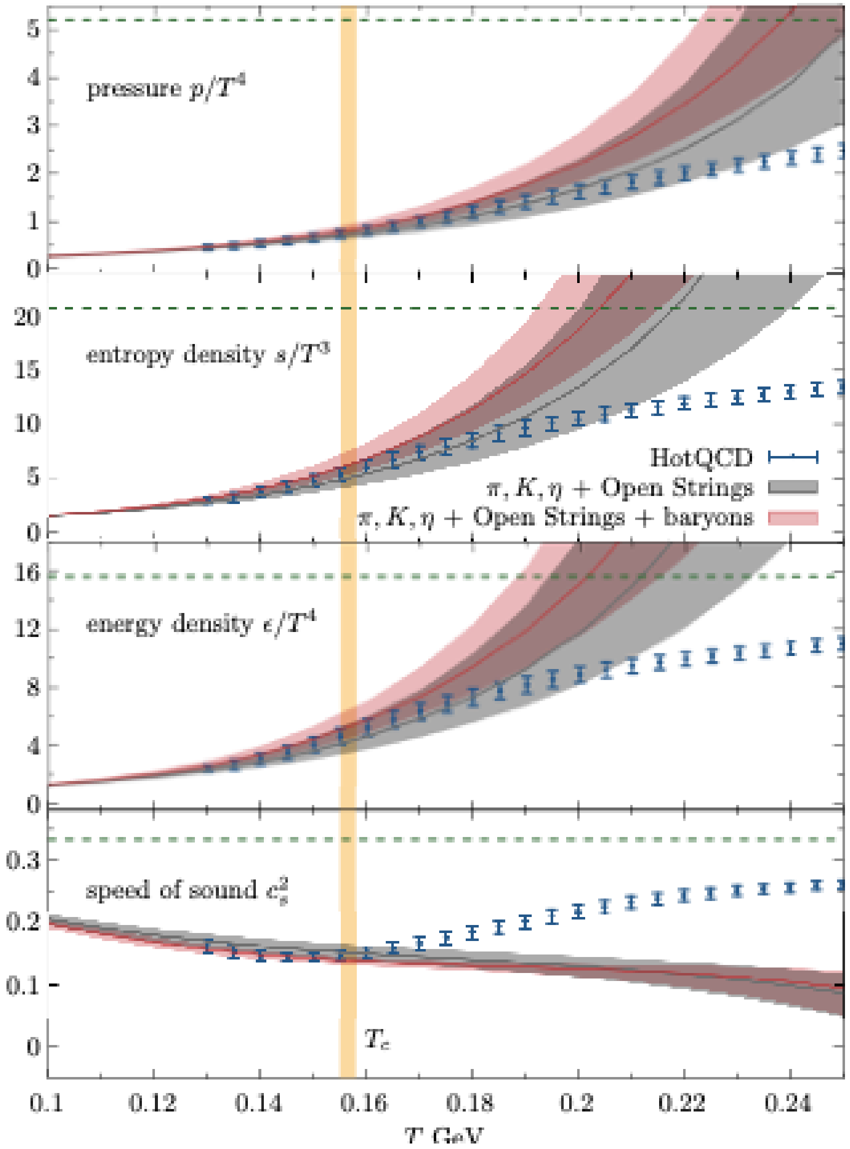
With the string theory resonance model and a Hagedorn temperature of about 300 MeV, describe the lattice data on entropy up until the chiral temperature, where presumably the degrees of freedom become quarks



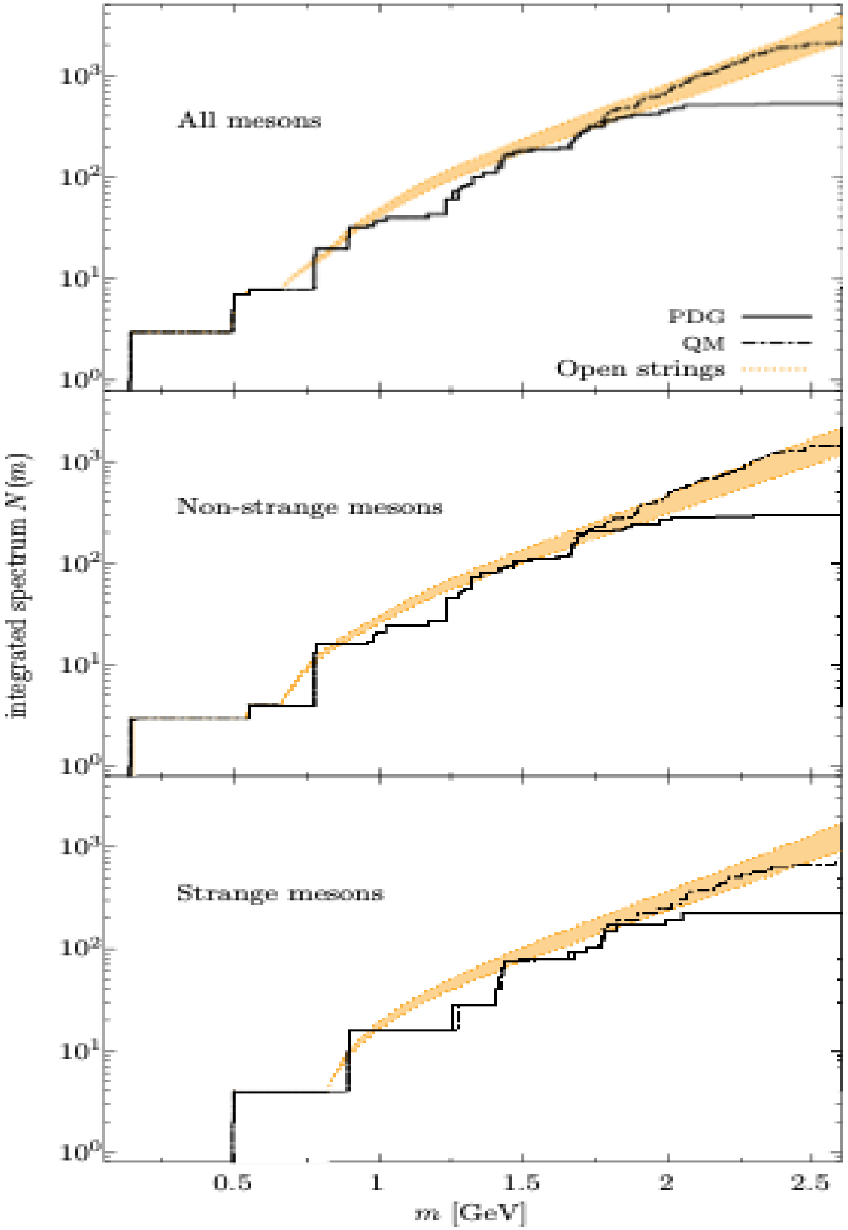
Any quasi-particle model ignoring interactions will have big problems with trace anomaly below 250 MeV

