Glasma as a fluid

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Glasma – the earliest phase of ultrarelativistic heavy-ion collisions

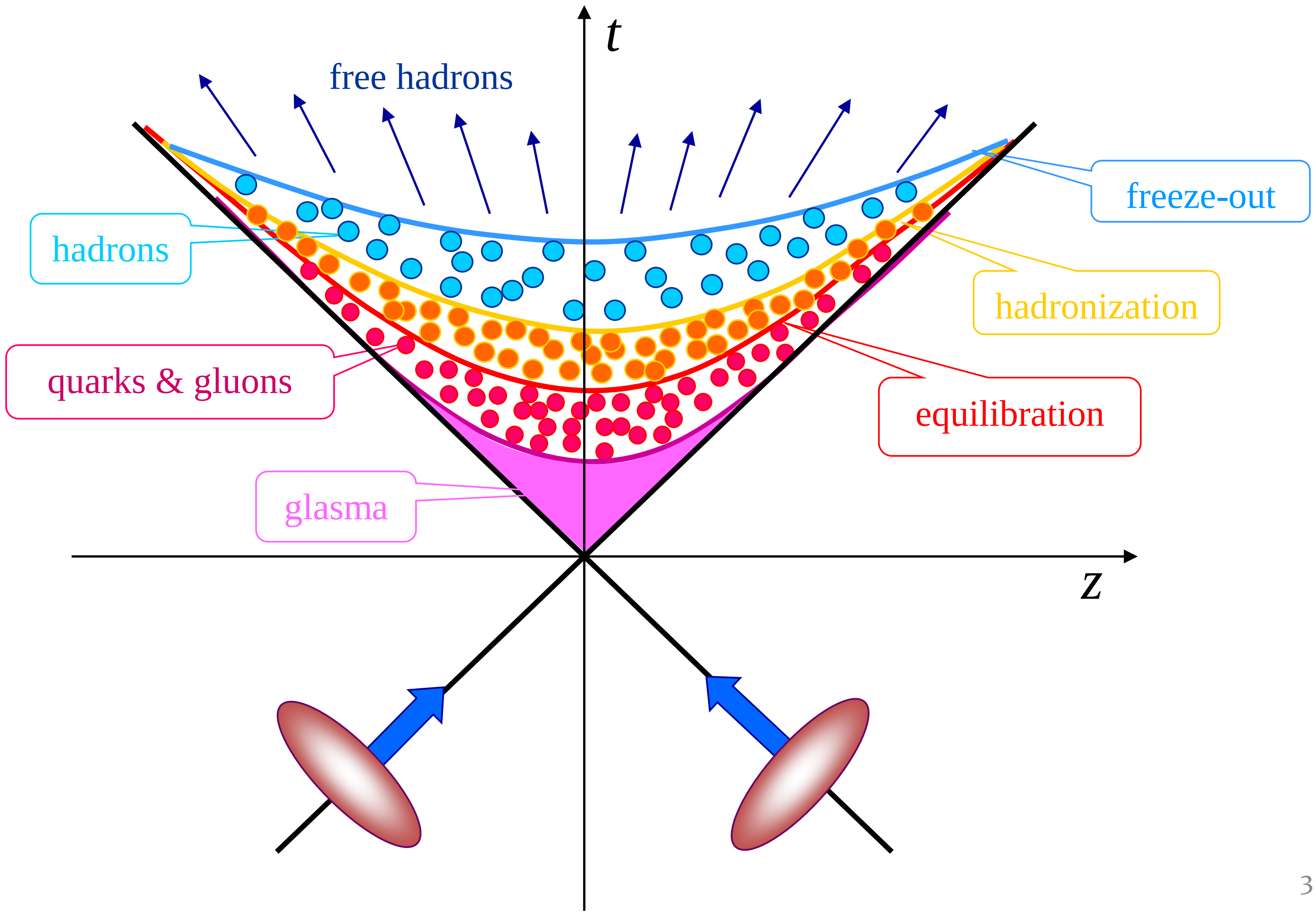
We have studied:

- energy density, pressures, pressure anisotropy;
- collective motion, elliptic flow vs. eccentricity;
- angular momentum and vorticity;
- jet quenching - $\hat{q}$ & $\frac{dE}{dx}$.

Phenomena observed in collisions of relativistic ions have their origins in the earliest phase of the collision.

M. Carrington & St. Mrówczyński, Acta Physica Polonica B **55**, 4-A3 (2024) review article

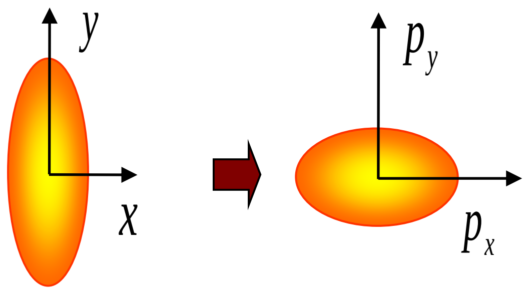
Scenario of ultrarelativistic heavy-ion collisions



Success of hydrodynamic description of heavy-ion collisions

Two conflicting requirements:

- Hydrodynamics needs local thermodynamical equilibrium.
- Hydrodynamic evolution must start very early, $\tau \sim 0.6$ fm/c.



Success of hydrodynamic description of heavy-ion collisions

Possible resolutions:

- Matter is strongly interacting and equilibrates fast.

  AdS/CFT: D.T. Son & A. O. Starinets, Ann. Rev. Nucl. Part. Sci. **57**, 95 (2007).

- Pre-quilibrium matter evolves along the hydrodynamic attaractor.

  M.P. Heller & M. Spaliński, Phys. Rev. Lett. **115**, 072501 (2015);
  J. Jankowski & M. Spaliński, Prog. Part. Nucl. Phys. **132**, 104048 (2023).

- Pre-quilibrium matter – glasma – behaves as a fluid.

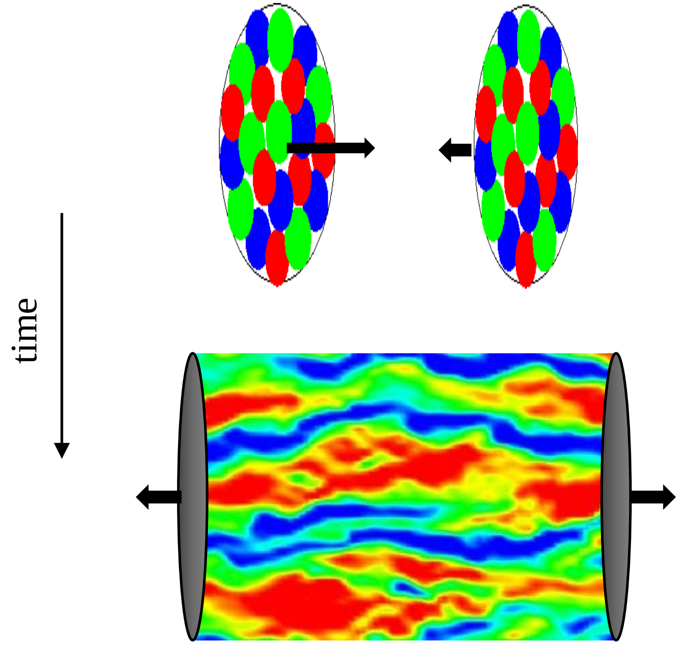
  M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110,** 054903 (2024),
  M. Carrington & St. Mrówczyński, arXiv:2505.07324

Outline of the discussion on glasma as a liquid

- Formalism of the classical Color Glass Condensate approach.
  - Justification of classical approximation.
  - Pre- and post-collision potentials.
  - Proper time expansion
  - Averaging over color configurations.
- Numerical results on the glasma energy-momentum tensor.
  - Energy density, pressures, pressure anisotropy.
  - Collective expansion, azimuthal anisotropy
  - Elliptic flow vs. Eccentricity.
  - Vorticity.
- Glasma fluid behavior.

