Zakopane
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Quark Spaghetti with Glue Balls (SQGB)

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(A simple story that sadly could have been told years ago....)

Scientific Question:

Is there a phase of matter between the Quark Gluon Plasma and the Hadron Gas?

Work in part motivated by some observations of Lenya Glozman, Tom Cohen, and colleagues

Motivation from lattice computations:

Chiral symmetry is restored about 160 MeV, but confinement as measured by Polyakov loop appears to disappear at about 300~Mev. This is the temperature of deconfinement in the pure glue theory

In this intermediate temperature range, entropy is smaller than expected

Old Problem:

How to characterize confinement:

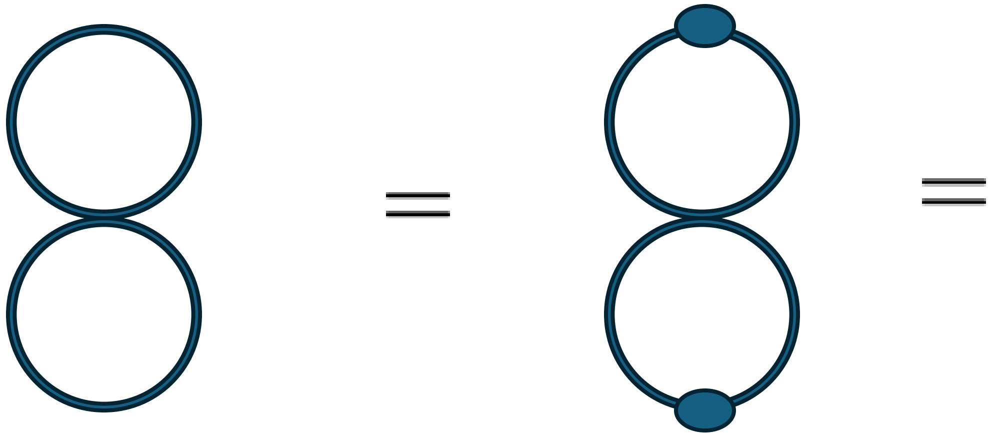
When dynamical quarks are present confinement at finite temperature or density is only approximately characterized by the Polyakov line because strings always break and the free energy of an isolated quark is always finite.

If we go to large N_c, or in pure gauge theory, there is truly confinement. In pure gauge theory, the deconfinement temperature is about 300 MeV, which is much different from the chiral transition in a theory with dynamical quarks.

Perhaps the region between 160 MeV and 300 MeV is distinct from that at higher or lower T.
Some evidence from lattice data showing that stingy vortices percolate at around 270 MeV. In this vortex picture, this should be the temperature of deconfinement.

How to characterize these separate regions.

Contribution to the pressure are densities times an amplitude:



$$= \int dp dp' \; n(p) A(p, p') n(p')$$

Kinetic energy of hadrons ~ 1, quarks ~N_c, gluons ~ N_c^2

Amplitude for meson-meson ~ $1/N_c$, glueball-glueball ~ $1/N_c^2$

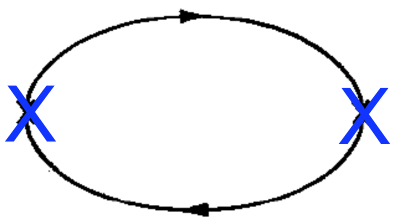
For pressure of hadrons to be of order N_c the
hadron density should be of order N_c
Pressure for glueballs to be of order N_c^2 the
density of glueballs should be of order N_c^2

Mesons and glueballs, density ~1
Quarks and glueballs. density ~ N_c
Quarks and gluons, density ~ N_c^2

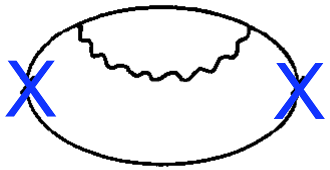
Form factors at large N_c

J ~ (gauge invariant) mesonic current

$$< J(x)J(0) > \sim N_c$$



Infinite # of planar diagrams for $< J\,J >$:



Confinement => sum over mesons, form factors ~ $N_c^{1/2}$

$$< J(x)J(0) > \sim \int d^4p\, e^{ip\cdot x} \sum_n < 0|J|n > \frac{1}{p^2 + m_n^2} < n|J|0 >$$

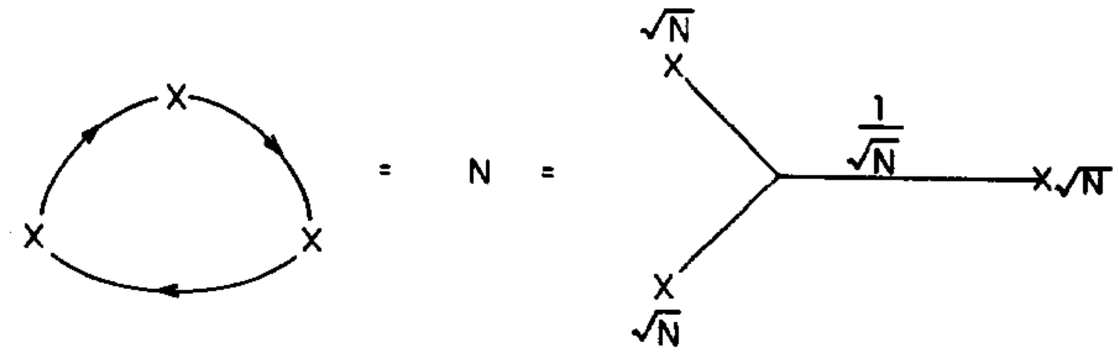
$$< J(x)J(0) > \sim \; N_c \quad \Rightarrow \; < 0|J|n > \sim \sqrt{N_c} \;\; \text{if} \;\; m_n \sim 1$$