Kovacs, Marczenko, Redlich, LM

Thermodynamic Contributions with and without baryons included, T_H =275-340 MeV



Comparison to experimental and theory extrapolated spectrum of meson states



The quark phase with energy density of order N_c has to a good approximation, no glueballs, and hence no gluons, in it. This is because glueballs are massive. The Debye screening mass is

$$M_{Debye}^2 = g^2(N_c/3 + N_f/6)T^2$$

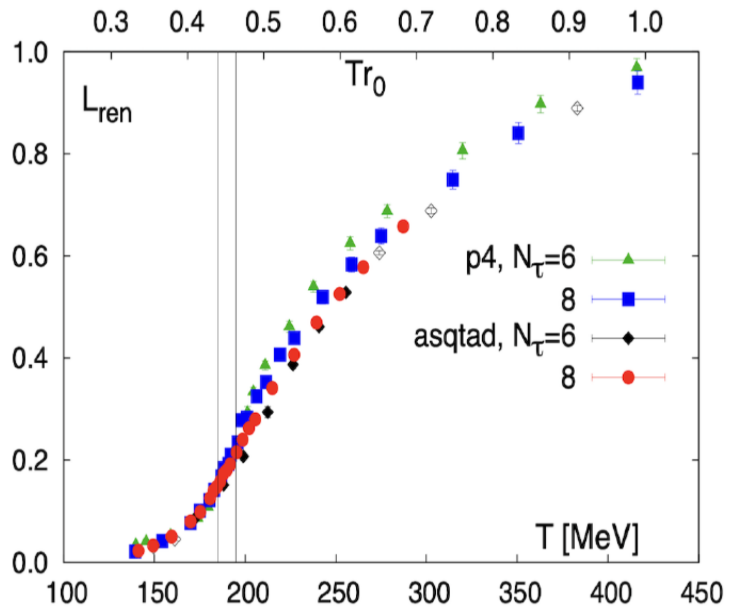
The first term is from gluons and the second from quarks. In the large N_c limit, one holds the 'tHooft coupling fixed,

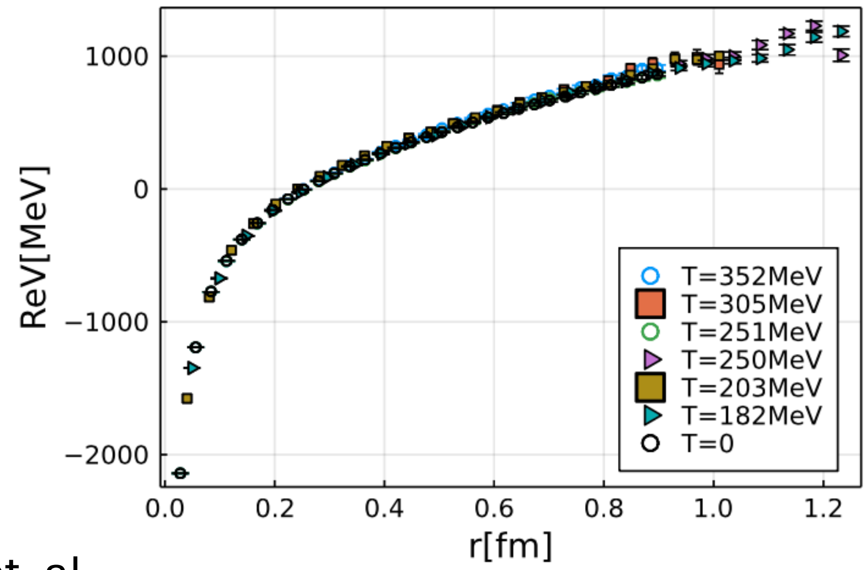
$$g_{'tHooft}^2 = g^2 N_c$$

IBecause glueball contribution is small, we ignore the first term, the second term vanishes in the large N_c limit, so that the new phase of quarks is confined. The quarks are stringy. We call this phase

Spaghetti of Quarks with Glueballs (SQGB)

Karsch et. al.





Bazavov et. al

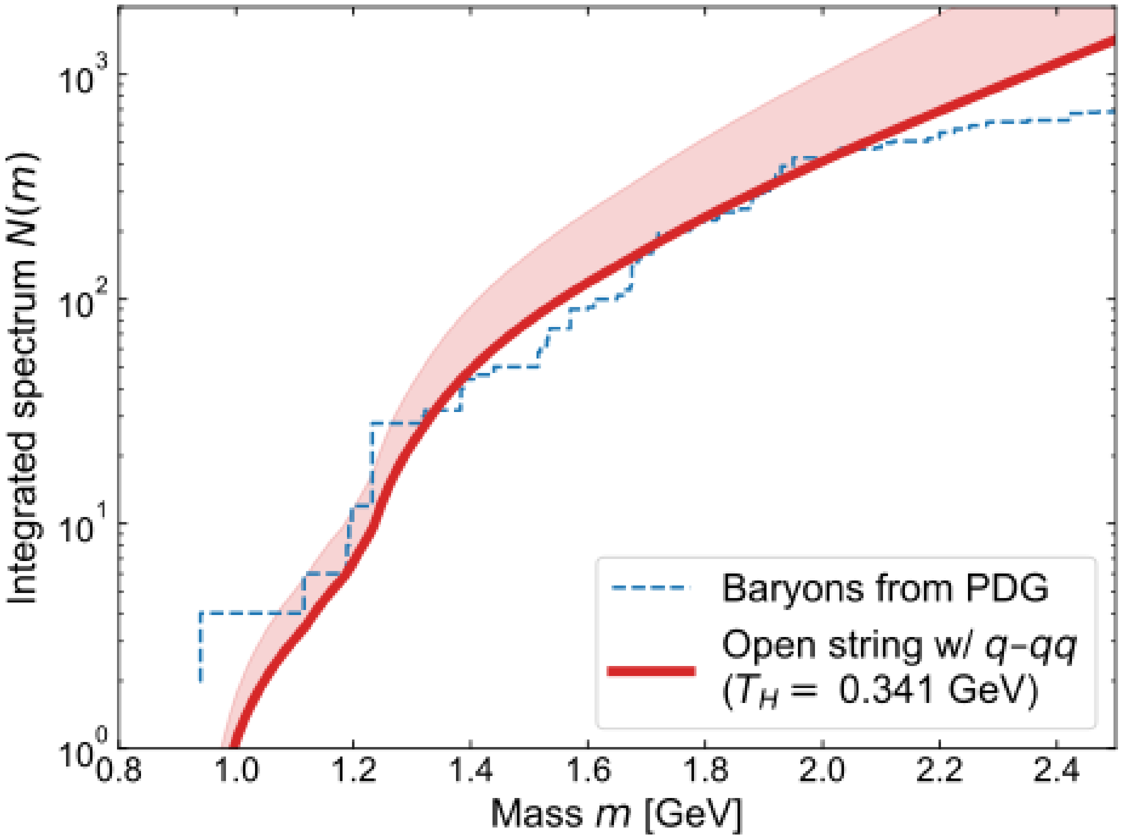
Baryons:
(work by Yuki Fujimoto)

Two types of configurations for strings in baryons:

Baryon junction or quark-diquark
In string theory, the string junction solution is unstable

Fujimoto computed the integrated density of states for quark-diquark pair using the Hagedorn temperature for 3 spatial dimensional string theory:

See also very recent work on susceptibilities by Marczenko and Redlich



The Chiral Transition

See Petreczky et. al.

$$< \overline{\psi}\psi >_T = < \overline{\psi}\psi >_0 - \frac{\partial P}{\partial m_q}$$

Take P(M) for a free resonance gas of quarks.