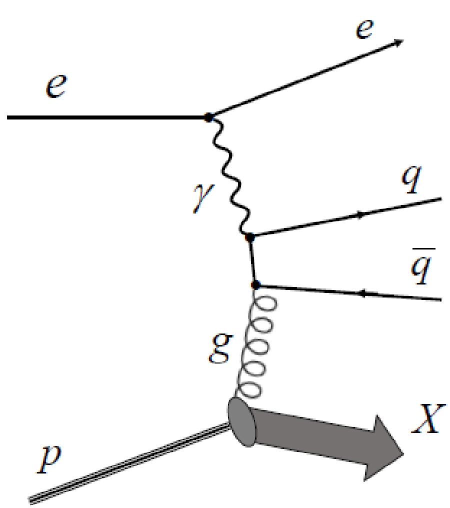
Glasma & Color Glass Condensate

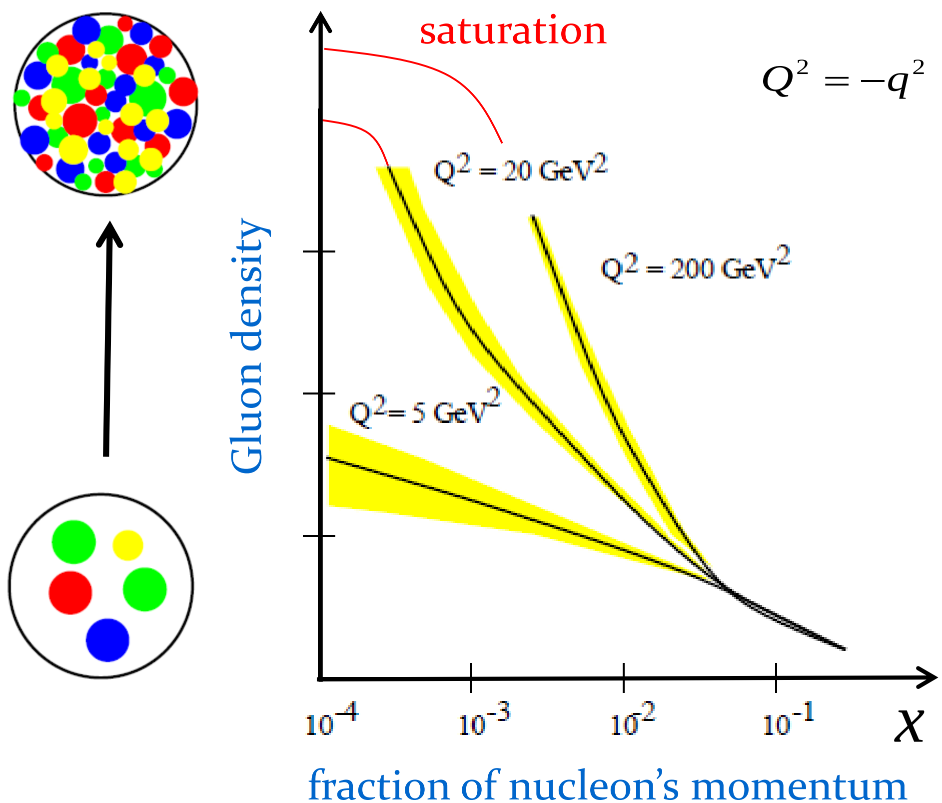
Color charges confined in the colliding nuclei generate **glasma** – the system of strong mostly classical chromodynamic fields which evolves towards equilibrium.



Saturation

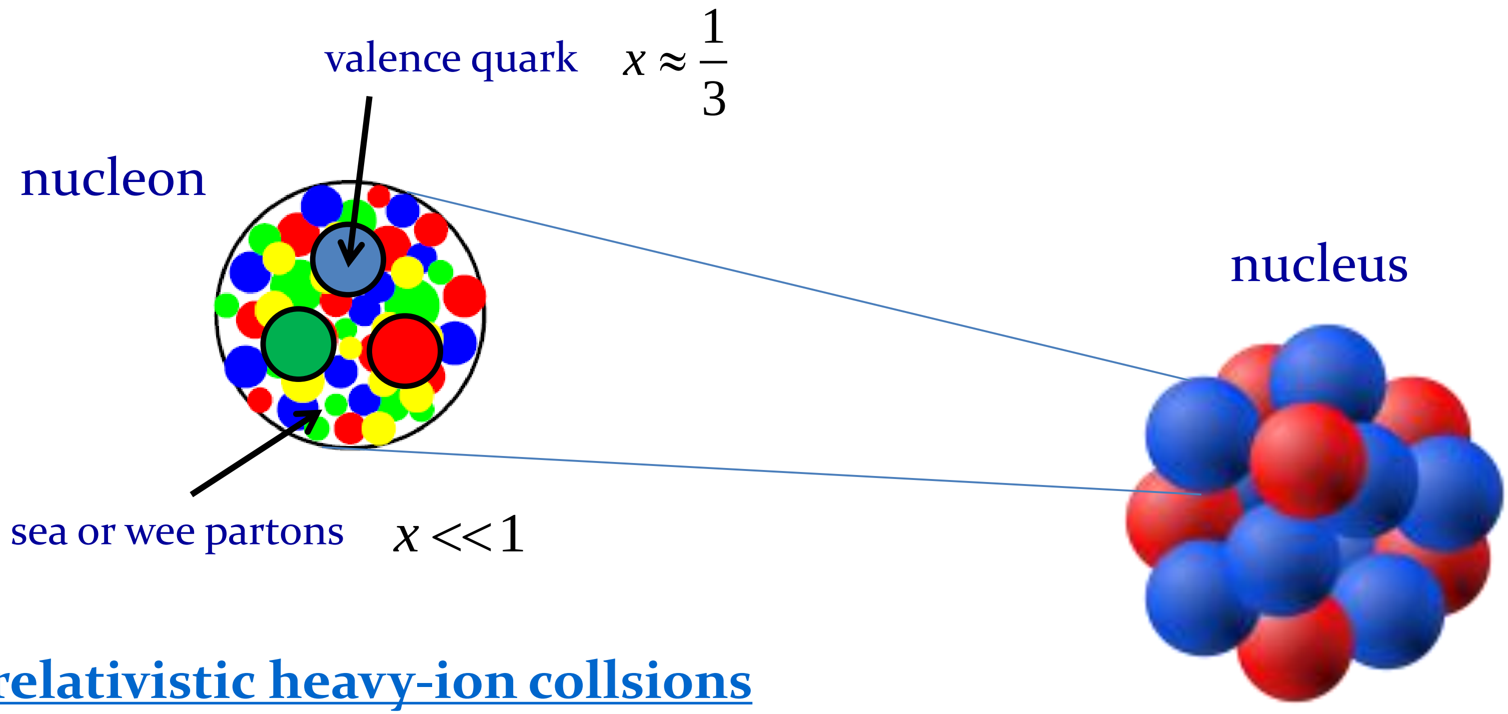
Deep Inelastic Scattering





Saturated gluon system can be described in terms of classical chromodynamic fields.

Scale separation between wee partons & valence quarks



In relativistic heavy-ion collsions

- Saturated wee partons – classical chromodynamic fields
- Valence quarks – classical sources of chromodynamic fields
- Saturation scale for A-A at LHC: $Q_s \approx 2\,\text{GeV} >> \Lambda_{\text{QCD}} \approx 0.2\,\text{GeV} \Rightarrow \alpha_s(Q_s) << 1$

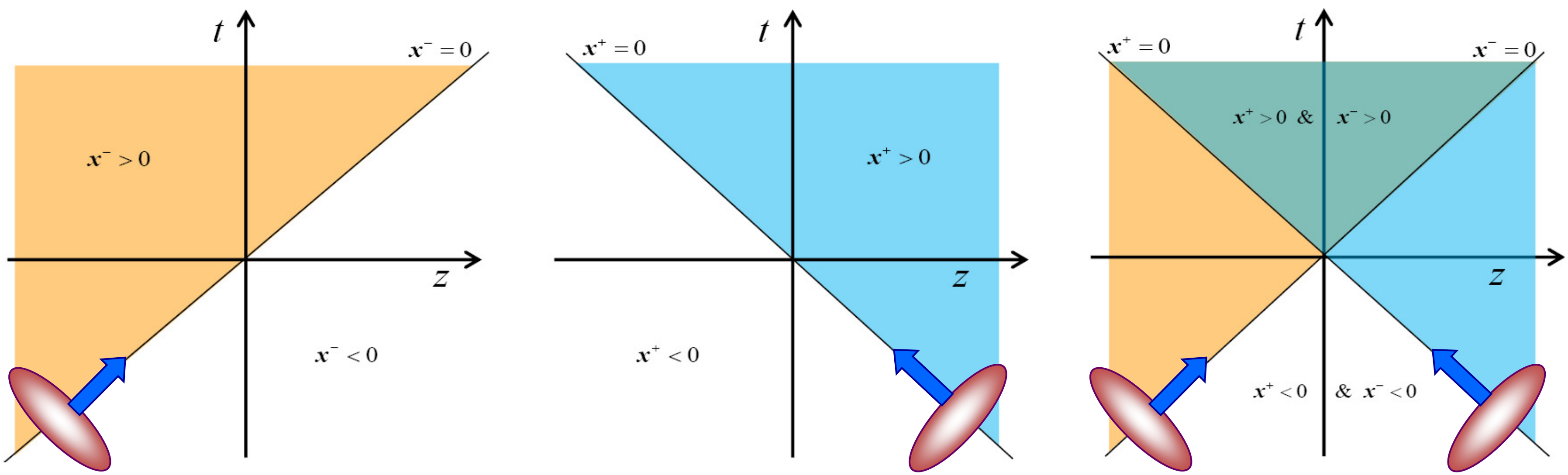
Ultrarelativistic heavy-ion collisions in light-cone variables

natural units $c = 1$

$$x^{\pm} \equiv \frac{t \pm z}{\sqrt{2}} \qquad x_{\mu} y^{\mu} = x_{+} y^{+} + x_{-} y^{-} - \vec{x} \cdot \vec{y} = x^{-} y^{+} + x^{+} y^{-} - \vec{x} \cdot \vec{y}$$

$$x^{\pm} = 0 \quad \Rightarrow \quad z = \pm t$$

$\Theta(x^{-}) > 0$ $\qquad$ $\Theta(x^{+}) > 0$ $\qquad$ $\Theta(x^{-})\Theta(x^{+}) > 0$



