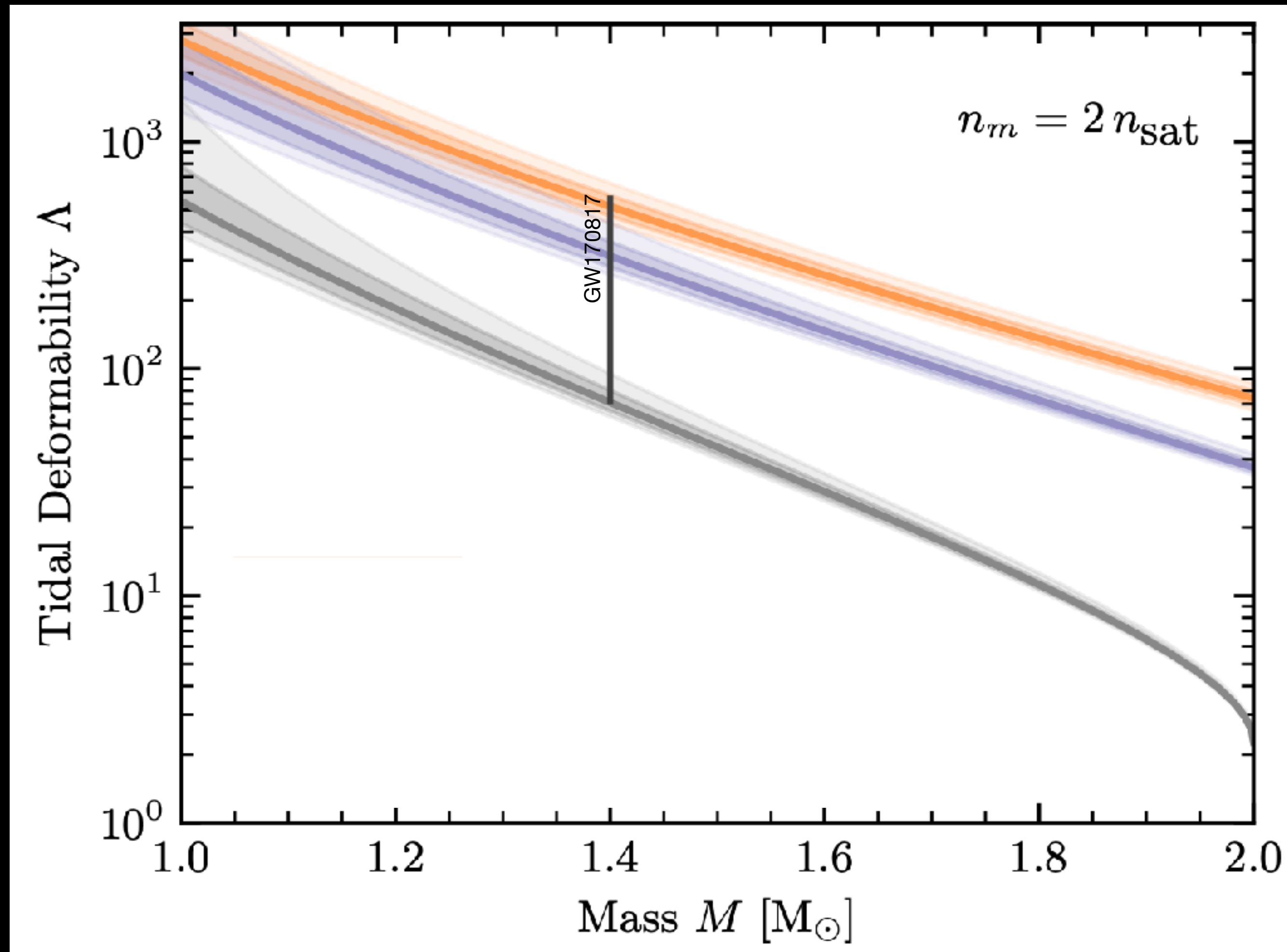
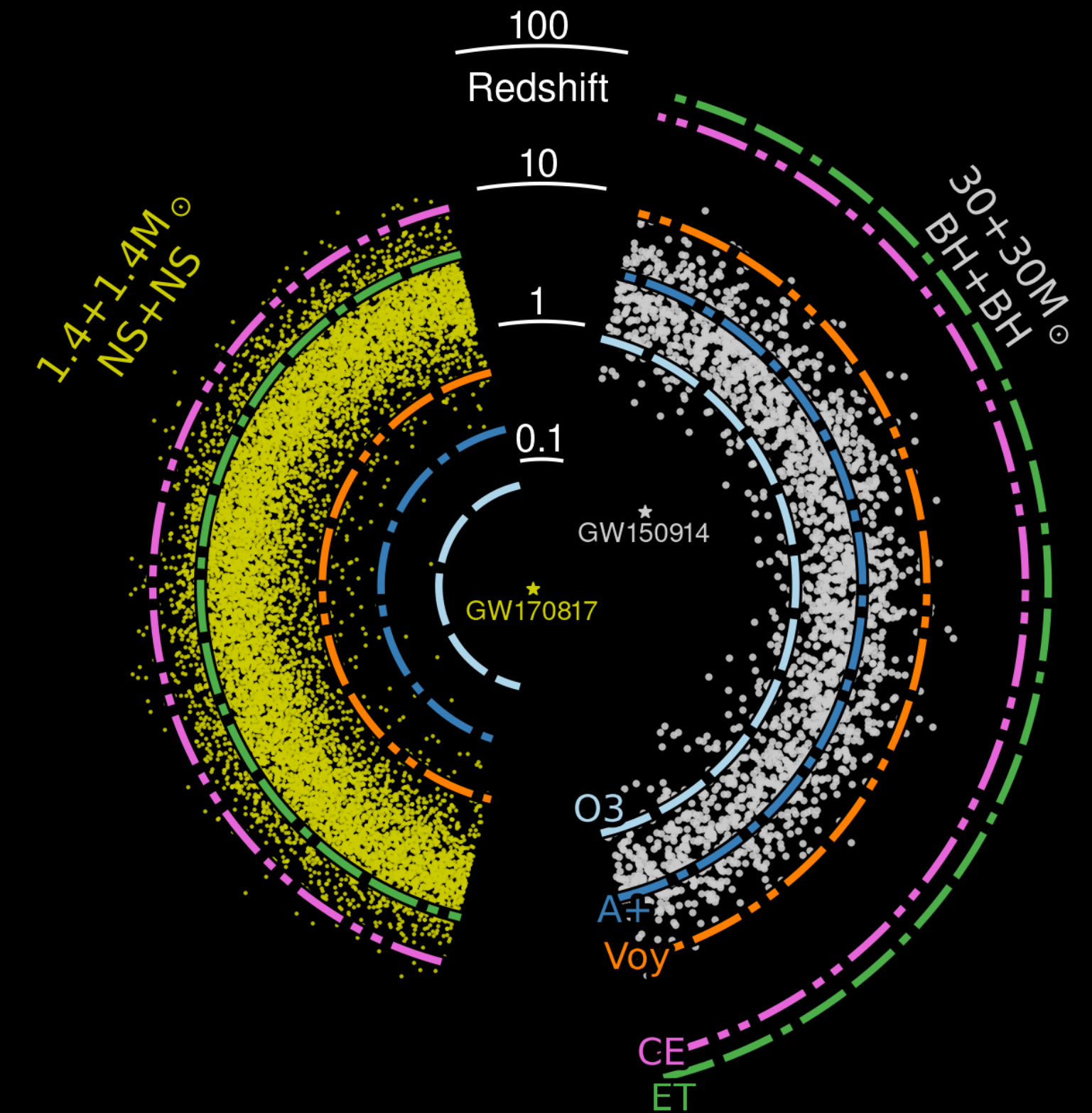


Tidal Deformability & Compact Object Populations



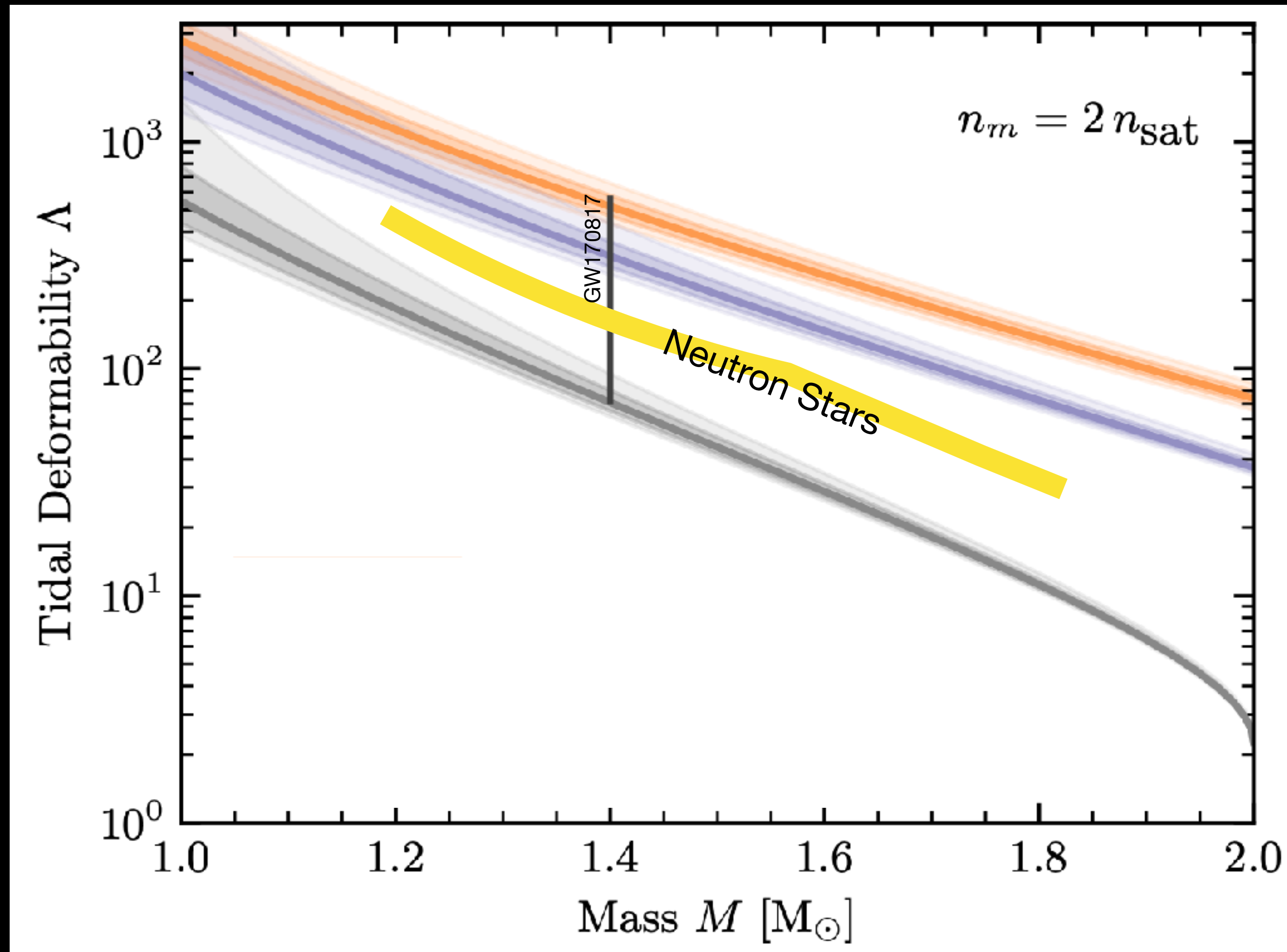
Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)

Large sample size: $>20,000$ BNS/year.



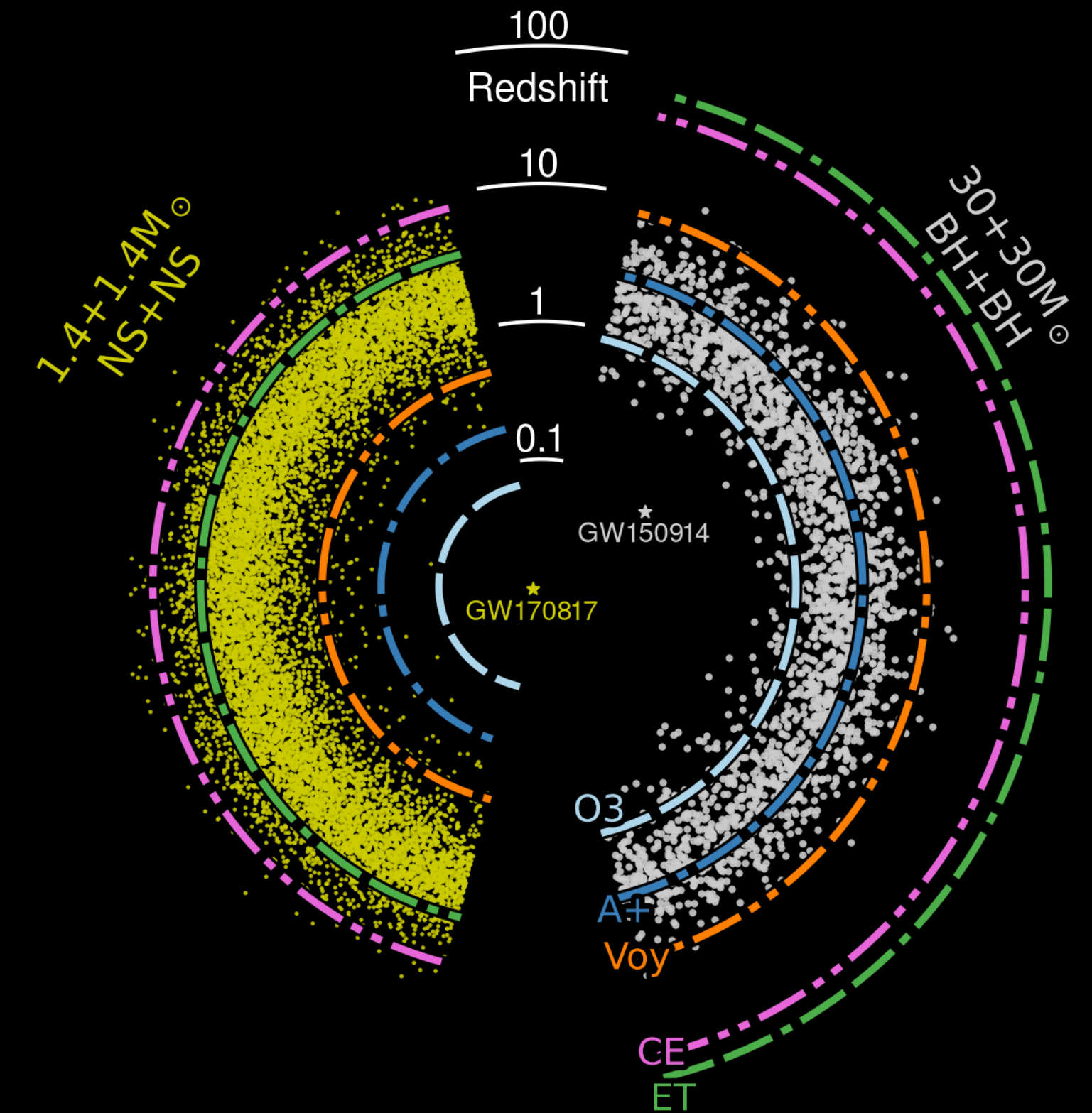
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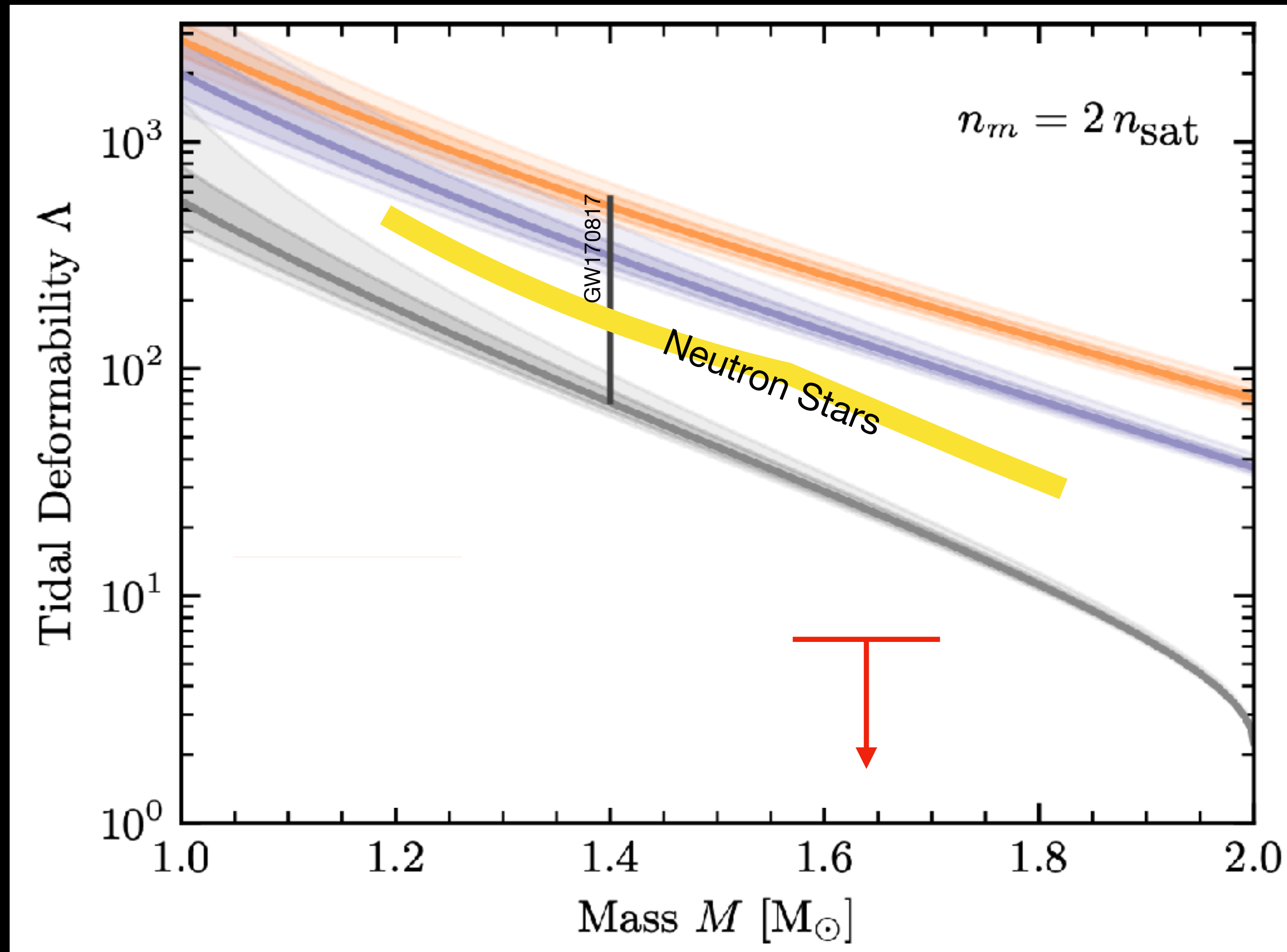
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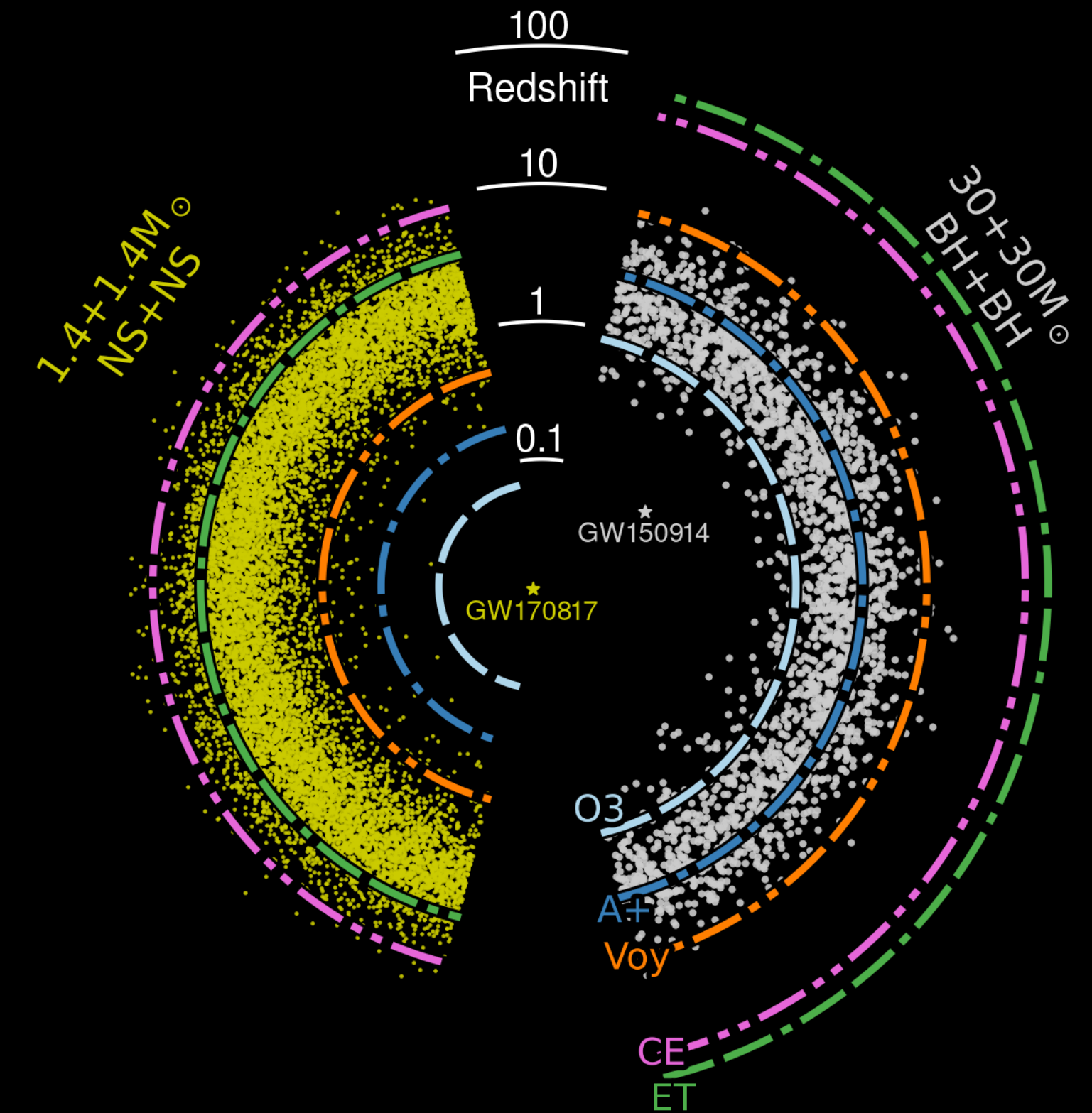
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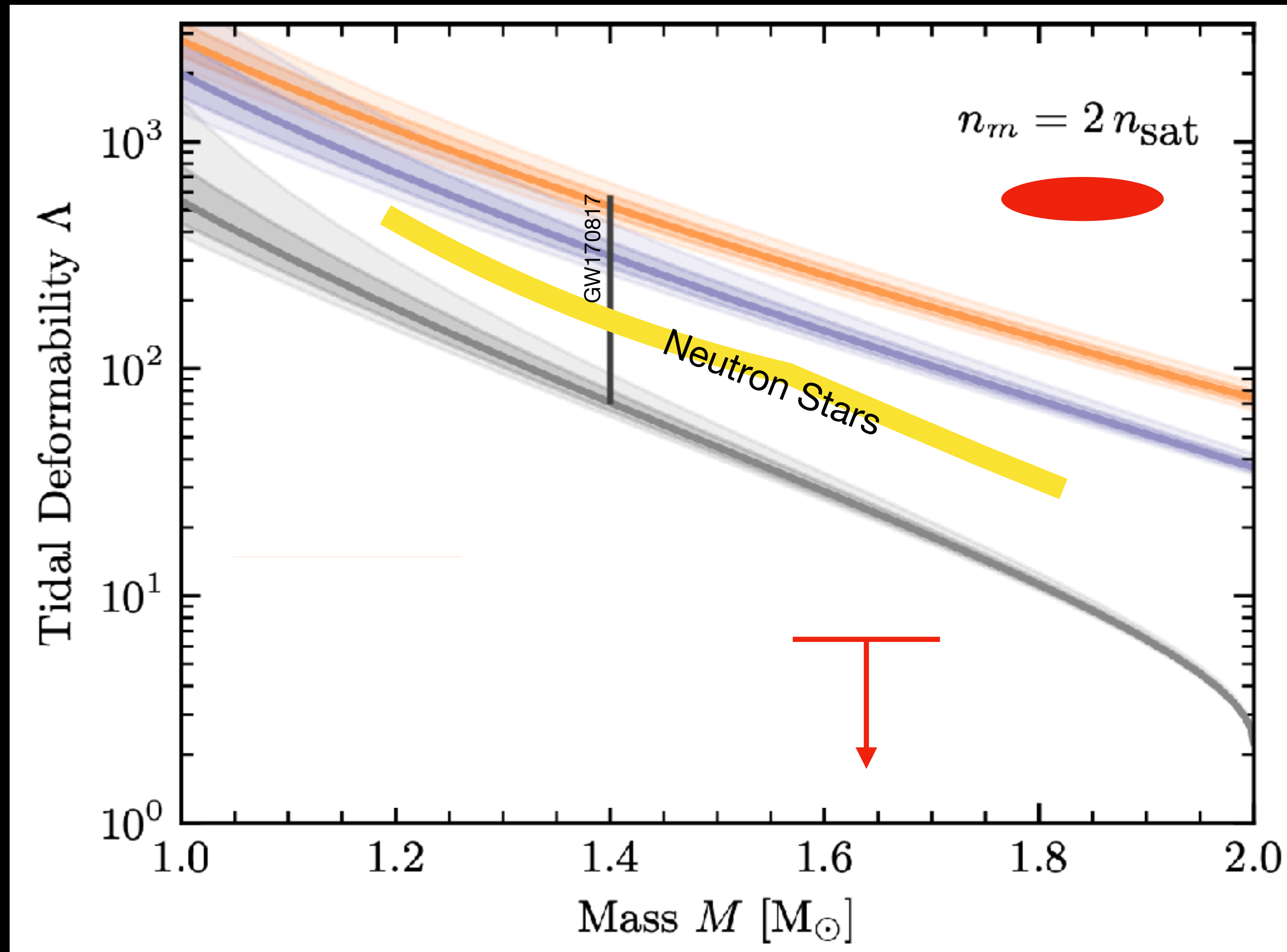


Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)



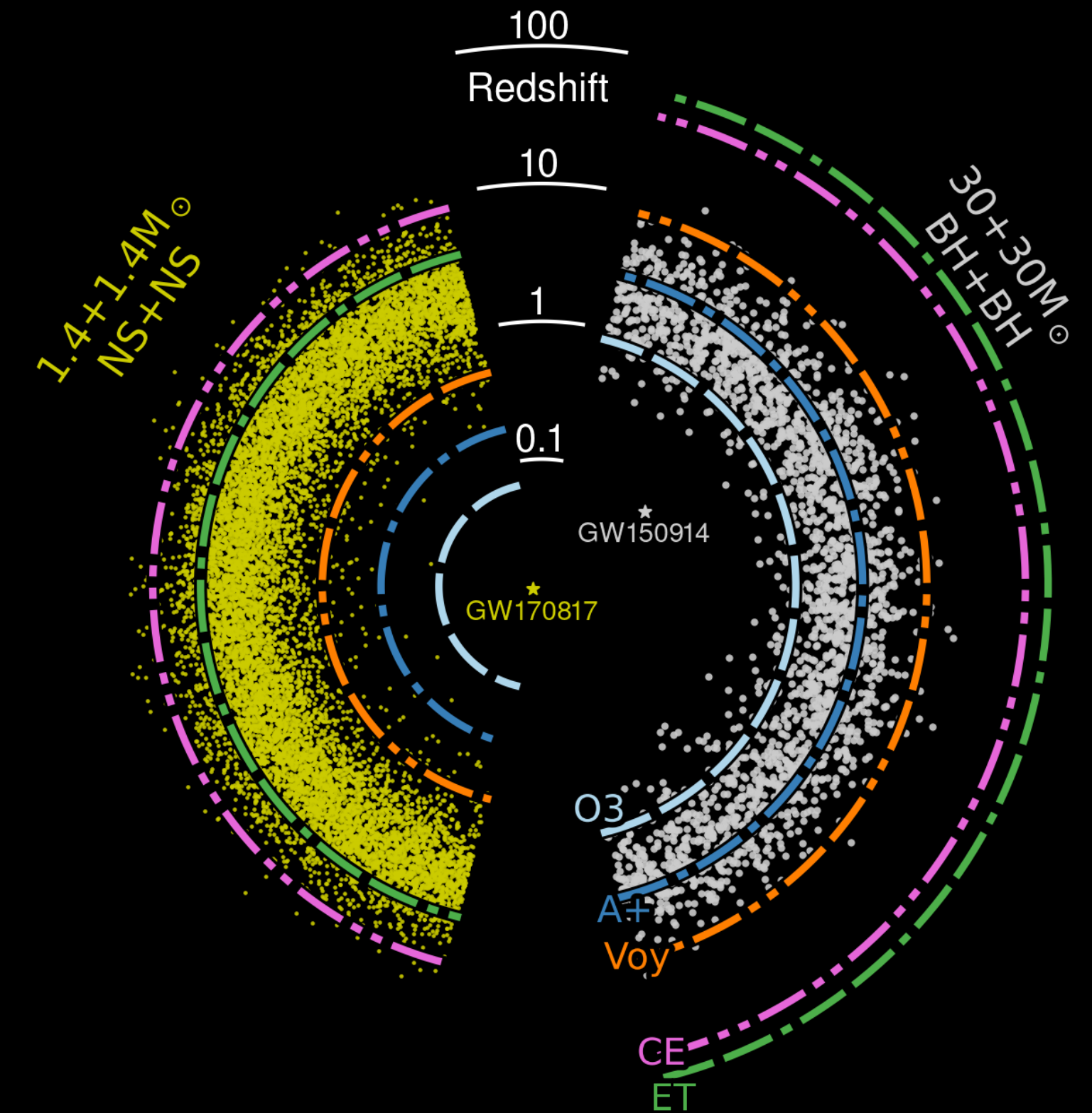
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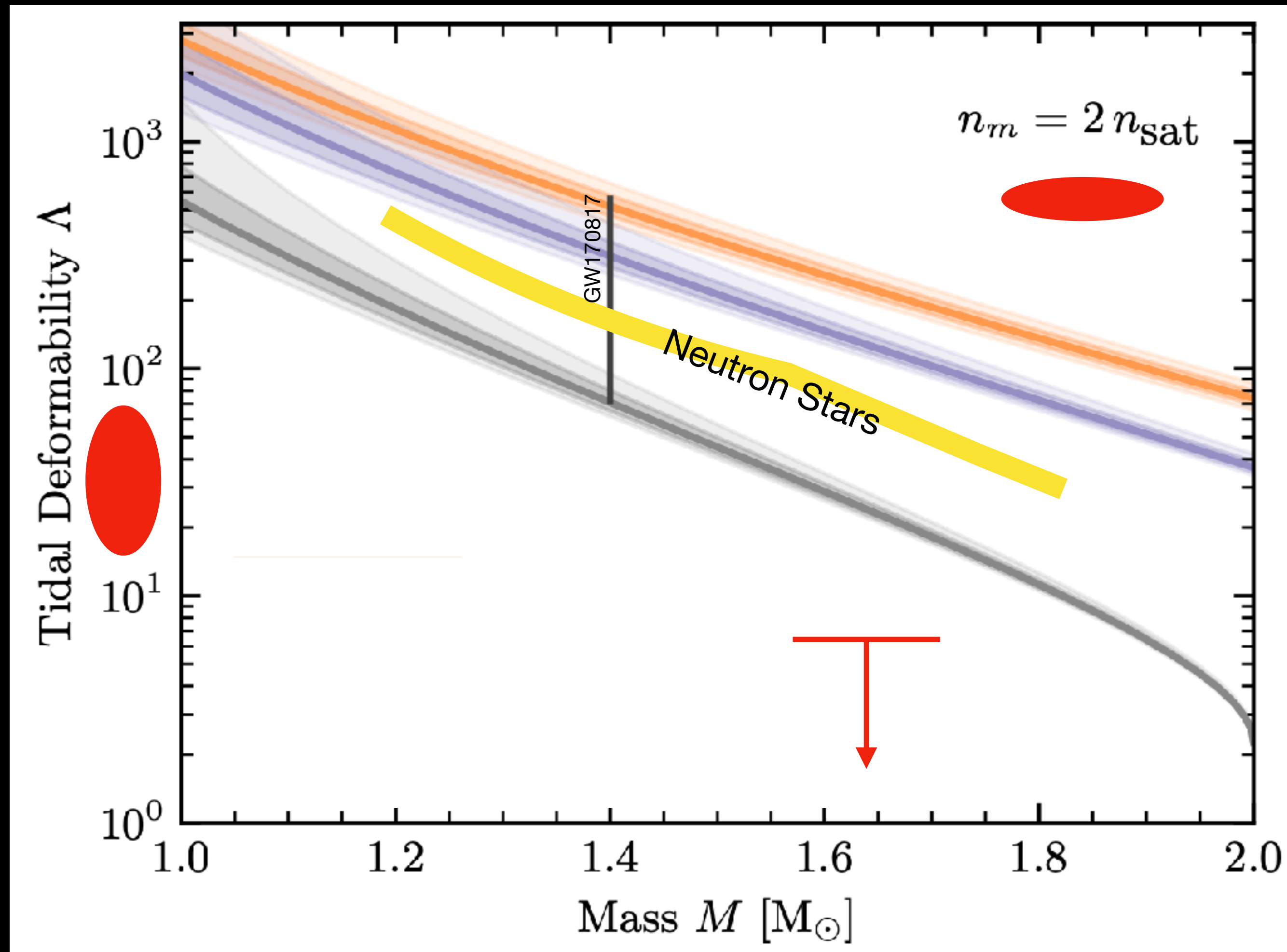
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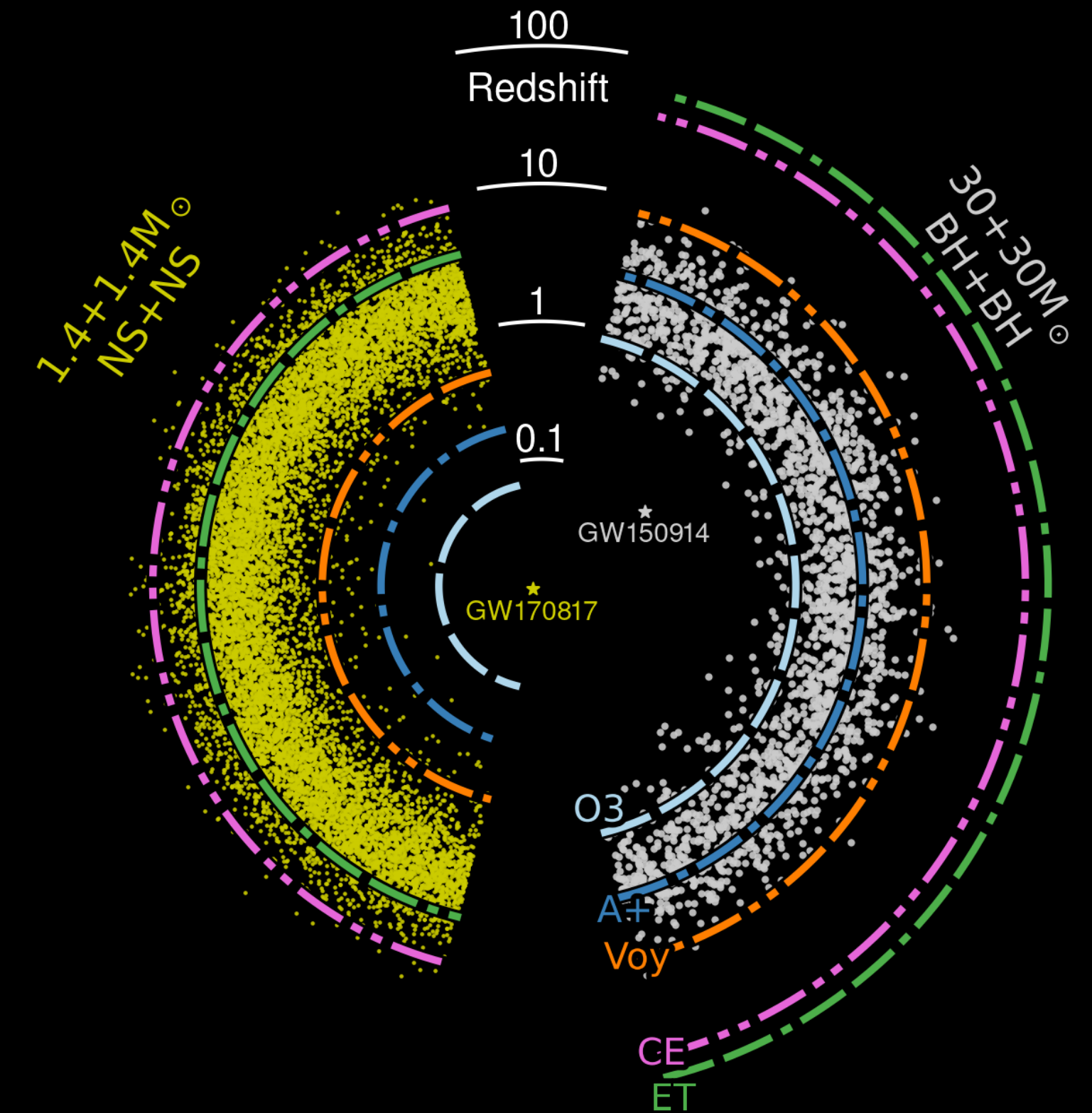
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The Dark Side of Neutron stars

Neutron stars are great places to look for dark matter:

- They accrete and trap dark matter.

$$M_\chi < 10^{-14} M_\odot \left(\frac{\rho_\chi}{1 \text{ GeV/cm}^3} \right) \frac{t}{\text{Gyr}}$$

- Produce “baryonic” or “leptonic” dark matter due to its high density

$$M_\chi \lesssim M_\odot \text{ for } m_\chi \lesssim 2 \text{ GeV}$$

- Produce thermal dark matter due to high temperatures at birth or during mergers.

$$M_\chi \lesssim 10^{-1} M_\odot \text{ for } m_\chi < 100 \text{ MeV}$$



Black-Holes in the Neutron Star Mass-Range

Idea: Accretion of asymmetric bosonic dark matter can induce the collapse of an NS to a BH.

Goldman & Nussinov (1989)

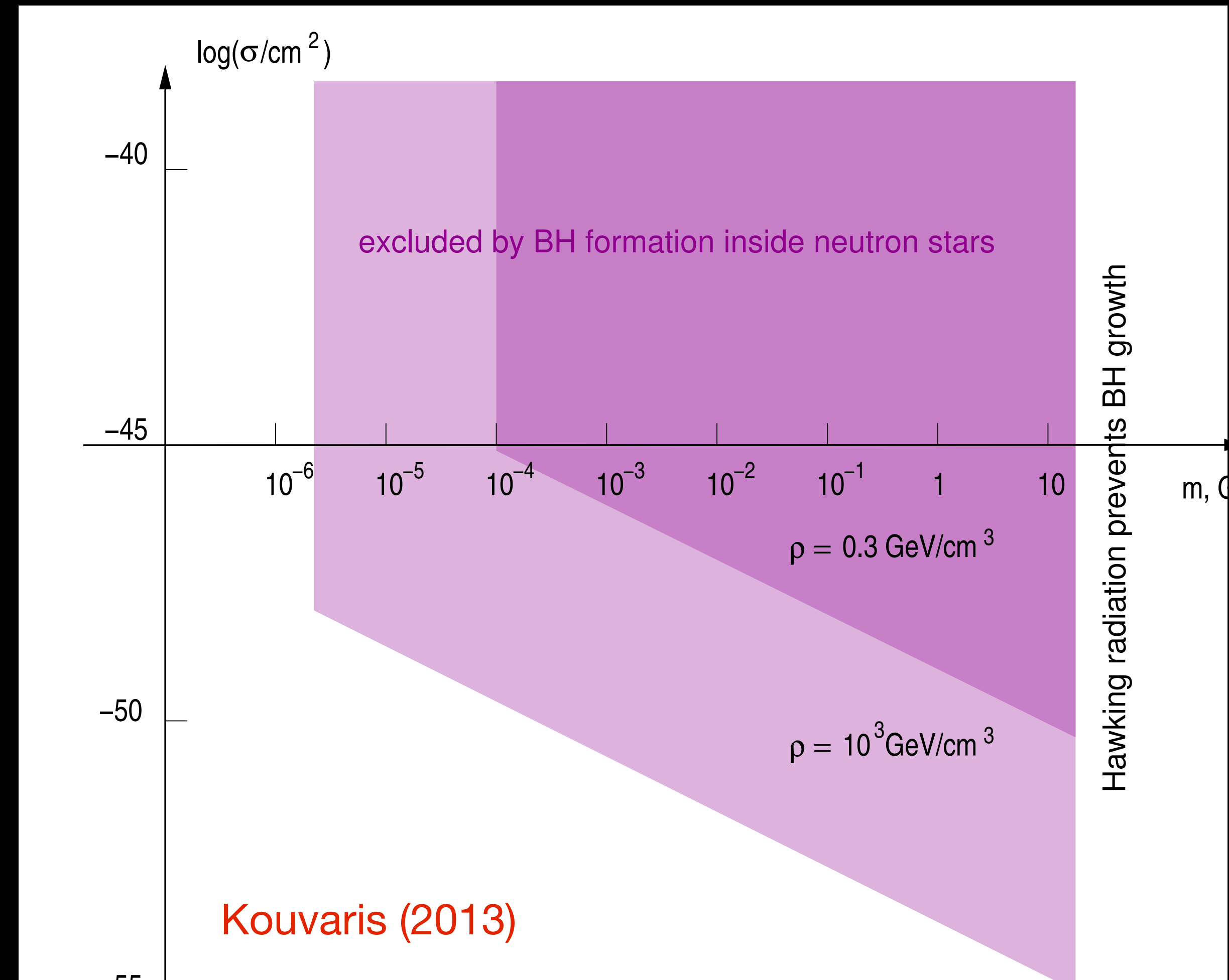
$$M_\chi \approx 10^{-14} M_\odot \text{ Min } \left[\frac{\sigma}{2 \times 10^{-45} \text{cm}^2}, 1 \right] \left(\frac{\rho_\chi}{1 \text{ GeV/cm}^3} \right) \frac{t}{\text{Gyr}}$$

The maximum mass of weakly Interacting bosons is negligible:

$$M_{\text{Bosons}} \approx 10^{-18} M_\odot \left(\frac{\text{GeV}}{m_\chi} \right)$$

The existence of old neutron stars in the Milkyway with estimated age $\sim$ Gyr provides strong constraints on asymmetric DM.

For a concise reviews see Kouvaris (2013) and Zurek (2013)



Time Scale for Converting NSs into BHs

For dark matter in the $1\text{--}10^6$ GeV mass range, black hole formation is complex and involves several timescales.

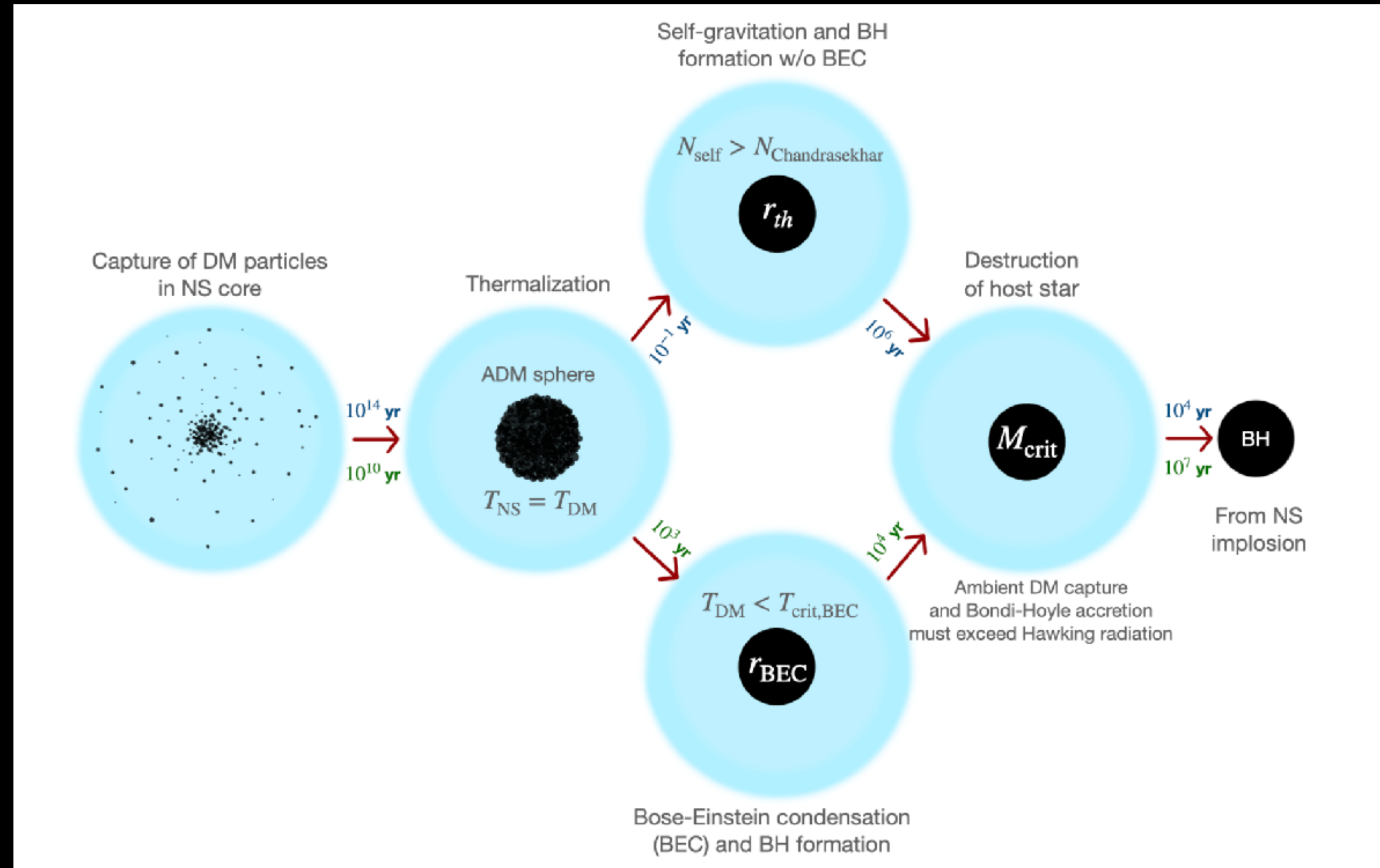
Capture time is typically the limiting step. But, thermalization can be slow in exotic superfluid phases and depends on processes in the inner core!

C. Kouvaris and P. Tinyakov (2011)

S. D. McDermott, H.-B. Yu, and K. M. Zurek, (2012)

B. Bertoni, A. E. Nelson, and S. Reddy (2013)

+ many more, more refined recent analyses.

