Wigner function of spin-1/2 particles in equilibrium

Valeriya Mykhaylova

Jagiellonian University

Based on arXiv:2505.02657, arXiv:2506.01537

In collaboration with Samapan Bhadury, Zbigniew Drogosz, Sudip Kumar Kar and Wojciech Florkowski

65 Cracow School of Theoretical Physics 2025

Describing particles with spin 1/2

Spin density matrices $f^{\pm}(x,p)$ (2×2) & spinor density matrices $X^{\pm}(x,p)$ (4×4):

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p), \qquad r,s = \{1,2\}$$

Relativistic fluid of spin-1/2 particles:

- *Classical treatment:* kinetic theory + $\int \mathrm{d}S$
- *Quantum treatment:* Wigner function + tr (spinor space)

$$W^{\pm}(x,k) = \frac{1}{4m}\int \mathrm{d}P\,\delta^{(4)}(k\mp p)(\not{p}\pm m)X^{\pm}(\not{p}\pm m) \tag{1}$$

[De Groot, Relativistic Kinetic Theory. Principles and Applications (1980)]

Spinor density matrix $X^{\pm}$

Commonly used form of $X^{\pm}$ in local equilibrium

[Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{2}$$

$$\Sigma^{\mu\nu} = \tfrac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$$

$\omega_{\mu\nu} = \Omega_{\mu\nu}/T$ – spin polarization tensor, $\Omega_{\mu\nu}$ – spin chemical potential

Spinor density matrix $X^{\pm}$

Commonly used form of $X^{\pm}$ in local equilibrium

[Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{2}$$

$$\Sigma^{\mu\nu} = \tfrac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$$

$\omega_{\mu\nu} = \Omega_{\mu\nu}/T$ – spin polarization tensor, $\Omega_{\mu\nu}$ – spin chemical potential

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix} \tag{3}$$

$\boldsymbol{e} = (e^1, e^2, e^3)$, $\boldsymbol{b} = (b^1, b^2, b^3)$ – electric- and magnetic-like vectors

Motivation

Mean spin polarization of spin-1/2 particles in equilibrium, $\mu = 0$

[Florkowski et al. PRD 97, 116017 (2018)]

$$\boldsymbol{P} = \frac{1}{2}\frac{\mathrm{tr}_2\left(f^{\pm}\boldsymbol{\sigma}\right)}{\mathrm{tr}_2\left(f^{\pm}\right)} = -\frac{1}{2}\tanh\left[\frac{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}}{2}\right]\frac{\boldsymbol{b}_*}{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}} \tag{4}$$

$$f^{+}_{rs}(x,p) = \tfrac{1}{2m}\,\bar{u}_r(p)X^{+}u_s(p), \qquad f^{-}_{rs}(x,p) = -\tfrac{1}{2m}\,\bar{v}_s(p)X^{-}v_r(p)$$

Mean spin polarization of spin-1/2 particles in equilibrium, $\mu = 0$

[Florkowski et al. PRD 97, 116017 (2018)]

$$\boldsymbol{P} = \frac{1}{2}\frac{\mathrm{tr}_2\left(f^{\pm}\boldsymbol{\sigma}\right)}{\mathrm{tr}_2\left(f^{\pm}\right)} = -\frac{1}{2}\tanh\left[\frac{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}}{2}\right]\frac{\boldsymbol{b}_*}{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}} \tag{4}$$

$$f^+_{rs}(x,p) = \tfrac{1}{2m}\,\bar{u}_r(p) X^+ u_s(p), \qquad f^-_{rs}(x,p) = -\tfrac{1}{2m}\,\bar{v}_s(p) X^- v_r(p)$$

This spinor density matrix [Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^\mu u_\mu}{T} \pm \frac{\mu}{T}\right]\exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{5}$$

Mean spin polarization of spin-1/2 particles in equilibrium, $\mu = 0$

[Florkowski et al. PRD 97, 116017 (2018)]

$$\boldsymbol{P} = \frac{1}{2}\frac{\mathrm{tr}_2\left(f^{\pm}\boldsymbol{\sigma}\right)}{\mathrm{tr}_2\left(f^{\pm}\right)} = -\frac{1}{2}\tanh\left[\frac{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}}{2}\right]\frac{\boldsymbol{b}_*}{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}} \tag{4}$$

