Spin hydrodynamics with dissipation and at higher orders in spin polarization

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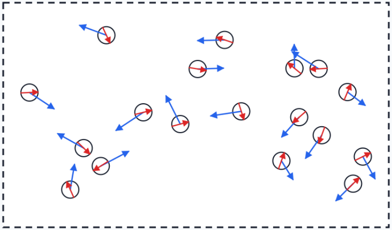
Spin hydrodynamics at higher orders in spin polarization

Perfect spin hydrodynamics

- Gas of particles with spin

$$\partial_\mu T^{\mu\nu}_{\rm eq} = 0, \qquad (1)$$
$$\partial_\mu N^{\mu}_{\rm eq} = 0,$$
$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu}$$



$$\partial_\lambda J^{\lambda\mu\nu}_{\rm eq} = 0 \Leftrightarrow \partial_\lambda S^{\lambda\mu\nu}_{\rm eq} = T^{\nu\mu}_{\rm eq} - T^{\mu\nu}_{\rm eq}$$
$$= 0 \text{ for symmetric } T^{\mu\nu}$$

- Nontrivial forms of energy–momentum and spin tensors
  For spin 1/2

$$\begin{cases} T^{\mu\nu}_{\rm eq} = T^{\mu\nu}_{\rm GLW} \\ S^{\lambda\mu\nu}_{\rm eq} = S^{\lambda\mu\nu}_{\rm GLW} \end{cases}$$

- In general, six new degrees of freedom in addition to T, μ, $\vec{u} = \gamma\vec{v}$.
  We can parametrize the spin polarization tensor $\omega_{\alpha\beta}$ as

$$\omega_{\alpha\beta} = k_\alpha u_\beta - k_\beta u_\alpha + \epsilon_{\alpha\beta\gamma\delta} u^\gamma \omega^\delta, \qquad (2)$$

with $k \cdot u = \omega \cdot u = 0$.

Classical spin description

- Spin 1/2

$$N^{\mu}_{\rm eq} = \int \mathrm{d}P\,\mathrm{d}S\, p^{\mu} \,\left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right], \tag{3}$$

$$T^{\mu\nu}_{\rm eq} = \int \mathrm{d}P\,\mathrm{d}S\, p^{\mu} p^{\nu} \,\left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right], \tag{4}$$

$$S^{\lambda\mu\nu}_{\rm eq} = \int \mathrm{d}P\,\mathrm{d}S\, p^{\lambda}\, s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]. \tag{5}$$

- The equilibrium distribution functions are
  Fermi–Dirac $f^{\pm}_{\rm eq}(x,p,s) = \left[\exp\left(\mp\xi(x) + p_{\mu}\beta^{\mu}(x) - \frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right) + 1\right]^{-1}$
  or Boltzmann $f^{\pm}_{0,\rm eq}(x,p,s) = \exp\left(\pm\xi(x) - p_{\mu}\beta^{\mu}(x) + \frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right)$,

- $\mathrm{d}P = \frac{\mathrm{d}^3p}{(2\pi)^3 E_p}$, $\mathrm{d}S = \frac{m}{\pi\mathfrak{s}}\mathrm{d}^4 s\,\delta(s\cdot s + \mathfrak{s}^2)\delta(p\cdot s)$,
  $\xi \equiv \mu/T$ and $\beta^{\mu} \equiv u^{\mu}/T$.

W. Florkowski, M. Hontarenko, Phys.Rev.Lett. 134 (2025) 8, 082302.
ZD, W. Florkowski, M. Hontarenko, (2024), Phys.Rev.D, **110**(9), 096018.

