Hadron yields in central nucleus-nucleus collisions, the statistical hadronization model and the QCD phase diagram

A. Andronic

- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- "Small systems" (pp, p-Pb collisions)

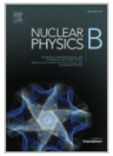
A.Andronic, P.Braun-Munzinger, K.Redlich, J.Stachel, Nature 561 (2018) 321, arXiv:2606.14799 (and ref.therein)

Hadron multiplicities, anno 1976

ELSEVIER

Nuclear Physics B

Volume 111, Issue 3, 6 September 1976, Pages 461-476



Multiplicity distributions in nucleus-nucleus collisions at high energies

A. Białłas [a], M. Bleszyński [a], W. Czyż [b]

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Abstract

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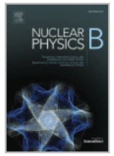

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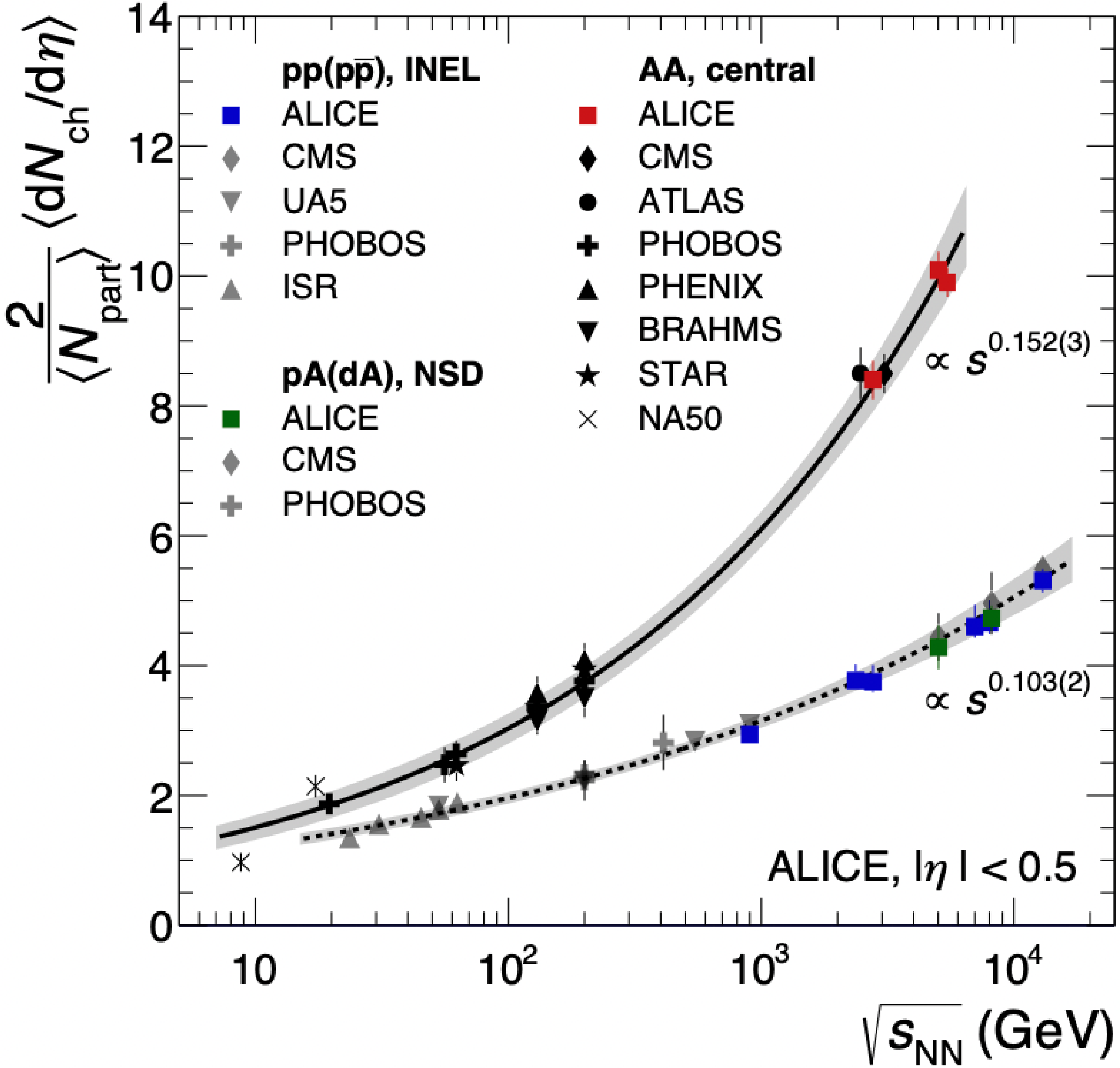
Happy Birthday Andrzej! ...and wounded nucleon model!

Abstract

The average and the dispersion of multiplicity distributions in nucleus-nucleus collisions are calculated assuming that the inelastic collision of two nuclei is an incoherent composition of collisions of individual nucleons. The average multiplicity is assumed to be proportional to the number of "wounded nucleons" i.e. the nucleons which underwent at least one inelastic collision. For the sake of comparison the average and the dispersion of the number of collisions is also discussed. Our calculations indicate that in nucleus-nucleus collisions, the amplification of various characteristics of the nucleon-nucleon interaction is far grater than in hadron-nucleus collisions.

Hadron multiplicities, anno 2026



ALICE, EPJC 84 (2024) 813

Scaled by $N_{part}/2$

$=1$ in pp

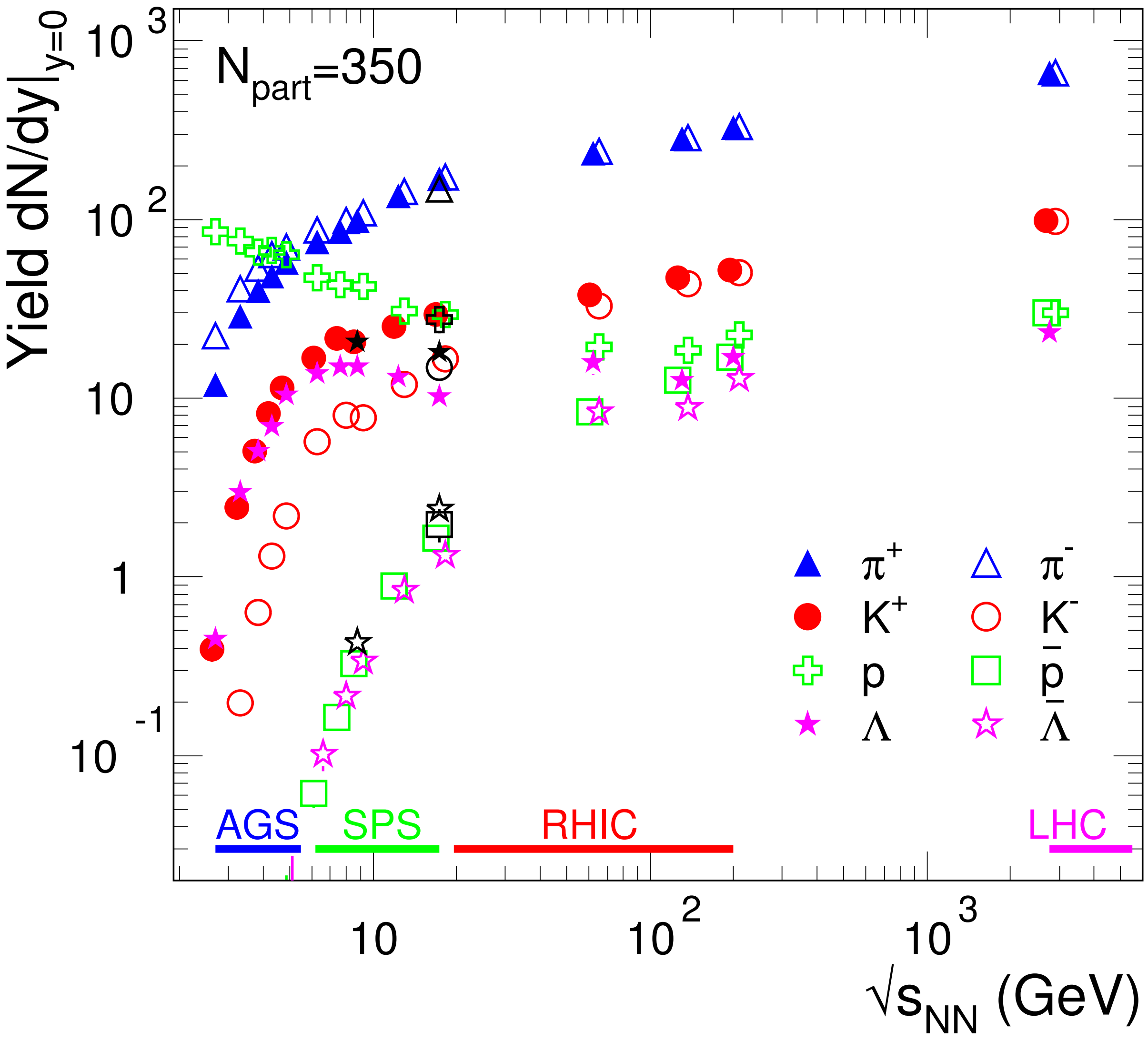
$\simeq 4$ in p–Pb

Particle production more "efficient" in AA collisions ("amplified"), $\sim \sqrt{s}^{0.1}$

reason is rather clear (NB: +flow)

effect of parton shadowing in p(d)-A collisions seems small

Hadron yields at midrapidity (central collisions)



A.Andronic, arXiv:1407.5003

- lots of particles, mostly newly created ($m = E/c^2$)
- a great variety of species:

  $\pi^\pm$ ($u\bar{d}$, $d\bar{u}$), m=140 MeV
  $K^\pm$ ($u\bar{s}$, $\bar{u}s$), m=494 MeV
  p (uud), m=938 MeV
  Λ (uds), m=1116 MeV
  also: $\Xi(dss)$, $\Omega(sss)$...

- mass hierarchy in production (u, d quarks: remnants from the incoming nuclei)

...natural to think of the thermal (statistical) model ($\mathrm{e}^{-m/T}$)

The statistical hadronization model

also known as: thermal / hadron resonance gas model

...is in a way the simplest model

the analysis of hadron yields within the thermal model provides a "snapshot" of a nucleus-nucleus collision at *chemical freeze-out*
(the earliest in the collision timeline we can look with hadronic observables)
test hypothesis of hadron abundancies in equilibrium (chemical)

...but the devil is in the details (sources of syst. uncert.) ...one needs :

- a complete hadron spectrum (all species of hadrons, see PDG, extra states?)
- canonical approach at low energies (and smaller systems)
- treatment of interactions
- to understand the data well (Ex.: control fractions from weak decays)

Reminder about ensembles

microcanonical: describes an isolated system (E, V, T)

canonical and *grand canonical*: suppose a heat reservoir (temperature T)
the system (our collection of hadrons) exchange energy with that

canonical: no particles are exchanged (N, V, T)

grand canonical: the system exchanges particles ($\langle N \rangle$, V, T)
chemical potentials ($\mu \rightarrow$ fugacities $\mathrm{e}^{\mu/T}$), are introduced to ensure conservation, on average, of particle numbers

The statistical (thermal) model

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 \mathrm{d}p \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;
$E_i = \sqrt{p^2 + m_i^2}$ total energy; $(+)$ for fermions (–) for bosons
$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by "initial conditions"

i) isospin: $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot}/N_B^{tot}$,
I_3^{tot}, N_B^{tot} isospin and baryon number of the system (=0 at high energies)

ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Brief summary of cases

General ($\epsilon = +1$ for bosons, $\epsilon = -1$ for fermions; particle index i omitted):

$$N = -T\frac{\partial \ln Z}{\partial \mu} = \frac{gV}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E-\mu)/T] \pm 1}$$

$$N = \frac{g}{2\pi^2} TVm^2 \epsilon \sum_{k=1}^{\infty} \frac{\epsilon^k}{k} \mathrm{e}^{\frac{\mu}{T}k} K_2\left(\frac{m}{T}k\right)$$

Classical statistics:

$$N = \frac{g}{2\pi^2} TVm^2 \mathrm{e}^{\frac{\mu}{T}} K_2\left(\frac{m}{T}\right) = gV\mathrm{e}^{\frac{\mu}{T}}\left(\frac{mT}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{m}{T}}\left(1 + \frac{15T}{8m} + O(T^2/m^2)\right)$$

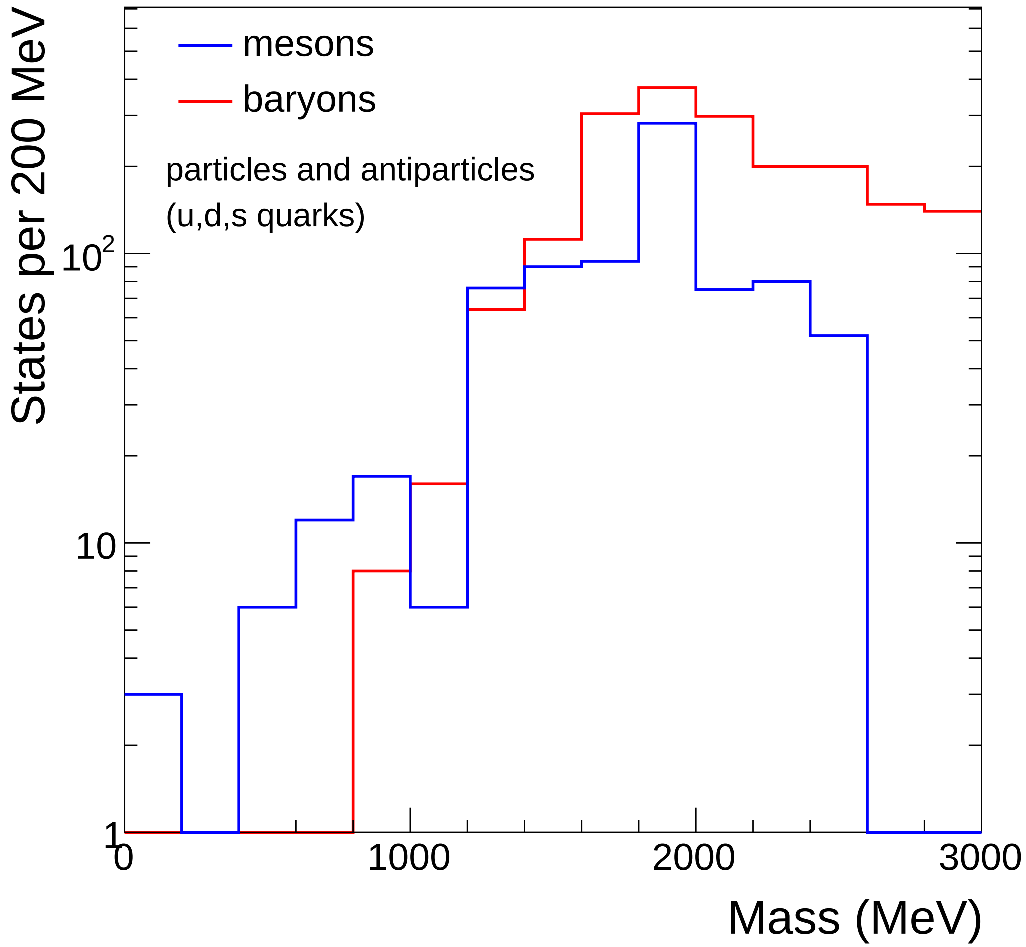
$$K_2(x) = \sqrt{\frac{\pi}{2x}}\mathrm{e}^{-x}\left(1 + \frac{15}{8x} + O(1/x^2)\right)$$

W.Florkowski, Phenomenology of Heavy-Ion Collisions (WS, 2010)

Model input: hadron spectrum

...embodies low-energy QCD ...*vacuum masses*

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



$(2J+1)$ counted in

for high m, BR not well known, but can be reasonably guessed

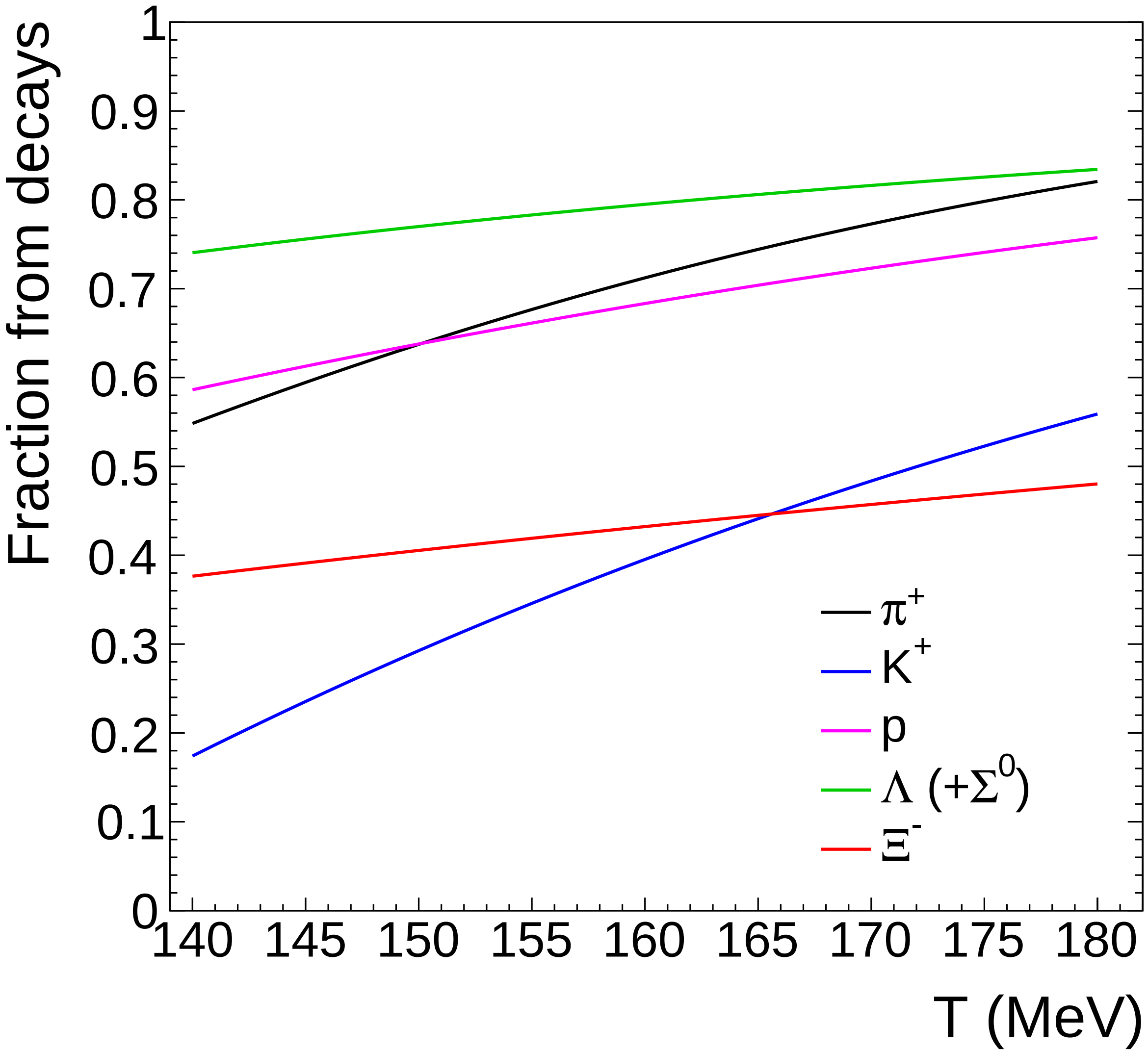
T found to be robust in fits with spectrum truncated above 1.8 GeV

σ [$f_0(500)$] meson proposed to be discarded (reduction of π densities by 3-4%)

Giacosa, Begun, Broniowski, APPS 9 (2016) 213

additional states (LQCD, RQM)?
...no change in results (w. S-matrix)
NPA 1010 (2021) 122176

(almost all) hadrons are subject to strong and electromagnetic decays



$\Delta \rightarrow p(n) + \pi$, $\rho \rightarrow \pi + \pi$

$\Sigma^0 \rightarrow \Lambda + \gamma$

weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for μ_B=0)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

$$n_i = \frac{g_i}{2\pi^2}\frac{1}{N_{BW}}\int_{M_{thr}}^\infty \mathrm{d}m \int_0^\infty \frac{\Gamma_i^2}{(m-m_i)^2+\Gamma_i^2/4} \cdot \frac{p^2 \mathrm{d}p}{\exp[(E_i^m-\mu_i)/T] \pm 1}$$

M_{thr} threshold mass for the decay channel.

Example: for $\Delta^{++} \rightarrow p + \pi^+$, M_{thr}=1.068 GeV ($m_{\Delta^{++}}$=1.232 GeV)

Important mainly at “low” temperatures ($T \lesssim$150 MeV)

Canonical treatment ("canonical suppression")

needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation in AA collisions: strangeness at low energies; (mid)peripheral collisions

full treatment is laborious, see Braun-Munzinger, Redlich, Stachel, nucl-th/0304013

approximation often used:

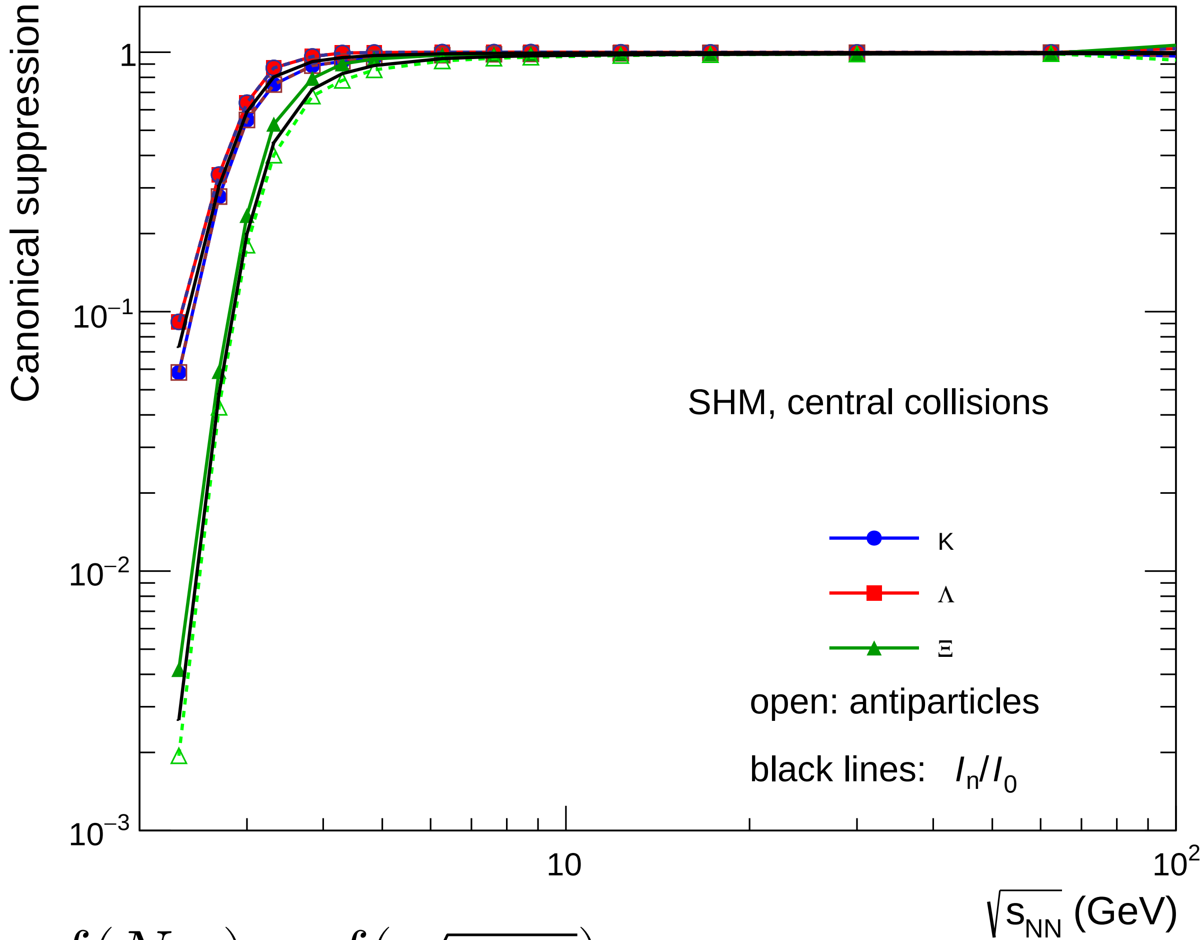
$$n^C_{i,S} = n^{GC}_{i,S} \cdot \frac{I_s(x)}{I_0(x)}$$

$$x = 2\sqrt{N_S N_{\overline{S}}}$$

$N_S = V_c \cdot \sum S \cdot n_{i,S}$, total amount of strangeness-carrying hadrons (part., antipart.)