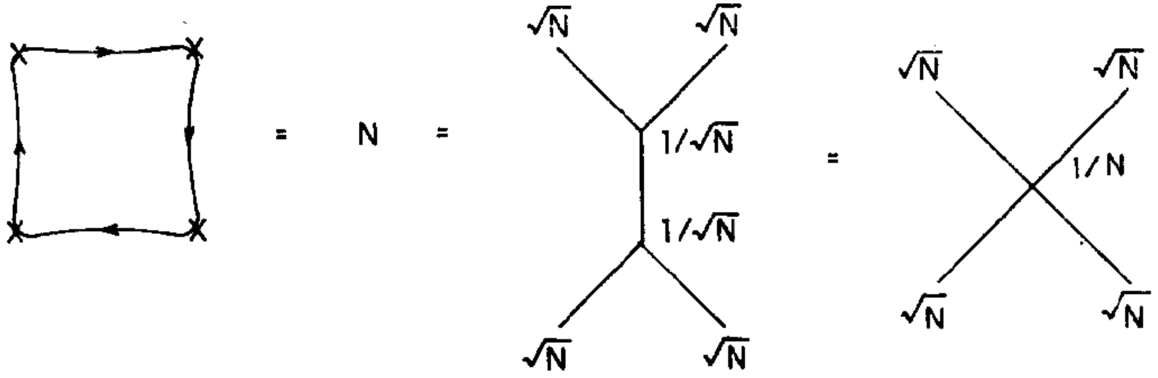
Mesons & glueballs *free* at $N_c = \infty$

With form factors $\sim N_c^{1/2}$, 3-meson couplings $\sim 1/N_c^{1/2}$; 4-meson, $\sim 1/N_c$
For glueballs, 3-glueball couplings $\sim 1/N_c$, 4-glueball $\sim 1/N_c^2$

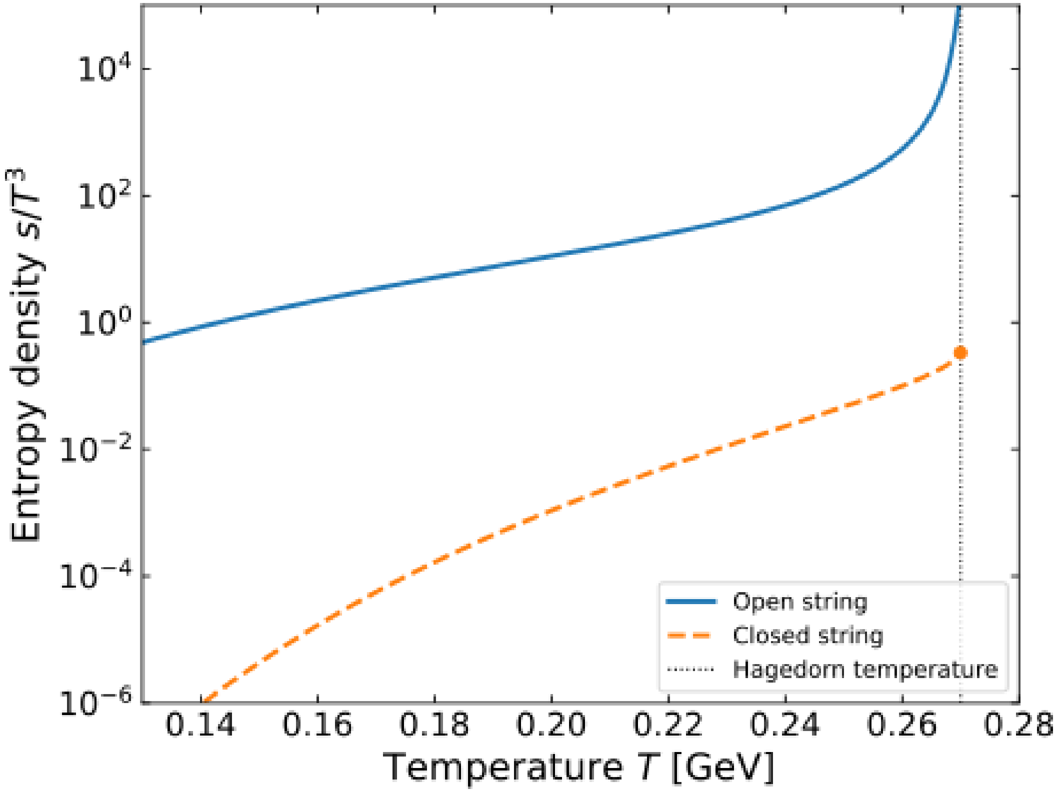
Mesons and glueballs don't interact at $N_c = \infty$.

Large N limit *always* (some) classical mechanics Yaffe '82





Simple model:



Contribution of glueballs is always very very small. The closed strong, glueball contribution, is non-divergent at the Hagedorn temperature

Mesons: open strings
Glueballs: closed strings
Quarks: Free quarks with mass between constituent mass and current quarks mass
Gluons: Mass between massless and ½ of glueball mass

Take closed and open string spectrum from string theory. Glueball and mesons must have the same Hagedorn temperature which we take to be

