Fitting Experimental Data from DIS and Single Inclusive Hadron Production at RHIC and LHC Using the CGC Framework

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Outline

1. Background
2. Technique and Results
3. Planned future

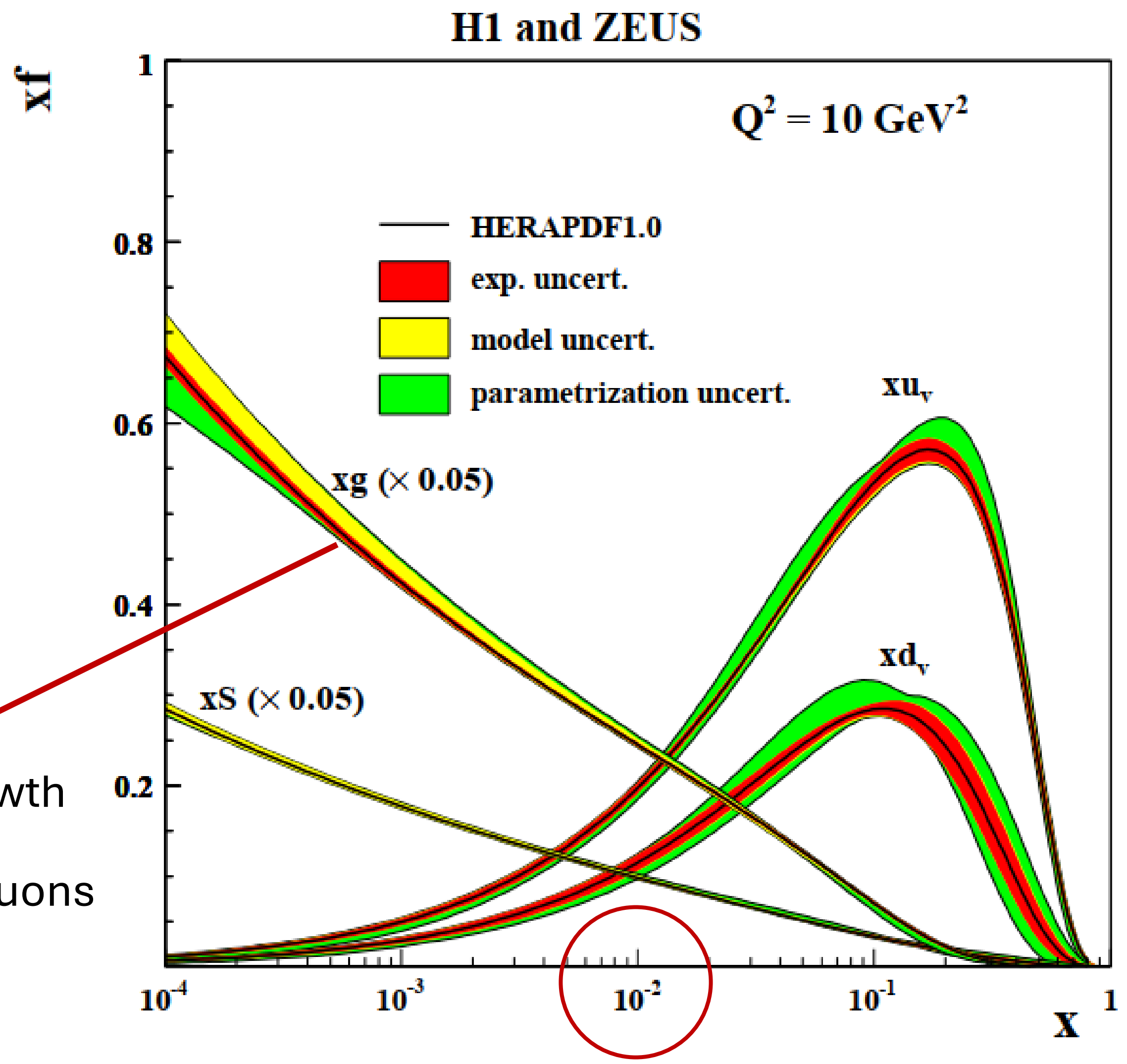
Small x-physics

0911.0884

Bjorken x = momentum of parton/momentum of hadron

Linear growth

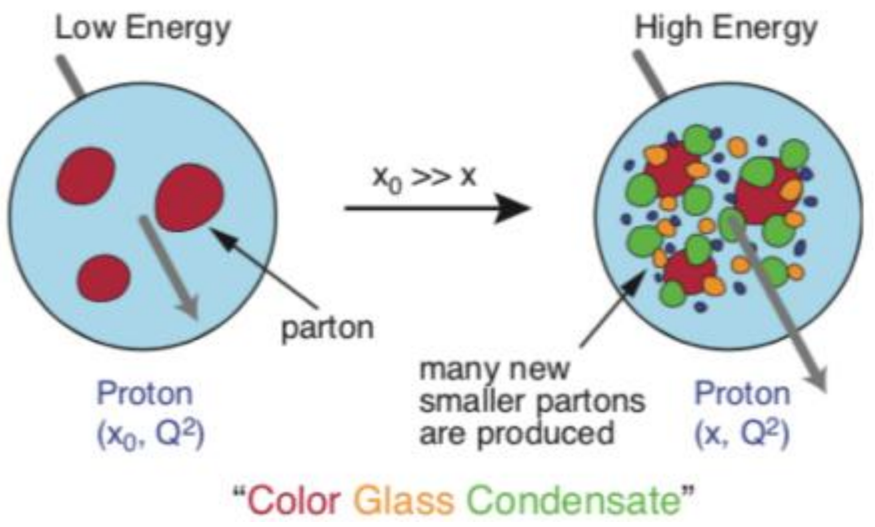
Parton splitting to gluons



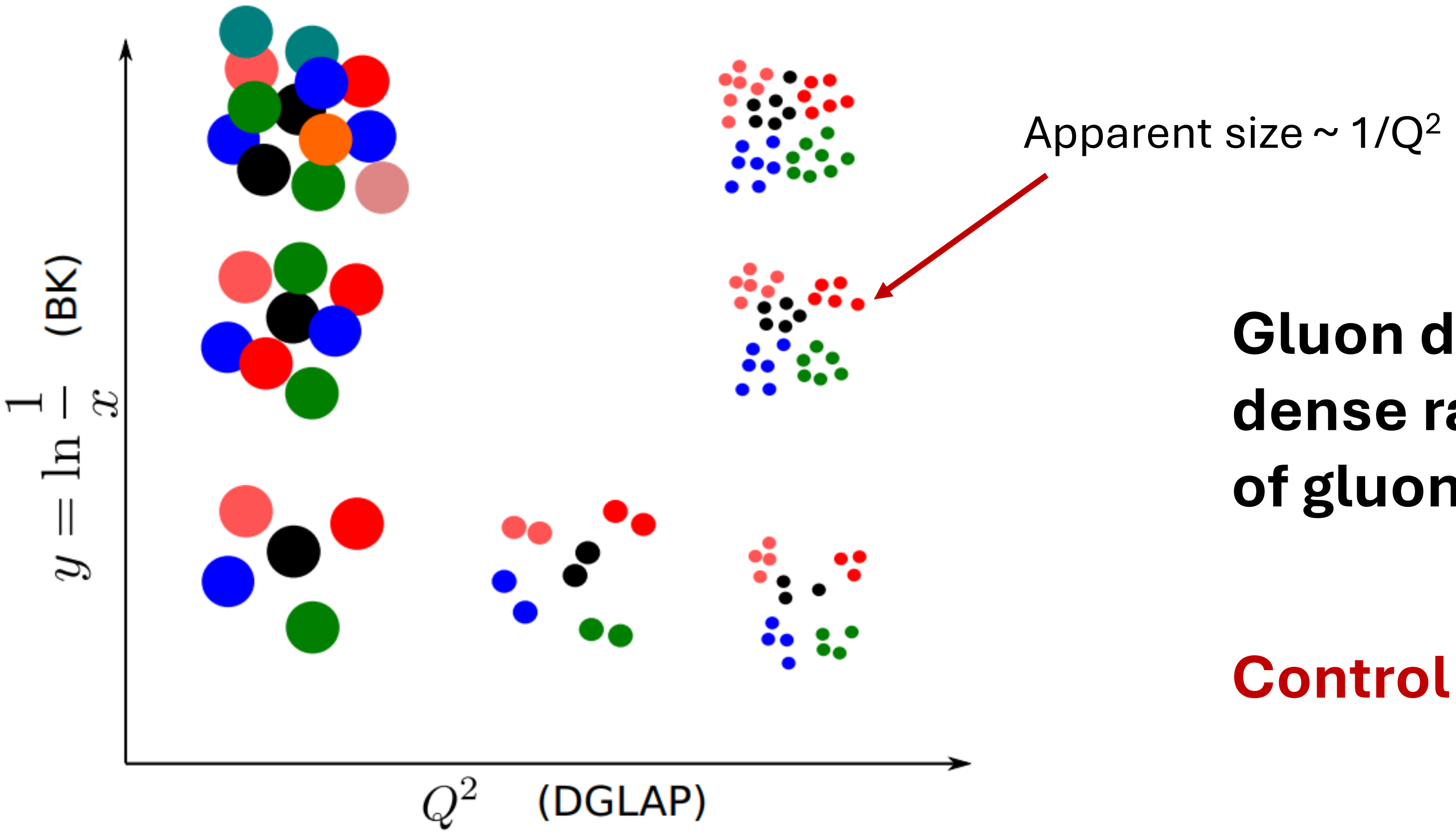
Color Glass Condensate (CGC)

New form of matter

- **Color**: gluons are colored
- **Glass**: “frozen” random color source, evolve slowly compared to scale time of hadron