We need to know

$$\sigma = m_q \frac{\partial M}{\partial m_q}$$

For pions:

$$f_\pi^2 m_\pi^2 = -m_q < \overline{\psi}\psi >_0$$

$$\sigma_\pi = \frac{m_q}{2m_\pi} \frac{dm_\pi^2}{dm_q} = -\frac{m_q}{2m_\pi f_\pi^2} < \overline{\psi}\psi >_0 = m_\pi/2$$

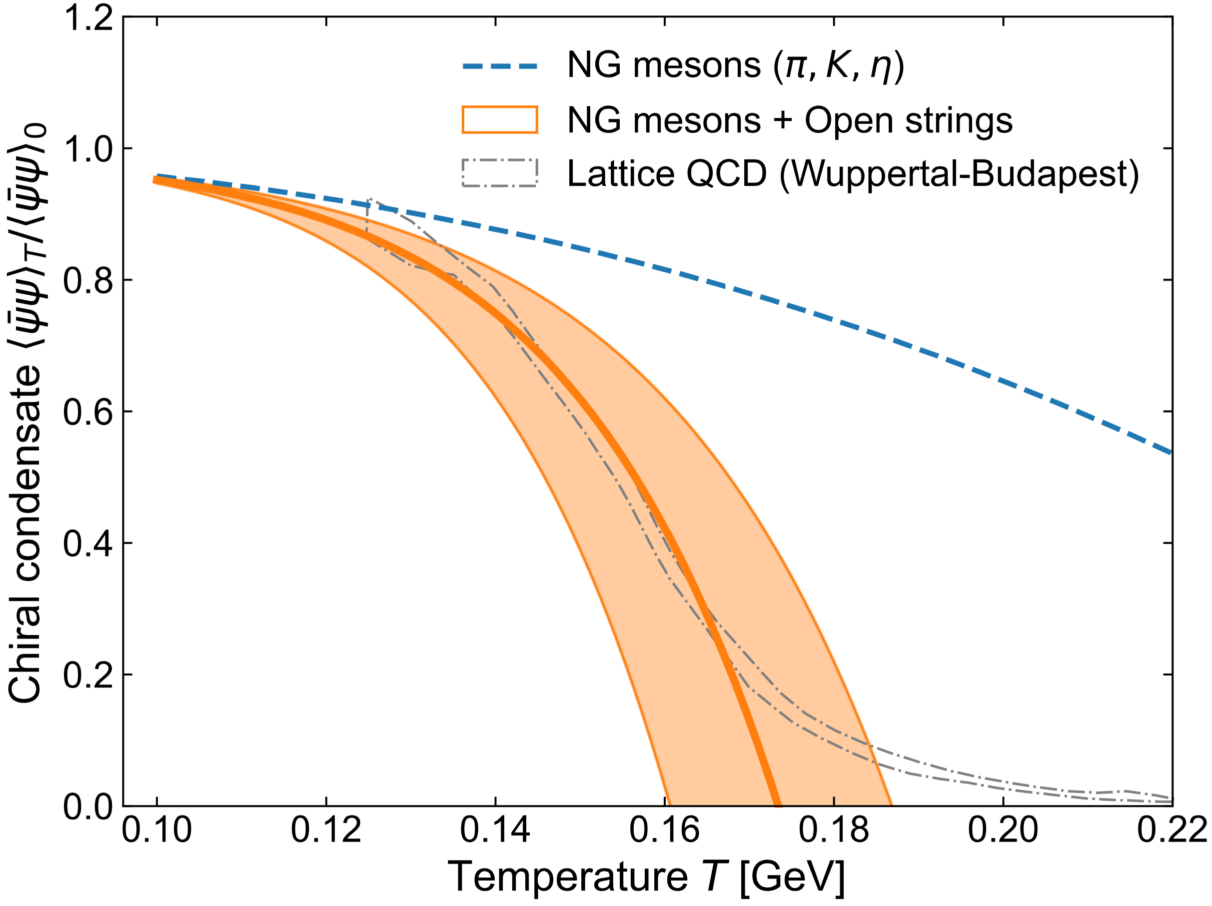
For massive mesons, we fit the sigma value to data on finite temperature values for chiral symmetry. We assume sigma arises only from low mass quarks, so that