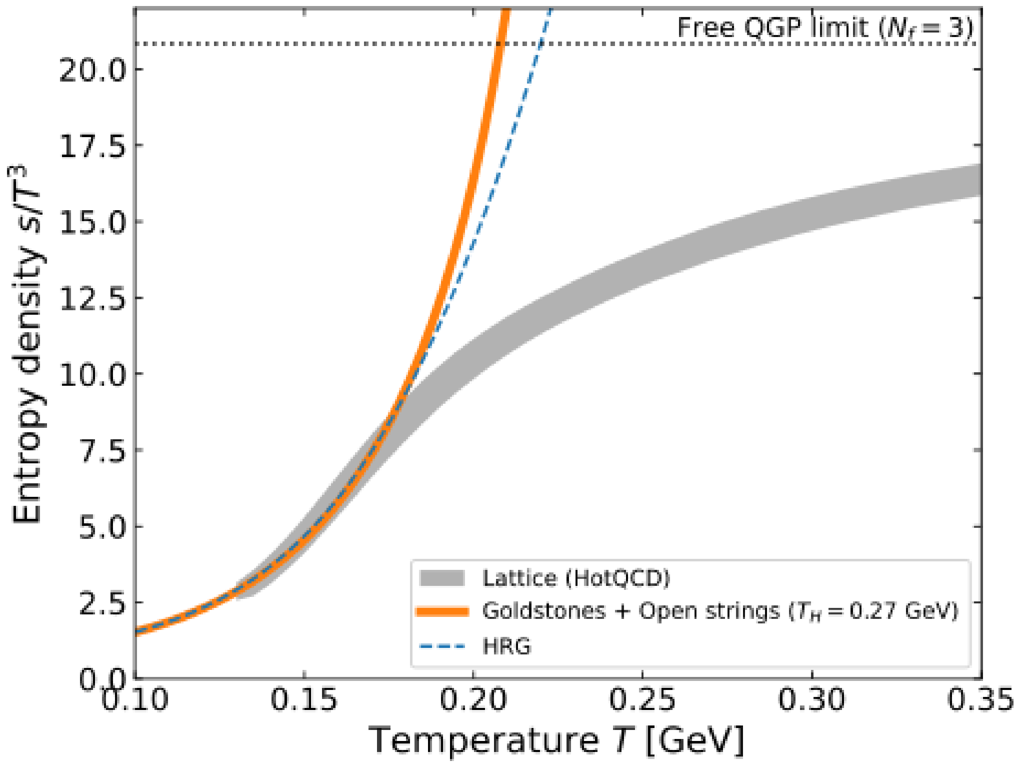
T_H \sim 300 MeV

We handle the contribution of lowest mass glueball states, and Goldstone boson meson states separately, and then treat higher mass states by a continuum integral. The meson integration begins at .6 GeV, and the glueball at about 3~GeV

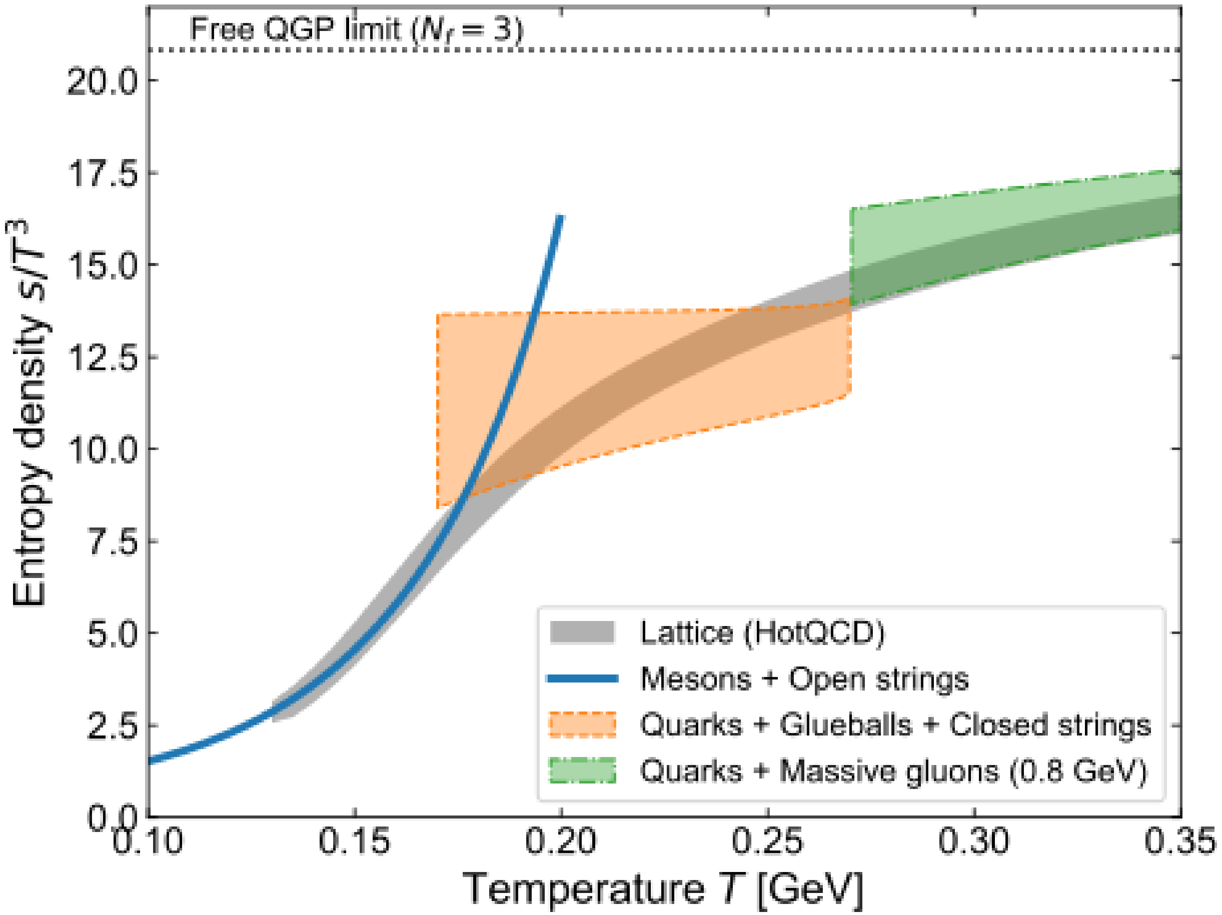
$$T_H^{string} = \sqrt{\frac{3\sigma}{2\pi}} = 304MeV$$

for

$$\sqrt{\sigma} = 440MeV$$



With the string theory resonance model and a Hagedorn temperature of 285 MeV, describe the lattice data on entropy up until the chiral temperature, where presumably the degrees of freedom become quarks

