Andreev reflection and Josephson current at hadron-quark interface

-A field theoretical approach to transport phenomena in dense matter-

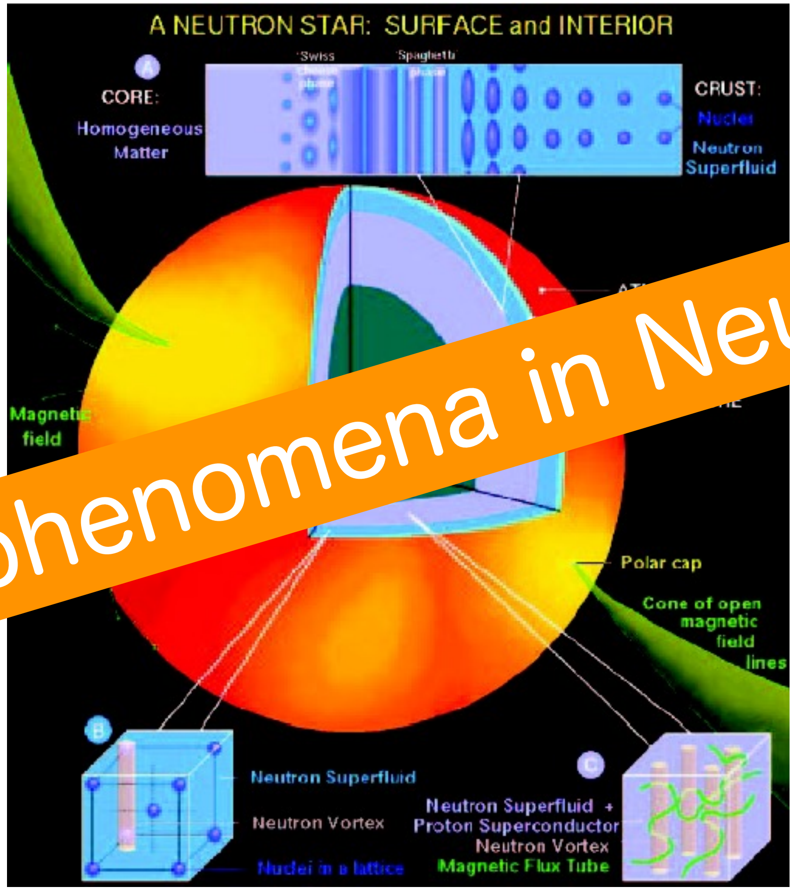
A series of works with
Mariusz Sadzikowski (Jagiellonian U.)
Yuki Juzaki (Saga U.)
Tingyu Zhang (Tokyo U.)
Hiroyuki Tajima (Tokyo U.)
Motoi Tachibana (Saga U.)

2506.09725 (latest result)

June 15, 2025
65th Cracow School
“Fundamental interactions”
@Zakopane, Poland

Highlight of this talk

-Physics of interface-



Transport phenomena in Neutron Stars

Memories of Zakopane and Krakow

(From right)

Sadzikowski

Reddy

Gupta

Me

(From right)

Ruiz Arriola

Broniowski

Koch

Sadzikowski

Me

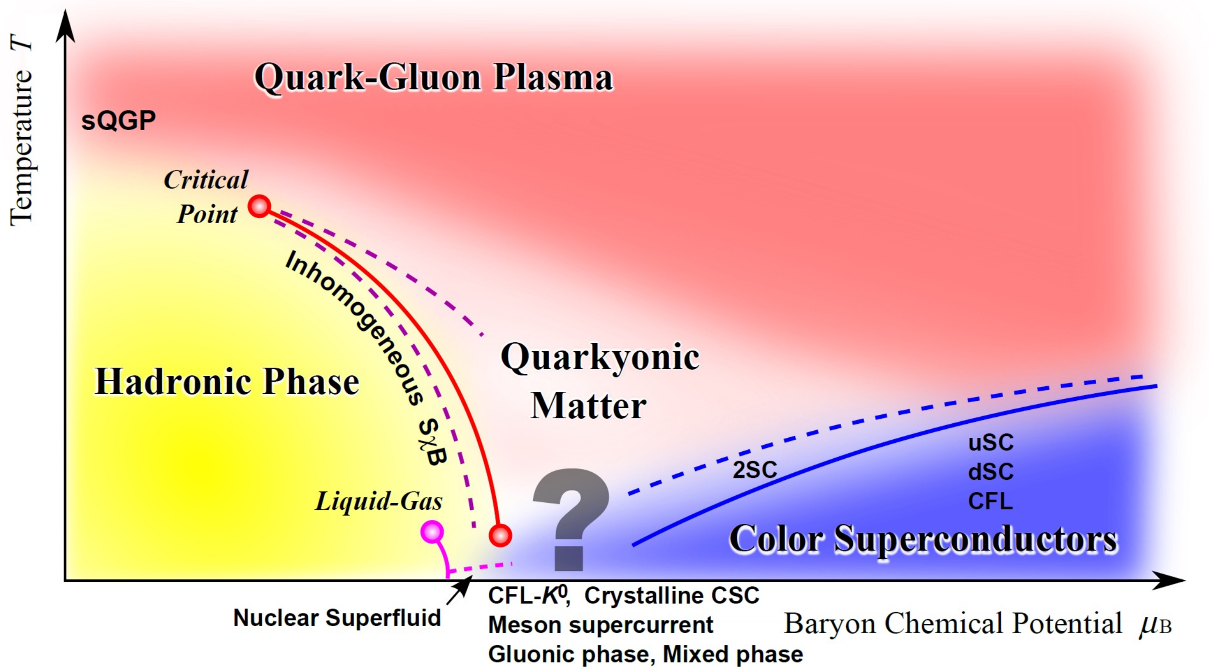
@Jagiellonian U.

(old campus)

- Introduction 4pages
- Neutron star as cipolla 4pages
- Color superconductivity 5pages
- Quark-hadron continuity 6pages
- Physics of interface 20pages

Introduction

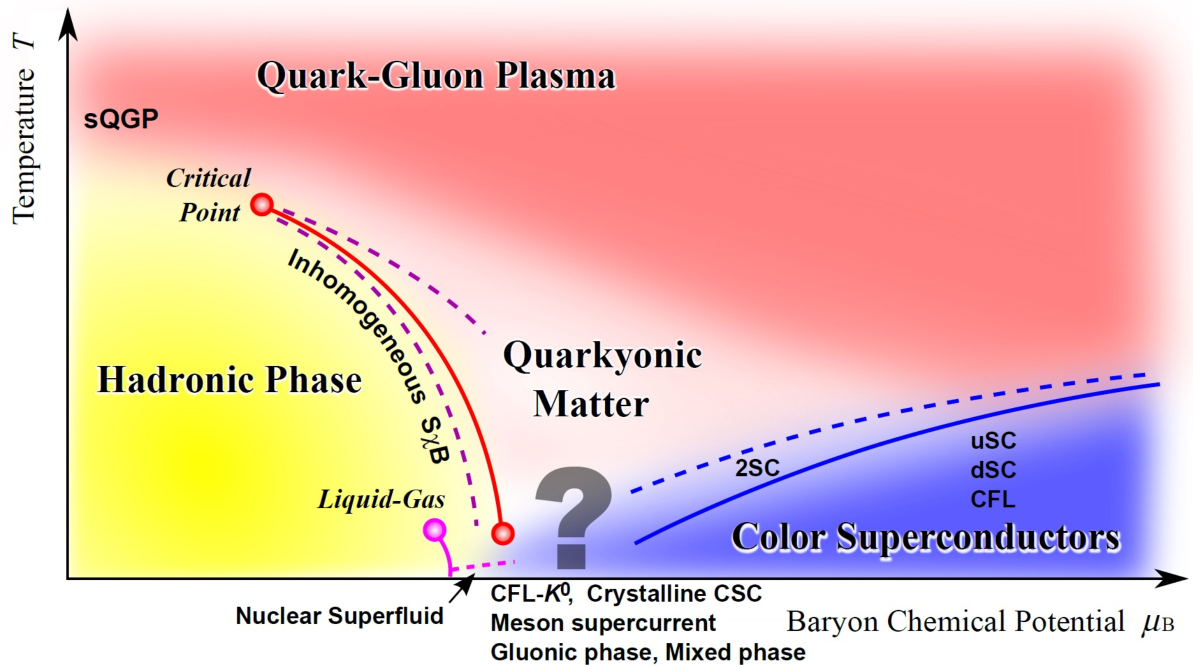
-A Big Challenge-
Exploring QCD phase diagram at nonzero T & μ



A schematic phase diagram of QCD
(taken from Kenji and Tetsuo's paper)

- ✅ Big bang, Little bang and Compact stars
- ✅ Condensed matter

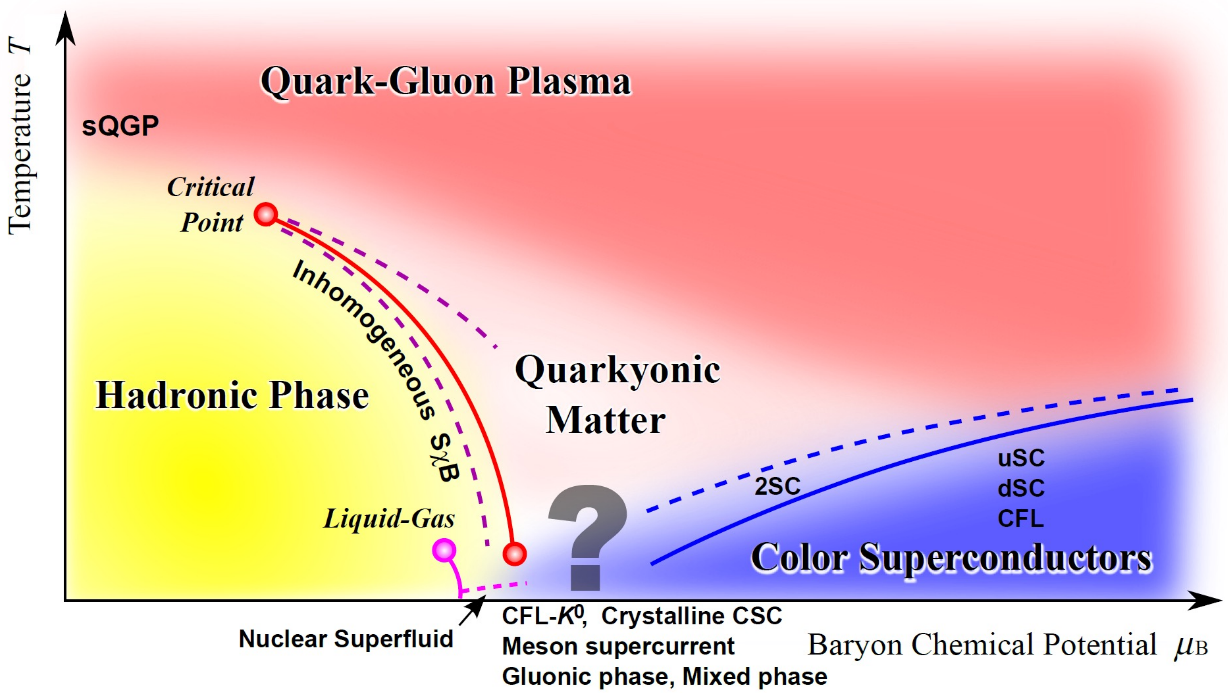
-Terra firma, Terra incognita-
Why finite density QCD is hard



✅ Sign problem

✅ Strong coupling

-A unified description of QCD matter-
Eventually lack of understanding confinement?