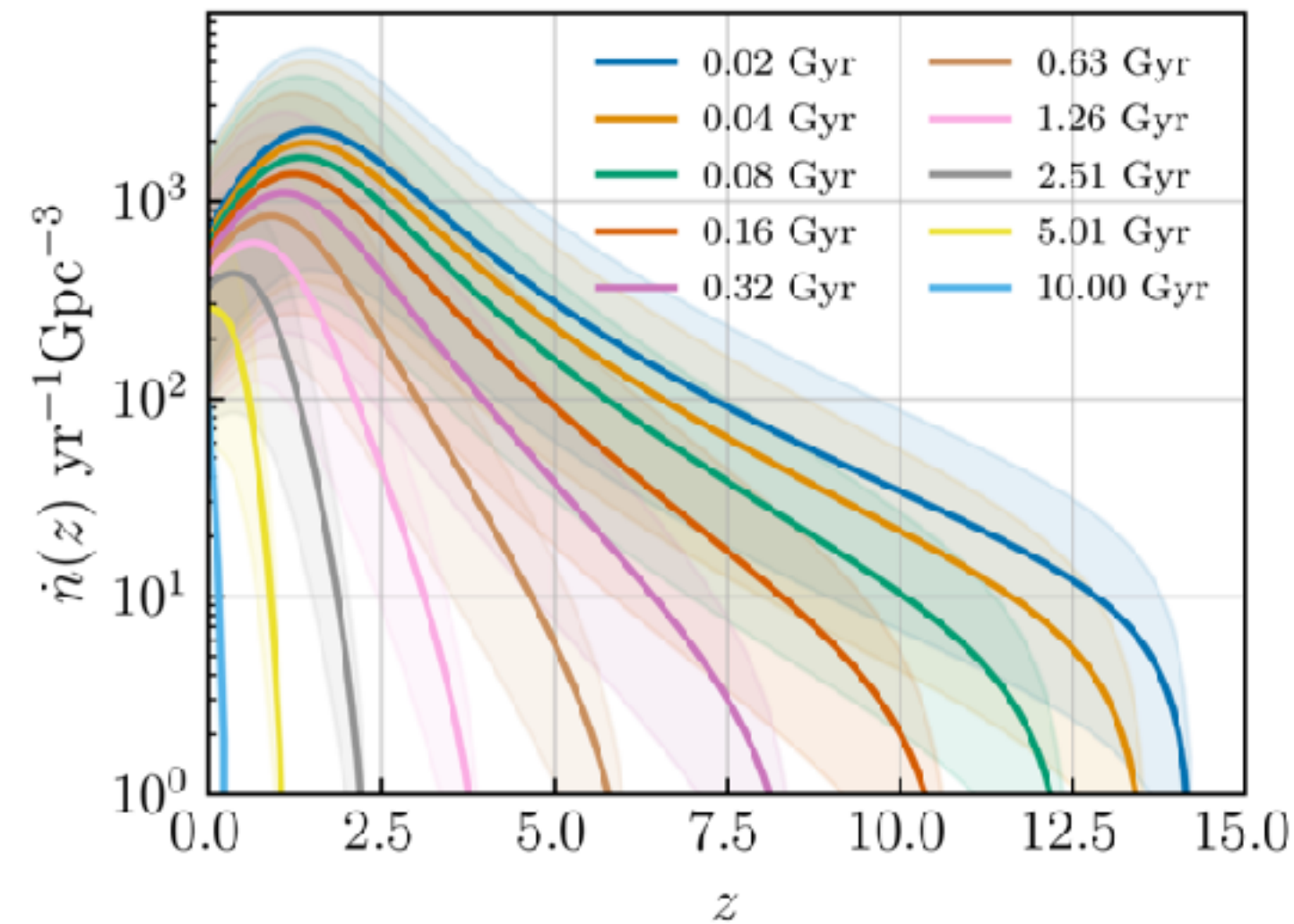


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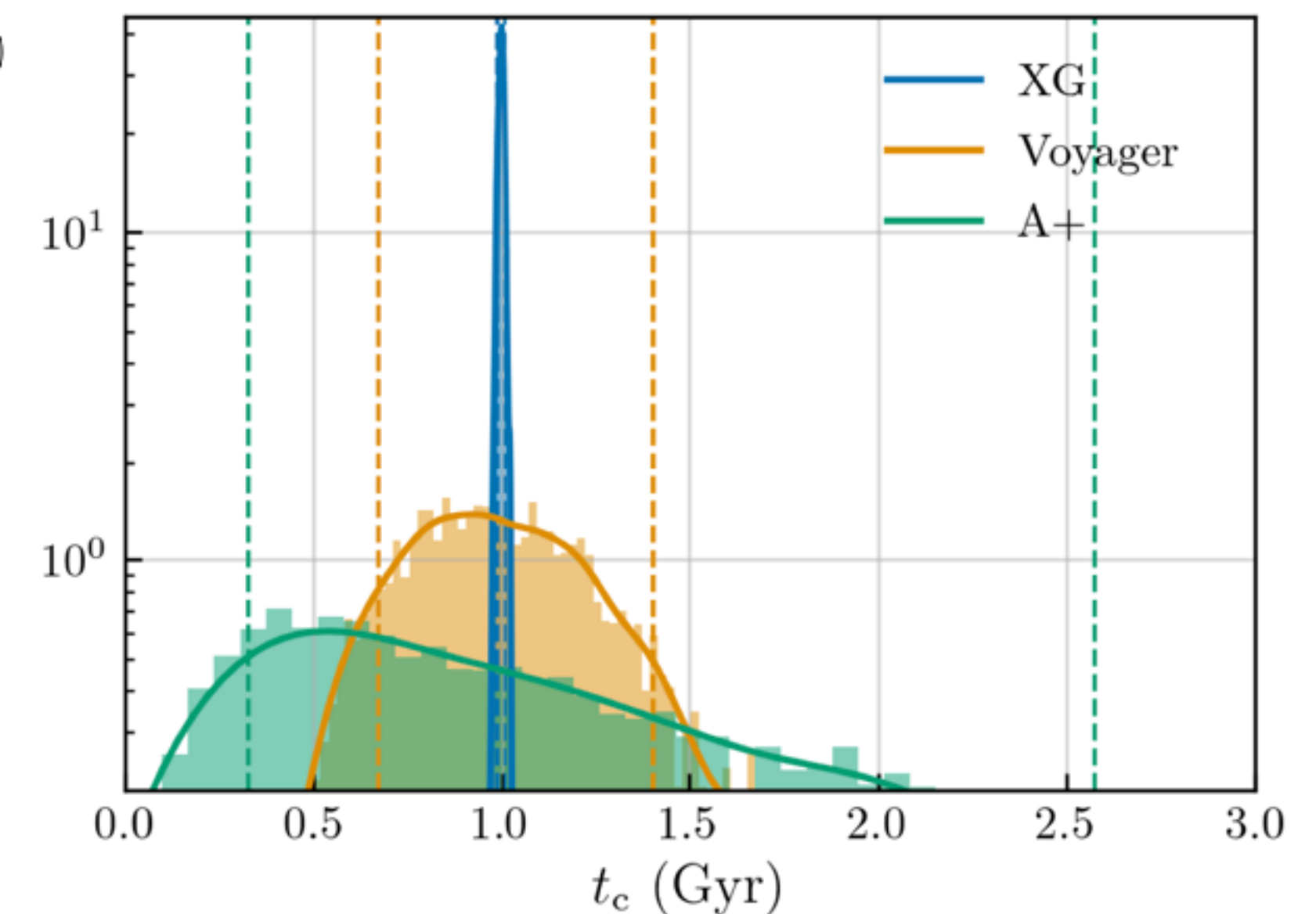
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A measurement of the tidal deformability will allow us to distinguish BBH from BNS to infer the collapse time in next generation detectors.

The number of merging black holes formed from NS implosion grows rapidly when the collapse time is less than a Gyr.



Constraining Dark Baryons

Dark sectors could contain particles in the MeV-GeV mass range that mix with baryons.

There was speculation that a dark baryon with mass m_χ between 937.76 - 938.78 MeV might explain the discrepancy between neutron lifetime measurements. Fornal & Grinstein (2018)

$$n \rightarrow \chi + \dots$$

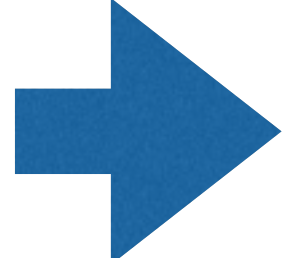
$$\tau_n^{\text{bottle}} = 879.6 \pm 0.6 \text{ s} \quad \text{--- counts neutrons}$$

$$\text{Br}_{n \rightarrow \chi} = 1 - \frac{\tau_n^{\text{bottle}}}{\tau_n^{\text{beam}}} = (0.9 \pm 0.2) \times 10^{-2}$$

$$\tau_n^{\text{beam}} = 888.0 \pm 2.0 \text{ s} \quad \text{--- counts protons}$$

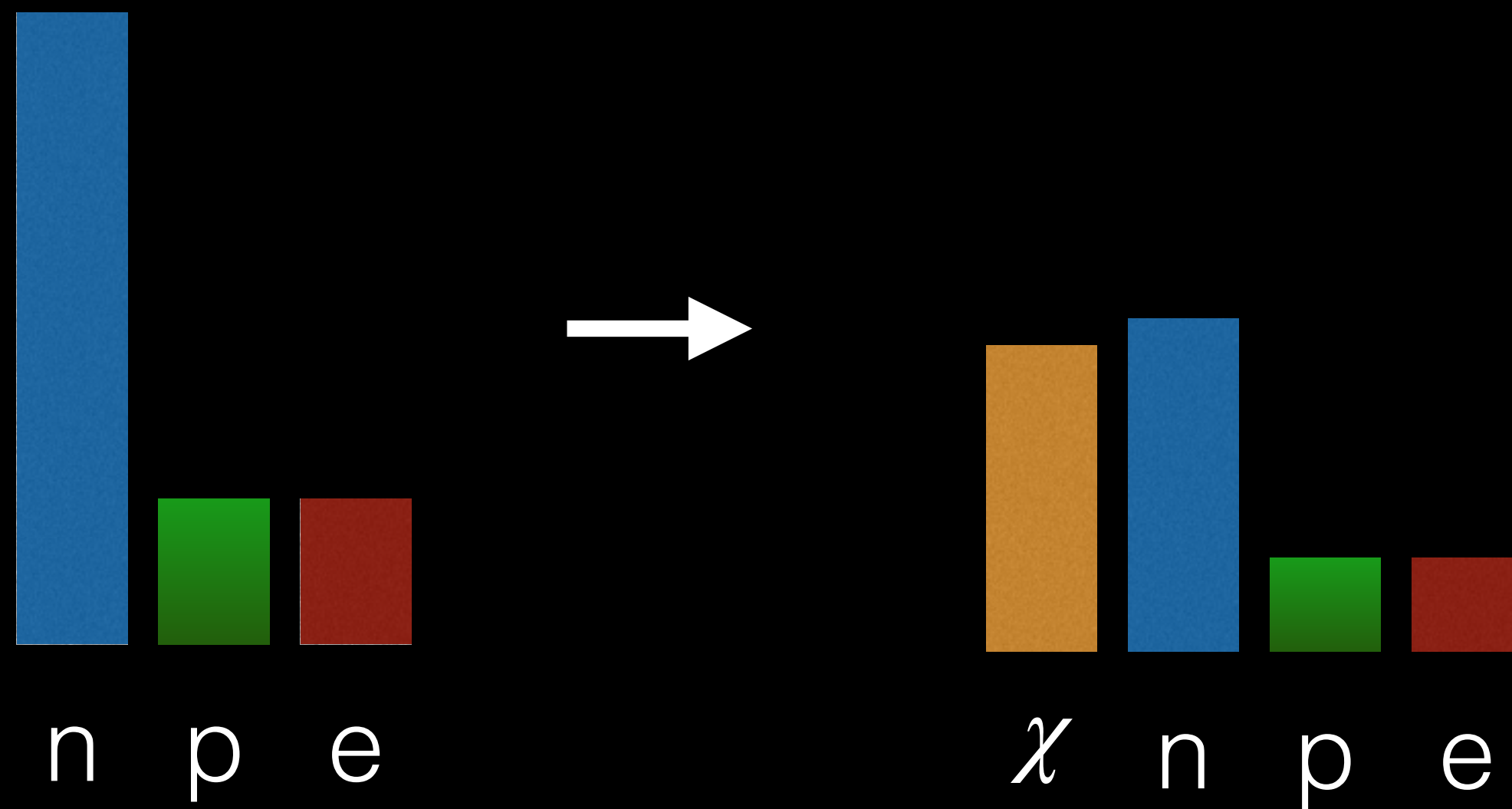
A model for hidden baryons that mix with the neutron:

$$\mathcal{L}_{\text{eff}} = \bar{n} (i \not{\partial} - m_n) n + \bar{\chi} (i \not{\partial} - m_\chi) \chi - \delta (\bar{\chi} n + \bar{n} \chi)$$

Mixing angle: $\theta = \frac{\delta}{\Delta M}$  An explanation of the anomaly requires $\theta \simeq 10^{-9}$

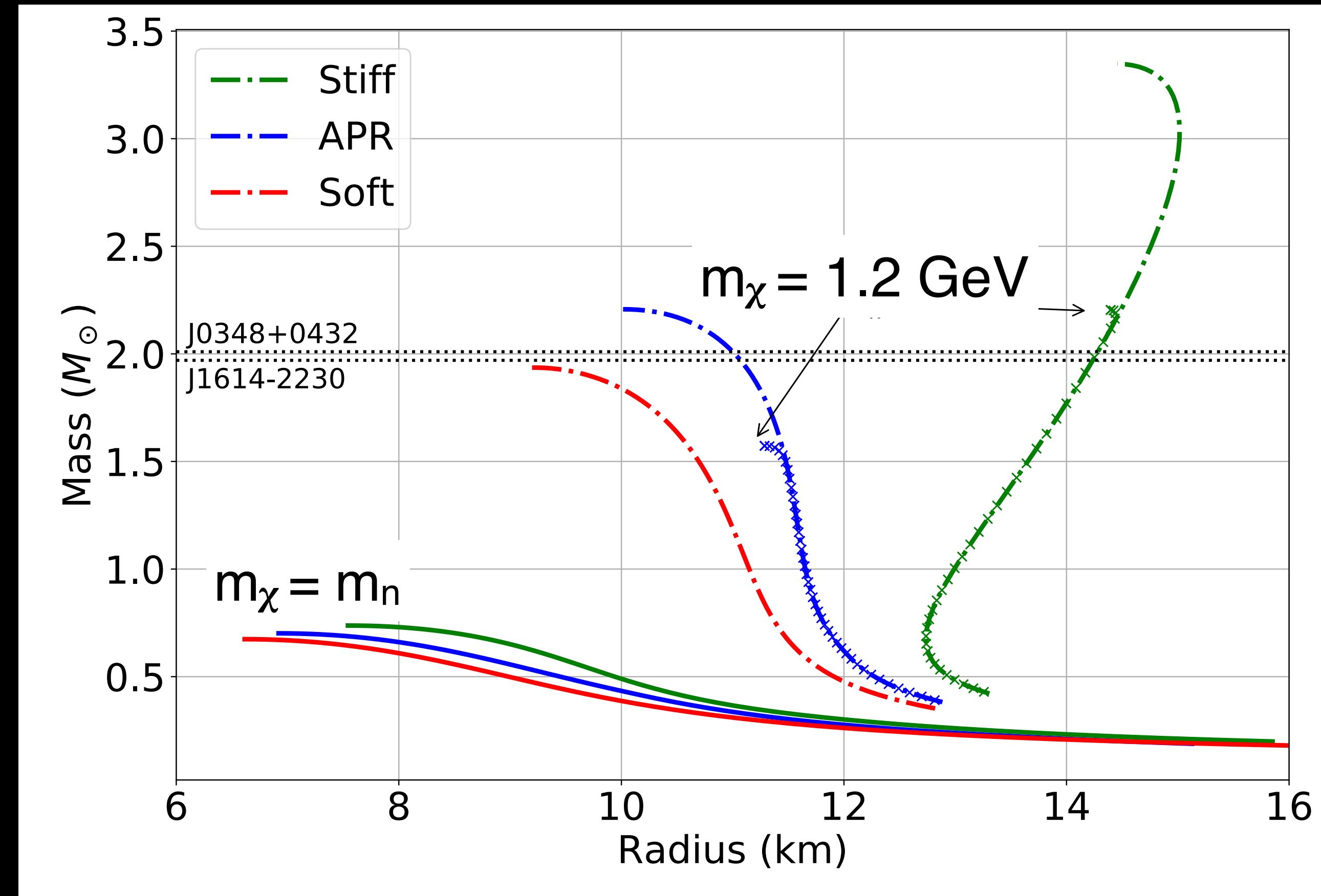
Neutron stars can probe much smaller mixing angles: $\theta \simeq 10^{-18}$

Weakly Interacting Dark Baryons Destabilize Neutron Stars



Neutron decay lowers the nucleon density at a given energy density.

When dark baryons are weakly interacting the equation of state is soft ~ similar to that of a free fermi gas.



This lowers the maximum mass of neutron stars.

Self-interacting Dark Matter

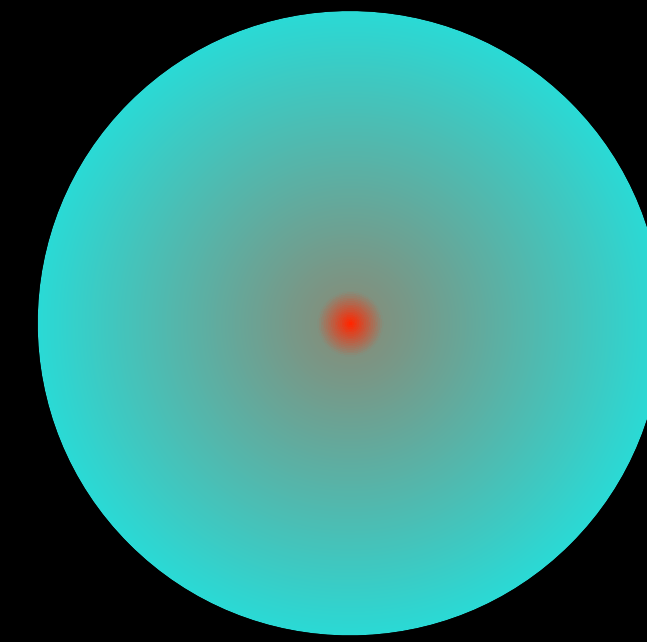
Using Gravitational Waves to Discover Hidden Sectors

Self-interacting dark matter can be stable and bound to neutron stars - a new class of compact dark objects.

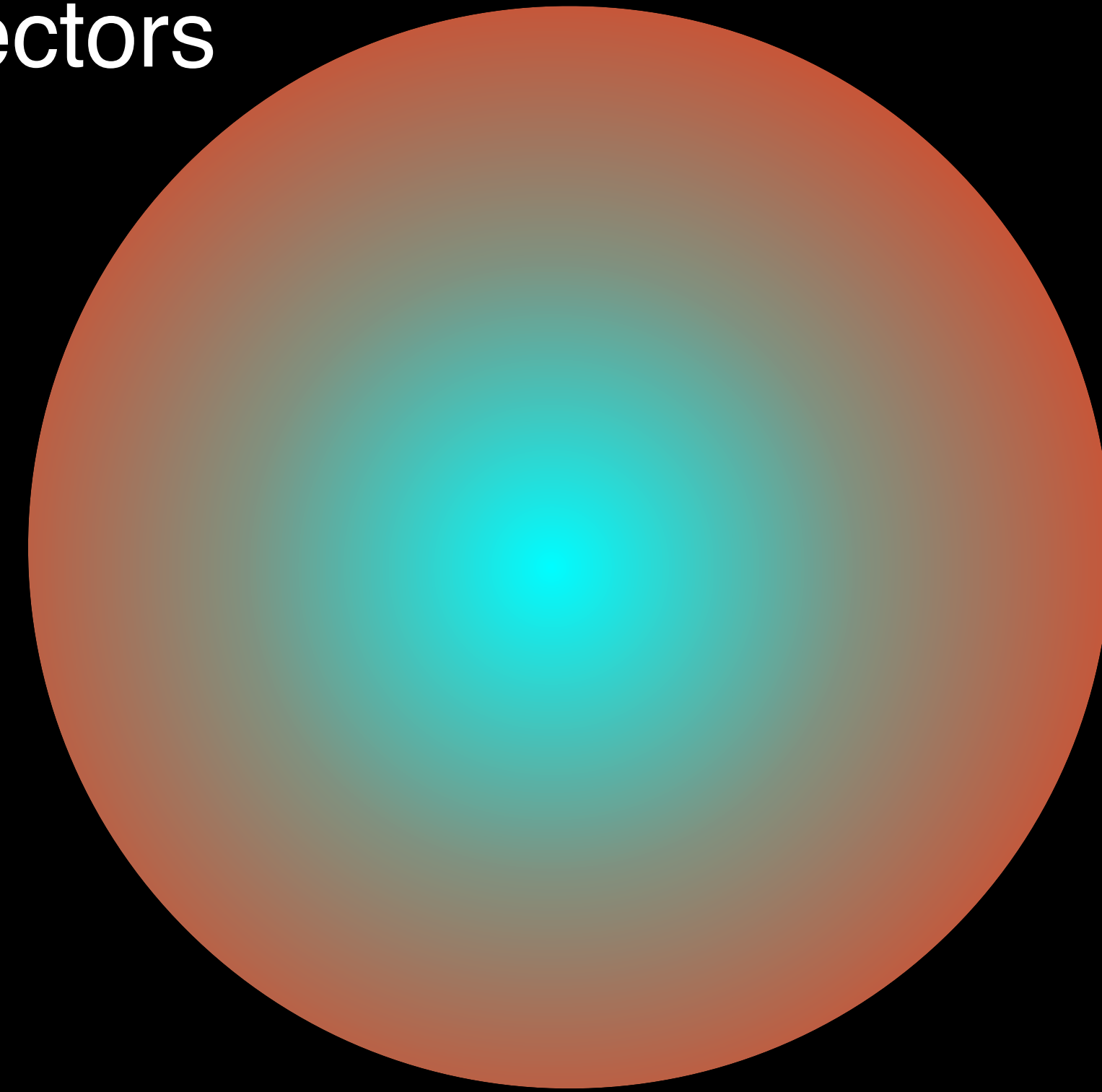
Gravitational wave observations of binary compact objects whose masses and tidal deformability's differ from those expected from neutron stars and stellar black holes would provide conclusive evidence for a strongly self-interacting dark sector:

Mass $< 0.1 M_{\text{solar}}$

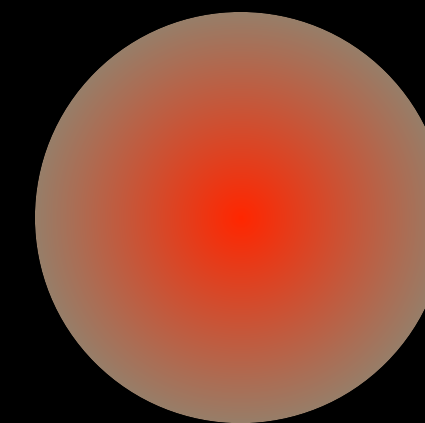
Tidal Deformability > 600



NS + dark-core



NS + dark-halo



Compact Dark Objects

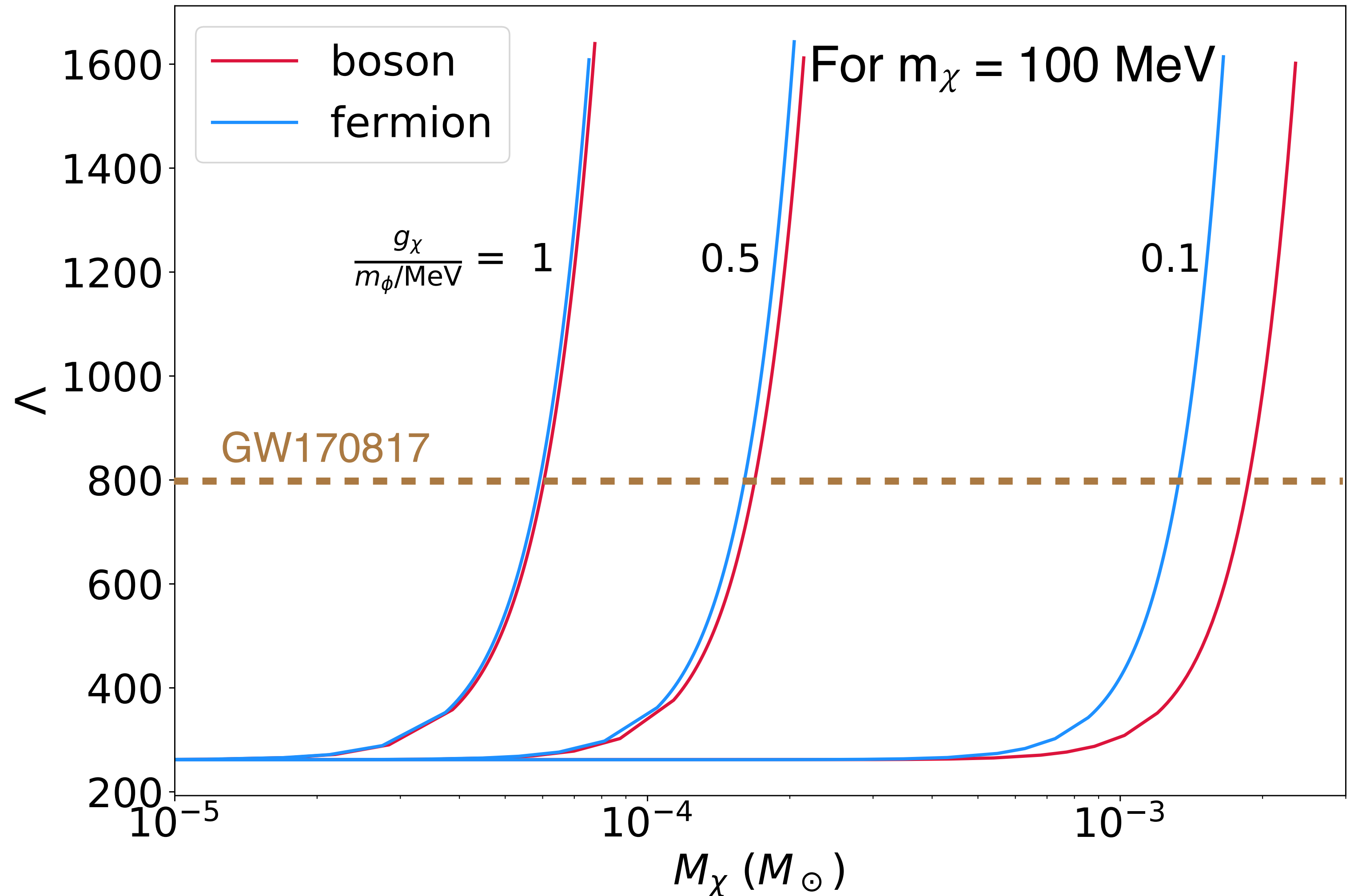
Dark Halos Alter Tidal Interactions

Trace amount of light dark matter $\sim 10^{-4}$ - $10^{-2} M_{\text{solar}}$ is adequate to enhance the tidal deformability $\Lambda > 800$!