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p)$$

This spinor density matrix [Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right]\exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{5}$$

violates the normalization of the polarization vector, $0 \leq |\boldsymbol{P}| \leq 1$

(from no polarization to pure state)

Motivation

Mean spin polarization of spin-1/2 particles in equilibrium, $\mu = 0$

[Florkowski et al. PRD 97, 116017 (2018)]

$$\boldsymbol{P} = \frac{1}{2}\frac{\mathrm{tr}_2\left(f^{\pm}\boldsymbol{\sigma}\right)}{\mathrm{tr}_2\left(f^{\pm}\right)} = -\frac{1}{2}\tanh\left[\frac{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}}{2}\right]\frac{\boldsymbol{b}_*}{\sqrt{\boldsymbol{b}_*\cdot\boldsymbol{b}_* - \boldsymbol{e}_*\cdot\boldsymbol{e}_*}} \tag{4}$$

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p)$$

This spinor density matrix [Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right]\exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{5}$$

violates the normalization of the polarization vector, $0 \leq |\boldsymbol{P}| \leq 1$

(from no polarization to pure state)

+ *other issues*

Deriving $X^{\pm}$ from the spin density matrices

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p) \tag{6}$$

Deriving $X^{\pm}$ from the spin density matrices

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p) \tag{6}$$

Decomposition of 2×2 Hermitian matrices $f^{\pm}_{rs}(x,p)$ [Florkowski et al., PRD97 (2018)]:

$$f^{\pm}_{rs}(x,p) = f^{\pm}_0(x,p)\left[\delta_{rs} + \boldsymbol{\zeta}^{\pm}_{*}(x,\boldsymbol{p}) \cdot \boldsymbol{\sigma}_{rs}\right] \tag{7}$$

- $f^{\pm}_0(x,p)$ – spin-averaged phase-space density
- $\zeta^{\pm\mu}_{*} = \left(0, \boldsymbol{\zeta}^{\pm}_{*}\right)$ – mean polarization vector in PRF, $\quad 0 \le |\boldsymbol{\zeta}^{\pm}_{*}| \le 1$

Deriving $X^{\pm}$ from the spin density matrices

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p) \tag{6}$$

Decomposition of 2×2 Hermitian matrices $f^{\pm}_{rs}(x,p)$ [Florkowski et al., PRD97 (2018)]:

$$f^{\pm}_{rs}(x,p) = f^{\pm}_{0}(x,p)\left[\delta_{rs} + \boldsymbol{\zeta}^{\pm}_{*}(x,\boldsymbol{p}) \cdot \boldsymbol{\sigma}_{rs}\right] \tag{7}$$

- $f^{\pm}_{0}(x,p)$ – spin-averaged phase-space density
- $\zeta^{\pm\mu}_{*} = \left(0, \boldsymbol{\zeta}^{\pm}_{*}\right)$ – mean polarization vector in PRF, $\quad 0 \leq |\boldsymbol{\zeta}^{\pm}_{*}| \leq 1$

Canonical boost to LAB frame, particle moves with $\boldsymbol{v}_p$:

$$\Lambda^{\mu}{}_{\nu}(\boldsymbol{v}_p)\zeta^{\nu}_{\pm *} \equiv \zeta^{\mu}_{\pm} \tag{8}$$

From Lorentz covariance we get

$$\bar{u}_r(p)\gamma_5\zeta^+_\mu\gamma^\mu u_s(p) = 2m\,\boldsymbol{\zeta}^+_* \cdot \boldsymbol{\sigma}_{rs} \tag{9}$$

$$\bar{v}_s(p)\gamma_5\zeta^-_\mu\gamma^\mu v_r(p) = -2m\,\boldsymbol{\zeta}^-_* \cdot \boldsymbol{\sigma}_{rs} \tag{10}$$

$$\Longrightarrow \boxed{X^{\pm}_s(x,p) = f^{\pm}_0(x,p)\left[1 + \gamma_5 \not{\zeta}^{\pm}(x,p)\right]} \tag{11}$$

