Loop calculations in light cone perturbation theory

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Outline

Lecture 1: basics

- Dipole picture of DIS
- Light cone quantization
- CGC
- BK equation

Lecture 2: NLO

- Dipole picture DIS at NLO
- Dipole picture DIS with quark masses: Mass renormalization LCPT
- Diffractive structure function at NLO

Summarizing work done by several people over the last $\lesssim$10 years:

(often not including me)

G. Beuf, C. Casuga, H. Hänninen, M. Karhunen, Y. Mulian, H. Mäntysaari, R. Paatelainen, J. Penttala

Dipole picture DIS at NLO

DIS at NLO: Fock state expansion

- Fock state decomposition of $|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$ (and ${}_i\langle\gamma_\lambda(\vec{q}', Q^2)|$) up to g^2:

$$|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = \sqrt{Z_{\gamma^*}}\Big[|\gamma_\lambda(\vec{q}, Q^2)\rangle + \sum_{q\bar{q}} \Psi^{\gamma^*\to q\bar{q}}|q(\vec{k}_0)\bar{q}(\vec{k}_1)\rangle + \sum_{q\bar{q}g} \Psi^{\gamma^*\to q\bar{q}g}|q(\vec{k}_0)\bar{q}(\vec{k}_1)g(\vec{k}_2)\rangle + \cdots\Big]$$

with **Light Cone Wave Functions** $\Psi^{\gamma^*\to q\bar{q}}$ **and** $\Psi^{\gamma^*\to q\bar{q}g}$

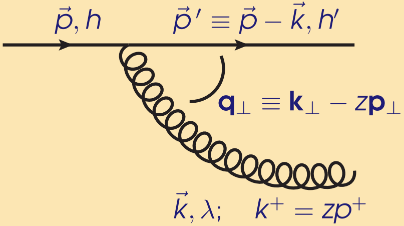
- Fourier-transform to $\perp$ coordinate: (eikonal scattering)
- Scattering off target: Wilson line correlators

$$\begin{aligned}
\hat{S}_E|q(\mathbf{x}_\perp)\bar{q}(\mathbf{y}_\perp)\rangle &= V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)|q(\mathbf{x}_\perp)\bar{q}(\mathbf{y}_\perp)\rangle \\
\hat{S}_E|q(\mathbf{x}_\perp)\bar{q}(\mathbf{y}_\perp)g(\mathbf{z}_\perp)\rangle &= V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)V_{\mathrm{adj}}(\mathbf{z}_\perp)|q(\mathbf{x}_\perp)\bar{q}(\mathbf{y}_\perp)g(\mathbf{z}_\perp)\rangle
\end{aligned}$$

DIS at NLO: procedure

Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

1. Evaluate LCPT diagrams
   - $\psi^{\gamma^* \to q\bar{q}}$ to 1 loop
   - $\psi^{\gamma^* \to q\bar{q}g}$ at tree level
2. Fourier-transform to transverse coordinate
3. "Square" i.e. ${}_i\langle \gamma_\lambda(\vec{q}\,', Q^2)|(\hat{S}_E - \mathbf{1})|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$



LCPT rules:

- Intermediate ($\ni$ "final") state k^- denominators
- On-shell vertices, most importantly $q\bar{q}g$

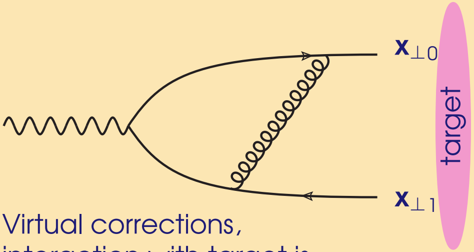
$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] = \frac{-2}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta_{h',h}\delta^{ij} + \frac{z}{2}ih\delta_{h',h}\varepsilon^{ij}\right]\mathbf{q}_\perp{}'^i \varepsilon_\perp{}^{*j}_\lambda,$$

(This is in $d = 4$, generalize for $d < 4$)

Note 2 index structures for massless quarks.

DIS at NLO: real and virtual corrections

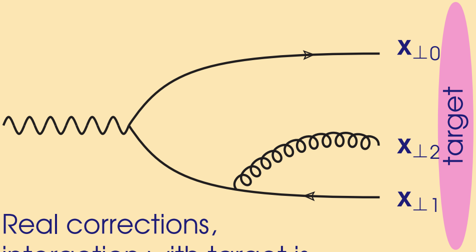
Here example diagams only



Virtual corrections,
interaction with target is

$$\mathcal{N}_{q\bar{q}}(\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1})$$

\+ UV divergence in loop



Real corrections,
interaction with target is

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2})$$

UV divergence in $\mathbf{x}_{\perp 2}$-integral

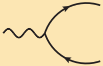
These UV-divergences cancel because for Wilson lines $\in SU(N_c)$

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2} \to \mathbf{x}_{\perp 0}) = \mathcal{N}_{q\bar{q}g}(\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2} \to \mathbf{x}_{\perp 1}) = \mathcal{N}_{q\bar{q}}(\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1})$$

DIS at NLO: subtracting BK equation

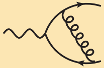
B. Ducloué, H. Hänninen, T. L. and Y. Zhu, (arXiv:1708.07328 (hep-ph)).

