Hadron spectra and thermodynamics for all quark flavors from a universal Hagedorn temperature

Michał Marczenko

SKCM²
WPI HIROSHIMA UNIVERSITY

with Győző Kovács, Larry McLerran, Krzysztof Redlich

- MM, Kovács, McLerran, Redlich, PRD 112 (2025) 9, 096010
- MM, Redlich, 2601.22902 [hep-ph] (2026)
- MM, McLerran, Redlich, 2603.28668 [hep-ph] (2026)

66. Cracow School of Theoretical Physics, Kraków, Poland, 16/06/2026

The Hagedorn Hypothesis

$$\rho(m) \simeq m^{-a} \exp(m/T_H)$$

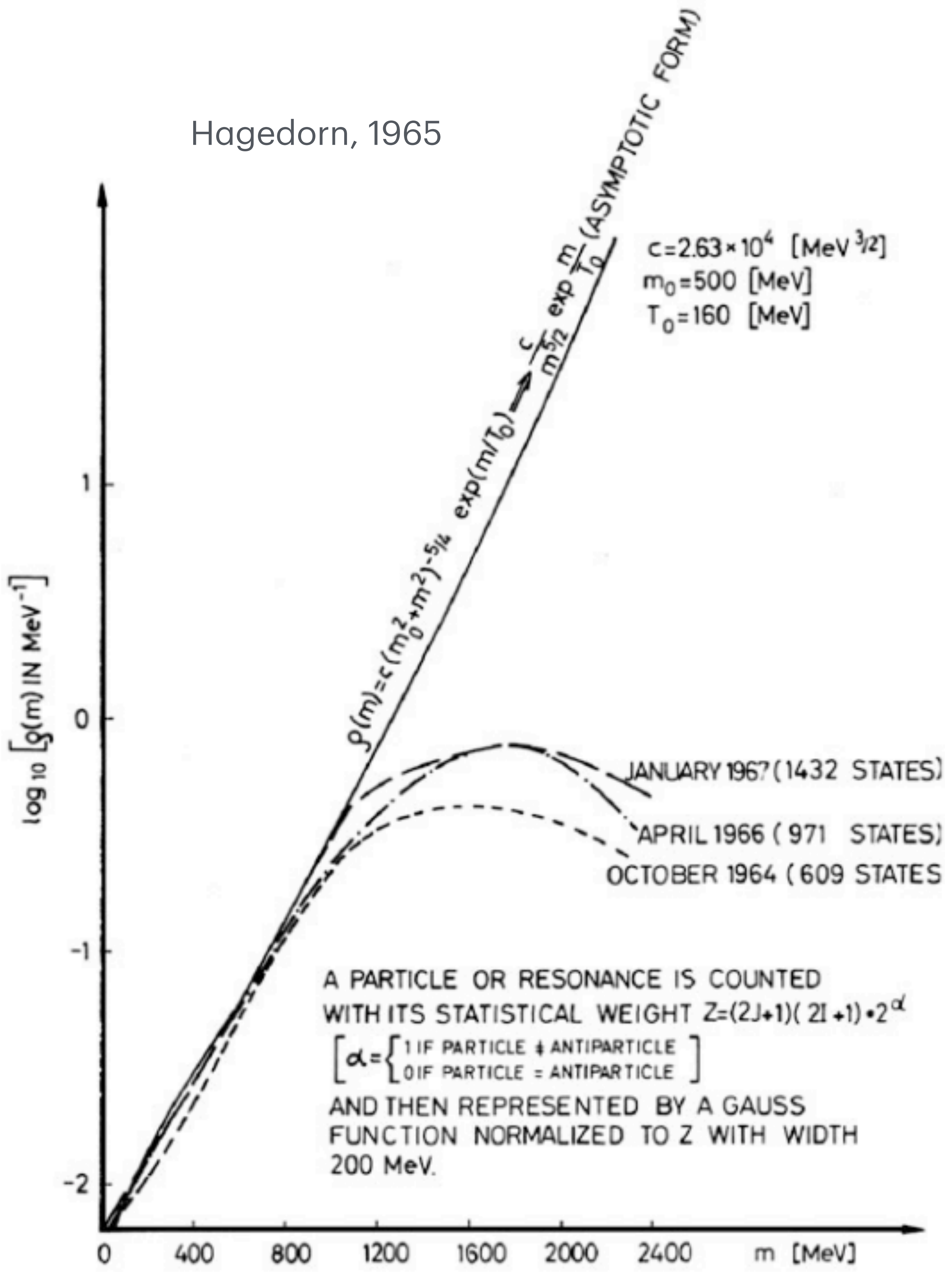
scale controlling the asymptotic growth of resonances

Singularity at T_H

$$Log\mathcal{Z} \sim \int \rho(m) e^{-m/T}$$

Hagedorn (1965): Limiting T for strong systems

Hagedorn, 1965



The Hagedorn Hypothesis

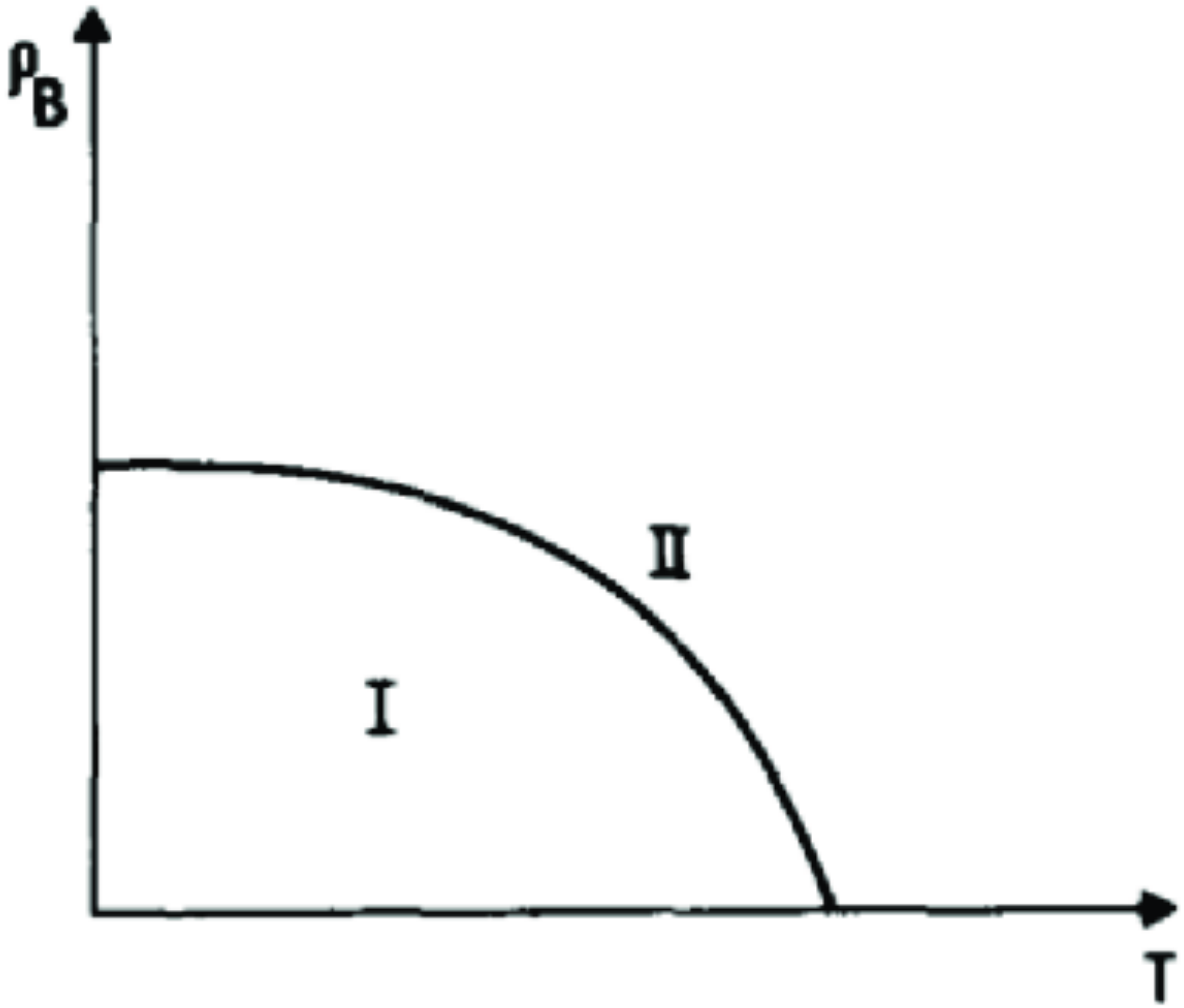
$$\rho(m) \simeq m^{-a} \exp(m/T_H)$$

scale controlling the asymptotic growth of resonances

Singularity at T_H

$$Log\mathcal{Z} \sim \int \rho(m) e^{-m/T}$$

Cabibbo, Parisi (1975): Transition to QGP



The Hagedorn Hypothesis

$$\rho(m) \simeq f(m)\ \exp(m/T_H)$$

Typical Prefactor

$$f(m) = A/(m^2 + m_0^2)^{5/4} \sim m^{-5/2}$$

But depends on Bootstrap Condition

Typical Hagedorn Temperature

$$T_H \approx 150 - 180\,\mathrm{MeV}$$

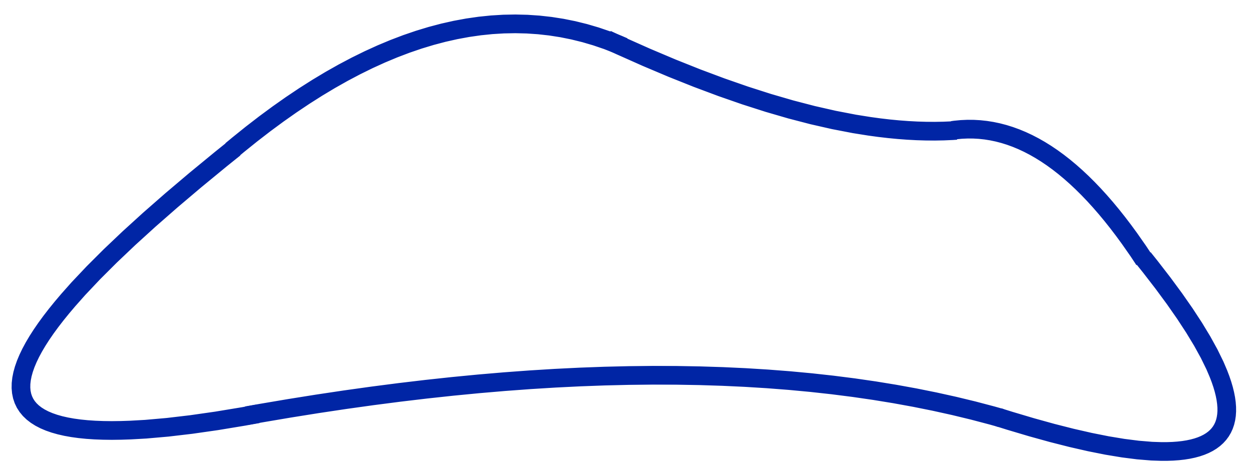
Associated with Deconfinement

see, e.g., Hagedorn, 1965, 1971; Satz, 1979; Satz, Redlich, 2016; Broniowski, Florkowski, 2000

Hagedorn Spectrum from d=4 Relativistic String

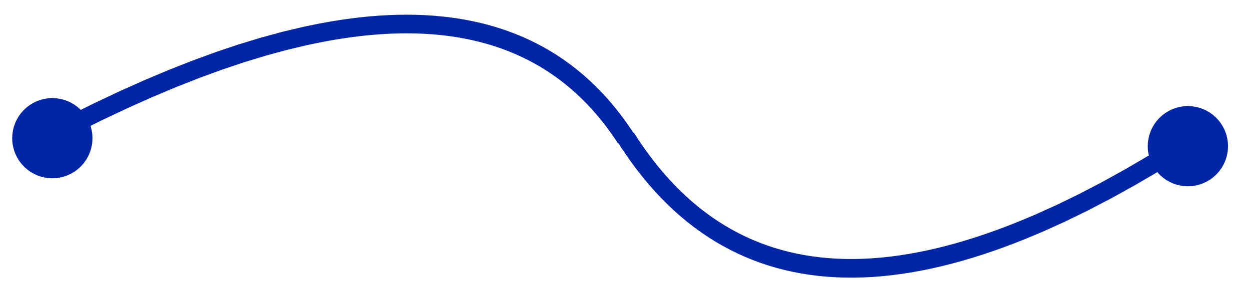
see Fujimoto, 2025; Green et al 1988; Meyer, 2009

GLUEBALLS AS CLOSED STRINGS



$$\rho_{\rm cl}(m) = \frac{1}{T_H}\left(\frac{2\pi}{3}\right)^3\left(\frac{m}{T_H}\right)^{-4} e^{m/T_H}$$

MESONS AS OPEN STRINGS



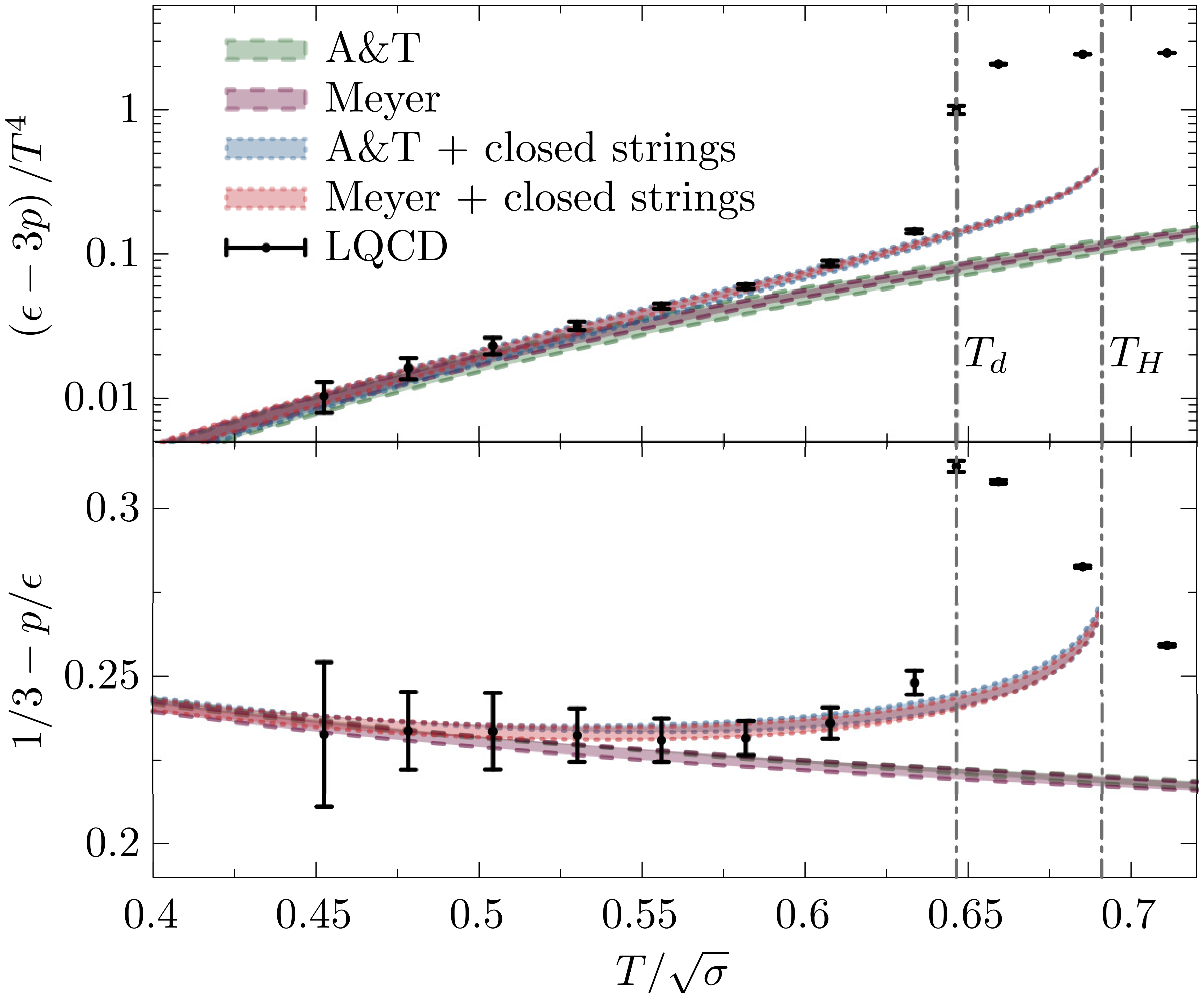
$$\rho_{\rm op}(m) = \frac{\sqrt{2\pi}}{6T_H}\left(\frac{m}{T_H}\right)^{-3/2} e^{m/T_H}$$

Common Hagedorn Temperature: $T_H = \sqrt{\frac{3}{2\pi}}\sqrt{\sigma}$ → Effective scale set by string tension