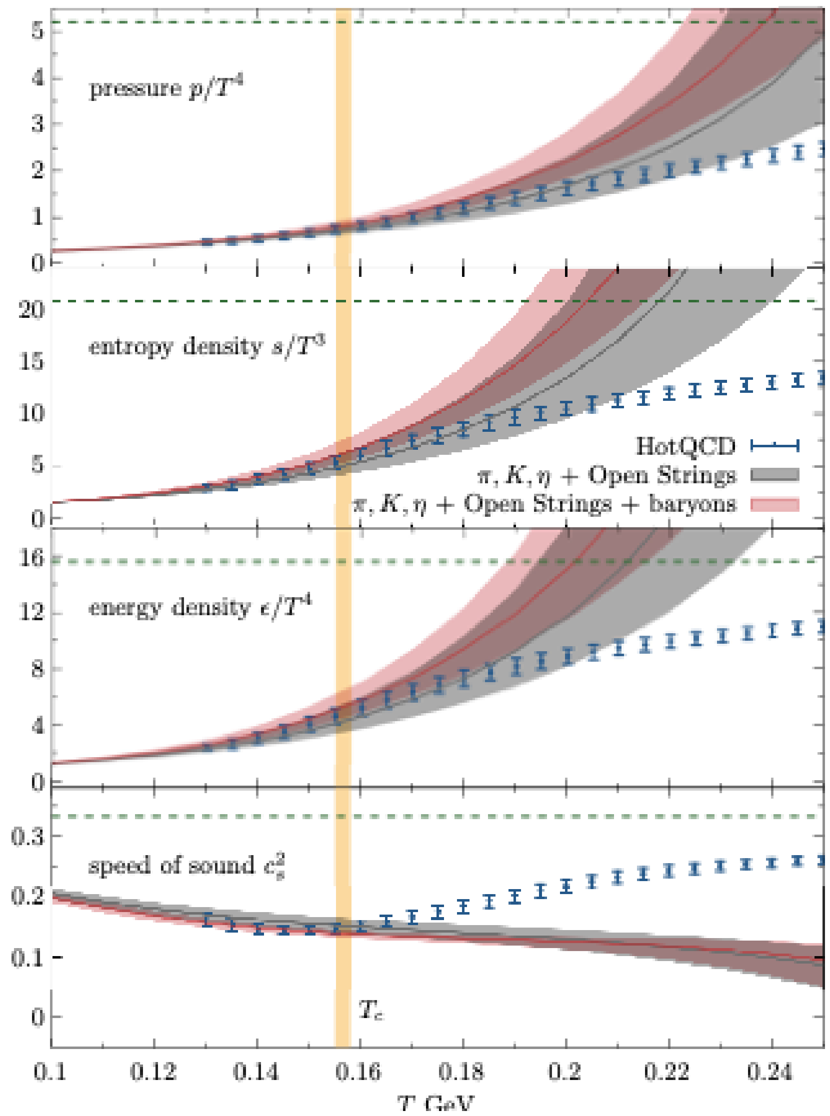


Matching on to quarks plus glueball at 160 MeV, and to quarks plus gluons at the Hagedorn temperature, the entropy is reasonably well described

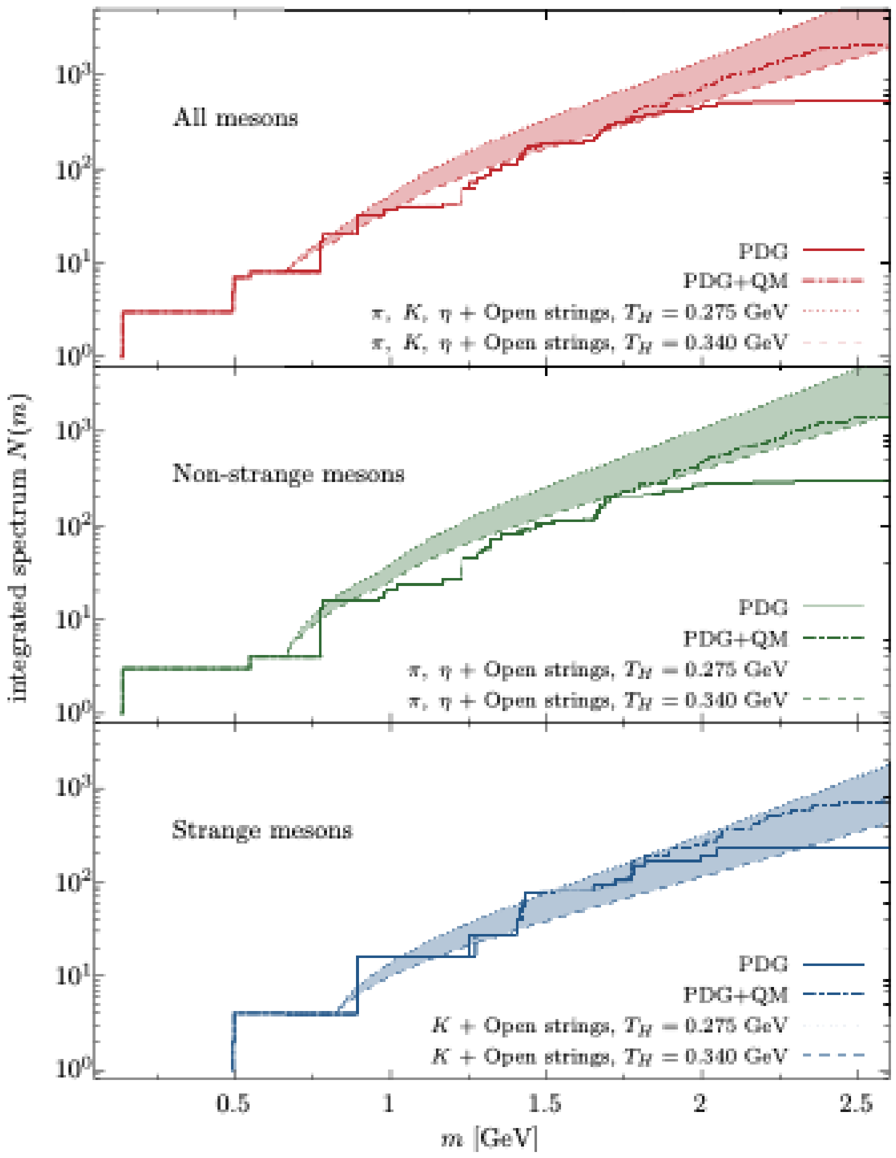
There is a correction to this from subtracting off the baryon contributions from lattice data, which makes a good fit at 310 MeV:
Redlich, Marczenko, Kovacs LM

Kovacs, Marczenko, Redlich, LM

Thermodynamic Contributions with and without baryons included, T_H =275-340 MeV



Comparison to experimental and theory extrapolated spectrum of mesons states



The quark phase with energy density of order N_c has to a good approximation, no glueballs, and hence no gluons, in it. This is because glueballs are massive. The Debye screening mass is

$$M^2_{Debye} = g^2(N_c/3 + N_f/6)T^2$$

The first term is from gluons and the second from quarks. In the large N_c limit, one holds the ‘tHooft coupling fixed,