Evaluate cross section as $\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{LO}}_{L,T} + \sigma^{\mathrm{dip}}_{L,T} + \sigma^{qg}_{L,T,\mathrm{sub.}}$



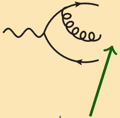
$\Rightarrow$

$$\sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1}} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1})|^2 \mathcal{N}_{01}(x_{Bj})$$



$- * \Rightarrow$

$$\sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1},z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$+ * \Rightarrow$

- LL

$k_g^+ \sim z_2$

$$\sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1},\mathbf{x}_{\perp 2}} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

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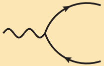
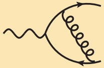
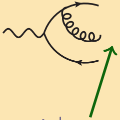


$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1}} |\psi^{\mathrm{LO}}_{\gamma^* \to q\bar{q}}(z_1, \mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1})|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1},z_1} \left|\psi^{\mathrm{LO}}_{\gamma^* \to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_{\perp 0},\mathbf{x}_{\perp 1},\mathbf{x}_{\perp 2}} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^* \to q\bar{q}g}(z_1, z_2, \{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2))$$

- LL

$k_g^+ \sim z_2$

$$- \left|\psi_{\gamma^* \to q\bar{q}g}(z_1, 0, \{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

LL: subtract leading log, already in BK-evolved $\mathcal{N}$

DIS at NLO: subtracting BK equation

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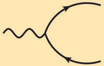
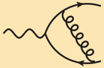
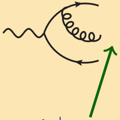


$$\Rightarrow \quad \sigma^{\text{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}} \left|\psi^{\text{LO}}_{\gamma^* \to q\bar{q}}(z_1, \mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1})\right|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\text{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}, z_1} \left|\psi^{\text{LO}}_{\gamma^* \to q\bar{q}}\right|^2 \left[\frac{1}{2} \ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



- LL

$k_g^+ \sim z_2$

$$+ * \Rightarrow \quad \sigma^{qg}_{\text{sub.}} \sim \alpha_s C_F \int_{z_1, z_2, \mathbf{x}_{\perp 0}, \mathbf{x}_{\perp 1}, \mathbf{x}_{\perp 2}} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^* \to q\bar{q}g}(z_1, z_2, \{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2))$$
$$- \left|\psi_{\gamma^* \to q\bar{q}g}(z_1, 0, \{\mathbf{x}_{\perp i}\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence LL: subtract leading log, already in BK-evolved $\mathcal{N}$

- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!
  ("k_T-factorization" with fixed rapidity scale is unstable @ NLO. Analogous probem in $p + A \to h + X$)

Confronting with HERA data

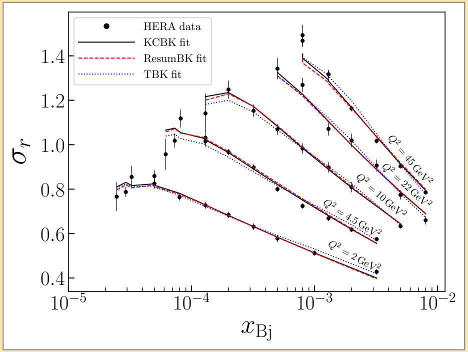
G. Beuf, H. Hänninen, T. L. and H. Mäntysaari, (arXiv:2007.01645 (hep-ph)).

Different resummations of collinear logs in BK

1. KCBK Beuf (arXiv:1401.0313 (hep-ph))
2. ResumBK E. Iancu et al (arXiv:1502.05642 (hep-ph)).
3. TBK B. Ducloue et al (arXiv:1902.06637 (hep-ph)) ,

Free parameters:

- σ_0: proton area
- Q_{s0}: initial saturation scale
- γ shape of initial condition as function of r
- C^2: scale of α_s as function of r (fit α_s or Λ_{QCD})



Main conclusions

- Fits are very good, χ^2/N varies 1.03 . . . 2.77
- Difference between BK resummation schemes absorbed in initial conditions.
  Similar as Albacete 2015 Visible only at LHeC kinematics

Quark masses in dipole picture DIS at NLO

Heavy quarks, motivation, issues

- Data
  - HERA F_2^c
  - Charm big part of EIC program
- LO F_2^c problematic in existing fits
  Dirty little secret: heavy quarks in LO rcBK fits do not actually work!

LCPT loops with massive quarks are fun!

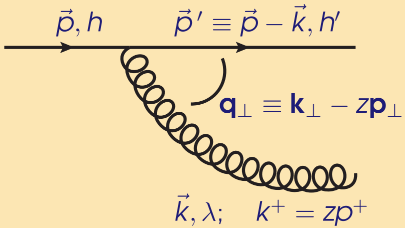
- Working with fixed helicity states (not Dirac traces=sums) : physics very explicit
- New Lorentz structures $\Longrightarrow$ rotational invariance constraints

Approach for this talk: same regularization as in massless case

- Cutoff in k^+
- $\perp$ dim. reg.