Derived with no assumptions on equilibrium properties

of the spin density matrix $f^{\pm}(x,p)$

New spinor density $X_s^{\pm}$

$$X_s^{\pm}(x,p) = f_0^{\pm}(x,p)\left[1+\gamma_5 \not{\zeta}^{\pm}(x,p)\right] \tag{12}$$

For general spacelike four-vectors $a_{\pm}^{\mu}$ satisfying $a_{\pm}^2 < 0$

$$\exp\left(\gamma_5 \not{a}_{\pm}\right) = \cosh\sqrt{-a_{\pm}^2}\left[1+\frac{\gamma_5 \not{a}_{\pm}}{\sqrt{-a_{\pm}^2}}\tanh\sqrt{-a_{\pm}^2}\right] \tag{13}$$

New spinor density $X_s^{\pm}$

$$X_s^{\pm}(x,p) = f_0^{\pm}(x,p)\left[1+\gamma_5 \not\zeta^{\pm}(x,p)\right] \qquad (12)$$

For general spacelike four-vectors $a_{\pm}^{\mu}$ satisfying $a_{\pm}^2 < 0$

$$\exp\left(\gamma_5 \not a_{\pm}\right) = \cosh\sqrt{-a_{\pm}^2}\left[1+\frac{\gamma_5 \not a_{\pm}}{\sqrt{-a_{\pm}^2}}\tanh\sqrt{-a_{\pm}^2}\right] \qquad (13)$$

Square brackets are equal if

$$\zeta_{\pm}^{\mu} = \frac{a_{\pm}^{\mu}}{\sqrt{-a_{\pm}^2}}\tanh\sqrt{-a_{\pm}^2} \quad \longrightarrow \quad \zeta_{\pm}^{\mu} = a_{\pm}^{\mu} \qquad (14)$$

New spinor density $X_s^{\pm}$

$$X_s^{\pm}(x,p) = f_0^{\pm}(x,p)\exp\left(\gamma_5 \not{a}_{\pm}\right) \tag{15}$$

New spinor density $X_s^{\pm}$

$$X_s^{\pm}(x,p) = f_0^{\pm}(x,p)\exp\left(\gamma_5 \not{a}_{\pm}\right) \tag{15}$$

Equilibrium spin density for Boltzmann statistics

$$X_s^{\pm}(x,p) = \exp\left[-\frac{p_\mu u^\mu}{T} \pm \frac{\mu}{T}\right]\exp\left[\gamma_5 \not{a}\right] \tag{16}$$

$$a_\mu(x,p) = -\frac{1}{2m}\tilde{\omega}_{\mu\nu}(x)p^\nu, \qquad \tilde{\omega}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\alpha\beta}\,\omega^{\alpha\beta} \tag{17}$$

Comparison of spinor densities

Previously known expression [Becattini, Chandra, Del Zanna, Grossi, Annals Phys. 338 (2013)]

$$X^{\pm}(x,p) = \exp\left[-\frac{p^\mu u_\mu}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{i}{8}\omega_{\mu\nu}(x)[\gamma^\mu,\gamma^\nu]\right] \tag{18}$$

Our new finding [Bhadury, Drogosz, Florkowski, Kar, VM, arXiv:2505.02657]

$$X_s^{\pm}(x,p) = \exp\left[-\frac{p^\mu u_\mu}{T} \pm \frac{\mu}{T}\right] \exp\left[-\frac{1}{4m}\gamma_5\,\epsilon_{\mu\nu\alpha\beta}\,\omega^{\alpha\beta}(x)\,p^\nu\,\gamma^\mu\right] \tag{19}$$

Comparison of spinor densities

Previously known expression [Becattini, Chandra, Del Zanna, Grossi, Annals Phys. 338 (2013)]

$$X^{\pm}(x,p) = \exp\left[-\frac{p^\mu u_\mu}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{i}{8}\omega_{\mu\nu}(x)[\gamma^\mu,\gamma^\nu]\right] \tag{18}$$