Self-Interactions of “natural-size” can provide adequate repulsion.

$g_{\chi}/m_{\phi} = (0.1/\text{MeV})$ or $(10^{-6}/\text{eV})$

Nelson, Reddy, Zhou (2019)



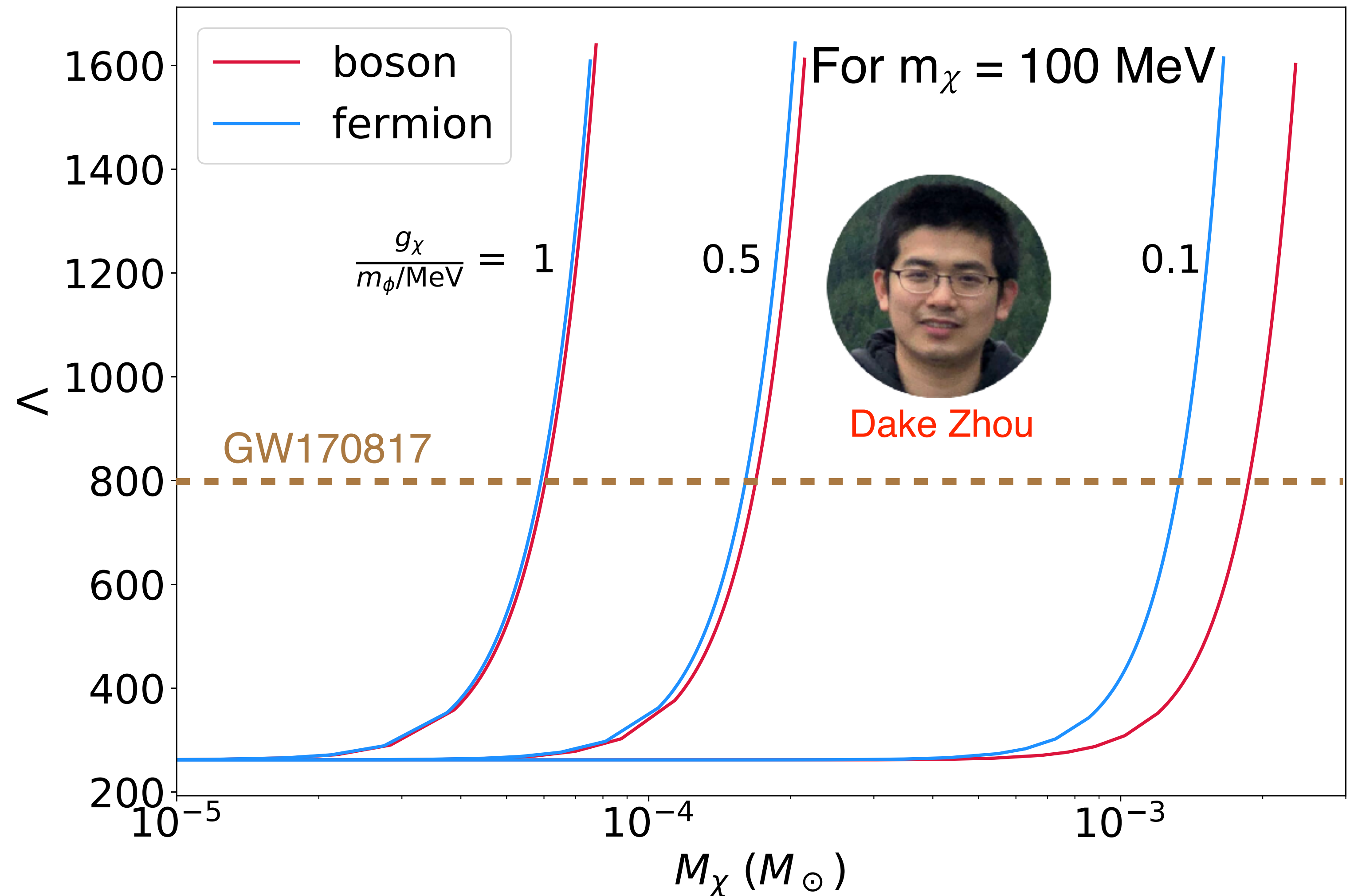
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Axion Condensation in Neutron Stars

R. Kumamoto, J. Huang, C. Drischler, M. Baryakthar, and S. Reddy (2024) [arXiv:2410.21590](#)



Mia Kumamoto
Grad. Student
U of Washington



Christian Drischler
Asst. Professor
Ohio U



Masha Baryakthar
Asst. Professor
U of Washington



Junwu Huang
Research Faculty
Perimeter Inst.

QCD

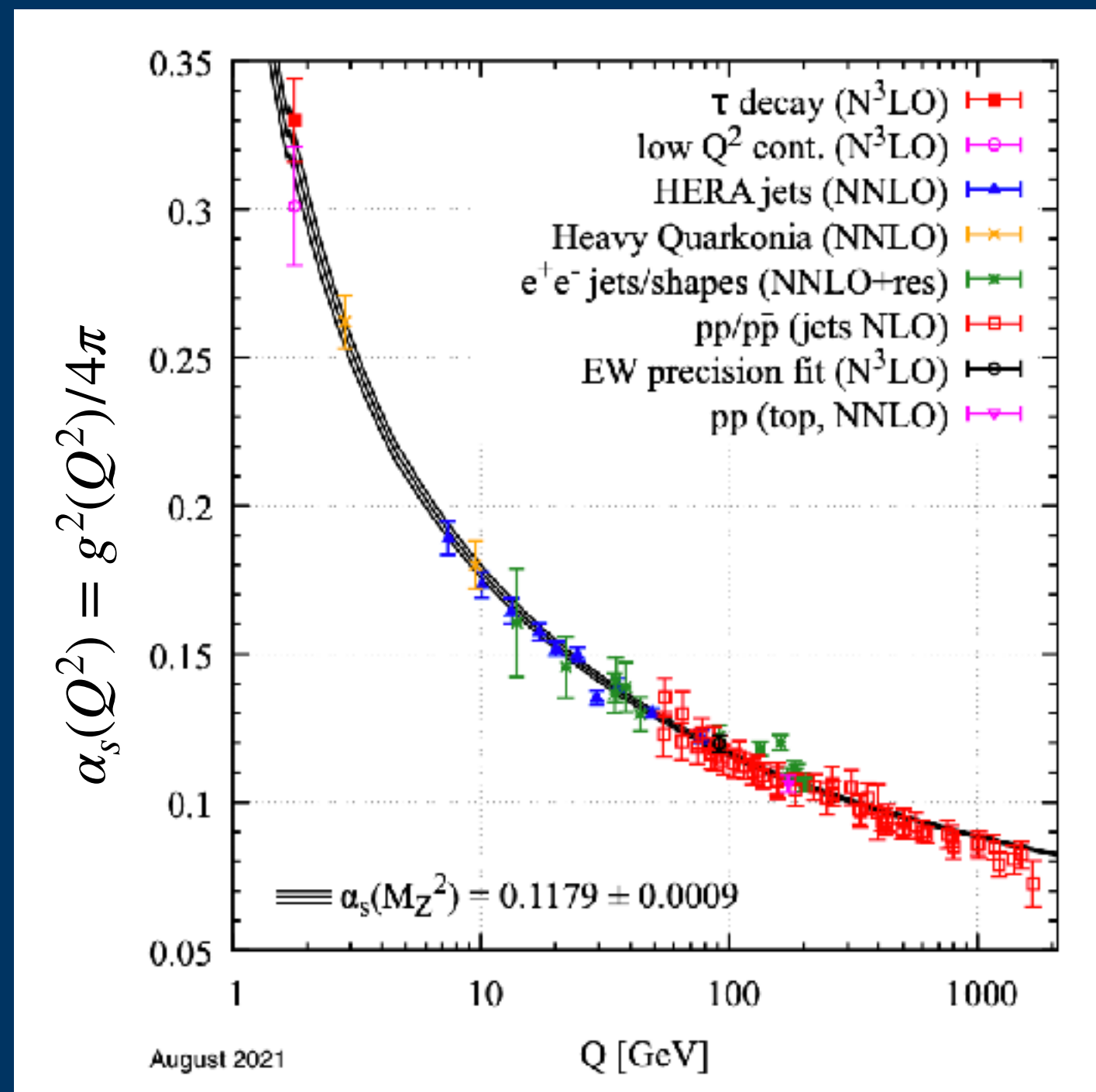
A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathcal{L} = \sum_f \bar{\psi}_{\alpha f} \left(i\gamma^\mu (\delta_{\alpha\beta} \partial_\mu - g (T_a G_\mu^a)_{\alpha\beta}) + m_f \right) \psi_{\beta f} - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

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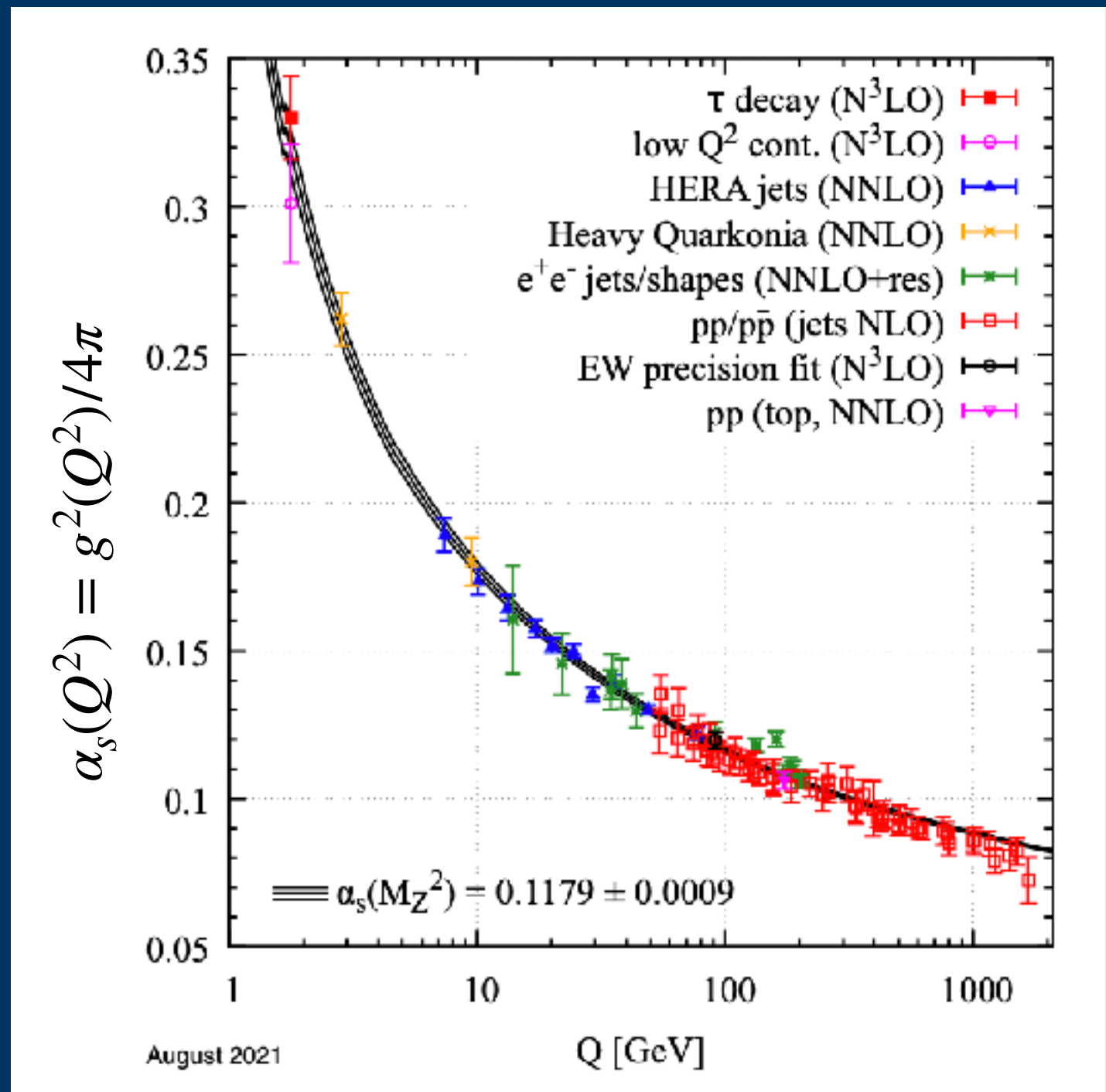
Running Coupling



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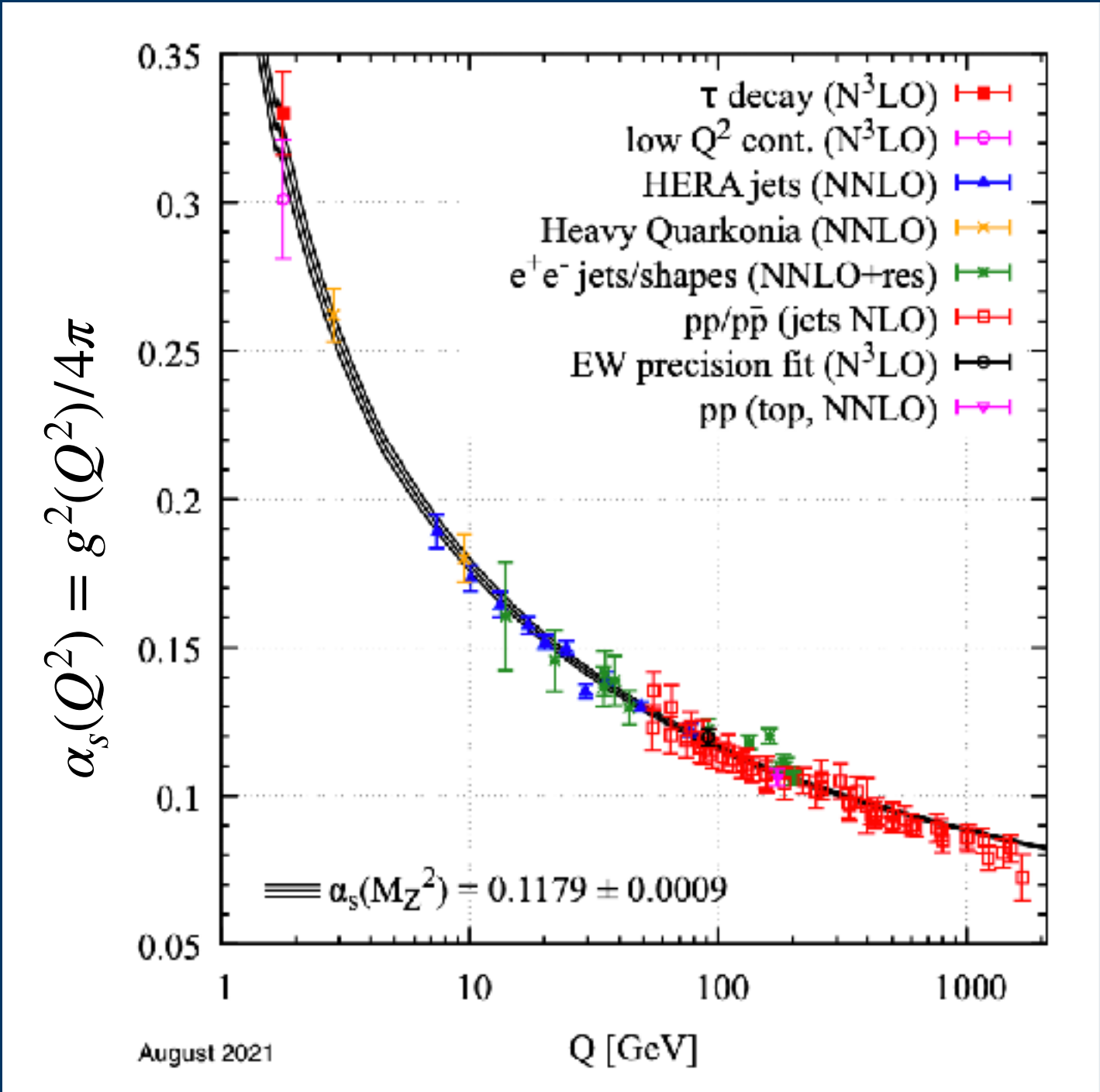
Quark Mass Matrix

$$\begin{pmatrix} m_u \approx 2.5 \text{ MeV} & 0 & 0 \\ 0 & m_d \approx 5 \text{ MeV} & 0 \\ 0 & 0 & m_s \approx 100 \text{ MeV} \end{pmatrix}$$

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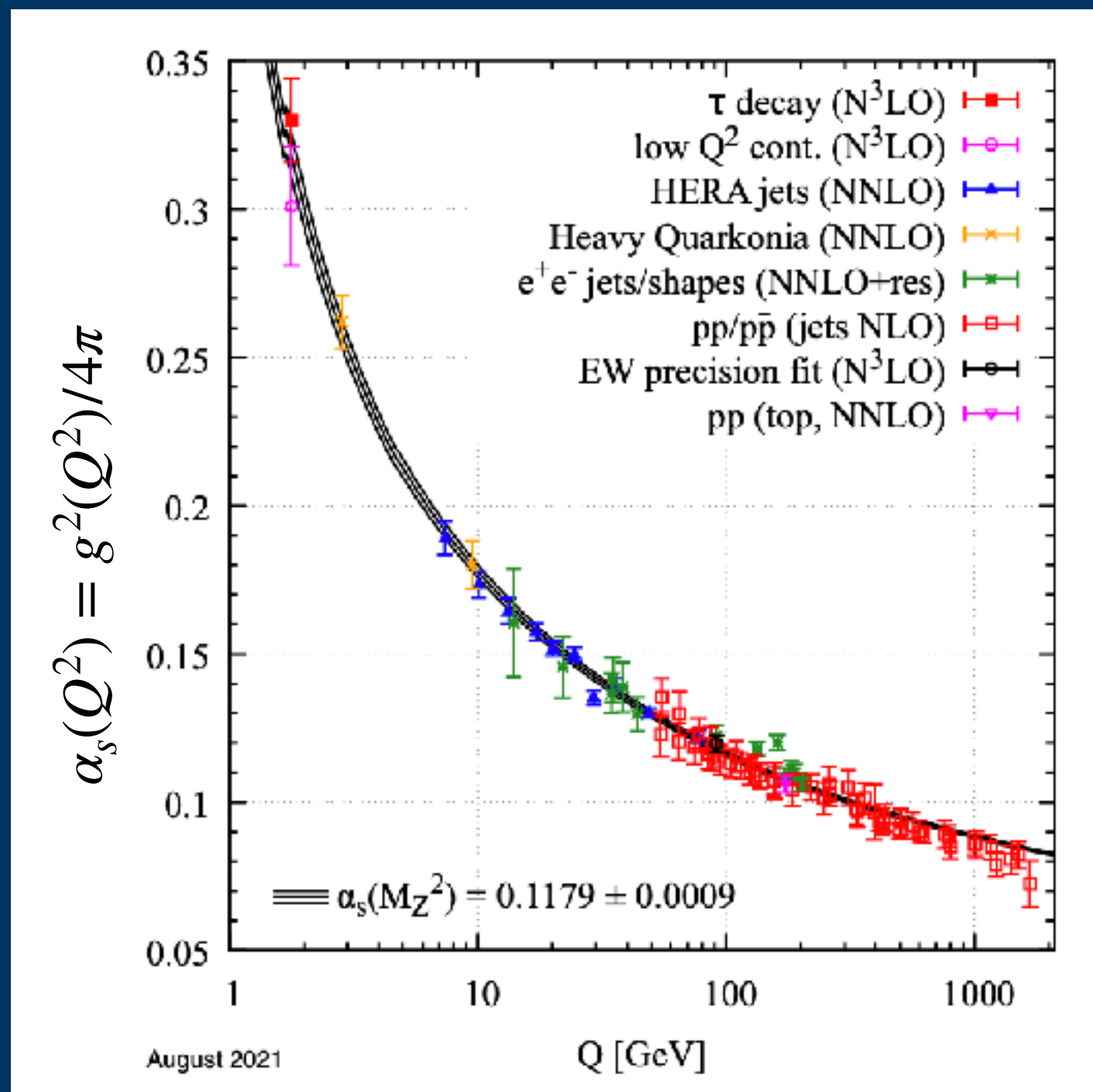
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θ -QCD

- Source of CP violation
- Induces neutron EDM:
 $d_n \approx 3 \times 10^{-16} \theta \text{ e cm}$
- Experimental bound:
 $d_n \lesssim 10^{-26} \text{ e cm}$
or $\theta \lesssim 10^{-10}$

θ and Axions

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$$

To explain $\theta < 10^{-10}$, θ was promoted to a dynamical quantity. New physics at a high scale introduced a new low energy field that relaxes to zero to minimize the free energy:

$$\theta = \frac{a}{f_a}$$

← Axion field

← A new high energy scale

The axion is a pseudo-scalar particle that arises as a Goldstone boson from the breaking of a new U(1) symmetry introduced by Peccei and Quinn.

At low energy,

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{4} a g_{a\gamma\gamma}^0 F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a} j_{a,0}^\mu$$

$$\text{where } g_{a\gamma\gamma}^0 = \frac{\alpha_{\text{em}}}{2\pi f_a} \frac{E}{N} \quad \text{and} \quad j_{a,0}^\mu = C_{q,0} \bar{q} \gamma^\mu \gamma_5 q$$

The θ -term can be encoded in a complex quark mass matrix

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \xrightarrow[\text{Tr}(Q_a) = 1]{\begin{bmatrix} u \\ d \\ s \end{bmatrix} \rightarrow \exp\left(i\frac{\theta}{2}Q_a\right) \begin{bmatrix} u \\ d \\ s \end{bmatrix}} M_q \rightarrow M_q \exp(i\theta Q_a)$$

where a convenient choice is

$$Q_a = \frac{M_q^{-1}}{\text{Tr } M_q^{-1}} = \frac{1}{m_u + m_d} \begin{pmatrix} m_d & & \\ & m_u & \\ & & 0 \end{pmatrix} + \mathcal{O}[m_u/m_s, m_d/m_s]$$

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Mass Terms in the Chiral Lagrangian

$$\sum_i m_i \bar{q}_i q_i \rightarrow \sum_{i,j} \bar{q}_{Ri} M_{ij} q_{Lj} + \text{h.c.}$$

Since chiral symmetry is broken spontaneously, in the low energy EFT, chiral invariance is implemented by (treating M_{ij} as a spurion field):

$$M_{ij} \rightarrow R M_{ij} L^\dagger$$

Parametrizing the excitations (Goldstone bosons/pions) by $\Sigma(x) \equiv \exp\left(\frac{2i\pi(x)}{f_\pi}\right)$

The mass term in chiral perturbation theory is $\mathcal{L}_M = c\Lambda f_\pi^2 \text{Tr}[M\Sigma] + \text{h.c.}$

Mass Terms in the Chiral Lagrangian

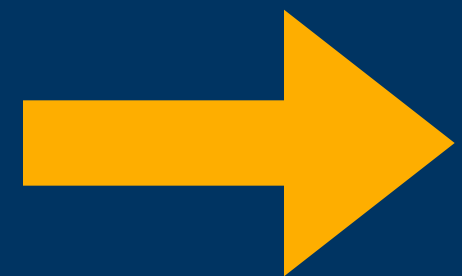
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$$m_\pi^2 = c\Lambda(m_u + m_d) = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2} (m_u + m_d)$$

Axion Mass and Energy

Incorporating the transformed quark mass matrix M_q in Chiral Perturbation Theory we can study the impact of axions on low energy nuclear and axion physics.