(Recall: Hamiltonian perturbation theory, k^--integrals already done)

Elementary vertex with masses



- h, h': light cone (z-axis) helicities
- $\mathbf{q}_\perp$: center-of-mass $\perp$ momentum in splitting
- polarization λ, with $\perp$ polarization vector $\varepsilon_{\perp\lambda}^{*j}$

$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k)u_h(p)\right] \sim \overbrace{\bar{u}_{h'}\gamma^+u_h}^{\sim\delta_{h,h'}}\,\delta^{ij}q^i\varepsilon_{\perp\lambda}^{*j} + \overbrace{\bar{u}_{h'}\gamma^+[\gamma^i,\gamma^j]u_h}^{\sim\delta_{h,h'}}\,q^i\varepsilon_{\perp\lambda}^{*j} + \overbrace{\bar{u}_{h'}\gamma^+\gamma^j u_h}^{\sim\delta_{h,-h'}}\,m_q\varepsilon_{\perp\lambda}^{*j}$$

- New 3rd light-cone-helicity-flip structure $\sim m_q$
- Note: $\perp$ momentum in non-flip, but not in flip vertex $\Longrightarrow$ less UV-divergent
- Loops: also generate 4th structure $\bar{u}_{h'}\gamma^+\gamma^i u_h\varepsilon_{\perp\lambda}^{*j}q^iq^j$
- In principle same as massless, but a lot more algebra ...
- Apart from quark mass renormalization (more in a minute)

First: result for $\gamma^* \to q\bar{q}$ with massive quarks

$\gamma^* \to q\bar{q}$ and DIS cross section Beuf, Paatelainen, T.L. 2103.14549, 2112.03158, 2204.02486

$$\widetilde{\psi}_{\mathrm{NLO}}^{\gamma_T^* \to q\bar{q}} = -\frac{ee_f}{2\pi}\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\Bigg\{\Bigg[\left(\frac{k_0^+ - k_1^+}{q^+}\right)\delta^{ij}\bar{u}(0)\gamma^+ v(1) + \frac{1}{2}\bar{u}(0)\gamma^+[\gamma^i,\gamma^j]v(1)\Bigg]\mathcal{F}\left[\mathbf{P}_\perp{}^i\mathcal{V}^T\right] + \bar{u}(0)\gamma^+ v(1)\mathcal{F}\left[\mathbf{P}_\perp{}^j\mathcal{N}^T\right]$$
$$+ m\bar{u}(0)\gamma^+\gamma^i v(1)\mathcal{F}\left[\left(\frac{\mathbf{P}_\perp{}^i\mathbf{P}_\perp{}^j}{\mathbf{P}_\perp{}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] - m\bar{u}(0)\gamma^+\gamma^j v(1)\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right]\Bigg\}\varepsilon_{\perp\lambda}^j.$$

$$\mathcal{F}\left[\mathbf{P}^i\mathcal{V}^T\right] = \frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}\kappa_z K_{\frac{D}{2}-1}(|\mathbf{x}_{01}|\kappa_z) + \frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left\{\left[\frac{5}{2} - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega_{\mathcal{V}}^T + L\right]\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I_{\mathcal{V}}^T\right\}$$

$$\mathcal{F}\left[\mathbf{P}^j\mathcal{N}^T\right] = \frac{i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|}\left\{\Omega_{\mathcal{N}}^T\,\kappa_z K_1(|\mathbf{x}_{01}|\kappa_z) + I_{\mathcal{N}}^T\right\}$$

$$\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2} - \frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] = \frac{(1-z)}{2}\left[\frac{\mathbf{x}_{01}^i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|^2} - \frac{\delta^{ij}}{2}\right]\int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{(u+1)^2}|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}$$
$$\times K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) + [z \leftrightarrow 1-z].$$

$$\mathcal{F}\left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2}\right] = \left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right) + \log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left. + 2\gamma_E\right\} + \frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}K_{\frac{D}{2}-2}(|\mathbf{x}_{01}|\kappa_z) + \left\{3 - \frac{\pi^2}{3} + \log^2\left(\frac{z}{1-z}\right) - \Omega_{\mathcal{V}}^T + L\right\}K_0(|\mathbf{x}_{01}|\kappa_z) + I_{\mathcal{VMS}}^T,$$

$$\Omega_{\mathcal{V}}^T = -\left(1 + \frac{1}{2z}\right)\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] + \frac{1}{2z}\left[\left(z+\frac{1}{2}\right)(1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) + [z \leftrightarrow 1-z]$$

$$I_{\mathcal{V}}^T = \int_0^1\frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$
$$- \int_0^1\mathrm{d}\xi\left(\frac{\log(\xi)}{(1-\xi)^2} + \frac{z}{(1-x)} + \frac{z}{2}\right)\frac{(1-z)m^2}{\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$- \int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{u(u+1)}\frac{m^2}{\kappa_\chi^2}\left[2\chi + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)(1-2\chi)\right]$$
$$\times\left\{\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}\right) - [u \to 0]\right\}$$
$$- \int_0^z\frac{\mathrm{d}\chi}{(1-\chi)^2}\int_0^\infty\frac{\mathrm{d}u}{(u+1)}(z-\chi)\left[1 - \frac{2u}{1+u}(z-\chi) + \left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)^2\right]$$
$$\times\frac{m^2}{\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) \quad + \quad [z \leftrightarrow 1-z].$$

$$\Omega_{\mathcal{N}}^T = \frac{z+1-2z^2}{z}\left[\log(1-z) + \gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right] - \frac{(1-z)}{z}\left[\frac{2z+1}{2}(1-\gamma) + \frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right) - [z \leftrightarrow 1-z] \tag{9}$$

$$I_{\mathcal{N}}^T = \frac{2(1-z)}{z}\int_0^z\mathrm{d}\chi\int_0^\infty\frac{\mathrm{d}u}{(u+1)^3}\Bigg\{\left[(2+u)uz + u^2\chi\right]\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) \tag{10}$$
$$+ \frac{m^2}{\kappa_\chi^2}\left(\frac{z}{1-z} + \frac{\chi}{1-\chi}[u - 2z - 2u\chi]\right)\left[\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right]\Bigg\} - [z \leftrightarrow 1-z].$$