$$\sigma_M = n_q \overline{\sigma}$$

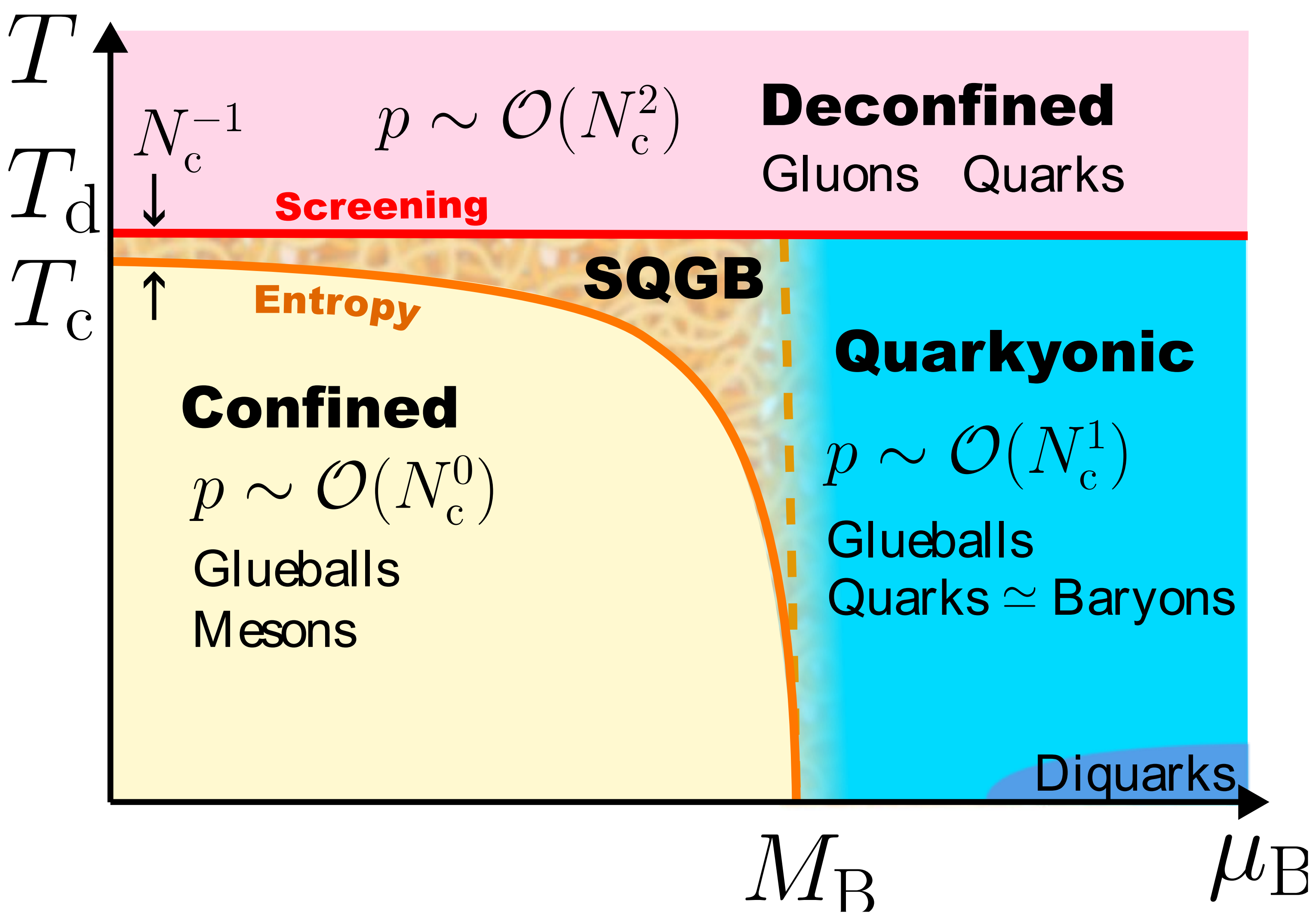
We find best fit is

$$\overline{\sigma} = 60 MeV$$

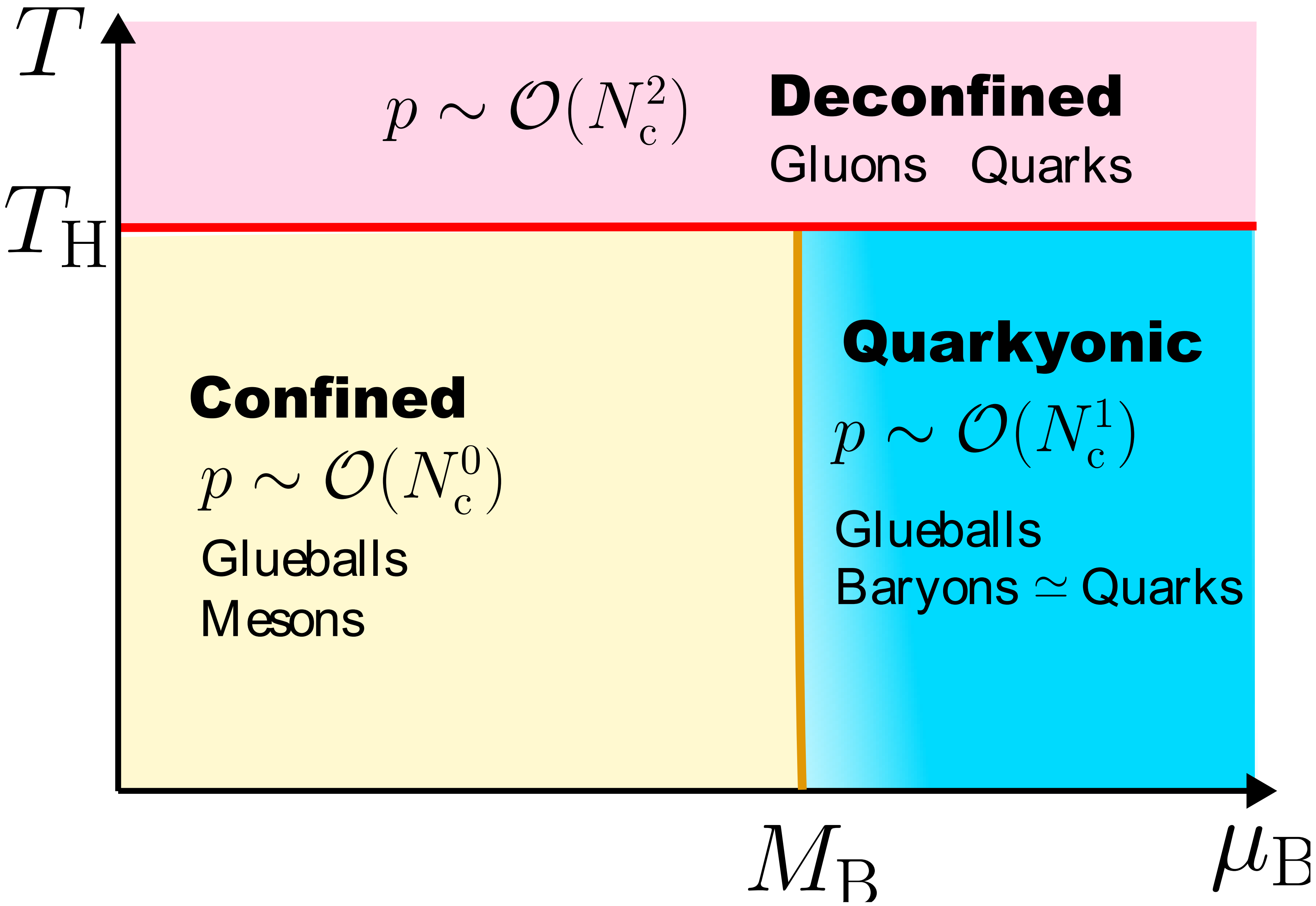
For Goldstone bosons, we use current algebra value



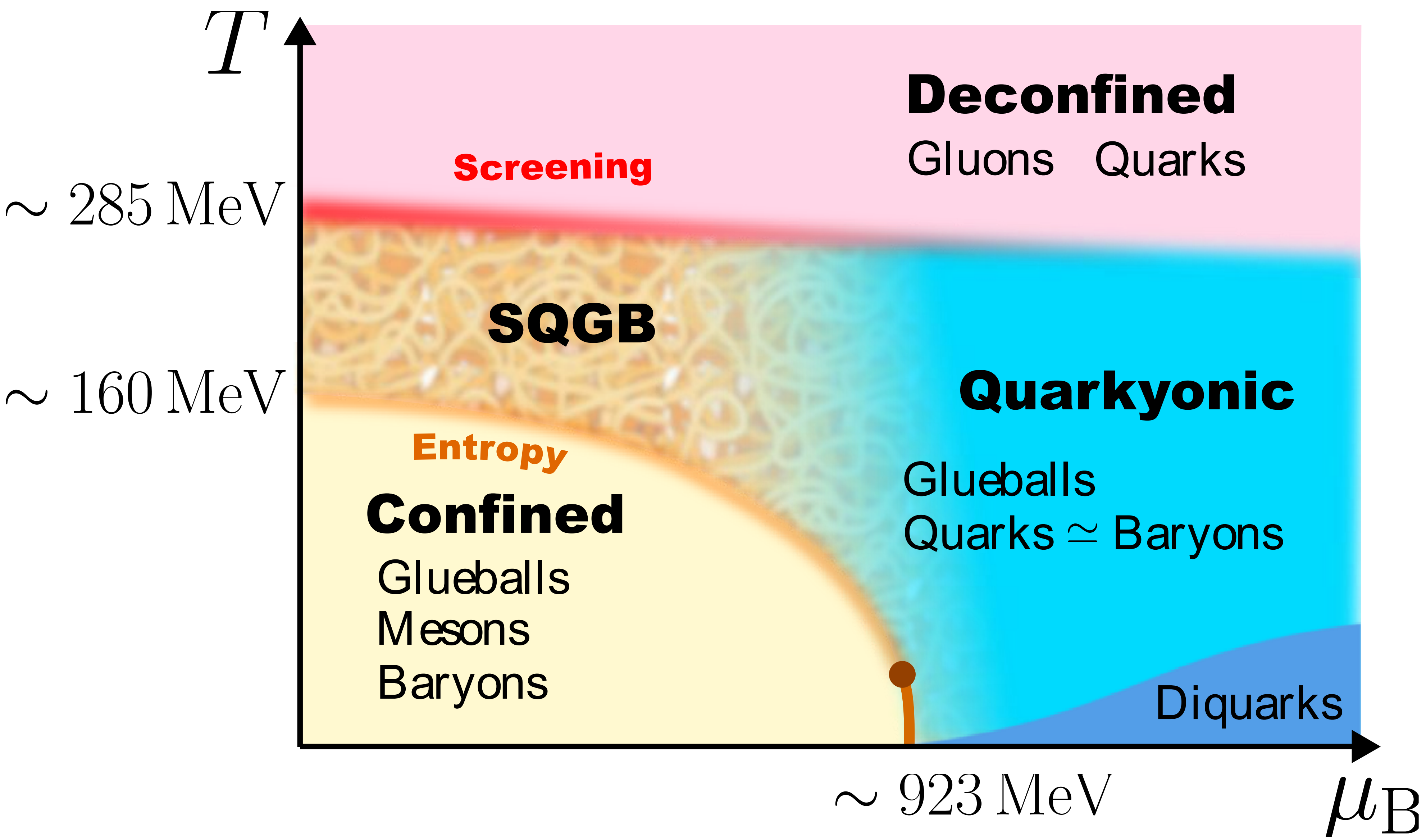
In the limit of a very large number of colors, because the density of quarks diverges, the QGP and SQGB temperatures should both become the Hagedorn temperature