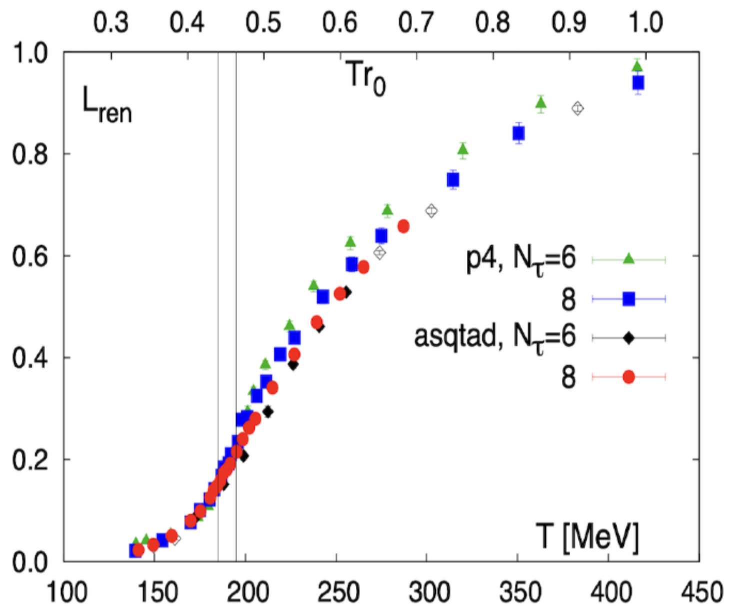
$$g^2_{'tHooft} = g^2 N_c$$

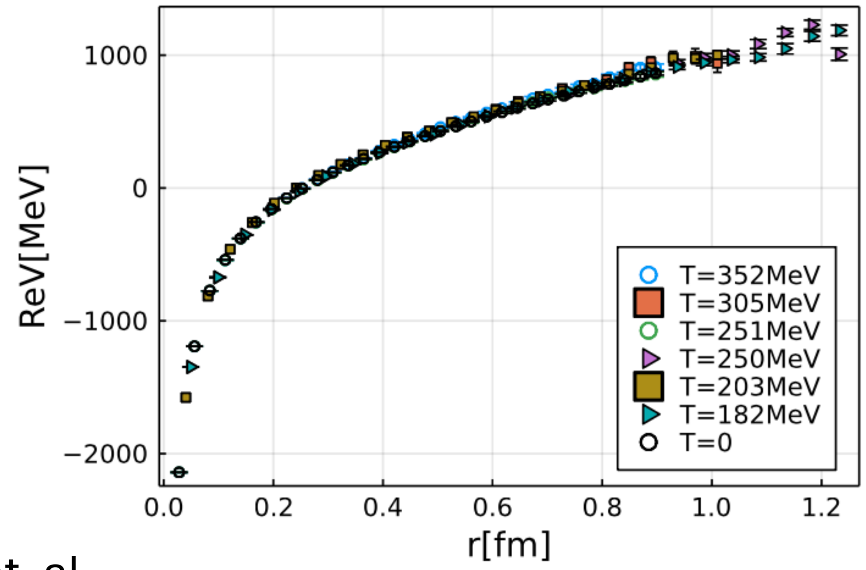
If because glueball contribution is small, we ignore the first term, the second term vanishes in the large N_c limit, so that the new phase of quarks is confined. The quarks are stringy. We call this phase,

Spaghetti of Quarks with Glueballs (SQGB)

But it is not very expensive **Quark Spaghetti with Glue Balls** since there is much spaghetti and few glueballs!

Karsch et. al.





Bazavov et. al

The Chiral Transition

$$< \overline{\psi}\psi >_T = < \overline{\psi}\psi >_0 - \frac{\partial P}{\partial m_q}$$

Take P(M) for a free resonance gas of quarks.

We need to know

$$\sigma = m_q \frac{\partial M}{\partial m_q}$$

For pions:

$$f_\pi^2 m_\pi^2 = -m_q < \overline{\psi}\psi >_0$$

$$\sigma_\pi = \frac{m_q}{2m_\pi}\frac{dm_\pi^2}{dm_q} = -\frac{m_q}{2m_\pi} < \overline{\psi}\psi >_0 / 2f_\pi^2 = m_\pi/2$$

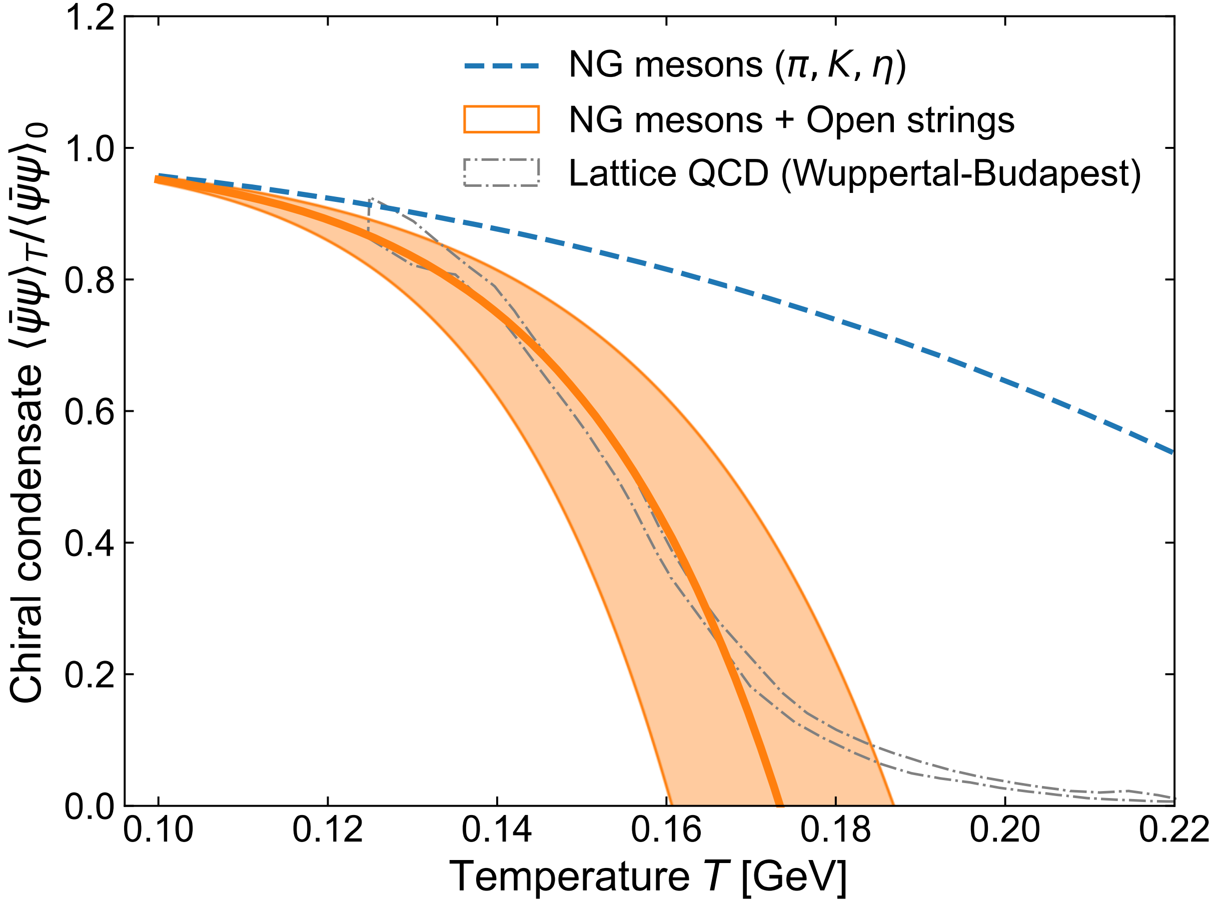
For massive mesons, we fit the sigma value to data on finite temperature values for chira symmetry.
We assume sigma arises only from low mass quarks, so that