✅ Nuclear matter

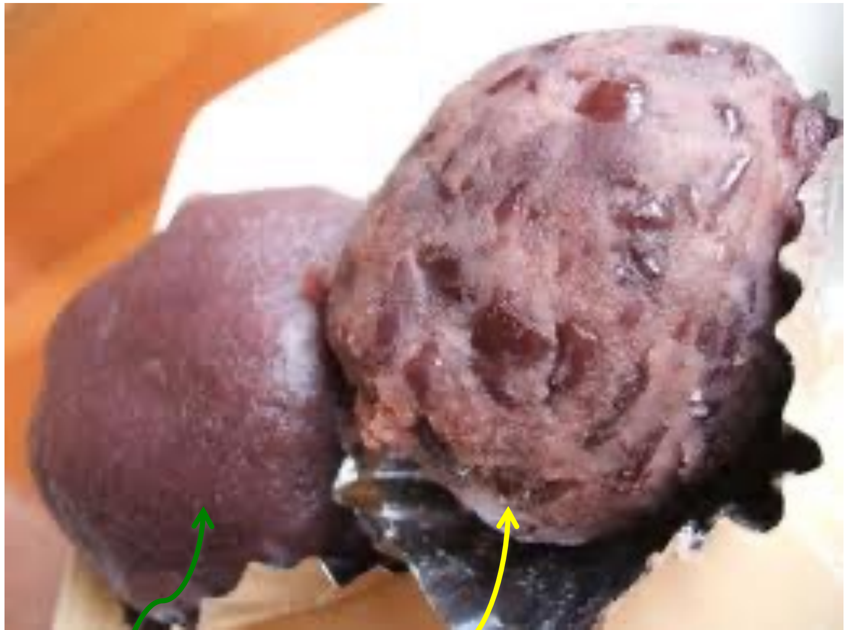
Low density
Quarks confined into hadrons

✅ Quark matter

High density
Quarks liberated from hadrons

(Japanese) nucleons, nuclear matter and quark matter

Nucleons
(AZUKI=Sweet beans)



Quark matter
(KOSHI-AN)

Nuclear matter
(TSUBU-AN)

Neutron Star as cipolla

-Physics with spatial multi-scale in 10km-

Mass-radius (M-R) relation

Fruitful phase structure of QCD can be discussed from neutron star properties

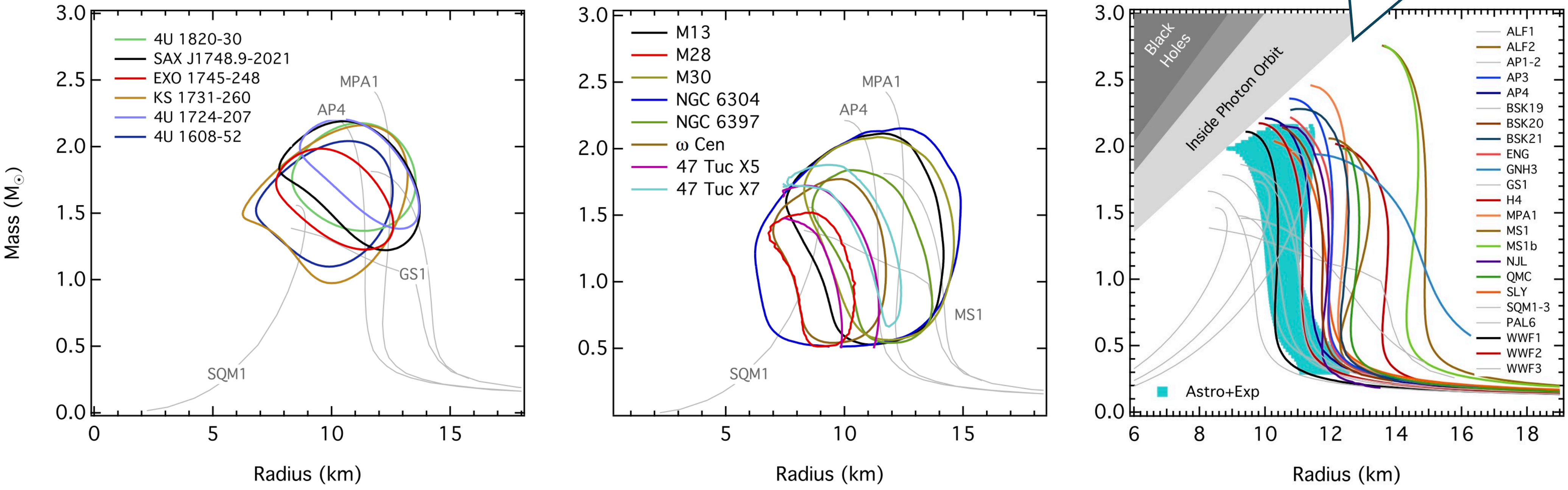
Neutron star mass and radius

For example,

F. Ozel, D. Psaltis, T. Guver, G. Baym, C. Heinke, S. Guillot, APJ 820 (2016) 28

http://xtreme.as.arizona.edu/NeutronStars/

Some EoSs are rejected

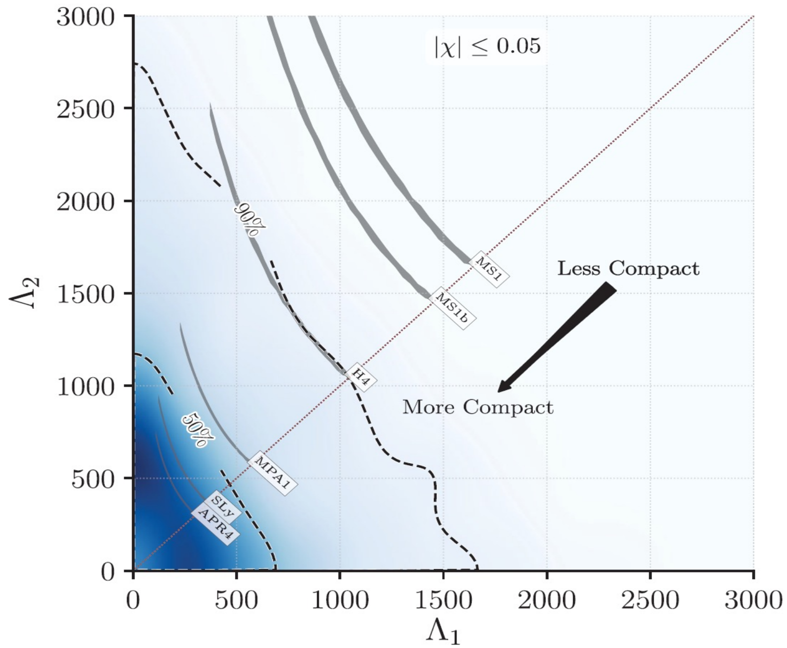


EoS has one to one correspondence with the neutron star M-R relation (via TOV equation)

GW from tidal force

There are several constraints
coming from neutron star observations

Restriction: **Gravitational wave signal from binary neutron star merger**



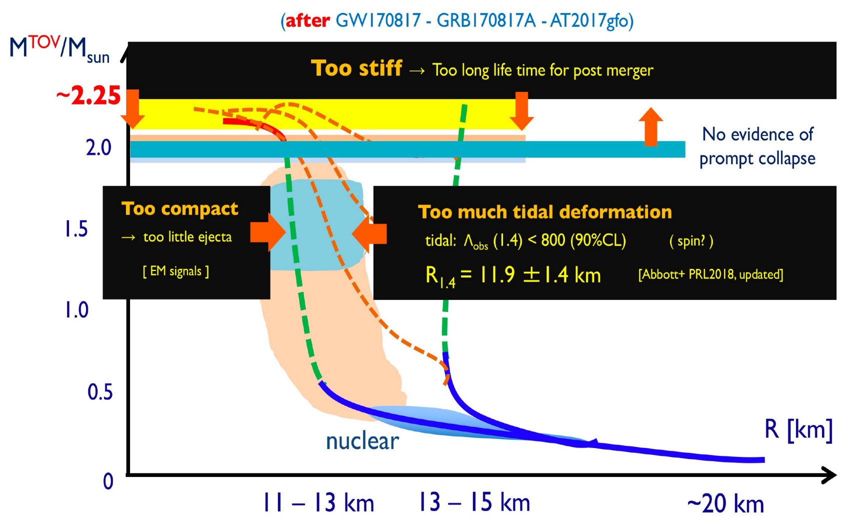
Abotto et al.(LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119 (2017) 161101

Maximal mass of neutron star

$\sim 2.15\text{–}2.25\ M_{\odot}$

M. Shibata, S. Fujibayashi, K. Hotokezaka, K. Kiuchi, K. Kyutoku, Y. Sekiguchi, and M. Tanaka, Phys. Rev. D 96 (2017)123012