$x^{-} = 0$ $\qquad$ $x^{+} = 0$

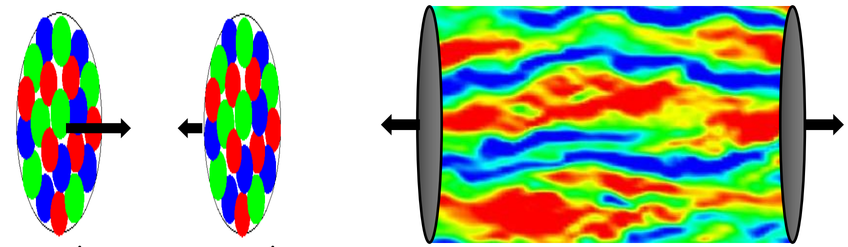
Color Glass Condensate

Classical Yang-Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x)$$

$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\pm}\delta(x^\mp)\rho_{1,2}(\mathbf{x}_\perp)$$

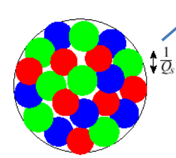


β_1^i β_2^i $\alpha, \alpha_\perp^i$

pre-collision potentials post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau, \mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau, \mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau, \mathbf{x}_\perp) \\ \qquad + \Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

$$\begin{cases} \alpha(0, \mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0, \mathbf{x}_\perp) = -\frac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

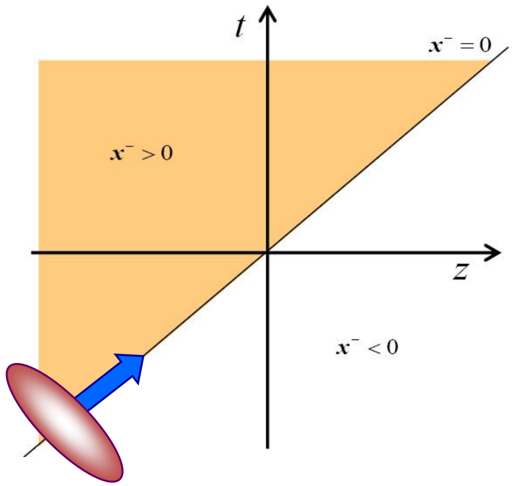
Gauge condition

$$x^+A^- + x^-A^+ = 0$$

$$\beta_{1,2}^\pm = 0$$

A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **52**, 3809 (1995).

Pre-collision potentials



$$j^\mu(x^-, \mathbf{x}_\perp) = \delta^{\mu+}\delta(x^-)\rho(\mathbf{x}_\perp)$$

Gauge condition: $A^i(x^-, \mathbf{x}_\perp) = 0$

$$D_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \begin{cases} A^-(x^-, \mathbf{x}_\perp) = 0 \\ A^+(x^-, \mathbf{x}_\perp) = \delta(x^-)\Lambda(\mathbf{x}_\perp) \end{cases}$$

$$E^z = B^z = 0$$
$$\mathbf{E}_\perp, \mathbf{B}_\perp \sim \delta(x^-)$$

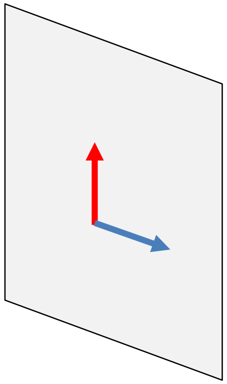
Poisson equation

$$\nabla_\perp^2 \Lambda(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp)$$

$$\Lambda(\mathbf{x}_\perp) = \int d^2x'_\perp\, G(\mathbf{x}_\perp - \mathbf{x}'_\perp)\rho(\mathbf{x}'_\perp)$$

$$G(\mathbf{x}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{\mathbf{k}_\perp^2 + m^2} = \frac{1}{2\pi} K_0(m|\mathbf{x}_\perp|)$$

IR regulator $m = \Lambda_{QCD}$



$x^- = 0$

Pre-collision potentials cont.

Gauge transformation: $A^{\mu}(x^-, \mathbf{x}_\perp) \quad \rightarrow \quad \beta^{\mu}(x^-, \mathbf{x}_\perp)$

gauge condition $A^i(x^-, \mathbf{x}_\perp) = 0$

light-come gauge $\beta^+(x^-, \mathbf{x}_\perp) = 0$

$$\beta^{\mu}(x) = U(x) A^{\mu}(x) U^{\dagger}(x) + \frac{i}{g} U(x) \partial^{\mu} U^{\dagger}(x)$$

$$U(x^-, \mathbf{x}_\perp) A^+(x^-, \mathbf{x}_\perp) U^{\dagger}(x^-, \mathbf{x}_\perp) + \frac{i}{g} U(x^-, \mathbf{x}_\perp) \partial^+ U^{\dagger}(x^-, \mathbf{x}_\perp) = 0$$

$$\begin{cases} U(x^-, \mathbf{x}_\perp) = P \exp\left(ig \int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_\perp) \right) \\ \beta^i(x^-, \mathbf{x}_\perp) = \frac{i}{g} U(x^-, \mathbf{x}_\perp) \partial^i U^{\dagger}(x^-, \mathbf{x}_\perp) \end{cases}$$

pure gauge except $x^- = 0$

J. Jalilian-Marian, A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{\perp(n)}^i(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha_{\perp(n)}^i=0 \quad \text{for} \quad n=1,3,5,\ldots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha_{\perp(0)}^i=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha_{\perp(2)}^i=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

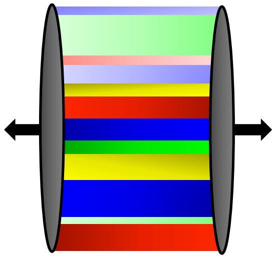
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0,0,E), \qquad \mathbf{B}_{(0)} = (0,0,B)$$

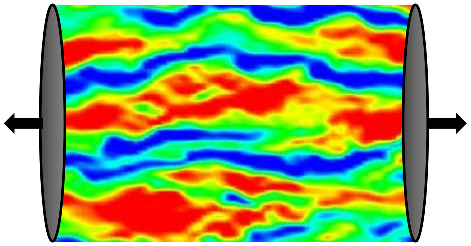
$$E^z_{(0)}(\mathbf{x}_\perp) = -ig[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta_1^i(\mathbf{x}_\perp), \beta_2^j(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up



R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Energy-momentum tensor

- $$T^{\mu\nu} = 2\mathrm{Tr}[F^{\mu\rho}F_{\rho}^{\ \nu} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}]$$

- $$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} - ig[A^{\mu}, A^{\nu}]$$

The energy-momentum tensor is symmetric, gauge invariant and obeys

- $$\partial_{\mu}T^{\mu\nu} = 0$$

T^{00} - energy density

T^{0i} - energy flux, Poynting vector

T^{xx}, T^{yy}, T^{zz} - pressures

T^{ij} - momentum flux

Averaging over collisions

$$T^{\mu\nu} \sim \sum \partial^i \partial^j \beta^k \beta^l \dots \beta^m \quad \Rightarrow \quad \left\langle T^{\mu\nu} \right\rangle \sim \sum \partial^i \partial^j \left\langle \beta^k \beta^l \dots \beta^m \right\rangle$$

Wick theorem – Gaussian averaging

$$\left\langle \rho_a^k(\mathbf{x}_\perp) \rho_b^l(\mathbf{y}_\perp) \dots \rho_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \rho_a^i(\mathbf{x}_\perp) \rho_b^j(\mathbf{y}_\perp) \right\rangle$$

Glasma graph approximation

$$\left\langle \beta_a^k(\mathbf{x}_\perp) \beta_b^l(\mathbf{y}_\perp) \dots \beta_c^m(\mathbf{z}_\perp) \right\rangle = \sum \prod \left\langle \beta_a^i(\mathbf{x}_\perp) \beta_b^j(\mathbf{y}_\perp) \right\rangle = \sum \prod B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Basic correlator

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv \left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho_a^i(\mathbf{x}'_\perp)\rho_b^j(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle = g^2\mu(\mathbf{x}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp-\mathbf{y}_\perp)$$

color charge surface density

$$\bar{\mu} = g^{-4}Q_s^2$$

- System uniform in the transverse plane $\mu(\mathbf{x}_\perp)=\bar{\mu}$

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{x}_\perp-\mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{r})$$

- System nonuniform in the transverse plane

Projected Woods-Saxon distribution

$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1+e^{R_A/a})}\int_{-\infty}^{\infty}\frac{dz}{1+\exp\left[\left(\sqrt{\mathbf{x}_\perp{}^2+z^2}-R_A\right)/a\right]}$$

$$\begin{cases} \mathbf{R} = \frac{1}{2}(\mathbf{x}_\perp+\mathbf{y}_\perp) \\ \mathbf{r} = \mathbf{x}_\perp-\mathbf{y}_\perp \end{cases}$$

$$B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R},\mathbf{r}) \approx \text{``}\textit{gradient expansion in } \mathbf{R}\text{''}$$