- **Condensate**: dense



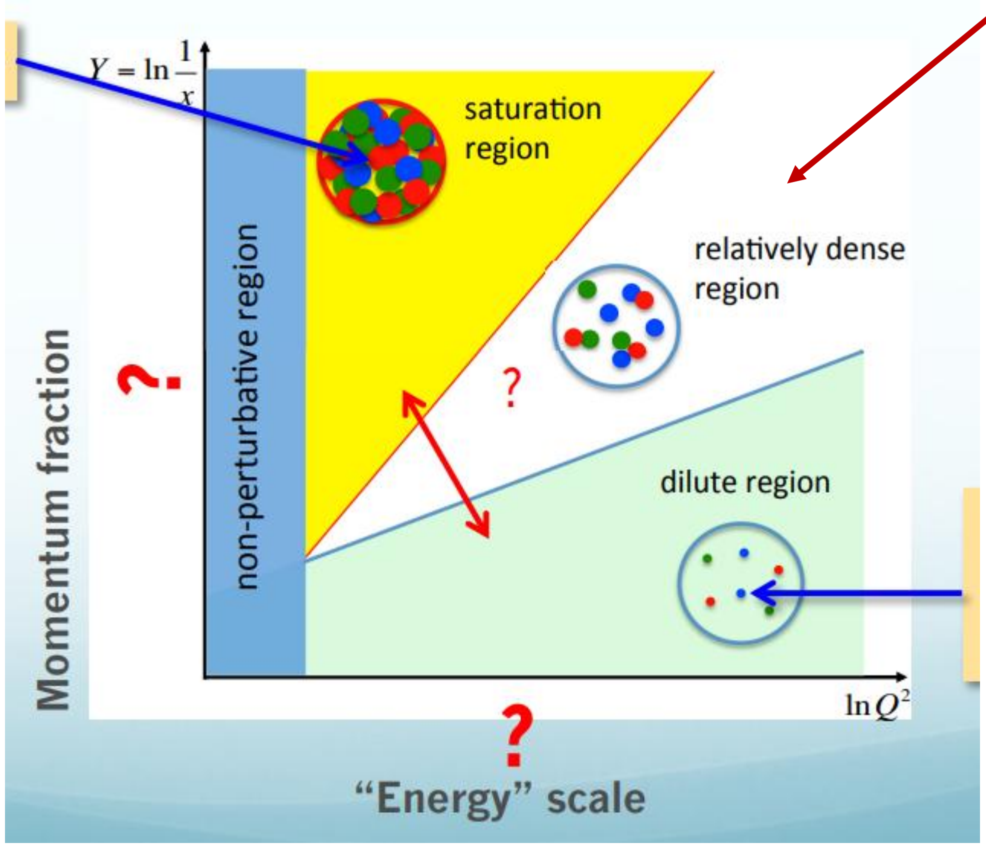
How does CGC happen in theory?



Gluon density is condition for dense rather than the number of gluon!

Control by BK equation

Gluon saturation

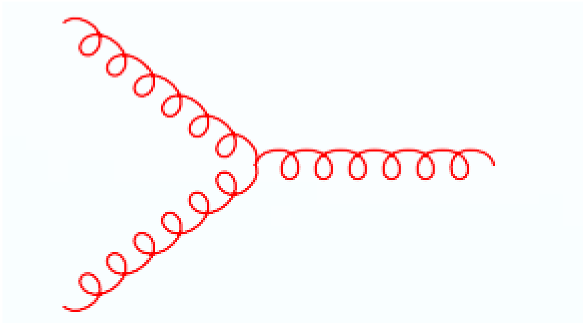


Study transition region:

$Q <\sim Q_s$

Unitarity of the dipole amplitude

Gluon recombination process:



Gluon saturation

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Heavy nuclei collision will be easy to find saturation more than proton collision

Require high energy experiment

BK equation at LO

$$\alpha_s(r) = \frac{12\pi}{(33-2N_f)\log\left(\frac{4C^2}{r^2\Lambda^2_{QCD}}\right)}$$

