Probing quark confinement through energy correlations

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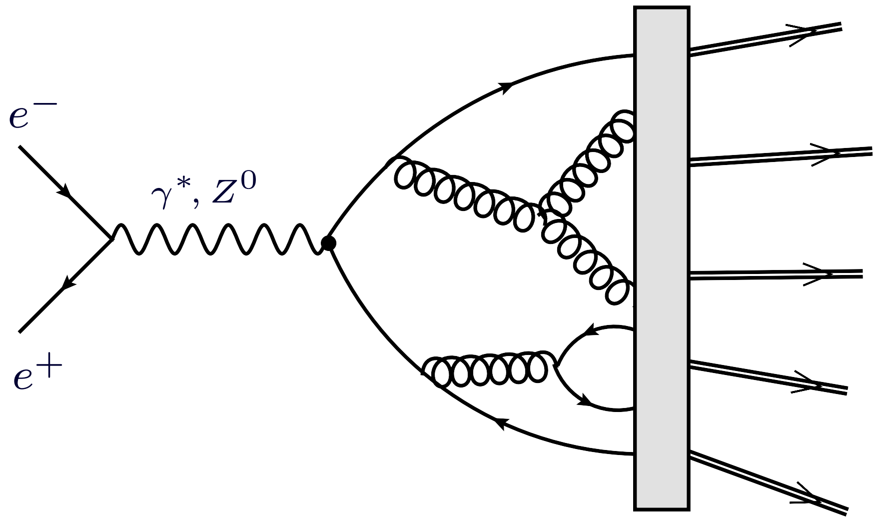
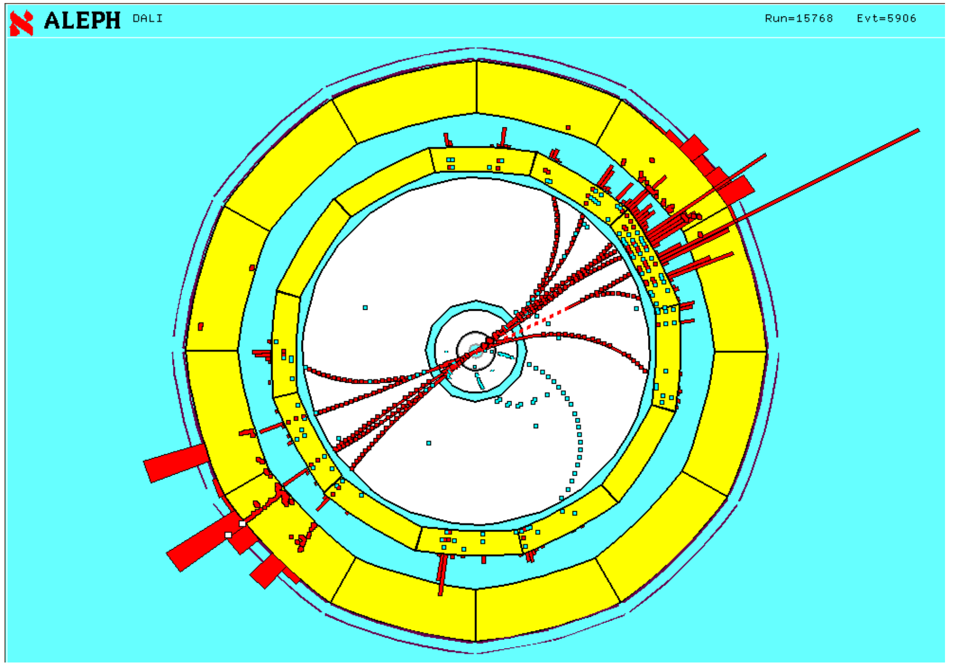
e^+e^- annihilation in QCD

- PETRA (1978-1986) and LEP (1989-2010)





- γ^* / Z^0 decay into quarks and gluons that undergo a hadronization process into hadrons
- Final states can be described using the class of *infrared finite* observables (event shapes):

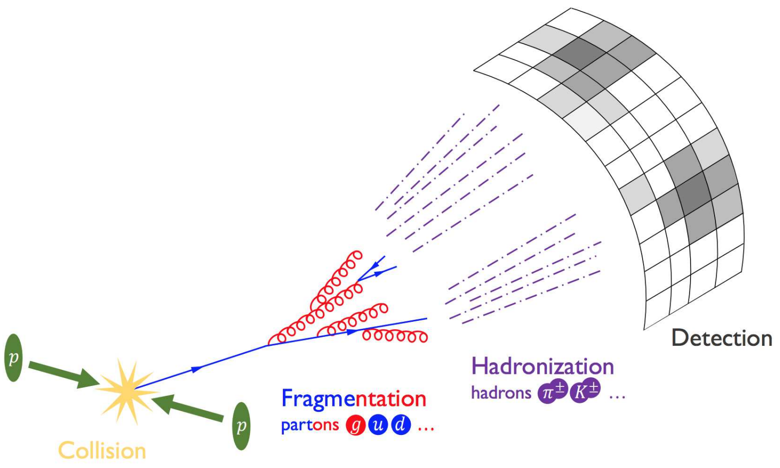
  energy-energy correlations (EEC), thrust, heavy mass, . . .
- Can be computed in perturbative QCD, hadronisation corrections are ‘small’ at high energy

$$\text{EEC} = \text{EEC}_{\text{PT}}(\alpha_s(Q)) + (\Lambda_{\text{QCD}}/Q)^p$$

Hadronization effects in QCD

Theoretical calculations in QCD are done in terms of quarks and gluons

But what is actually produced and detected are hadrons (pions, kaons, protons, etc.)



How to relate theoretical predictions (quarks, gluons) and experimental quantities (hadrons)?