Taken from **T. Kojo, arXiv:1904.05080**



An example : quarkyonic matter

L. McLerran, R. Pisarski, NPA 796 (2007) 83

High density confined matter

Quarkyonic = Quark + Baryonic

Excitations are mesonic and baryonic

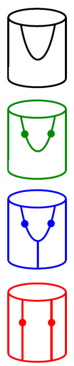
There are some attempts to investigate the quarkyonic matter from **holographic models**

Quarks and baryons must be considered at the same time

They consider the deconfined background

N. Kovensky, A. Schmitt, JHEP 09 (2020) 112

Top-down (Sakai-Sugimoto model)



mesonic

baryonic

quarkyonic (LTQy)

quarkyonic (HTQy)

Contributions from string sources
(It induces the **baryon number density** created by **quarks**)

$$S = S_{\mathrm{DBI}} + S_b + S_s + S_m$$

Dirac-Born-Infeld action

Quark mass contributions

Contributions from point-like baryons
(It induces the **baryon number density** created by **baryons**)

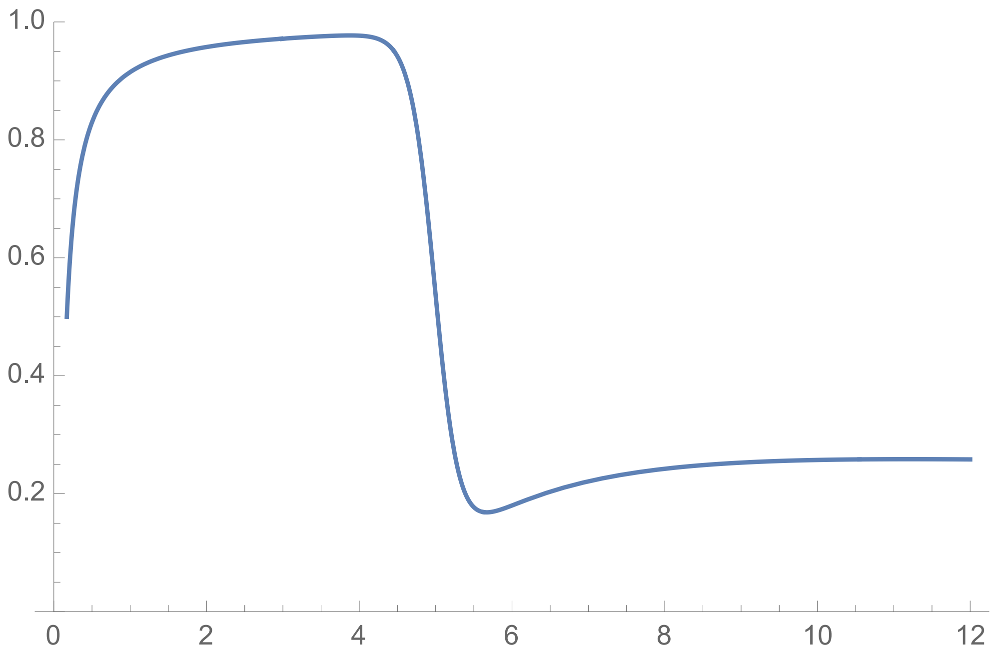
In some works, authors only investigated the chiral symmetric confined matter, but it is not enough to understand the quarkyonic matter.

-Holography as a solution-

Free from sign problem

Strongly coupled system

Unified description



PRD100(2019)106011,PRD104(2021)126002
PRD107(2023)06364

- ☐ Holographic EoS for nuclear and quark matter
- ☐ Mass-radius(M-R) curve
- ☐ C_s and trace anomaly

Quick tour of color superconductivity

(not general, fairly my personal point of view)

Superconducting Quark Matter (QM)

Barrois (77), Frautschi (77)
Bailin-Love (84), Iwasaki-Iwado (95)
Alford-Rajagopal-Wilczek, (98)
Rapp-Schafer-Shuryak-Velkovsky (98)

Quark Cooper paring (diquark)

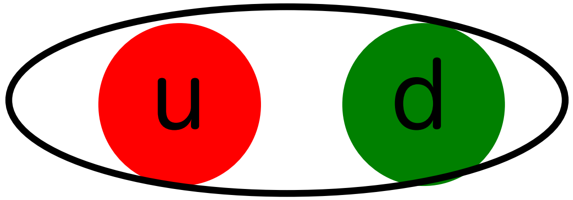
2-flavor color superconductivity [2SC]

$$\left\langle q_{\alpha i} C \gamma_5 q_{\beta j} \right\rangle \propto \Delta_{2SC} \varepsilon^{\alpha\beta 3} \varepsilon^{ij}$$

Color flavor locking [CFL]

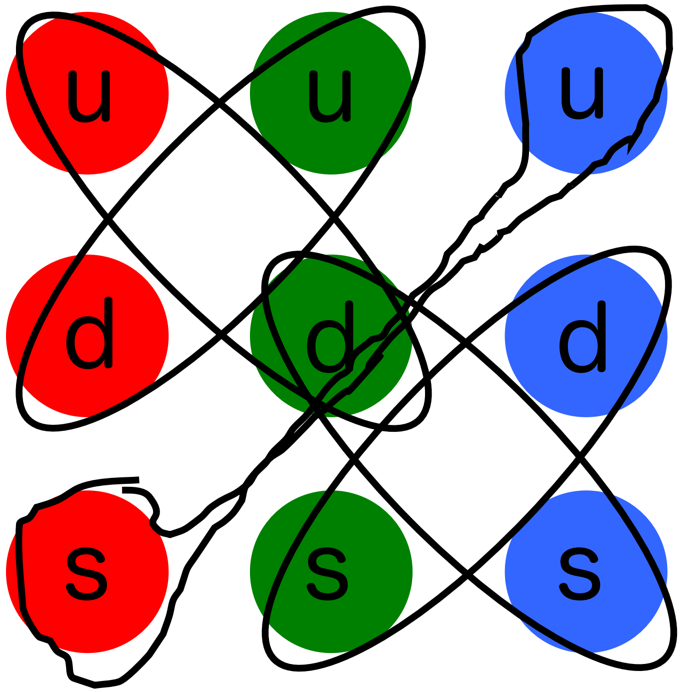
$$\left\langle q_{\alpha i} C \gamma_5 q_{\beta j} \right\rangle = \Delta_{CFL} \varepsilon_{\alpha\beta I} \varepsilon^{ijI} \propto \Delta_{CFL} \left(\delta^i_\alpha \delta^j_\beta - \delta^j_\alpha \delta^i_\beta \right)$$

2SC



strange quark,
blue quarks
unpaired

CFL



All the quarks
are paired
↓
the most sym.
ground state

2SC : $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_B \rightarrow SU(2)_c \times SU(2)_L \times SU(2)_R \times U(1)_{\bar{B}}$

Chiral ○ Baryon ○

Strange and/or blue quarks are dominant!
(other quarks are Boltzmann-suppressed)

CFL : $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2$

Chiral × Baryon ×

Nambu-Goldstone bosons are dominant!
(all quarks are Boltzmann-suppressed)

CFL effective Lagrangian

Casalbuoni-Gatto(1999), Hong-Rho-Zahed (1999)
Son-Stephanov (2000), Bedaque-Schafer (2002)

$$L_{eff} = \frac{f_\pi^2}{4}\left[Tr\nabla_0\Sigma\nabla_0\Sigma^* - v^2 Tr\vec{\nabla}\Sigma\cdot\vec{\nabla}\Sigma^*\right]$$

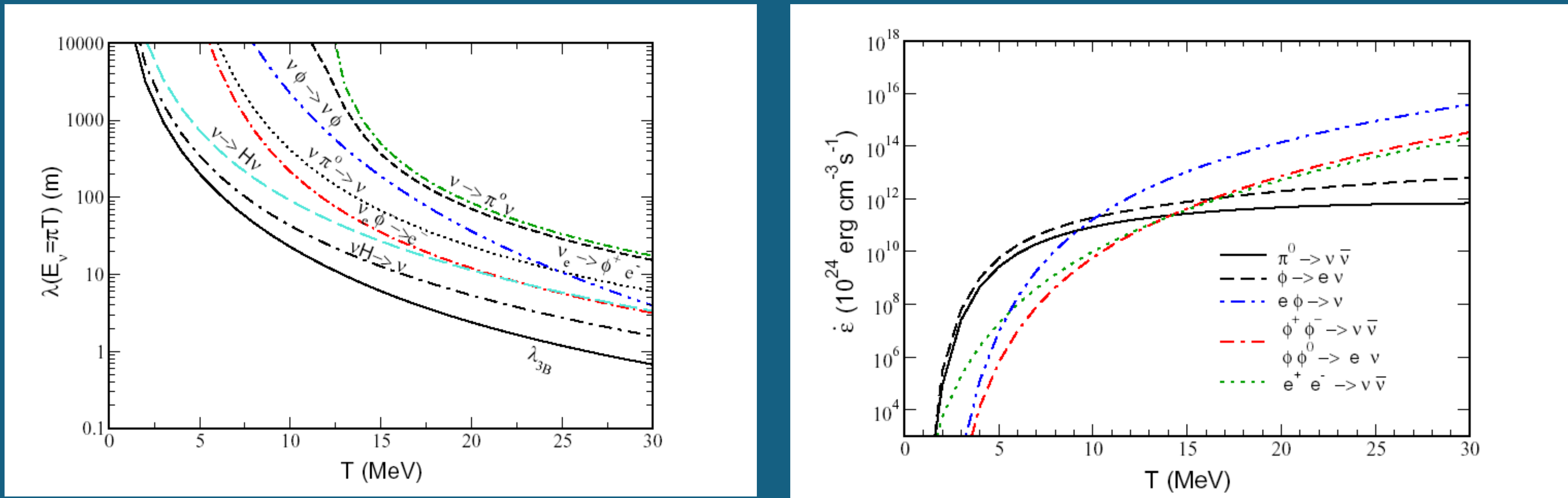
$$+f_\pi^2\left[\frac{a}{2}Tr\tilde{M}(\Sigma+\Sigma^*)+\frac{\chi}{2}TrM(\Sigma+\Sigma^*)\right]$$

$$\Sigma \equiv \exp(2i\Pi / f_\pi)$$ (Π : NG bosons)

ν rates in CFL quark matter

Jaikumar-Prakash-Schafer (2002)