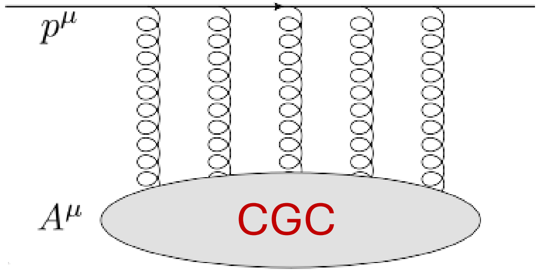
Linear term: Splitting

Non-linear term: Recombination

$$\frac{\partial N(r,Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int d^2 r_1 K(r,r_1,r_2)[N(r_1,Y)+N(r_2,Y)-N(r,Y)-N(r_1,Y)N(r_2,Y)]$$

$$N_{q\bar{q}} = 1 - \frac{1}{N_c}\mathrm{Tr}\, V(x_T)V^\dagger(y_T), \qquad V(x_T) = \mathrm{P}\exp\left[-ig\int_{-\infty}^{\infty} \mathrm{d}z^+ A^-(z^+, x^-=0, x_T)\right],$$

Wilson line, all dynamic of multiple scattering



Improved BK equation 1507.03651, 1912.09196

Running coupling

$$\alpha_s(r) = \frac{1}{b_{N_f} \ln\left[4C_\alpha^2/(r^2\Lambda_{N_f}^2)\right]}$$

$$b_{N_f} = (11N_c - 2N_f)/12\pi.$$

$$\frac{\partial N(r,Y)}{\partial Y} = \frac{\alpha_s}{2\pi}\int d^2r_1 K(r,r_1,r_2)\left[S_{\boldsymbol{xz}}(\eta-\delta_{\boldsymbol{xz};r})S_{\boldsymbol{zy}}(\eta-\delta_{\boldsymbol{zy};r}) - S_{\boldsymbol{xy}}(\eta)\right]$$

$$\bar{\alpha}_{\rm Bal} = \bar{\alpha}_s(|\boldsymbol{x}-\boldsymbol{y}|)\left[1 + \frac{\bar{\alpha}_s(|\boldsymbol{x}-\boldsymbol{z}|) - \bar{\alpha}_s(|\boldsymbol{y}-\boldsymbol{z}|)}{\bar{\alpha}_s(|\boldsymbol{x}-\boldsymbol{z}|)\bar{\alpha}_s(|\boldsymbol{y}-\boldsymbol{z}|)} \times \frac{\bar{\alpha}_s(|\boldsymbol{x}-\boldsymbol{z}|)(\boldsymbol{y}-\boldsymbol{z})^2 - \bar{\alpha}_s(|\boldsymbol{y}-\boldsymbol{z}|)(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\right],$$

Balitsky scheme

$$\delta_{\boldsymbol{xz};r} \equiv \max\left\{0, \ln\frac{r^2}{|\boldsymbol{x}-\boldsymbol{z}|^2}\right\}$$

Kinematic constraint

CGC characteristic

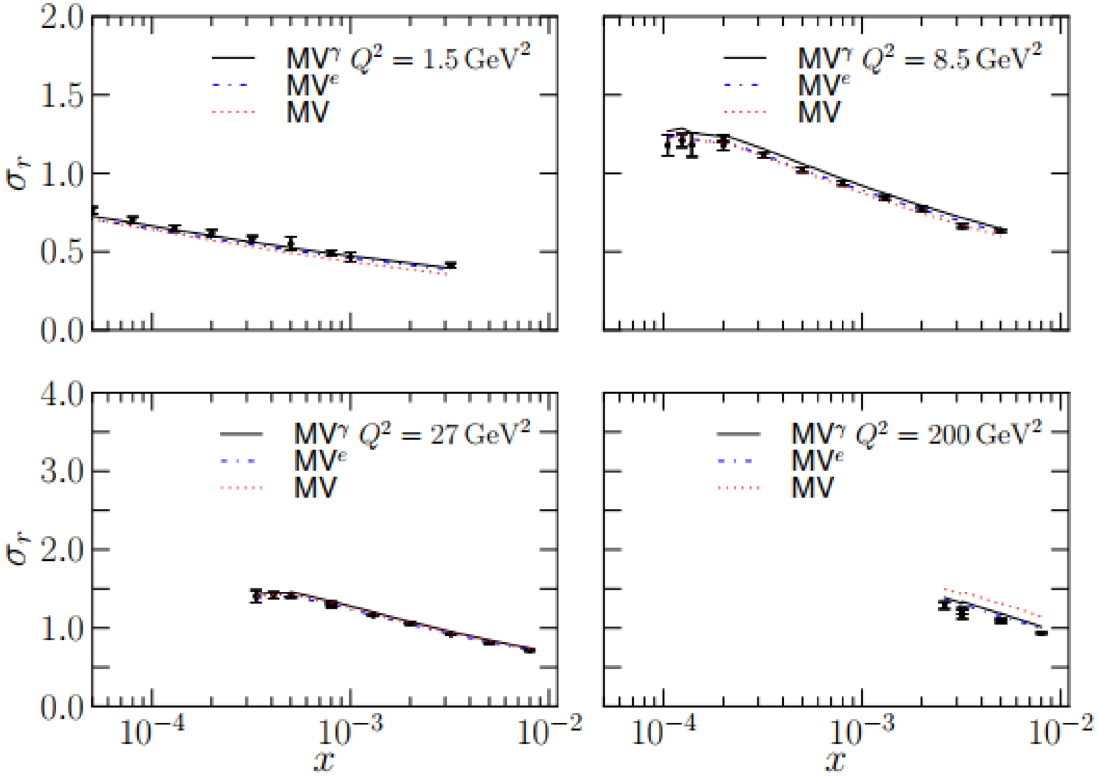
- At small x ($x < 0.01$) in saturation regime
- Coherent multiple scatterings instead of a single scattering
- Governed by BK/JIMWLK equation
- Universal property

Why fitting ?

- Show the correctness of CGC and the universal property
- From the result explain other observables in experiment.