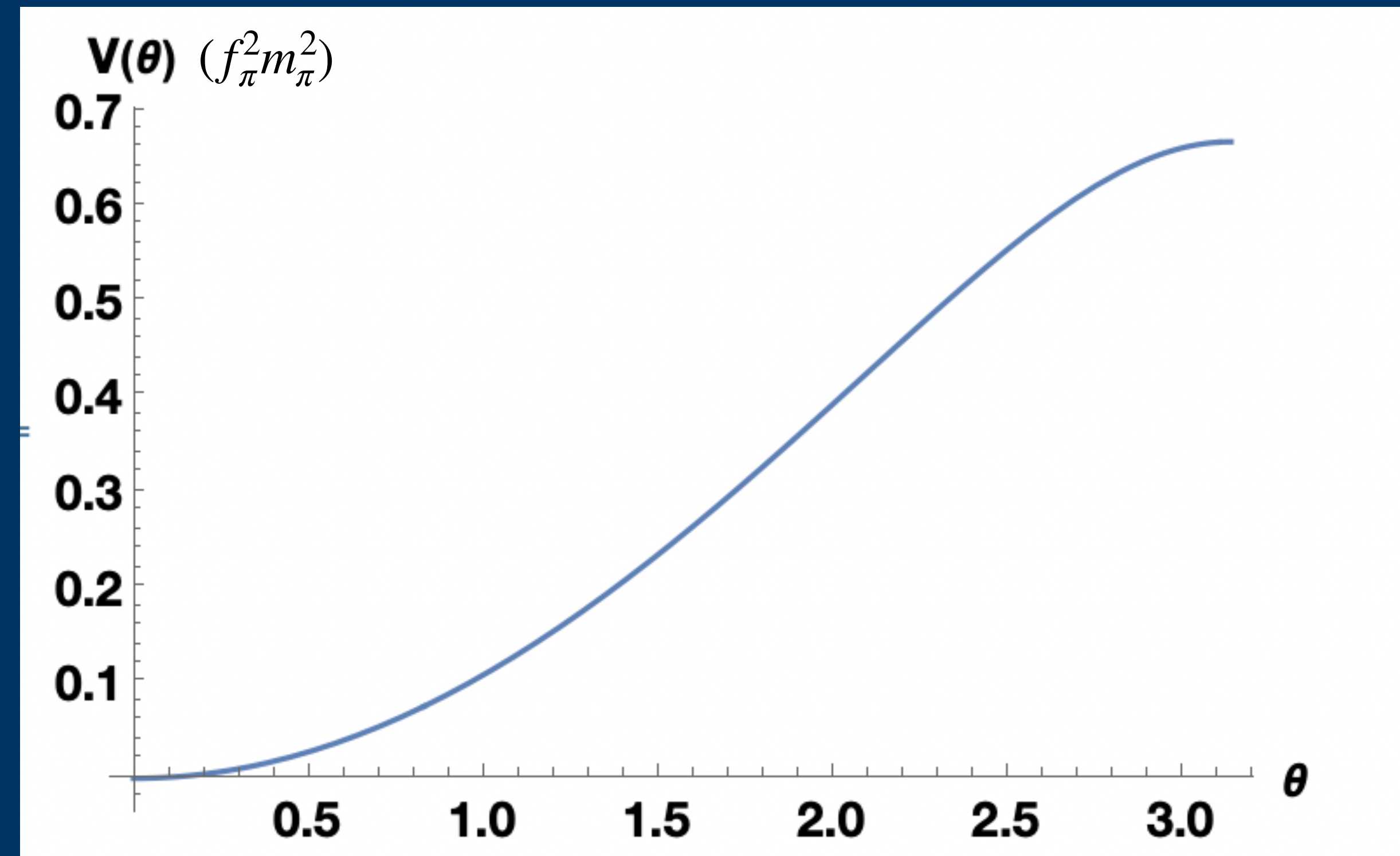
This leads to an axion mass which can be calculated from Chiral Perturbation Theory

$$m_a^2 = \frac{f_\pi^2}{f_a^2} \left(\frac{m_u m_d}{(m_u + m_d)^2} \right) m_\pi^2$$

And a corresponding contribution to the energy density or an axion potential

$$V\left(\theta = \frac{a}{f_a}\right) = f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left[\frac{\theta}{2} \right]} \right]$$
$$= \frac{1}{2} f_a^2 m_a^2 \theta^2 + \dots$$

Which is minimized at $\theta = 0$.



Exceptionally light QCD axions

There has been recent interest in more exotic scenarios involving a large number of BSM gauge fields that also couple to the QCD axion.

In these scenarios the axion potential takes the form

$$V\left(\theta = \frac{a}{f_a}\right) = \epsilon f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left[\frac{\theta}{2} \right]} \right]$$

where the new parameter $\epsilon < 1$ leads to a lighter axion

$$m_a^2 = \epsilon \frac{f_\pi^2}{f_a^2} \left(\frac{m_u m_d}{(m_u + m_d)^2} \right) m_\pi^2$$

$\epsilon \ll 1$ can be realized in some BSM axion models.

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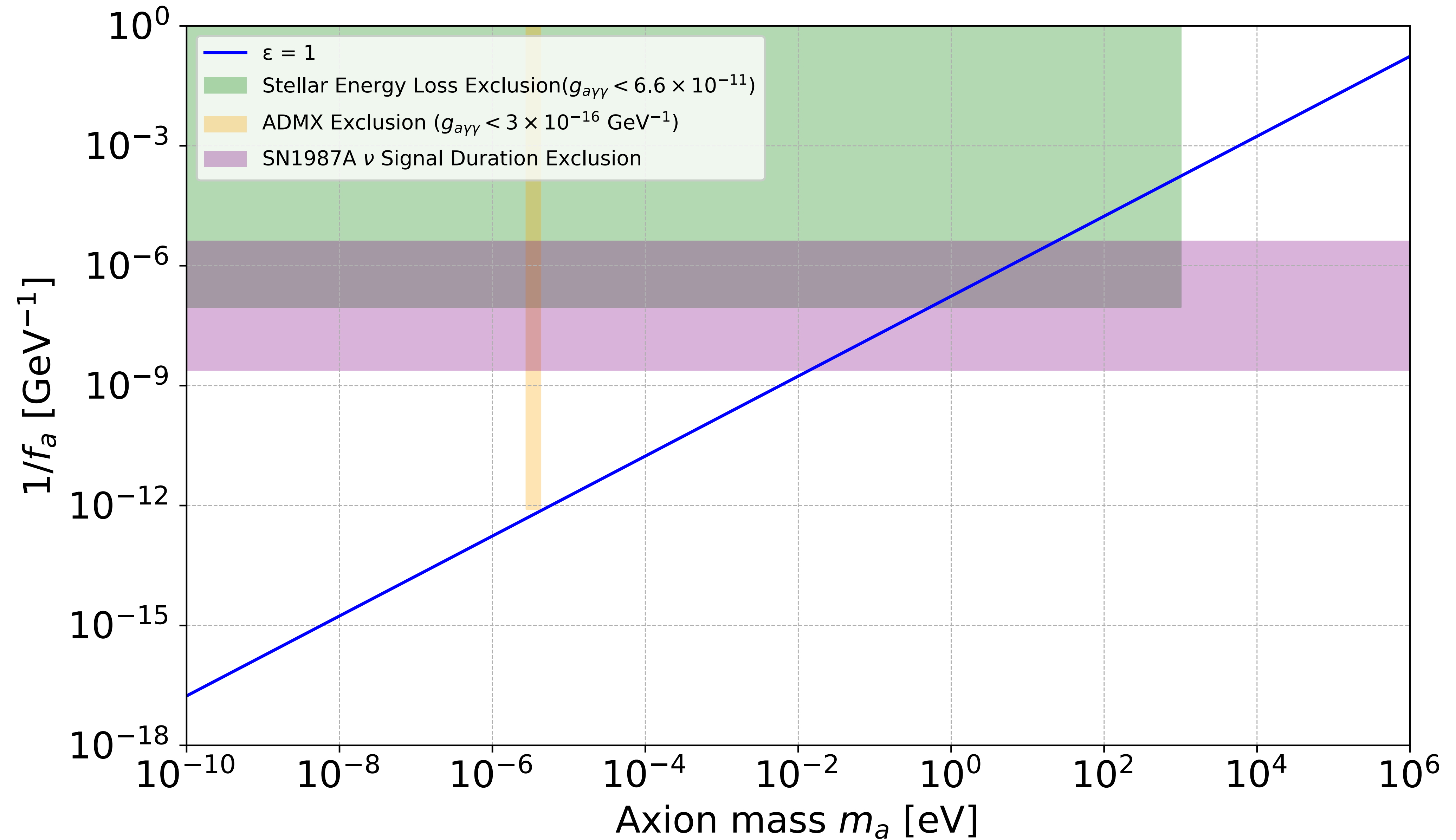
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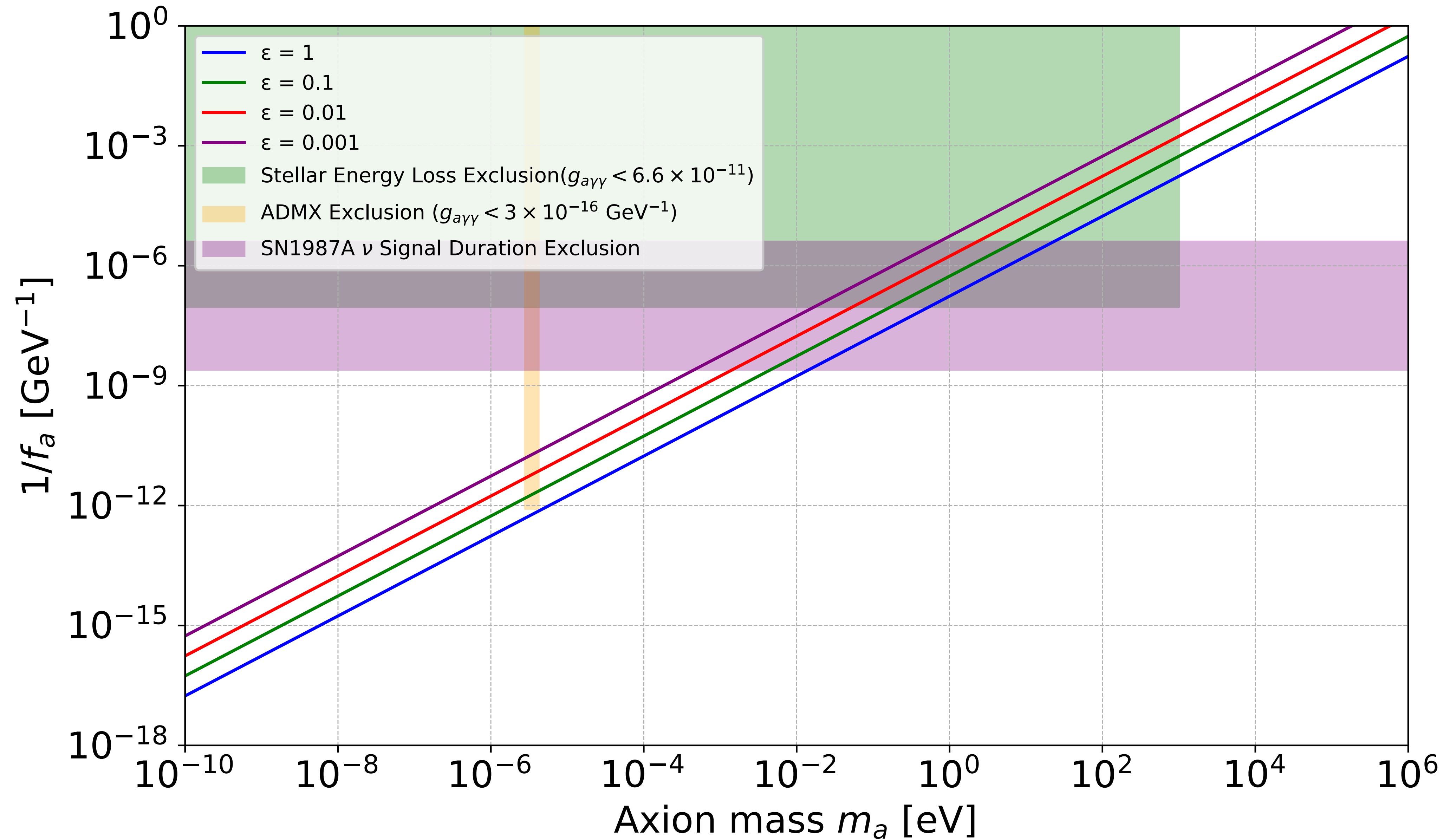
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QCD Axion & Lighter Cousins: Parameter Space



QCD Axion & Lighter Cousins: Parameter Space



Renewed Interest in Axions

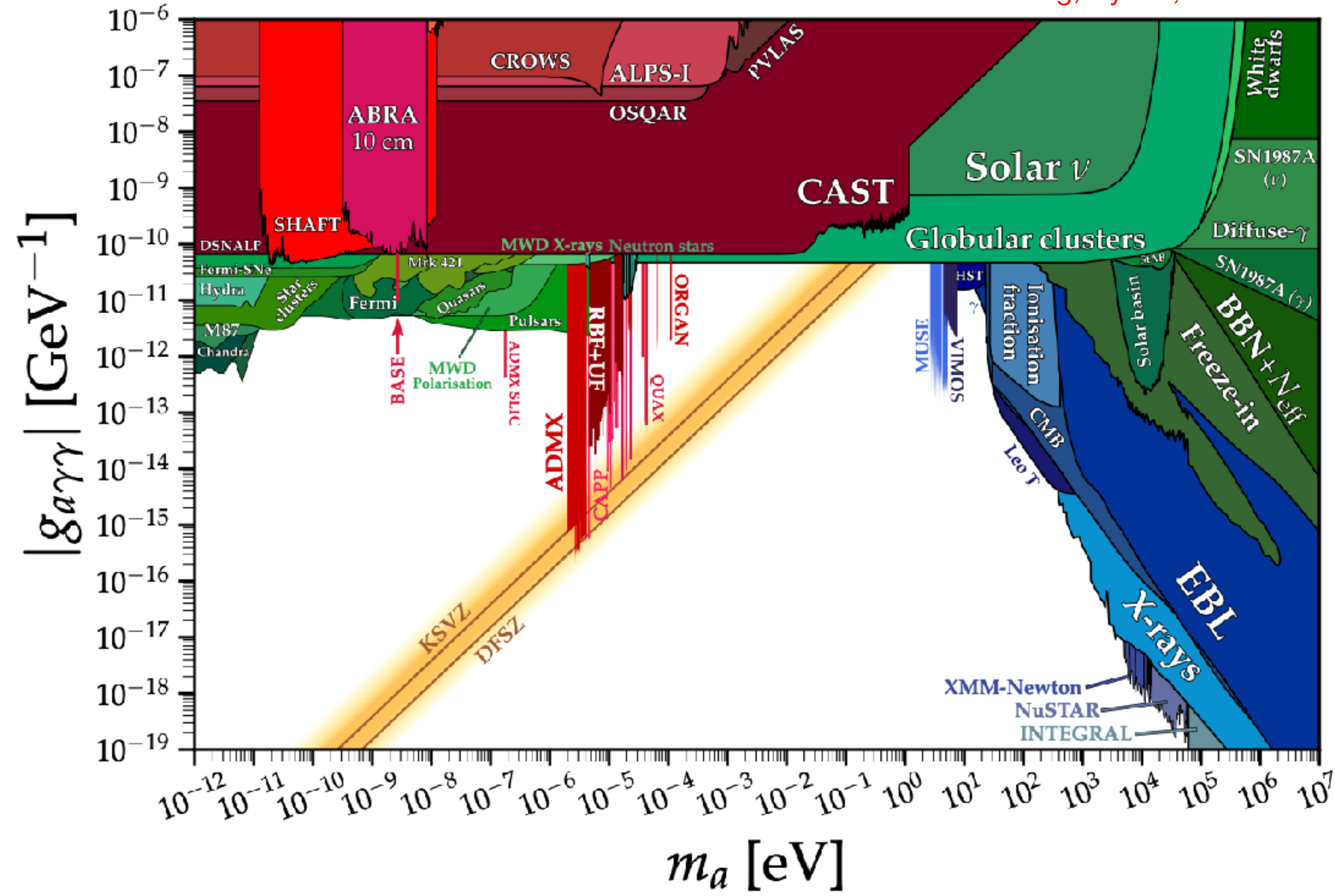
Experiments are mostly sensitive to

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi} \frac{1}{f_a} \left(\frac{E}{N} - 1.92 \right) \approx \frac{10^{-3}}{f_a}$$

E/N , a model-dependent parameter, can range from 0 to $44/3$.

Model	E/N
KSVZ	0
DFSZ	$8/3$

PDG 2023: Rosenberg, Rybka, Safdi.



Quark masses dependence of hadrons

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2} (m_u + m_d) + \dots \quad \longrightarrow$$

$$K_{m_\pi} = \frac{m_q}{m_\pi} \frac{\delta m_\pi}{\delta m_q} \simeq 0.5$$

$$m_n = m_0 + \underset{\substack{\uparrow \\ \simeq 50 \pm 10 \text{ MeV}}}{\sigma_{\pi n}} \frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}} + \dots \quad \longrightarrow$$

$$K_{m_n} = \frac{m_q}{m_n} \frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n} \frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05$$

Quark masses dependence of hadrons

$$\begin{aligned}
 m_\pi^2 &= -\frac{\langle \bar{q}q \rangle}{3f_\pi^2} (m_u + m_d) + \dots \quad \longrightarrow \quad K_{m_\pi} = \frac{m_q}{m_\pi} \frac{\delta m_\pi}{\delta m_q} \simeq 0.5 \\
 m_n &= m_0 + \underset{\substack{\uparrow \\ \simeq 50 \pm 10 \text{ MeV}}}{\sigma_{\pi n}} \frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}} + \dots \quad \longrightarrow \quad K_{m_n} = \frac{m_q}{m_n} \frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n} \frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05
 \end{aligned}$$

Masses of heavier vector mesons are relatively insensitive to the quark mass.

$$K_{m_\rho} = \frac{m_q}{m_\rho} \frac{\delta m_\rho}{\delta m_q} \simeq 0.05$$

Quark masses dependence of hadrons

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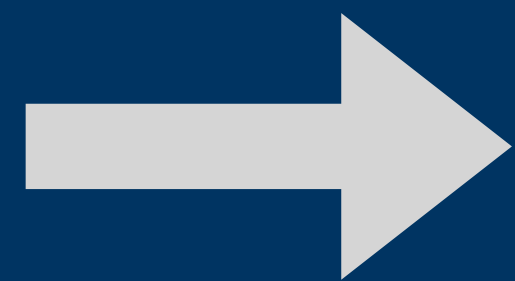
Mass of the scalar sigma meson

$$K_{m_\sigma} = \frac{m_q}{m_\sigma} \frac{\delta m_\sigma}{\delta m_q} \gtrsim 0.1$$

Hadrons at $\theta \neq 0$

Because $M_q \rightarrow M_q \exp(2i\theta Q_a)$
the pion mass decreases with θ :

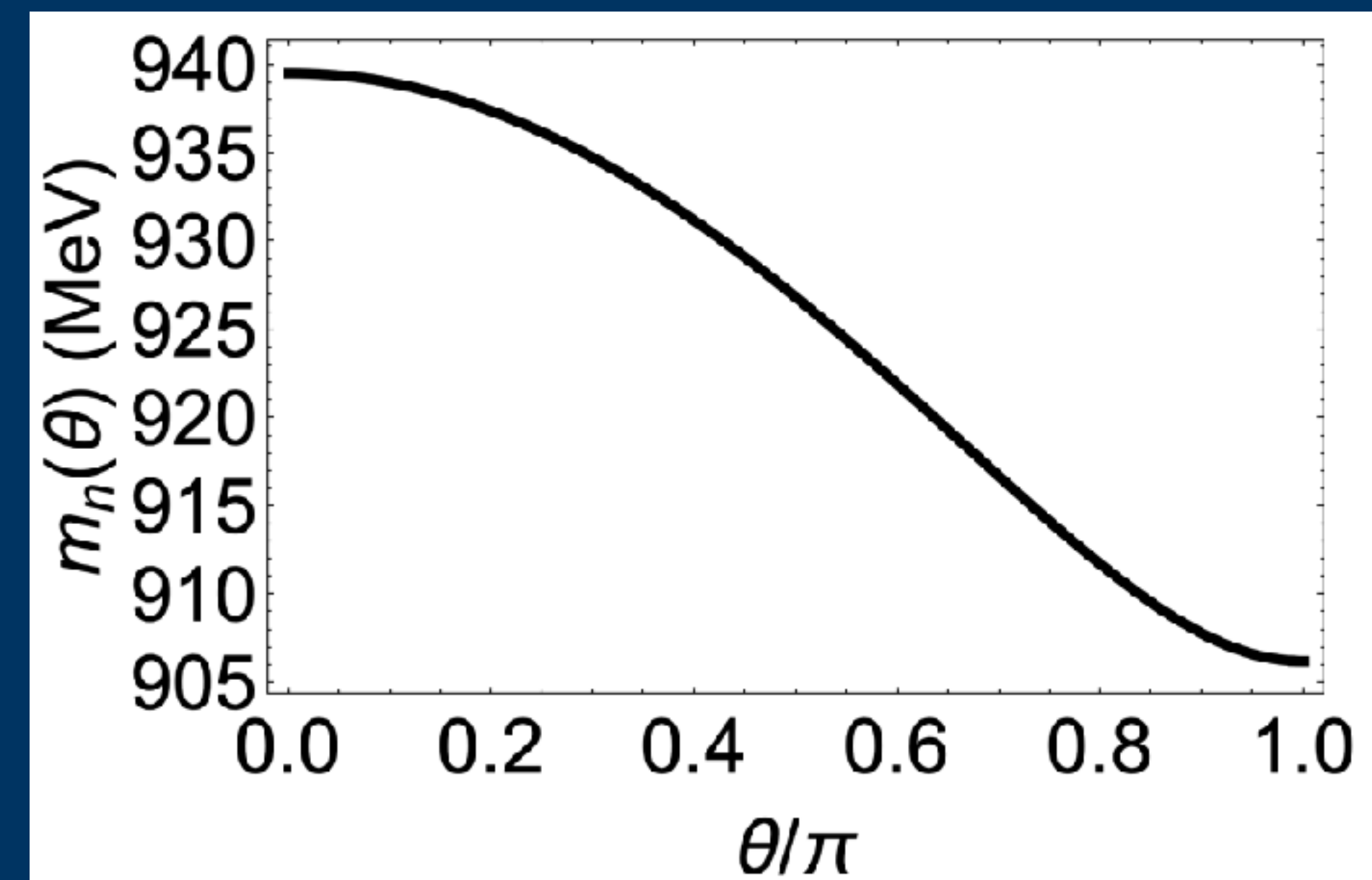
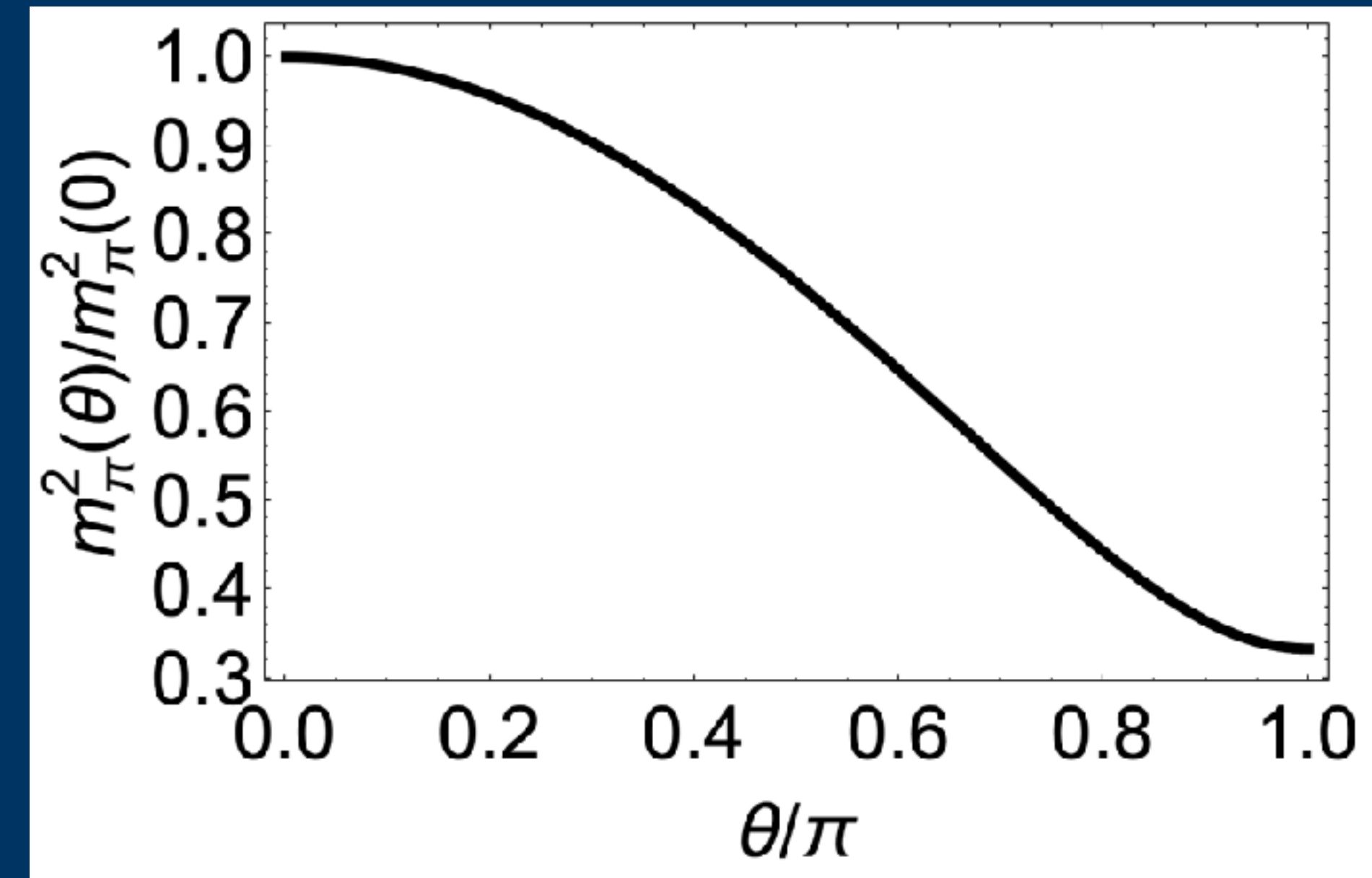
$$m_\pi^2(\theta) = m_\pi^2(\theta = 0) \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left[\frac{\theta}{2} \right]}$$



$$\frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} = \frac{m_d - m_u}{m_d + m_u} \approx \frac{1}{3}$$