Reddy-Sadzikowski-Tachibana (2002)



Quark-hadron continuity

-A new critical point in dense QCD-

Quark-hadron continuity

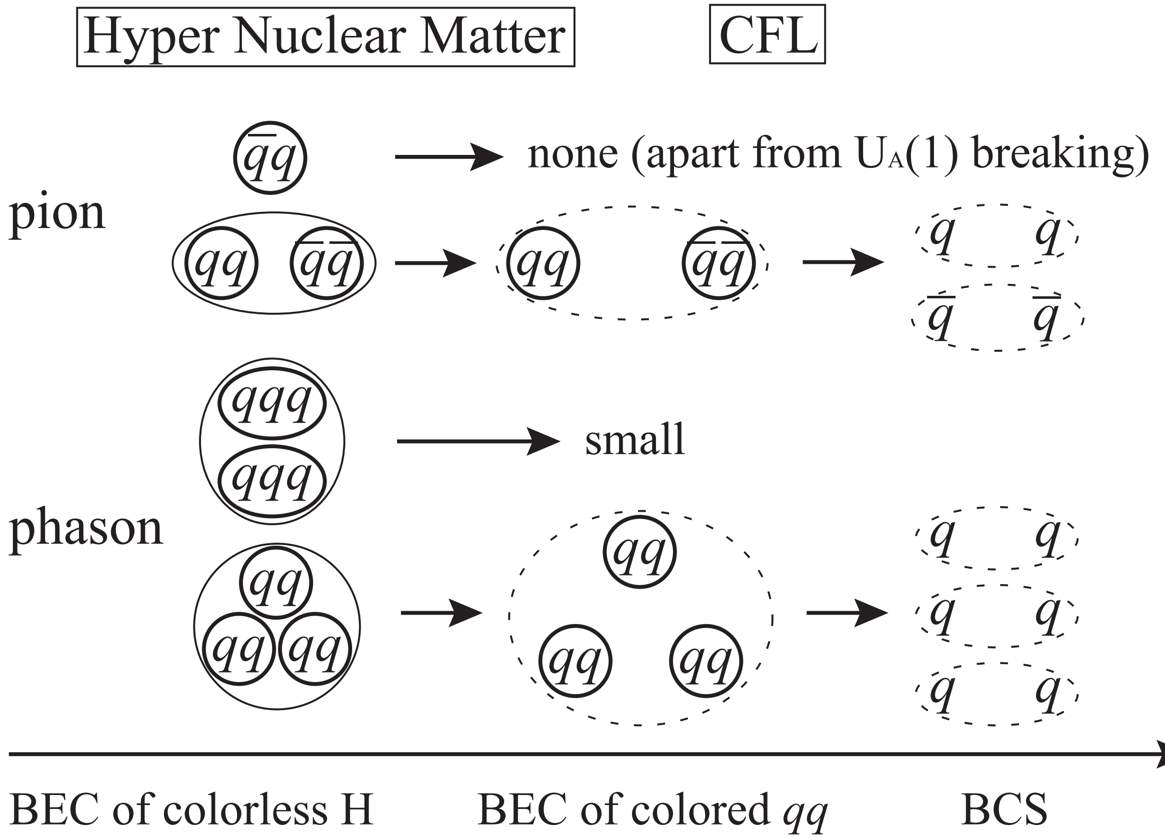
T. Schaefer, F. Wilczek (2000)

Phase	Hadronic (confinement)	Color-flavor locked(Higgs)
Symmetry breaking Pattern	$SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{L+R}$	$SU(3)_L \times SU(3)_R \times SU(3)_C \times U(1)_B \rightarrow SU(3)_{L+R+C}$
Order parameter	chiral condensate	diquark condensate
$U(1)_B$	broken in the dibaryon channels	broken by d
Elementary Excitations	Pseudo-scalar mesons (**π** etc)	NG bosons
	vector mesons (**ρ** etc)	massive gluons
	baryons	massive quarks (CFL gap)

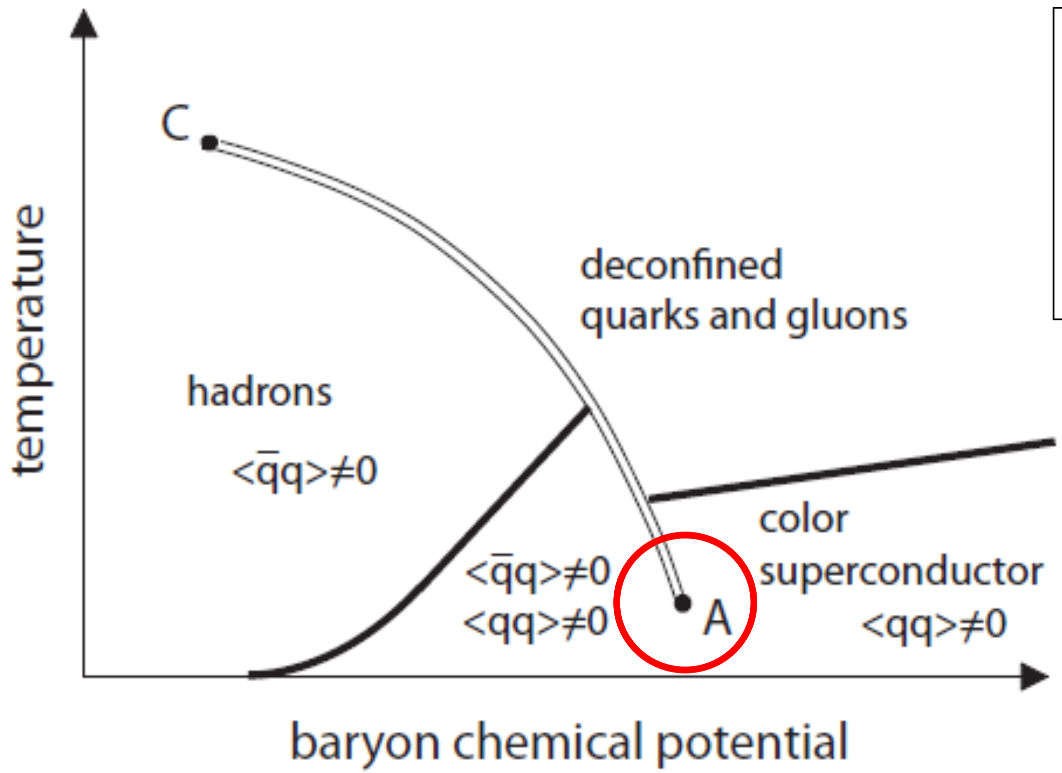
A realization of Fradkin-Shenker complementarity

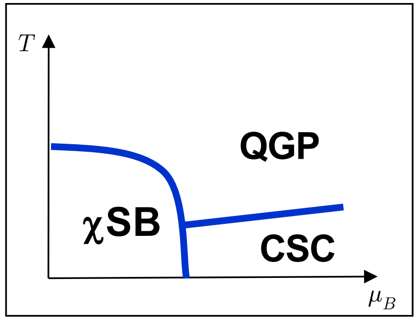
Structural change of dense strongly-interacting matter

K. Fukushima (2004)



“Anomaly driven critical point in high density QCD”





Yamamoto, Hatsuda, Baym & Tachibana (2006)

PRL 97, 122001 (2006) PHYSICAL REVIEW LETTERS week ending 22 SEPTEMBER 2006

New Critical Point Induced By the Axial Anomaly in Dense QCD

Tetsuo Hatsuda,[1] Motoi Tachibana,[2] Naoki Yamamoto,[1] and Gordon Baym[3]
[1]*Department of Physics, University of Tokyo, Japan*
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[3]*Department of Physics, University of Illinois, 1110 W. Green St., Urbana, Illinois 61801, USA*
(Received 10 May 2006; published 18 September 2006)

We study the interplay between chiral and diquark condensates within the framework of the Ginzburg-Landau free energy, and classify possible phase structures of two and three-flavor massless QCD. The QCD axial anomaly acts as an external field applied to the chiral condensate in a color superconductor and leads to a crossover between the broken chiral symmetry and the color superconducting phase, and, in particular, to a new critical point in the QCD phase diagram.

DOI: 10.1103/PhysRevLett.97.122001 PACS numbers: 12.38.−t, 26.60.+c

PRL 99, 130406 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

Superfluidity and Magnetism in Multicomponent Ultracold Fermions

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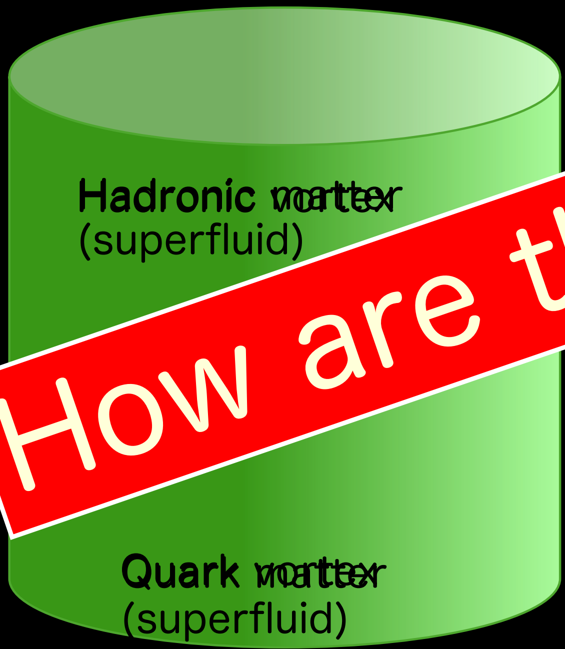
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
(Received 2 May 2007; published 28 September 2007)

We study the interplay between superfluidity and magnetism in a multicomponent gas of ultracold fermions. Ward-Takahashi identities constrain possible mean-field states describing order parameters for both pairing and magnetization. The structure of global phase diagrams arises from competition among these states as functions of anisotropies in chemical potential, density, or interactions. They exhibit first and second order phase transition as well as multicritical points, metastability regions, and phase separation. We comment on experimental signatures in ultracold atoms.

DOI: 10.1103/PhysRevLett.99.130406 PACS numbers: 05.30.Jp, 03.75.Mn, 03.75.Ss

Thought experiment



Pour quarks
into

adronic matter
ower: Quark matter

Then, rotate the bucket

Upper: Hadronic vortex
Lower: Quark vortex

How are they connected?