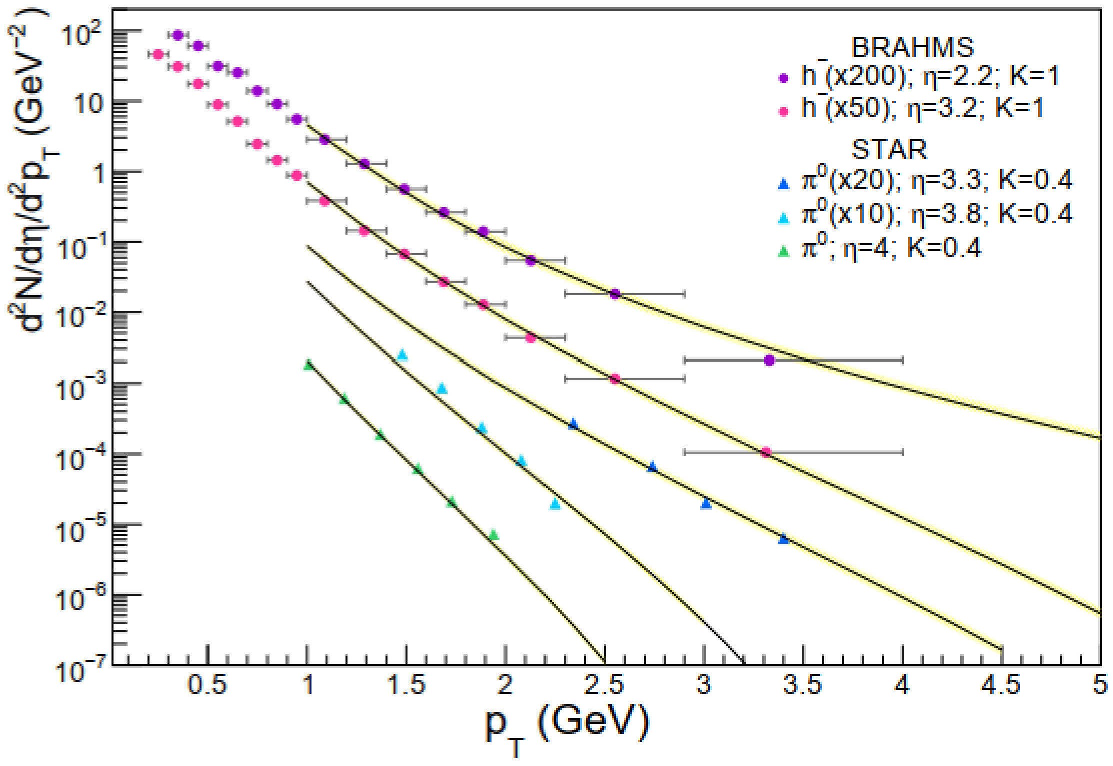
Current Goal

1309.6963, 1001.1378



DIS

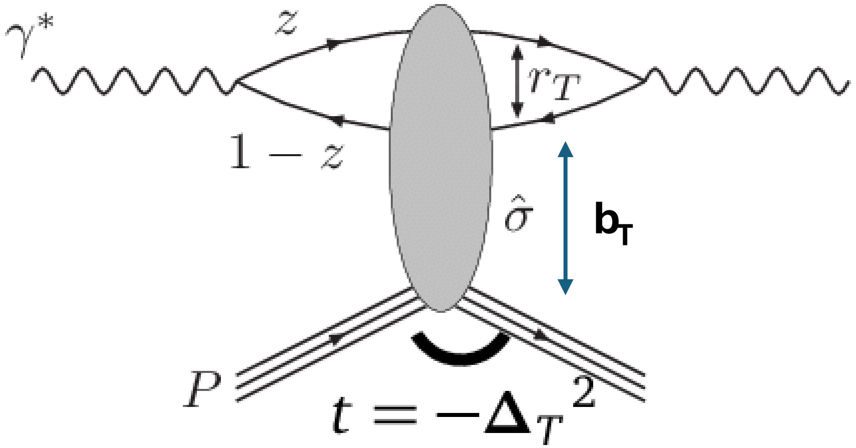
T. Lappi, H. Mäntysaari, 2013



Negatively charged hadron in pp collision

Javier L. Albacete, Cyrille Marquet, 2010

Deep Inelastic Scattering



Photon wave function QED:
$\gamma^* \to q\bar{q}$

$$\sigma_{T,L}^{Y*P}(x,Q^2) = 2 \sum_{f=u,d,s} \int_0^1 dz \int \underbrace{d^2\mathbf{b_T}}_{\sigma_0/2} \int d^2\mathbf{r_T}\, |\Psi_{T,L}^{Y*\to f\bar{f}}|^2\, N_F(\mathbf{b_T},\mathbf{r_T},x)$$

$\sigma_0/2$

Dipole Amplitude:
QCD dynamics

Single Inclusive Hadron Production

LHAPDF library

PDF

Fragmentation function

$$\frac{dN_h}{dy_h\, d^2p_t} = \frac{K}{(2\pi)^2} \sum_q \int_{x_F}^{1} \frac{dz}{z^2} \Big[x_1 f_{q/p}(x_1, p_t^2)\, \tilde{N}_F\left(x_2, \frac{p_t}{z}\right) D_{h/q}(z, p_t^2) + x_1 f_{g/p}(x_1, p_t^2)\, \tilde{N}_A\left(x_2, \frac{p_t}{z}\right) D_{h/g}(z, p_t^2) \Big]$$

Fourier transform of dipole amplitude
in position space: **FFTW3 library**

Dipole Amplitude

DIS: $x_0 = 0.01$
RHIC: $x_0 = 0.015$

Initial BK equation

$$N(r_T, x = x_0) = 1 - \exp\left[-\frac{(r^2 Q_{s,0}^2)^\gamma}{4}\ln\left(\frac{1}{r\Lambda_{\rm QCD}} + e_c \cdot e\right)\right]$$

2 parameters: **$Q_{S,0}$, γ**

BK evolution 1 parameter: **C**

$N(r_T, x)$

2 observables 2 parameter: **σ_0, m_f** in DIS

Technique

1. **Fitting: Levenberg-Marquardt (LevMar) algorithm**

Minimize $$\chi^2(p) = \sum_{i=1}^{n} \left[y_i - f(x_i, p) \right]^2$$

Residual **r**

Theory prediction of cross-section

Data experiment

2. **BK evolution: Automatic Differentiation algorithm**

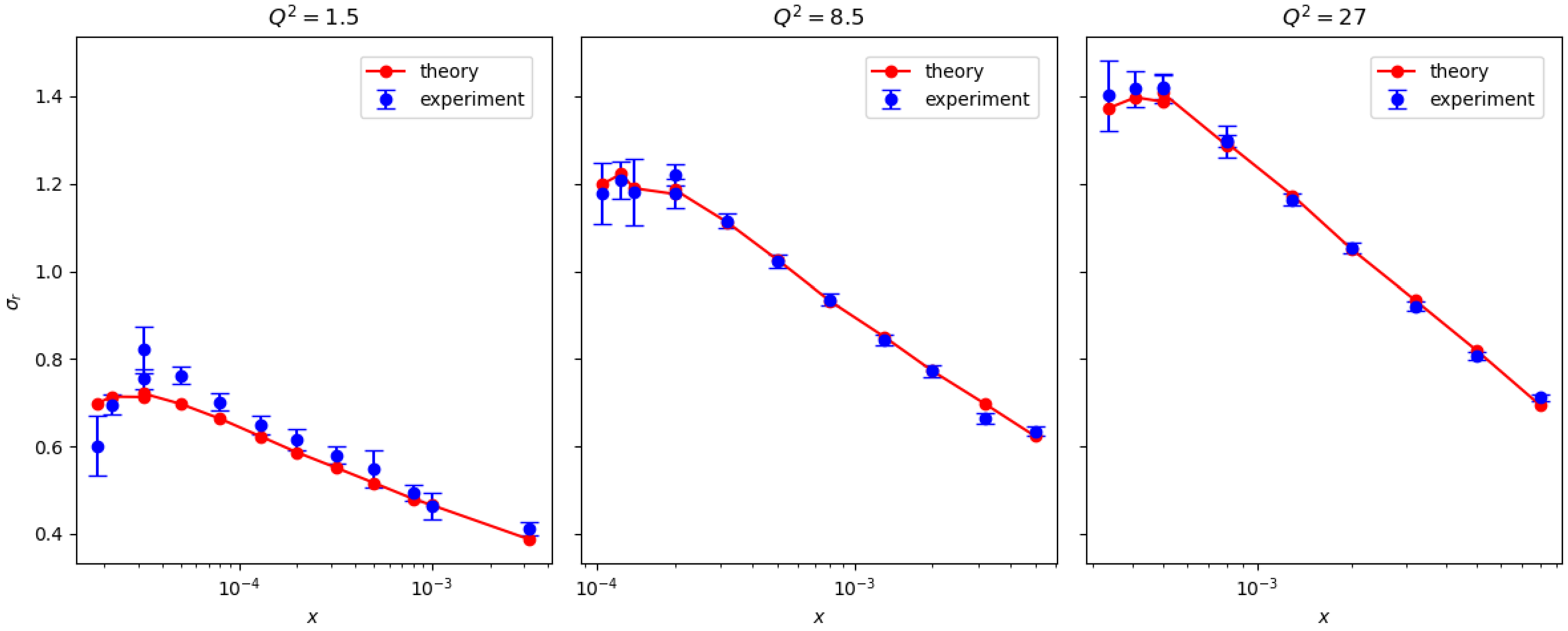
   Allow to compute “analytic” derivative of cross-section respect to parameters