For infinite N_c:



For three colors and realistic parameters



Very Recent Results:

Marczenko-Redlich: Baryon susceptibilities well described for $T < 170$ MeV

Marczenko-McLerran-Redlich (in preparation)

Excellent description of lattice results for charm contribution to thermodynamics for $T < 170$ MEV

In addition to excellent description for light quark quantities, get an excellent description of charmed and and bottom quark meson and baryon abundances

Global best Hagedorn temperature is about 320 MeV, within the limits predicted by the string theory and experimental measures of the string tension

Quarkyonic Matter at Finite Baryon Density or Finite Isospin Density

Finite isospin density, zero baryon density, or zero isospin density and finite baryon number density

Quarkyonic, because

$$M^2_{Debye} \sim \alpha_{'tHooft}\ \mu^2/N_c$$

Low temperatures: pion condensate

$$\mu \leq \Lambda_{QCD}$$

Very High Temperatures: Fermi gas with cooper pairing

$$\mu \gg \sqrt{N_c}\Lambda_{QCD}$$

Quarkyonic Region

$$\Lambda_{QCD} \ll \mu \ll \sqrt{N_c}\Lambda_{QCD}$$

At finite baryon density, the high density phase is an underoccupied gas of nucleons with fully occupied thin shell at the surface, corresponding to a filled Fermis sea of quarks with a finite width Fermi surface

Duality relation between quark and baryon for phase space densities

$$\rho^Q_{PS}(k) = \int \frac{d^3p}{(2\pi)^3} K(k - p/N_c)\rho^N_{PS}$$

The quark fermi sea corresponds to a flat phase space density of baryons of height 1/N_c^3

$$p \leq p^N_F = 2p^Q_F$$

For an initial phase space distribution at low density for a Fermi gas yields a threshold density (for non singular K)

There is a limiting density

$$\rho^0 = 1/K(0)$$

This density is low, of the order of nuclear matter density

Koch, Vovchenko, Miller and LM

For finite Isospin quarkyonic region should be a Fermi sea with a Fermi shell composed of either bosons or Cooper pairs. Scale for cooper pairing would be the QCD scale and is not computable by weak coupling methods

Duality relation between quark up quark and charged meson for phase space densities

$$\rho^Q_{PS}(k) = \frac{1}{2N_c} \int \frac{d^3p}{(2\pi)^3} K(k - p/2) \rho^M_{PS}(p)$$

The quark fermi sea corresponds to a flat phase space density of mesons of height N_c/4

$$p < p^M_F = 2k^Q_F$$

For an initial phase space distribution at low density (corresponding to a constant scalar field) which is,

$$\rho^0_{PS} = \rho^0 (2\pi)^3 \delta^{(3)}(\vec{p})$$

There is a limiting density

$$\rho^0 < 2N_c/K(0)$$

At low densities there is a condensate, but at some density the phase space density for bosons occupies momentum states up to some maximum momentum, corresponding to a Fermi surface for quarks

On the “Fermi” surface there is a condensate of bosons and possibly Cooper pairs

Why does this happen?

If there is some limiting density, then in a scalar field theory the repulsive quartic scalar field interactions cease to keep up with the negative mass squared term

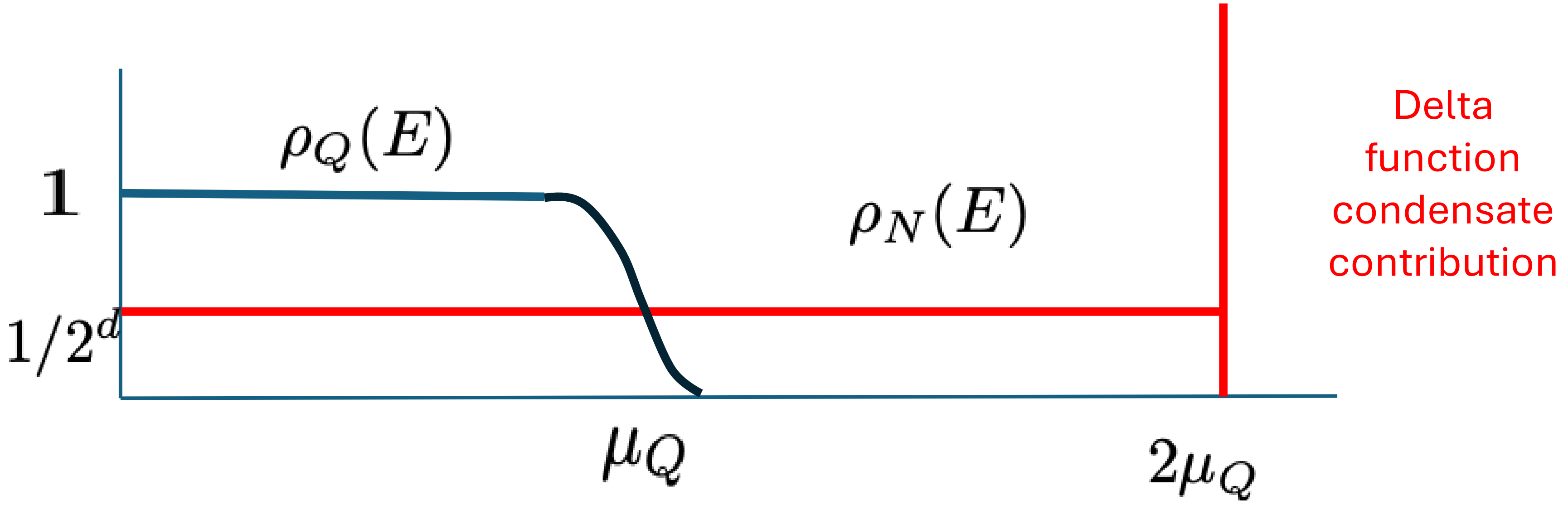
$$\frac{1}{2}\phi \left\{k^2 - \mu_I^2\right\} \phi + \frac{\Lambda}{4N_c}\phi^4 \sim \frac{1}{2}\phi \left\{k^2 - \mu_I^2\right\} \phi$$

For small k, this term is unstable and states occupy until the Fermi exclusion principle of the underlying quark degrees of freedom becomes important

Can this be seen using Monte Carlo methods and composite operators?