PHYSICAL REVIEW D **99,** 036004 (2019)

Continuity of vortices from the hadronic to the color-flavor locked phase in dense matter

Mark G. Alford,[1] Gordon Baym,[2,3] Kenji Fukushima,[4] Tetsuo Hatsuda,[3,5] and Motoi Tachibana[6]

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[3]*iTHES Research Group and iTHEMS Program, RIKEN, Wako, Saitama 351-0198, Japan*
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(Received 21 March 2018; published 7 February 2019)

We study how vortices in dense superfluid hadronic matter can connect to vortices in superfluid quark matter, as in rotating neutron stars, focusing on the extent to which quark-hadron continuity can be maintained. As we show, a singly quantized vortex in three-flavor symmetric hadronic matter can connect smoothly to a singly quantized non-Abelian vortex in three-flavor symmetric quark matter in the color-flavor locked phase, without the necessity for boojums appearing at the transition.

Physics of interface

-Andreev reflection & Josephson current in QCD-

Andreev reflection in QCD

BCS Hamiltonian

$$H = \int d^3x \Bigg[\sum_{\alpha=\uparrow,\downarrow} \psi^\dagger_\alpha(t,\vec{r}) \left(-\frac{\nabla^2}{2m} - E_\mathrm{F} \right) \psi_\alpha(t,\vec{r}) + \Delta(\vec{r}) \psi^\dagger_\uparrow(t,\vec{r}) \psi^\dagger_\downarrow(t,\vec{r}) + \Delta^*(\vec{r}) \psi_\downarrow(t,\vec{r}) \psi_\uparrow(t,\vec{r}) \Bigg]$$

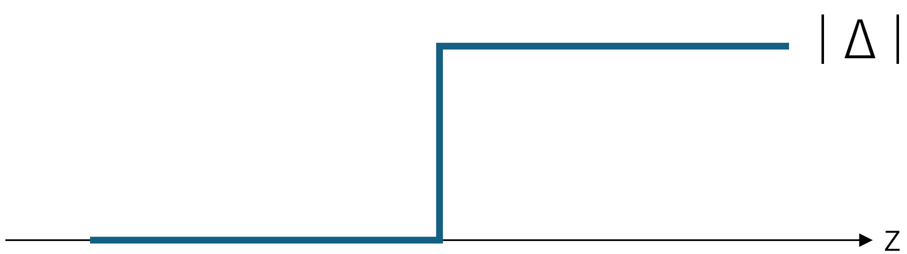
Bogoliubov –de Gennes (B-dG) equation

$$i\partial_t \begin{pmatrix} \psi_\uparrow(t,\vec{r}) \\ \psi^\dagger_\downarrow(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} -\frac{\nabla^2}{2m} - E_\mathrm{F} & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & \frac{\nabla^2}{2m} + E_\mathrm{F} \end{pmatrix} \begin{pmatrix} \psi_\uparrow(t,\vec{r}) \\ \psi^\dagger_\downarrow(t,\vec{r}) \end{pmatrix}$$

Andreev reflection in condensed matter system

Andreev, Zh. Eksp. Teor. Fiz. 46 (19… …23

An incident electron from normal metal hits the NS interface.Then what happens?



B-dG eq. (Δ=0)

$$\begin{aligned} i\dot{\psi}_\uparrow(t,\vec{r}) &= \left(-\frac{\nabla^2}{2m} - E_\mathrm{F}\right)\psi_\uparrow(t,\vec{r}) \\ i\dot{\psi}^*_\downarrow(t,\vec{r}) &= \left(\frac{\nabla^2}{2m} + E_\mathrm{F}\right)\psi^*_\downarrow(t,\vec{r})\,. \end{aligned}$$

$$\phi \equiv \begin{pmatrix} \psi_\uparrow(t,\vec{r}) \\ \psi^*_\downarrow(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} f\exp(-iEt + i\vec{q}\vec{r}) \\ g\exp(-iEt + i\vec{q}\vec{r}) \end{pmatrix}$$

$$\begin{aligned} E &= \varepsilon_q && \text{for particles} \\ E &= -\varepsilon_q && \text{for holes,} \end{aligned}$$

B-dG eq. (Δ=Const.)

$$\begin{aligned} \phi &= D\begin{pmatrix} \sqrt{\frac{1}{2}(1+\xi/E)}\exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1-\xi/E)}\exp(-i\delta/2) \end{pmatrix}\exp(i\vec{q}_+\vec{r} - iEt) \\ &\quad + F\begin{pmatrix} \sqrt{\frac{1}{2}(1-\xi/E)}\exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1+\xi/E)}\exp(-i\delta/2) \end{pmatrix}\exp(i\vec{q}_-\vec{r} - iEt). \end{aligned}$$

$$\xi = \sqrt{E^2 - |\Delta|^2}$$

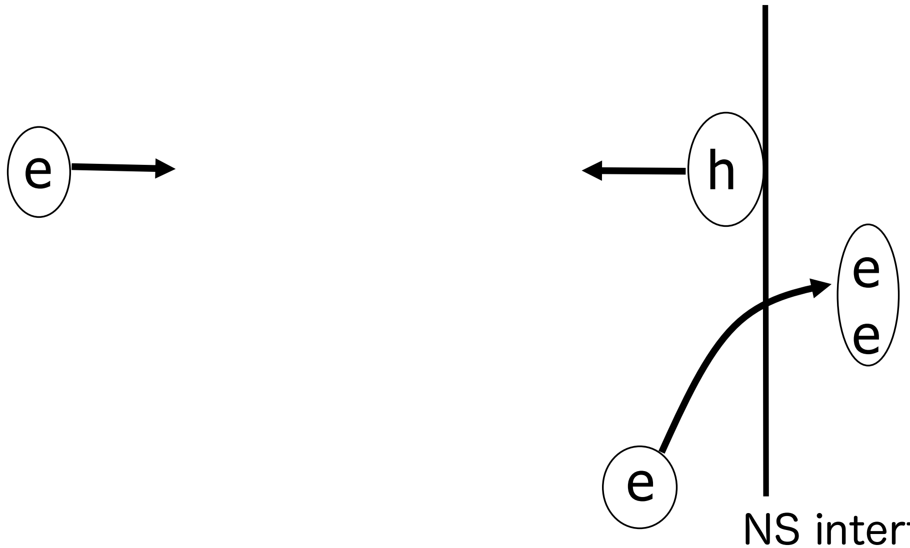
$$\frac{\vec{q}^{\,2}_\pm}{2m} = E_\mathrm{F} \pm \xi$$

Normal

$$\phi_< = \begin{pmatrix} \exp(ikz) + B\exp(-ikz) \\ C\exp(ipz) \end{pmatrix}$$

Super

$$\phi = D\begin{pmatrix} \sqrt{\frac{1}{2}(1+\xi/E)}\exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1-\xi/E)}\exp(-i\delta/2) \end{pmatrix}\exp(i\vec{q}_+\vec{r} - iEt) + F\begin{pmatrix} \sqrt{\frac{1}{2}(1-\xi/E)}\exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1+\xi/E)}\exp(-i\delta/2) \end{pmatrix}\exp(i\vec{q}_-\vec{r} - iEt).$$



$$B = O\left(\frac{1}{E_{\mathrm{F}}}\right)$$

suppresed

$$C = \sqrt{\frac{E-\xi}{E+\xi}} + O\left(\frac{1}{E_{\mathrm{F}}}\right)$$

dominant

$$D = \sqrt{\frac{2E}{E+\xi}} + O\left(\frac{1}{E_{\mathrm{F}}}\right)$$

dominant

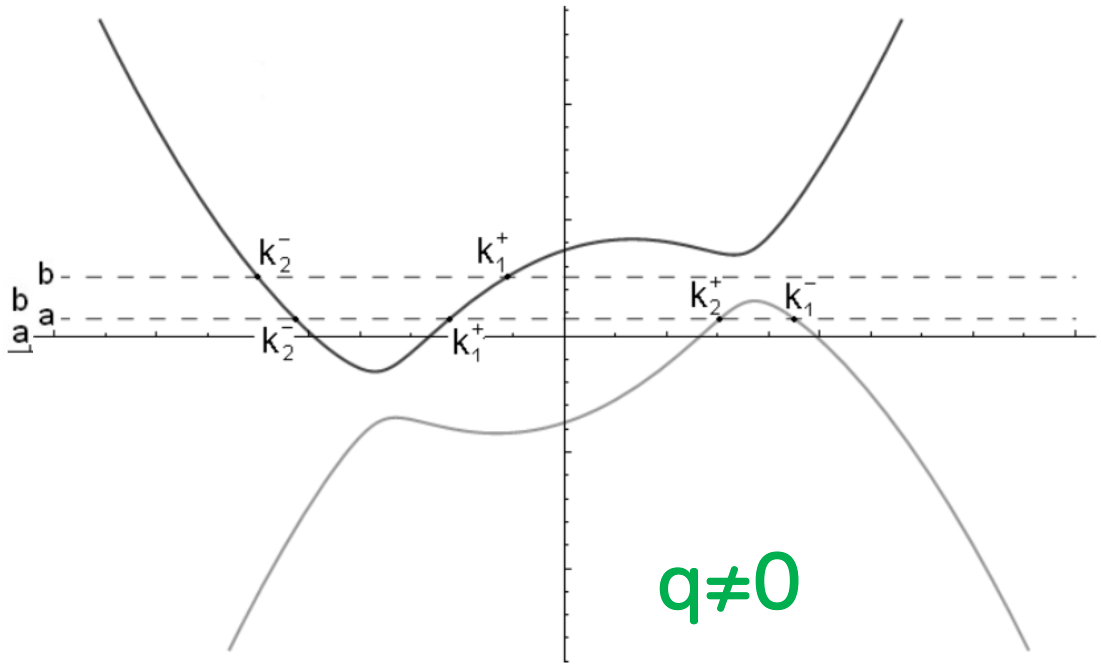
$$F = O\left(\frac{1}{E_{\mathrm{F}}}\right)$$

suppresed

Andreev reflection in inhomogeneous superconductor

T. Partyka, M. Sadzikowski, M.T. (2009)

$$\Delta(z) = \Delta_0 e^{2iqz}$$



$$E_{\pm} = \frac{1}{2}\left(\epsilon_{k+q} - \epsilon_{k-q} \pm \sqrt{(\epsilon_{k-q} + \epsilon_{k+q})^2 + 4|\Delta_0|^2}\right)$$