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Numerical results

Pb-Pb collisions at LHC

$N_c = 3$

$g = 1$

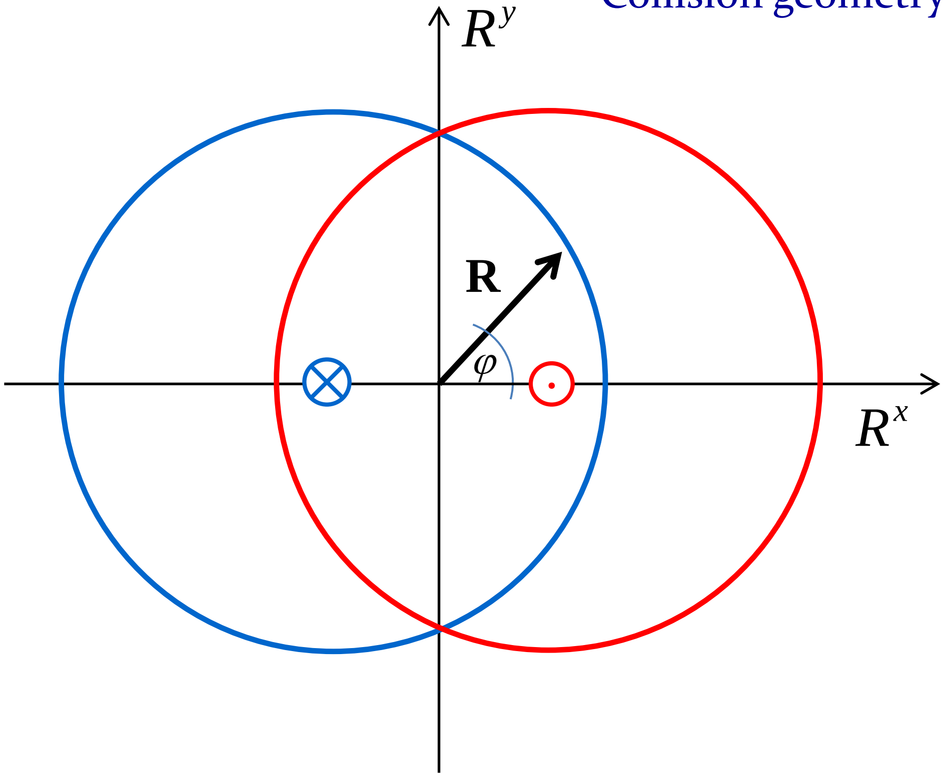
$Q_s = 2$ GeV

$m = 0.2$ GeV

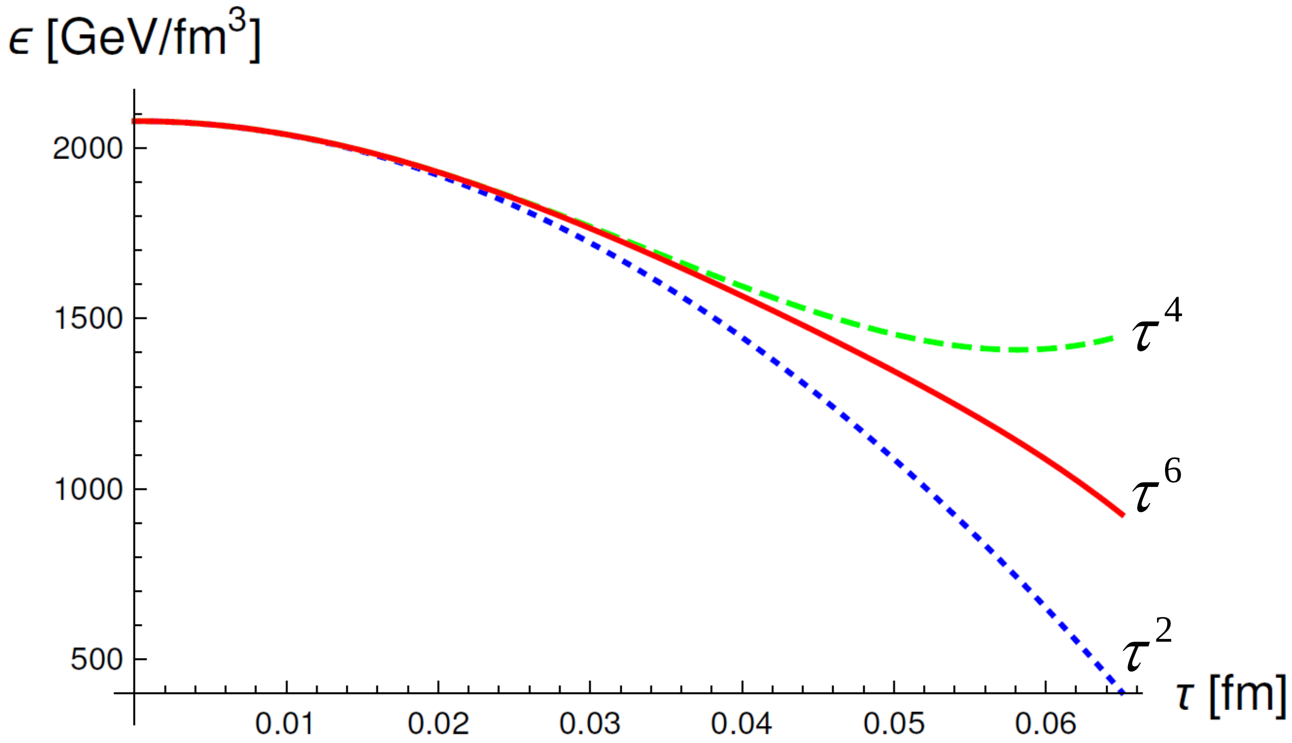
$R_A = 7.4$ fm

$a = 0.5$ fm

Collision geometry



Energy density



$$\varepsilon_0 = \varepsilon_{\rm eq} \quad \Rightarrow \quad T \approx 1.3\,{\rm GeV}$$

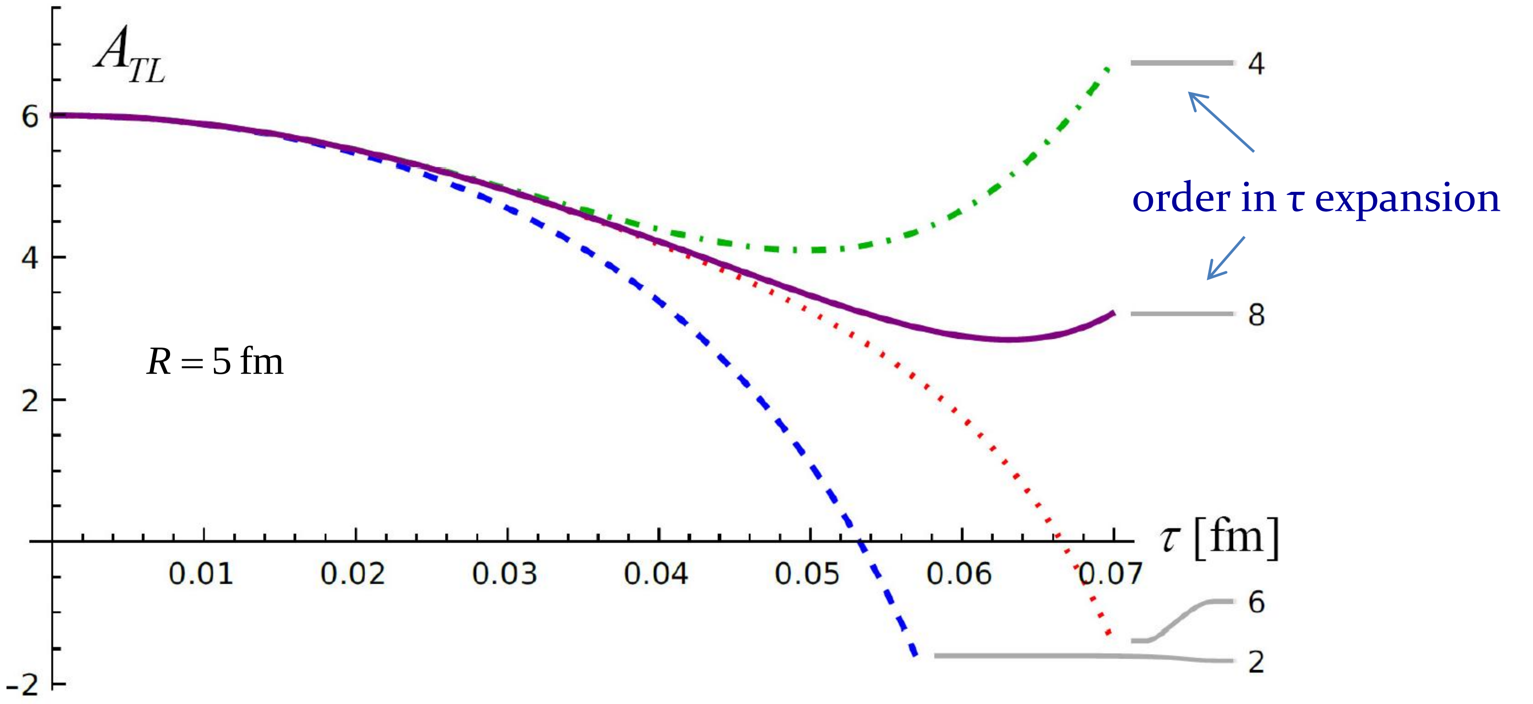
M. Carrington, A. Czajka & St. Mrówczyński, Eur. Phys. J. A **58**, 5 (2022)