Our new finding [Bhadury, Drogosz, Florkowski, Kar, VM, arXiv:2505.02657]

$$X_s^{\pm}(x,p) = \exp\left[-\frac{p^\mu u_\mu}{T} \pm \frac{\mu}{T}\right] \exp\left[-\frac{1}{4m}\gamma_5\, \epsilon_{\mu\nu\alpha\beta}\, \omega^{\alpha\beta}(x)\, p^\nu\, \gamma^\mu\right] \tag{19}$$

$$W^{\pm}(x,k) = \frac{1}{4m}\int \mathrm{d}P\, \delta^{(4)}(k\mp p)(\not{p}\pm m) X_s^{\pm}(\not{p}\pm m) \rightarrow \text{thermodynamics}$$

Thermodynamics from Wigner function

$$N^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P \, p^{\mu} \left[f^{+}_{rr}(x,p) - f^{-}_{rr}(x,p) \right] - \textit{baryon current} \tag{20}$$

Thermodynamics from Wigner function

$$N^\mu(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu \left[f^+_{rr}(x,p) - f^-_{rr}(x,p)\right] - \textit{baryon current} \tag{20}$$

$$\mathcal{N}^\mu(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu \left[f^+_{rr}(x,p) + f^-_{rr}(x,p)\right] - \textit{particle current} \tag{21}$$

Thermodynamics from Wigner function

$$N^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} \left[f^{+}_{rr}(x,p) - f^{-}_{rr}(x,p) \right] - \textit{baryon current} \tag{20}$$

$$\mathcal{N}^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} \left[f^{+}_{rr}(x,p) + f^{-}_{rr}(x,p) \right] - \textit{particle current} \tag{21}$$

$$T^{\mu\nu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} p^{\nu} \left[f^{+}_{rr}(x,p) + f^{-}_{rr}(x,p) \right] - \textit{energy-momentum tensor} \tag{22}$$

Thermodynamics from Wigner function

$$N^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} \left[f^{+}_{rr}(x,p) - f^{-}_{rr}(x,p) \right] - \textit{baryon current} \tag{20}$$

$$\mathcal{N}^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} \left[f^{+}_{rr}(x,p) + f^{-}_{rr}(x,p) \right] - \textit{particle current} \tag{21}$$

$$T^{\mu\nu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} p^{\nu} \left[f^{+}_{rr}(x,p) + f^{-}_{rr}(x,p) \right] - \textit{energy-momentum tensor} \tag{22}$$

$$S^{\lambda,\mu\nu}(x) = \frac{1}{2} \sum_{r,s=1}^{2} \int \mathrm{d}P\, p^{\lambda} \left[\sigma^{+\mu\nu}_{sr}(p) f^{+}_{rs}(x,p) + \sigma^{-\mu\nu}_{sr}(p) f^{-}_{rs}(x,p) \right] - \textit{spin tensor}$$

Thermodynamics from Wigner function

$$N^\mu(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu \left[f^+_{rr}(x,p) - f^-_{rr}(x,p)\right] - \textit{baryon current} \tag{20}$$

$$\mathcal{N}^\mu(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu \left[f^+_{rr}(x,p) + f^-_{rr}(x,p)\right] - \textit{particle current} \tag{21}$$

$$T^{\mu\nu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^\mu p^\nu \left[f^+_{rr}(x,p) + f^-_{rr}(x,p)\right] - \textit{energy-momentum tensor} \tag{22}$$

$$S^{\lambda,\mu\nu}(x) = \frac{1}{2}\sum_{r,s=1}^{2} \int \mathrm{d}P\, p^\lambda \left[\sigma^{+\mu\nu}_{sr}(p) f^+_{rs}(x,p) + \sigma^{-\mu\nu}_{sr}(p) f^-_{rs}(x,p)\right] - \textit{spin tensor}$$

$$S^\mu(x) = -\frac{1}{2}\int \mathrm{d}P\, p^\mu \left\{\mathrm{tr}_4\left[X^+_s\left(\ln X^+_s - 1\right)\right] + \mathrm{tr}_4\left[X^-_s\left(\ln X^-_s - 1\right)\right]\right\} - \textit{entropy}$$