Andreev reflection in QCD

M. Sadzikowski, M.T. (2002, 2003)

$$H = \int d^3x \left[\sum_{a,i} \psi_a^{i\,\dagger}(-i\vec{\alpha}\cdot\vec{\nabla} - \mu)\psi_a^i + \sum_{a,b,i,j} \Delta_{ab}^{ij\,*}(\psi_a^{i\,\mathrm{T}} C\gamma_5 \psi_b^j) + \mathrm{h.c.} \right] \qquad \left\langle \psi_a^{i\,\mathrm{T}} C\gamma_5 \psi_b^j \right\rangle = \Delta_{ab}^{ij}$$

$$(u_{\mathrm{red}}, d_{\mathrm{green}}, s_{\mathrm{blue}}, d_{\mathrm{red}}, u_{\mathrm{green}}, s_{\mathrm{red}}, u_{\mathrm{blue}}, s_{\mathrm{green}}, d_{\mathrm{blue}})$$

$$\Delta_{ij}^{ab} = \begin{pmatrix} 0 & \Delta_{ud} & \Delta_{us} & & & & & & \\ \Delta_{ud} & 0 & \Delta_{ds} & & & & & & \\ \Delta_{us} & \Delta_{ds} & 0 & & & & & & \\ & & & 0 & -\Delta_{ud} & & & & \\ & & & -\Delta_{ud} & 0 & & & & \\ & & & & & 0 & -\Delta_{us} & & \\ & & & & & -\Delta_{us} & 0 & & \\ & & & & & & & 0 & -\Delta_{ds} \\ & & & & & & & -\Delta_{ds} & 0 \end{pmatrix}$$

$$\begin{aligned} \Delta_{us} &= \Delta_{ds} = 0,\ \Delta_{ud} = \tilde{\Delta}, && \text{for 2SC} \\ \Delta_{us} &= \Delta_{ds} = \Delta_{ud} = \Delta, && \text{for CFL} \end{aligned}$$

Probability current

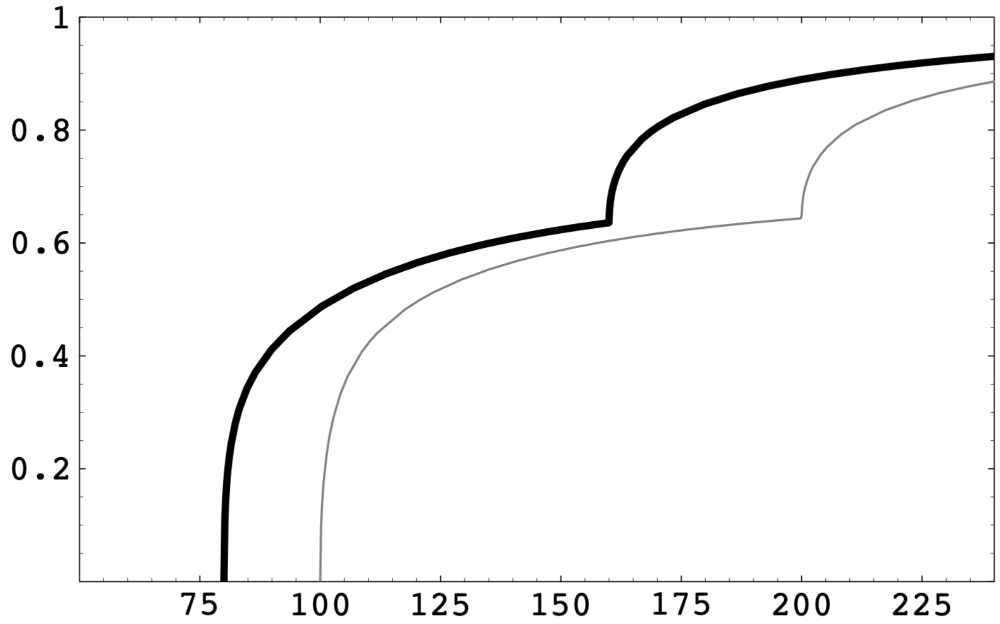


Fig. 1. Dependence of the probability current (35) as a function of energy E [MeV] for two sets of parameters: $\Delta = 80$ MeV, $\tilde{\Delta} = 60$ MeV (black curve) and $\Delta = 100$ MeV, $\tilde{\Delta} = 60$ MeV (gray curve).

Comment by Igor Shovkovy

Why don’t you think about the interface between hadronic and quark matter?

Andreev reflection at Hadron/Color superconductor

Y. Juzaki, M.T. (2020)

Q. How to incorporate the confinement of colors?

1. Quarks inside hadrons are free (constituent picture)

2. They are always in the color-singlet states

3. Each quark inside the hadron is Andreev-reflected and the only color-singlet combination is possible out of the reflected quarks

Andreev reflection of quarks.

Incident quark	Reflected hole
u_{R}	$d^{\mathrm{H}}_{\mathrm{G}}$ or $s^{\mathrm{H}}_{\mathrm{B}}$
u_{G}	$d^{\mathrm{H}}_{\mathrm{R}}$
u_{B}	$s^{\mathrm{H}}_{\mathrm{R}}$
d_{R}	$u^{\mathrm{H}}_{\mathrm{G}}$
d_{G}	$u^{\mathrm{H}}_{\mathrm{R}}$ or $s^{\mathrm{H}}_{\mathrm{B}}$
d_{B}	$s^{\mathrm{H}}_{\mathrm{G}}$
s_{R}	$u^{\mathrm{H}}_{\mathrm{B}}$
s_{G}	$d^{\mathrm{H}}_{\mathrm{B}}$
s_{B}	$u^{\mathrm{H}}_{\mathrm{R}}$ or $d^{\mathrm{H}}_{\mathrm{G}}$

Define "meson"

$$M_a \equiv i_a \bar{j}_a \,, \qquad \text{(no sum)}$$

Ex. pion

$$\boxed{\begin{aligned} \pi^+_{\mathrm{R}} &= u_{\mathrm{R}}\bar{d}_{\mathrm{R}} \to (d_{\mathrm{G}}\bar{u}_{\mathrm{G}})^{\mathrm{H}} \equiv (\pi^-_{\mathrm{G}})^{\mathrm{H}} \\ \pi^+_{\mathrm{G}} &= u_{\mathrm{G}}\bar{d}_{\mathrm{G}} \to (d_{\mathrm{R}}\bar{u}_{\mathrm{R}})^{\mathrm{H}} \equiv (\pi^-_{\mathrm{R}})^{\mathrm{H}} \end{aligned}}$$

$\pi^+_{\mathrm{B}} \to \pi^+_{\mathrm{B}}$ w.o/ s quark

$\pi^+_{\mathrm{B}} \to$ None w/ s quark

Andreev reflection of K mesons.

Incident particle	Incident (color)	Reflected (color)	Reflected particle
K^+	K_{R}^+	$(K_{\mathrm{B}}^-)^{\mathrm{H}}$	K^+
	K_{G}^+	$\times$	
	K_{B}^+	$(K_{\mathrm{R}}^-)^{\mathrm{H}}$	
K^-	K_{R}^-	$(K_{\mathrm{B}}^+)^{\mathrm{H}}$	K^-
	K_{G}^-	$\times$	
	K_{B}^-	$(K_{\mathrm{R}}^+)^{\mathrm{H}}$	
K^0	K_{R}^0	$\times$	K^0
	K_{G}^0	$(\bar{K}_{\mathrm{B}}^0)^{\mathrm{H}}$	
	K_{B}^0	$(\bar{K}_{\mathrm{G}}^0)^{\mathrm{H}}$	
$\bar{K}^0$	$\bar{K}_{\mathrm{R}}^0$	$\times$	$\bar{K}^0$
	$\bar{K}_{\mathrm{G}}^0$	$(K_{\mathrm{B}}^0)^{\mathrm{H}}$	
	$\bar{K}_{\mathrm{B}}^0$	$(K_{\mathrm{G}}^0)^{\mathrm{H}}$	

Baryons with two different flavors.

Incident particle	Constitution	Reflected particles
p	uud	$p\,,\,(\Xi^-)^{\mathrm{H}}$
n	udd	$n\,,\,(\Xi^0)^{\mathrm{H}}$
Δ^+	uud	$\Delta^+\,,\,(\Xi^-)^{\mathrm{H}}$
Δ^0	udd	$\Delta^0\,,\,(\Xi^0)^{\mathrm{H}}$
Σ^+	uus	$\Sigma^+\,,\,(\Sigma^-)^{\mathrm{H}}$
Σ^-	dds	$\Sigma^-\,,\,(\Sigma^+)^{\mathrm{H}}$
Ξ^0	uss	$\Xi^0\,,\,(\Delta^0)^{\mathrm{H}}$ or $(n)^{\mathrm{H}}$
Ξ^-	dss	$\Xi^-\,,\,(\Delta^+)^{\mathrm{H}}$ or $(p)^{\mathrm{H}}$

Baryons with three different flavors.