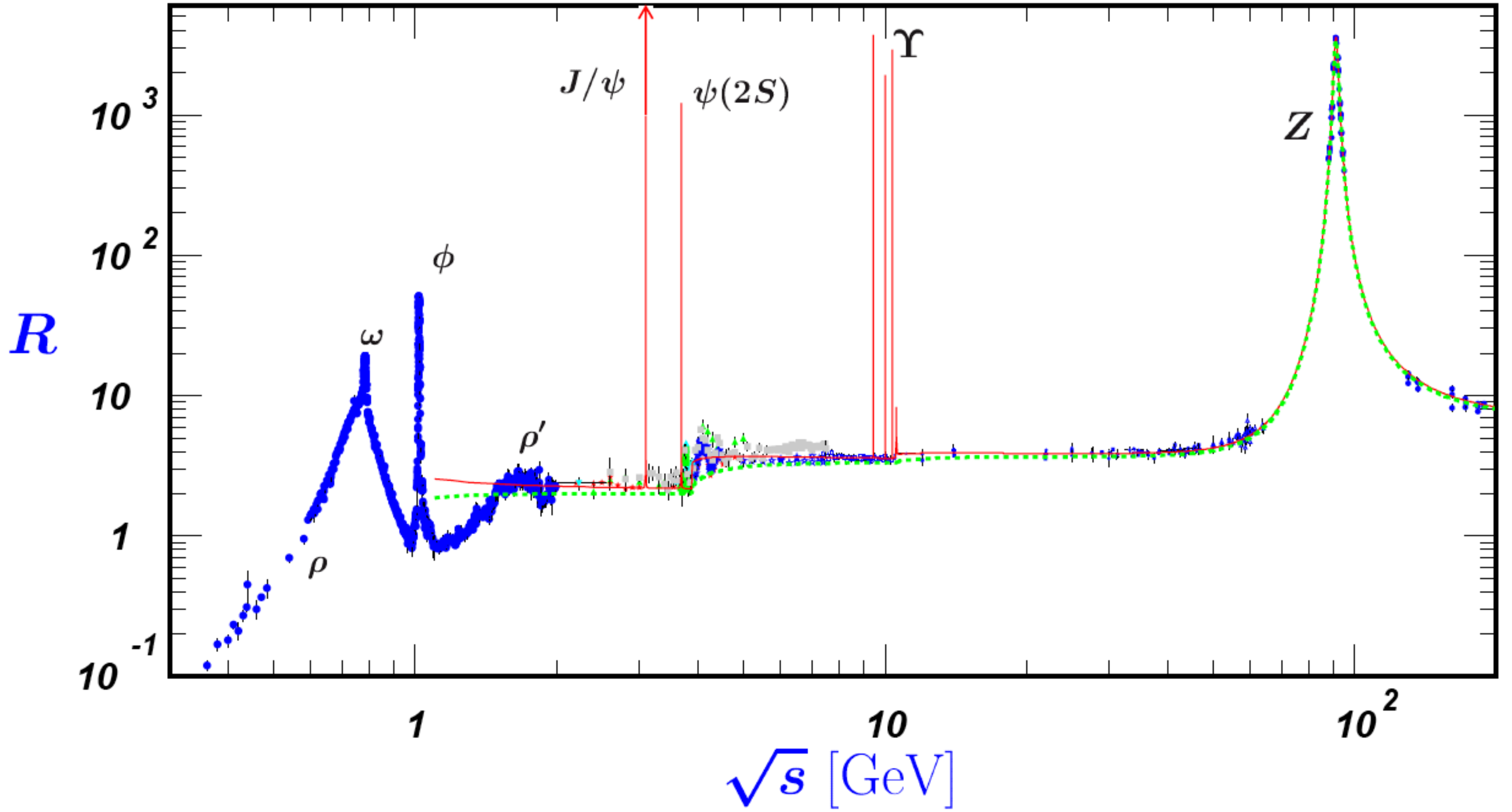
Solve confinement problem (too hard)

Quark-hadron duality: [Poggio, Quinn and Weinberg'76]

Certain inclusive hadronic observables have to (approximately) coincide at high energies