The resulting decrease in the nucleon mass

$$m_n(\theta) = m_0 + \underset{\substack{\uparrow \\ \simeq 50 \text{ MeV}}}{\sigma_{\pi n}} \frac{m_\pi^2(\theta)}{m_\pi^2(\theta = 0)} + \dots$$



Axion Condensation

The decrease in the nucleon mass $m_n(\theta) = m_0 + \sigma_{\pi n} \frac{m_\pi^2(\theta)}{m_\pi^2(\theta=0)} + \dots$

favors a first-order transition to a ground state with $\theta = \pi$

Neglecting interactions, the energy gain per nucleon is

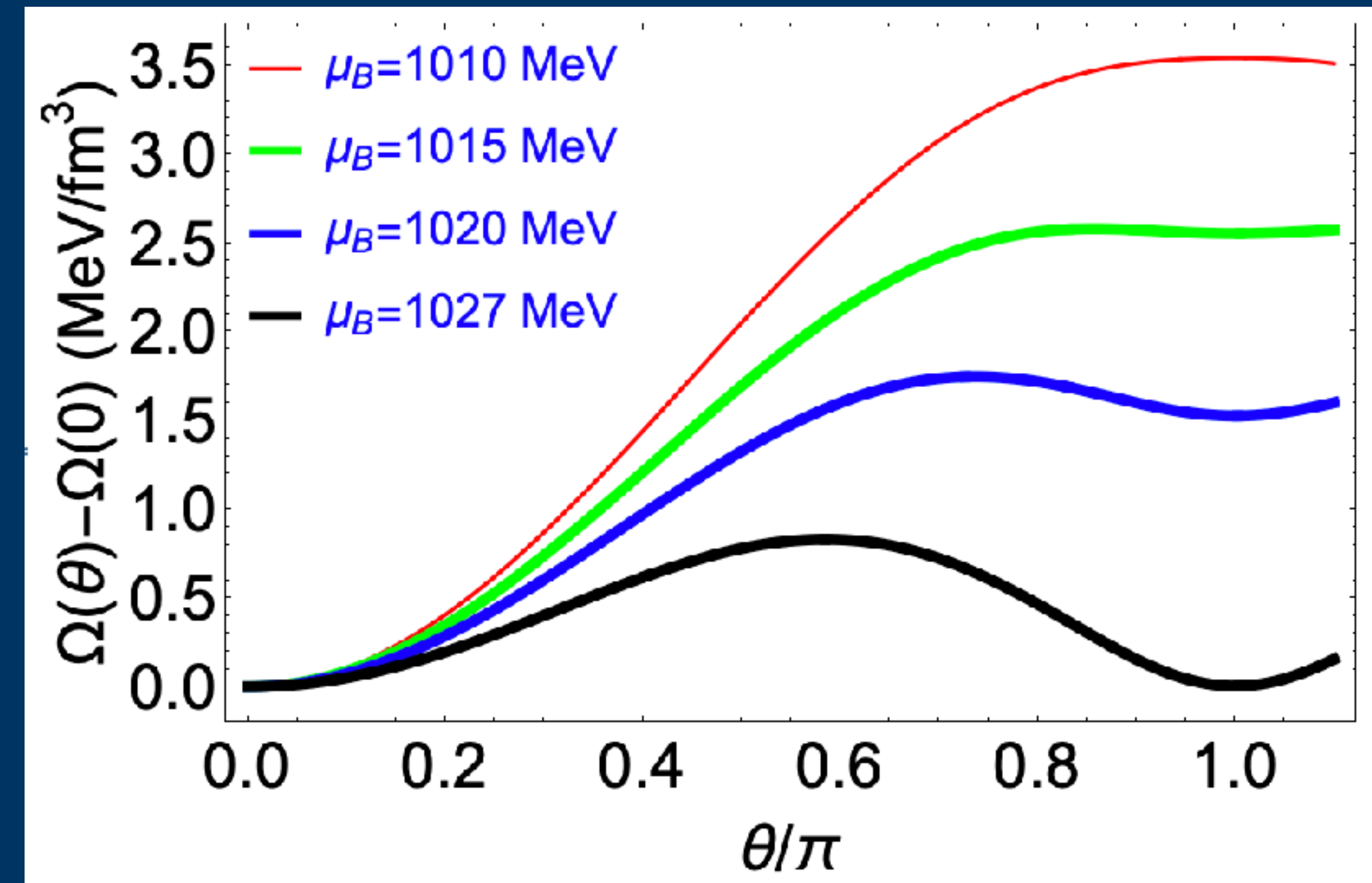
$$\Delta E \simeq \sigma_{\pi n} \left(1 - \frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} \right) \simeq \frac{2}{3} \sigma_{\pi n}$$

The energy cost (due to axion potential) per nucleon is

$$\Delta E = \frac{V(\theta = \pi)}{n_B} \simeq \frac{2}{3} \frac{f_\pi^2 m_\pi^2}{n_B}$$

The Condensation occurs when $\sigma_{\pi N} n_B \gtrsim f_\pi^2 m_\pi^2$

For $\sigma_{\pi n} = 50 \text{ MeV}$ $\longrightarrow$ $n_B^c \simeq 1.9 n_{\text{sat}}$ (First-order) $n_B^c \simeq 2.6 n_{\text{sat}}$ (second-order)



But, nuclear interactions are important
at $n_B \simeq 2n_{\text{sat}}$

If nuclear interactions become more attractive at
 $\theta = \pi$, when $m_\pi \simeq 82$ MeV:

$$n_B^c < 2 n_{\text{sat}}$$

Axions would condense inside neutron stars.
For any value of f_a !

Condensation of Light QCD axions is Robust

Light axions would condense when $\sigma_{\pi N} n_B \gtrsim \epsilon f_\pi^2 m_\pi^2$

Or when $n_B^c = \epsilon \frac{f_\pi^2 m_\pi^2}{\sigma_{\pi n}} \longrightarrow n_B^c \simeq 2\epsilon n_{\text{sat}}$

For $\epsilon \ll 1$ condensation happens at low baryon density —one can neglect the role of nuclear interactions.

For $\epsilon < 0.1$ ordinary matter (nuclei) are only metastable.

Stability of the earth and sun require $\epsilon > 10^{-13}$, and

White Dwarfs need $\epsilon > 10^{-7}$

Back to QCD axions ($\epsilon = 1$)

Can we do nuclear physics at $m_\pi \simeq 82$ MeV?

What is the sign of

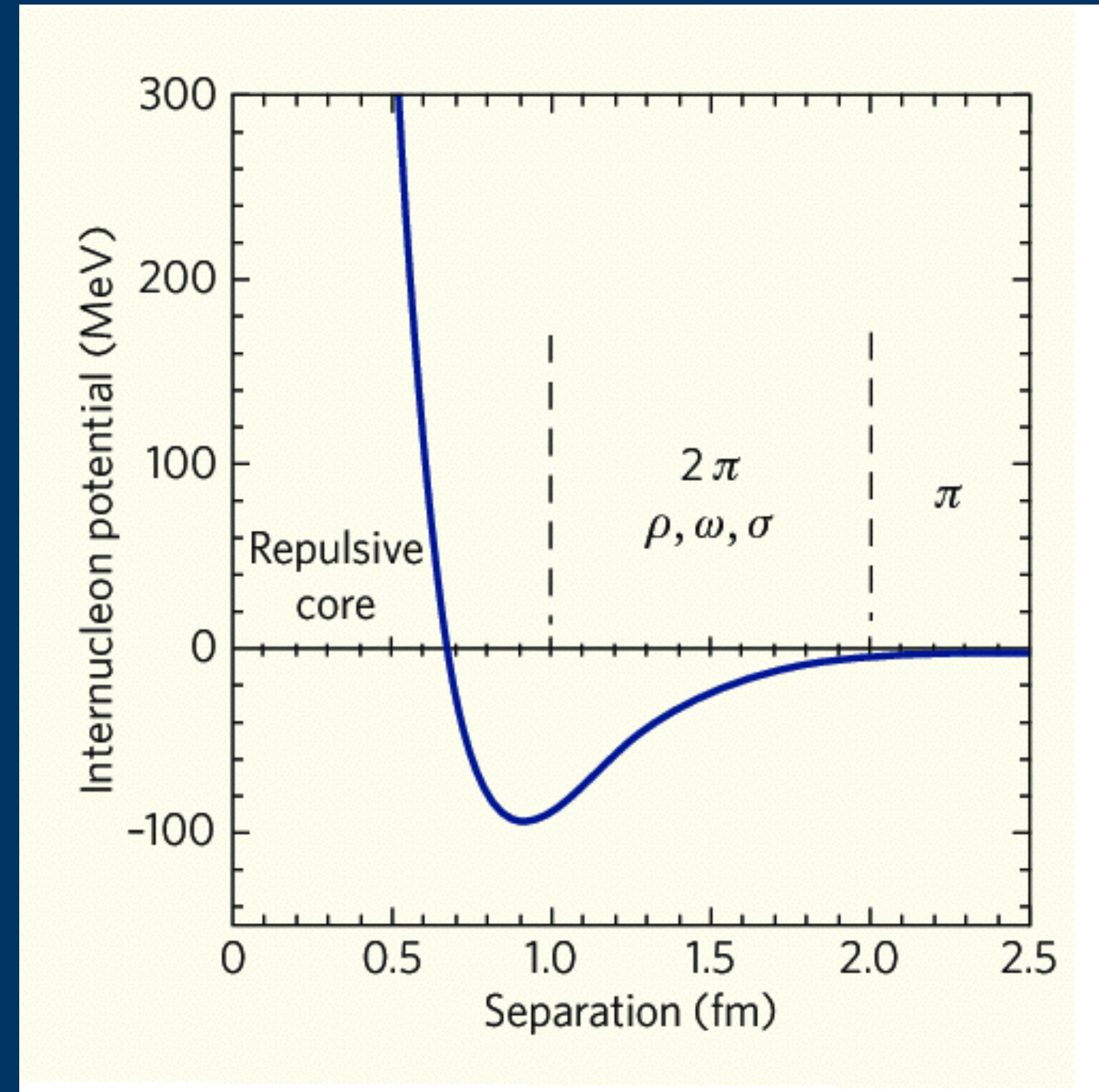
$$\Delta E_{\text{int}} = E_{\text{int}}(m_\pi = 82 \text{ MeV}) - E_{\text{int}}(m_\pi^{\text{phys}})$$

at $n_B \lesssim 2 n_{\text{sat}}$?

How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

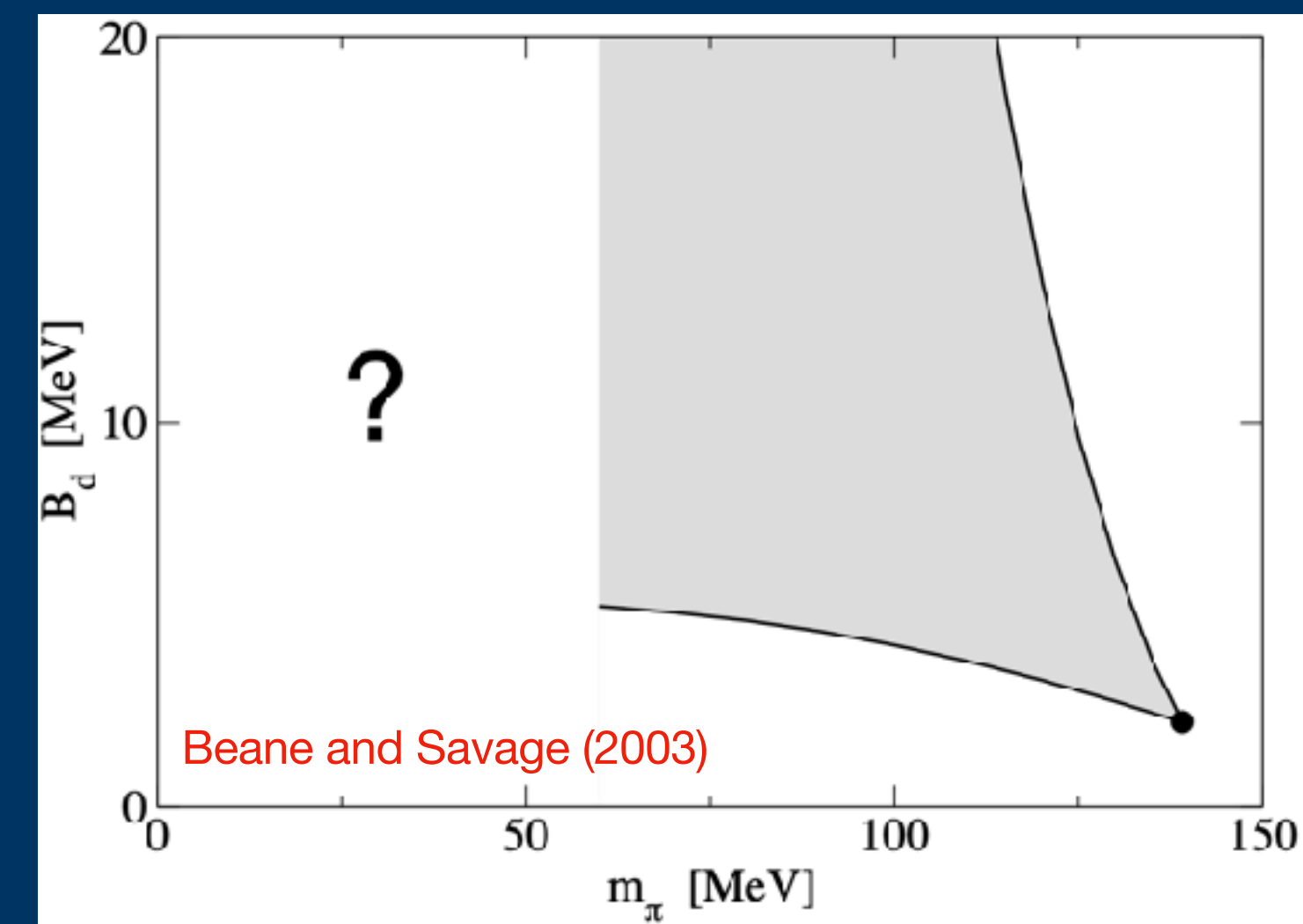
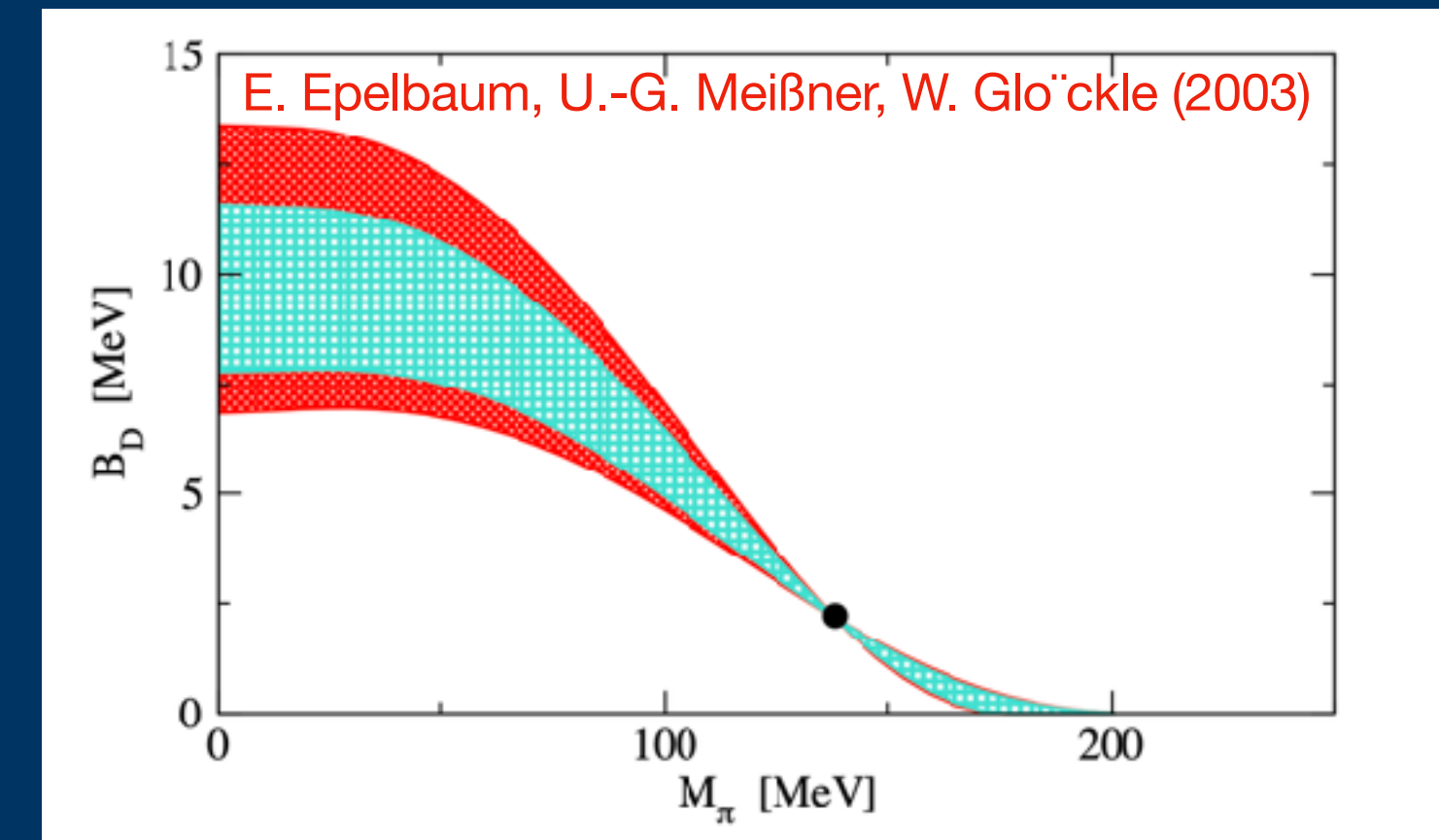
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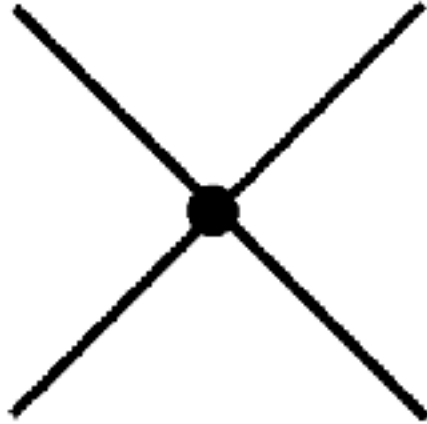
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- The effect on pion-exchange is easy to implement, but effects at short distances are not.
- Models with reasonable assumptions suggest that the deuteron binding energy increases with decreasing pion mass.

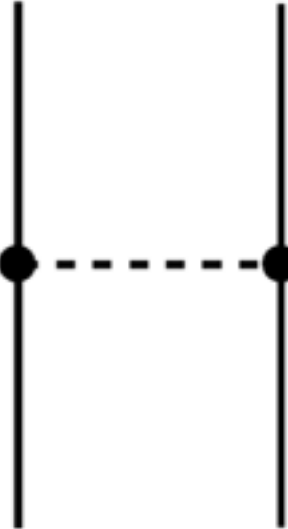


Quark (pion) mass-dependence of NN interaction in EFT

$$V_{\text{LO}}(q) =$$


 $C_0 + D_2 m_\pi^2$

+


 $\frac{g_A^2}{4f_\pi^2} \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{\mathbf{q}^2 + m_\pi^2} \tau_1 \cdot \tau_2$

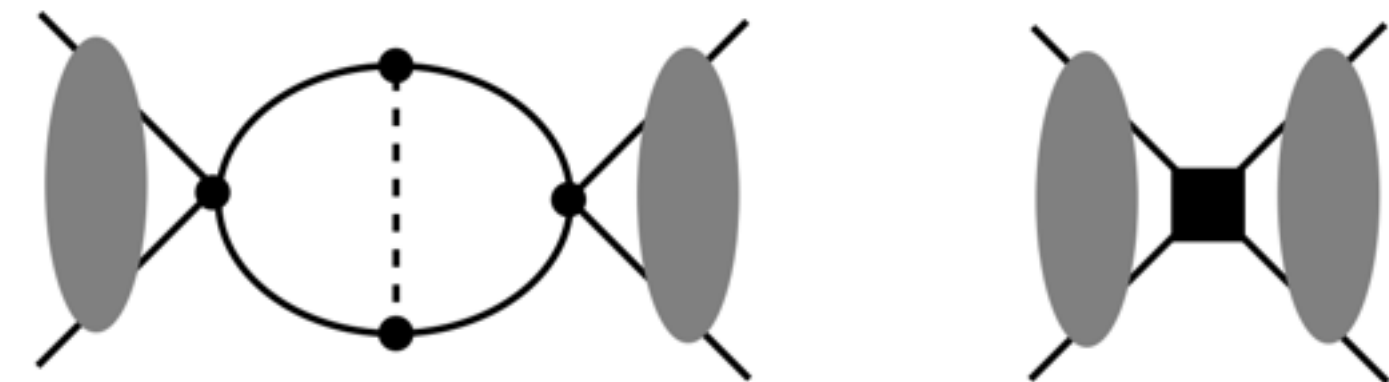
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$$V_{\text{LO}}(q) = \begin{array}{c} \text{Diagram: Four lines meeting at a central point} \\ C_0 + D_2 m_\pi^2 \end{array} + \begin{array}{c} \text{Diagram: Two vertical lines connected by a horizontal dashed line} \\ \frac{g_A^2}{4f_\pi^2} \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{\mathbf{q}^2 + m_\pi^2} \tau_1 \cdot \tau_2 \end{array}$$

Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:



$$\Lambda \frac{d}{d\Lambda} \left(\frac{D_2}{C_0^2} \right)_{\text{KSW}} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \longrightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \approx \frac{1}{4}$$

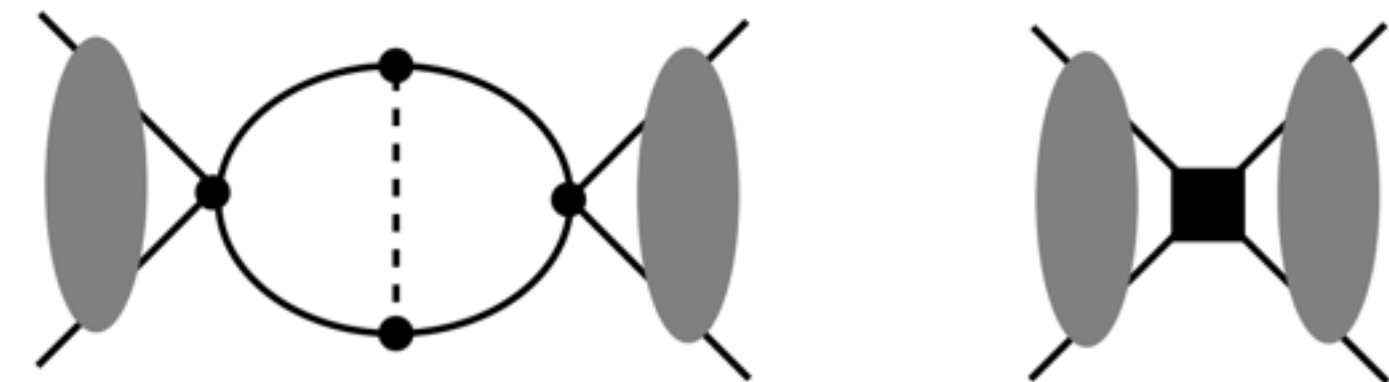
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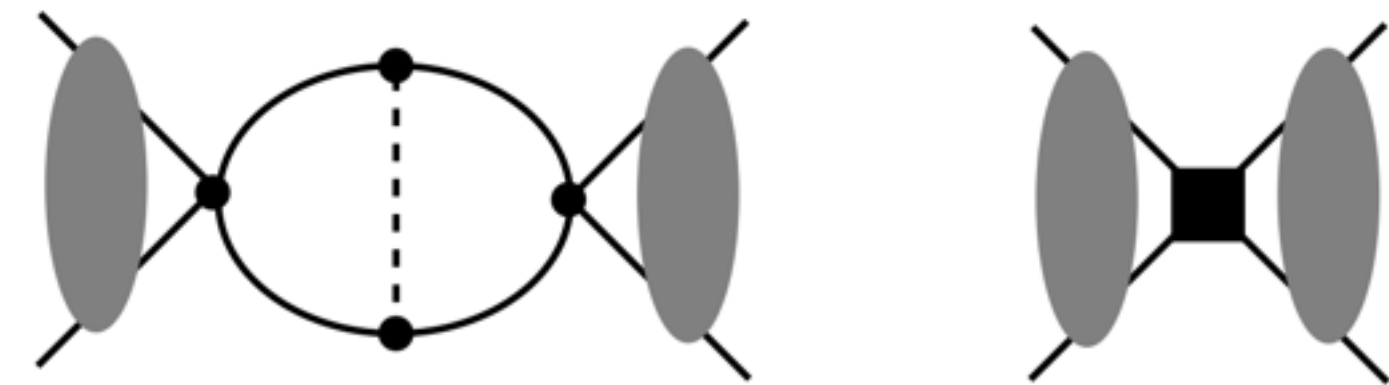
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Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration),

D_2 can be important.