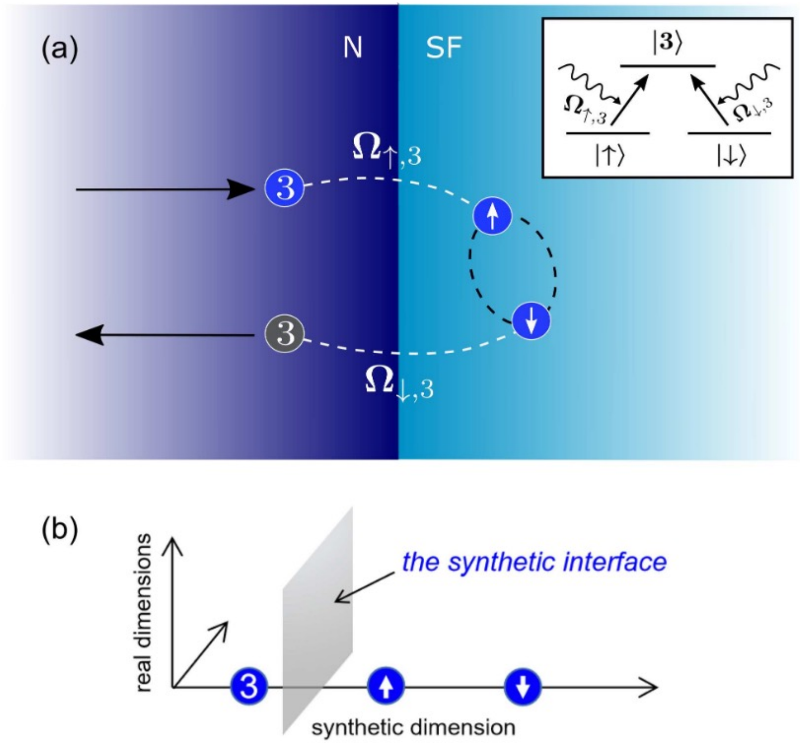
Incident particle	Constitution	Reflected particles
Λ	uds	$\Lambda\,,\,(\Lambda)^{\mathrm{H}}$
Σ^0	uds	$\Sigma^0\,,\,(\Sigma^0)^{\mathrm{H}}$

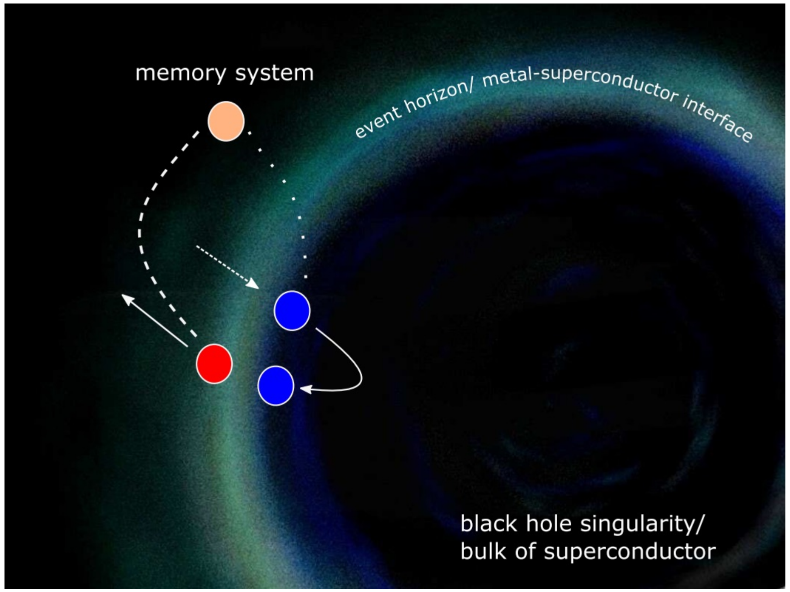
Comment by Deog-Ki Hong

Why don’t you think about the quantum field theory of quark & hadron system?

Quantum tunneling transport at Hadron-quark interface

T. Zhang, H. Tajima, M.T.
2506.09725[nucl-th]





AR as an analog of Hawking radiation

Manikandan, Jordan (2017-20)

Hadron-quark matter model

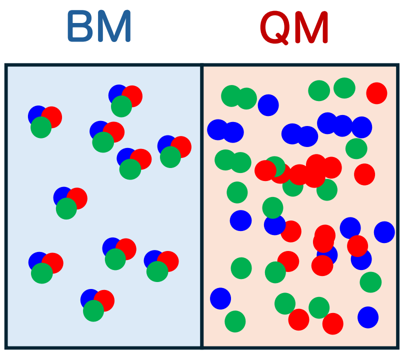
$$H = H_B + H_Q + H_T$$

Baryon Matter (BM) Hamiltonian

$$H_B = \sum_{\mathrm{B}} \left(\xi_{\boldsymbol{K},\mathrm{B}} B^{\dagger}_{\boldsymbol{K}\sigma_B\tau_B} B_{\boldsymbol{K}\sigma_B\tau_B} - \xi_{\boldsymbol{K},\bar{\mathrm{B}}} \bar{B}^{\dagger}_{\boldsymbol{K}\sigma_B\tau_B} \bar{B}_{\boldsymbol{K}\sigma_B\tau_B} \right) + V_{BB} + V_{B\bar{B}} + V_{\bar{B}\bar{B}},$$

Quark Matter (QM) Hamiltonian

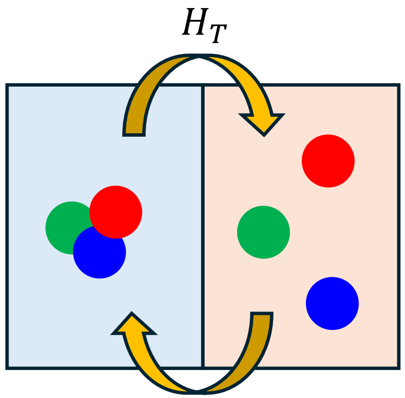
$$H_Q = \sum_{\mathrm{Q}} \left(\xi_{\boldsymbol{k},Q} q^{\dagger}_{\boldsymbol{k}\sigma\tau a} q_{\boldsymbol{k}\sigma\tau a} - \xi_{\boldsymbol{k},\bar{\mathrm{Q}}} \bar{q}^{\dagger}_{\boldsymbol{k}\sigma\tau a} \bar{q}_{\boldsymbol{k}\sigma\tau a} \right) + V_{QQ} + V_{Q\bar{Q}} + V_{\bar{Q}\bar{Q}},$$



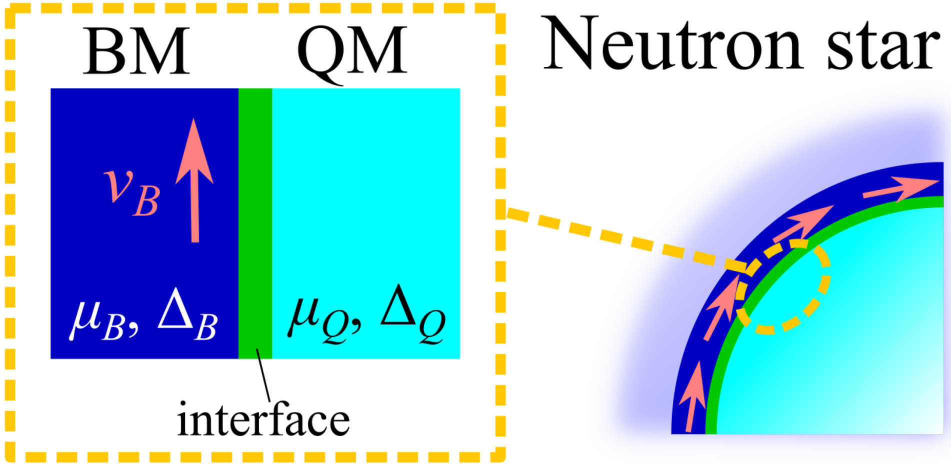
Tunneling Hamiltonian for baryon-multi-quark conversion

$$H_T = \sum_{\mathrm{B},\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3} \mathcal{T}_{(\boldsymbol{K},\sigma_B,\tau_B,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\})} B^\dagger_{\boldsymbol{K}\sigma_B\tau_B} \times \varepsilon_{a_1a_2a_3} q_{\boldsymbol{k}_1\sigma_1\tau_1a_1} q_{\boldsymbol{k}_2\sigma_2\tau_2} q_{\boldsymbol{k}_3\sigma_3\tau_3} + \mathrm{h.c.}$$

$$+ \sum_{\mathrm{B},\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3} \bar{\mathcal{T}}_{(\boldsymbol{K},\sigma_B,\tau_B,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\})} \bar{B}^\dagger_{\boldsymbol{K}\sigma_B\tau_B} \times \varepsilon_{a_1a_2a_3} \bar{q}_{\boldsymbol{k}_1\sigma_1\tau_1a_1} \bar{q}_{\boldsymbol{k}_2\sigma_2\tau_2} \bar{q}_{\boldsymbol{k}_3\sigma_3\tau_3} + \mathrm{h.c.}$$



Tunneling current and friction



$$\hat{I} = -\frac{d}{dt}\rho_B = i\left[\rho_B, H\right]$$

Current operator

$$\hat{\boldsymbol{F}} = -\frac{d\boldsymbol{P}_B}{dt} = i[\boldsymbol{P}_B, H]$$

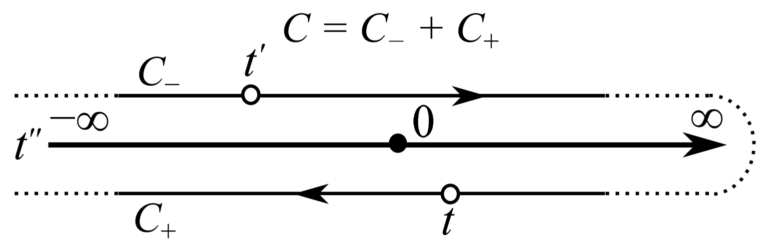
Friction operator

Schwinger-Keldysh approach for tunneling transport

S-matrix operator

$$\hat{S}_C = \exp\left[-i\int_C dt'' H_T^{(H)}(t'')\right]$$

Keldysh contour



Tunneling current

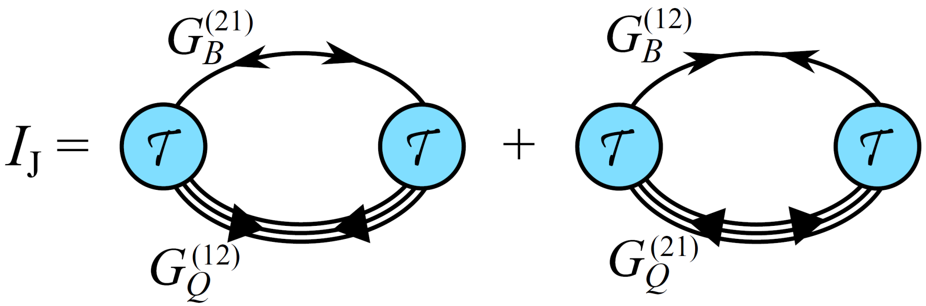
$$\langle \hat{I}(t,t')\rangle = i \sum_{\mathrm{B},\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3} \mathcal{T}_{(\boldsymbol{K},\sigma_B,\tau_B,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\})}\varepsilon_{a_1a_2a_3} e^{i(\mu_\mathrm{B}+\boldsymbol{v}_\mathrm{B}\cdot\boldsymbol{K})t} e^{-3i\mu_\mathrm{Q}t'}$$
$$\times \left\langle T_C\left[\hat{S}_C B^{\dagger(K)}_{\boldsymbol{K}\sigma_B\tau_B}(t) q^{(K)}_{\boldsymbol{k}_1\sigma_1\tau_1a_1}(t') q^{(K)}_{\boldsymbol{k}_2\sigma_2\tau_2}(t') q^{(K)}_{\boldsymbol{k}_3\sigma_3\tau_3}(t')\right]\right\rangle_0 + \mathrm{h.c.},$$