Anisotropy

Central Pb-Pb collsions

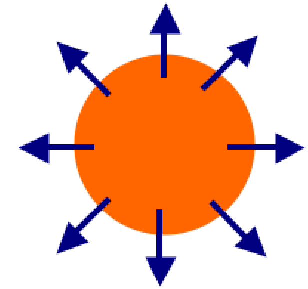
$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L} \qquad p_T \equiv \langle T^{xx} \rangle, \quad p_L \equiv \langle T^{zz} \rangle$$

$$\tau = 0 \quad \Rightarrow \quad p_T = -p_L = \varepsilon \quad \Rightarrow \quad A_{TL} = 6$$



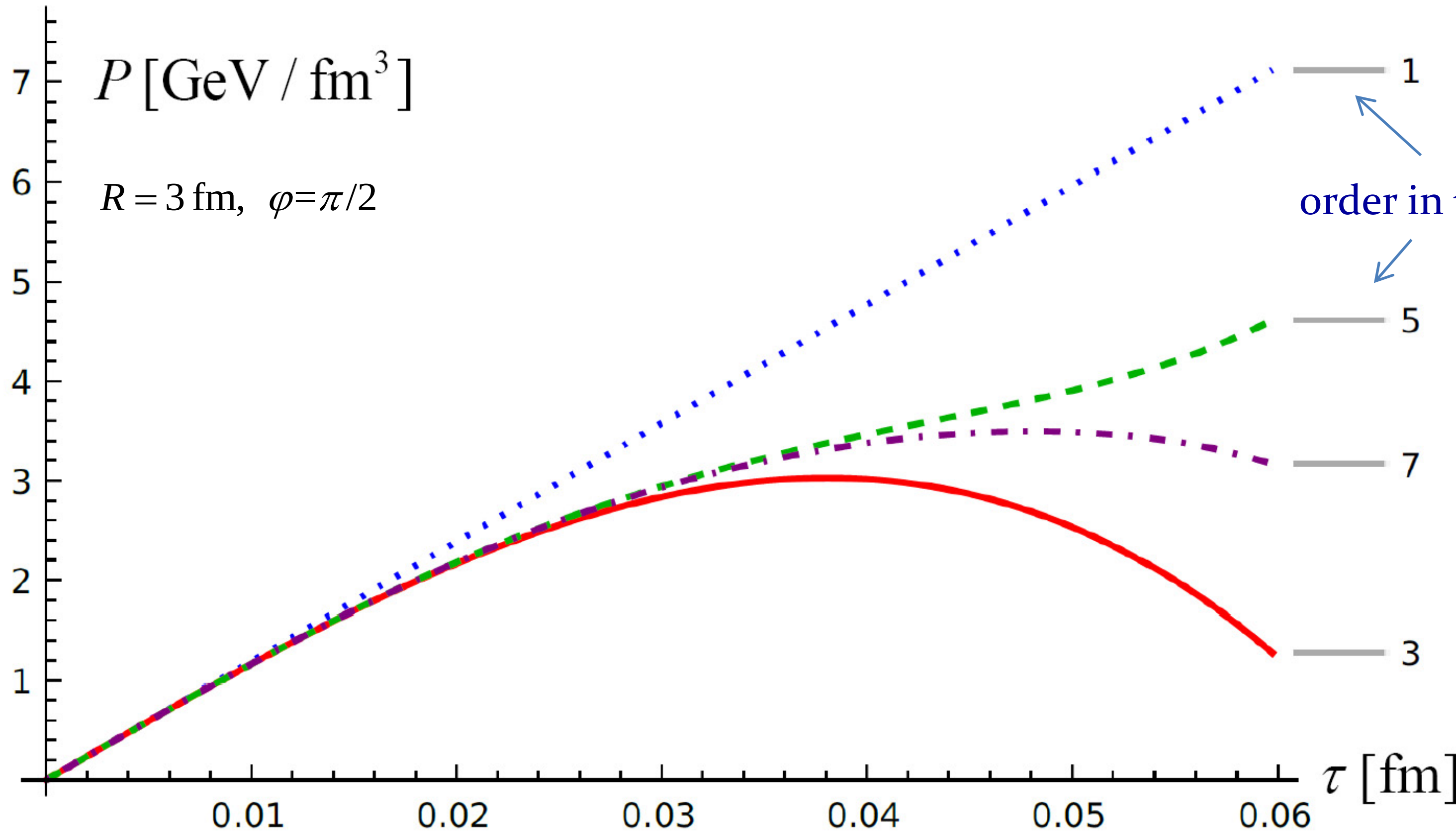
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow



Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$

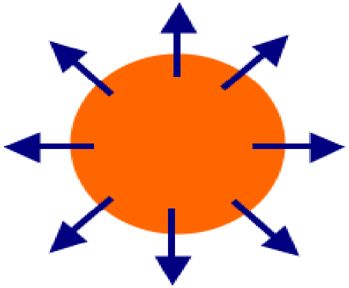


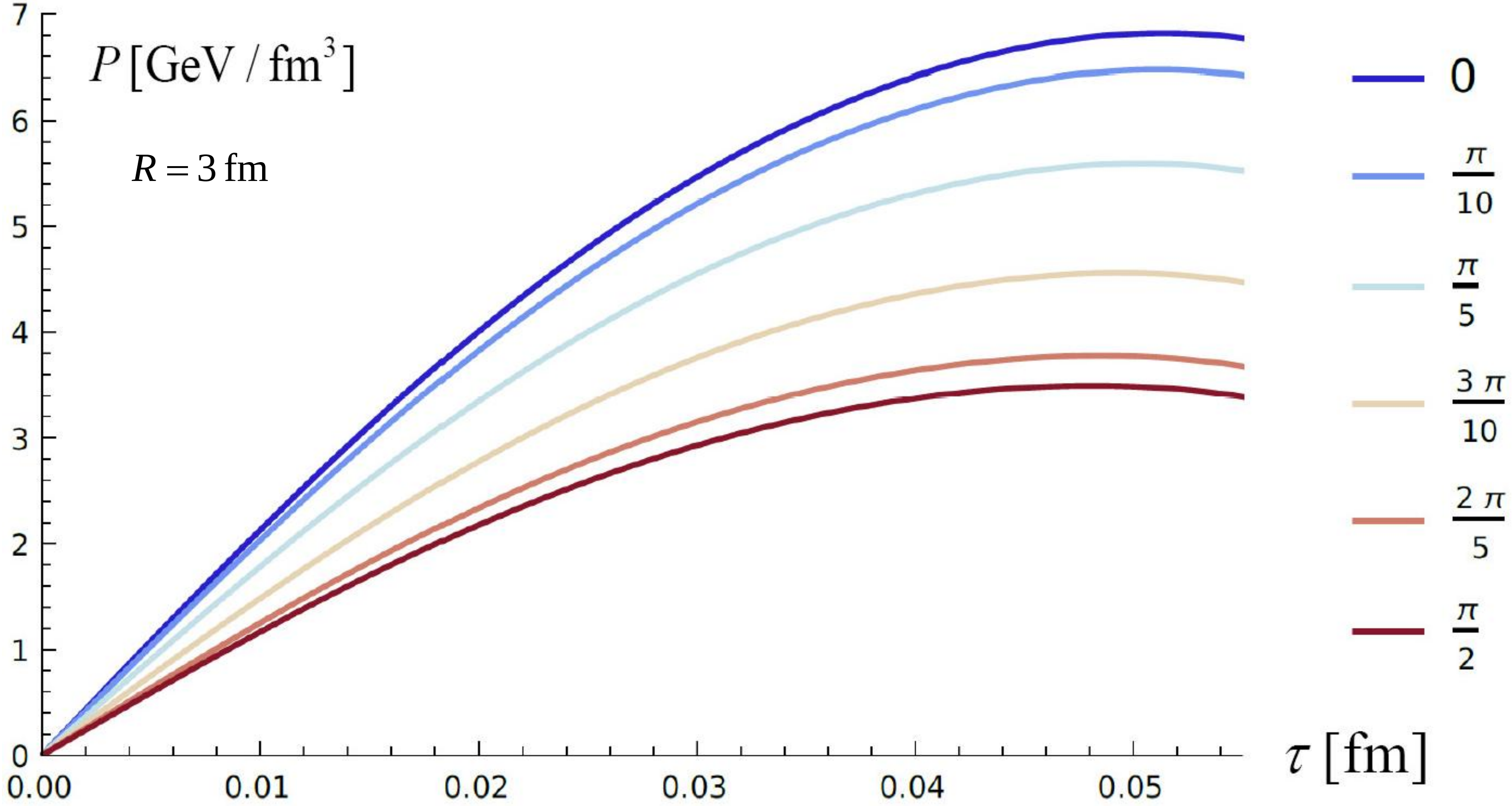
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow cont.