Fourth-order expansion in ω

- The integrals can be computed. Up to the fourth order in ω:

$$N^{\mu}_{\rm eq} = (n_0 + n_2 + n_4)u^{\mu} + (n_{t0} + n_{t2})t^{\mu}, \tag{6}$$

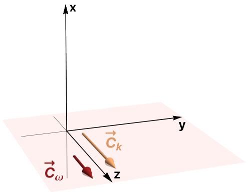
$$T^{\mu\nu}_{\rm eq} = (\varepsilon_0 + \varepsilon_2 + \varepsilon_4)u^{\mu}u^{\nu} - (P_0 + P_2 + P_4)\Delta^{\mu\nu} + (P_{k\omega} + P_{k\omega 2})(k^{\mu}k^{\nu} + \omega^{\mu}\omega^{\nu}) + (P_t + P_{t2})(t^{\mu}u^{\nu} + t^{\nu}u^{\mu}) + P_{\omega k}(k^{\mu}\omega^{\nu} + \omega^{\mu}k^{\nu}), \tag{7}$$

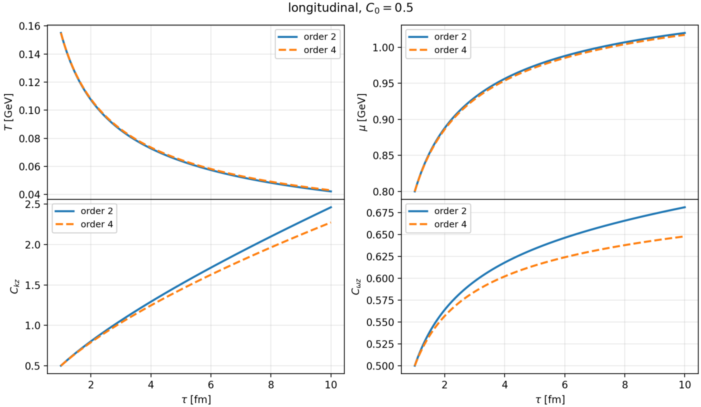
$$S^{\lambda\mu\nu} = u^{\lambda}\left[F_k\left(k^{\mu}u^{\nu} - k^{\nu}u^{\mu}\right) + F_{\omega}\left(\omega^{\mu}u^{\nu} - \omega^{\nu}u^{\mu}\right) + F_{kt}\left(t^{\mu}k^{\nu} - t^{\nu}k^{\mu}\right) + F_t t^{\mu\nu}\right] + F_{\Delta k}(\Delta^{\lambda\mu}k^{\nu} - \Delta^{\lambda\nu}k^{\mu}) + F_{\Delta\omega}(\Delta^{\lambda\mu}\omega^{\nu} - \Delta^{\lambda\nu}\omega^{\mu}) + F_{\omega k}\omega^{\lambda}(k^{\mu}\omega^{\nu} - k^{\nu}\omega^{\mu}) + F_{tk}t^{\lambda}(k^{\mu}u^{\nu} - k^{\nu}u^{\mu}) + F_{tu}(t^{\lambda\mu}u^{\nu} - t^{\lambda\nu}u^{\mu}) + F_{tt}t^{\lambda}t^{\mu\nu}, \tag{8}$$

where $t^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}u_{\alpha}\omega_{\beta}, \quad t^{\mu} = t^{\mu\nu}k_{\nu} = \epsilon^{\mu\nu\alpha\beta}k_{\nu}u_{\alpha}\omega_{\beta}$.

