The $\rho\pi$ puzzle and vector glueball mixing

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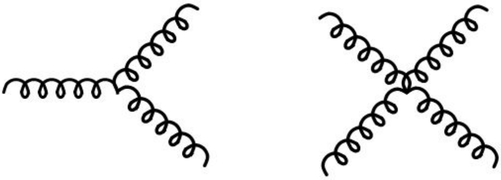
Introduction

The QCD lagrangian contains gluon self-interaction



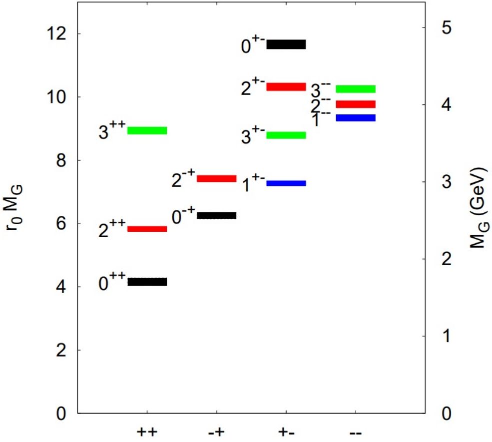
- Is there a bound state of only gluons?
- Theory says yes! Experimentally not verified yet
- There are various theoretical models on glueballs e.g. lattice QCD, sum rules, functional methods, holographic QCD, and <u>effective models</u>
- Glueballs will mix with conventional mesons, hard to identify. Need both experiment and theory to find them

Glueball spectrum

Lattice calculations have found a large spectrum of pure gluon states.

The two lightest ones are the scalar ($J^{PC} = 0^{++}$) and the tensor ($J^{PC} = 2^{++}$) and as such they are some of the best candidates for experimental verification.

We will focus on the vector glueball, which is also the lowest mass $C = -1$ glueball. In chiral models, interactions and decays are easier to implement than on the lattice.



Chen et al, 2005

The vector glueball

- For the vector glueball, with mass 3.8 - 4 GeV, there are currently no candidates
- But the vector glueball may play a role in the decay of known states due to its mixing
- The J/ψ and $\psi(2S)$ are bound $c\bar{c}$ states that are quite stable, and primarily decay through a 3-gluon channel, making them sensitive to glueballs
- They are supposed to follow the "13% rule", but this rule is broken for yet unknown reasons

The 13% rule

In perturbative, non-relativistic QCD the partial decay width to some final state X is proportional to the wave function at the origin

$$\Gamma(J/\psi \to X) = |\psi(\mathbf{r} = 0)|^2 |M_X|^2$$

Similar expression for $\psi(2S)$, neglecting phase space, M only depends on the final state so it divides out in

$$Q_h = \frac{\mathcal{B}(\psi(2S) \to h)}{\mathcal{B}(J/\psi \to h)} \approx \frac{\mathcal{B}(\psi(2S) \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} \approx 13\%$$

This is the "13% rule", the ratio of branching fractions is independent of the final state and equals approximately 13%.

The $\rho\pi$ puzzle

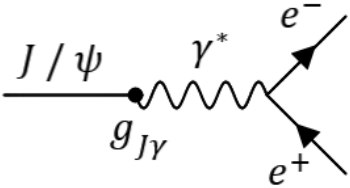
The 13% rule is severely violated in some decay channels, in particular for the $\rho\pi$ channel

$K^0\bar{K}^{0*}$ + c.c	$2.59 \pm 0.54\%$
K^+K^{-*} + c.c	$0.48 \pm 0.10\%$
$\eta\omega$	$< 6.3\%$
$\omega\eta'$	$16.93^{+13.33}_{-11.23}\%$
$\phi\eta$	$4.19 \pm 0.54\%$
$\phi\eta'$	$3.35 \pm 0.57\%$
$\rho\pi$	$0.19 \pm 0.073\%$
$\eta\rho$	$11.4 \pm 3.39\%$
$\rho\eta'$	$23.46^{+21.12}_{-14.99}\%$
$\omega\pi^0$	$4.67 \pm 1.43\%$

Q_h in VP channel, PDG values

From 13% rule to coupling constant

The 13% rule is based on the decay into electron-positron, through the diagram



From this we can find the $J/\psi - \gamma$ coupling and the ratio r between J/ψ and $\psi(2S)$ couplings (wavefunctions)

$$\frac{\mathcal{B}(\psi(2S) \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} \approx 13\% \Longrightarrow r^2 \approx 0.35.$$

We assume this same ratio r holds for every other coupling too.