Pb-Pb collisions at b = 6 fm

$$P \equiv R^i T^{0i}$$



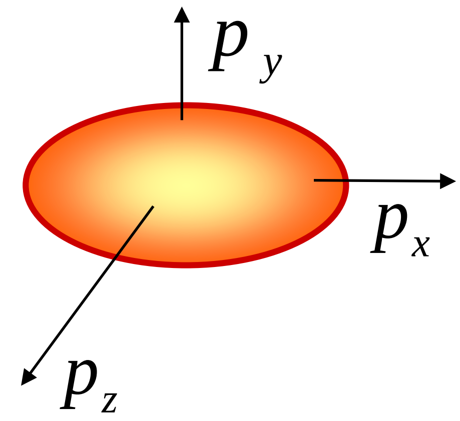


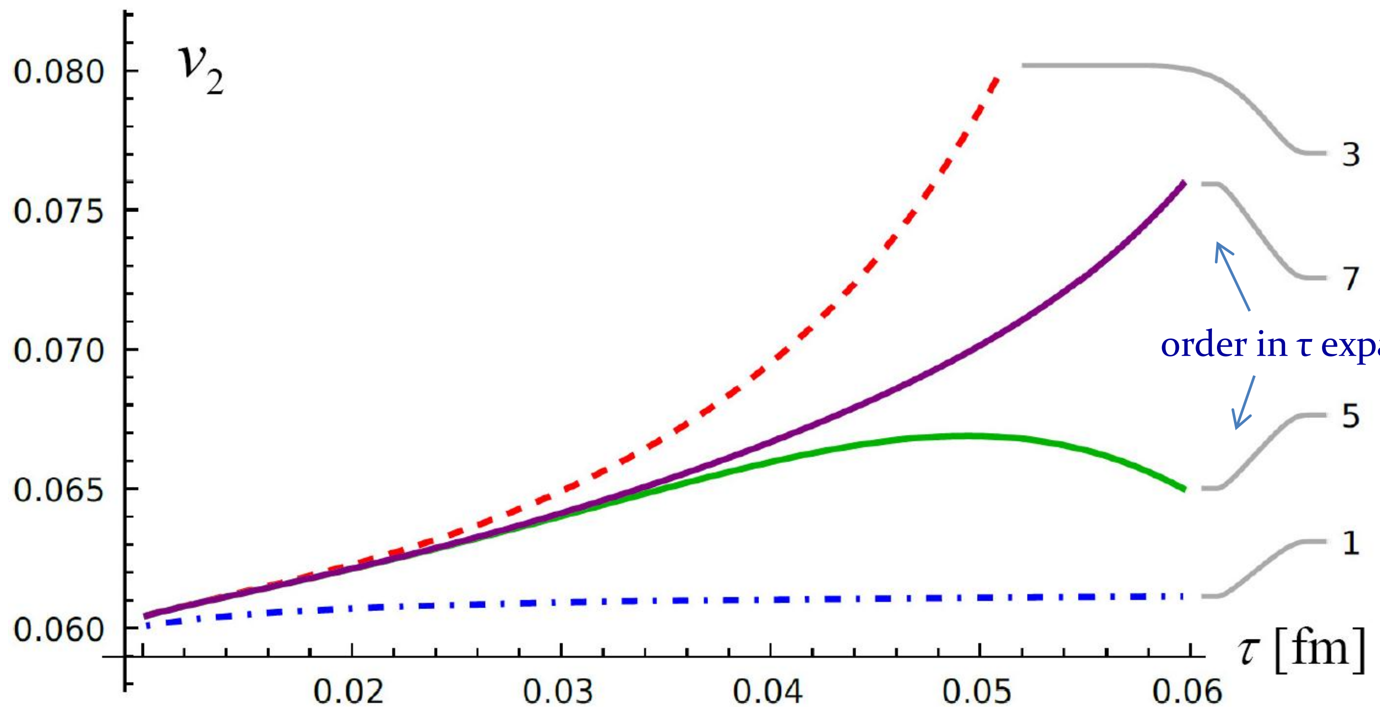
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Elliptic flow

Pb-Pb collisions at b = 2 fm

$$v_2 = \frac{\int d^2R \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$



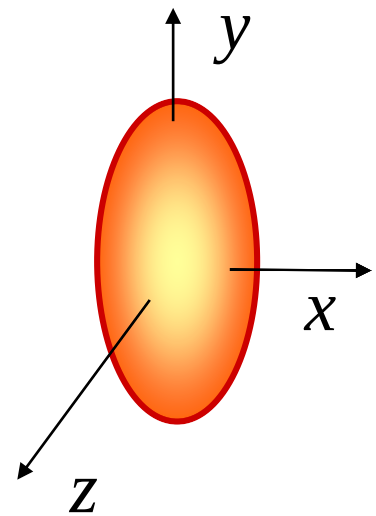


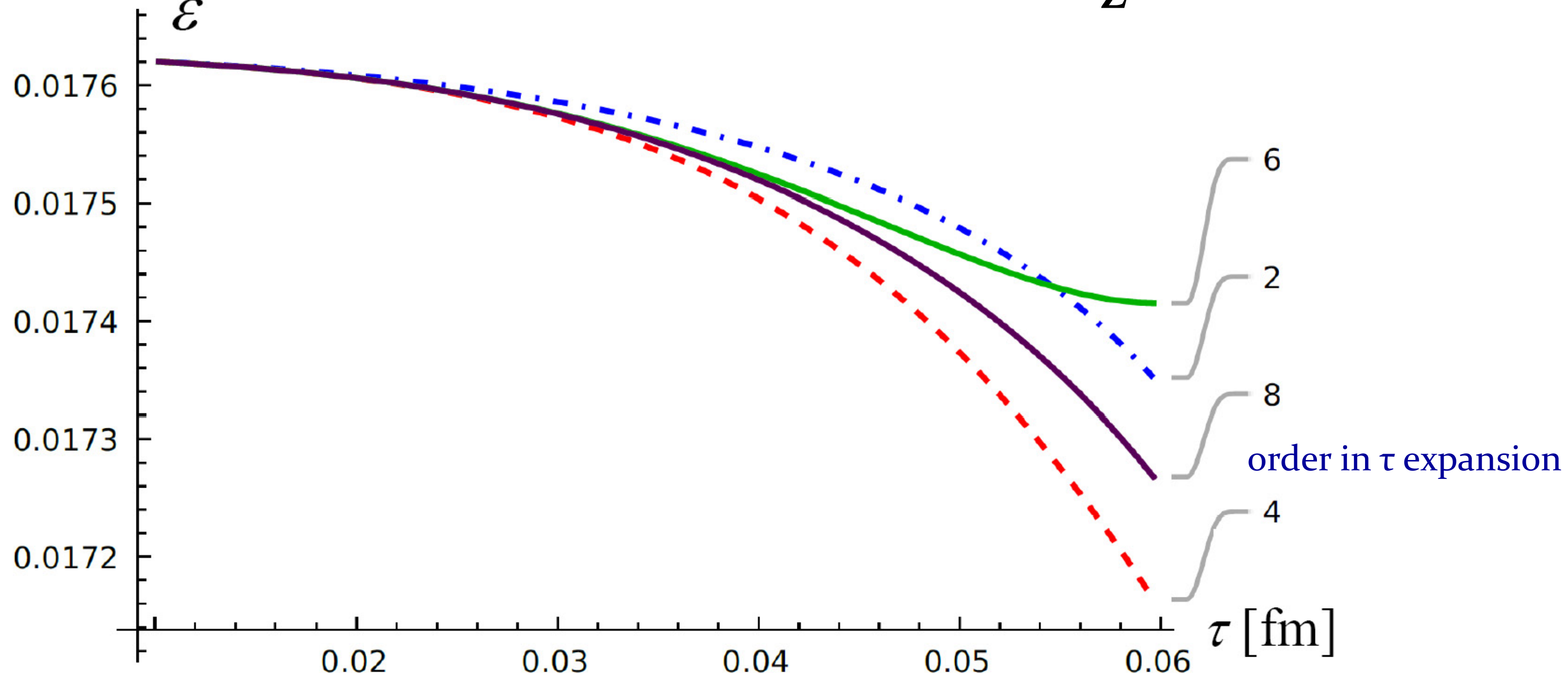
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Eccentricity

Pb-Pb collisions at b = 2 fm

$$\varepsilon = \frac{\int d^2R \, \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} \, T^{00}}$$