$$\sigma_M = n_q \overline{\sigma}$$

We find best fit is

$$\overline{\sigma} = 60 MeV$$

For Goldstsone bosons, we use current algebra value



SQGB and Quarkyonic Matter

Quarkyonic matter is for low temperatures and finite baryon number density. Because there are no gluons the matter is confined. In ordinary hadronic matter there are no baryons for temperature because

$$e^{-M_B/T} \,|_{T \sim \Lambda_{QCD}} \sim e^{-N_c}$$

Baryons exist as quark-like degrees of freedom because of a chemical potential

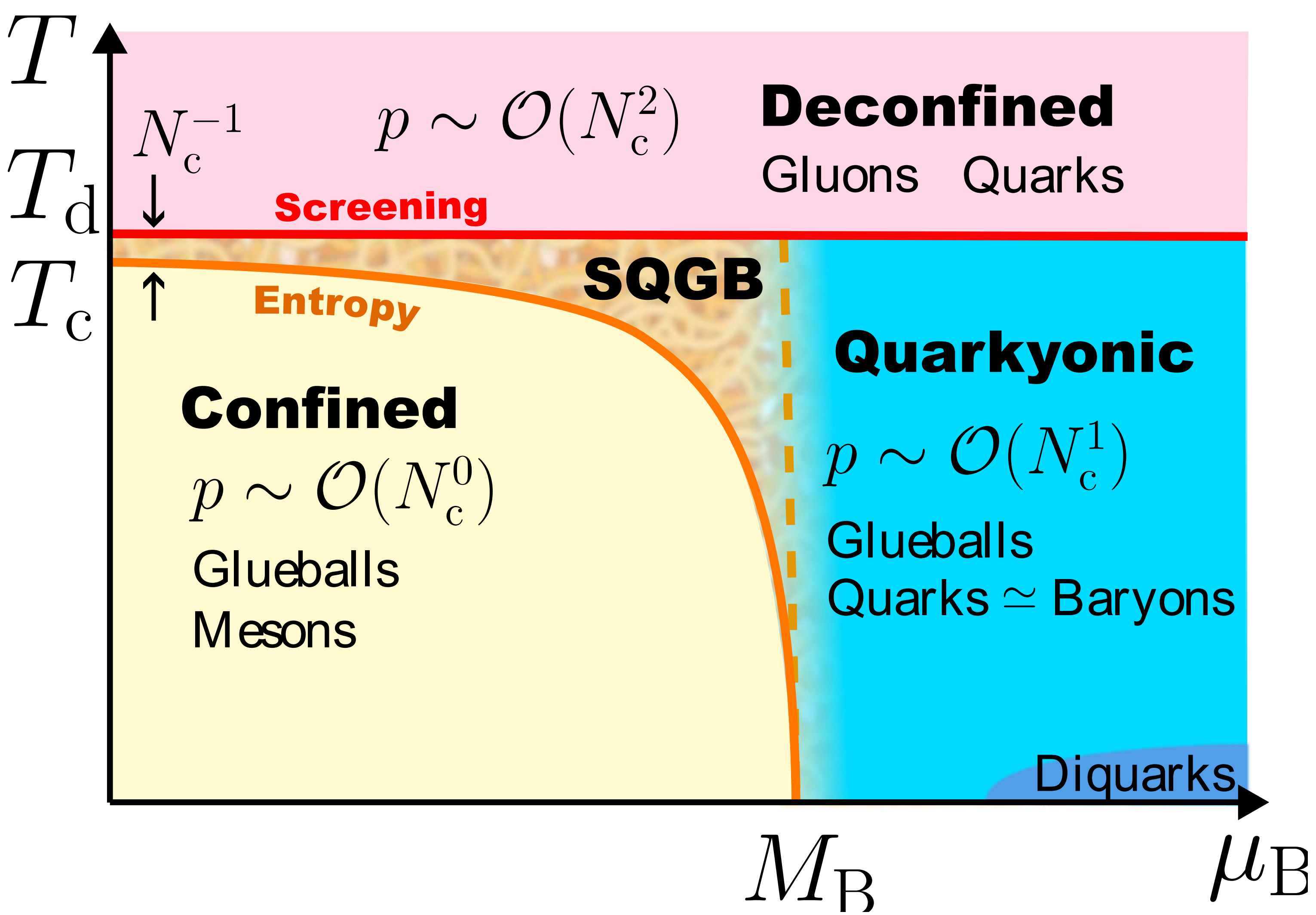
$$\mu_B - M = \kappa T, \kappa \sim 1$$

The density is quarks is of order N_c. in Quarkyonic matter.