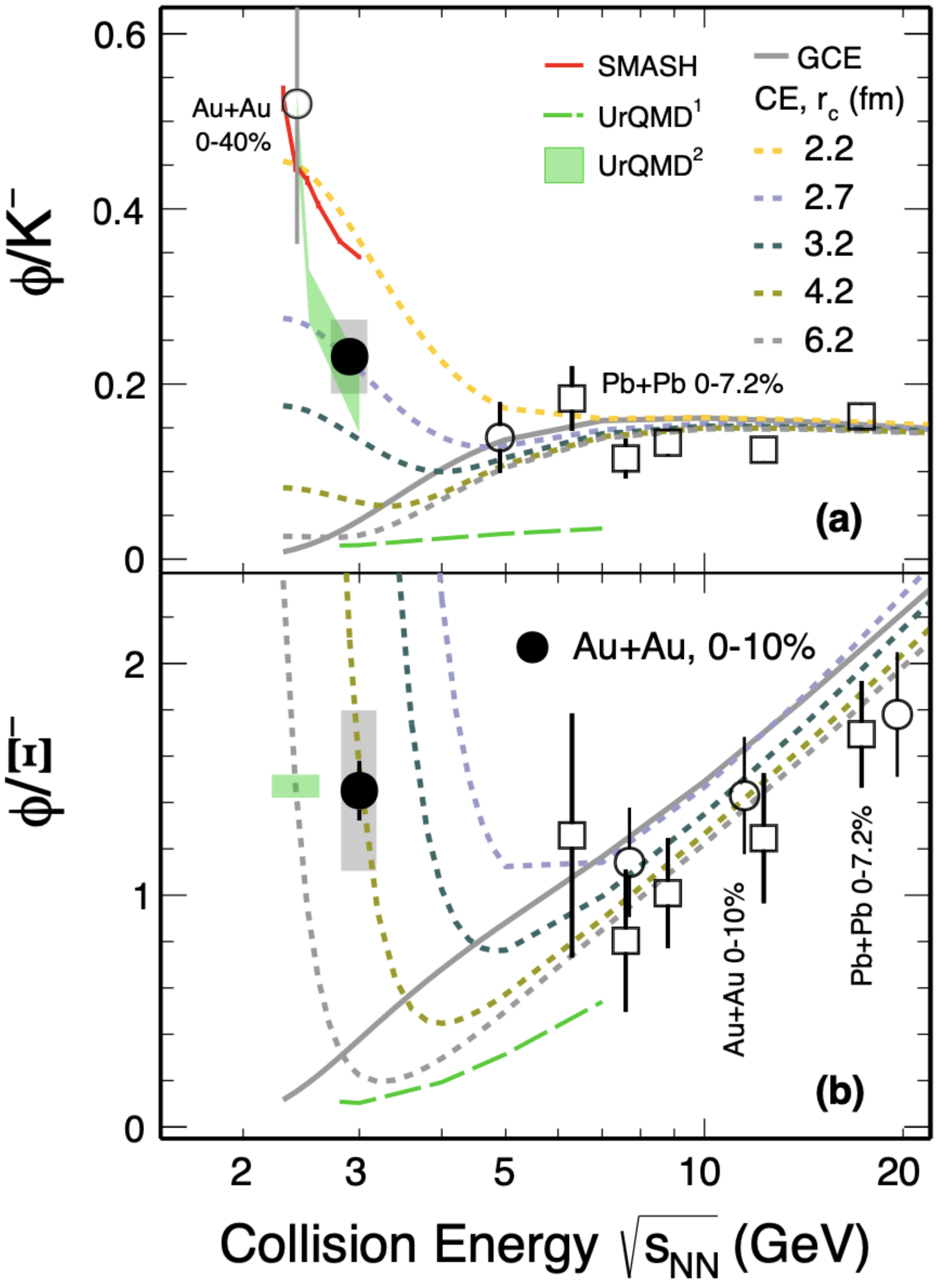
$$n^C_{K,\Lambda} = n^{GC}_{K,\Lambda} \cdot \frac{I_1(x)}{I_0(x)},$$

$$n^C_{\Xi} = n^{GC}_{\Xi} \cdot \frac{I_2(x)}{I_0(x)}, \; n^C_{\phi} = n^{GC}_{\phi}$$



V_c canonical (correl.) volume, $V_c = f(N_{ch}) = f(\sqrt{s_{NN}})$

Canonical treatment ("canonical suppression")



STAR, PLB 831 (2022) 137152

r_c radius of canonical (correl.) volume

to constrain V_c a large set of hadron yields is needed (STAR, CBM)

V_c: strangeness correlation volume

Strangeness suppression factor, γ_s

...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying “absolute” strangeness $s = |S - \bar{S}|$: $n_i \to n_i \gamma_s^s$

Examples: $K^{\pm}$ ($u\bar{s}$, $\bar{u}s$): $n_K \gamma_s$, Λ (uds): $n_\Lambda \gamma_s$,

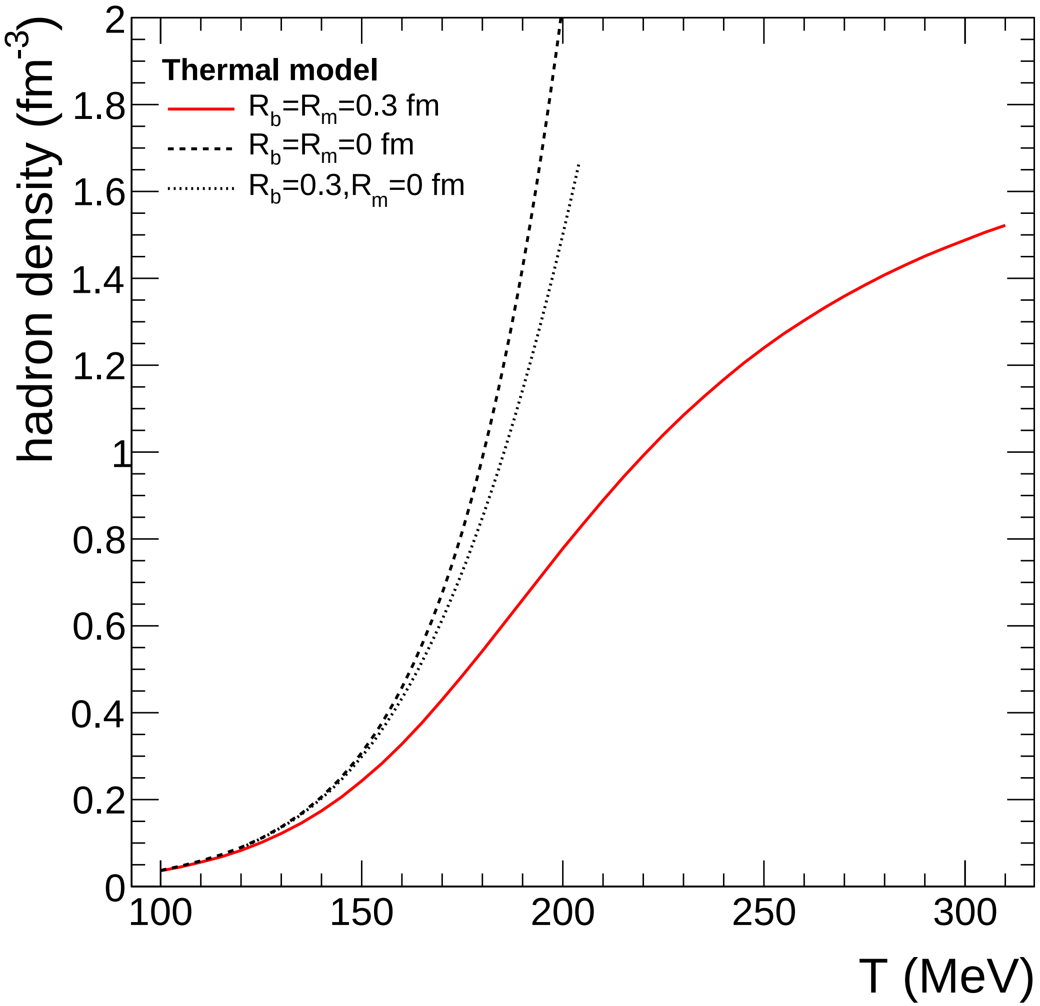
$\Xi(dss)$: $n_\Xi \gamma_s^2$, $\Omega(sss)$: $n_\Omega \gamma_s^3$, $\phi(s\bar{s})$: $n_\phi \gamma_s^2$

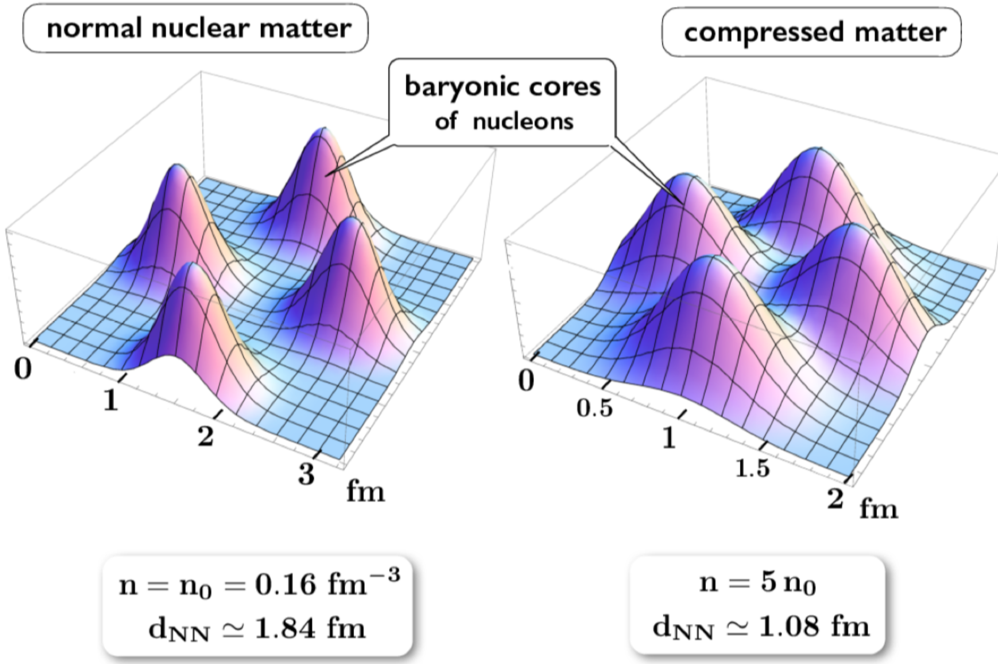
in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix γ_s=1

Hadron densities





Weise, arXiv:1811.09682

(baryons: gaussians, r=0.5 fm)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3$ fm ...hard-sphere repulsion)

Air at NTP: intermolecule distance ≃50 × molecule size

Considering interactions

hadron eigenvolumes ...to mimick interactions (beyond low-density, Dashen-Ma)

$R_{meson} = 0.3, R_{baryon} = 0.3$ *fm was used for long time*

point-like hadrons lead to same T, but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3$ fm leads to
$T = 161.0 \pm 2.0$ MeV, $\mu_B = 0$ *fixed*, $V = 3470 \pm 280$ fm^3

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω, cannot constrain T (unphysically large)

Vovchenko, Stöcker (et al.), arXiv:1512.08046,

...and anything else can be imagined, see (R dependent on mass & strangeness)

Alba, Vovchenko, Gorenstein, Stöcker, NPA 974 (2018) 22, etc.

mass-dependent Breit-Wigner resonance widths:

Vovchenko, Gorenstein, Stöcker, PRC 98 (2018) 034906

Interactions, rightly done

...for now only at the LHC

non-strange baryon sector treated in S-matrix formalism
(πN scattering phase shifts, *including non-resonant contributions*)

PLB 792 (2019) 304 (+P.M. Lo, B.Friman)

solved the so-called "proton puzzle" (too many protons in the statistical model)

for T=156 MeV, proton yield decreased by 17% comp. to point-like (non-int.)