$$I_{\mathcal{VMS}}^T = \int_0^1\frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2}\right)\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right) - [\xi \to 0]\right\}$$
$$+ \int_0^1\mathrm{d}\xi\left(-\frac{3(1-z)}{2(1-\xi)} + \frac{(1-z)}{2}\right)K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + \frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$+ \int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{(u+1)^2}\left\{-z - \frac{u}{(1+u)}\frac{(z+u\chi)}{z}(\chi - (1-z))\right\}K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)$$
$$+ \int_0^z\mathrm{d}\chi\int_0^\infty\frac{\mathrm{d}u}{(u+1)^3}\left\{\frac{\kappa_z^2}{\kappa_\chi^2}\left[1 + u\frac{\chi(1-\chi)}{z(1-z)}\right] - \frac{m^2}{\kappa_\chi^2}\frac{\chi}{(1-\chi)}\left[2\frac{(1+u)^2}{u} + \frac{u}{z(1-z)}(z-\chi)^2\right]\right\}$$
$$\times\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2 + u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right) - [u \to 0]\right\} + [z \leftrightarrow 1-z].$$

Comparison to HERA data

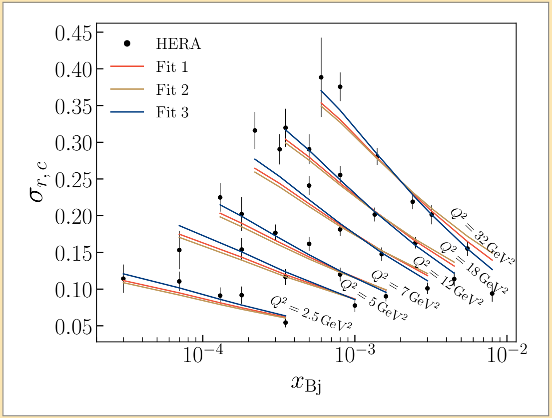
H. Hänninen, H. Mäntysaari, R. Paatelainen and J. Penttala, PRL **130** (2023) no.19, 19 [arXiv:2211.03504 [hep-ph]]

- Some massless fits also work for $F_2^{c\bar{c}}$
- Some don't $\Longrightarrow$ constraining power

At NLO: dipole picture with BK evolution describes both F_2 and $F_2^{c\bar{c}}$

Very recent combined fit

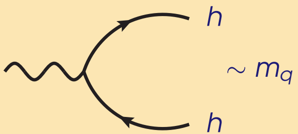
C. Casuga et al [arXiv:2506.00487 [hep-ph]].



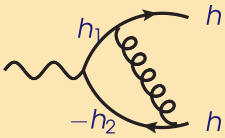
Mass renormalization

Vertex corrections to LC helicity flip vertex

Look at LC helicity flip part of LO vertex



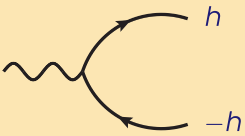
Vertex corrections from diagrams like

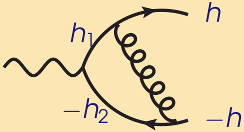


3 vertices $\Longrightarrow$ 2 options

- 1 flip vertex: $h_1 \neq h$, $h_2 \neq h_1$ or $h_2 \neq h$
  $\Longrightarrow$ log-divergent $\sim m_q \frac{1}{\varepsilon}$ (2 ED's $\sim \mathbf{k}_\perp{}^2$ each, 2 vertices $\mathbf{k}_\perp$ each, measure $d^2\mathbf{k}_\perp$)
  $\Longrightarrow$ absorb into **vertex mass** counterterm δm_V,
  same as δm_q in conventional perturbation theory
- 3 flip vertices: $h_1 \neq h$, $h_2 \neq h_1$ and $h_2 \neq h$
  $\Longrightarrow$ finite NLO contribution

Vertex corrections to non-flip vertex

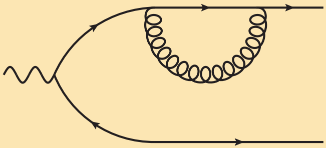
Non-flip part of LO vertex 

Corrections from 

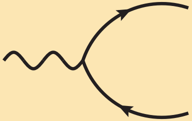
$\Longrightarrow$ again 2 options

- no flip vertex: $h_1 = h$, $h_2 = h_1$ and $h_2 \neq -h$
  vertices as in massless theory $\Longrightarrow$ not new contribution
- 2 flip + 1 non-flip $h_1 = h$ or $h_2 = h_1$ or $h_2 = -h$
  $\Longrightarrow$ again finite NLO contribution
  (2 ED's $\sim \mathbf{k}_\perp{}^2$ each, 1 vertex $\sim \mathbf{k}_\perp$, finite integral $\sim \int \mathrm{d}^2\mathbf{k}_\perp \frac{\mathbf{k}_\perp}{((\mathbf{k}_\perp - \dots)^2 + \dots)((\mathbf{k}_\perp - \dots)^2 + \dots)}$)

Quark propagator corrections



can have 0 or 2 flip vertices (1 gives zero by symmetry)

- Loops give m_q-dependent divergence $\sim$  $\times \dfrac{m_q^2}{\Delta k^-_{\mathrm{LO}}} \dfrac{1}{\varepsilon}$
- Can absorb into a renormalization of m_q^2 in ED of LO LCWF $(k_q^- = (\mathbf{k}_{\perp q}^2 + m_q^2)/(2k_q^+))$
- But now the problem, known since 90's e.g. Haridranath, Zhang, also Burkardt in Yukawa th.
- In our regularization: k^+ cutoff, $\perp$ dim. reg.
  this **kinetic mass** counterterm δm_k is **not** same as the vertex correction δm_v
- In fact δm_v is same as in covariant theory, δm_k different
- So how to determine finite part of δm_v and δm_k?