   Done by Prof. Piotr Korcyl

3. **Theory Uncertainty: Hessian Method & Monte Carlo Method**

Preliminary results

DIS (271 data points) (Balitsky scheme + Kinematic constraint)



Preliminary results

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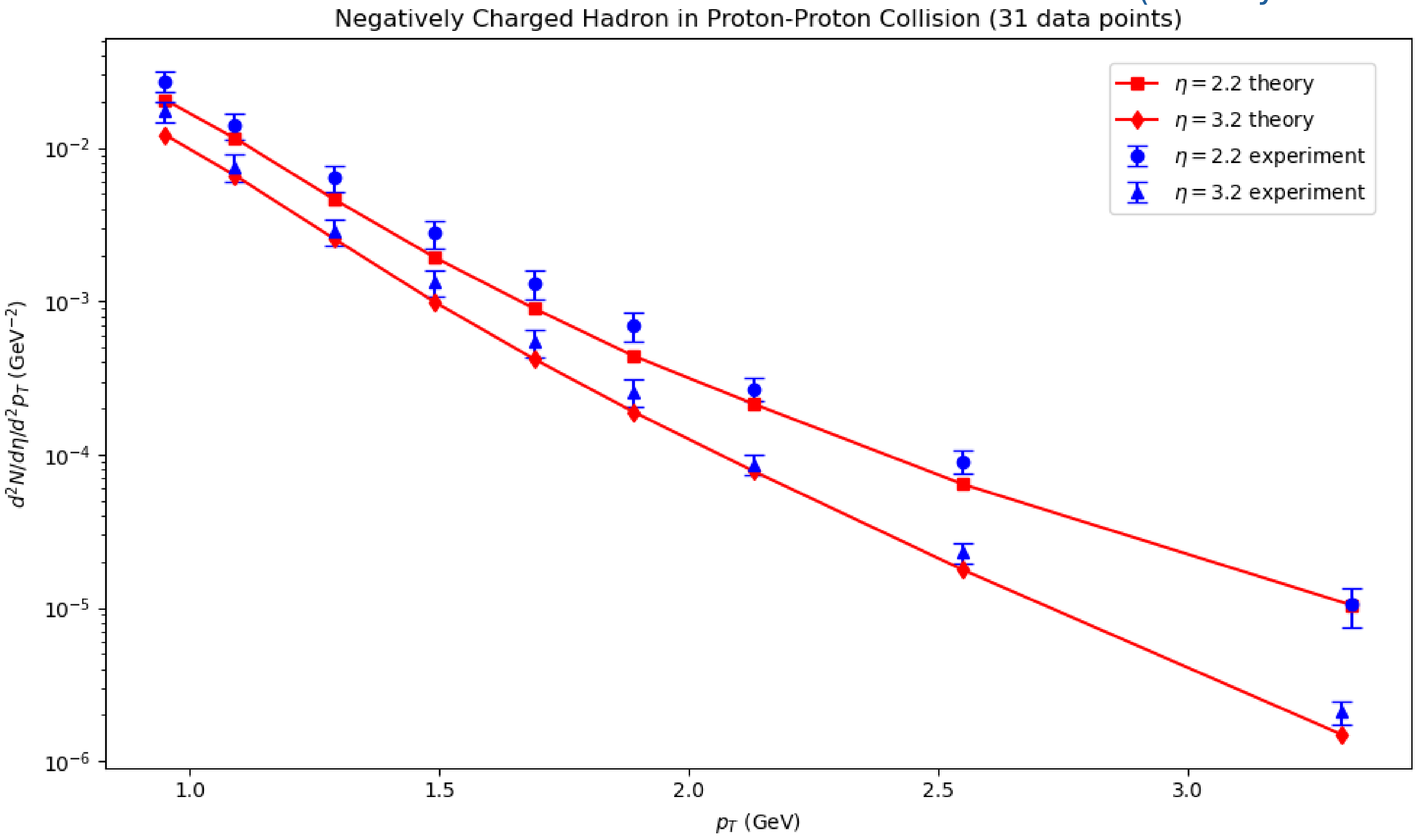
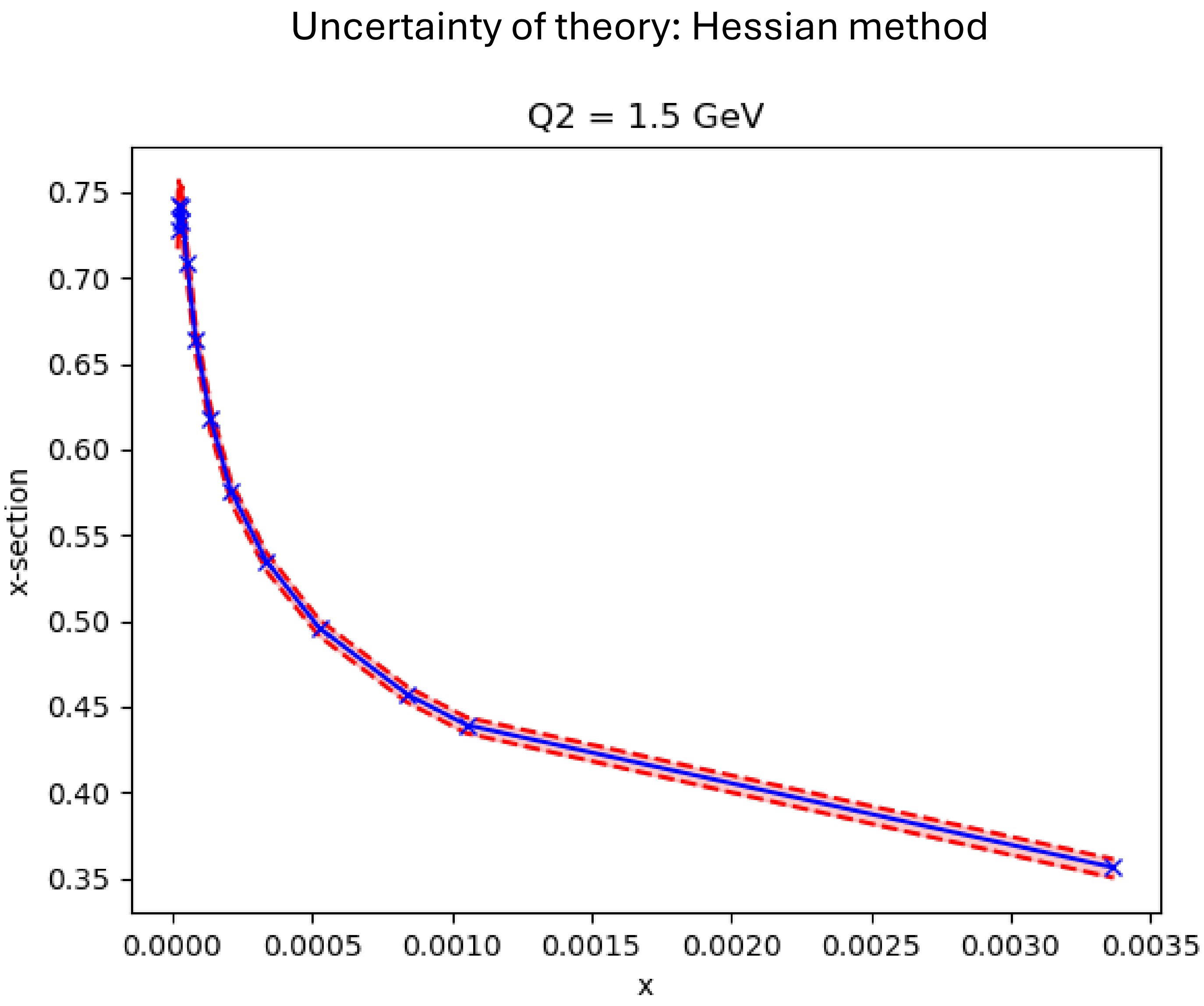