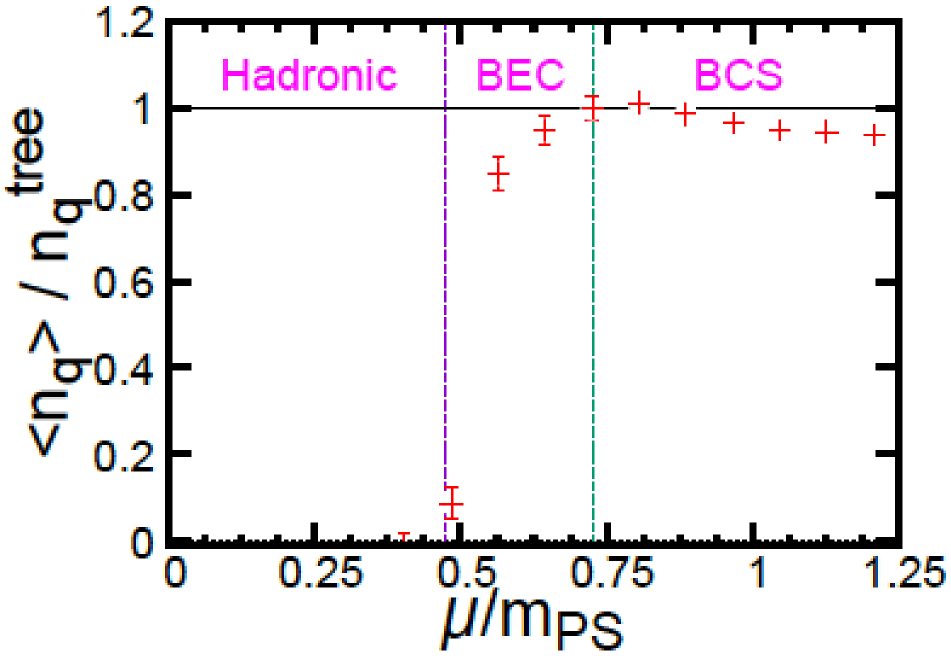
For N_c = 2 QCD: Situations should be similar

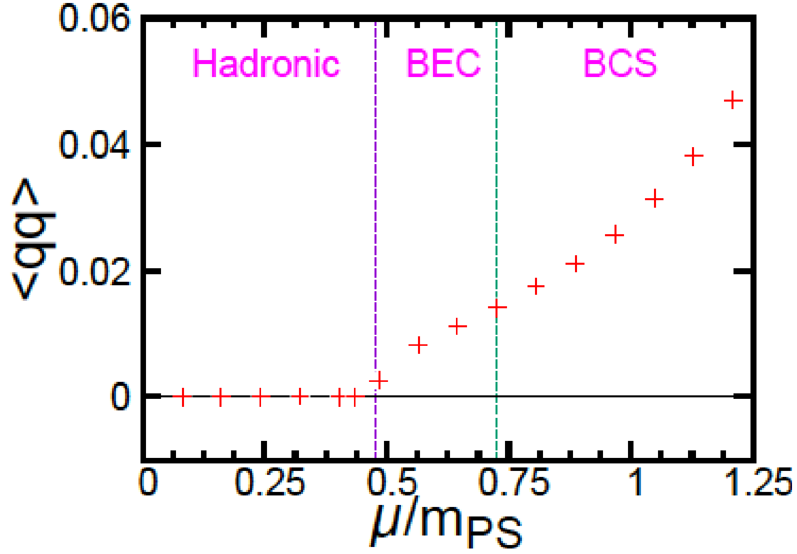
When quarks are fully occupied, the bosonic baryons look like a Fermi gas with occupation number 1/N_c^d



Quark Fermi surface broadened from the surface structure of the baryon number distribution:
It is a small correction to the total density at large density, and in 1+1 d generates a constant contribution to the trace
Surface can arise for either Cooper pair or chiral spiral like effects

Even and odd N_c are continuous because surface distribution is approximately delta function. Height of meson region is different for finite isospin density