S-matrix for $\Lambda(+\Sigma^0)$ hyperons: correction $\simeq$+20% (code-dep., hyperon states)

Cleymans et al., PRC 103 (2021) 014904

NB: presence of resonances implies interaction
($R = 0.3$ fm was a reasonable choice)

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to m=2 GeV;
our code: ∼600 species (including fragments, charm and bottom hadrons)

for resonances, the width is considered in calculations

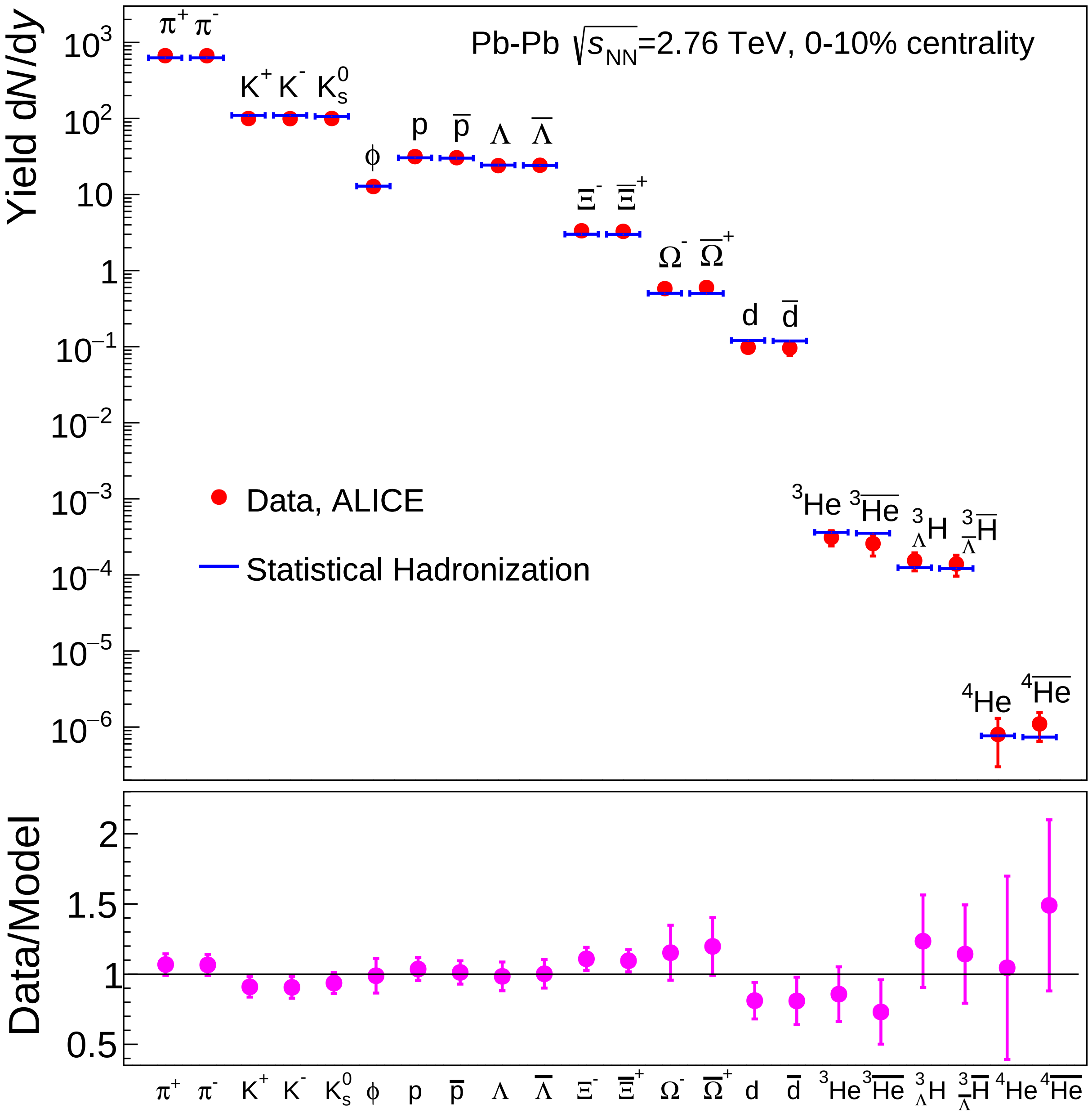
canonical treatment whenever needed (small abundances)

Minimize: $\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$

N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

⇒ (T, μ_B, V) ...*tests chemical freeze-out* (chemical equilibrium)

Thermal fit – LHC, Pb–Pb, 0-10%



matter and antimatter produced in equal amounts

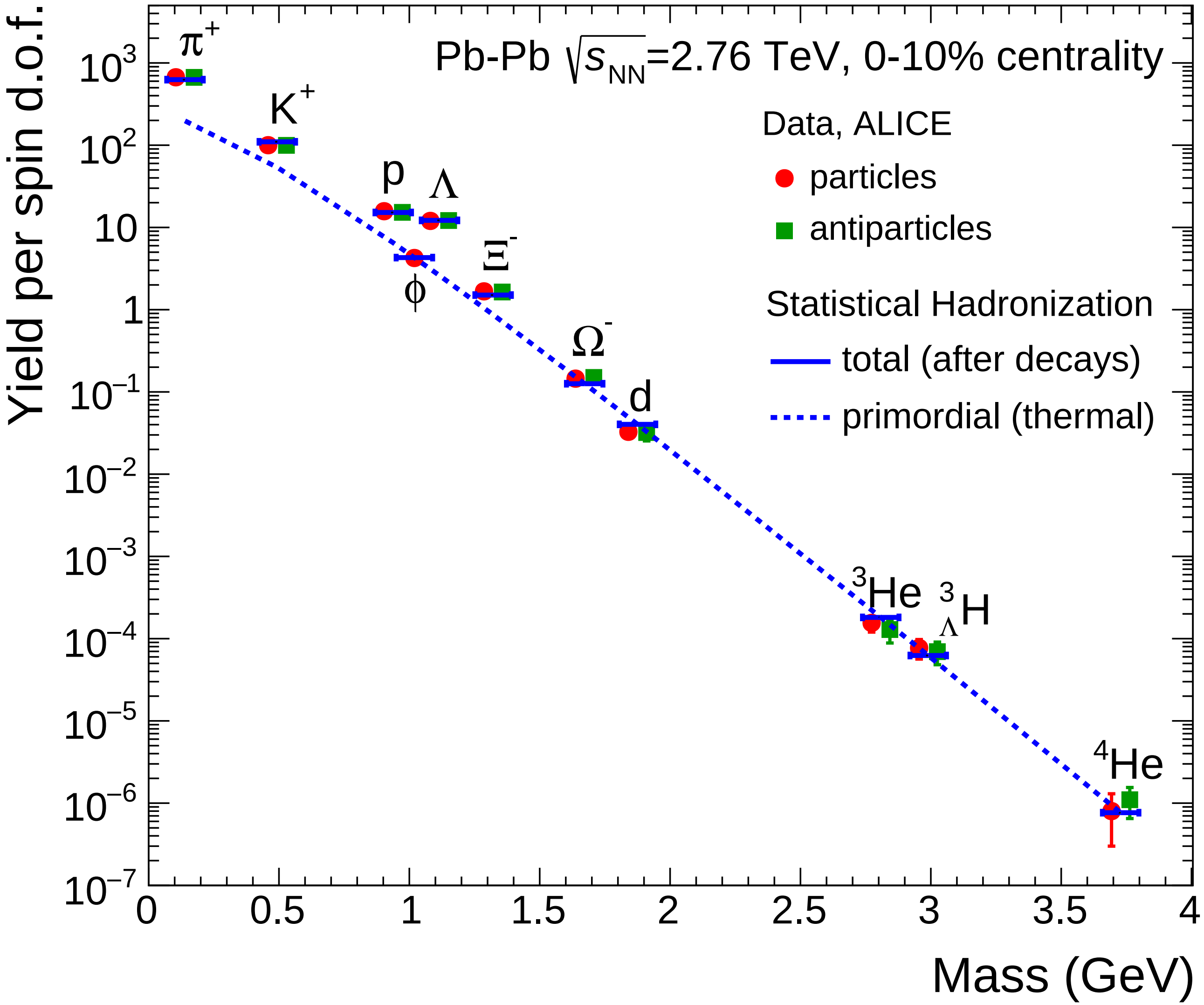
$T_{CF} = 156.6 \pm 1.7$ MeV
$\mu_B = 0.7 \pm 3.8$ MeV
$V_{\Delta y=1} = 4175 \pm 380$ fm^3

$\chi^2/N_{df} = 16.7/19$

chemical equilibrium seems to be realized (in the QGP phase)

remarkably, loosely-bound objects (d, E_b=2.2 MeV) also well described

Model uncertainties: hadron spectrum



contribution of resonances is significant (and particle-dependent)

Fit of ϕ, Ω, d, ^{3}He, $^3_\Lambda$H, ^{4}He:

$T_{CF} = 156 \pm 2.5$ MeV

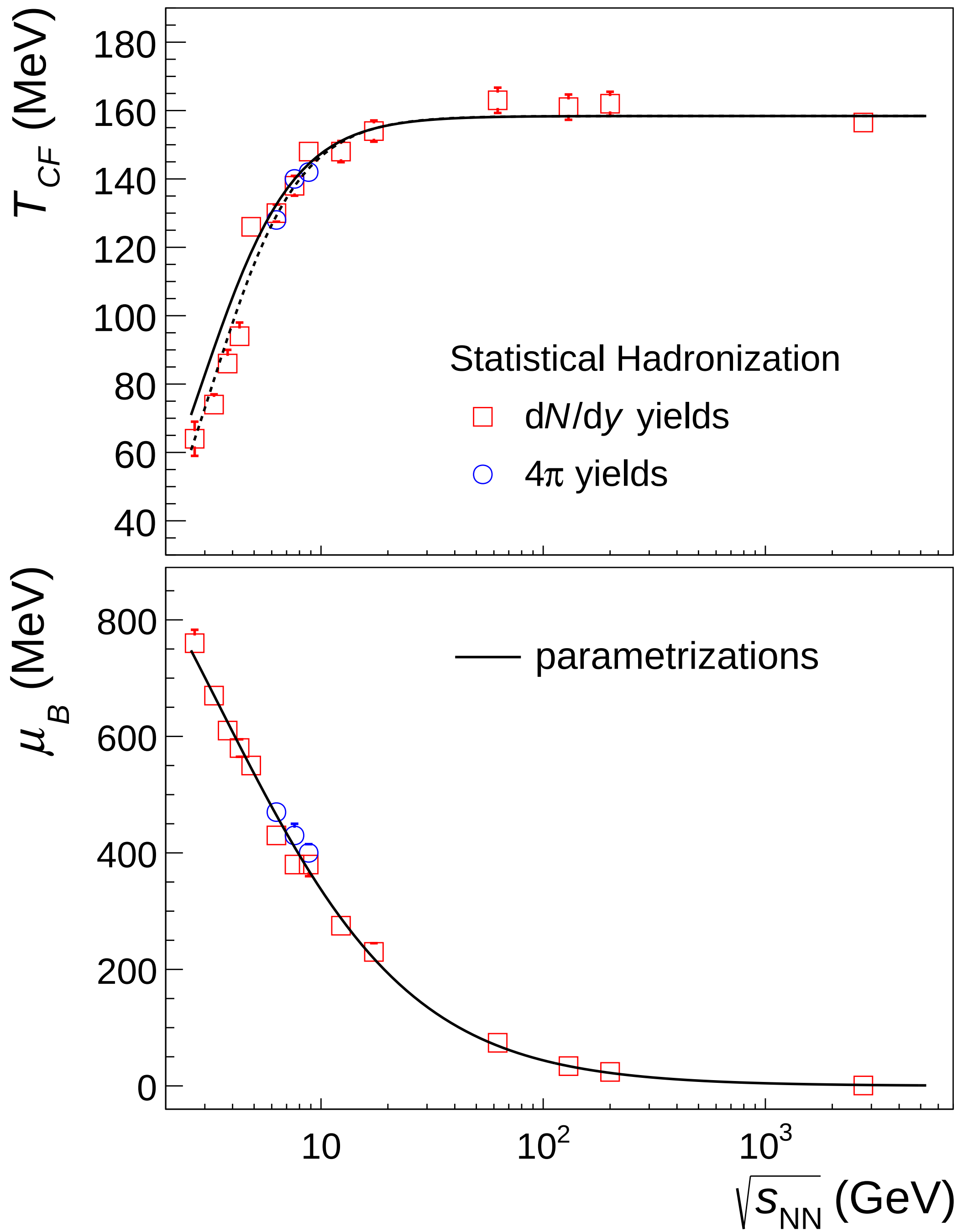
$(\chi^2/N_{df} = 7.4/8)$

Fit of nuclei (d, ^{3}He, ^{4}He):

$T_{CF} = 159 \pm 5$ MeV

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

Energy dependence of T, μ_B (central collisions)



thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1+\exp(2.20 - \ln(\sqrt{s_{NN}}(\mathrm{GeV}))/0.48)}$$

slightly updated (old param: dashed line), with V_c

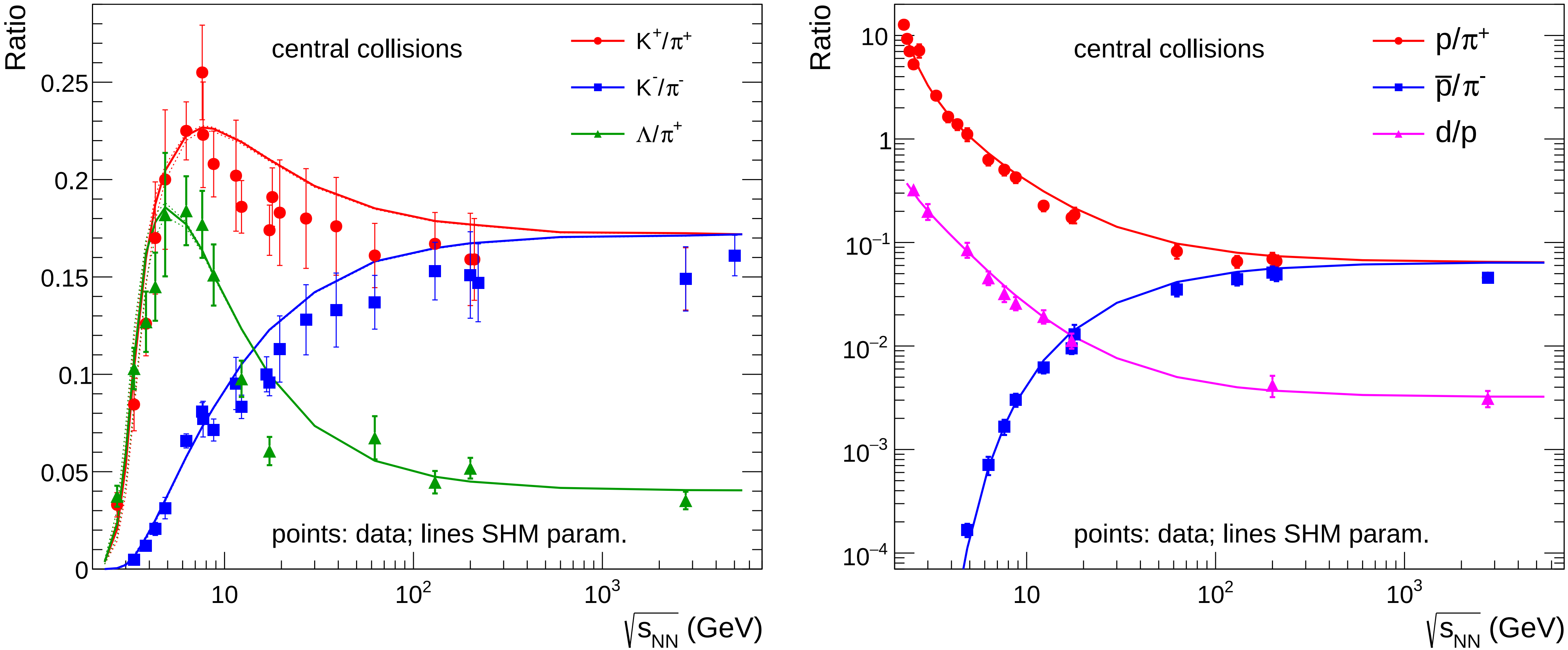
$$\mu_B[\mathrm{MeV}] = \frac{1307.5}{1+0.288\sqrt{s_{NN}}(\mathrm{GeV})}$$

NPA 772 (2006) 167, PLB 673 (2009) 142

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

The grand (albeit partial) view



"structures" described by SHM very well

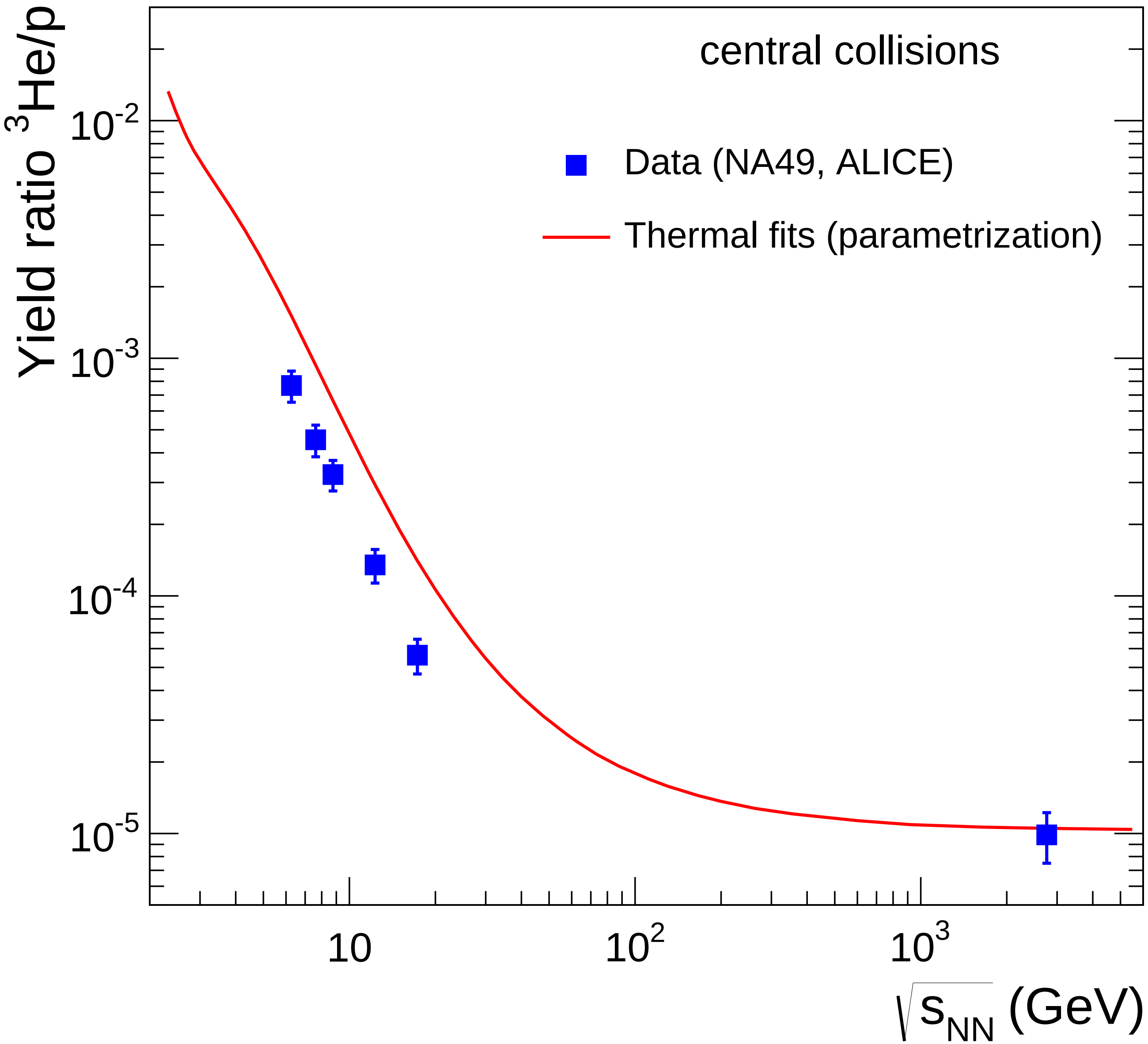
...determined by strangeness conservation (and T,μ_B)

NB: no S-matrix treatment (available only for $\mu_B \simeq 0$)

Data: AGS: E895, E864, E866, E917, E877; SPS: NA49, NA44; RHIC: STAR, BRAHMS; LHC: ALICE

Something that doesn't work so well ?