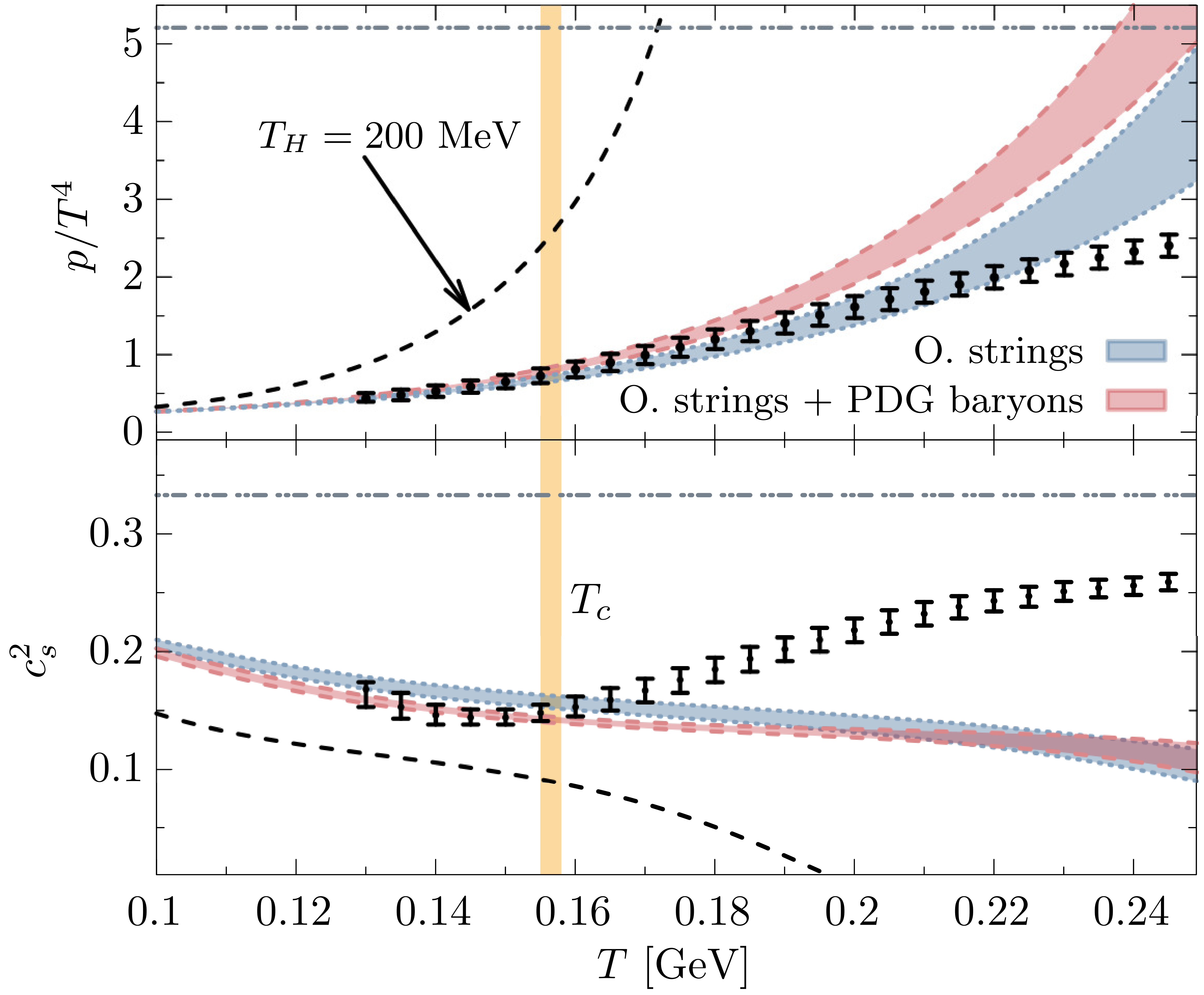
- Recent estimates on $\sqrt{\sigma} \approx 480\,\mathrm{MeV}$ give $T_H \approx 330\,\mathrm{MeV}$ (e.g. Brambilla et al, 2023)
- Baryons described as quark-diquark open strings (Fujimoto, 2026)

Confined LQCD EoS described by strings

SU(3) pure gauge theory



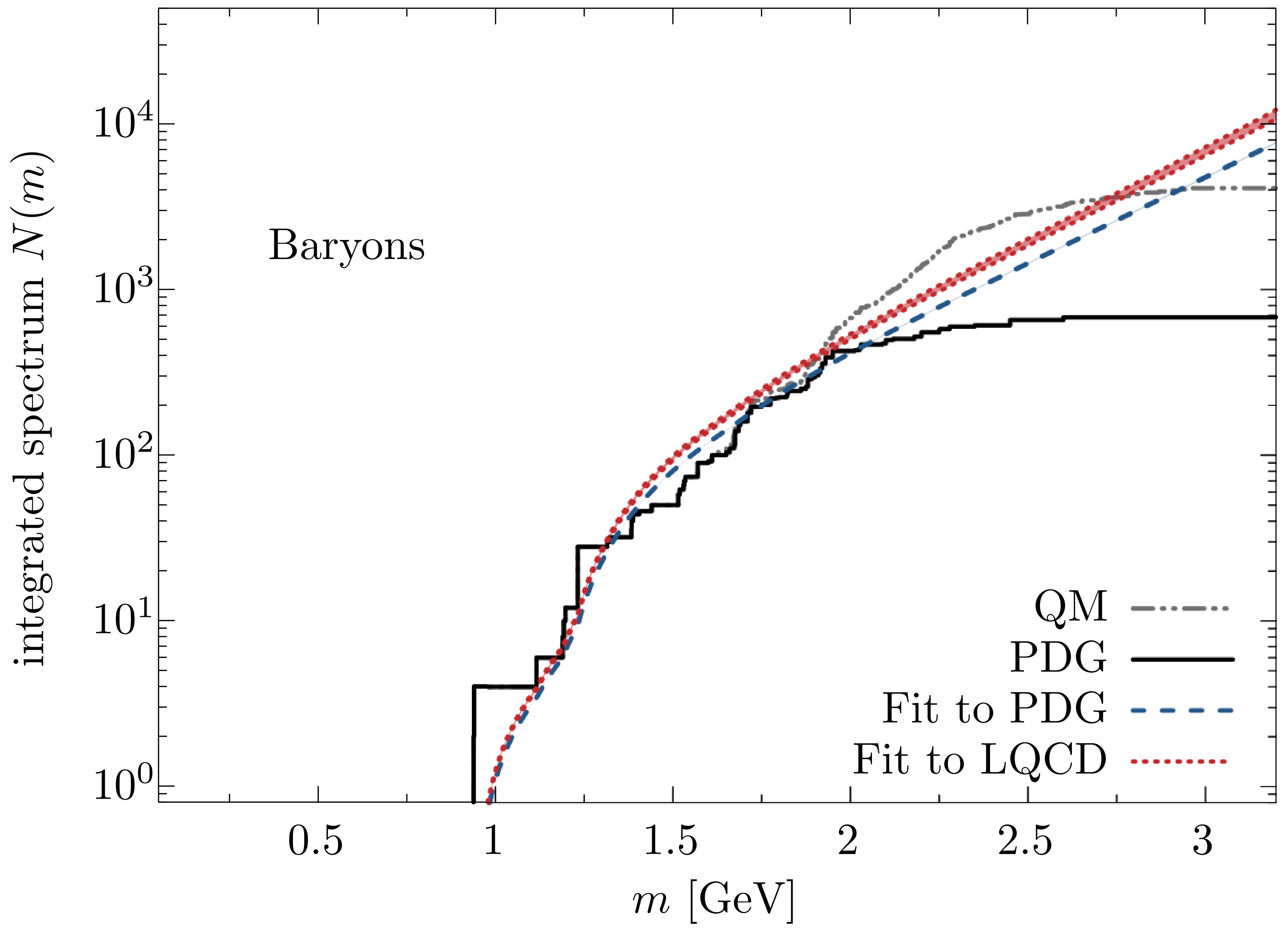
2+1-flavor LQCD



Thermodynamics exposes the full spectrum

Spectrum from experimental data

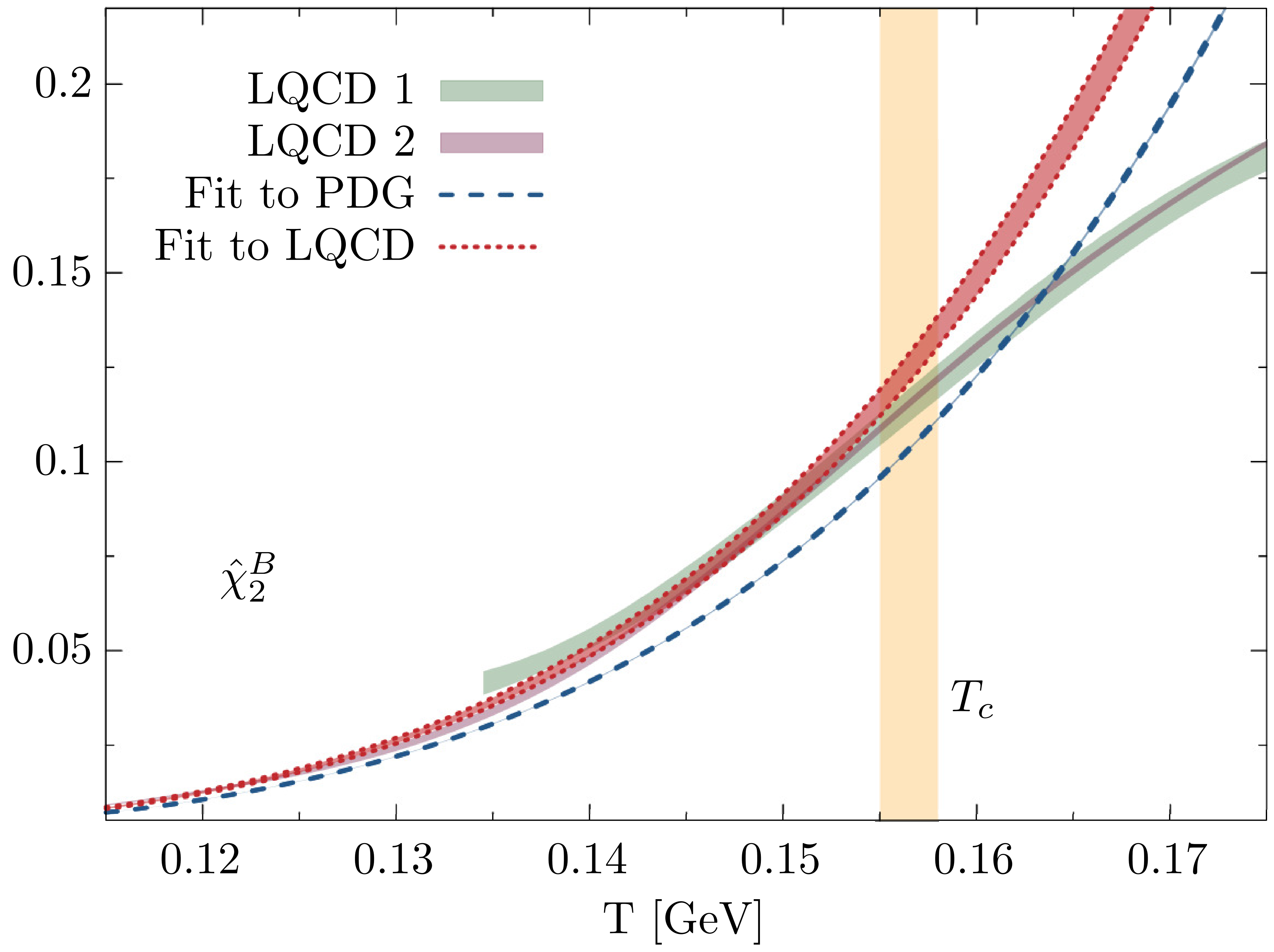
- Fit to spectrum gives: $T_H \simeq 340\,\mathrm{GeV}$
- Underestimates LQCD thermodynamics



Cumulative spectrum: $N = \int_0^m dm' \rho(m')$

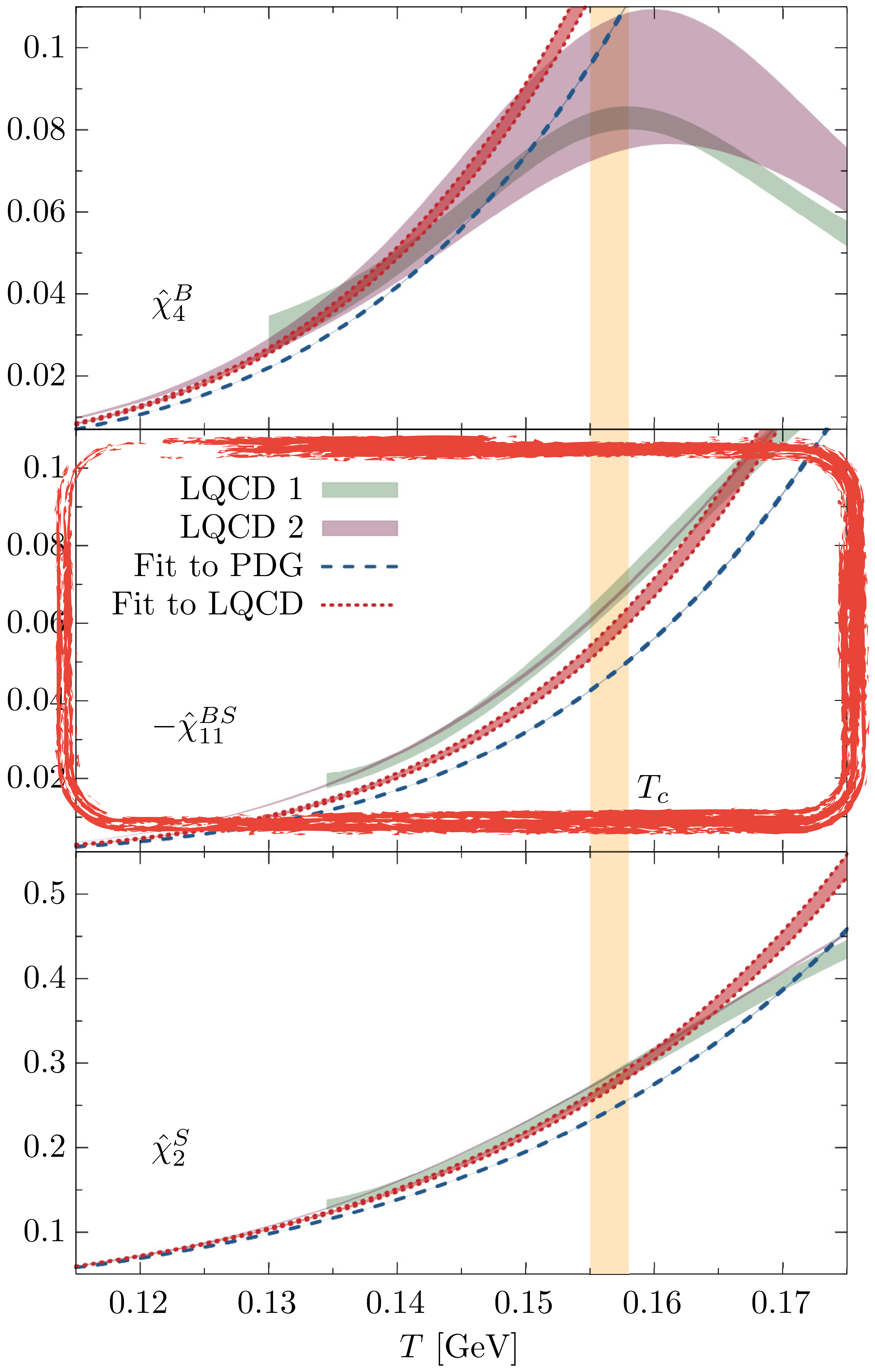
Spectrum from LQCD fluctuations

- Fit to $\hat{\chi}_2^B$ gives: $T_H \simeq 323\,\mathrm{GeV}$
- Spectrum consistent with PDG



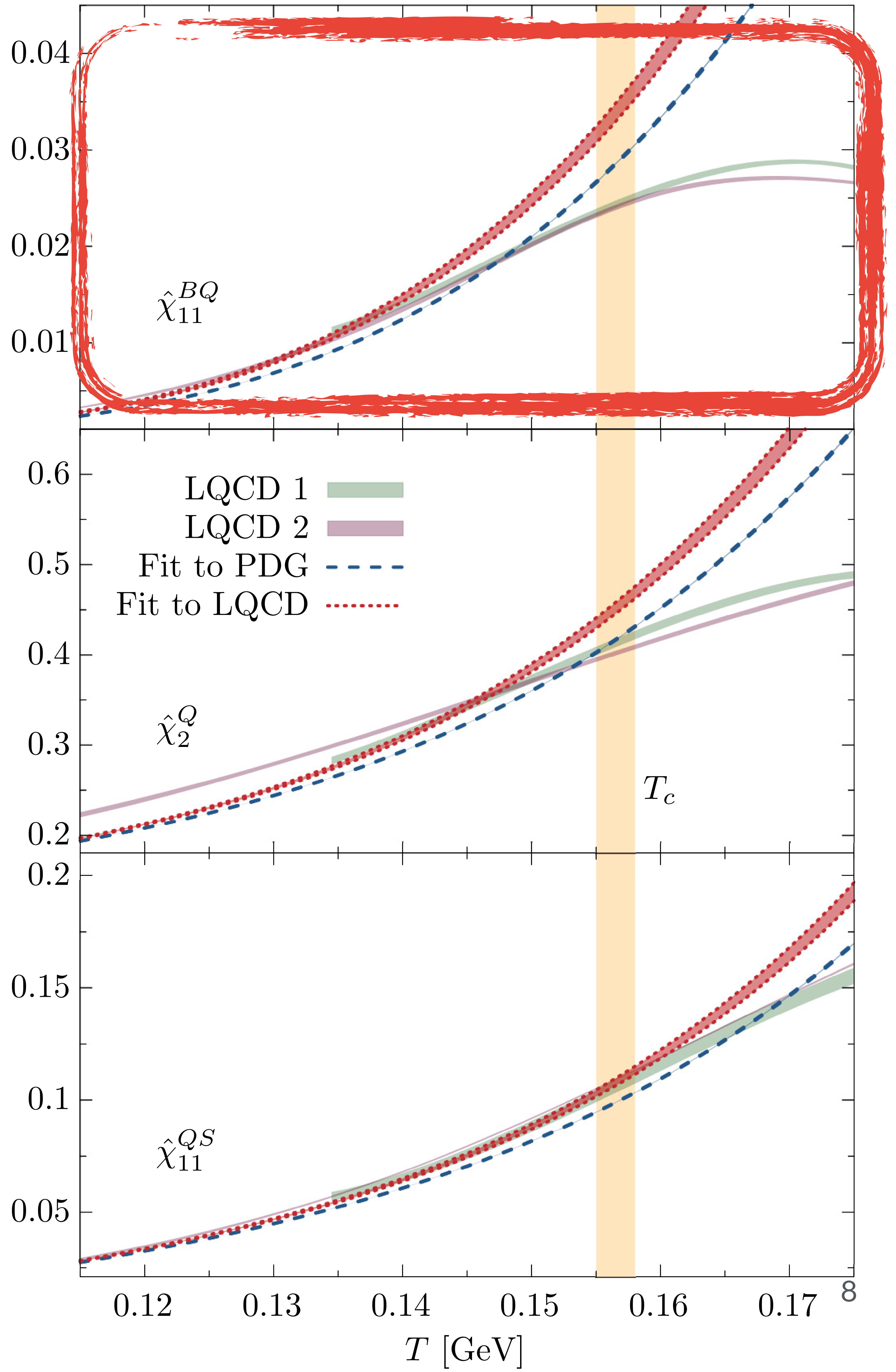
Generalised susceptibilities: $\hat{\chi}_{jkl}^{BQS} = \frac{\partial^n \hat{P}(T,\mu)}{\partial \hat{\mu}_B^j \partial \hat{\mu}_Q^k \partial \hat{\mu}_S^l}$

Correlations in the Baryonic Sector



Overall agreement is good

Residual discrepancies point toward missing repulsive/non-resonant interactions.