Preliminary results

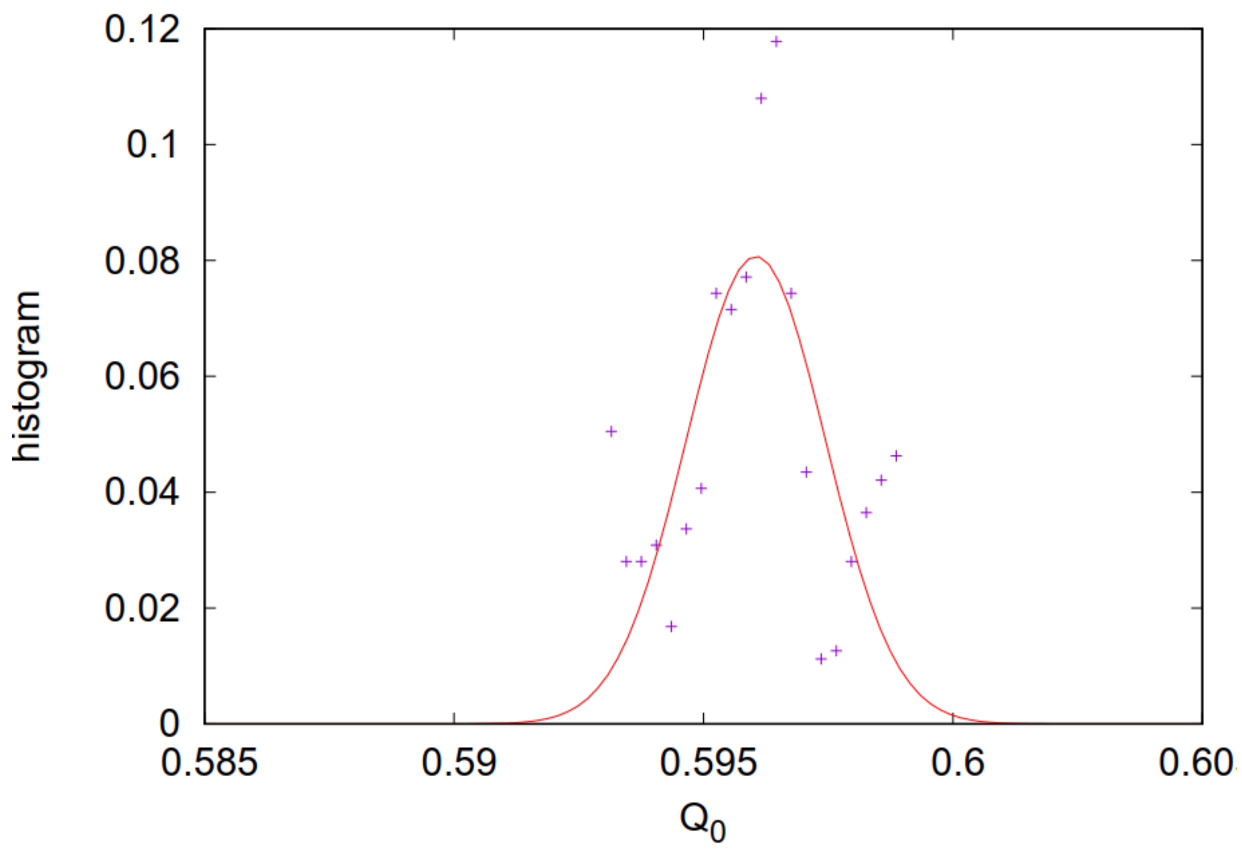
Scheme	Qs,0 [GeV]	sigma0 [mb]	C	gamma	mf [GeV]	LambdaQCD [Gev]	chi2/d.o.f
Mother scheme	0.4044278	29.49385	21.45029	1.109577	0.0837771		1.88
Balitsky + Kinematic	0.3536551	35.89932	1.092072	1.08882	0.1180737		2.175
Balitksy + Kinematic	0.3734437	36.35449	0.588563	1.114598	0.117476	0.323887	2.026