RG suggests:

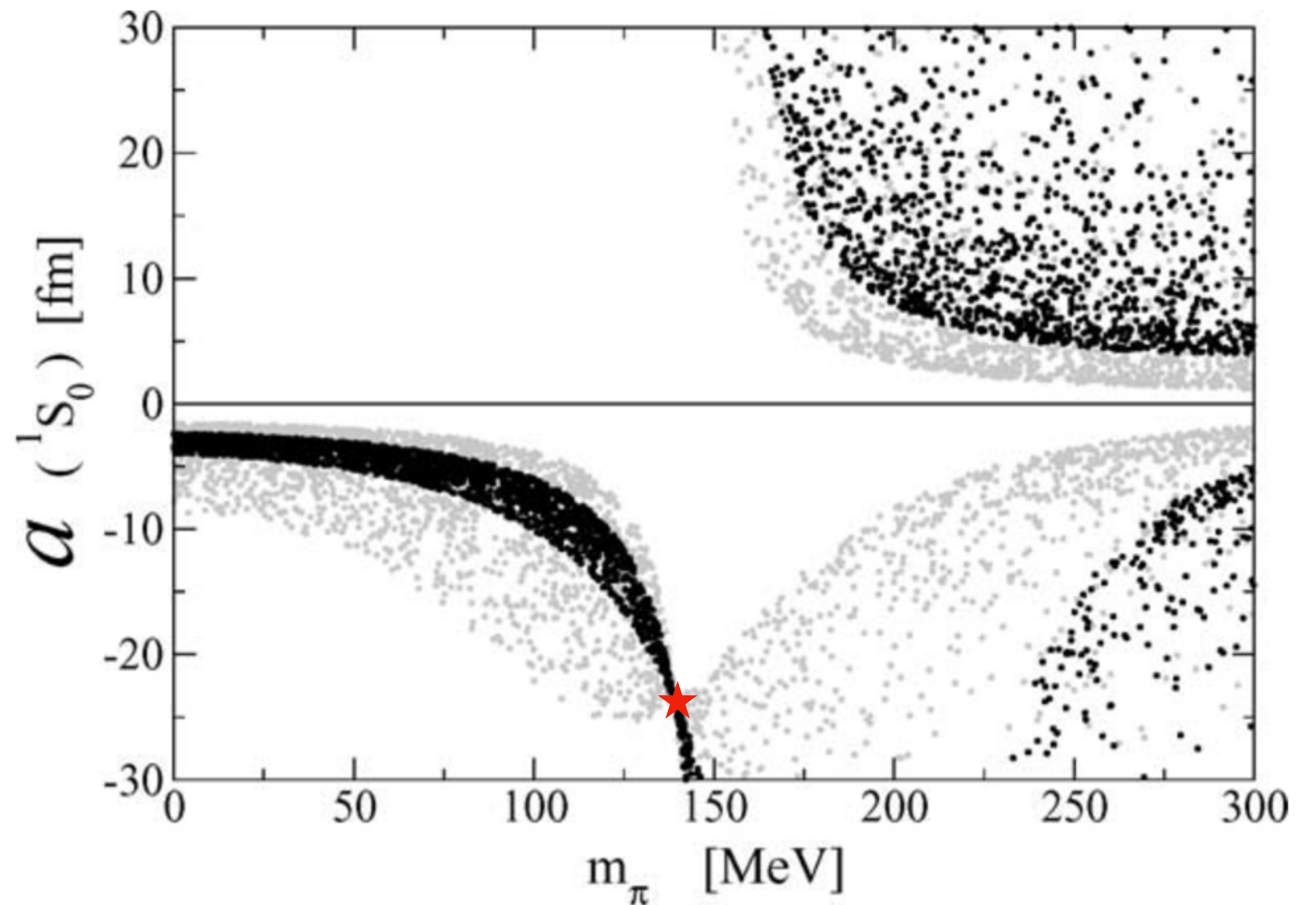
$$|D_2| \simeq \frac{C_0^2}{4} \approx \frac{1}{5f_\pi^4}$$

Variation over a smaller range:

$$-\frac{1}{5} < \eta = \frac{D_2 m_\pi^2}{C_0} < \frac{1}{5}$$

has a significant impact on s-wave observables.

S.R. Beane, M.J. Savage / Nuclear Physics A 717 (2003) 91–103



Pion Mass Dependence of Binding Energies

From resonance saturation and matching ChiEFT to the Bonn. potential model.

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Peláez (2013)

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A modest increase in the binding of nuclear matter at $\theta = \pi$!

Can interactions favor $\theta = \pi$ in neutron matter ?

- ChiEFT at N²LO, with simple assumptions about short-distance forces.

- $$m_n = m_0 + \sigma_{\pi n} \frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}}$$

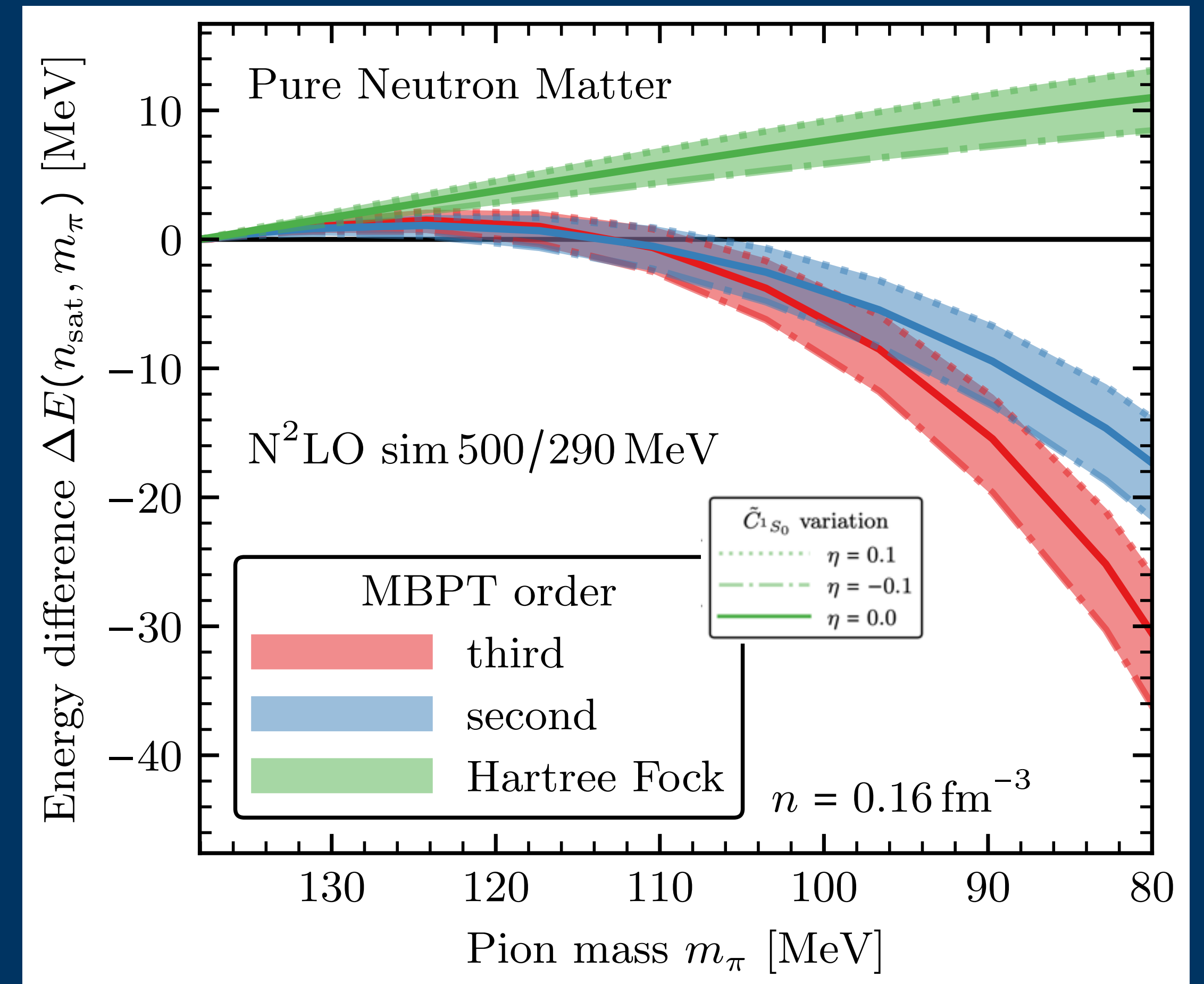
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$$-0.1 < \eta = \frac{D_2 m_\pi^2}{\tilde{C}_{1S_0}} < 0.1$$

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$$\Delta E_{\text{int}} = E_{\text{int}}(m_\pi) - E_{\text{int}}(m_\pi^{\text{phys}})$$