Correlations in the Baryonic Sector

Charm Lattice QCD Equation of State

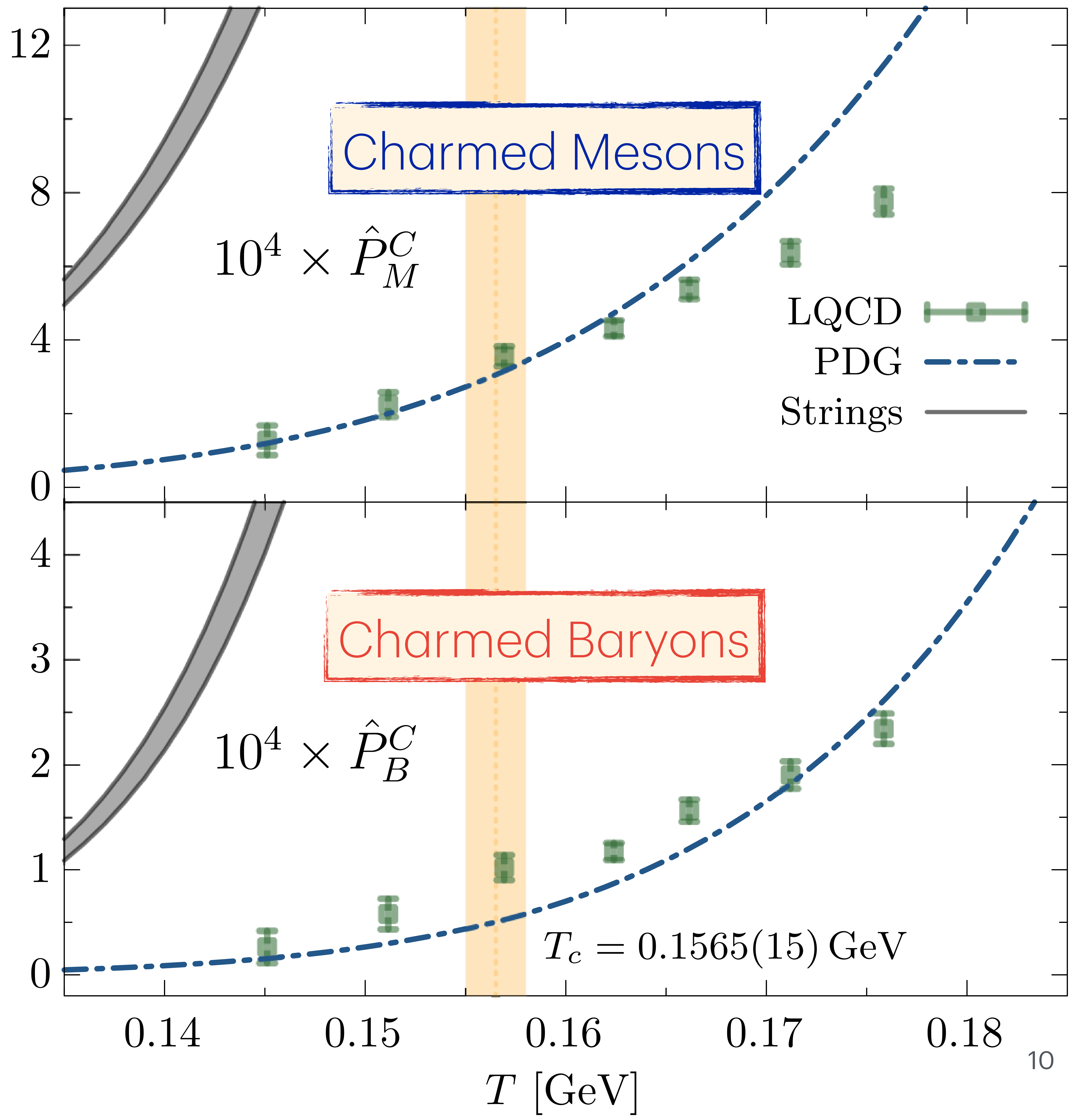
LQCD thermodynamics (Kaczmarek et al, 2025):

- Consistent with PDG mesons
- Inconsistent with PDG baryons (factor of 2 at T_c)

Hagedorn spectrum overpredicts the EOS

String spectrum governs excitations above the current quark mass

$$\rho(m) \longrightarrow \rho\left(E = m - \sum n_q m_q\right)$$



Parameter-free Equation of State

Quark masses from PDG

$m_s = 0.093\,\mathrm{GeV},\ m_c = 1.273\,\mathrm{GeV}$

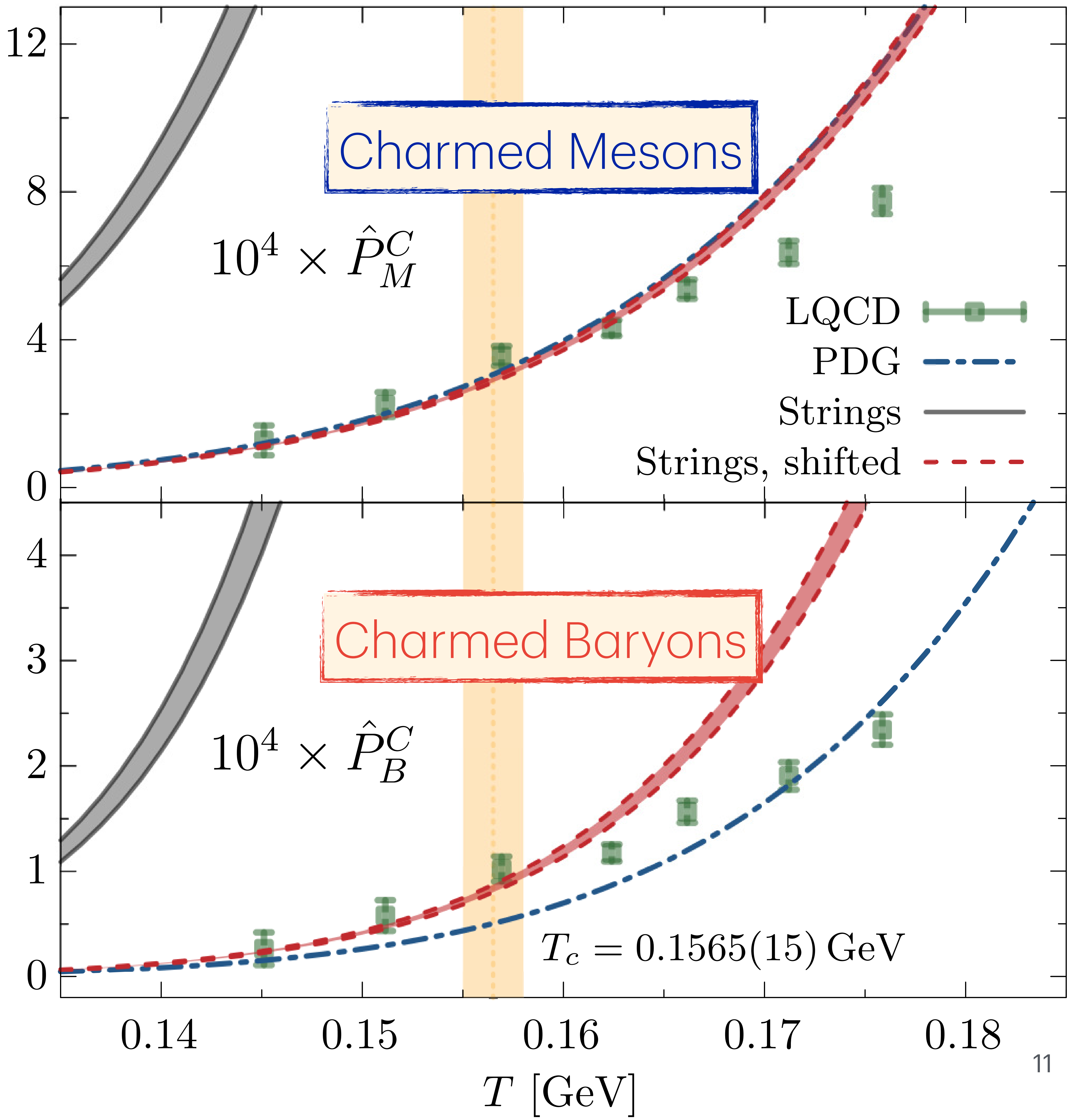
$+$

No (re)fitting

$T_H \approx 323\,\mathrm{MeV}$ fixed from light sector

Parameter-free agreement with LQCD EoS

Hadronic spectra are controlled by excitation energy above the current quark mass



Charm Quark Susceptibilities up to 4th Order

Overall description of LQCD fluctuations quantitatively consistent across quantum number channels

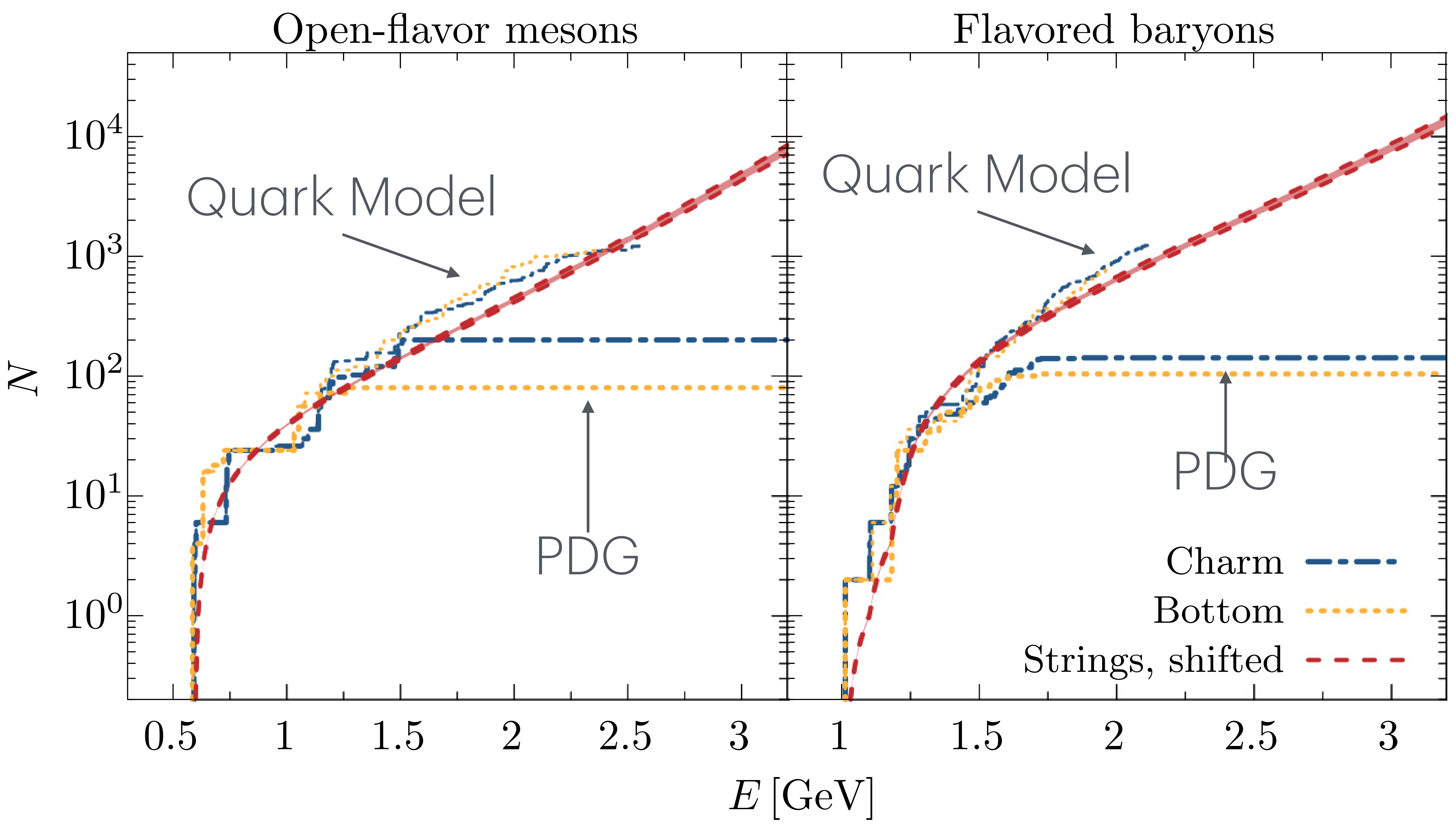
Charm vs Bottom Hadron Spectra

Universal Excitation Energy Threshold

$$E_{\text{thr}} = m_{\text{thr}} - \sum n_q m_q$$

Single excitation spectrum for all flavors

$$\rho_{\text{op}}(E) = \frac{\sqrt{2\pi}}{6T_H}\left(\frac{E}{T_H}\right)^{-3/2} e^{E/T_H}$$