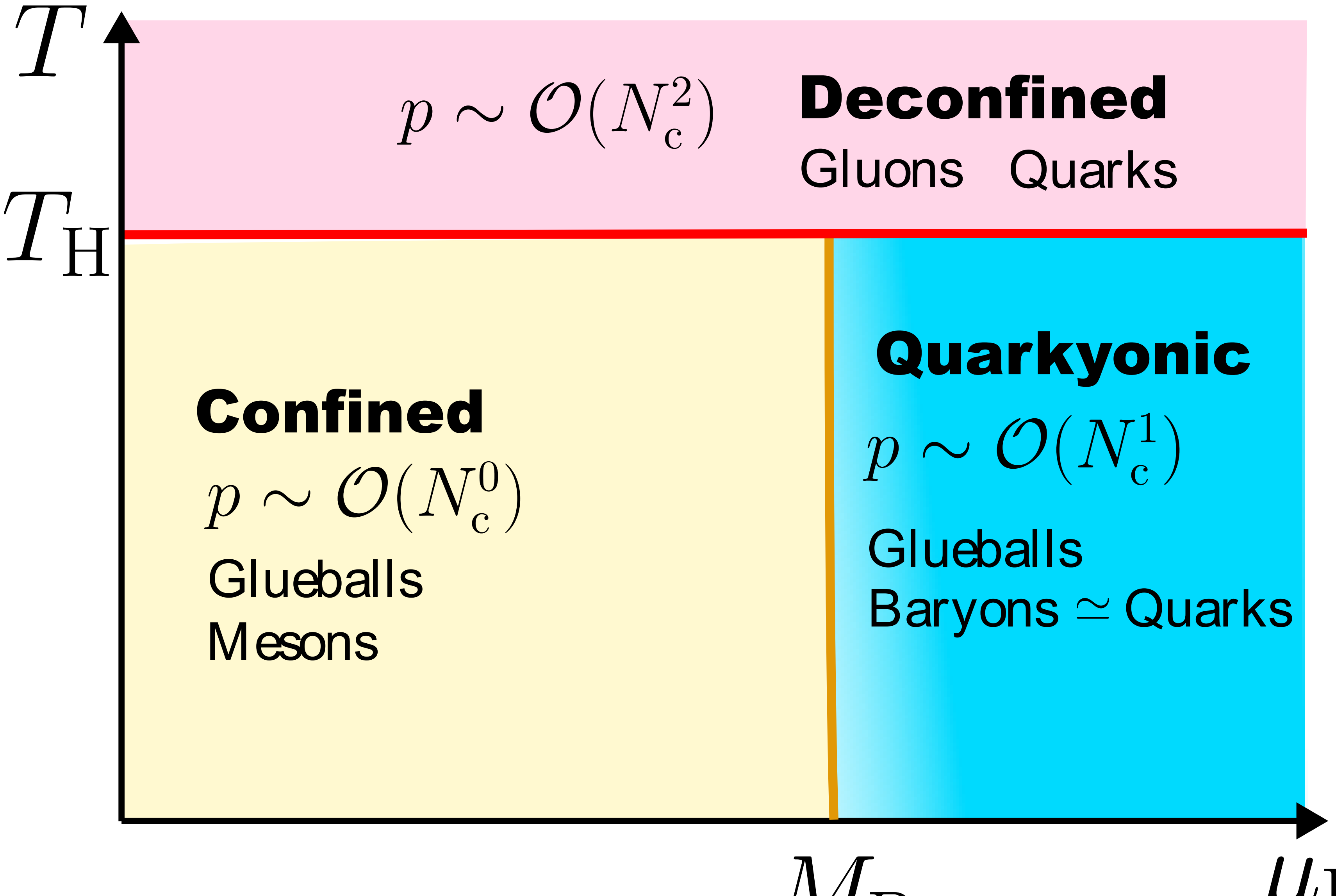
This is very similar to the SQGB, except in the SQGP there are antiquarks.

Naively, Quarkyonic Matter has restored chiral symmetry, but condensate form at the fermi surface that break chiral symmetry. The weak chiral symmetry breaking effects differentiate the SQGP from Quakryonic Matter

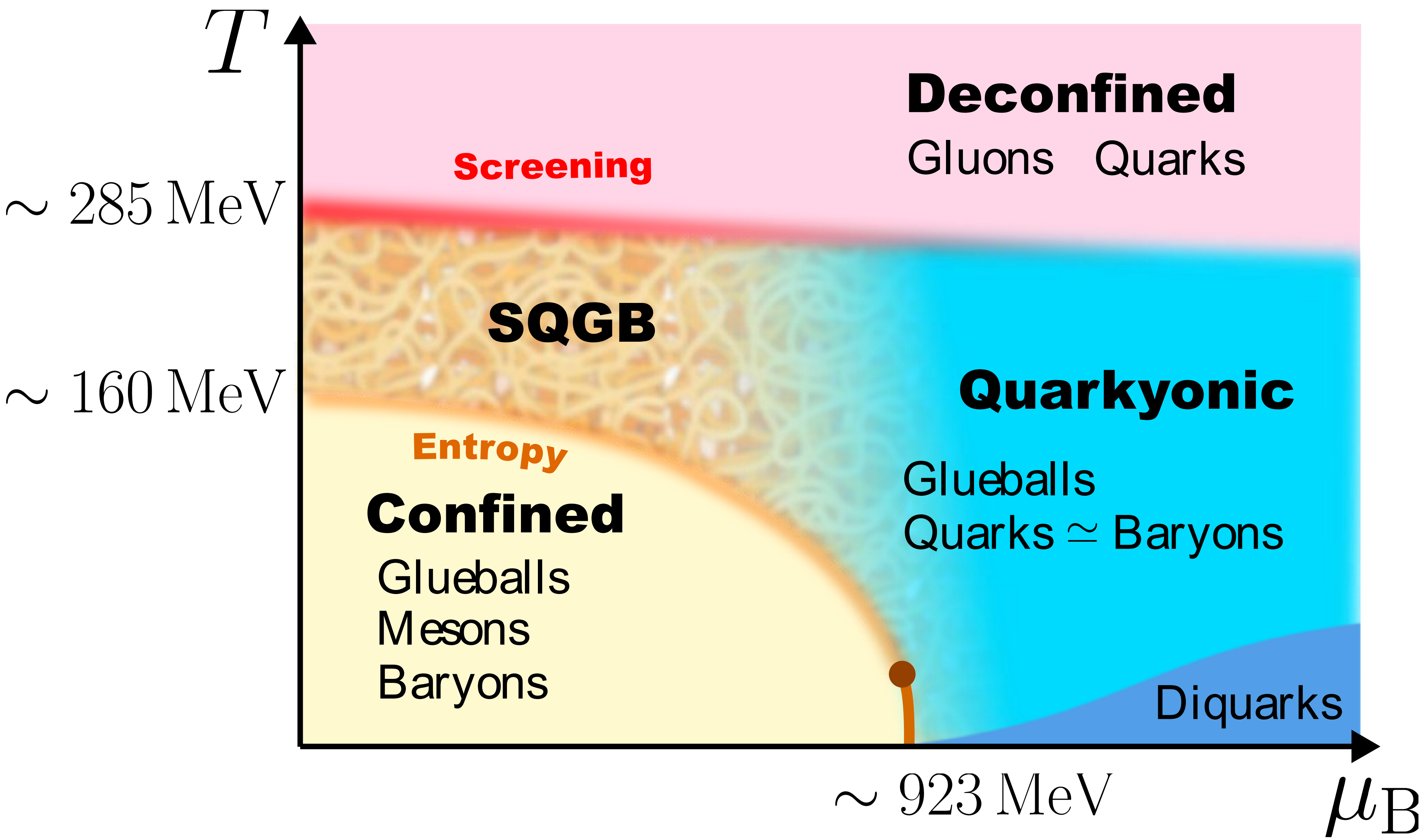
In the limit if a very large number of colors, because the density of quarks diverges, the QGP and SQGB temperatures should both become the Hagedorn temperature