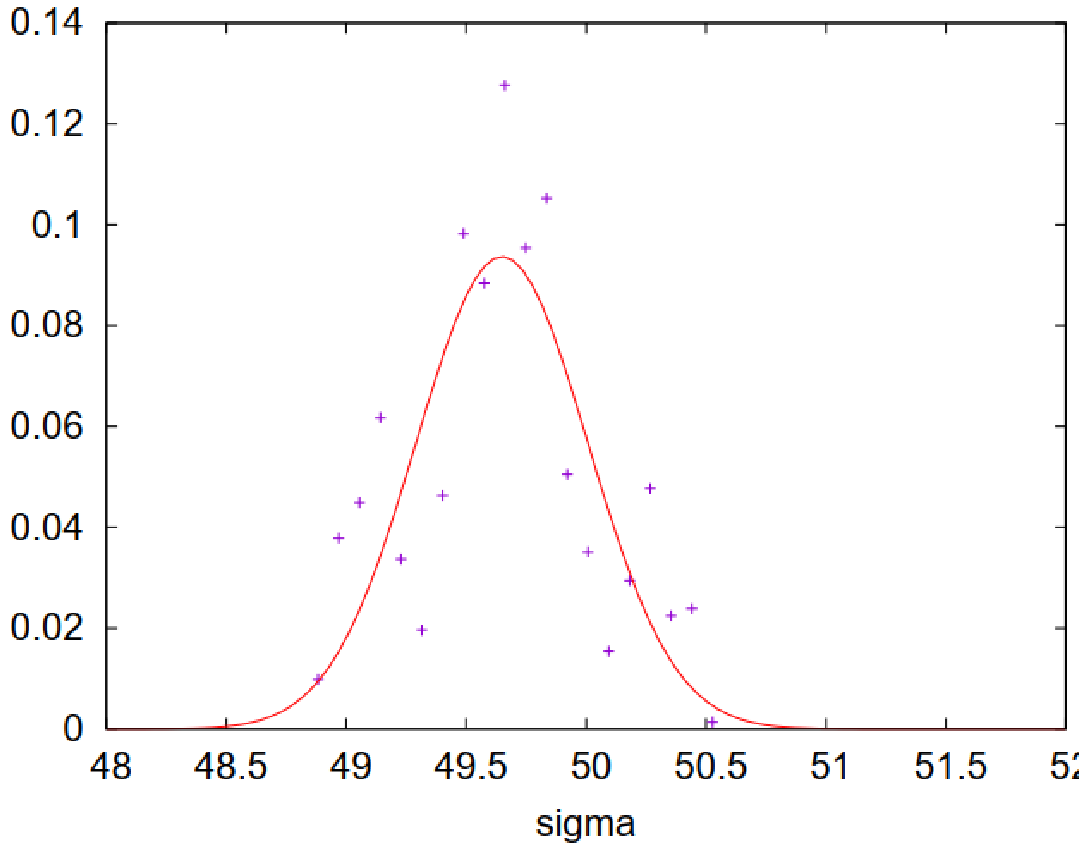
Preliminary results



Preliminary results

Monte Carlo method





Preliminary results

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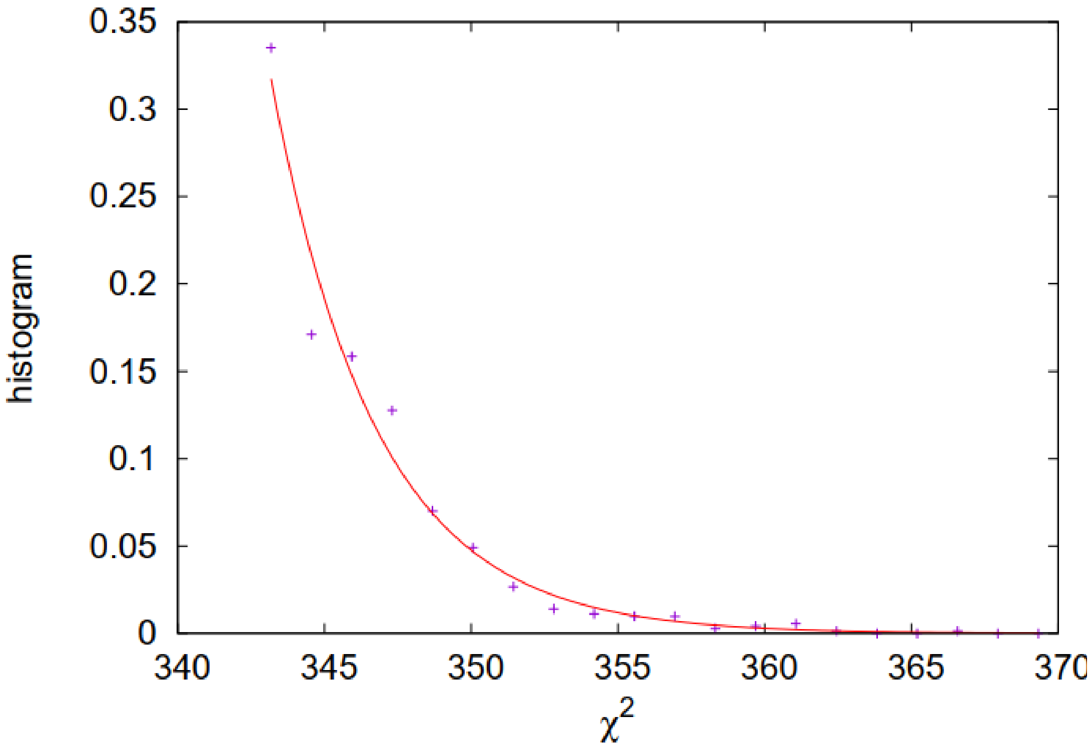
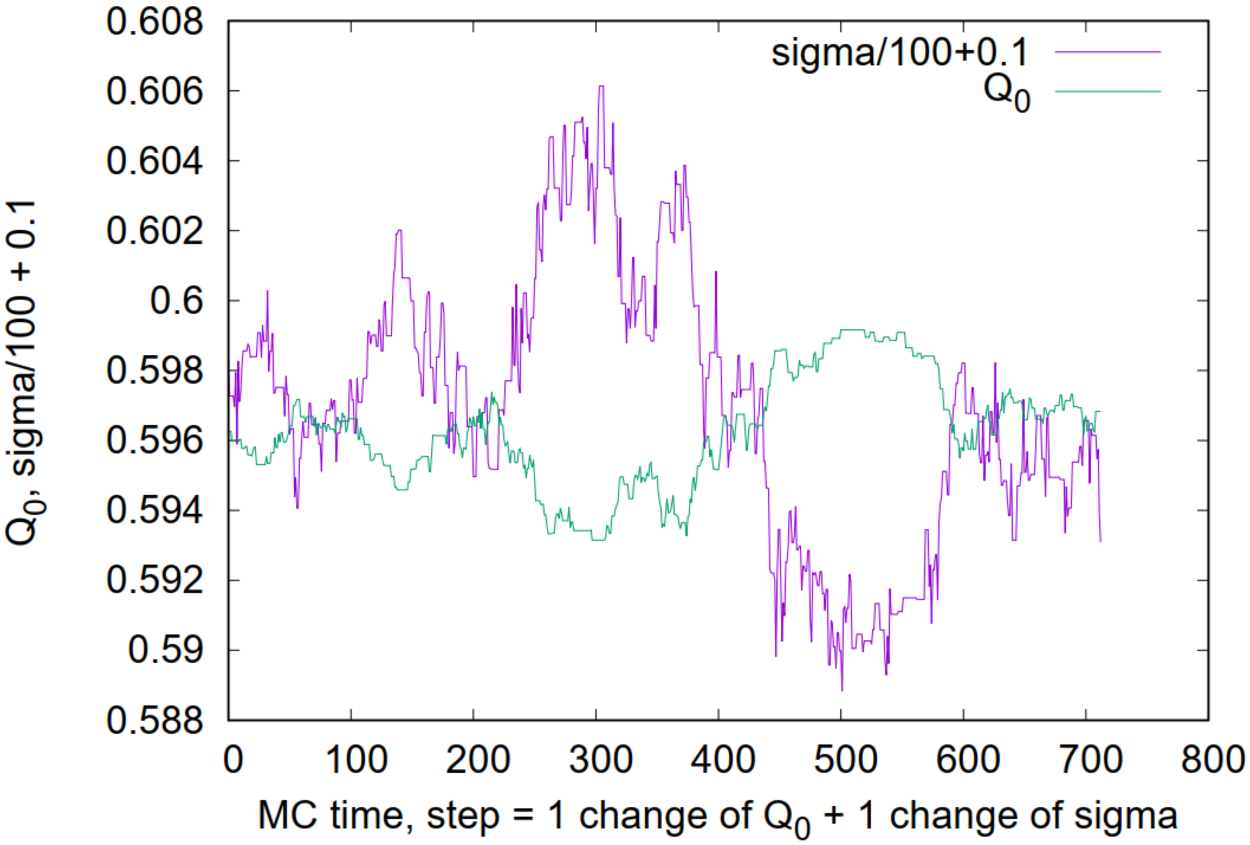