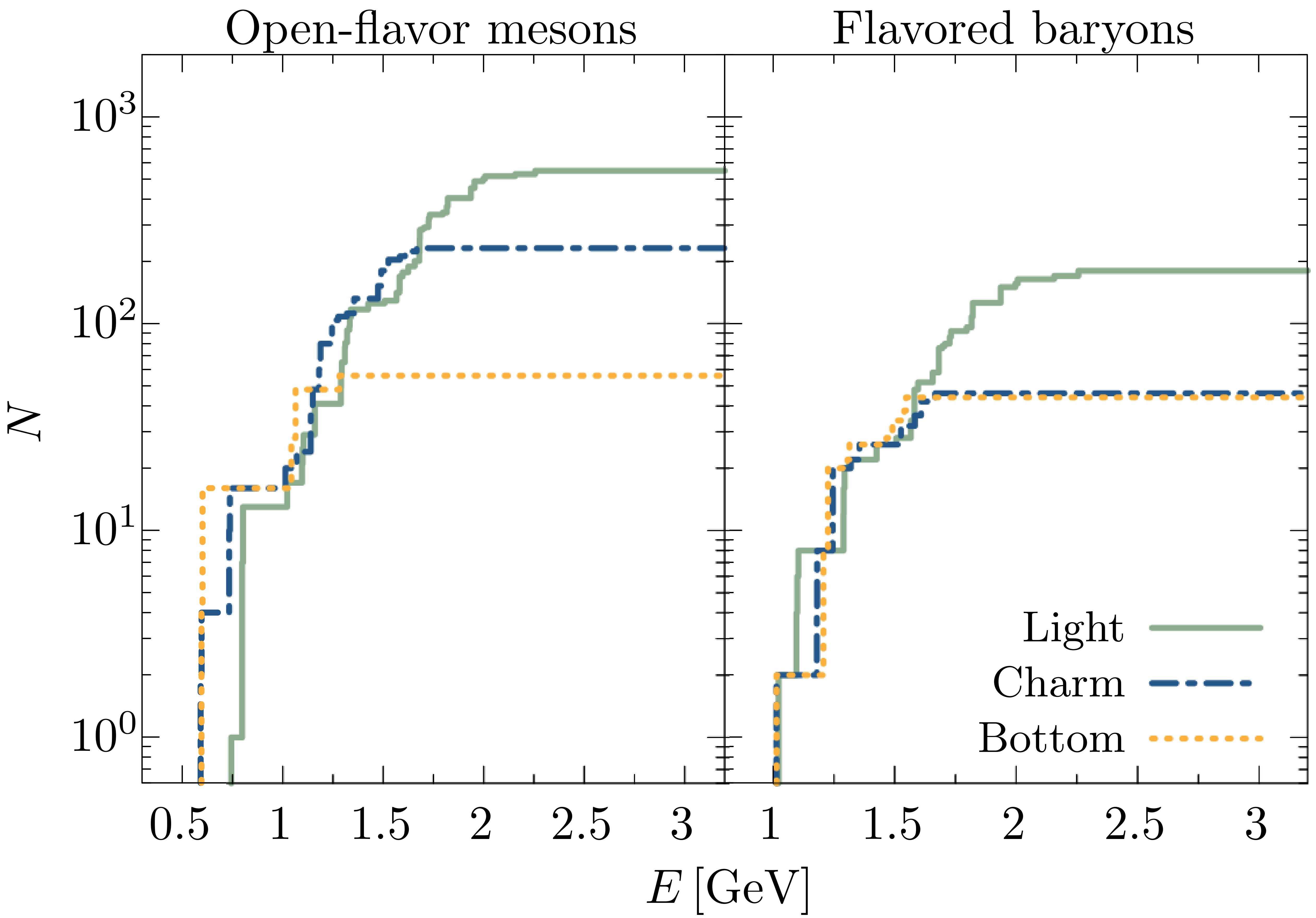


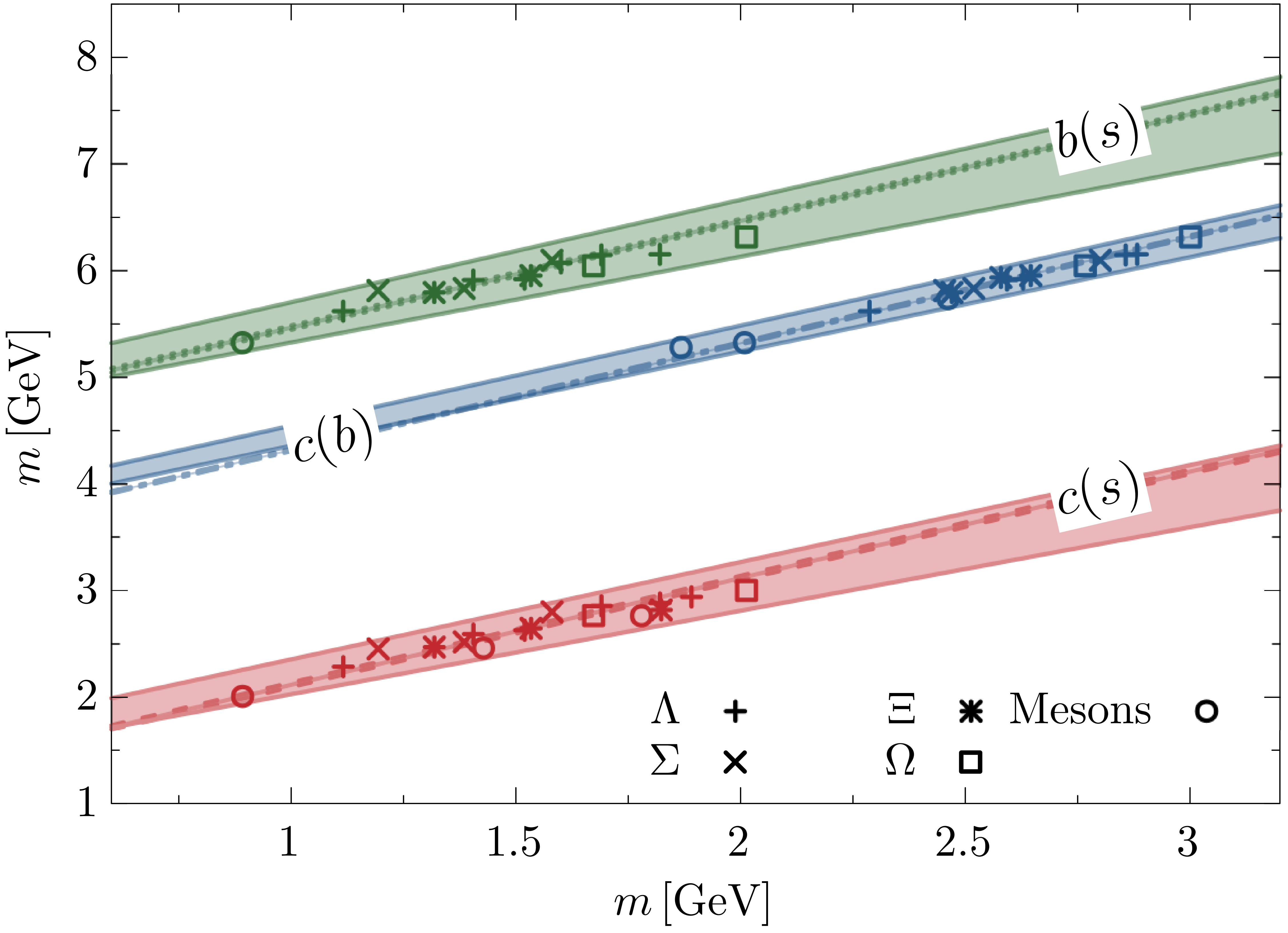
$$\rho^{\text{charm}}(E) \simeq \rho^{\text{bottom}}(E)$$

within uncertainties

Charm and bottom follow the same excitation pattern

Universal growth of states across quark flavors





Exponential growth of states above the energy threshold is universal and governed by T_H linked to the fundamental string tension

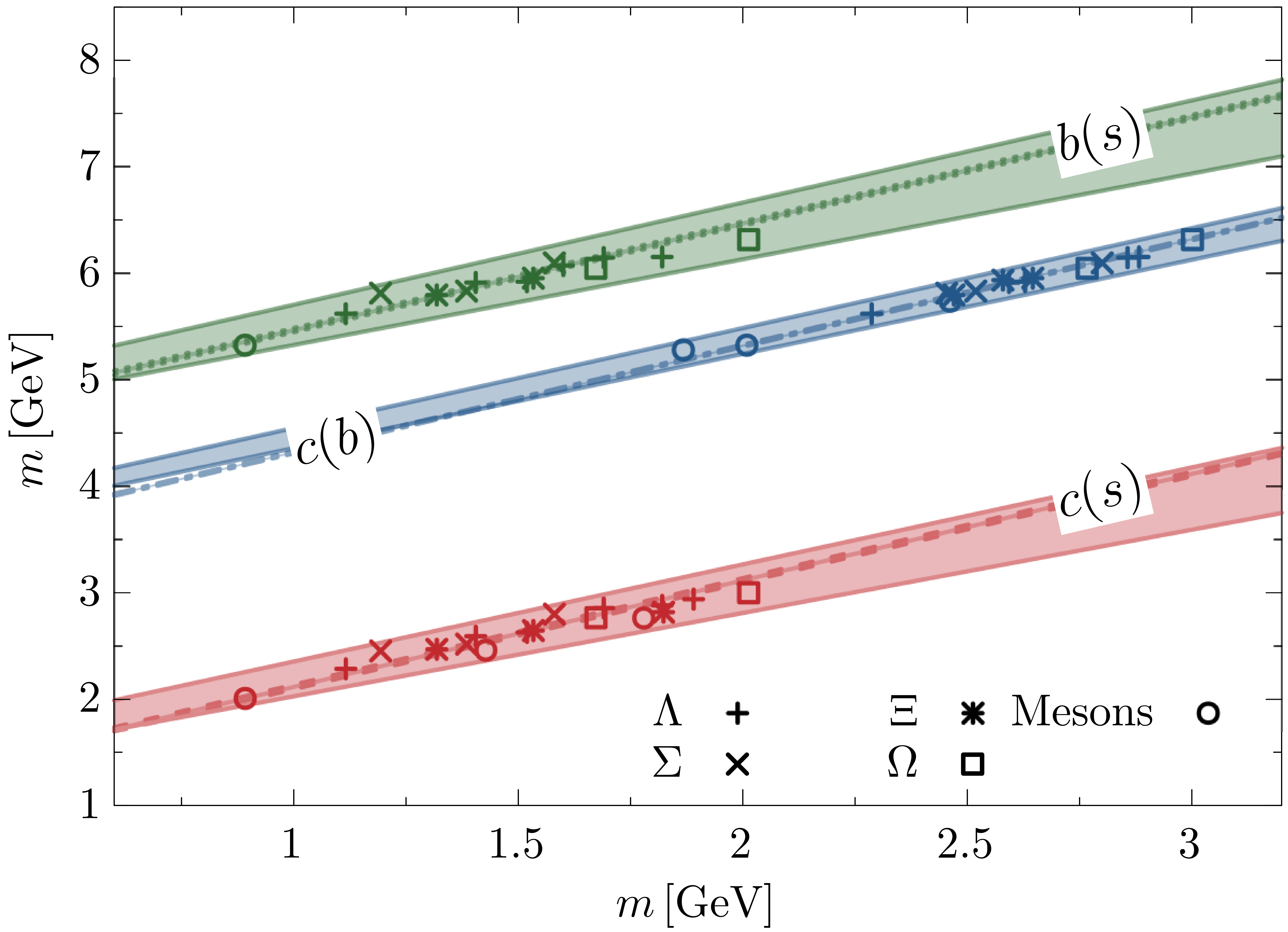
Hadrons emerge naturally as strings

Exponentially growing spectra are governed by a universal T_H

Scale set by the string tension

Thank You

Universality across quark flavors



Thermodynamics

$$\rho_{\text{glue}} = \theta(m - m_{\text{thr}})\rho_{\text{cl}}(m)$$

$$\rho_M = \sum_{i=\pi,K,\eta} d_i \delta(m - m_i) + \sum_{i=\rho,K^*,\phi} d_i \theta(m - m_i)\rho_{\text{op}}(m)$$

$$\rho_B = \sum_{i=N,\Delta,\Lambda,\ldots} d_i \theta(m - m_i)\rho_{\text{op}}(m)$$

$$\hat{P}(m) = \frac{1}{2\pi^2}\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right) e^{\mu/T}$$

$$\hat{P} = \int_0^\infty dm\, \rho(m)\, \hat{P}(m)$$

Baryons as quark-diquark strings

Sharov, 2000

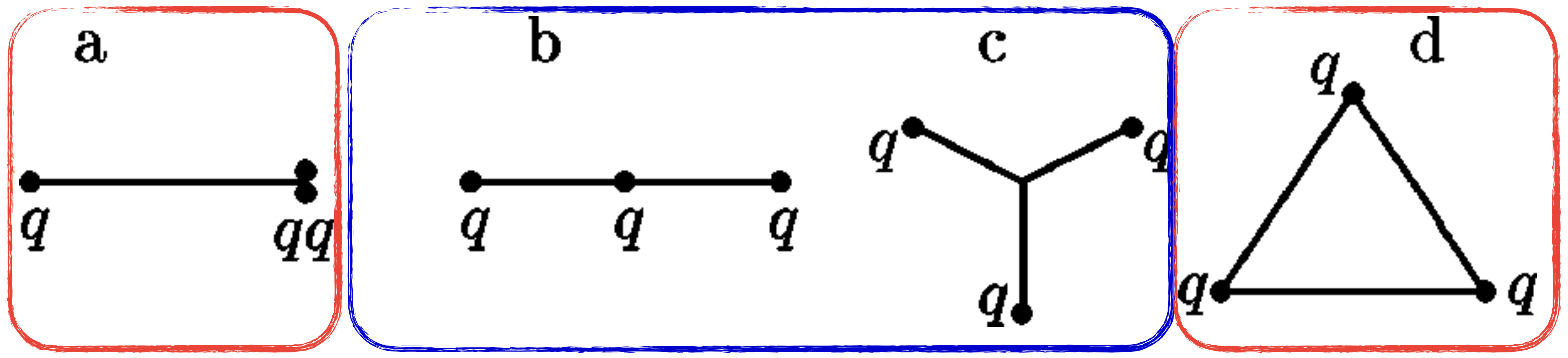
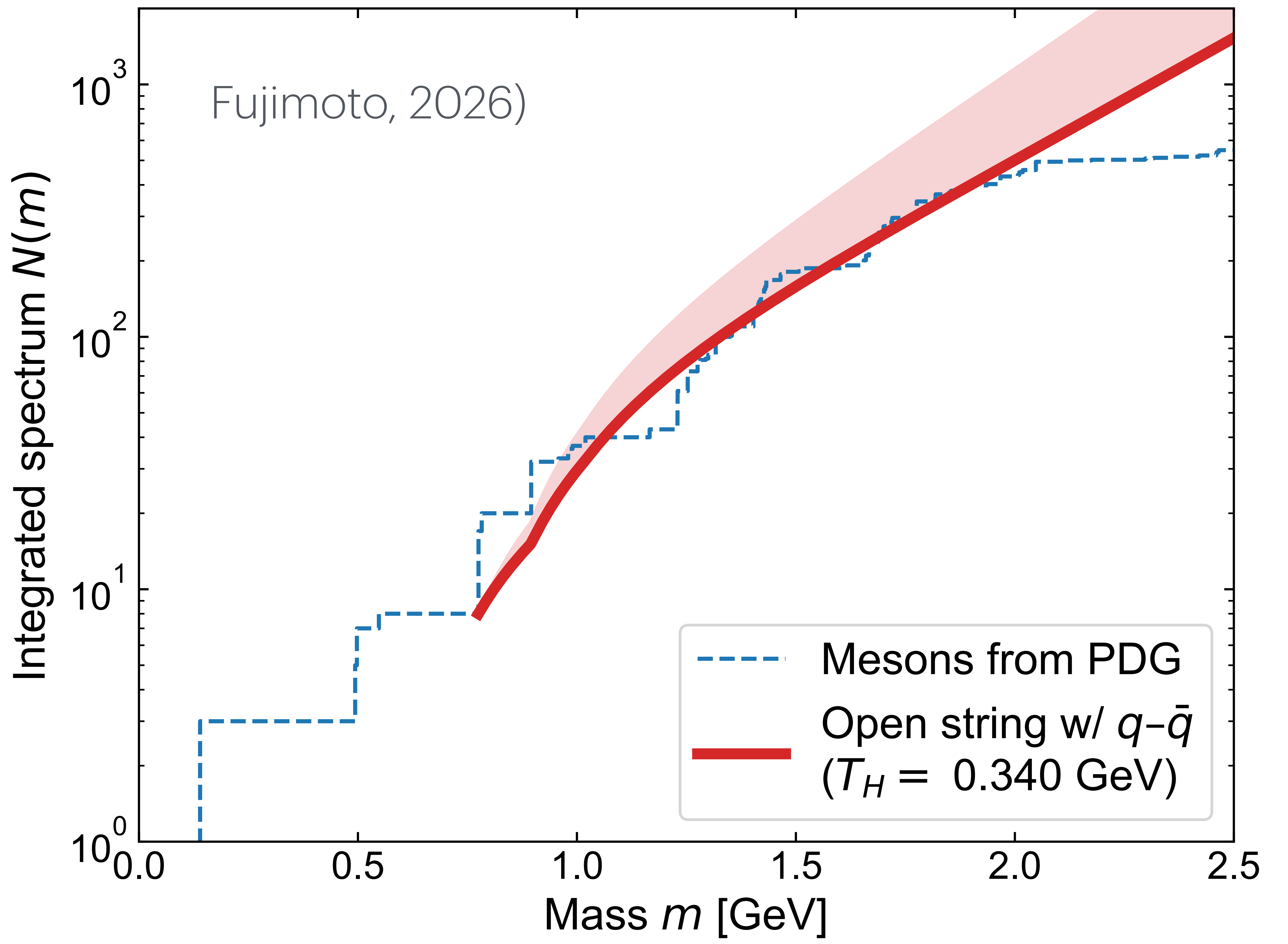


FIG. 1. String baryon models.

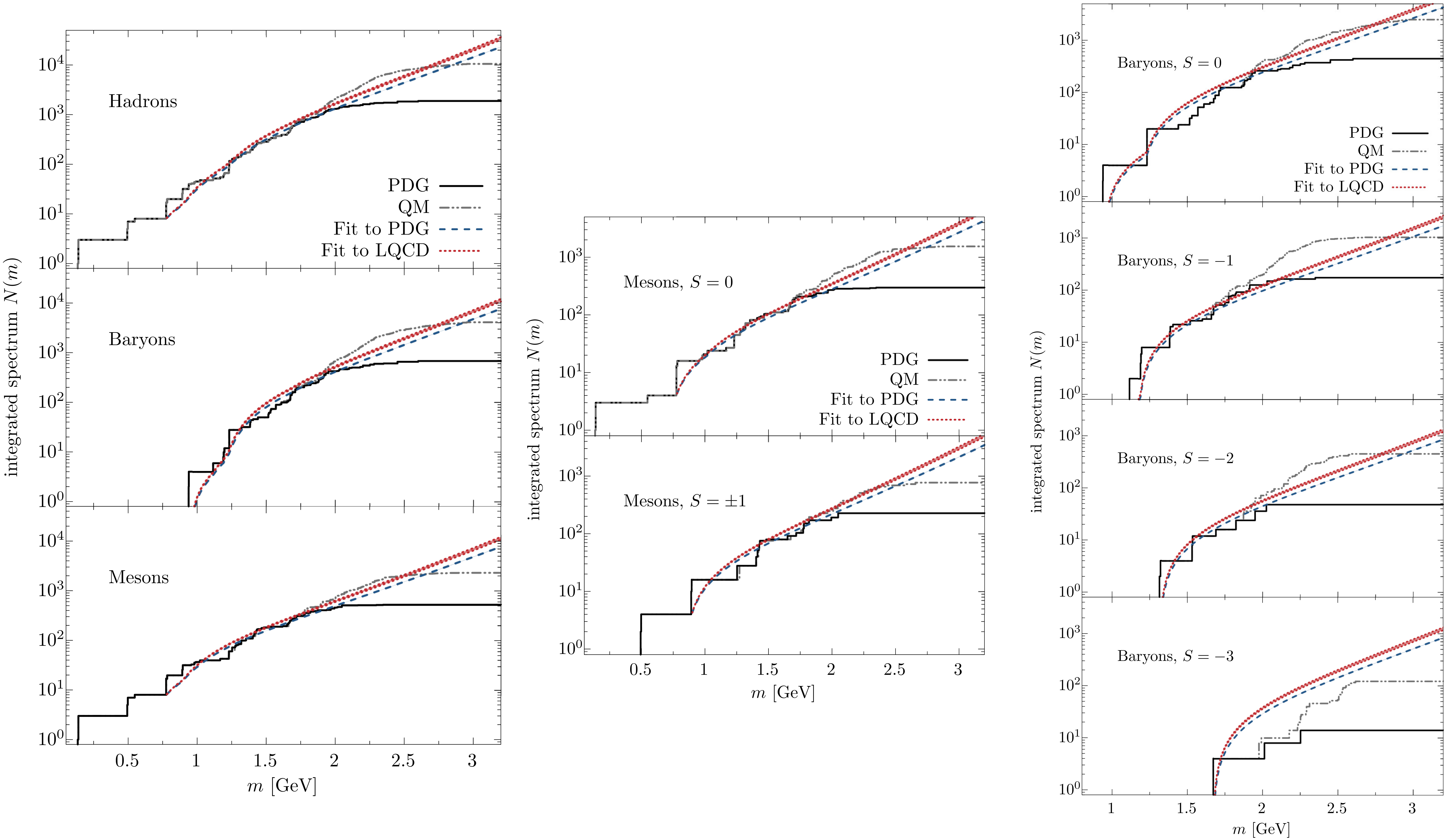
Classical rotational motion

- qqq and $\mathbf{Y}$ are unstable
- $q(qq)$ and $\boldsymbol{\Delta}$ are stable