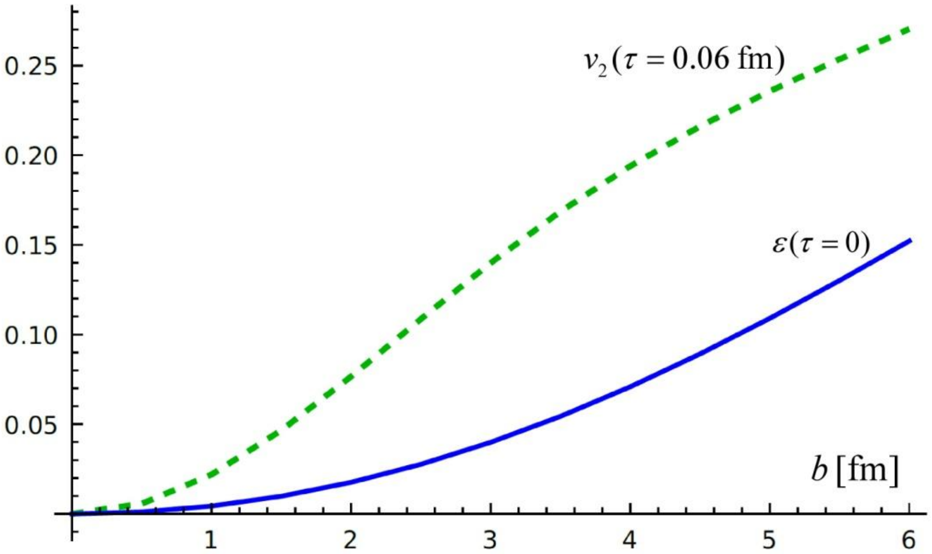


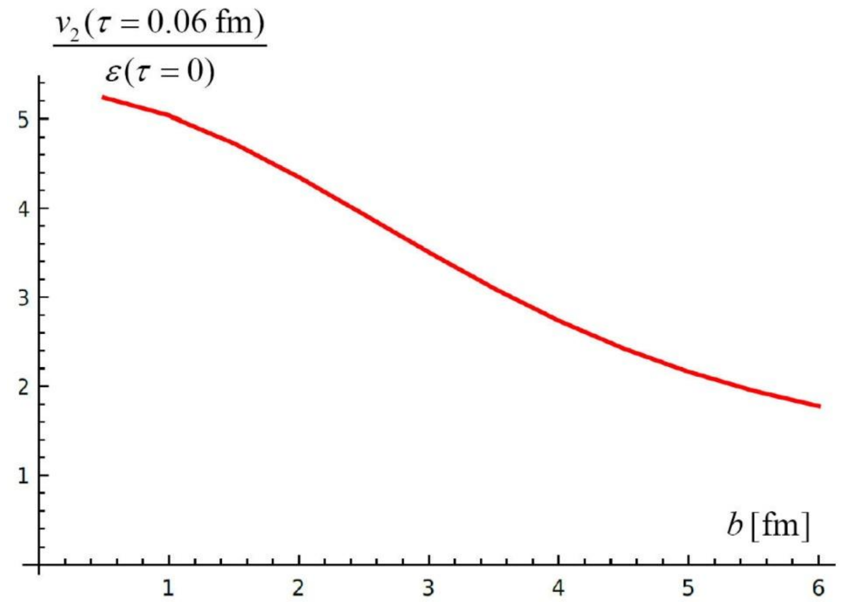


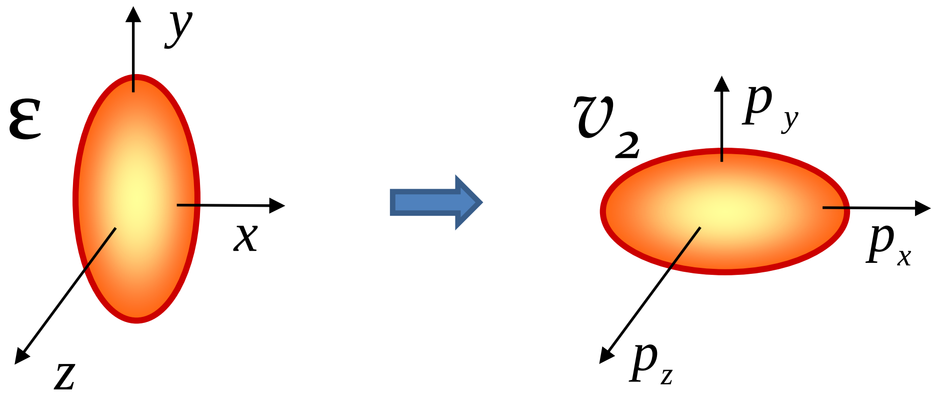
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hydrodynamic-like behavior

Pb-Pb collisions





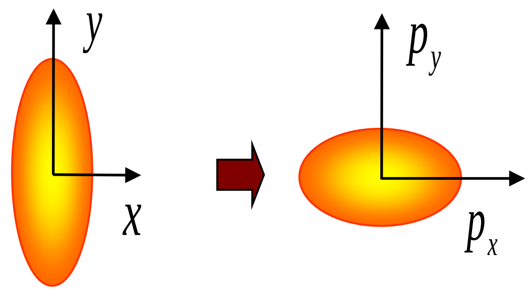


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Equation of universal flow

$$T^{tx} \approx -\frac{1}{2} t \frac{\partial T^{tt}}{\partial x}$$

Energy flow is generated by gradient of energy density.



Assumptions:

- $\partial_\mu T^{\mu\nu} = 0$
- Energy momentum tensor is mostly diagonal.
- The system is boost-invariant.

Glasma in ultrarelativistic collisions

$$T^{\mu\nu}(t=0) = \mathrm{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$$

short-time evolution

J. Vredevoogd & S. Pratt, Phys. Rev. C **79**, 044915 (2009);
M. Carrington, St. Mrówczyński &J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Equation of universal flow cont.

Derivation in Milne coordinates $\tau \equiv \sqrt{t^2 - z^2}, \quad \eta \equiv \frac{1}{2}\ln\frac{t+z}{t-z}$

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\rho} T^{\rho\nu} + \Gamma^\nu_{\mu\rho} T^{\mu\rho} = 0$$

$\nu = x$

$$\left(\frac{\partial}{\partial\tau} + \frac{1}{\tau}\right) T^{\tau x} + \frac{\partial T^{xx}}{\partial x} + \underbrace{\frac{\partial T^{yx}}{\partial y}}_{\approx 0 \text{ mostly diagonal } T^{\mu\nu}} + \overbrace{\frac{\partial T^{\eta x}}{\partial \eta}}^{= 0 \text{ boost invariance}} = 0$$

$\eta \approx 0 \quad \Rightarrow \quad z = 0 \ \ \& \ \ \tau = t$

$$\left(\frac{\partial}{\partial t} + \frac{1}{t}\right) T^{tx} + \frac{\partial T^{xx}}{\partial x} = 0$$

$T^{xx} \approx T^{tt} \approx \text{const} \quad \& \quad T^{tx}(t=0) = 0$

$$T^{tx} = -\frac{1}{2} t \frac{\partial T^{tt}}{\partial x}$$

Universal flow
in proper time expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \tau^n T_n^{\mu\nu}$$

$$T_{n+1}^{tx} = -\frac{1}{2}\tau\frac{\partial T_n^{tt}}{\partial x}$$

$n = 1, 2, \ldots 7$

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Mapping on hydrodynamic $T^{\mu\nu}$

$$T^{\mu\nu}_{\text{glasma}}(\tau,\mathbf{x}_T) \quad vs. \quad T^{\mu\nu}_{\text{hydro}}(\tau,\mathbf{x}_T)$$

Eigenvalue problem:
$$T^{\mu\nu}_{\text{glasma}} w_\nu = \lambda w^\mu$$

Ideal hydrodynamics

$$T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p = \frac{1}{3}\varepsilon$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu$$

Anisotropic hydrodynamics

$$T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p_T)u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L) z^\mu z^\nu$$

$$T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p_L = \varepsilon - 2 p_T$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu, \qquad T^{\mu\nu}_{\text{hydro}} z_\nu = -p_L z^\mu$$

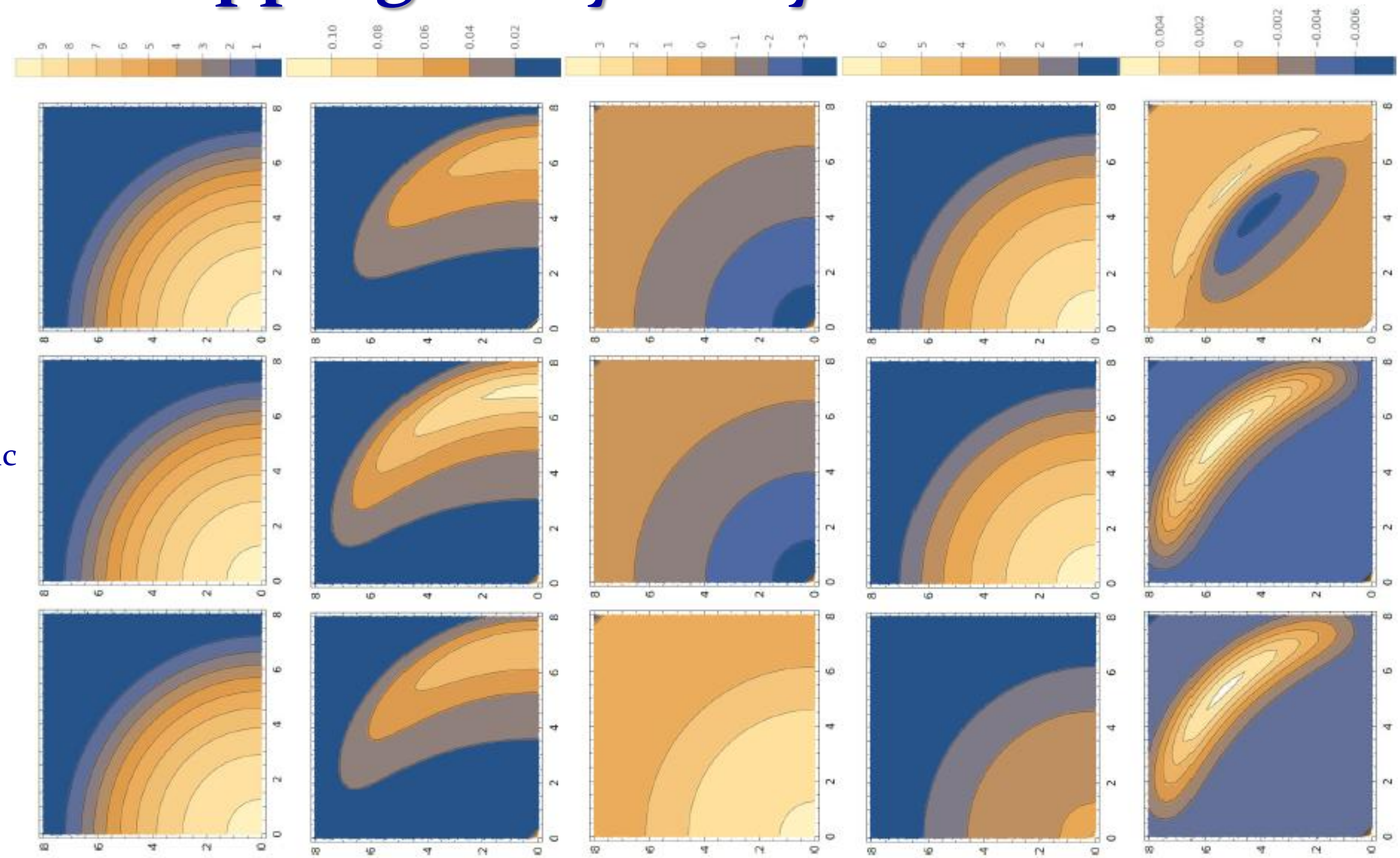
W. Florkowski & R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)
M. Martinez & M. Strickland, Nucl. Phys. A **848**, 183 (2010)