[Florkowski et al. PRD 97, 116017 (2018)]

Generalized thermodynamic relations for perfect spin hydrodynamics

$\beta_\alpha = \frac{u_\alpha}{T}, \qquad \xi = \frac{\mu}{T}$

$$\textit{Entropy}: \; S^\mu = T^{\mu\alpha}\beta_\alpha - \frac{1}{2}\omega_{\alpha\beta}S^{\mu,\alpha\beta} - \xi N^\mu + \mathcal{N}^\mu \tag{23}$$

$$\textit{Change of particle current}: \; d\mathcal{N}^\mu = N^\mu d\xi - T^{\lambda\mu}d\beta_\lambda + \tfrac{1}{2}S^{\mu,\alpha\beta}d\omega_{\alpha\beta} \tag{24}$$

$$\textit{Change of entropy}: \; dS^\mu = -\xi dN^\mu + \beta_\lambda dT^{\lambda\mu} - \tfrac{1}{2}\omega_{\alpha\beta}dS^{\mu,\alpha\beta} \tag{25}$$

[Florkowski, Hontarenko, PRL 134 (2025)]

Quantum vs. classical spin treatments

Quantum

$$X_s^{\pm} = [\cdots + \gamma_5 a_\mu \gamma^\mu]$$

$$a_\mu(x,p) = -\frac{1}{2m}\tilde{\omega}_{\mu\nu}(x)p^\nu$$

Classical

$$f^{\pm}(x,p,s) = \exp\left[\cdots + \frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right]$$

$$s^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\gamma\delta}p_\gamma s_\delta$$

$$\boxed{\frac{1}{2}\gamma_5\gamma^\mu \quad \longleftrightarrow \quad s^\mu}$$

Quantum vs. classical spin treatments

Quantum

$$X_s^{\pm} = [\cdots + \gamma_5 a_\mu \gamma^\mu]$$

$$a_\mu(x,p) = -\frac{1}{2m}\tilde{\omega}_{\mu\nu}(x)p^\nu$$

Classical

$$f^{\pm}(x,p,s) = \exp\left[\cdots + \frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right]$$

$$s^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\gamma\delta}p_\gamma s_\delta$$

$$\boxed{\frac{1}{2}\gamma_5\gamma^\mu \quad \longleftrightarrow \quad s^\mu}$$

N^μ and $T^{\mu\nu}$ modified by $\omega_{\mu\nu}$ in classical and quantum descriptions

Quadratic corrections:

$$\cosh\sqrt{-a^2} = 1 - \frac{1}{2}a^2$$

$$1 + \frac{1}{16}\int \mathrm{d}S\,\omega_{\mu\nu}s^{\mu\nu}\,\omega_{\rho\sigma}s^{\rho\sigma}$$

Applicability range

$$X_s^{\pm}(x,p,s) = \exp\left[-\frac{p_\mu u^\mu}{T} \pm \frac{\mu}{T} + \gamma_5 a_\mu(x,p)\gamma^\mu\right] \tag{26}$$

$$a_\mu(x,p) = -\tfrac{1}{2m}\tilde{\omega}_{\mu\nu}(x)p^\nu$$

Convergence criterion [Drogosz, Florkowski, VM, arXiv:2506.01537]

$$\boxed{\frac{1}{2}\sqrt{\boldsymbol{b}'^2 + \boldsymbol{e}'^2 + 2|\boldsymbol{e}' \times \boldsymbol{b}'|} < \frac{m}{T}} \tag{27}$$

$\frac{1}{2}$ - maximum spin polarization in quantum case

Our results are applicable within the range of hydrodynamic parameters used in current models of HIC

Summary

We have found:

- local equilibrium Wigner function for spin-1/2 particles with proper normalization of the mean polarization vector;
- agreement of the generalized thermodynamics in quantum and classical spin formalisms – employed in perfect spin hydrodynamics;
- applicability range of the spinor density $X_s^{\pm}$ in Wigner-function formalism and of the distribution function $f(x, p, s)$ in kinetic theory with spin.

Research supported by the Polish National Science Centre Grant No. 2022/47/B/ST2/01372