- Closed strings thermally suppressed: $n_{q(qq)} \gg n_{\Delta}$
- Quark-diquark picture supported by Regge trajectories (Selem, Wilczek 2006)

Baryonic spectrum described with the same T_H as mesons (Fujimoto, 2026)


Hadrons
Baryons
Mesons
Mesons, $S = 0$
Mesons, $S = \pm 1$
Baryons, $S = 0$
Baryons, $S = -1$
Baryons, $S = -2$
Baryons, $S = -3$
PDG
QM
Fit to PDG
Fit to LQCD
integrated spectrum $N(m)$
m [GeV]

Counting the States in Practice

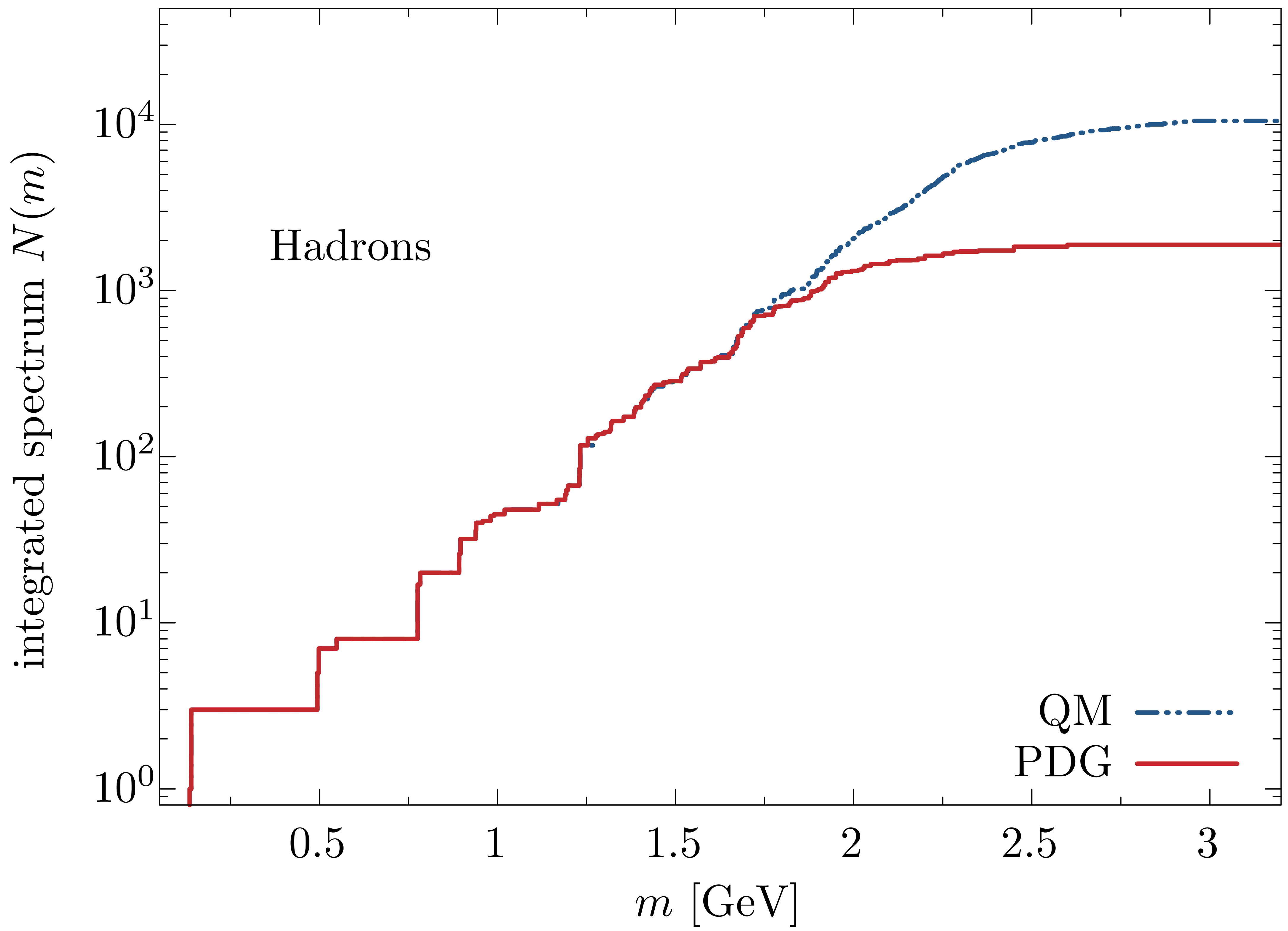
$$N(m) = \int_0^m dm' \rho(m') \quad \text{such that} \quad \rho(m) = \left.\frac{dN(m')}{dm'}\right|_m$$

Discrete spectrum

$$\rho(m) = \sum_i g_i \delta(m - m_i) \qquad N(m) = \sum_i g_i \theta(m - m_i)$$

Continuous spectrum

$$\rho(m) = f(m) e^{m/T_H} \qquad N(m) = \int_0^m dm' f(m') e^{m'/T_H}$$



Experimental spectrum → Particle Data Group

Theoretical spectrum → Relativistic Quark Model

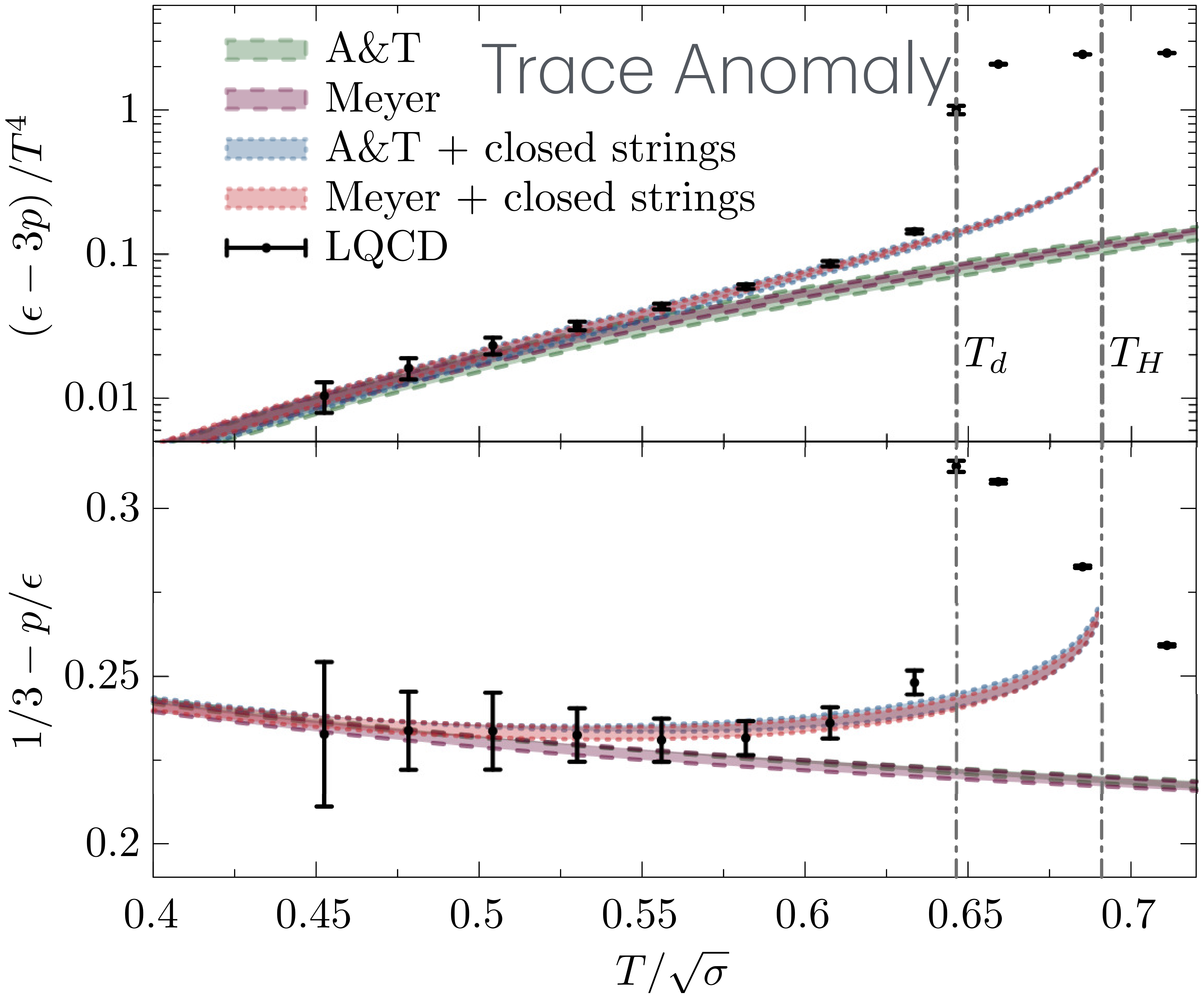
Thermodynamics of Glueballs

$$\hat{P}(m) = \frac{1}{2\pi^2}\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right) e^{\mu/T}$$

$$\hat{P} = \int_0^\infty dm\, \rho(m)\, \hat{P}(m)$$

Link to deconfinement in Pure Gauge Theory:

$T_H = 1.069(5)\, T_{\mathrm{dec}}$ (Lucini et al, 2004)



Thermodynamics described well with asymptotic Hagedorn tail (Meyer, 2004)

Glueball Density of States

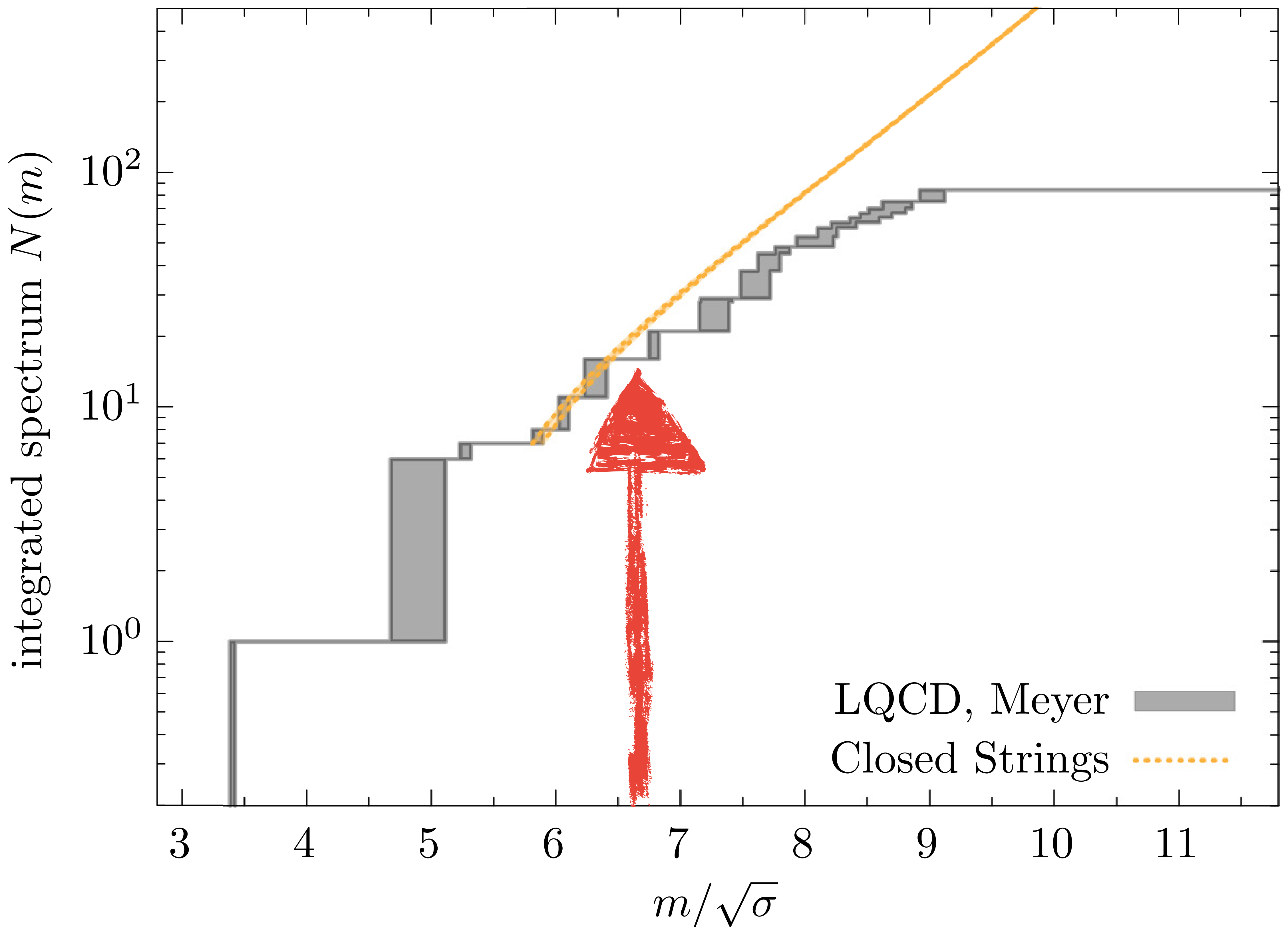
$$N(m) = \int_0^m dm' \rho(m') \quad \text{such that} \quad \rho(m) = \left. \frac{dN(m')}{dm'} \right|_m$$

In Pure Gauge Theory scale is set by $\sqrt{\sigma}$

Hagedorn Temperature: $\dfrac{T_H}{\sqrt{\sigma}} = \sqrt{\dfrac{3}{2\pi}}$

Spectrum is practically parameter-free

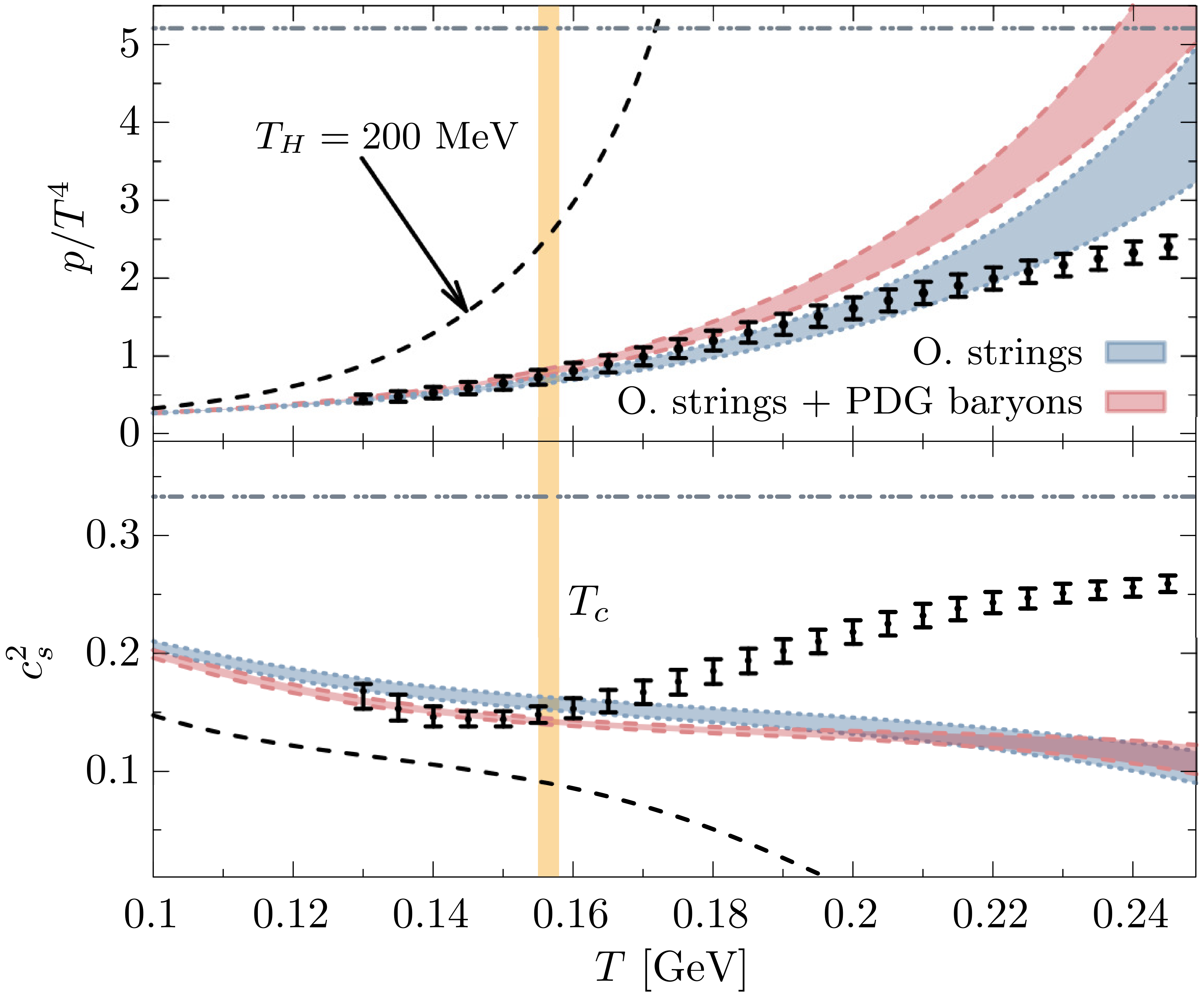
$$\rho_{\rm gb}(m) = \theta(m - m_{\rm thr})\rho_{\rm cl}$$