Mass renormalization

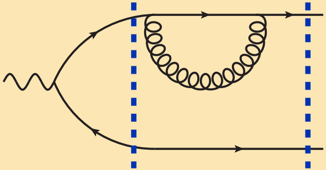
- Mass has 2 conceptually different roles here:
  - Kinetic mass: relates energy and momentum
  - Vertex mass: amplitude of helicity flip in gauge boson vertex
- 1 parameter in Lagrangian, but 2 parameters in LCPT Hamiltonian
- Lorentz-invariance requires they stay the same
- Both gauge condition $A^+ = 0$ and regularization (k^+-cutoff and $\perp$ dim. reg.) violate rotational invariance $\Longrightarrow$ $m_v \neq m_k$ at loop level $\Longrightarrow$ "textbook stuff"

There are 3 options to deal with this

1. Renormalization conditions to set separately m_v and m_k $\Longrightarrow$ discuss next
2. Smartly combine with instantaneous "normal ordering" diagrams before regularizing & integrating $\Longrightarrow$ can keep $m_k = m_v$ but cannot calculate blindly
   For details see Beuf @ Hard Probes 2018
3. Use some other regularization $\Longrightarrow$ finite parts hard!

Two mass renormalization conditions

- Pole mass/on shell renormalization point:
  - Timelike virtual $\gamma^* \to q\bar{q}$ with $q^2 = M^2$ (Same diagrams as for spacelike γ^*)
  - On shell final state $M^2 = (\mathbf{k}_{\perp q}^{\;2} + m_q^2)/(z(1-z))$ (i.e. $ED_{\text{LO}} \to 0$)
- One condition: propagator diagram



  is the most divergent at on-shell point $\Longrightarrow$ cancel this $\Longrightarrow$ kinetic mass
- Vertex mass (+ cross checks) from Lorentz-invariance
  1-loop vertex corrections: coefficients of 4 structures ($\mathbf{P}_\perp = (1-z)\mathbf{k}_{\perp q} - z\mathbf{k}_{\perp \bar{q}}$)

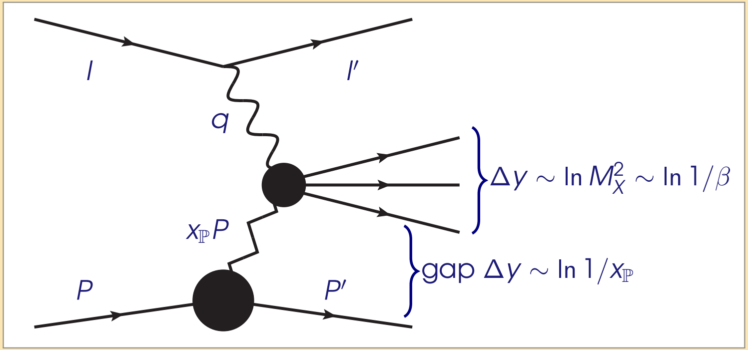
$$\bar{u}(0)\not{\varepsilon}_\lambda(q)v(1) \quad (\mathbf{P}_\perp \cdot \boldsymbol{\varepsilon}_{\perp\lambda})\bar{u}(0)\gamma^+ v(1) \quad \frac{(\mathbf{P}_\perp \cdot \boldsymbol{\varepsilon}_{\perp\lambda})}{\mathbf{P}_\perp^{\;2}}\mathbf{P}_\perp^{\;j}\bar{u}(0)\gamma^+\gamma^j v(1) \quad \boldsymbol{\varepsilon}_{\perp\lambda}^{\;j}\bar{u}(0)\gamma^+\gamma^j v(1)$$

  must reproduce 2 Lorentz-invariant form factors (Dirac & Pauli) @ on-shell point

Dipole picture diffractive DIS at NLO

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- **Diffractive system mass** M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2
- Lower $x_{\mathbb{P}}$ than dijets (e.g. at EIC)

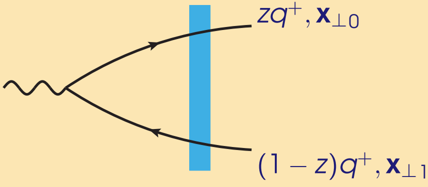
$$x_{Bj} = x_{\mathbb{P}} \beta$$

(This talk: $x_{\mathbb{P}}$ small, β not.)

Diffractive DIS at leading order

- Full kinematics and impact parameter dependence
  G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{\lambda,\, q\bar{q}}}{\mathrm{d}{M_X}^2\, \mathrm{d}|t|} = \frac{N_c}{4\pi} \int_0^1 \mathrm{d}z \int_{\mathbf{x}_{\perp 0}\mathbf{x}_{\perp 1}\bar{\mathbf{x}}_{\perp 1}\bar{\mathbf{x}}_{\perp 2}} \mathcal{I}^{(2)}_{\mathbf{\Delta}_\perp} \mathcal{I}^{(2)}_{M_X}$$
$$\times \sum_{f,h_0,h_1} \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_{\bar{0}} \bar{q}_{\bar{1}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_0 \bar{q}_1}\right) \boxed{\left[S^\dagger_{\bar{0}\bar{1}} - 1\right]\left[S_{01} - 1\right]}$$