to the same quantities calculated in terms of quarks and gluons

Total cross section $e^+e^- \to hadrons$

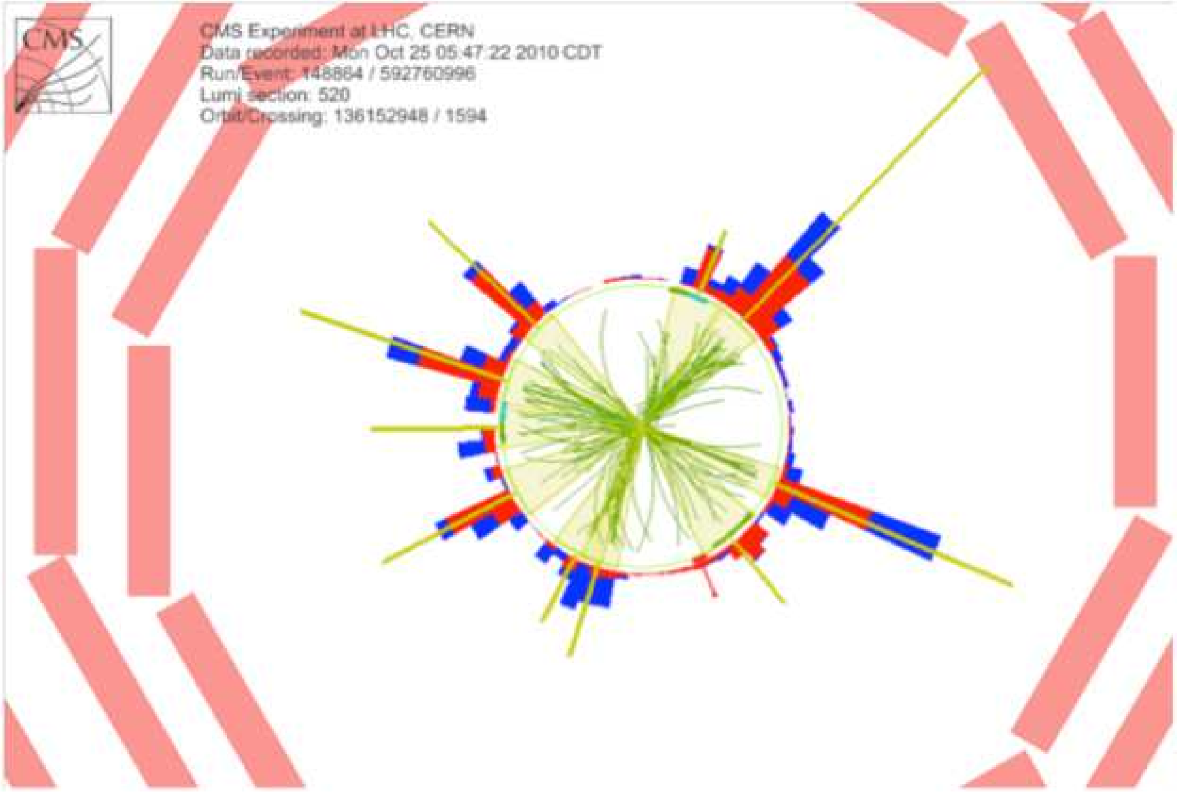


Quark-hadron duality:

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = R_{\text{PT}}(\alpha_s(s)) + O\left(\frac{\Lambda^4}{s^2}\right)$$

Hadronization corrections are parameterized by nonPT vacuum condensates $\langle \alpha_s F^2 \rangle,\ m\bar{\psi}\psi, \ldots$.

Multi-jet final states



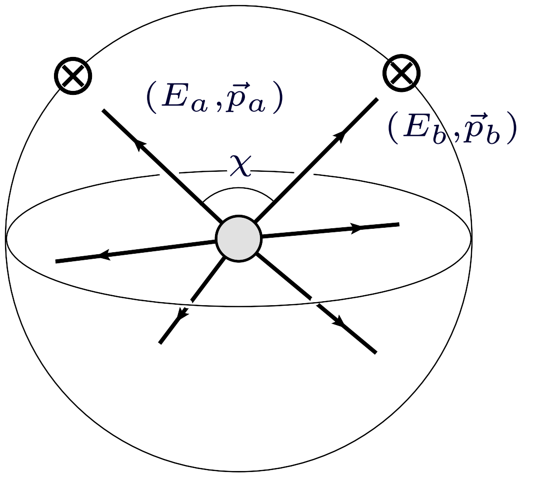
- ✔ A lot of particles produced
- ✔ Energy of particles is deposit at the calorimeters
- ✔ Admit description in terms of the energy distribution on the celestial sphere

Energy-energy correlation

✔ Function of the angle $0 \le \chi \le \pi$ between detected particles [Basham,Brown,Ellis,Love'78]

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)$$

Total energy $\sum_a E_a = Q$

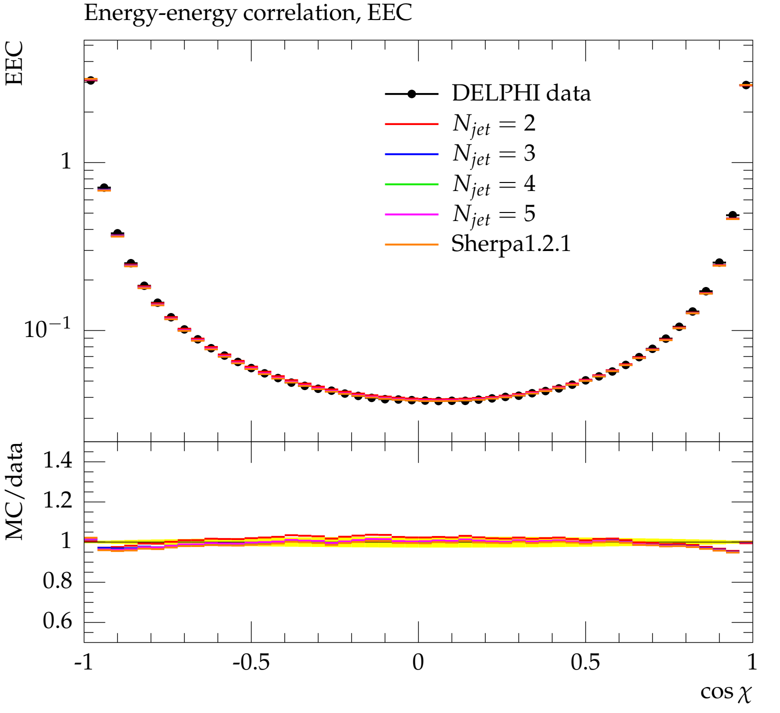


✔ EEC in e^+e^- final states (1978 – today):

- ✗ Very precise experimental data
- ✗ Slow progress on the theory side

$$\mathrm{EEC}(\chi) = \underbrace{\alpha_s(Q)A(\chi)}_{\text{Basham et al'78}} + \underbrace{\alpha_s^2(Q)B(\chi)}_{\text{Dixon et al'18}} + O(\alpha_s^3)$$

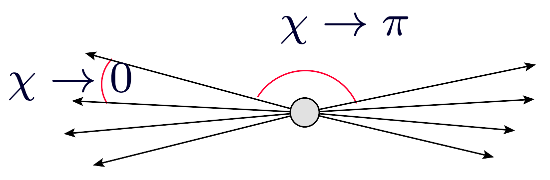
- ✗ Much faster progress in super-Yang-Mills theory (3 loops + strong coupling)



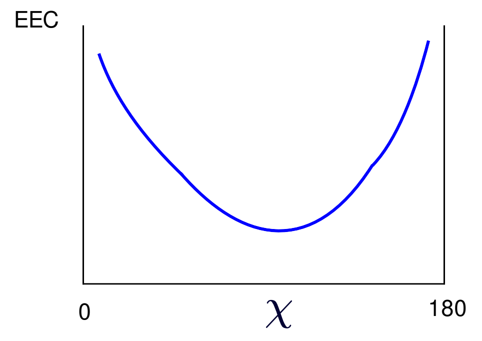
What we expect

Two different types of the final states:

- ✔ Two-jet final states

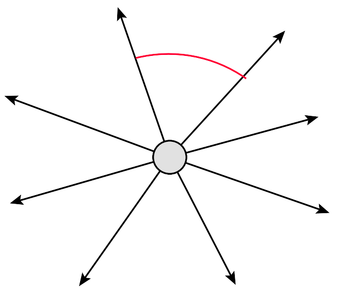


$\Longrightarrow$



The energy correlations are picked around the end-points

- ✔ No jets – homogenous distribution of the energy



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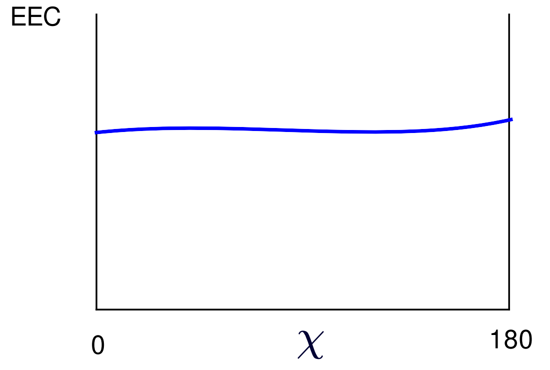


The energy correlations are independent on the angle

The choice depends on the underlying dynamics: the first scenario is realized in QCD, the second one in maximally super-Yang-Mills theory at strong coupling

The simplest gauge theory

Maximally supersymmetric Yang-Mills theory

- ✔ Most (super)symmetric theory possible (without gravity)
- ✔ Uniquely specified by local internal symmetry group - e.g. number of colors N_c for $SU(N_c)$
- ✔ Exactly scale-invariant field theory for any coupling (Green functions are powers of distances)
- ✔ Weak/strong coupling duality (AdS/CFT correspondence, gauge/string duality)

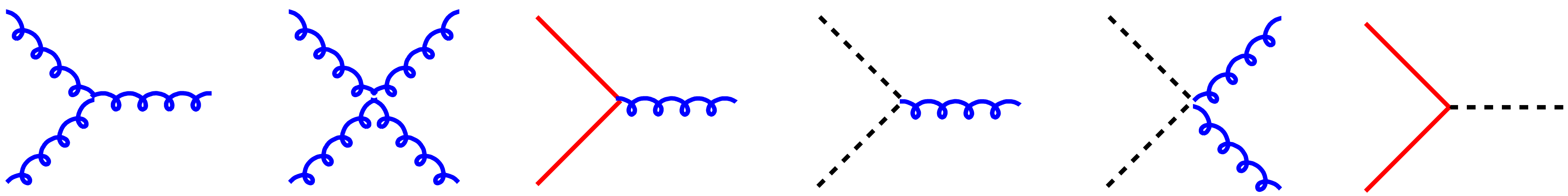
Particle content:

massless spin-1 gluon (= the same as in QCD)

4 massless spin-1/2 gluinos (= cousin of the quarks)

6 massless spin-0 scalars

Interaction between particles:



All proportional to same dimensionless coupling λ and related to each other by supersymmetry

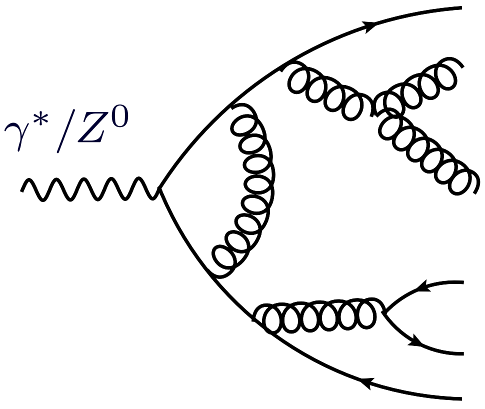
Conventional approach

✔ EEC as a weighted cross-section

$$\mathrm{EEC}(\chi) = \sum_{a,b,X} \int \mathrm{dLIPS}\, \left|\mathcal{A}_{a+b+X}\right|^2 \frac{E_a E_b}{Q^2} \delta(\cos\chi - \cos\theta_{ab})$$

The amplitude of creation of the final state $|a, b, X = \text{everything}\rangle$

$$\mathcal{A}_{a+b+X} = \int d^4x\, \mathrm{e}^{iQx} \langle a, b, X | J(x) | 0 \rangle$$



✔ Main disadvantages:

- ✗ presence of infrared divergences in transition amplitudes $\mathcal{A}_{a+b+X}$
- ✗ integration over the Lorentz invariant phase space of the final states $d\mathrm{LIPS}$
- ✗ necessity for summation over all final states $\sum_X$

✔ New approach: EEC can be computed from *correlation functions of energy flow operators*

EEC from correlation functions

- Total cross section from the optical theorem

$$\begin{aligned}\sigma_{\rm tot}(q) &= \sum_X (2\pi)^4 \delta^{(4)}(Q-p_X)|\mathcal{A}_{J\to X}|^2 \\ &= \int d^4x\ \mathrm{e}^{iQx} \sum_X \langle 0|J^\dagger(0)|X\rangle\, \mathrm{e}^{-ixp_X}\, \langle X|J(0)|0\rangle \\ &= \int d^4x\ \mathrm{e}^{iQx} \qquad \underbrace{\langle 0|J^\dagger(x)J(0)|0\rangle}_{\text{Wightman correlation function}}\end{aligned}$$

- Energy-energy correlation

$$\mathrm{EEC} \sim \sum_X \langle 0|J^\dagger(x)|X\rangle\, \underbrace{w_{\vec{n}_1}(X) w_{\vec{n}_2}(X)}_{\text{EEC weight factor}} \langle X|J(0)|0\rangle = \langle 0|J^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)J(0)|0\rangle$$