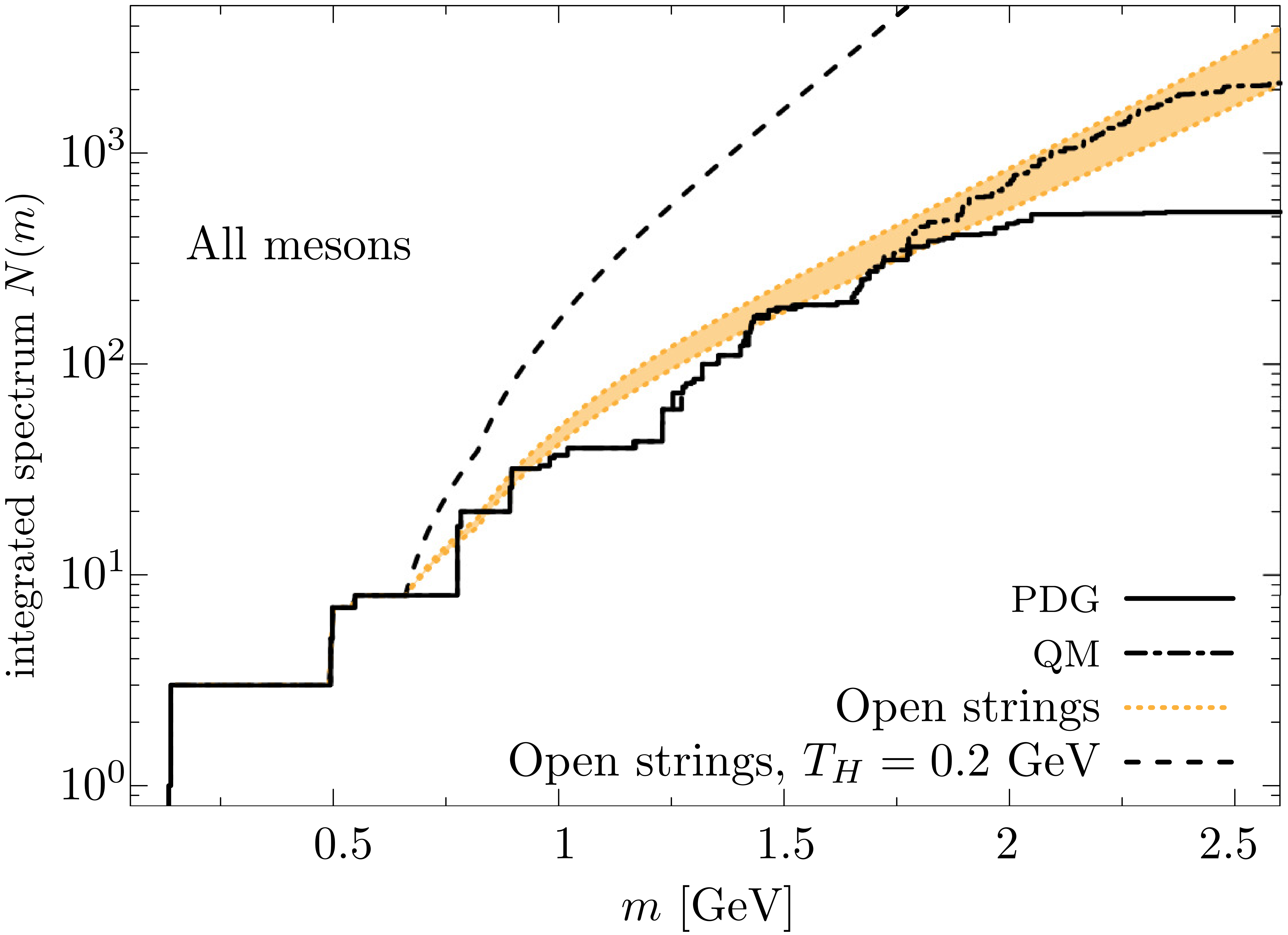


Above $2\,M_0$, LQCD spectrum saturates due to difficulties in extraction of the states

Open-String Spectrum and Thermodynamics

The same set of parameters describes mesonic sector too





Regge Phenomenology and Flavor Dependence

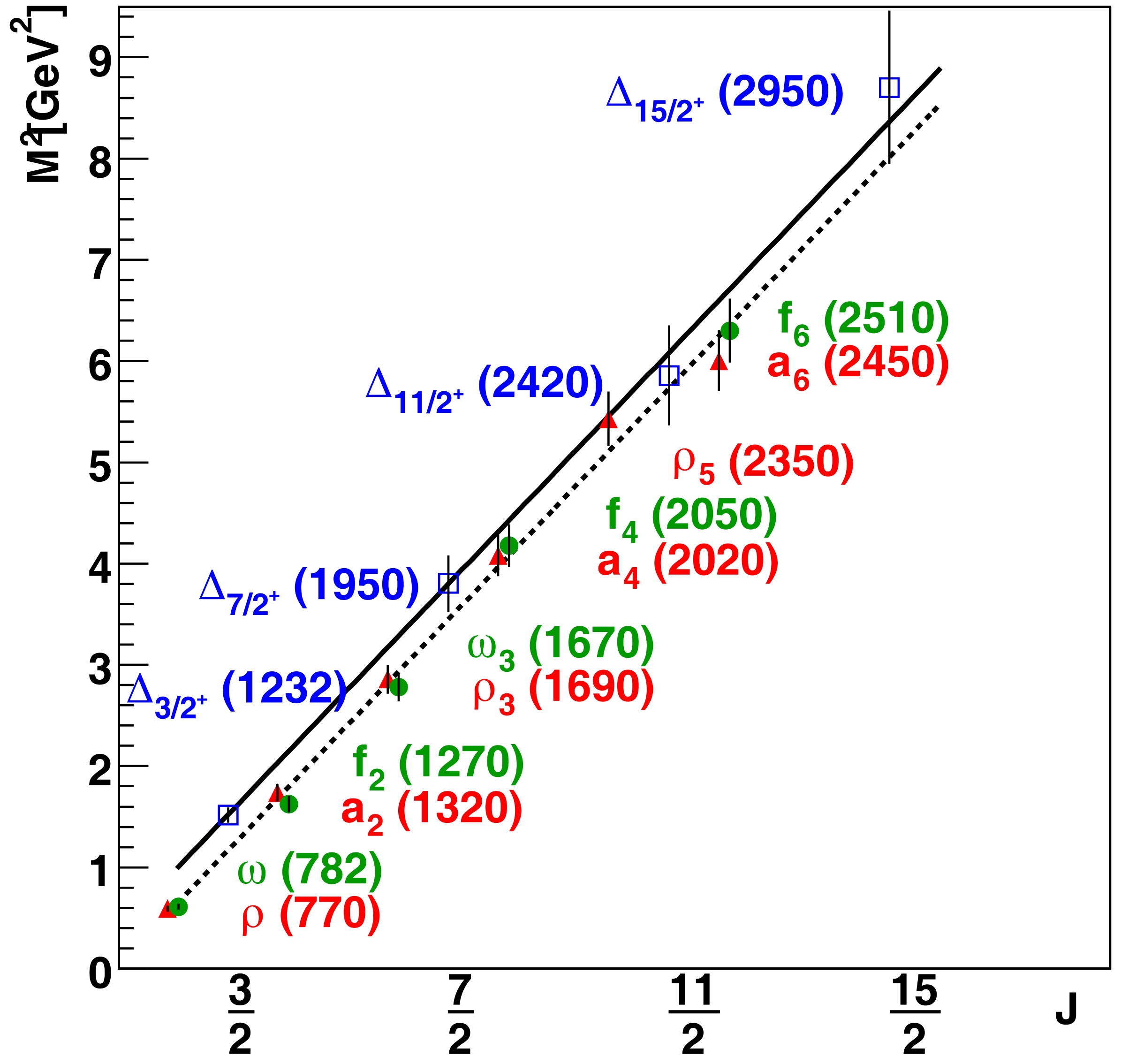
Experimental observation

Light hadrons follow approximately linear trajectories

$$J \simeq \alpha M^2 + \beta \quad \text{where} \quad \alpha \equiv \frac{dJ}{dM^2} = (2\pi\sigma)^{-1}$$

see Chew & Frautschi (1962), Collins (1977)

Klempt, Mensch (2012)

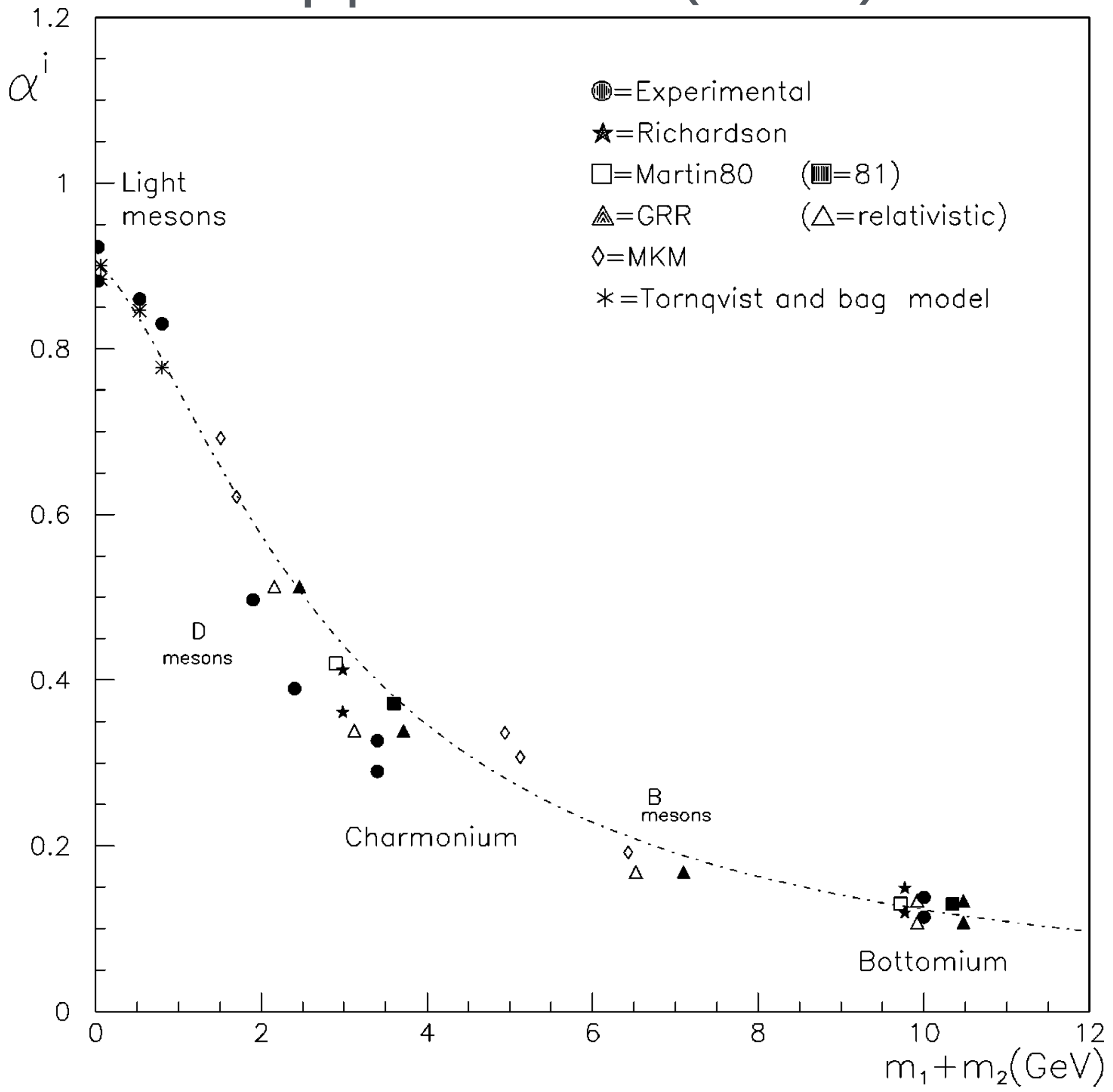


Heavy Flavor suppresses the slope

reduced effective slopes and apparent flavor dependence

see Filipponi & Srivastava (1998), Afonin (2007), Sonnenschein & Weissman (2014)

Filipponi et al (1998)



Is the flavor dependence dynamical or kinematic?

Excitation Energy Reveals Linearity

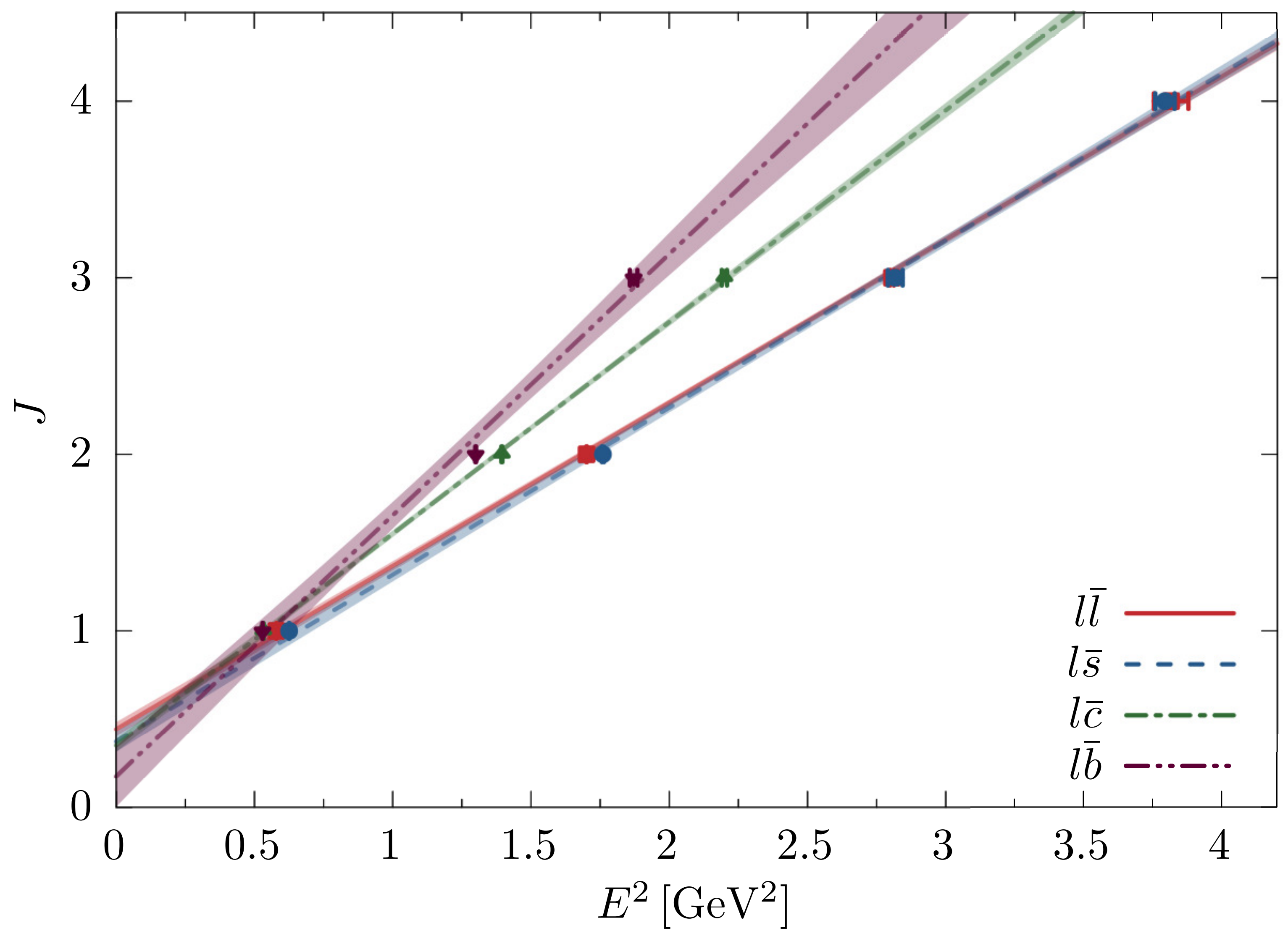
Excitation Energy Representation

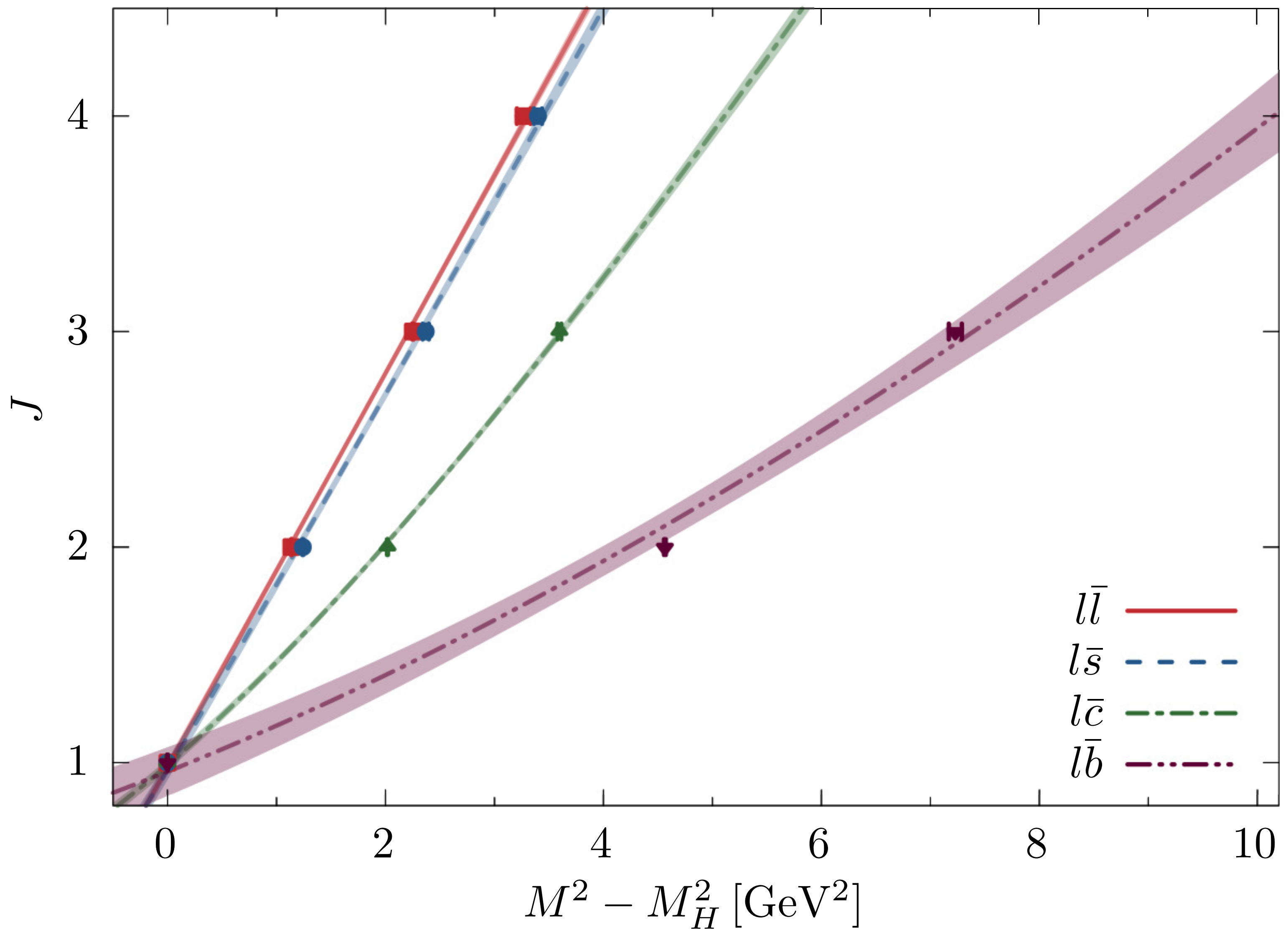
- $J(E^2) = \alpha_E E^2 + \beta_E$
- $\alpha_E = \frac{dJ}{dE^2} = const$