Friction force

$$\langle \hat{\boldsymbol{F}}(t,t')\rangle = i \sum_{\mathrm{B},\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3} \mathcal{T}_{(\boldsymbol{K},\sigma_B,\tau_B,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\})}\boldsymbol{K}\varepsilon_{a_1a_2a_3} e^{i(\mu_\mathrm{B}+\boldsymbol{v}_B\cdot\boldsymbol{K})t} e^{-3i\mu_\mathrm{Q}t'}$$
$$\times \left\langle T_C\left[\hat{S}_C B^{\dagger(K)}_{\boldsymbol{K}\sigma_B\tau_B}(t) q^{(K)}_{\boldsymbol{k}_1\sigma_1\tau_1a_1}(t') q^{(K)}_{\boldsymbol{k}_2\sigma_2\tau_2a_2}(t') q^{(K)}_{\boldsymbol{k}_3\sigma_3\tau_3a_3}(t')\right]\right\rangle + \mathrm{h.c.}.$$

Josephson tunneling current at hadron-quark interface

$$I_{\mathrm{J}}(t,t') = -\sum_{\mathrm{B}} \sum_{\mathrm{Q}_1,\mathrm{Q}_2,\mathrm{Q}_3} \int_C dt_1\, \mathcal{T}^2 \times e^{i(\mu_B+\boldsymbol{v}_{\mathrm{B}}\cdot\boldsymbol{K})t} e^{-3i\mu_Q t'} e^{i(\mu_B+\boldsymbol{v}_{\mathrm{B}}\cdot\boldsymbol{K}-3\mu_Q)t_1} \times G^{(12)}_{Q,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\}}(t',t_1) G^{(21)}_{B,\boldsymbol{K},\sigma_B,\tau_B}(t_1,t) + \mathrm{h.c.}$$

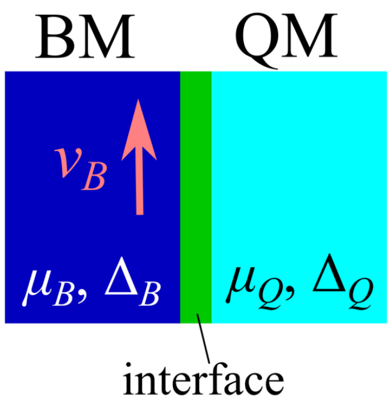


G_{B} : Contour-ordered baryon Green's function

G_Q : Contour-ordered three-quark Green's function

Explicit form of the Josephson tunneling current with retarded Green's functions

$$
\begin{aligned}
I_{\mathrm{J}}(t) = 4 \sum_{\boldsymbol{K},\sigma_B,\tau_B} \sum_{\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \Big\{ & \mathcal{T}^2 \,\mathrm{Im}\, G^{(12)\mathrm{ret.}}_{Q,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\}}(\omega-\Delta\mu)\,\mathrm{Im}\, G^{(21)\mathrm{ret.}}_{B,\boldsymbol{K},\sigma_B,\tau_B}(\omega) \\
& \times [f(\omega-\Delta\mu)-f(\omega)] \cos(2\Delta\mu t+\Delta\phi) \\
& - \mathcal{T}^2 \Big[\mathrm{Re}\, G^{(12)\mathrm{ret.}}_{Q,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\}}(\omega-\Delta\mu)\,\mathrm{Im}\, G^{(21)\mathrm{ret.}}_{B,\boldsymbol{K},\sigma_B,\tau_B}(\omega) f(\omega) \\
& + \mathrm{Im}\, G^{(12)\mathrm{ret.}}_{Q,\{\boldsymbol{k}_i,\sigma_i,\tau_i,a_i\}}(\omega-\Delta\mu)\,\mathrm{Re}\, G^{(21)\mathrm{ret.}}_{B,\boldsymbol{K},\sigma_B,\tau_B}(\omega) f(\omega-\Delta\mu) \Big] \sin(2\Delta\mu t+\Delta\phi) \Big\}
\end{aligned}
$$

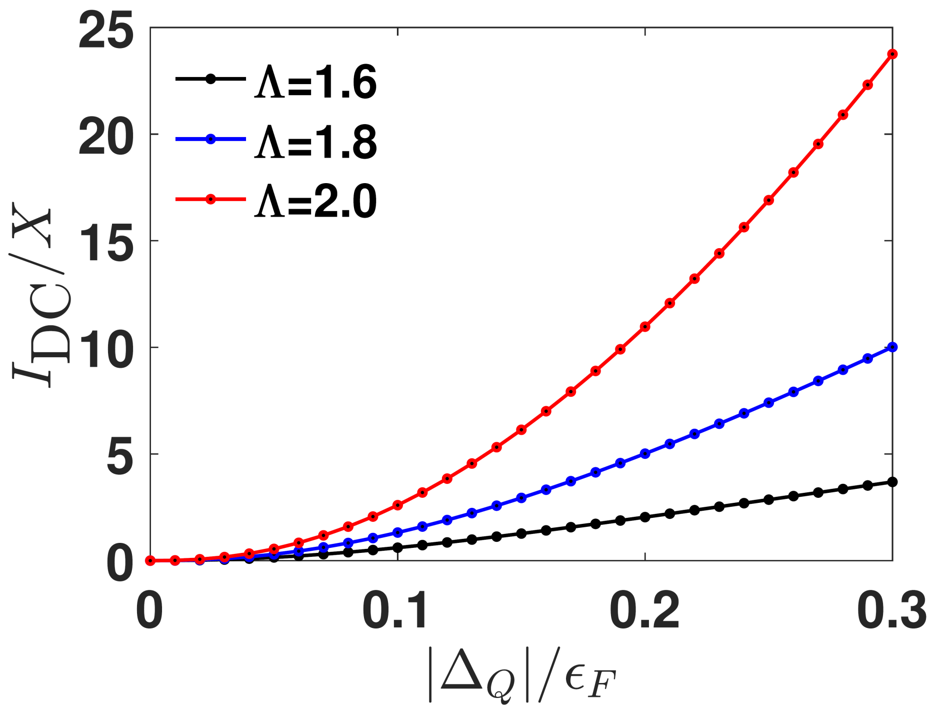


Bias parameters

$$\Delta\mu = \mu_B + \boldsymbol{v}_B \cdot \boldsymbol{K} - 3\mu_Q$$

$$\Delta\phi = \phi_B - 3\phi_Q$$

DC Josephson tunneling current



$$I_{\mathrm{DC}} \propto |\Delta_B||\Delta_Q|^3 \sin(\phi_B - 3\phi_Q)$$

Current occurs even at $\Delta\mu = 0 \rightarrow$ DC Josephson current

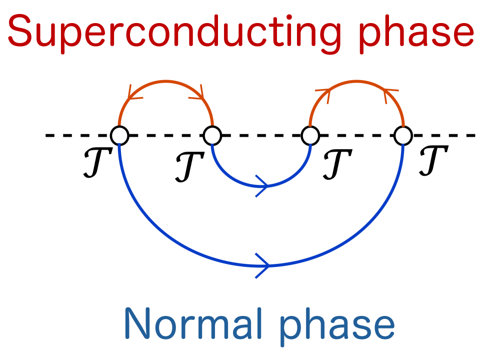
Comment: Work by Sedrakian and Rau

"Josephson currents in neutron stars"
(*Phys.Rev.D* 111 (2025) 2, 023044, arXiv:2407.13686)

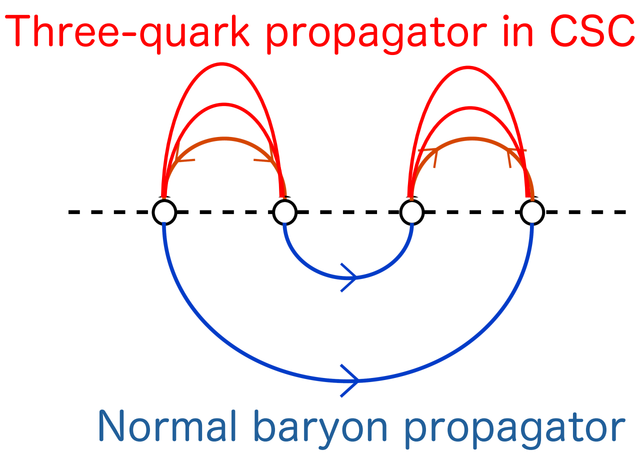
Both AC and Josephson currents considered
in a different context of us

Towards the Andreev reflection at hadron-quark interface

In the conventional normal-superfluid junction,
the Andreev reflection appears
at the 4th order of tunneling coupling



$$I_{\mathrm{A}} = 8\,\mathcal{T}^4 \sum_{\mathbf{k}} \int \frac{d\omega}{2\pi} |G_{d,12}(\mathbf{k},\omega)|^2 \mathrm{Im}\, G_{c,11}(\mathbf{k},\omega-\Delta\mu)$$
$$\mathrm{Im}\, G_{c,22}(\mathbf{k},\omega+\Delta\mu)\left[f(\omega-\Delta\mu)-f(\omega+\Delta\mu)\right]$$



Summary

<u>Study of dense QCD matter</u>

0. Physics of interface
1. Dense QCD world as “Terra Incognita”
2. Andreev reflection in QCD
3. A field theoretical approach
   to quantum transport phenomena

-Josephson current and quantum friction-

Congratulations on

the 65th anniversary

of the Krakow School !

Dziękuję !

感謝