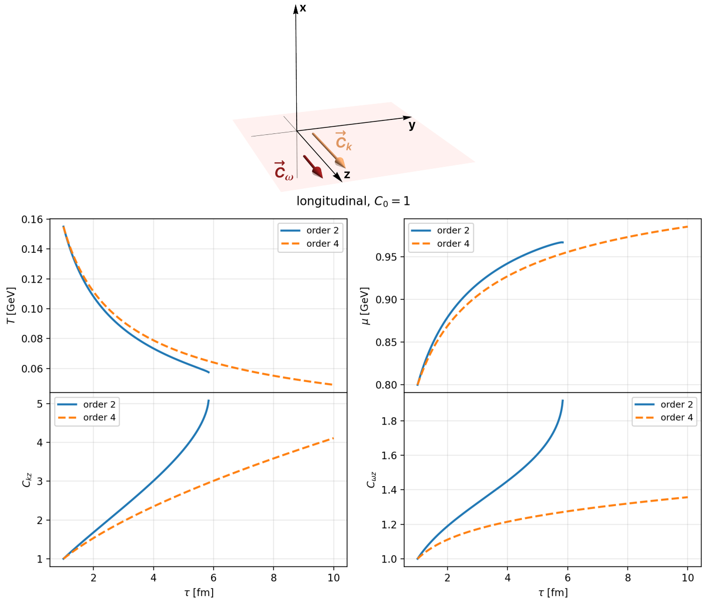
- The scalar coefficients multiplying the vectors and tensors on the RHS are functions of T, μ, k^2, ω^2, k^4, $k^2\omega^2$, $(k\cdot\omega)^2$, ω^4.
- We have computed them for both the Fermi–Dirac and the Boltzmann case.
- We used them in numerical simulations in a boost-invariant transversely homogeneous geometry and found the difference to matter in the case of large spin polarizations.
  ZD, N. Łygan, [in preparation]

Boost-invariant transversely homogeneous geometry: results

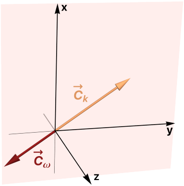




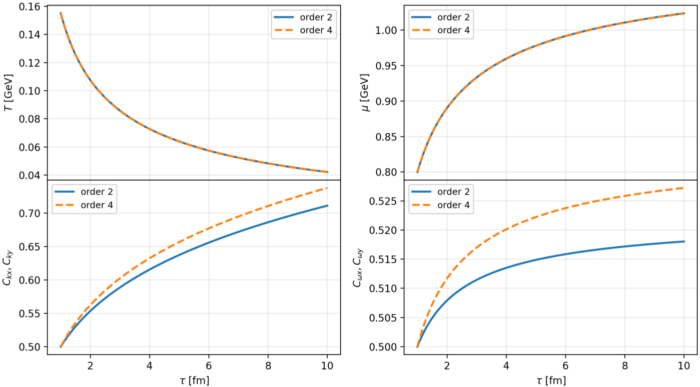
Boost-invariant transversely homogeneous geometry: results



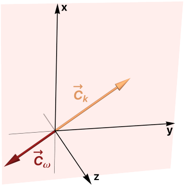
Boost-invariant transversely homogeneous geometry: results



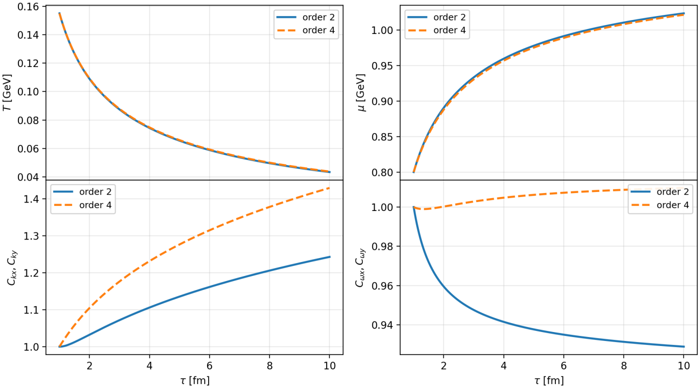
transverse, $C_0 = 0.5$



Boost-invariant transversely homogeneous geometry: results



transverse, $C_0 = 1$



Generating functions

- Spin 1/2, classical approach

$$n_{\rm cl}(x) = 2\int \mathrm{d}P \cosh\xi \exp\left(-p_\mu \beta^\mu\right) \int \mathrm{d}S \exp\left(\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right) \tag{9}$$

- Spin 1/2, quantum approach

$$n_{\rm qt}(x) = 4\int \mathrm{d}P \exp\left(-p\cdot\beta\right) \cosh\xi \, \cosh\sqrt{-a^2}, \tag{10}$$

S. Bhadury, ZD, W. Florkowski, S.K. Kar, V. Mykhaylova, arXiv:2505.02657 [hep-ph].
S.K. Kar, V. Mykhaylova, arXiv:2511.09580 [quant-ph].