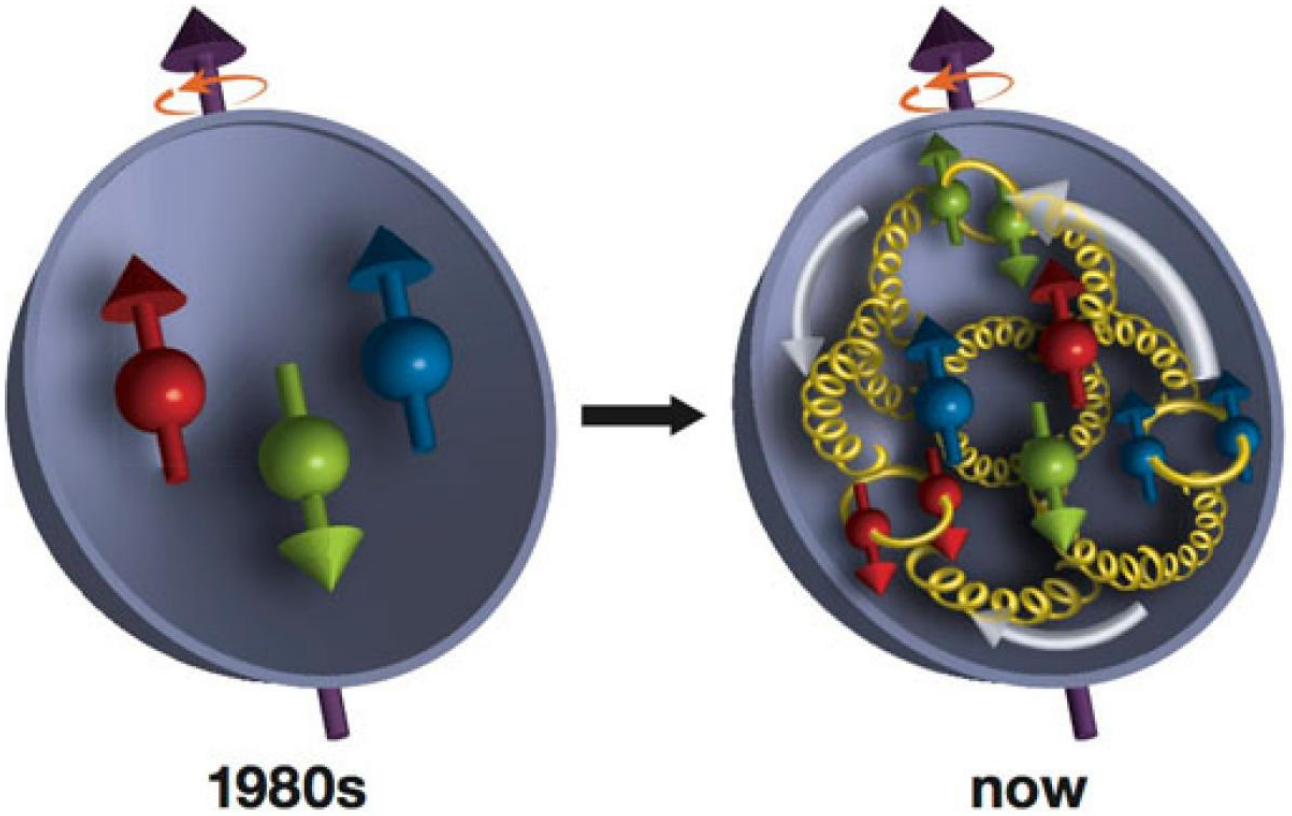
Planned goal

1. Produce the fit for other initial BK and BK
2. Add other observables at LHC
3. Incopporate theory uncertainty into results

Thank you!

Small x-physics

Bjorken x = momentum of parton/momentum of hadron



Technique

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Minimize $$\chi^2(p) = \sum_{i=1}^{n} \left[y_i - f(x_i, p) \right]^2$$

Residual $\mathbf{r}$

Data experiment

Theory prediction of cross-section

Update parameters

$$\mathbf{p}_{k+1} = \mathbf{p}_k - (\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I})^{-1}\mathbf{J}^T\mathbf{r}$$

- $\mathbf{J}$ is the Jacobian matrix of residuals r_i with respect to $\mathbf{p}$. ← Require 1st derivative
- $\mathbf{r}$ is the vector of residuals.
- λ is a damping factor (Levenberg's contribution), and $\mathbf{I}$ is the identity matrix.

Technique

2. BK evolution: Automatic Differentiation

Allows to evaluate 'analytic' derivatives of a computer program with respect to external parameters.

- numbers are promoted to vectors

$$x \to \begin{pmatrix} x \\ \partial_A \\ \partial_B \\ \partial_A^2 \\ \partial_A \partial_B \\ \vdots \end{pmatrix}$$

- all arithmetic operators are overloaded
- functions with derivatives have to be provided
- works for most algorithms

Benefits

- Faster convergence of the fit
- Provide Hessian matrix for estimation of uncertainties
- Test the sensitive of the parameters to the data

Technique

3. Theory Uncertainty: Hessian Method

Assume that χ^2_{global} is quadratic about the global minimum

$$\Delta\chi^2_{\mathrm{global}} \equiv \chi^2_{\mathrm{global}} - \chi^2_{\mathrm{min}} = \sum_{i,j=1}^{n} H_{ij}\left(a_i - a_i^0\right)\left(a_j - a_j^0\right),$$

where

$$H_{ij} = \frac{1}{2}\frac{\partial^2\chi^2_{\mathrm{global}}}{\partial a_i \partial a_j}\bigg|_{\mathrm{min}}$$

We can diagonalize the covariance matrix $C \equiv H^{-1}$,

$$\sum_{j=1}^{n} C_{ij} v_{jk} = \lambda_k v_{ik},$$

$$a_i - a_i^0 = \sum_{k=1}^{n}\left(\sqrt{\lambda_k}\, v_{ik}\right) z_k \quad \Rightarrow \quad \Delta\chi^2_{\mathrm{global}} = \sum_{k=1}^{n} z_k^2 \equiv T^2$$