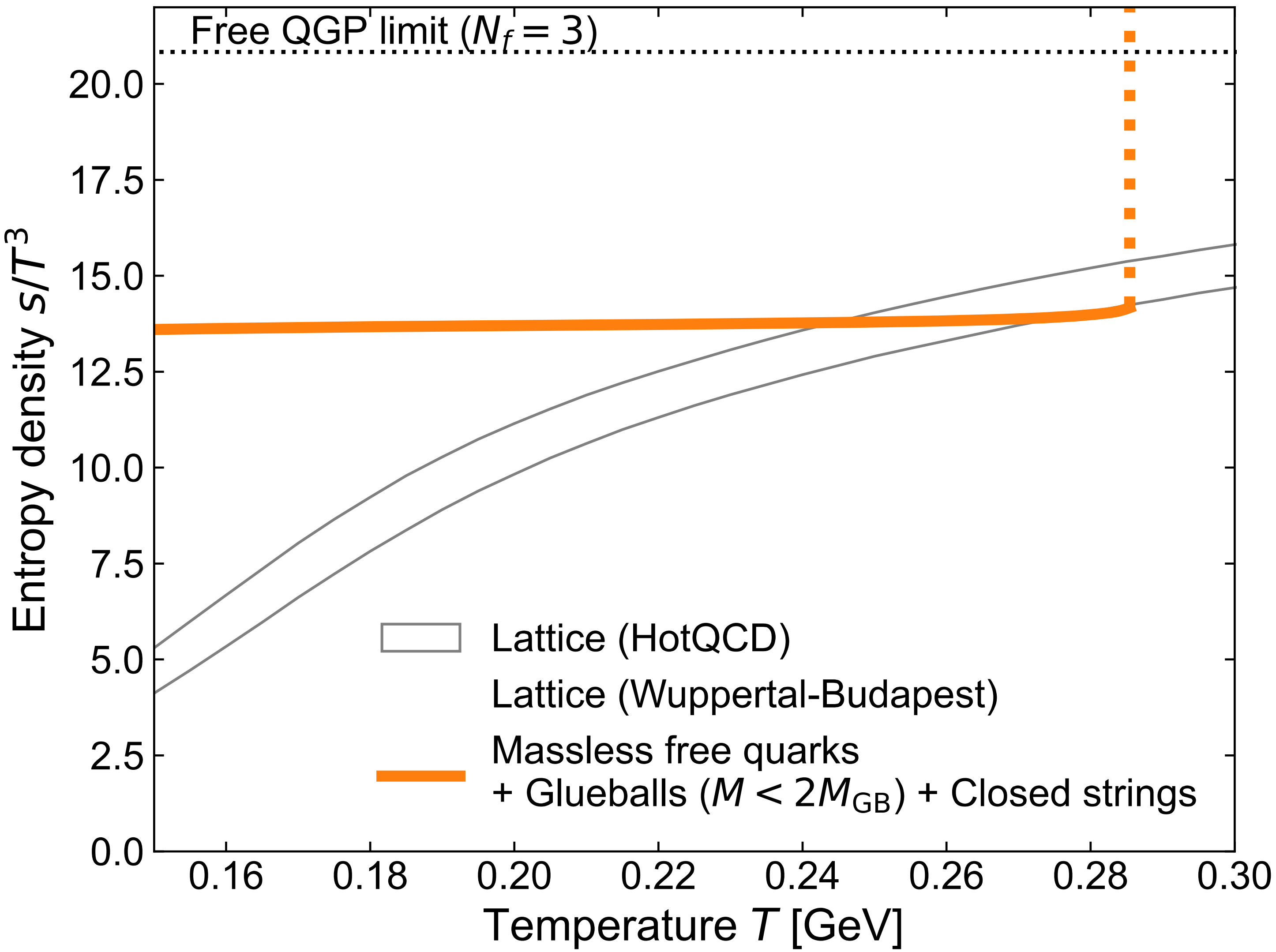
For infinite N_c:



For three colors and realistic parameters



What are the degrees of freedom of this new phase. Certainly quarks. Perhaps the long distance confining nature does not much affect the description of bulk properties of matter as quarks. Maybe there is some T dependent chirally symmetric mass term, but perhaps the massless description work well for some range of parameters. Perhaps the free quark description becomes better the further aways we are from the chiral temperature?



And this quasi free quark description would explain susceptibilities.

For Quarkyonic matter, there is an explicit Idylliq model that builds in this dual description either in terms of quarks or nucleons. Quarks remain confined in nucleon, the quarks have a filled Fermi sea with a small contribution from a broadened Fermi surface. At high density, therefore many quantities are correctly computed from a filled Fermi sea of non-interacting quarks.

But all of this involve more thinking!