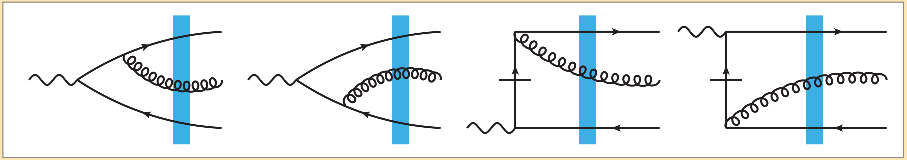
(Blue: shockwave target)

- $q\bar{q}$ crossing shockwave: dipole S_{01}
- Quadratic in dipole: sensitive to saturation
- "Transfer functions:" relate coordinates at shockwave to:
  - Momentum transfer $t = -{\mathbf{\Delta}_\perp}^2$ $\mathcal{I}^{(2)}_{\mathbf{\Delta}_\perp} = \frac{1}{4\pi} J_0\left(\sqrt{|t|}\, \|z\mathbf{x}_{\perp \bar{0}0} - (1-z)\mathbf{x}_{\perp \bar{1}1}\|\right)$
  - Invariant mass $\mathcal{I}^{(2)}_{M_X} = \frac{1}{4\pi} J_0\left(\sqrt{z(1-z)}M_X \|\bar{\mathbf{r}}_\perp - \mathbf{r}_\perp\|\right)$

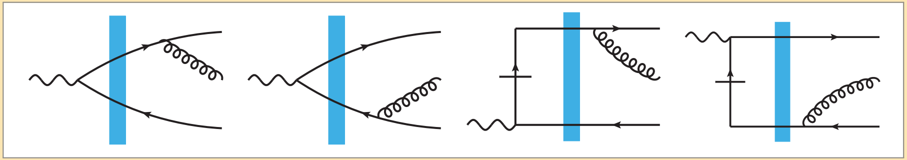
Now to NLO

NLO radiative corrections

- Emission before target

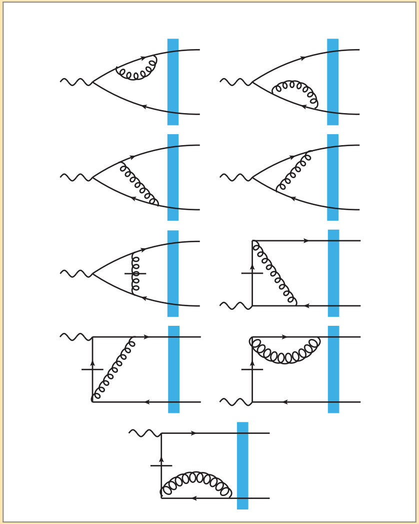


  - Squares already in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari `arXiv:2206.13161`
  - Contain leading $\ln Q^2$ contribution, which we rederive in `arXiv:2206.13161`
    (Mysterious "Wüsthoff" contribution, used in famous Golec-Biernat+Wüsthoff papers)
- Emission after target



- Interferences $\Longrightarrow$ simplify with some of the virtual corrections

NLO virtual

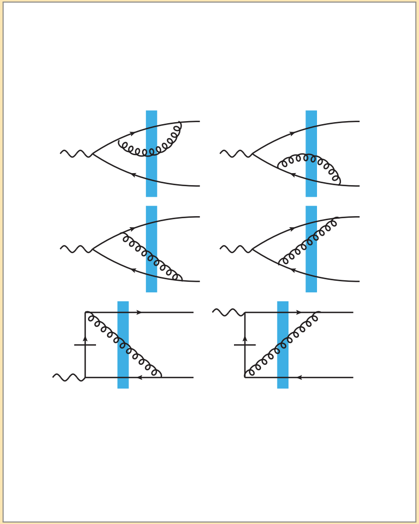


See also Boussarie et al 2014: diffractive jets,

Caucal et al 2021 inclusive

- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction

NLO virtual



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- Vertex corrections:
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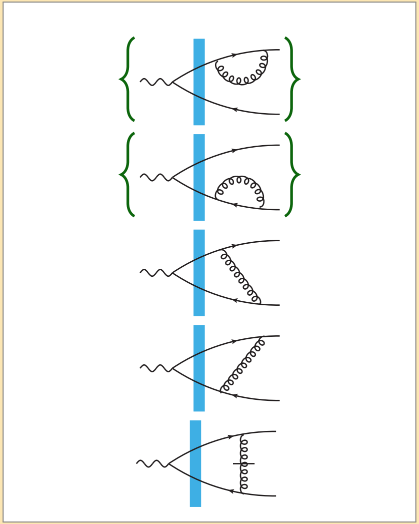
- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude

NLO virtual



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- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude
- Final state interactions
  (Propagator corrections $\{\}$ $\to$
  State renormalization, in fact = 0 in dim. reg.)