EM interactions via vector meson dominance

In the previous slide we introduced the $J/\psi - \gamma$ transition by

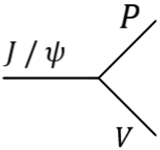
$$\mathcal{L} = g_{J\gamma} J_{\mu\nu} F^{\mu\nu}.$$

For the light mesons we can do this through vector meson dominance, in the limit of universality we can do this via a shift

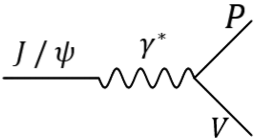
$$V_\mu \to V_\mu + \frac{e}{g_\rho} A_\mu Q$$

With $Q = $ diag(2/3,-1/3,-1/3) the charge matrix, and $g_\rho \simeq 5.0$ an universal dimensionless coupling for the light vectors.

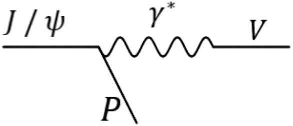
The interactions



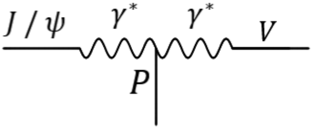
$$g_J \tilde{J}_{\mu\nu} Tr[\partial^\mu \mathcal{P} \phi_0 V^\nu \phi_0]$$



$$\frac{-e}{4g_\rho} g_{J\gamma} g_{PVV} \tilde{J}_{\mu\nu} Tr[P\{Q, V^{\mu\nu}\}]$$



$$-g_J (\frac{e}{g_\rho})^2 Tr[\partial^\mu \mathcal{P} \phi_0 Q \phi_0] Tr[V^\nu Q]$$



$$\frac{1}{4} g_{J\gamma} g_{PVV} (\frac{e}{g_\rho})^3 Tr[PQQ] Tr[VQ]$$

Only the strong coupling constant g_J is unknown here

We also include form factors for the virtual photons $F(q^2) = \frac{1}{1-q^2/\Lambda^2}$

Vector glueball mixing

The $\psi(2S)$ has a mass of 3.686 GeV, close to the vector glueball mass of $3.8 - 4$ GeV, so it is possible for them to mix

$$\begin{pmatrix} \psi(2S) \\ \mathcal{O}' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} c\bar{c} = \psi \\ ggg = \mathcal{O} \end{pmatrix}$$

Leads to interference term in amplitude that can give both suppression and enhancement for different channels

The vector glueball only contributes to the strong decay channel and not to any of the electromagnetic ones. We can easily compare both cases where the glueball mixes with the J/ψ or the $\psi(2S)$.

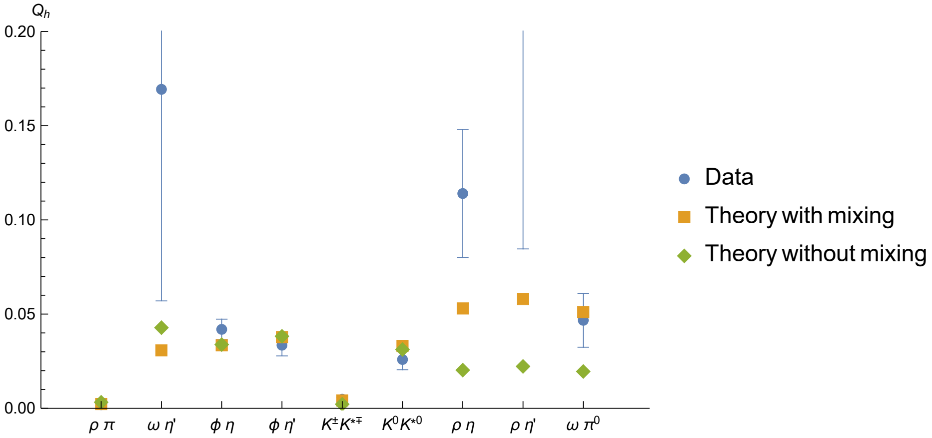
Fitting to data

From the previous diagrams we have our decays. We fit with a χ^2 method:

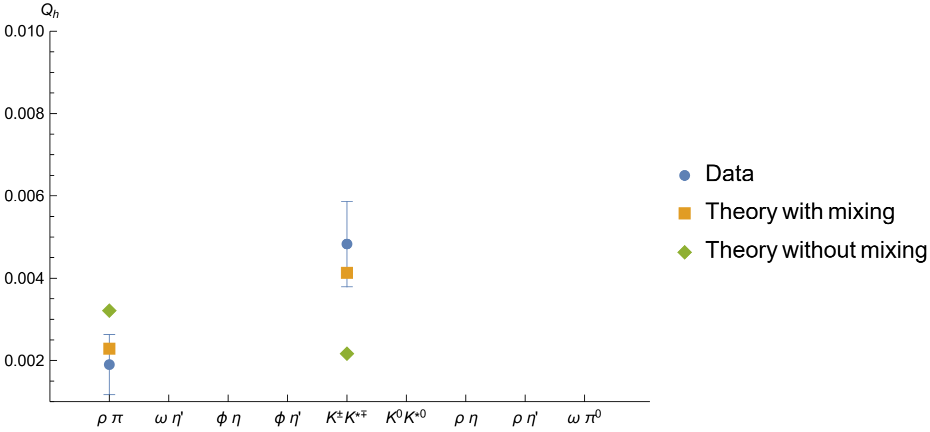
$$\chi^2 = \sum \left(\frac{1}{\Delta_i} (Q_h(gJ, gO, \Lambda) - \text{datapoint}) \right)^2$$

- 3 free parameters, strong coupling g_J, glueball coupling including mixing g_O, and momentum scale Λ in form factors.
- For no mixing: $\chi^2/d.o.f. \sim 5$
- For nonzero mixing angle: $\chi^2/d.o.f. \sim 2$
- The fit is slightly better if mixing is with $\psi(2S)$, but the difference is too small to draw conclusions

PV fit


Q_h
0.20
0.15
0.10
0.05
0.00
$\rho\ \pi$
$\omega\ \eta'$
$\phi\ \eta$
$\phi\ \eta'$
$K^{\pm}K^{*\mp}$
K^0K^{*0}
$\rho\ \eta$
$\rho\ \eta'$
$\omega\ \pi^0$
Data
Theory with mixing
Theory without mixing

PV fit zoomed in



Summary

- Glueballs are a yet undiscovered state of QCD, they are difficult to find but could show up indirectly in certain problems
- We used the extended Linear Sigma Model together with electromagnetic interactions given by vector meson dominance and a vector glueball in an attempt to resolve the $\rho\pi$ puzzle
- With few parameters, we find a reasonable agreement with experiment on the ratio between the two states. Including the vector glueball helps with this.
- This can be further studied in different decay channels where the 13% rule is broken differently or not broken at all, and we can extend it to a full description of both J/ψ and $\psi(2S)$ decays
- With a full description we can extract more information about the glueball mixing and use this to predict the glueball's decays

Backup slides

$f_J(2220)$

$f_J(2220)$ is historically seen as a good candidate for the tensor glueball

$f_J(2220)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	not seen
Γ_2	$\pi^+\pi^-$	not seen
Γ_3	$K\overline{K}$	not seen
Γ_4	$p\overline{p}$	not seen
Γ_5	$\gamma\gamma$	not seen
Γ_6	$\eta\eta'(958)$	seen
Γ_7	$\phi\phi$	not seen
Γ_8	$\eta\eta$	not seen

- Only $\eta\eta\prime$ is seen, but we find it is $\sim 10^{-3}$ times $\pi\pi$ mode.
- PDG lists decay ratio $\pi\pi/\bar{K}K = 1.0 \pm 0.5$, we find $\pi\pi/\bar{K}K \sim 2.5$

Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \longrightarrow P^{(1)} P^{(2)}} = \frac{\kappa_{gpp,i}\, \lambda^2\, |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60\, \pi\, m_{G_2}^2} \, ;$$

- while for two vector mesons

$$\Gamma_{G_2 \to V^{(1)} V^{(2)}} = \frac{\kappa_{gvv,i} \lambda^2 |\vec{k}_{v^{(1)},v^{(2)}}|}{120\, \pi\, m_{G_2}^2} \Big(15 + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(1)}}^2} + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(2)}}^2} + \frac{2|\vec{k}_{v^{(1)},v^{(2)}}|^4}{m_{v^{(1)}}^2\, m_{v^{(2)}}^2} \Big) \, ;$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \longrightarrow A_1 P} = \frac{\kappa_{gap,i}\, \lambda^2\, |\vec{k}_{a_1,p}|^3}{120\, \pi\, m_{G_2}^2} \big(5 + \frac{2\, |\vec{k}_{a_1,p}|^2}{m_{a_1}^2} \big) \, .$$