Mapping on hydrodynamic $T^{\mu\nu}$

glasma

anisotropic hydro

ideal hydro



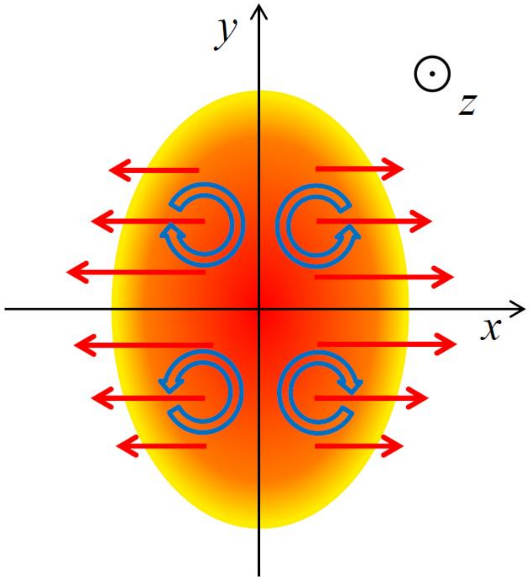
$\tau = 0.06$ fm

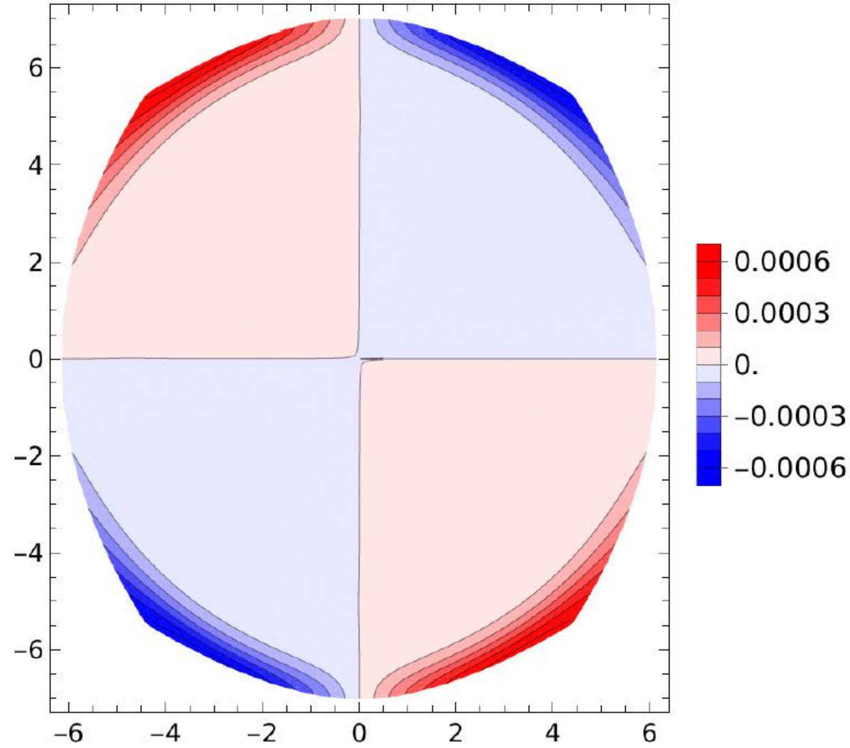
$b = 0,\ z = 0$

$T^{tt} = \varepsilon$ T^{tx} T^{zz} T^{xx} T^{xy}

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Vorticity





$$V^i(\vec{r}) \equiv \frac{T^{0i}(\vec{r})}{T^{00}(\vec{r})}$$

ω^z at $b = 2$ fm & $\tau = 0.06$ fm in τ^8 order

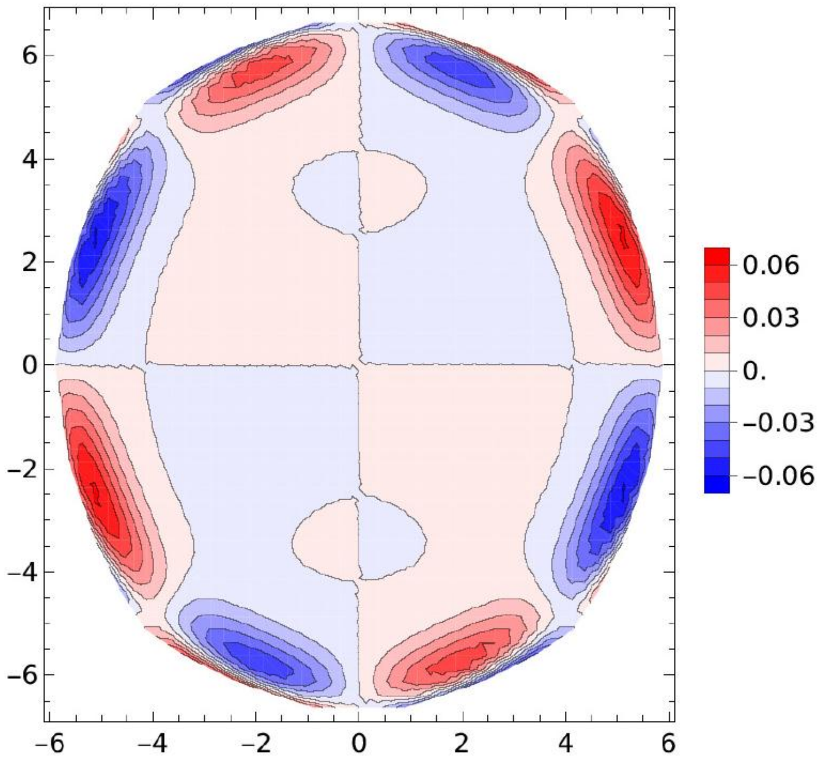
$$\omega(\vec{r}) = \nabla \times \vec{V}(\vec{r})$$

$$\omega_{\rm th}(\vec{r}) = \nabla \times \frac{\vec{u}(\vec{r})}{T(\vec{r})}$$

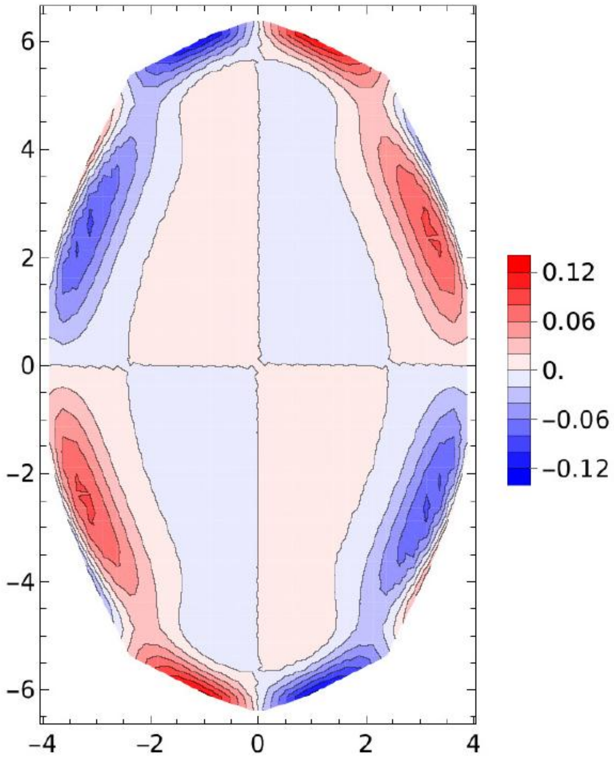
M. Carrington & St. Mrówczyński, arXiv:2505.07324

Local angular momentum

$$\frac{dL^z(\vec{r}_0)}{d\eta} = -\tau \int_{\Delta^2} d^2 r \left((r^y - r_0^y) T^{0x} - (r^x - r_0^x) T^{0y} \right)$$



$b = 2$ fm



$b = 6$ fm

M. Carrington & St. Mrówczyński, arXiv:2505.07324

Conclusion

Glasma behaves as a fluid.