$$N^\mu_{\rm eq} = -\frac{\partial^2 n}{\partial\beta_\mu \partial\xi}, \tag{11}$$

$$T^{\mu\nu}_{\rm eq} = \frac{\partial^2 n}{\partial\beta_\mu \partial\beta_\nu}, \tag{12}$$

$$S^{\lambda\mu\nu}_{\rm eq} = -\frac{\partial^2 n}{\partial\beta_\lambda \partial\omega_{\mu\nu}}. \tag{13}$$

- The integrals converge for physical values of spin polarization ω.
  ZD, W. Florkowski, V. Mykhaylova, Phys.Rev.D 112 (2025) 5, L051901

Comparison of the frameworks up to any order in ω

- It can be shown that

$$n_{\rm qt}(x) = 4 \int \mathrm{d}P \exp\left(-p\cdot\beta\right) \cosh\xi \sum_{n=0}^{\infty} (-1)^n \frac{(a^2)^n}{(2n)!}, \tag{14}$$

$$n_{\rm cl}(x) = 4 \int \mathrm{d}P \exp(-p\cdot\beta) \cosh\xi \sum_{n=0}^{\infty} (-1)^n \frac{\mathfrak{s}^{2n}(a^2)^n 2^{2n}}{(2n+1)!}, \tag{15}$$

where

$$a_\mu(x,p) \equiv -\frac{1}{4m}\epsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta}(x)p^\nu . \tag{16}$$

- To achieve an exact equivalence, the classical spin normalization $\mathfrak{s}$ would have to depend on the expansion order,

$$\mathfrak{s}_{2n} = \frac{1}{2}(2n+1)^{\frac{1}{2n}} . \tag{17}$$

- If $\mathfrak{s} = \sqrt{3/4}$, then the classical and quantum pictures exactly agree only at the lowest nontrivial order and exponentially diverge from each other at higher orders. The $2n$-th order contribution to the generating function is then

$$\left(\frac{\mathfrak{s}}{\mathfrak{s}_{2n}}\right)^{2n} = \frac{3^n}{2n+1} \tag{18}$$

times larger classically. However, the *structure* of the terms is the same.
ZD, Phys.Lett.B 873 (2026) 140205

Numerical results

- The perfect-fluid expressions have been worked out quite comprehensively.
- To perform numerical simulations of perfect spin hydrodynamics, we write down the conservation equations in a selected geometry – eleven first-order differential equations for eleven unknowns in general.
- In geometries with additional symmetries, the equations often disentangle.
- We may solve the equations from $\partial_\mu N^\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ without spin first, and then evolve spin on this background, as an approximation; or we may include spin feedback and solve all the equations at once.
- Boost-invariant, transversely homogeneous
  ZD, W. Florkowski, N. Łygan, R. Ryblewski, Phys.Rev.C 111 (2025) 2, 024909
  ZD, N. Łygan, 2604.17392 [hep-ph]
  ZD, N. Łygan, [in preparation]
- Boost-invariant, cylindrically symmetric
  ZD, W. Florkowski, J. Witkowski arXiv:2605.01857 [hep-ph]
- This showed the feasibility of the approach, although the geometries are too simple for comparison with experiment yet.
- Simulations in more general geometries and with dissipation (see the later part of the talk) come next!

Spin hydrodynamics with dissipation

Close-to-equilibrium dynamics

- General expansion to the second order in ω and to the first order in gradients:

$$
\begin{aligned}
N^\mu &= a u^\mu + b^\mu, \\
T^{\mu\nu} &= c u^\mu u^\nu + d_s^\mu u^\nu + d_s^\nu u^\mu + d_a^\mu u^\nu - d_a^\nu u^\mu + e_a^{\mu\nu} + e_s^{\mu\nu}, \\
S^{\lambda,\mu\nu} &= u^\lambda \left[(f^\mu u^\nu - f^\nu u^\mu) + \epsilon^{\mu\nu\rho\sigma} u_\rho w_\sigma \right] + i^{\lambda\mu} u^\nu - i^{\lambda\nu} u^\mu + j^{\lambda\mu\nu},
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
a &= \bar{n}(T,\xi,k^2,\omega^2), \quad b^\mu = \lambda \nabla^\mu \xi + n_t t^\mu, \\
c &= \bar{\varepsilon}(T,\xi,k^2,\omega^2), \quad d_s^\mu = -\kappa(Du^\mu - \beta \nabla^\mu T) + P_t t^\mu, \\
d_a^\mu &= \lambda_a \beta^{-1}(\beta D u^\mu + \beta^2 \nabla^\mu T - 2k^\mu), \quad e = \bar{P} - \zeta\theta - (1/3) P_{k\omega}(k^2 + \omega^2), \\
e_s^{\langle\mu\nu\rangle} &= 2\eta\sigma^{\mu\nu} + P_{k\omega}(k^{\langle\mu} k^{\nu\rangle} + \omega^{\langle\mu}\omega^{\nu\rangle}), \quad e_a^{\mu\nu} = \gamma\beta\nabla^{[\mu} u^{\nu]}, \quad f^\mu = A(T,\xi) k^\mu, \\
i^{\lambda\mu} &= -\chi_1 \Delta^{\lambda\mu} u^\beta \nabla^\alpha \omega_{\alpha\beta} - \chi_2 u_\nu \nabla^{\langle\lambda}\omega^{\mu\rangle\nu} - \chi_3 u_\nu \Delta^{[\mu}_\rho \nabla^{\lambda]}\omega^{\rho\nu} + \frac{A}{2} t^{\lambda\mu}, \\
j^{\lambda\mu\nu} &= \frac{\chi_4}{2} \nabla^\lambda \omega^{\langle\mu\rangle\langle\nu\rangle} + \frac{A}{2}(\Delta^{\lambda\mu} k^\nu - \Delta^{\lambda\nu} k^\mu), \quad w^\mu = A_1(T,\xi)\omega^\mu,
\end{aligned} \tag{20}
$$

with $D = u^\mu \partial_\mu$, $\theta = p_\mu u^\mu$, $\sigma^{\mu\nu} = p^{\langle\mu} u^{\nu\rangle}$, η, ζ – shear and bulk viscosity, $\lambda =$ the diffusion coefficient, κ – thermal conductivity, λ_a, γ – coefficients introduced in Hattori 2019, $\chi_1, \chi_2, \chi_3, \chi_4$ – coefficients from Biswas 2023.

K. Hattori et al., Phys.Lett.B **795** (2019) 100-106.

R. Biswas, A. Daher, A. Das, W. Florkowski, R. Ryblewski, Phys.Rev.D **108**, 014024 (2023).

- Let us focus on one dissipative term now:

$$\begin{aligned} N^\mu &= \bar{n} u^\mu + n_t t^\mu, \\ T^{\mu\nu} &= \bar{\varepsilon} u^\mu u^\nu + \bar{P} \Delta^{\mu\nu} + d_a^\mu u^\nu - d_a^\nu u^\mu \\ &\quad + P_{k\omega}(k^\mu k^\nu + \omega^\mu \omega^\nu) + P_t(t^\mu u^\nu + t^\nu u^\mu), \\ S^{\lambda\mu\nu} &= u^\lambda \left[A\left(k^\mu u^\nu - k^\nu u^\mu\right) + A_1 t^{\mu\nu}\right] \\ &\quad + \frac{A}{2}\left(t^{\lambda\mu} u^\nu - t^{\lambda\nu} u^\mu + \Delta^{\lambda\mu} k^\nu - \Delta^{\lambda\nu} k^\mu\right), \end{aligned} \tag{21}$$

where

$$d_a^\mu = \lambda_a \beta^{-1}(\beta D u^\mu + \beta^2 \nabla^\mu T - 2k^\mu) \tag{22}$$

with $D = u^\mu \partial_\mu$, λ_a – coefficient from Hattori 2019.
K. Hattori et al., Phys.Lett.B **795** (2019) 100-106.