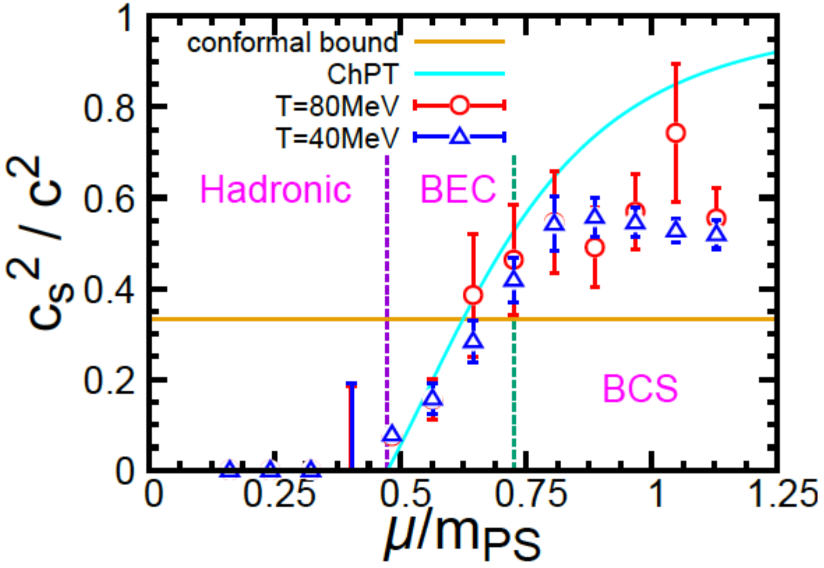




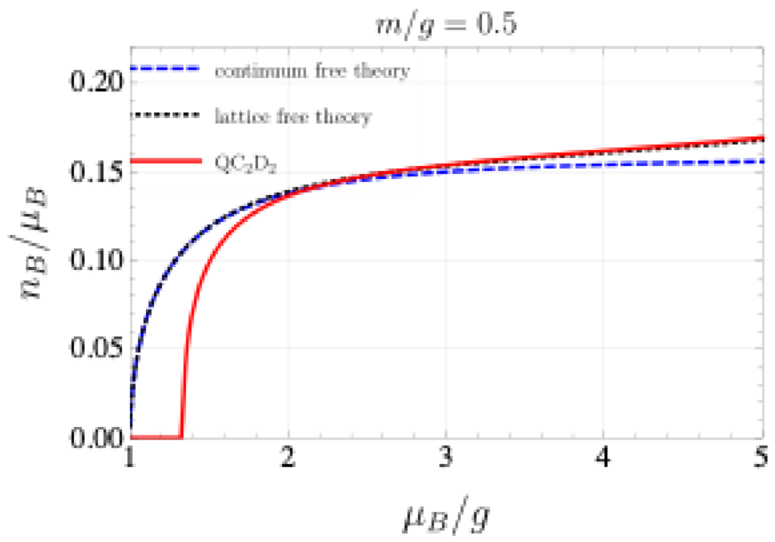
Iida, Itou, Murakami, Suenega

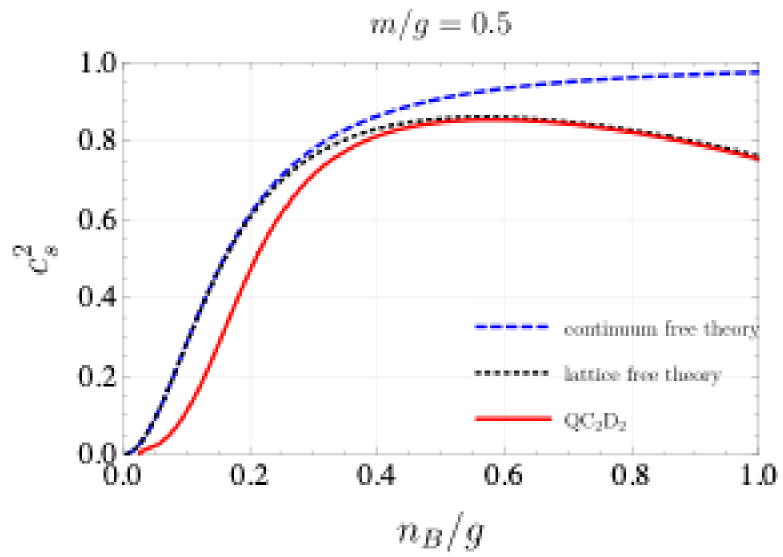
Correlation functions show confining behaviour at large distances;

Potential between quarks is linear



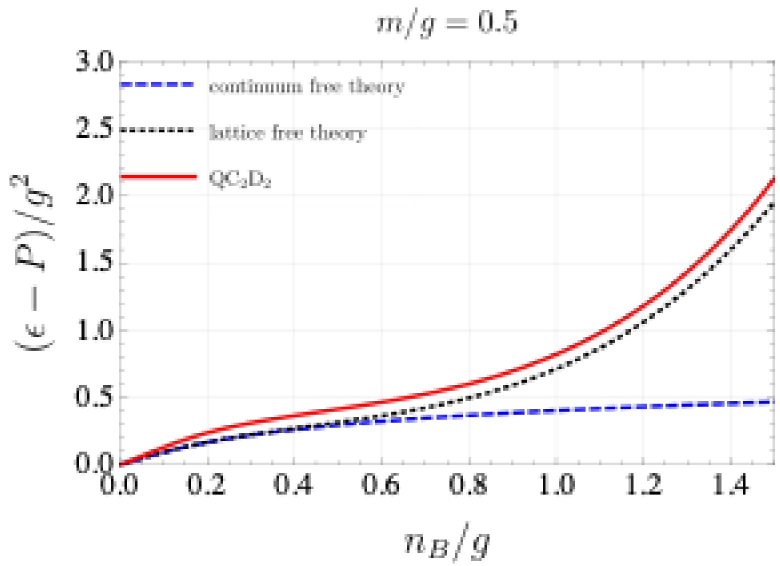
Condensate has surface area dependence on baryon chemical potential at large density, and do not much affect the density chemical potential dependence

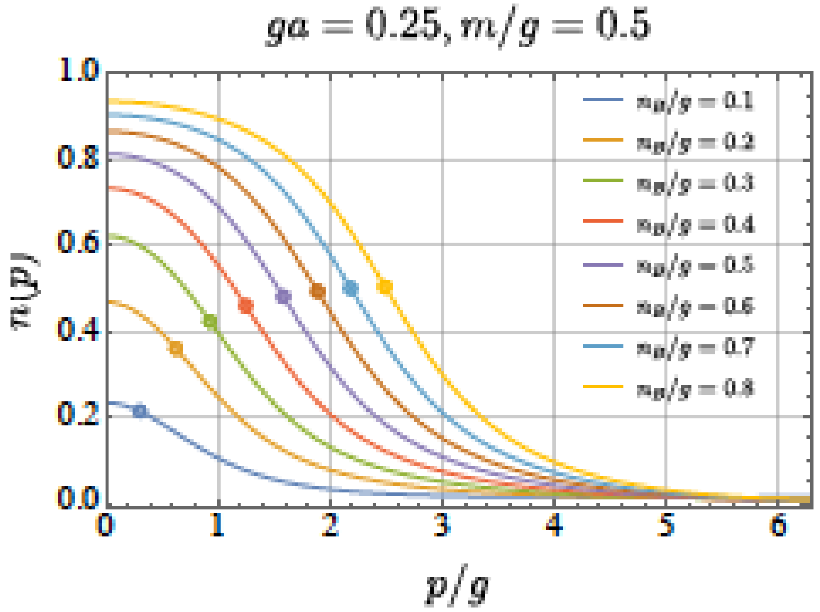




Small deviation of trace anomaly from free theory is probably a Fermi surface effect. Particle hole pairs or Cooper pairs which are diminished by fluctuations in 1+1 d?

Fujikura and Hidaka





Density is low where transition occurs,
About .2 fm^-1?