Energy flow operator

$$\mathcal{E}(\vec{n})|X\rangle = \underbrace{\sum\nolimits_a E_a\, \delta^{(2)}(\Omega_{\vec{p}_a} - \Omega_{\vec{n}})}_{w_{\vec{n}}(X)}\, |X\rangle$$

- Multiple energy flow correlations

$$\mathrm{E...EC} \sim \langle 0|J^\dagger(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots\mathcal{E}(\vec{n}_L)J(0)|0\rangle$$

Energy correlations in interacting theory

- ✔ Energy flow in the direction $\vec{n}$ on the celestial sphere [Sveshnikov,Tkachev],[GK,Oderda,Sterman]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} du\, n^i T_{0i}(t = r + u, r\vec{n})$$

Flux of the energy on the celestial sphere in the direction $\vec{n}$

- ✔ Energy flow correlations

$$\mathrm{EEC}(\chi) = \int d^4x\ \mathrm{e}^{iQx} \langle 0|J^\dagger(x)\, \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) J(0)|0\rangle$$

Generalization of the optical theorem for the weighted cross-sections

- ✔ Multi-fold integral of *Wightman* 4pt function

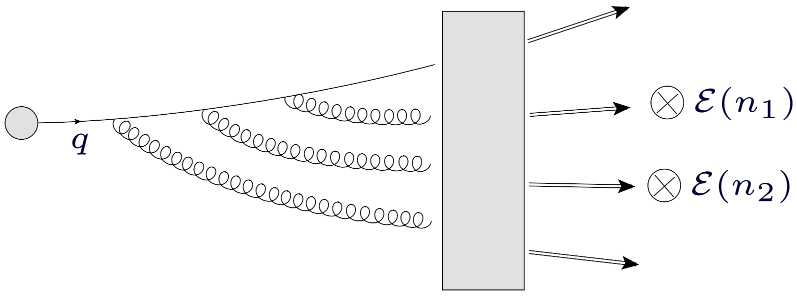
$$\mathrm{EEC} \sim \underbrace{\int d^4x\, \mathrm{e}^{iQx}}_{\text{Fourier}} \underbrace{\lim_{r_i\to\infty} r_1^2 r_2^2 \int_{-\infty}^{\infty} dt_1 dt_2}_{\text{Detector limit}} \underbrace{\langle 0|J^\dagger(x)\, T_{0\vec{n}_1}(x_1) T_{0\vec{n}_2}(x_2)\, J(0)|0\rangle}_{\text{Wightman corr. function}} \Big|_{x_i = (t_i + r_i, r_i\vec{n}_i)}$$

No IR divergences at intermediate steps, no need to go away from $d = 4$ dimensions

- ✔ Nonperturbative formulation of the energy correlations in terms of four-point correlation functions

Correlations at small angles

Energy correlations at small angles probe an internal structure of jets



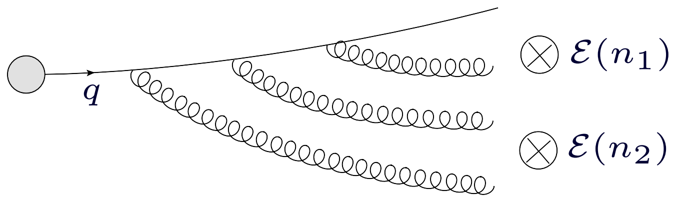
$$\tfrac{1}{2}(n_1 n_2) = \tfrac{1}{2}(1 - \cos\chi) \sim \chi^2$$

Can be computed in perturbative QCD, hadronisation corrections are expected to be ‘small’

$$\mathrm{EEC}(\chi) = \mathrm{EEC}_{\mathsf{PT}}(\chi) + \text{Hadronization corrections}$$

Inclusive decay of a quark with invariant mass $m_q^2 = Q^2\chi^2$

$$\mathrm{EEC}_{\mathsf{PT}}(\chi) =$$



$$\sim \ 1/\chi^{2-\gamma(\alpha_s(Q\chi))}$$

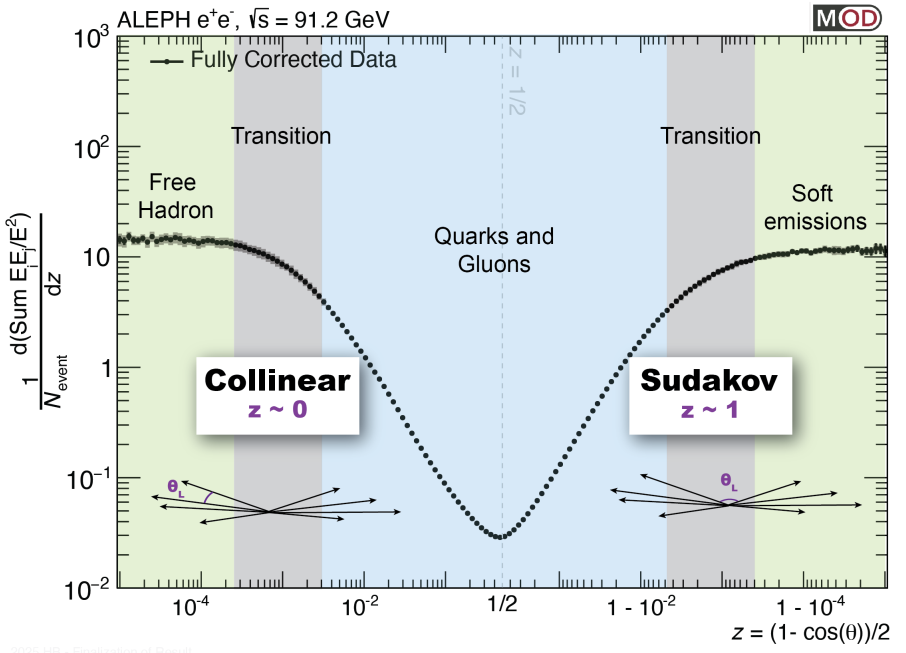
Hadronization corrections are small for $\Lambda_{\mathrm{QCD}}/(Q\chi) \ll 1$

What happens for $\chi = O(\Lambda_{\mathrm{QCD}}/Q)$?