^{3}He/p ratio well described at the LHC, but not at lower energies

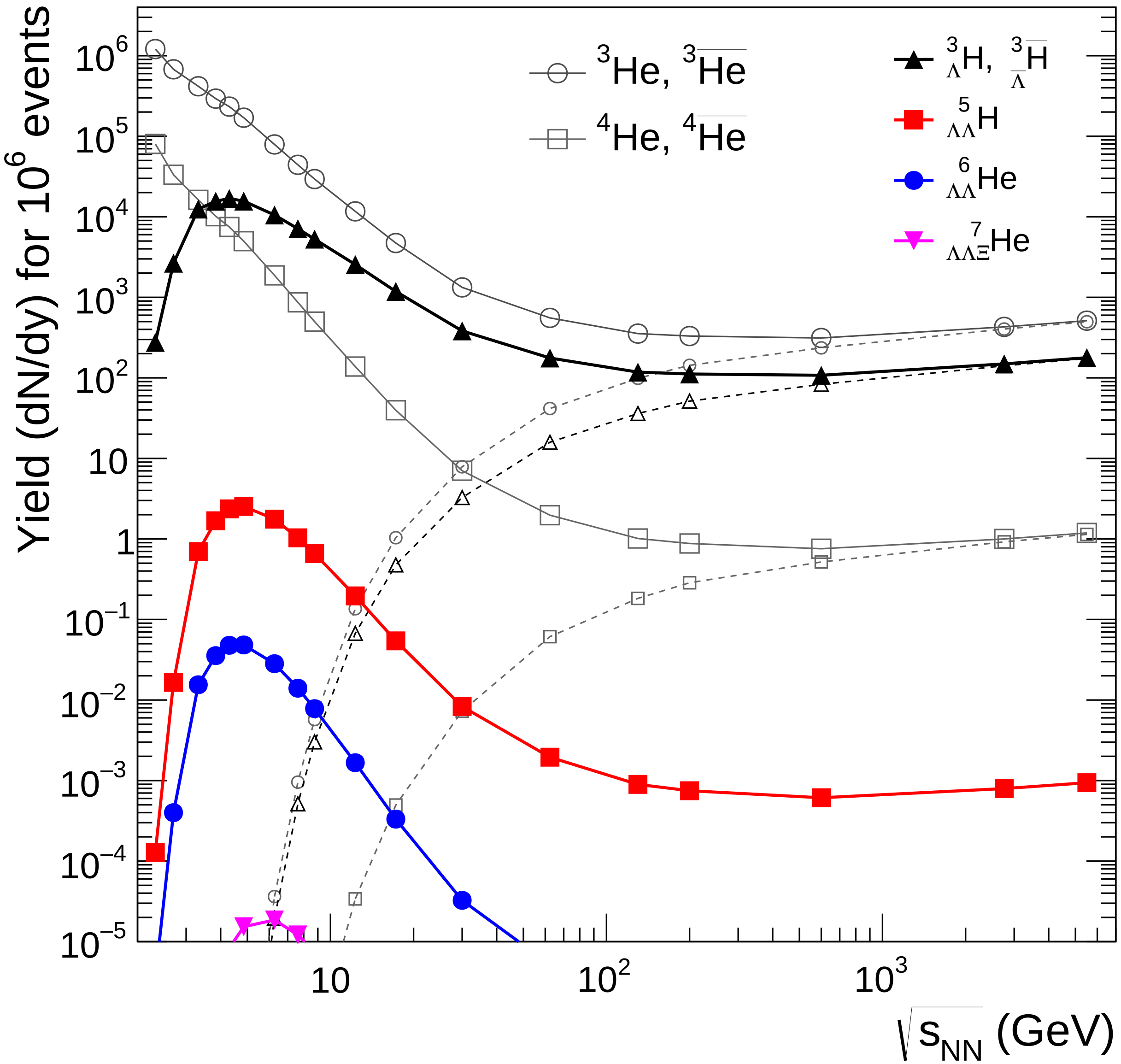


see talk by PBM (Thu)

ALICE 5.02 TeV:

Complex objects

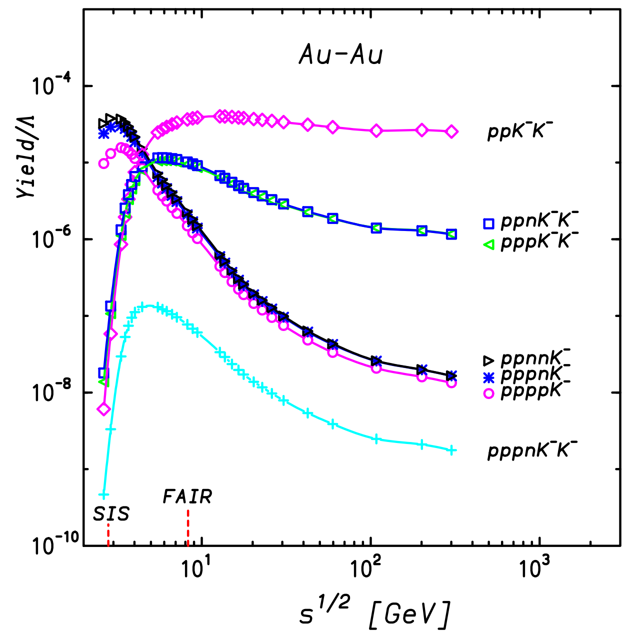
...are copiously produced at FAIR (RHIC BES) energies



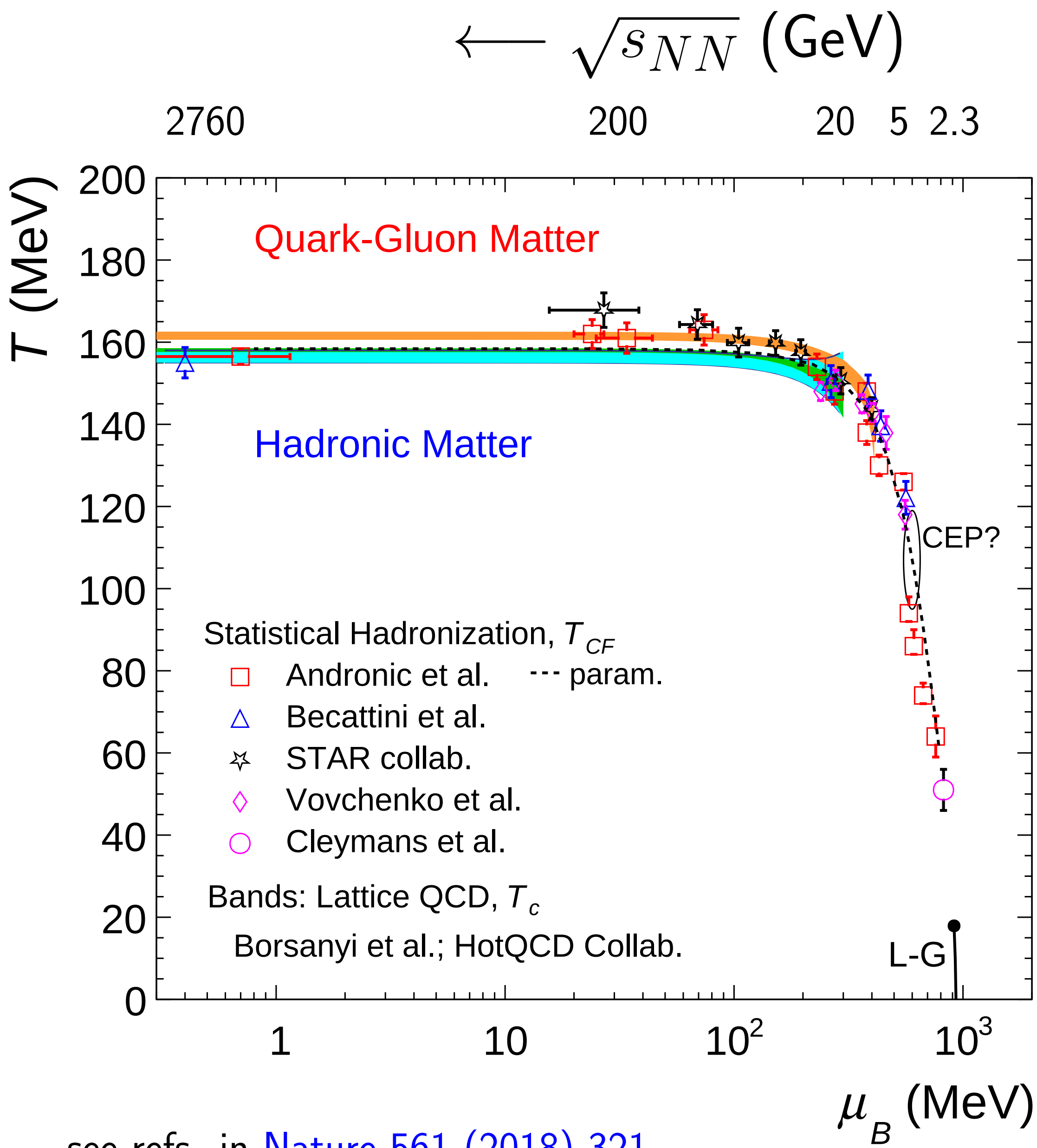
...some to be discovered

maybe also nucleon-K^- clusters?

AA, PBM, K.Redlich, NPA 765 (2006) 211



The phase diagram of QCD



see refs. in Nature 561 (2018) 321

low μ_B*: remarkable "coincidence" with Lattice QCD results*

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

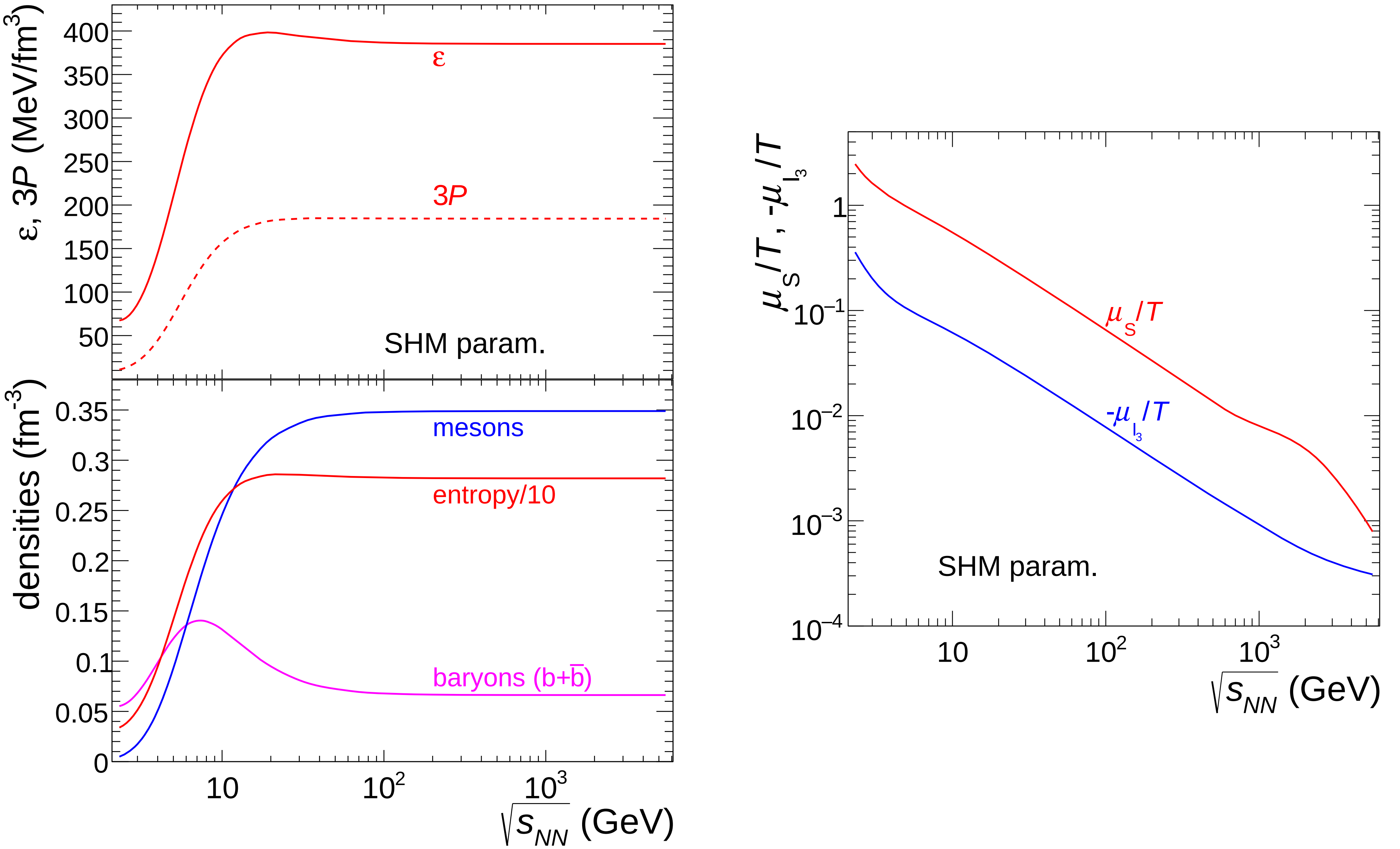
$\mu_B > 0$: more matter, from "remnants" of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery* (at FAIR?)