NLO diffractive DIS cross section

Calculation in `2401.17251 [hep-ph]`

Beuf, T.L. , Paatelainen, Mäntysaari, Penttala

We have calculated all these contributions

- Diffractive structure function:
  clean IR-safe, [perturbative = experimental] final state definition M_X!
  (No fragmentation function, jet definition)
  $\Longrightarrow$ Divergences must cancel
- Explicit expressions will not fit in the slides, but there in `2401.17251 [hep-ph]`
- Technically: $A^+ = 0$ gauge Hamiltonian perturbation theory = LCPT

Some features of the calculation:

- Divergence structure
- Treatment of energy denominators

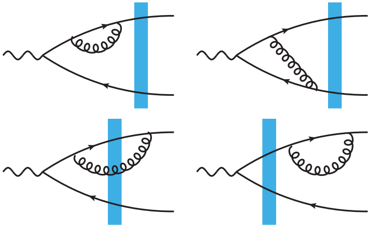
Regularization and divergences

Regularization procedure in LCPT

- Transverse momentum in $2-2\varepsilon$ dimensions $\Longrightarrow \frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow 1/\alpha$, $\ln^2 \alpha$, $\ln \alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon} \ln \alpha$ divergences:
   $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization

1. UV $\frac{1}{\varepsilon}$ e.g. in



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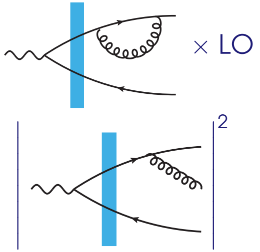
2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission

2. Collinear $\frac{1}{\varepsilon}$ e.g. in



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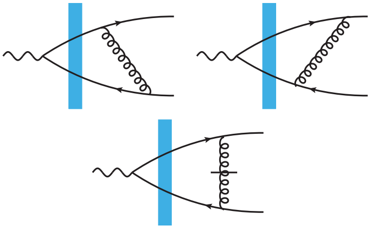
3. $1/\alpha$: normal and instantaneous exchange

3. $1/\alpha$ e.g. in



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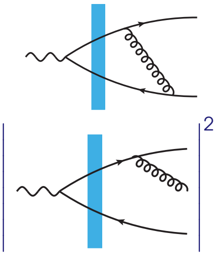
4. $\ln^2\alpha$ from final state exchange and emission
   (M_X restriction matters here!)

4. $\ln^2\alpha$ e.g. in



Regularization and divergences

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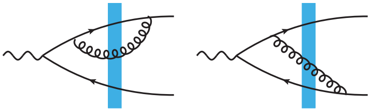
   wavefunction renormalization
2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission
3. $1/\alpha$: normal and instantaneous exchange
4. $\ln^2\alpha$ from final state exchange and emission
   (M_X restriction matters here!)
5. Remaining $\ln\alpha$ absorbed into BK/JIMWLK

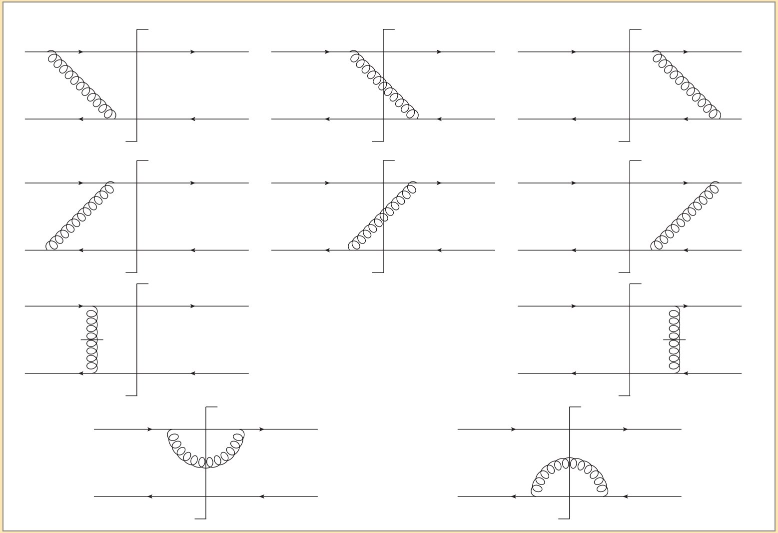
5. $\ln\alpha$ e.g. in



Challenge: how to dig these out from diagrams?

Final state corrections

How to dig out different types of divergences? Penttala



Final state corrections

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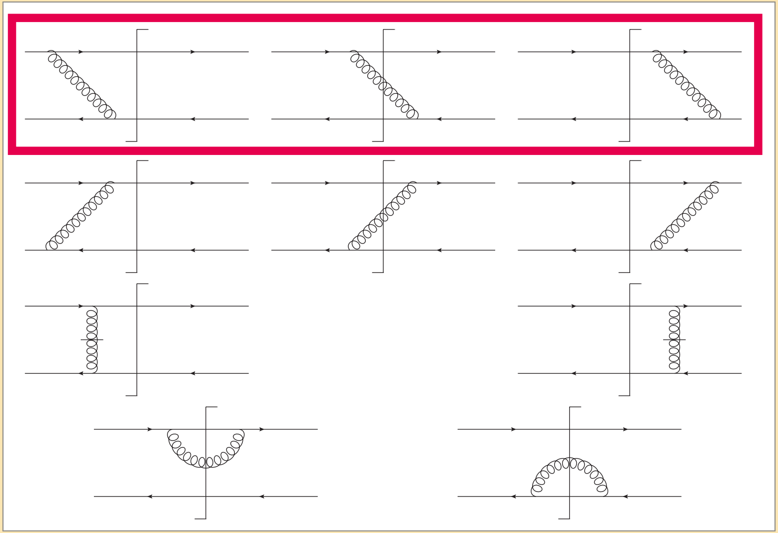
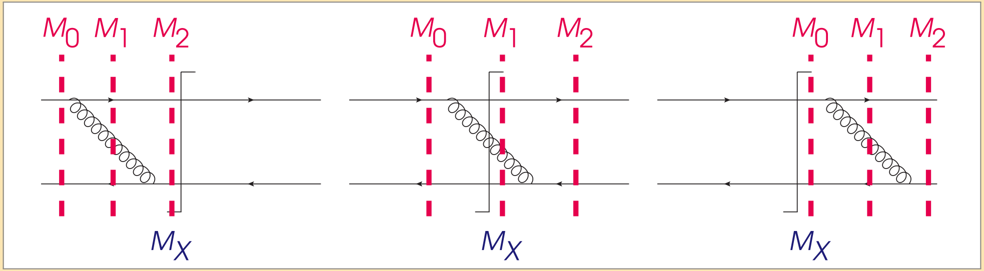