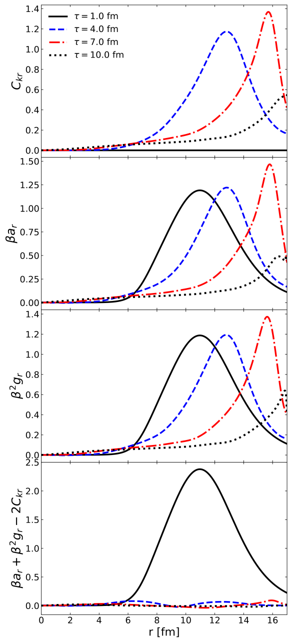
- Preliminary: results with dissipation in the boost-invariant cylindrically symmetric gemetry.
- ZD, W. Florkowski, J. Witkowski arXiv:2605.01857 [hep-ph] + dissipation
- Initial temperature: 0.3 GeV (Woods–Saxon profile in r), particle mass: 1 GeV
- Nonzero dissipative λ_a coefficient.
- Decomposition of $\omega_{\mu\nu}$:

$$\omega_{\alpha\beta} = k_\alpha u_\beta - k_\beta u_\alpha + t_{\alpha\beta},$$

$$k^\mu = C_{kr} R^\mu + C_{k\phi} \Phi^\mu + C_{kz} Z^\mu$$

$$\omega^\mu = C_{\omega r} R^\mu + C_{\omega\phi} \Phi^\mu + C_{\omega z} Z^\mu$$

- The system reaches global equilibrium in which the spin polarization tensor is given by thermal vorticity, $\omega_{\mu\nu} = \partial_{[\nu}\beta_{\mu]}$.
- The difference from thermal vorticity is proportional to the quantity in the last panel and decreases quickly.



Summary

Summary

- The equations of spin hydrodynamics have been worked out quite comprehensively; now it is time for numerical simulations.
- Classical and quantum spin approaches to spin hydrodynamics agree at the second order in spin polarization, and higher-order corrections differ by multiplicative numerical factors.
- We have preliminary results showing dissipation and the spin–orbit interaction, and they look promising.
- Simulations in more general geometries and with a greater variety of dissipative terms come next.

Thank you for your attention!

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Quantum spin description

- Start from the Wigner function

$$W^{\pm}(x,k) = \frac{1}{4m}\int \mathrm{d}P\, \delta^{(4)}(k \mp p)(\not{p} \pm m) X_s^{\pm}(\not{p} \pm m), \tag{23}$$

- Use the new equilibrium spin density

$$X_s^{\pm}(x,p) = \exp\left[\pm\xi(x) - \beta_\mu(x) p^\mu + \gamma_5 \not{a}\right]. \tag{24}$$

- The conserved currents and tensors are

$$N^\mu(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu \left[f_{rr}^{+}(x,p) - f_{rr}^{-}(x,p)\right], \tag{25}$$

$$T^{\mu\nu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu p^\nu \left[f_{rr}^{+}(x,p) + f_{rr}^{-}(x,p)\right], \tag{26}$$

$$S^{\lambda\mu\nu}(x) = \frac{1}{2}\sum_{r,s=1}^{2} \int \mathrm{d}P\, p^\lambda \sigma_{sr}^{+\mu\nu}(p) f_{rs}^{+}(x,p) + \sigma_{sr}^{-\mu\nu}(p) f_{rs}^{-}(x,p), \tag{27}$$

where
$\sigma_{sr}^{+\mu\nu}(p) = 1/(2m)\, \bar{u}_s(p)\sigma^{\mu\nu} u_r(p)$ and $\sigma_{sr}^{-\mu\nu}(p) = 1/(2m)\, \bar{v}_r(p)\sigma^{\mu\nu} v_s(p)$

S. Bhadury, ZD, W. Florkowski, S.K. Kar, V. Mykhaylova, arXiv:2505.02657 [hep-ph].
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