Different regimes

Recent re-analysis of LEP2 data

Bossi et al', 2505.11828 [hep-ex]



For small angle χ, the EEC describes the correlation between particles belonging to the same jet

Behaves differently at small angles depending on how χ compares with $\Lambda_{\rm QCD}/Q$

$$\textbf{Flat}: \quad \text{EEC} \sim \text{const}, \qquad\qquad \textbf{OPE}: \quad \text{EEC} \sim 1/\chi^{2-\gamma}$$

The same behaviour was observed for the energy correlations at LHC [Komiske,Moult,Thaler,Zhu'23]

What is physics mechanism of the transitioning between the two regimes?

Main idea: use multiplicity of particles $K \gg 1$ as a new expansion parameter

Warm up example: free scalar theory

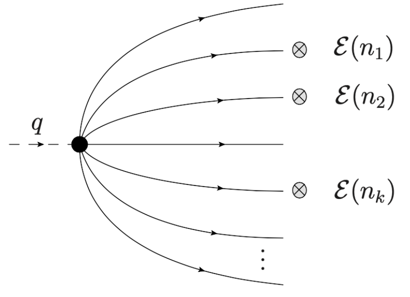
Multi-particle final state containing K massless scalar particles

$$|H(q)\rangle = \int d^4x\, e^{iqx} O_K(x)|0\rangle\,, \qquad O_K(x) = \phi^K(x)$$

Large multiplicity = Heavy source operator $K \gg 1$

Energy correlations at large K (= new parameter of the expansion)

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_M)\rangle =$$



$$= \int d\sigma_K\, w_{n_1}(p_1)\dots w_{n_M}(p_M)$$

The weight factor $w_n(p)$ selects the particle in the final state moving along $n^\mu = (1, \vec{n})$

$$w_n(p) = p^0 \delta^{(2)}\left(\frac{\vec{p}}{p^0} - \vec{n}\right)$$

The differential cross-section

$$d\sigma_K = (2\pi)^4 \delta^{(4)}(q - \sum_{i=1}^{K} p_i) \prod_{n=1}^{K} d\mathrm{LIPS}(p_n)\,, \qquad d\mathrm{LIPS}(p) = \frac{d^4p}{(2\pi)^4} 2\pi\delta_+(p^2)$$

Large K limit in a free theory

Energies of detected particle scale as $p_i^0/Q = \varepsilon_i/K$

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_M)\rangle = Q^M \int_0^\infty \prod_{i=1}^M \frac{d\varepsilon_i\,\varepsilon_i^2}{8\pi}\, e^{-\varepsilon_i}\left[1-\frac{1}{K}\sum_{i<j}\varepsilon_i\varepsilon_j(1-z_{ij})+\dots\right]$$

Ensemble of noninteracting particles with energies distributed according to $dP(\varepsilon) = d\varepsilon\,\varepsilon^2 e^{-\varepsilon}$

Dimensionless angular variables

$$z_{ij} = \frac{q^2(n_in_j)}{2(qn_i)(qn_j)} = \frac{1}{2}(1-\cos\theta_{ij})\,, \qquad q^\mu = (Q,\vec{0})$$

✔ The leading term is independent of the angles

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_M)\rangle_{\rm free} = \left(\frac{Q}{4\pi}\right)^K\left[1+\frac{1}{K}\sum_{1\le i<j\le M} 9z_{ij}+\dots\right]$$

Homogeneous distribution of the energy; the angular dependence is suppressed by $1/K$

✔ The same behaviour was observed in $\mathcal{N}=4$ SYM at strong coupling $\lambda\to\infty$ [Hofman,Maldacena]

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_M)\rangle_{\mathcal{N}=4} = \left(\frac{Q}{4\pi}\right)^K[1+O(1/\lambda)]$$

In both cases, the flat regime is associated with a large number of soft particles in the final state

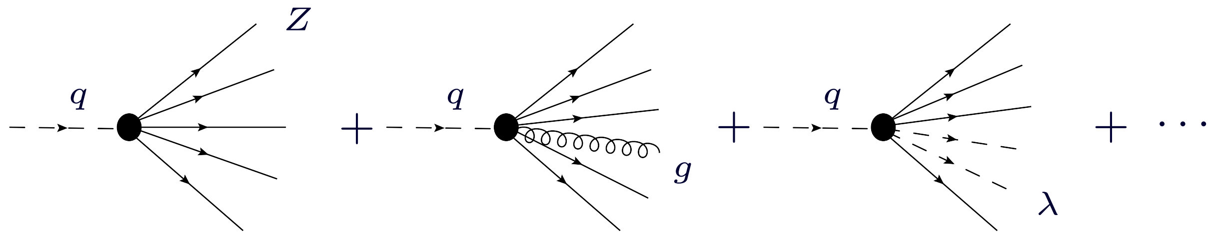
Large K limit in interacting theory

Energy correlation in multi-particle final state is flat in a free theory

How does the interaction between the particles in the final state affect this result?