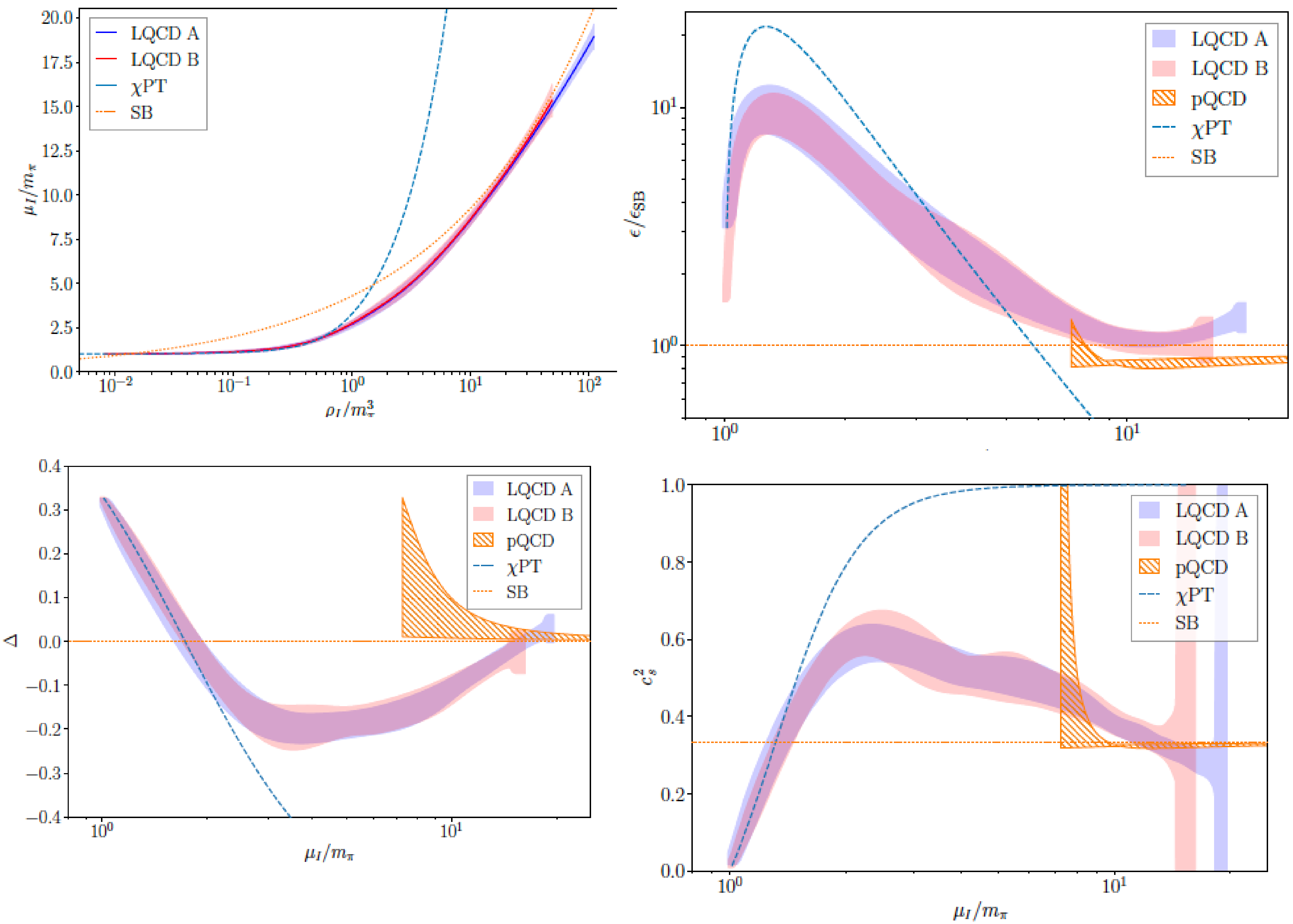
N_c = 3 at
Finite
Isospin:
NPLQCD

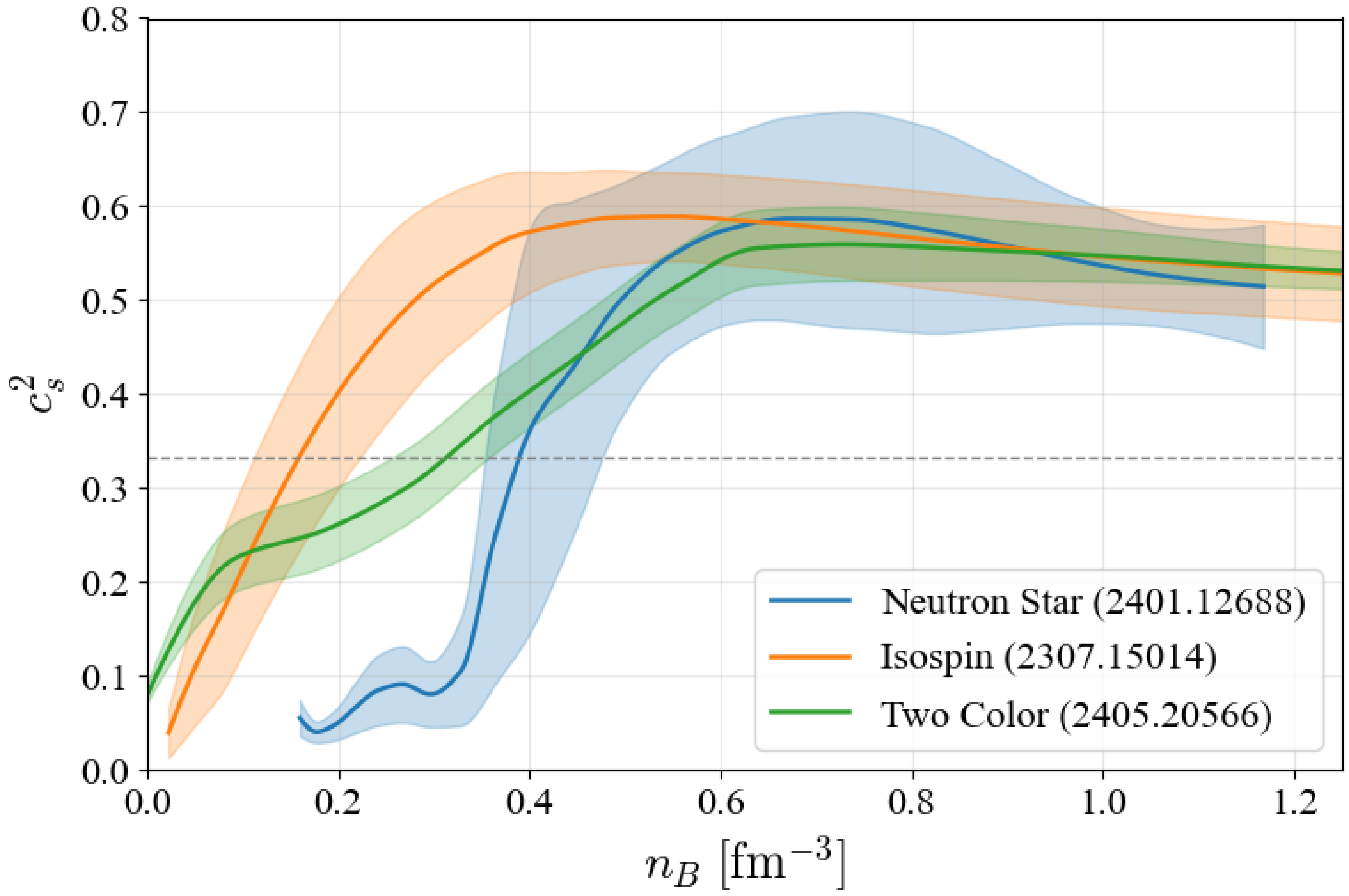
$$\mu_I \sim 700\ MeV$$
$$\sim 5/m_\pi$$

$$\mu_Q \sim 350 MeV$$

Quarkyonic behavior sets in at lowish density, but higher than for finite density

Lowest density and rise of sound velocity due to condensation





With Itou, Fujikura, Fukushima and Hidaka

My Opinions:

Finite Temperature

Finite temperature data consistent with a region T_chiral < T < T_Hagedorn

We now understand from first principles computations the thermodynamics below and near T_chiral and above T_Hagedorn

Lattice data shows onset of quark like fluid behavior for a temperature about midway between the upper and lower limit

In the intermediate region, long range correlations look confined

Not yet consensus on physics of this region

Finite Density

Data for N_c = 2 in 2 and 4 d at finite baryon number density, and for N_c = 3 in 4 d at finite isospin

VERY EXCITING

Results show transition at low density to behavior close to a weakly interacting gas of quarks.
For N_c =2, correlations at large distance are confined

Quarkyonic Matter