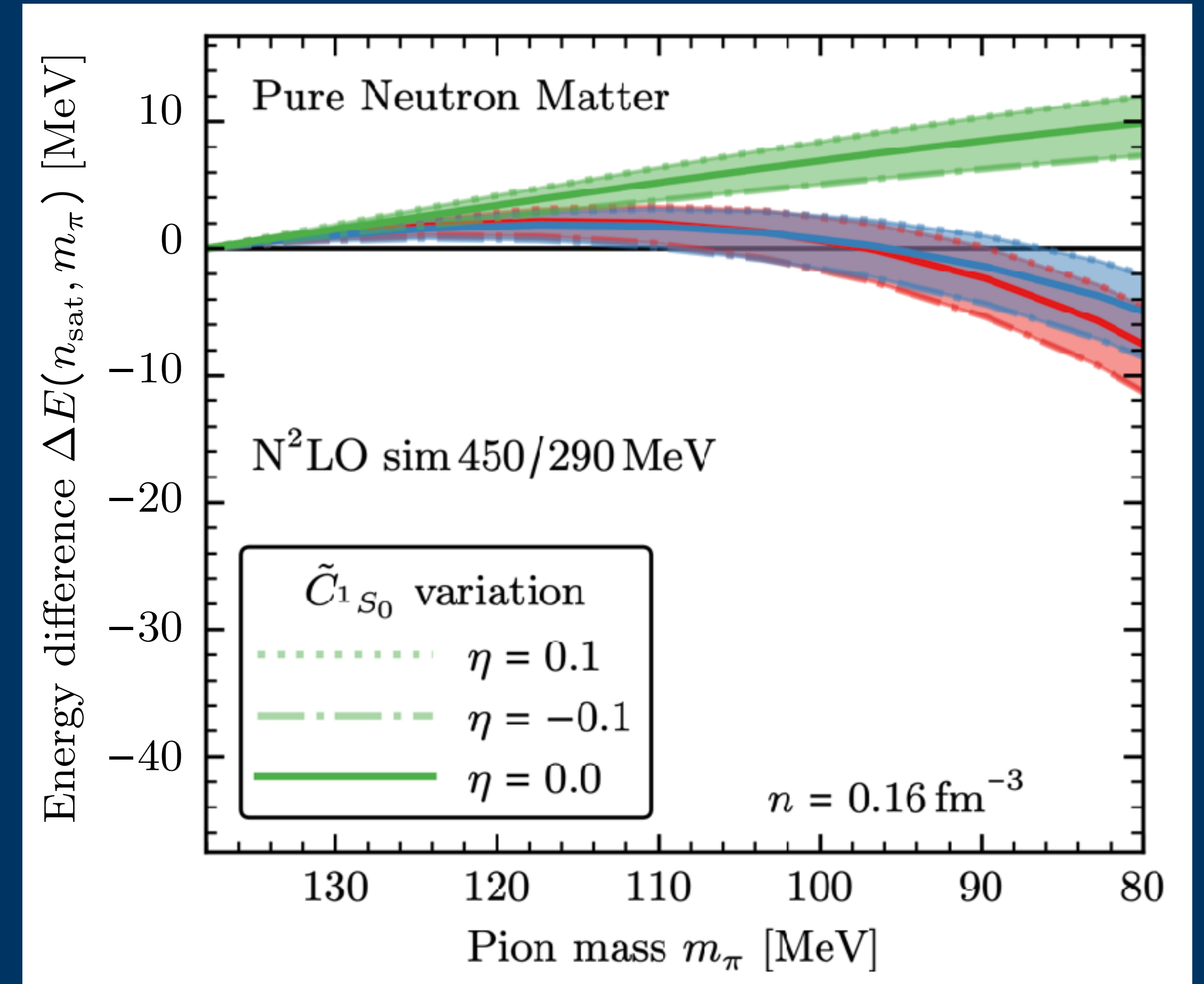


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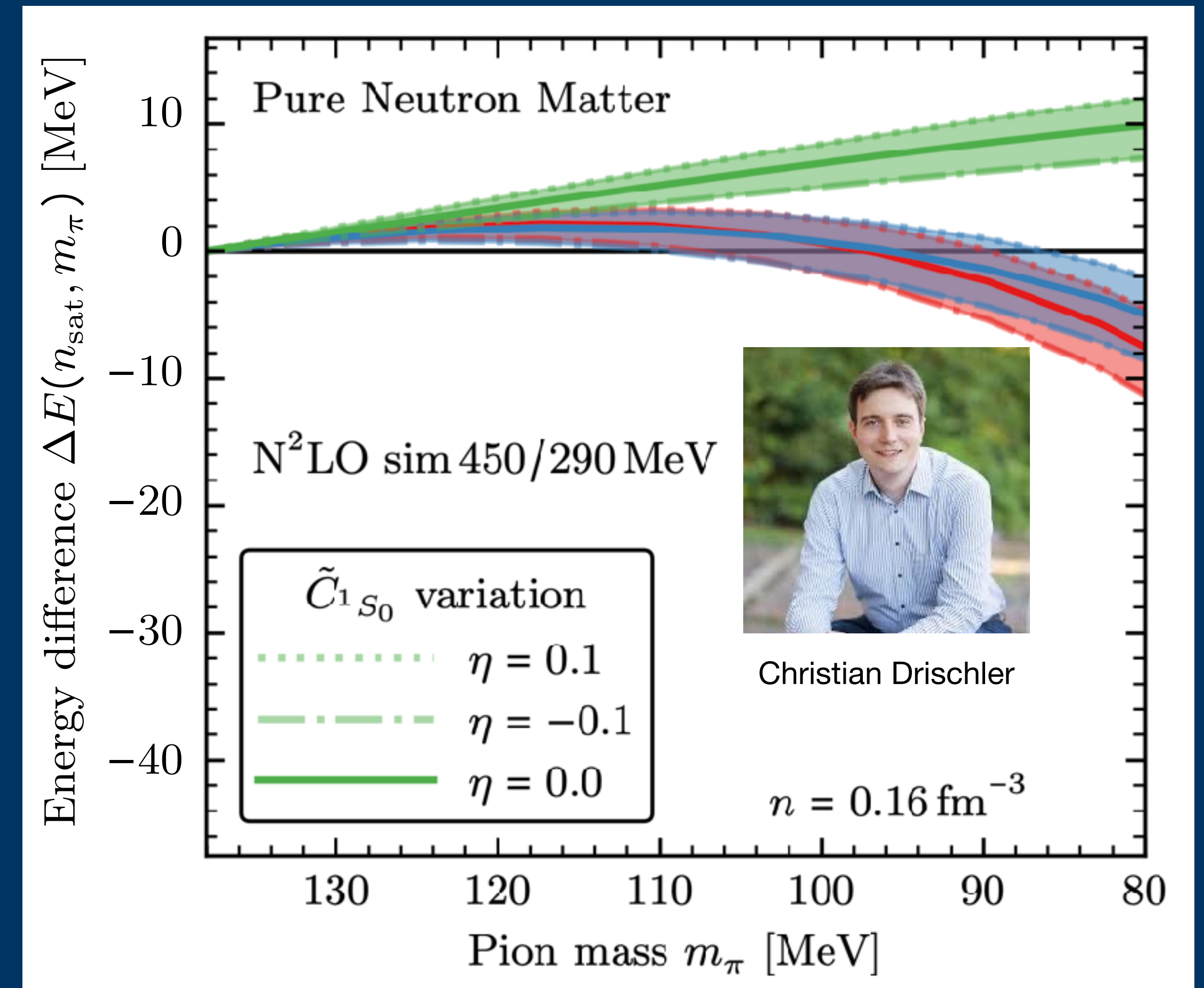
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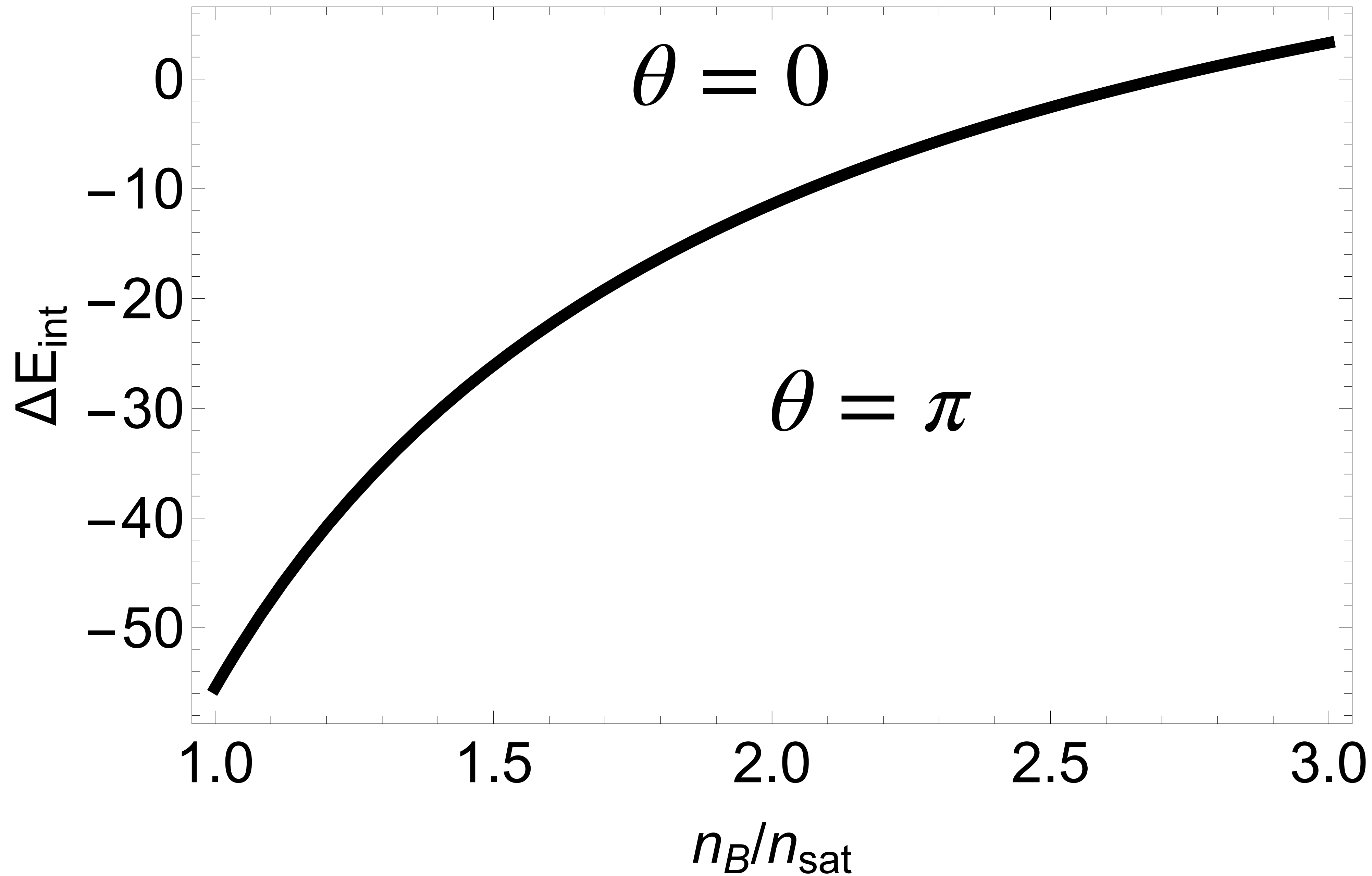
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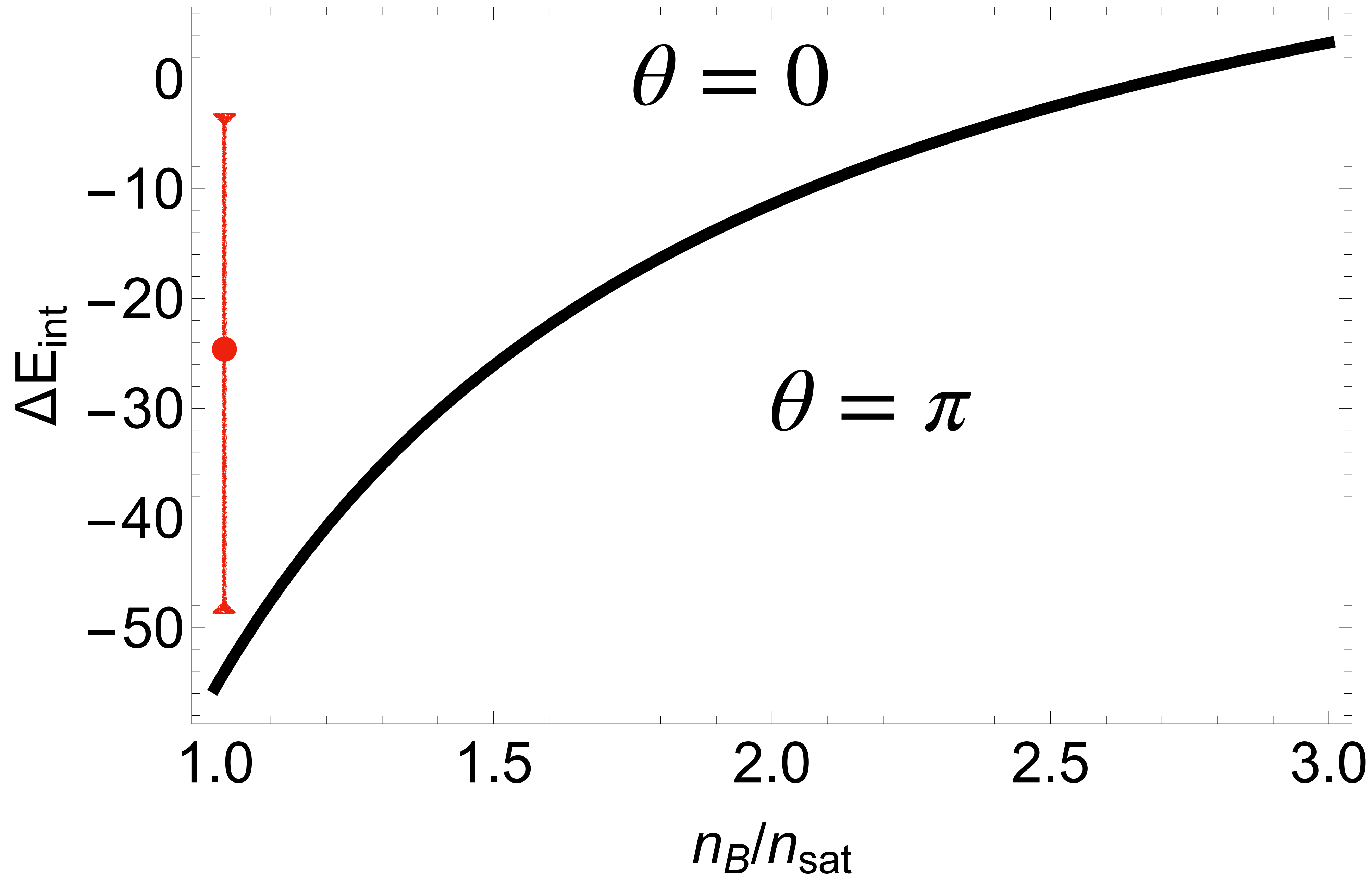
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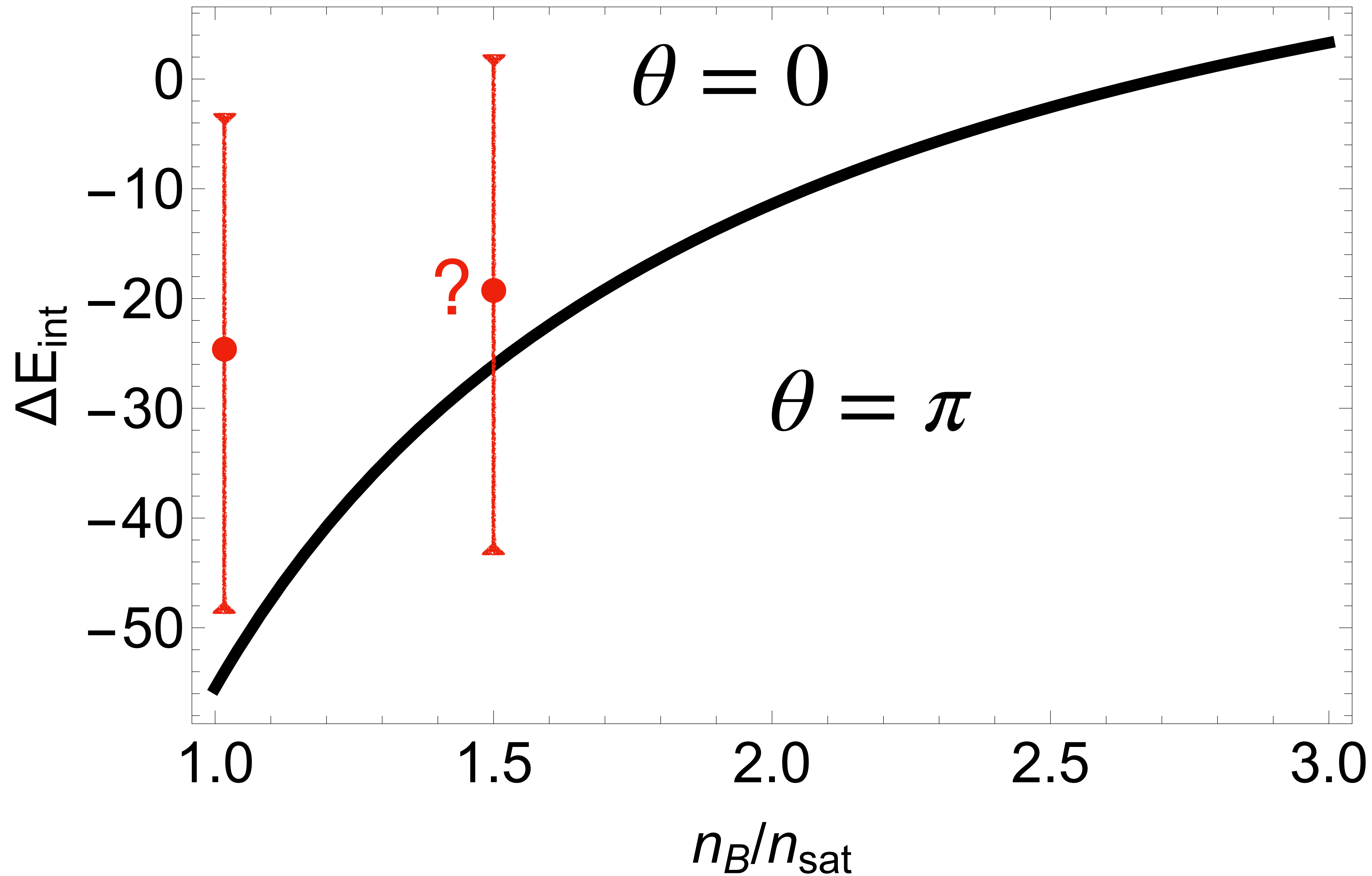
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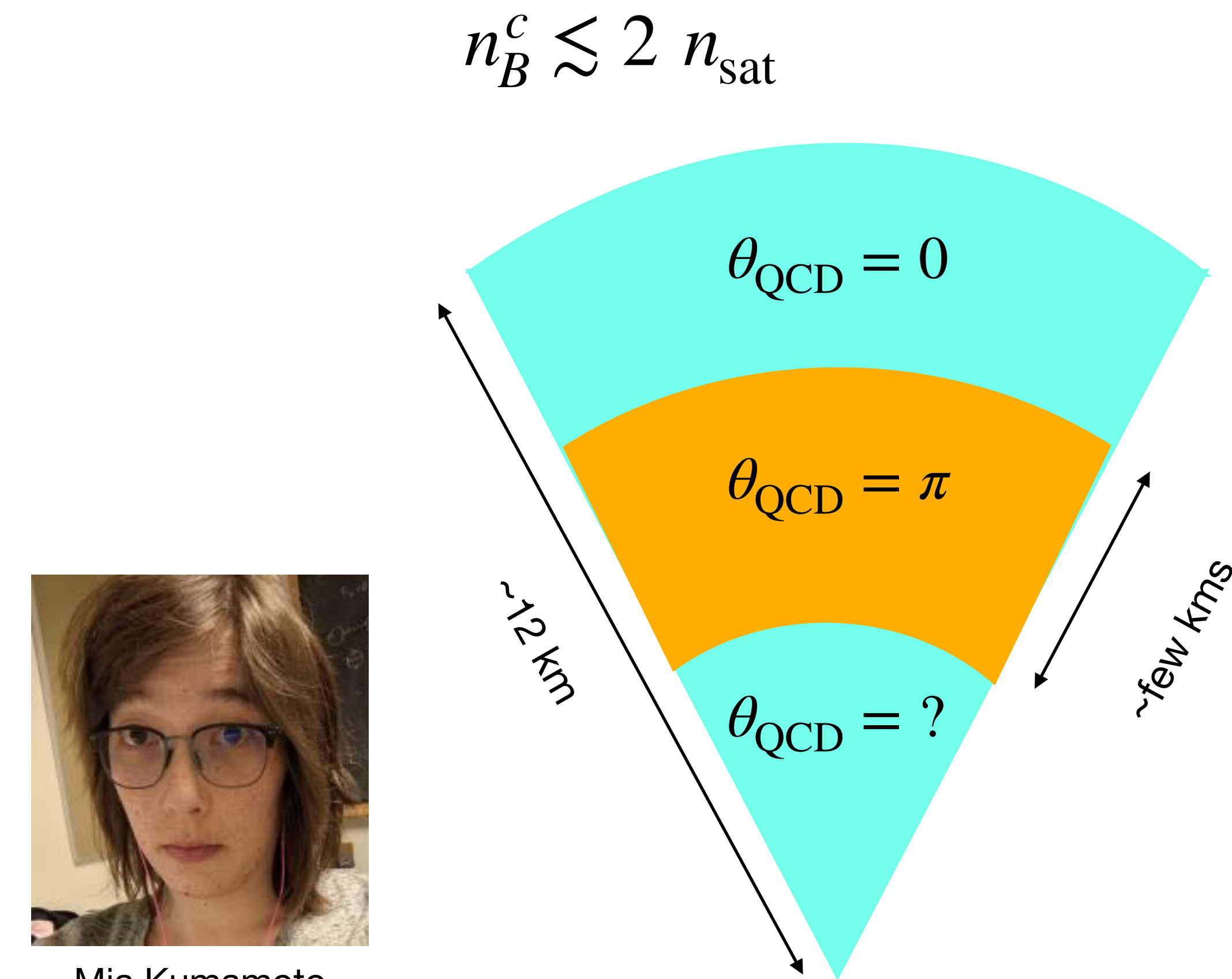
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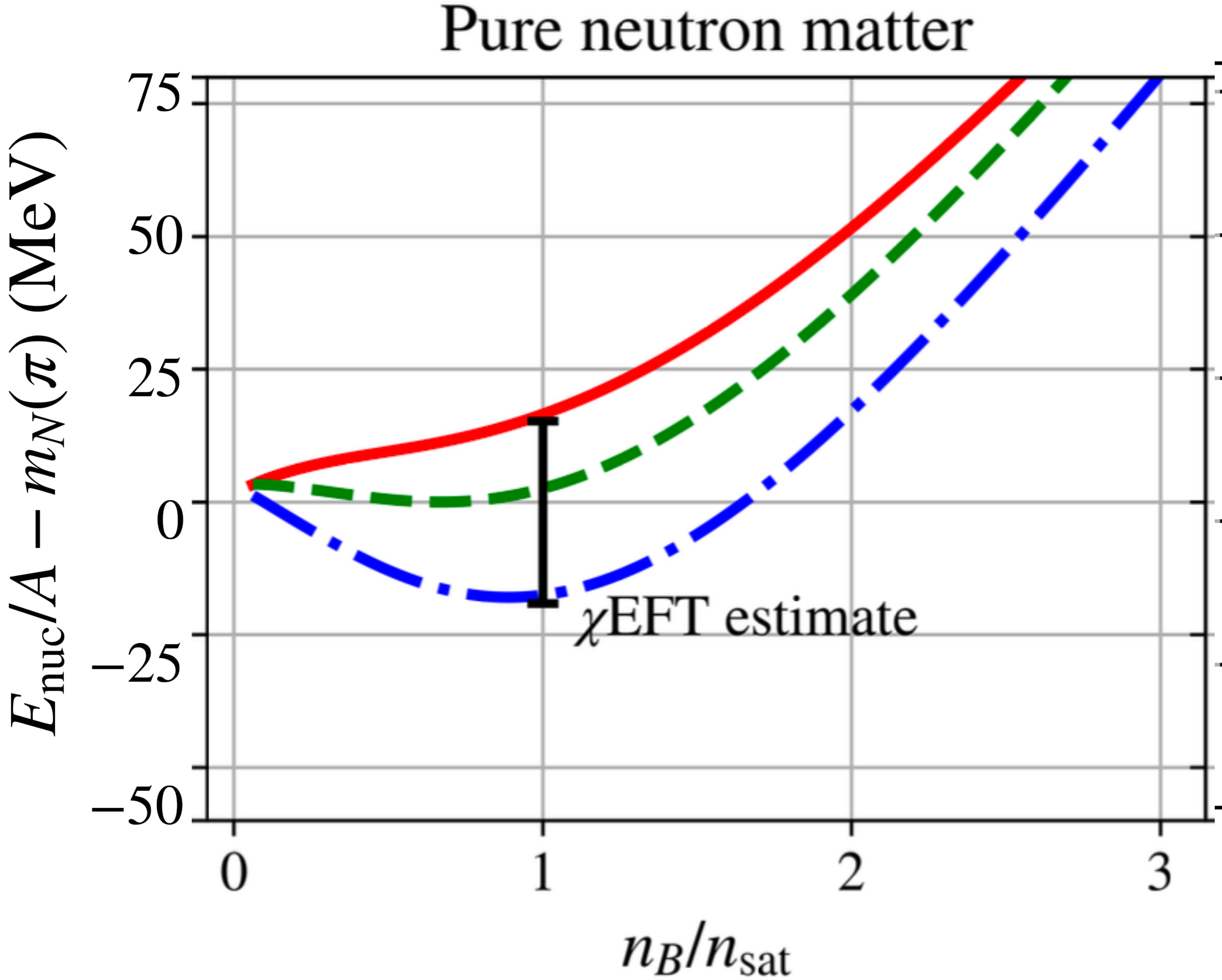
Axion Condensation in Mean Field Models

Using our preliminary ChiEFT results we constructed mean field models.

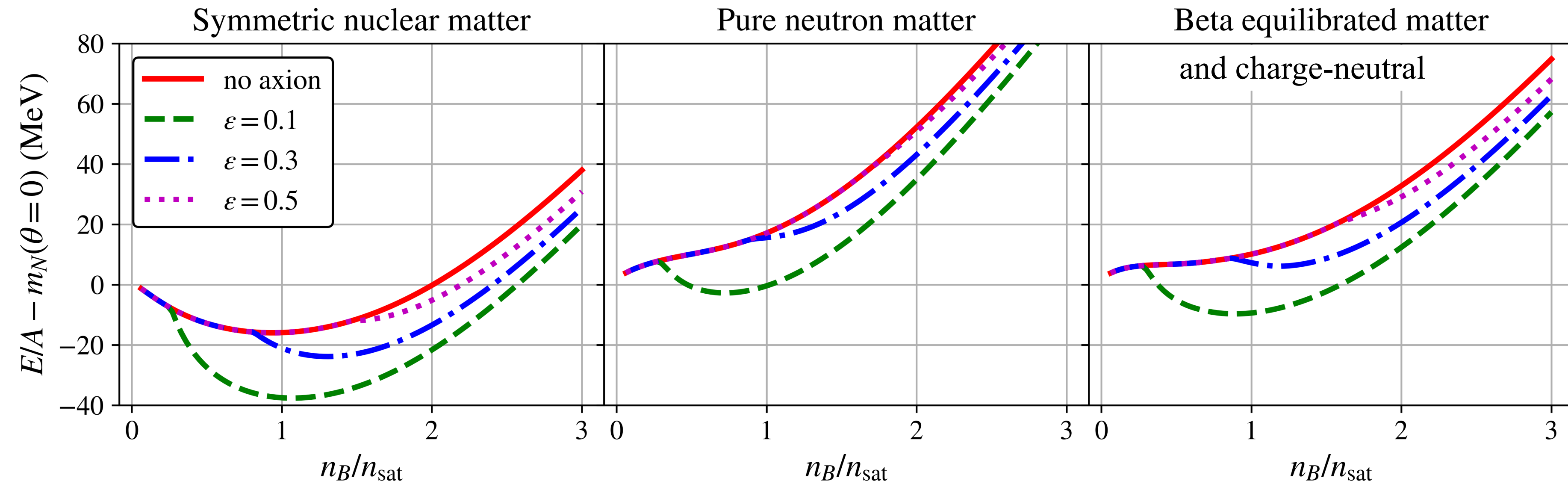
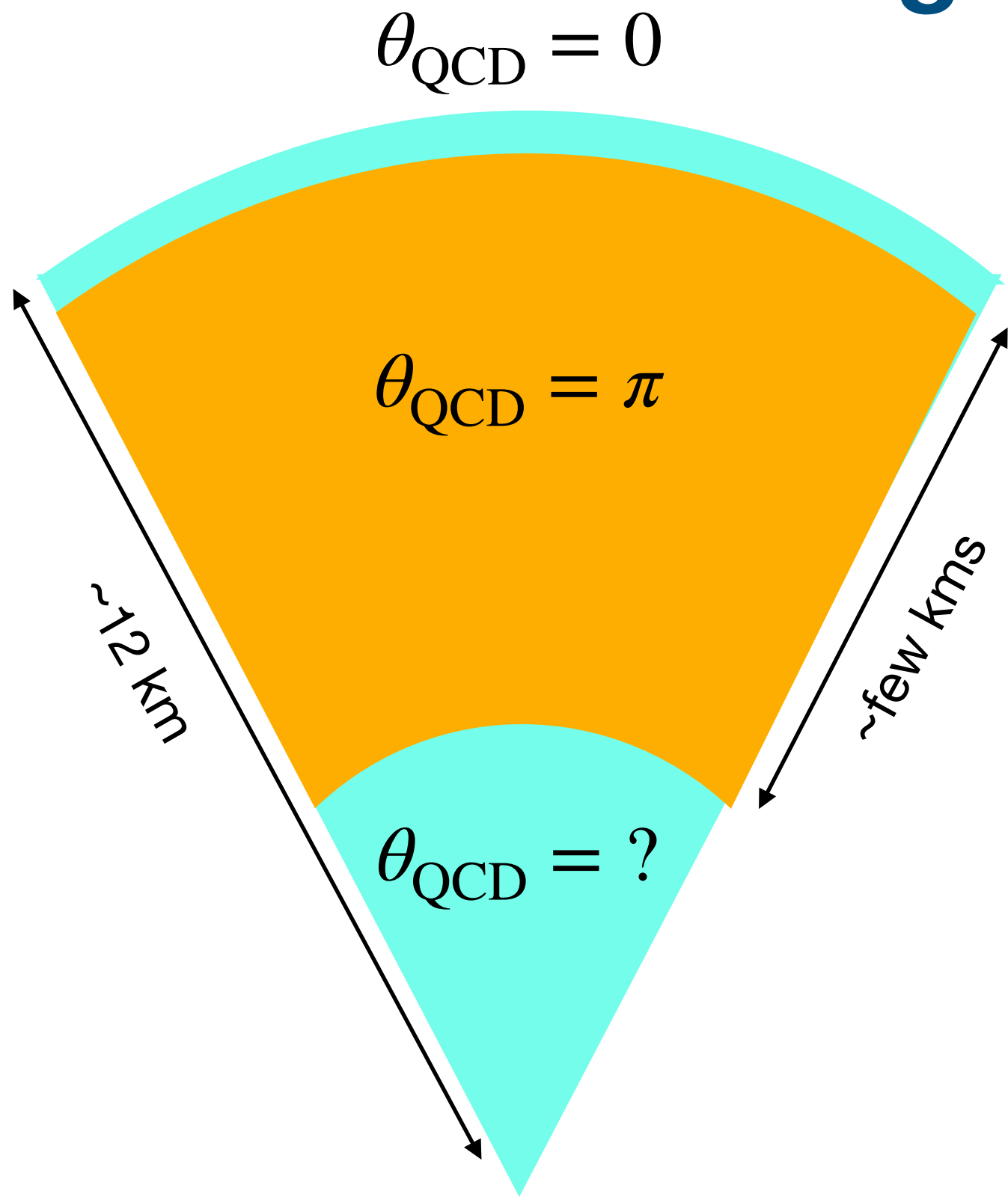
The modest increase in attraction at $\theta = \pi$ in mean field models favors axion condensation at



Mia Kumamoto



Light QCD Axions ($\epsilon < 1$) Condense in the Crust



M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar, S. Reddy (2024)

- For $\epsilon < 0.1$ ordinary matter is only metastable.
- Large objects, which we call π balls, can be favored at zero pressure.
- Equation of state of matter is qualitatively altered at low pressure.

Improved Constraints on ϵ from NS Glitches

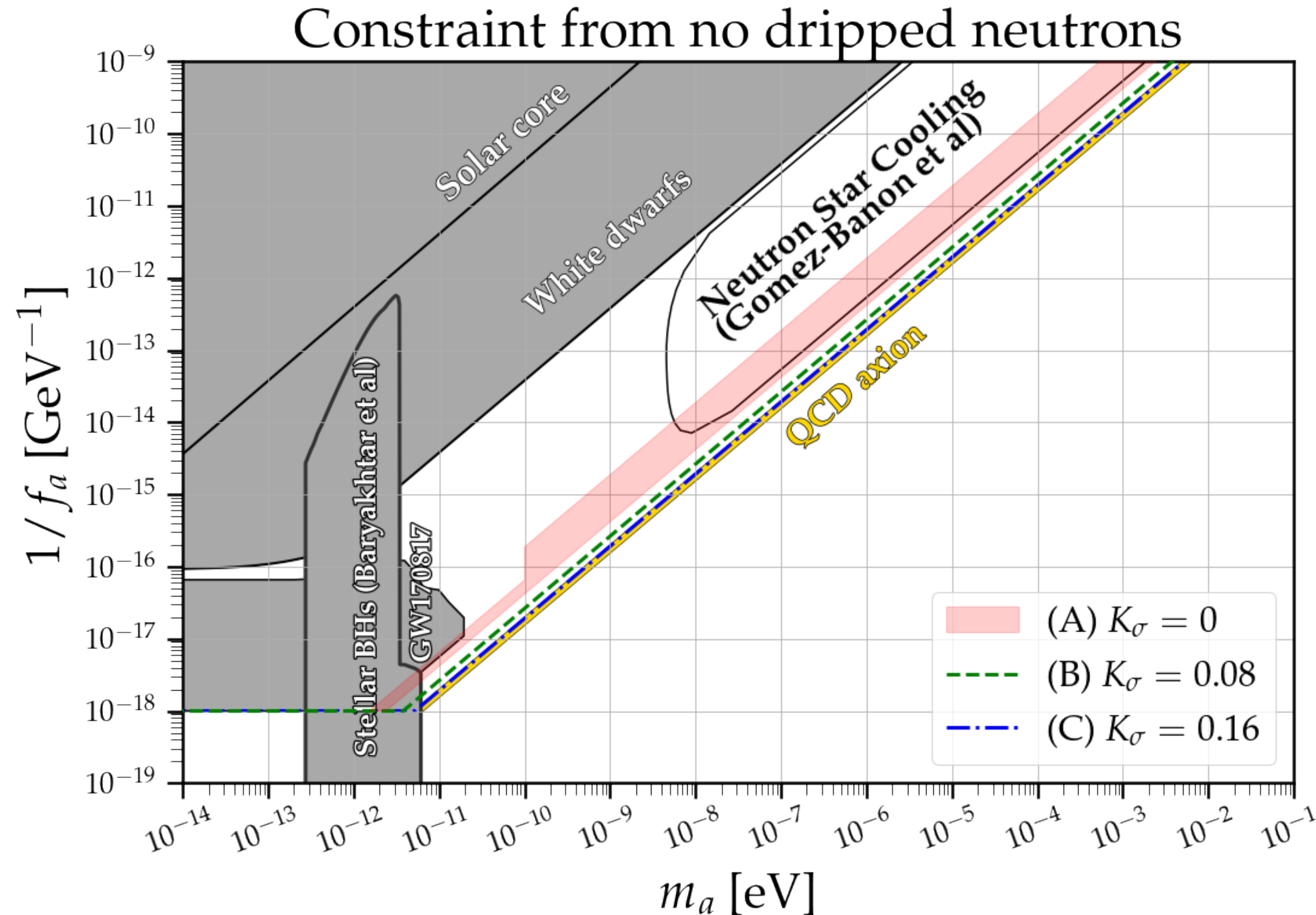
Glitches, rapid spin-up of neutron stars requires a NS region in which a superfluid coexists with a solid.

In the standard scenario, neutrons drip out of nuclei in the inner crust when

$$\frac{Z}{A} \simeq \frac{1}{2} \sqrt{1 - \frac{a_{\text{Bulk}}}{a_{\text{Sym}}}} \approx 0.3$$

When axion condense at inner crust densities, neutron drip is disfavored.

There is too little superfluid to explain glitched observed in the Vela pulsar.



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