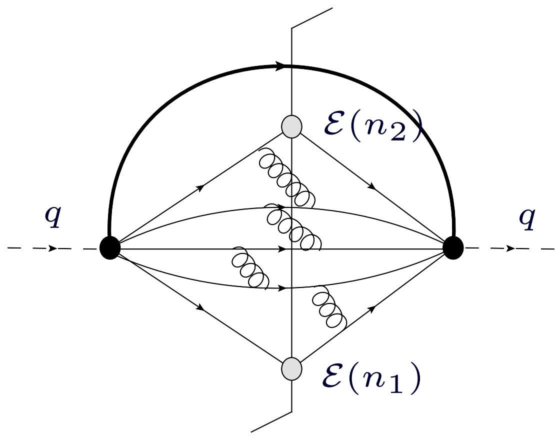
Consider the EEC in $\mathcal{N}=4$ SYM in the final state created by a half-BPS operator $O_K = \mathrm{tr}[Z^K(x)]$

$$|H(q)\rangle = \int d^4x\, e^{iqx} O_K(x)|0\rangle =$$



Energy-energy correlation

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_K = \frac{(q^2)^4}{2(qn_1)^3(qn_2)^3}\,\frac{\mathcal{F}_K(z)}{(4\pi)^2} =$$



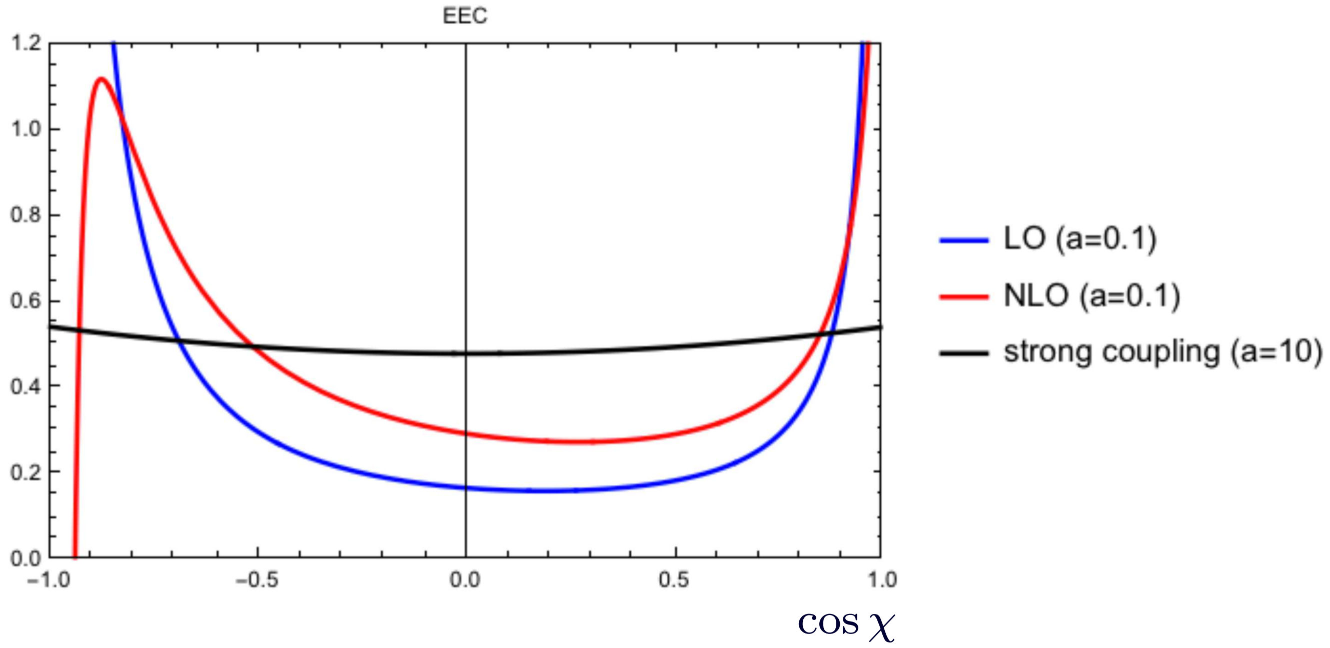
Sum over all possible final states containing many on-shell particles

Depends on 't Hooft coupling λ and angle between the detectors

$$z = \frac{q^2(n_1n_2)}{2(qn_1)(qn_2)} = \frac{1}{2}(1-(\vec{n}_1\vec{n}_2)) = \frac{1}{2}(1-\cos\theta_{12})$$

What we expect for $\mathcal{F}_K(z,\lambda)$

For $K=2$ the EEC is peaked around $z=0$ and $z=1$ at weak coupling (two jet final state!)



The EEC becomes flat at strong coupling $\lambda\to\infty$ and K fixed [Hofman,Maldacena]

$$\mathcal{F}_K(z) \overset{\lambda\to\infty}{\sim} 2+\frac{4\pi^2}{\lambda}\left(1-6z(1-z)\right)+O(\lambda^{-3/2})$$

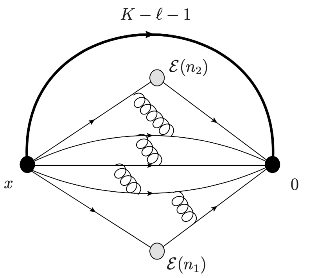
For $K\gg 1$ and finite λ we expect the EEC to be flat as well [Chicherin,GK,Sokatchev,Zhiboedov]

$$\mathcal{F}_{K\gg 1}(z)=2+O(1/K)$$

Examine the transition from $K=2$ to $K\to\infty$ at weak coupling

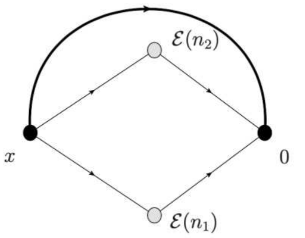
Weak coupling

$$\mathcal{F}_K(z,\lambda) = \mathcal{F}_K^{(0)}(z) + \sum_{\ell=1}^{\infty}\left(\frac{\lambda}{4\pi^2}\right)^{\ell}\mathcal{F}_K^{(\ell)}(z) =$$