Physical-Mass Representation

- $J(M^2) = \alpha_M M^2 + \beta_M$
- $\alpha_M = \frac{dJ}{dM^2} = \alpha_E \left(1 - \sum m_q/M\right)$

Trajectories for ρ, K^*, D^*, B^*



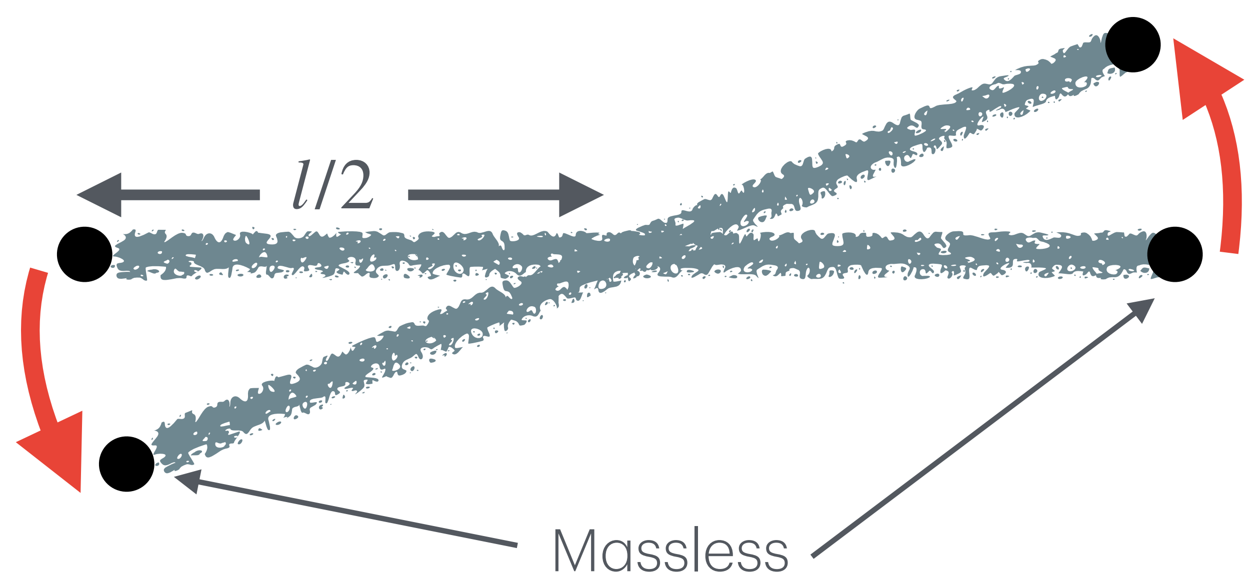


Slope hierarchy: $\alpha_E^{l\bar{l}} < \alpha_E^{l\bar{s}} < \alpha_E^{l\bar{c}} < \alpha_E^{l\bar{b}}$

Slope hierarchy: $\alpha_M^{l\bar{l}} > \alpha_M^{l\bar{s}} > \alpha_M^{l\bar{c}} > \alpha_M^{l\bar{b}}$

Geometry of Rotating String

Light-Light String: $m_1 = m_2 = 0$

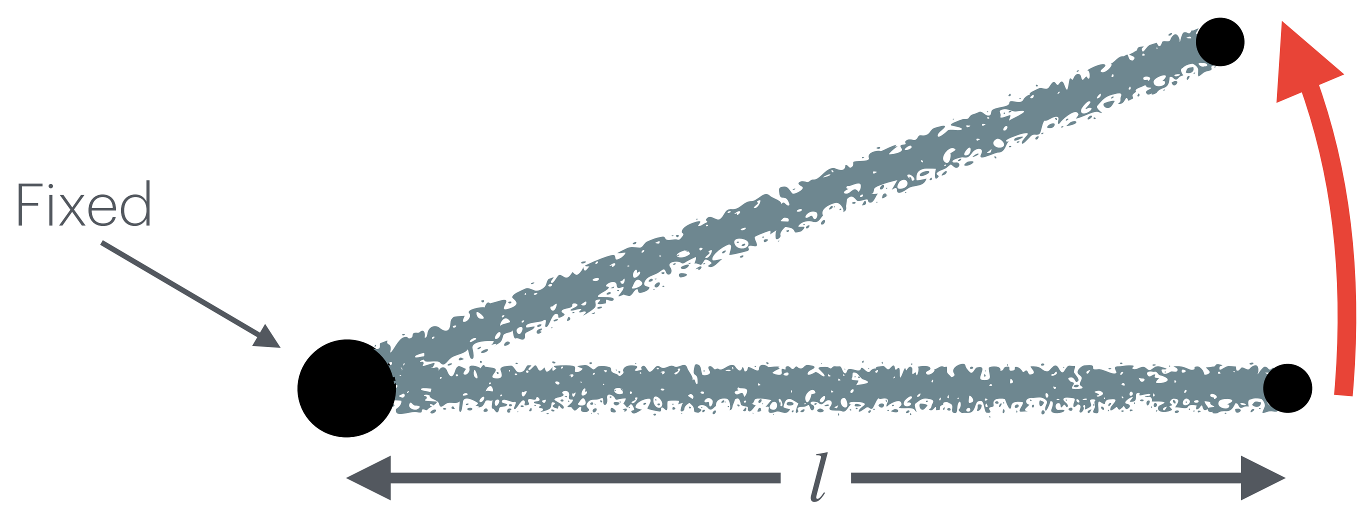


$$M = 2\int_0^{l/2} dr \frac{\sigma}{\sqrt{1 - v^2(r)}} = \frac{1}{2}\pi\sigma l$$

$$L = 2\int_0^{l/2} dr \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} = \frac{1}{8}\pi\sigma l^2$$

$$\longrightarrow \quad \alpha_{LL} = \frac{1}{2\pi\sigma}$$

Heavy-Light String: $m_1 = 0,\ m_2 = \infty$



$$M = 2\int_0^{l} dr \frac{\sigma}{\sqrt{1 - v^2(r)}} = \frac{1}{2}\pi\sigma l$$

$$L = 2\int_0^{l} dr \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} = \frac{1}{4}\pi\sigma l^2$$

$$\longrightarrow \quad \alpha_{HL} = \frac{1}{\pi\sigma}$$

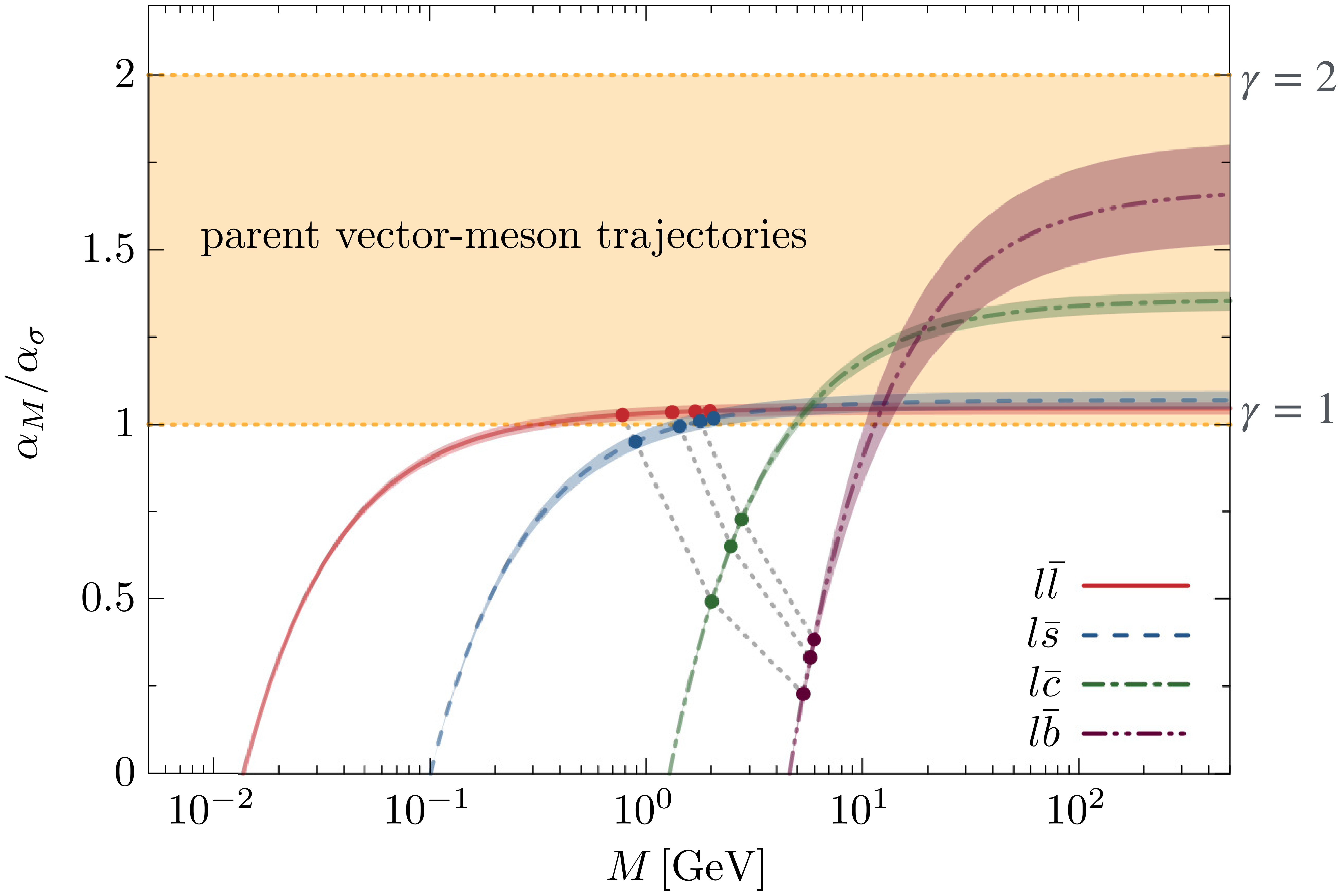
$$\alpha_{LL} = \frac{1}{2}\alpha_{HL}$$

Geometry changes the slope, but string tension is universal

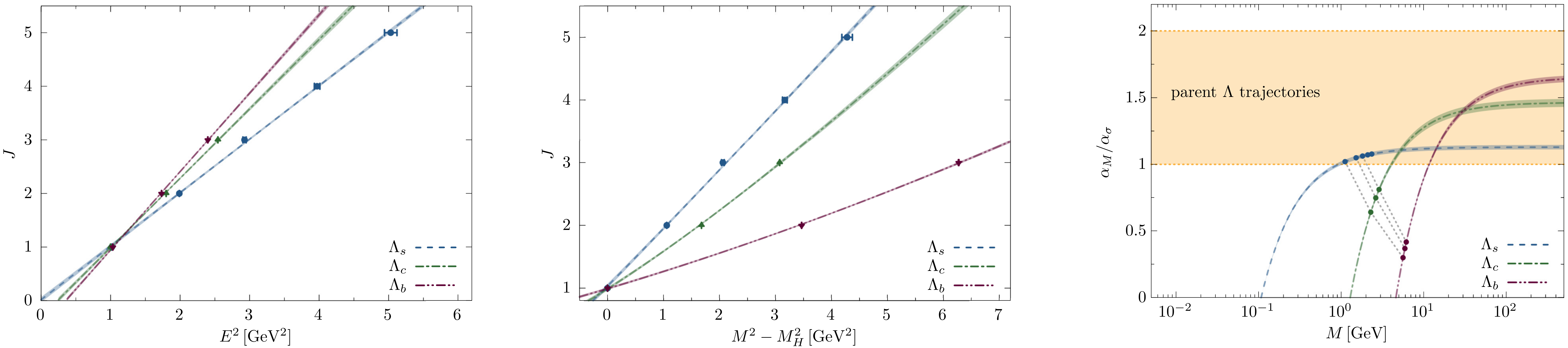
Geometrical Correction

Introduce correction: $\alpha_E \longrightarrow \gamma \cdot \alpha_E = \gamma \cdot (2\pi\sigma)^{-1}$

As $M \to \infty$ the ratio $\alpha_M / \alpha_\sigma \to \gamma$



Λ Baryons as quark-diquark strings



Vector Mesons

$\gamma_s = 1.070(25)$

$\gamma_c = 1.356(27)$

$\gamma_b = 1.673(145)$

Lambda Baryons

$\gamma_s = 1.129(17)$

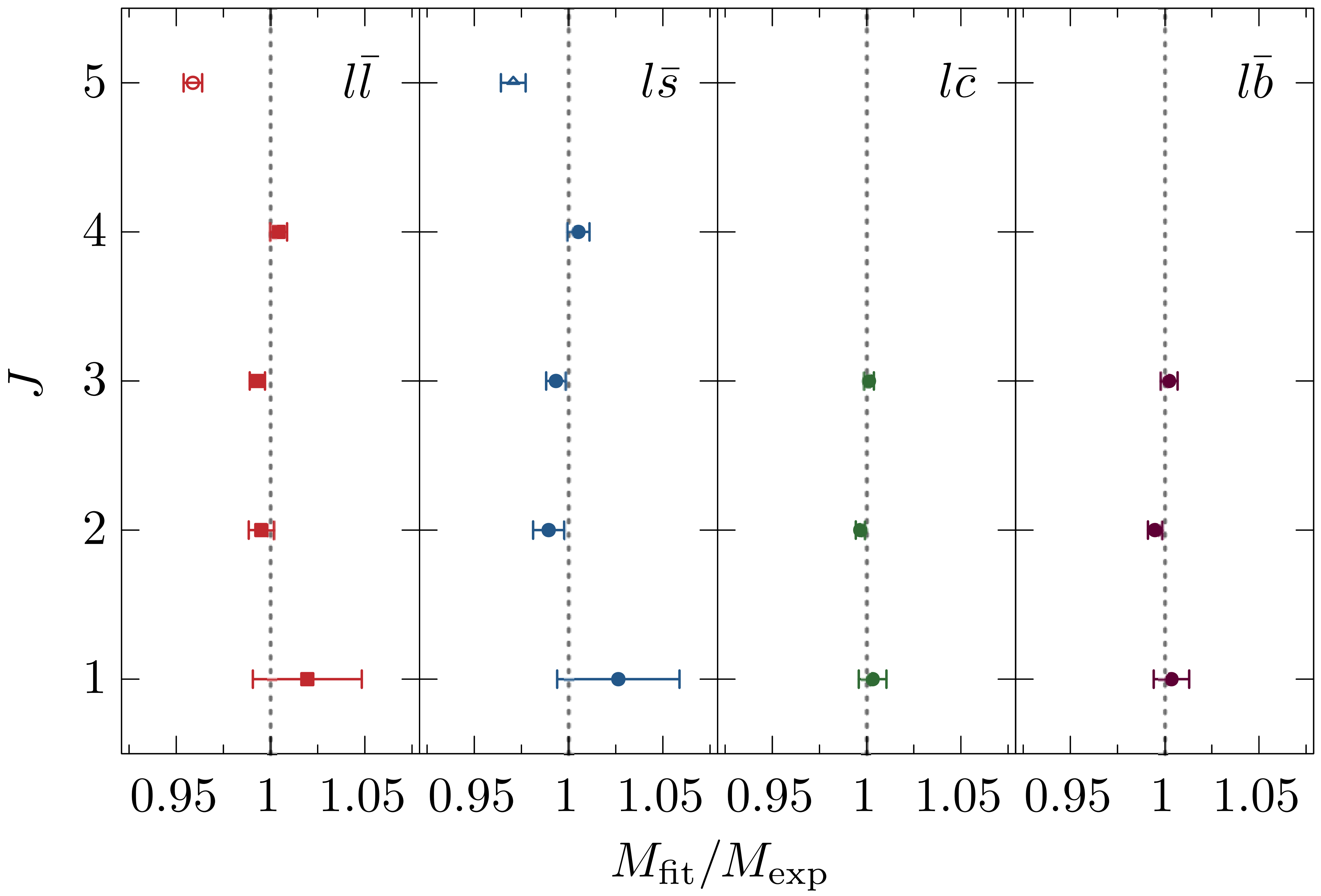
$\gamma_c = 1.464(30)$

$\gamma_b = 1.655(24)$

Similar enhancement factors for mesons and baryons support Hagedorn universality and the quark–diquark picture of baryons.