As an example: consider 1st row

"Beuf trick": $\delta({M_X}^2 - \dots)$ as imaginary part of "propagator"

Off-shell propagators = light cone energy denominators + M_X constraint



$$\frac{\delta({M_X}^2 - M_2^2)}{(M_2^2 - M_1^2 + i\delta)(M_2^2 - M_0^2 + i\delta)} + \frac{\delta({M_X}^2 - M_1^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta({M_X}^2 - M_0^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)}$$

$$= \frac{1}{2\pi i}\left[\frac{1}{({M_X}^2 - M_0^2 - i\delta)({M_X}^2 - M_1^2 - i\delta)({M_X}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

(Note: sign of $i\delta$ essential)

- Then express numerator (⊥ momentum dot products) in terms of M_0^2, M_1^2, M_2^2
- Combine before integration $\Longrightarrow$ separate different divergence types

Conclusions, lecture 2

Lecture 2: NLO

- Dipole picture of DIS: $\gamma^* \to q\bar{q}$
- Light cone quantization: partons in γ^*
- CGC: target is dense gluon field
- BK equation: add one soft gluon, absorb into redefinition of target

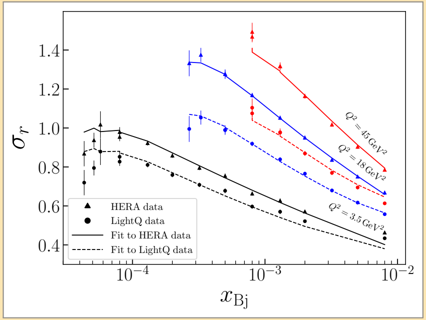
Lecture 2: NLO

- Dipole picture DIS total cross section available at NLO
- Being used to describe HERA data
- Quark masses included; longstanding issue of mass renormalization in LCPT
- Diffractive structure function at NLO calculated, implementation in progress.

Heavy and light quarks

Elephant in the room: heavy quarks, simultaneous F_2 & F_2^c difficult also at LO

- Only have NLO cross sections for $m = 0$
- Data is for total F_2 & F_2^c, not light-quark separately; charm is large fraction of F_2
- Here: interpolated "light quark-only" dataset.
  $\Longrightarrow$ fit with NLO dipole picture
  Errors not correct (experimentalists needed for this)
- Result: achieve good fits, typically with larger σ_0 and slower evolution speed
  (The α_s scale parameter C allows this to adjust.)
- Lesson: F_2 is not ideal for dipole picture
  (known: aligned jet configurations)
  $\Longrightarrow$ F_L and F_2^c for more reliable probes.



Dirac and Pauli form factors

$$\Gamma^\mu(q) = F_D(q^2/m^2)\,\gamma^\mu + F_P(q^2/m^2)\,\frac{q_\nu}{2m}\,i\sigma^{\mu\nu}$$

$$\Psi^{\gamma_T^*\to q\bar{q}}_{\mathrm{LO}} + \Psi^{\gamma_T^*\to q\bar{q}}_{\mathrm{NLO}} = \delta_{\alpha_0\alpha_1}\frac{ee_f}{\mathrm{ED}_{\mathrm{LO}}}\Big\{\bar{u}(0)\not{\varepsilon}_\lambda(q)v(1)\left[1+\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{V}^T\right] + \frac{q^+}{2k_0^+k_1^+}(\mathbf{P}_\perp\cdot\boldsymbol{\varepsilon}_{\perp\lambda})\bar{u}(0)\gamma^+v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{N}^T$$
$$+\frac{q^+}{2k_0^+k_1^+}\frac{(\mathbf{P}_\perp\cdot\boldsymbol{\varepsilon}_{\perp\lambda})}{\mathbf{P}_\perp{}^2}\mathbf{P}_\perp{}^j m\bar{u}(0)\gamma^+\gamma^j v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{S}^T + \frac{q^+}{2k_0^+k_1^+}m\bar{u}(0)\gamma^+\not{\varepsilon}_\lambda(q)v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{M}^T\Big\}.$$

$$-\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\frac{m^2}{\mathbf{P}_\perp{}^2}\,\mathcal{S}^T\Big|_{\mathbf{P}_\perp{}^2=-\overline{Q}^2-m^2} = F_P(q^2/m^2)$$

$$-\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\frac{1}{(2z-1)}\,\mathcal{N}^T\Big|_{\mathbf{P}_\perp{}^2=-\overline{Q}^2-m^2} = F_P(q^2/m^2)$$

$$\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{V}^T\Big|_{\mathbf{P}_\perp{}^2=-\overline{Q}^2-m^2} = -1 + F_D(q^2/m^2) + F_P(q^2/m^2)$$

$$\mathcal{M}^T\Big|_{\mathbf{P}_\perp{}^2=-\overline{Q}^2-m^2} = 0$$