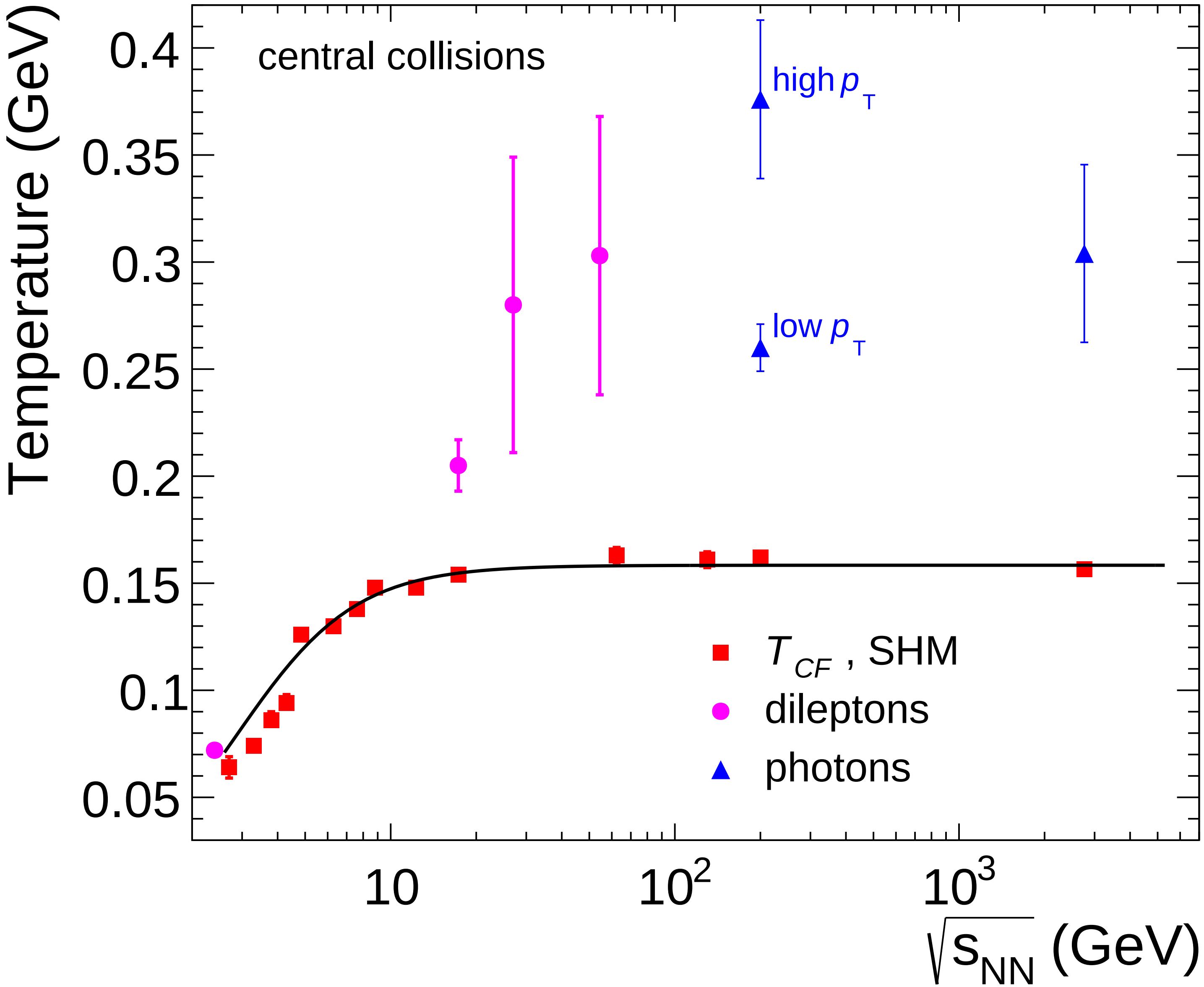
μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

points: independent analyses of same data $\rightarrow$ "model/code uncert." are small

The thermal properties at chemical freeze-out

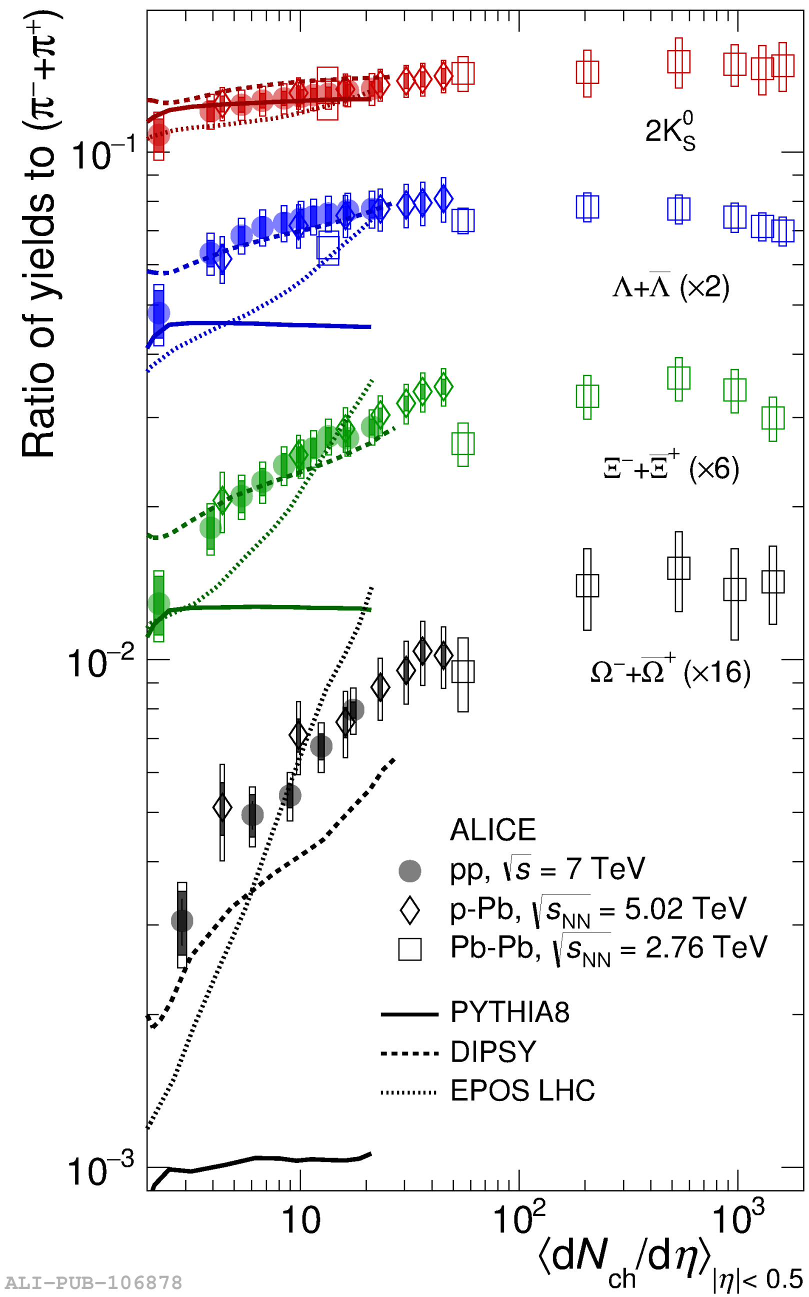


Chemical freeze-out in context



Strangeness production - from small to large systems

'ALICE, Nature Physics 13 (2017) 535



(big geometric) fireball in Pb–Pb reached with violent pp and p–Pb collisions

canonical to grand-canonical strangeness production regime Vislavicius, Kalweit, arXiv:1610.03001

is the same mechanism at work in small systems (at large multiplicities)?

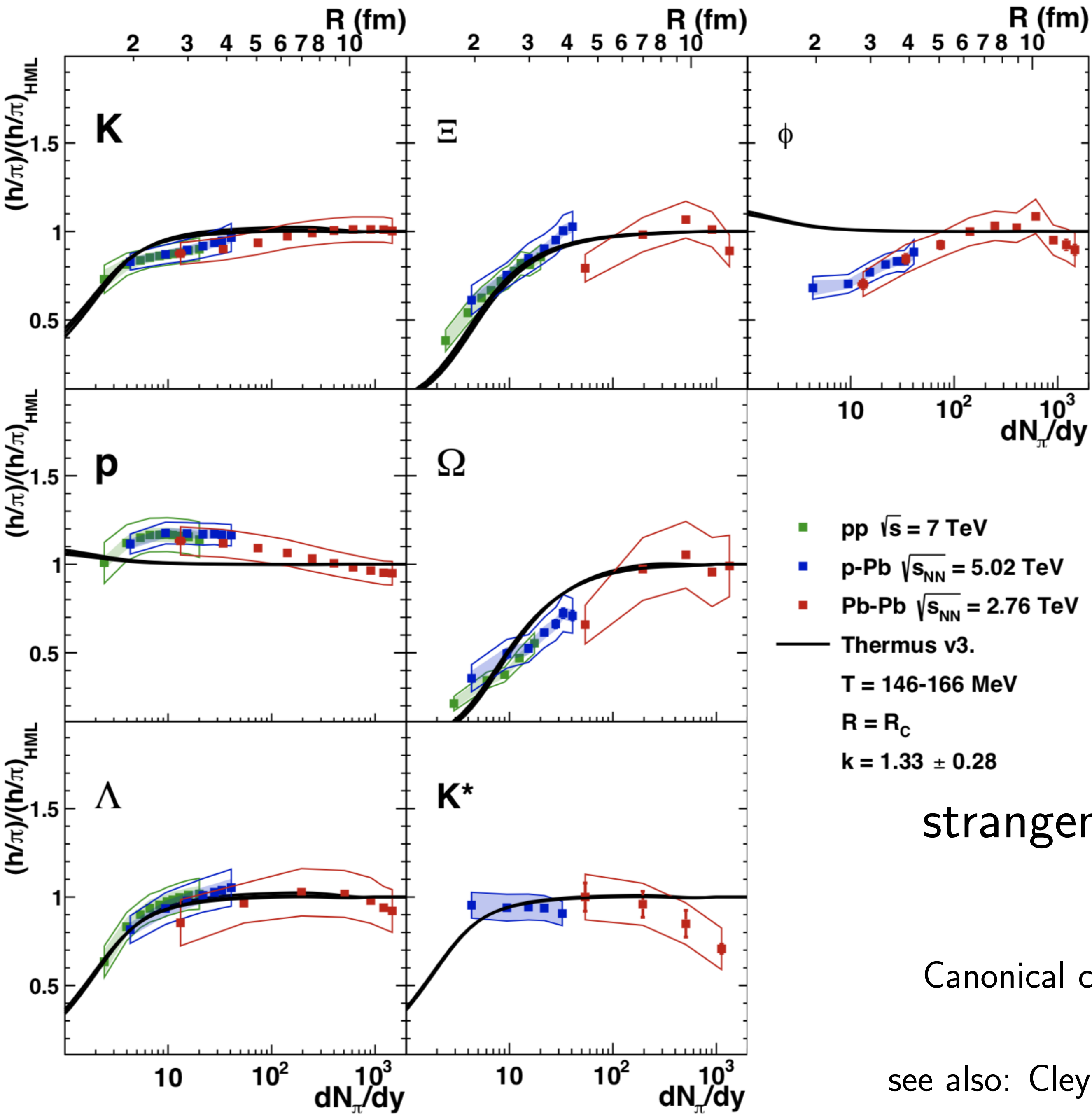
string hadronization (PYTHIA8) does not describe data