QCD spectroscopy governed by a universal excitation-energy string structure

Regge trajectories Fit



Mesons

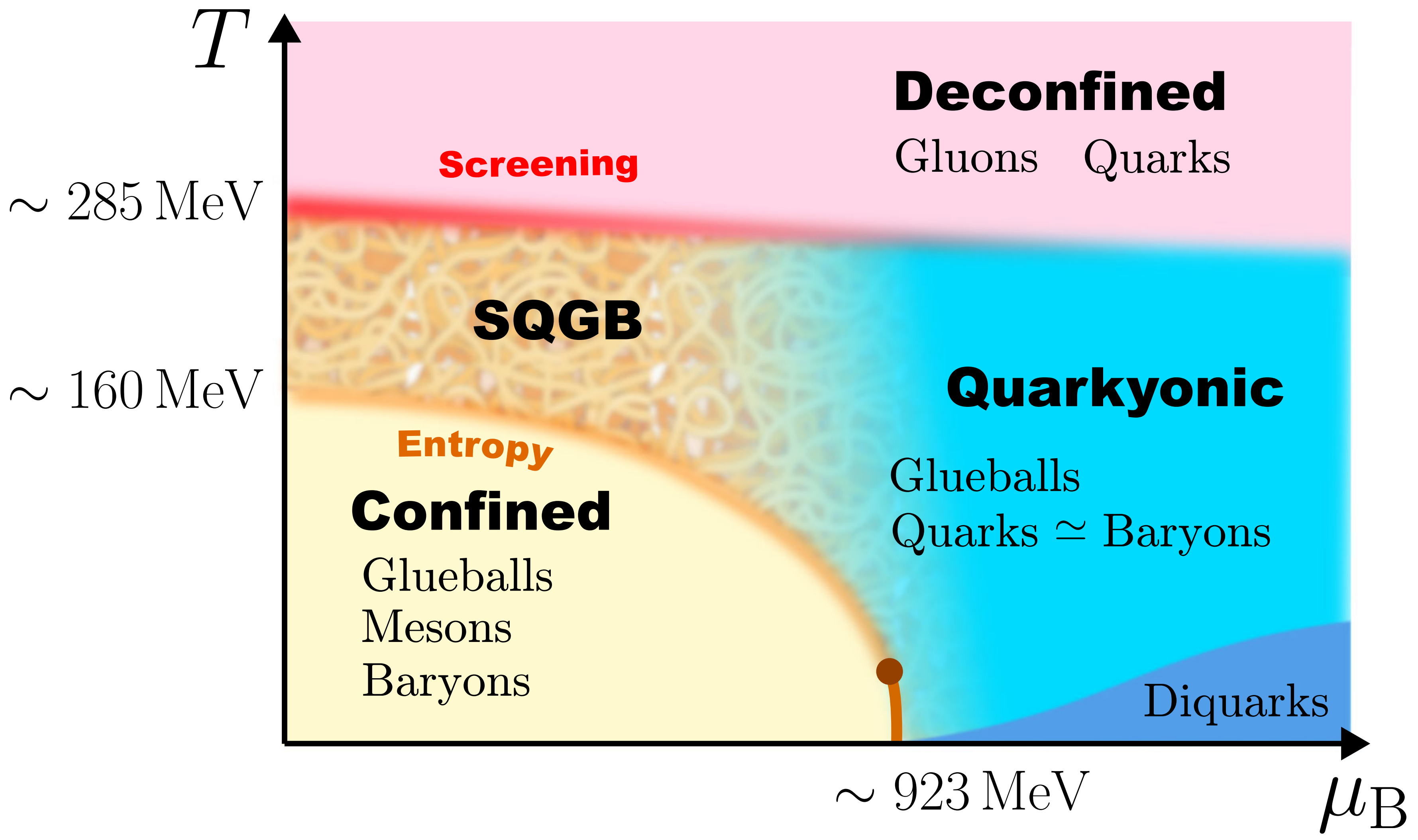
sector	α_E $[\text{GeV}^{-2}]$	β_E	$\gamma = \alpha_E/\alpha_\sigma$
$l\bar{l}$	0.924(16)	0.442(40)	1.046(18)
$l\bar{s}$	0.946(23)	0.372(57)	1.070(25)
$l\bar{c}$	1.199(25)	0.350(37)	1.356(27)
$l\bar{b}$	1.479(127)	0.174(172)	1.673(145)

Baryons

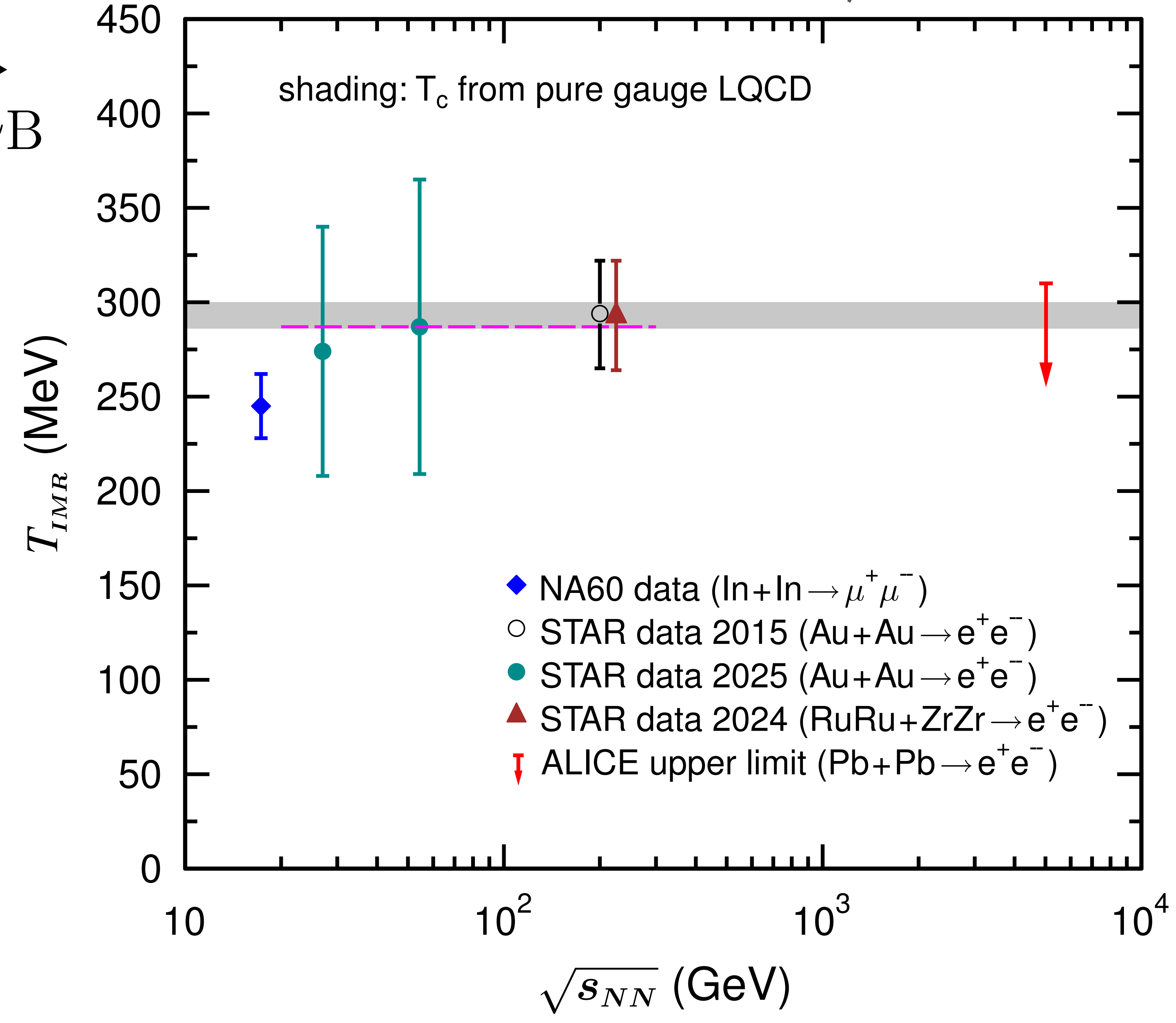
sector	α_E $[\text{GeV}^{-2}]$	β_E	$\gamma = \alpha_E/\alpha_\sigma$
Λ $([ud]s)$	0.998(15)	0.018(49)	1.129(17)
Λ_c $([ud]b)$	1.295(27)	-0.305(50)	1.464(30)
Λ_b $([ud]b)$	1.463(26)	-0.518(39)	1.655(24)

The $\sqrt{\sigma}$ scale Reappears

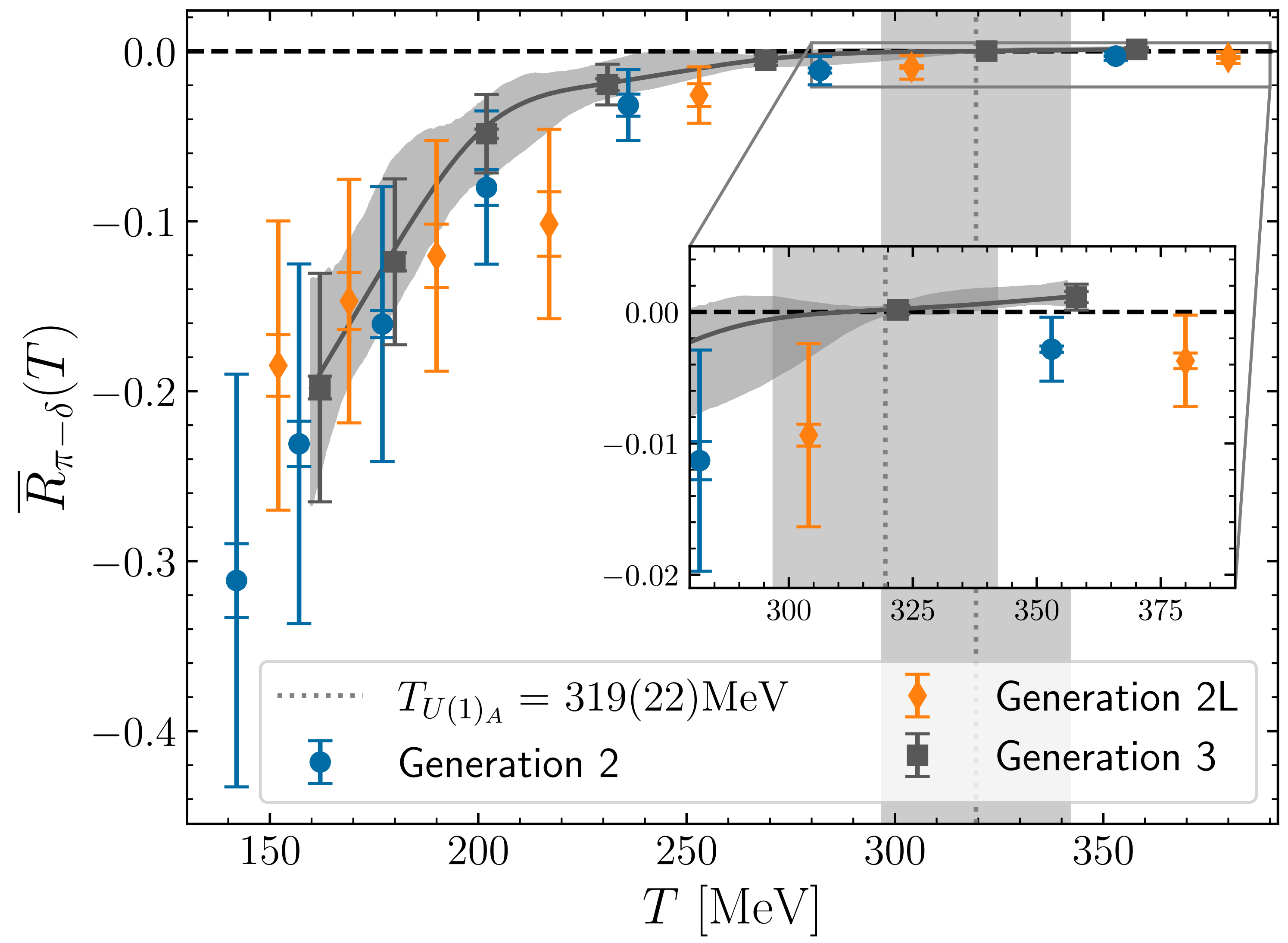
Fujimoto et al, 2025



Vovchenko et al, 2026



Aarts et al, 2025



Channel	mass [GeV]	I	Q	S	deg
ρ $(l\bar{l})$	0.770	1	$0, \pm1$	0	16
K^* $(l\bar{s},\ s\bar{l})$	0.896	1/2	$0, \pm1$	±1	16
ϕ $(s\bar{s})$	1.019	0	0	0	4

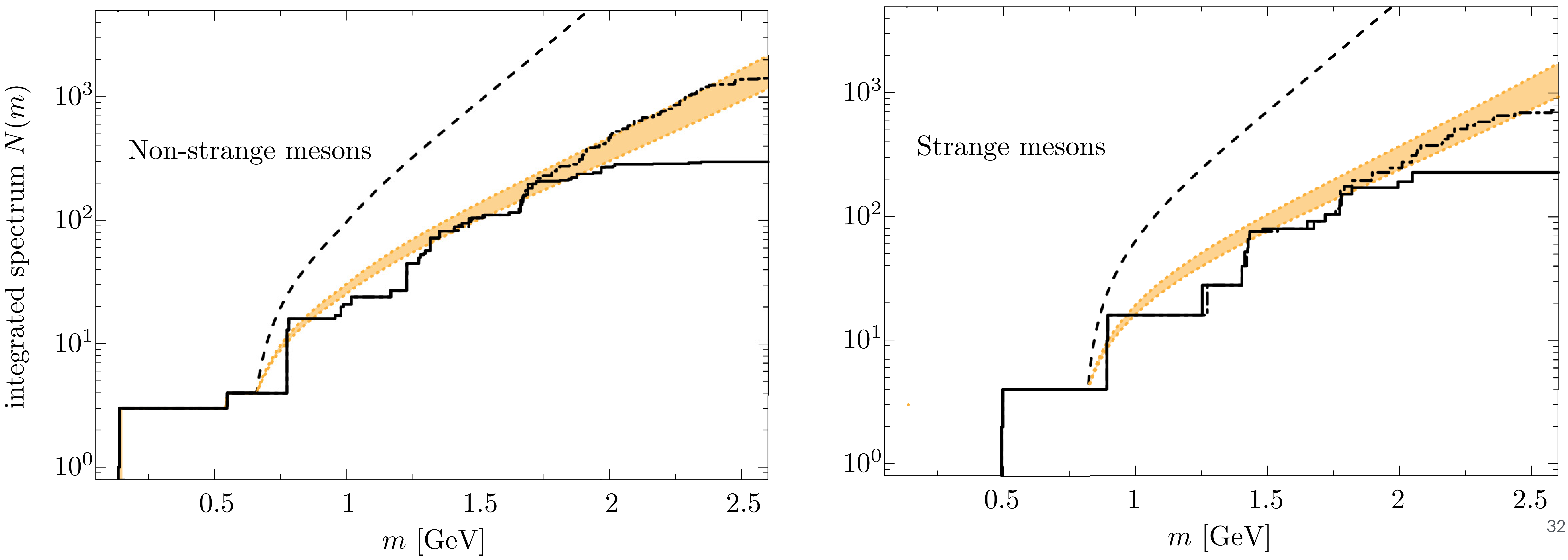
State	String	J	I	S	Mass [GeV] $Q=c$	Mass [GeV] $Q=b$	g
$D,\ B$	$Q\bar{l},\ \bar{Q}l$	0	1/2	0	1.867	5.279	16
$D_s,\ B_s$	$Q\bar{s},\ \bar{Q}s$	0	0	±1	1.968	5.367	8
$\eta_c,\ \eta_b$	$Q\bar{Q}$	0	0	0	2.984	9.398	4

Channel	mass [GeV]	I	Q	S	deg
N $(l[ll])$	0.938	1/2	$(0,\ 1)$	0	4
Λ $(s[ll])$	1.116	0	(0)	-1	2
Σ $(l[ls])$	1.189	1	$(0,\ \pm1)$	-1	6
Δ $(l\{ll\})$	1.232	3/2	$(0,\ \pm1,\ 2)$	0	16
Ξ $(s[ls])$	1.315	1/2	$(0,\ -1)$	-2	4
Ω $(s\{ss\})$	1.672	0	(-1)	-3	4

State	String	J	I	S	Mass [GeV] $Q=c$	Mass [GeV] $Q=b$	g
Λ_Q	$Q[ll]$	1/2	0	0	2.286	5.620	2
Σ_Q	$Q\{ll\}$	1/2, 3/2	1	0	2.453	5.811	18
Ξ_Q	$Q[ls]$	1/2	1/2	-1	2.468	5.793	4
Ξ'_Q	$Q\{ls\}$	1/2, 3/2	1/2	-1	2.576	5.935	12
Ω_Q	$Q\{ss\}$	1/2, 3/2	0	-2	2.695	6.046	6
Ξ_{QQ}	$l\{QQ\}$	1/2, 3/2	1/2	0	3.519	–	12
Ω_{QQ}	$s\{QQ\}$	1/2, 3/2	0	-1	3.778	–	6
Ω_{QQQ}	$Q\{QQ\}$	3/2	0	0	4.800	–	4

Definite quantum-number sectors

Density of state is additive: $\rho(m) = \rho^{S=0}(m) + \rho^{|S|=1}(m)$



Charm Spectral Abundances

Shifted Spectrum

$$\rho(m) \longrightarrow \rho(E = m - \sum n_q m_q)$$

Overall excitation-energy string spectrum captures global charm-hadron systematics