DIPSY (Mueller's dipole, BFKL evolution) fares better Dipole evolution in Impact Parameter Space and rapiditY Flensburg, Gustafson, Lonnblad, arXiv:1103.4321

EPOS LHC: core(QGP)-corona model

new ideas: ropes; thermodynamical string fragmentation

Particle production - from small to large systems



Vislavicius, Kalweit,

arXiv:1610.03001

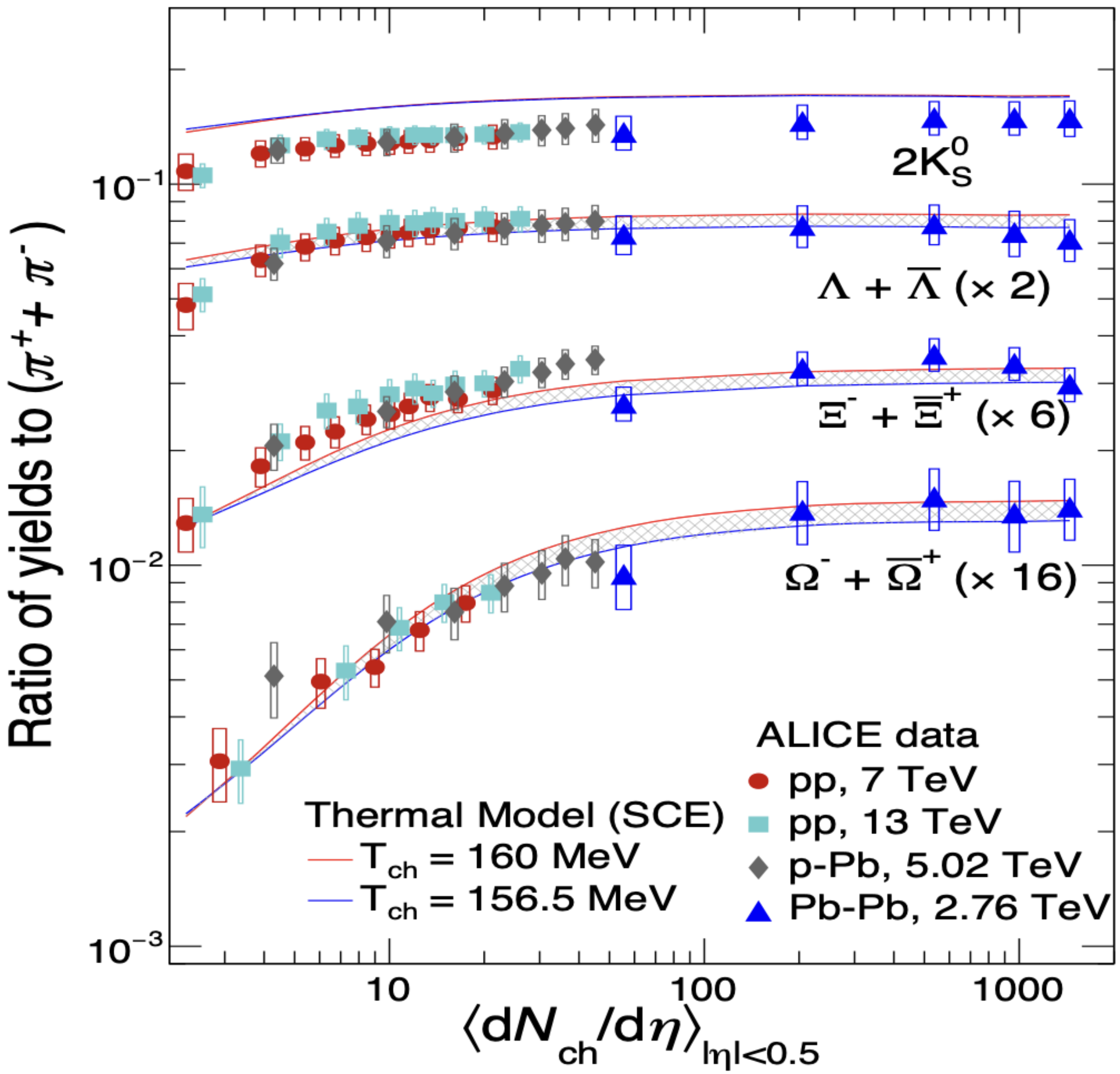
ratios to high multiplicity limit (HML)

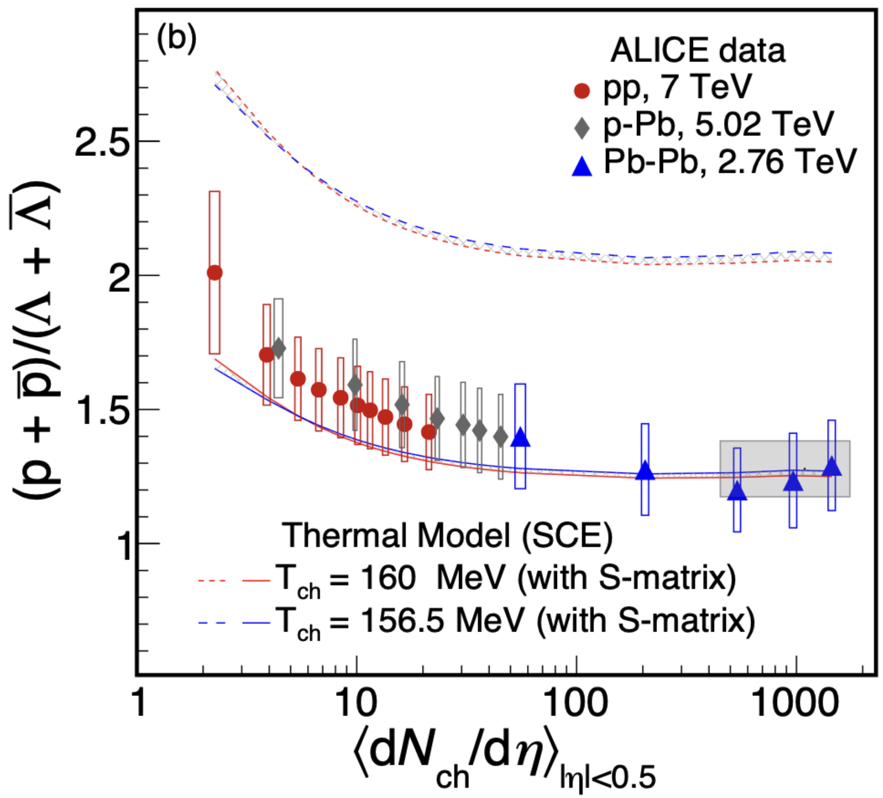
strangeness-canonical stat. hadr. describes data well

Canonical cons. volume is a parameter (k, here)

see also: Cleymans et al., PRC 103 (2021) 014904

Particle production - from small to large systems





strangeness-canonical SHM +S-matrix (p: -25%, Λ: +24%) describes data very well

Canonical-strangeness conservation (correlation) volume $V_c = 12.3 + 3.02 \times \frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}$ (fm^3)

Cleymans et al., PRC 103 (2021) 014904

What about heavier quarks?

up to now we only considered hadrons built with *up, down, strange* quarks
...these are light, masses from a few MeV (u, d) to ∼90 MeV (s)

what about heavier ones?
...for instance *charm*, which weights about 1.2 GeV

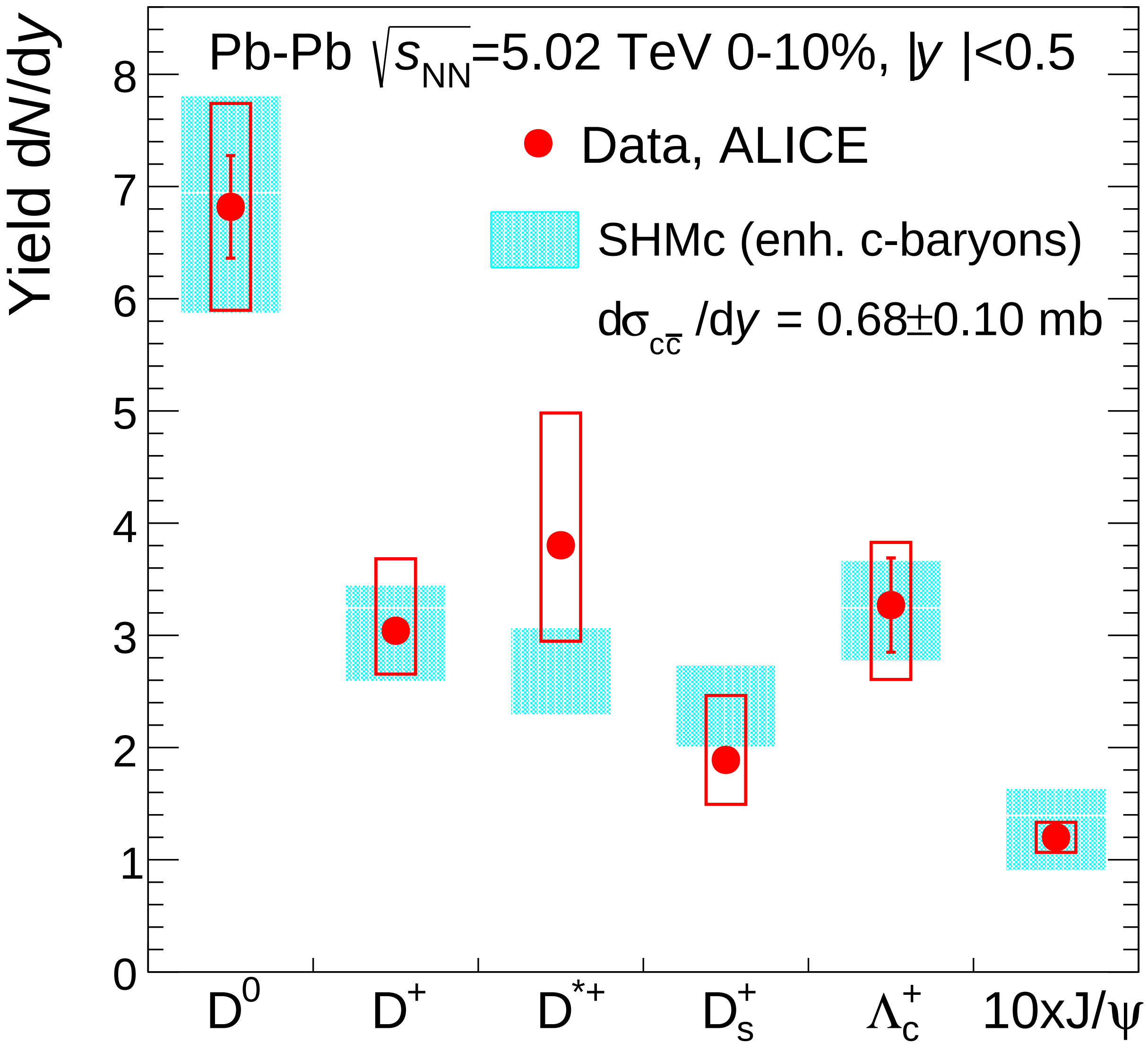
produced in pairs ($c\bar{c}$) in initial hard collisions, $t \sim 1/(2m_c) \leq 0.1$ fm/c

preserve their identity throughout the evolution of the fireball

...ideal messengers of the early stage

Statistical hadronization for charm

...as for u,d,s quarks, at T =156.5 MeV



see talk by J.Stachel (Fri)

Summary

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the "minimal model"*

  a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary

  ...at high energies (at low energies canonical suppression needs more care)

  ...but is it more than a 1st order description (of loosely-bound objects)?

  ...and what fundamental point does it make about hadronization?

  (as a dynamical process, understanding still missing)
- more insights from higher moments and from heavier (charm) quarks