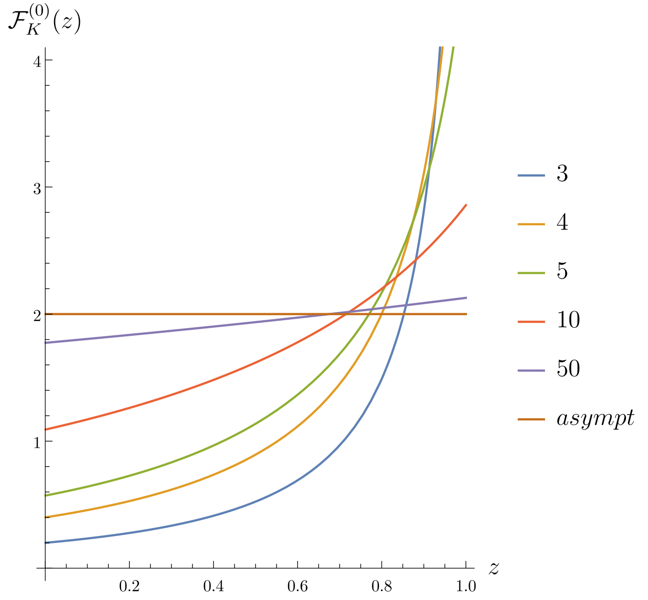


Born approximation for arbitrary weight K

$$\mathcal{F}_K^{(0)}(z) = \frac{2(K-2)(K-1)}{(K+1)(K+2)}\,{}_2F_1\left(3,3;K+3|z\right) =$$



The angular dependence flattens out for $K\to\infty$



$$\mathcal{F}_K^{(0)}(z) \overset{K\gg 1}{=} 2 + \frac{6}{K}(3z-2) + O(1/K^2)$$

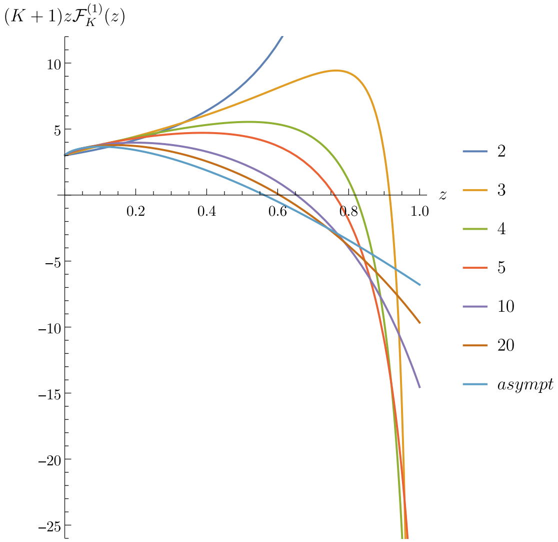
One and two loops

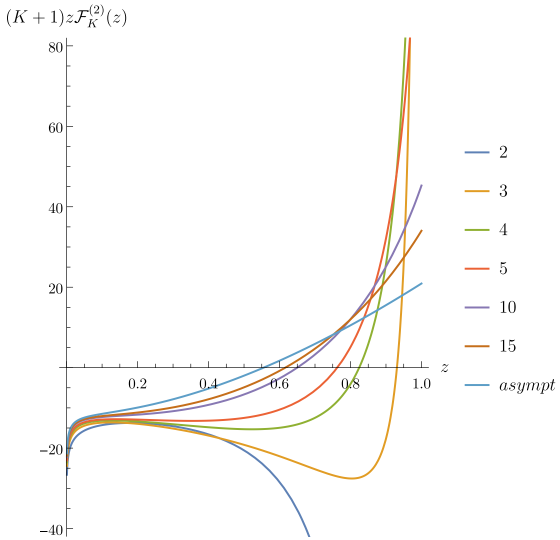
$$\mathcal{F}_K^{(1)}(z) = \frac{1}{z^{K+2}}\Big[c_1^{[K]}(z)L(z) + c_2^{[K]}(z)\mathrm{Li}_2(z) + c_3^{[K]}(z)\log(z)\log(1-z)$$

$$+ c_4^{[K]}(z)\log^2(1-z) + c_5^{[K]}(z)\log(1-z) + c_6^{[K]}(z)\log(z) + c_7^{[K]}(z)\Big]$$

$$\mathcal{F}_K^{(2)}(z) = \frac{1}{z^{K+2}}\Big[\text{Multi-linear combinations of HPLs with argument } \sqrt{z} \text{ of weight } w \leq 5\Big]$$

$c_m^{[K]}(z)$ are polynomials of degree $K-1$





The angular dependence flattens out at large K

For $K \gg 1$, the function $(K+1)z\mathcal{F}_K^{(\ell)}(z)$ has universal behaviour at small angles

Large K limit in AdS/CFT

A lot of progress in computing correlation functions in $\mathcal{N} = 4$ SYM

Superconformal Ward identity

$$\langle O_K T T O_K \rangle = P(\partial) \langle O_K O_2 O_2 O_K \rangle$$

This correlation function is known at weak and strong coupling

Energy correlations depend on how K (= multiplicity) compares to 't Hooft coupling λ

- ✔ At strong coupling, for $\lambda \gg K \gg 1$, the EEC does not depend on K

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle \overset{\lambda \gg 1}{\approx} 2 + \frac{4\pi^2}{\lambda}(1 - 6z(1-z)) + O(\lambda^{-3/2})$$

  Ceases to depend on the angular separation as $\lambda \to \infty$

- ✔ At large multiplicity, for $K \gg 1$ and finite λ,

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle \overset{K \gg 1}{\approx} 2 + \frac{1}{K}\mathcal{F}(\lambda, z)$$

  The angular dependence is suppressed by the factor of $1/K$

In both limits, the energy correlations are flat but the underlying mechanism is different

Back to LHC: correlations at small angles

Energy correlations for small angles $z \to 0$ and weak coupling $a = \lambda/(4\pi^2) < 1$

$$\begin{aligned}\mathcal{F}_K(z) &= \mathcal{F}_K^{(0)}(z) + a\mathcal{F}_K^{(1)}(z) + a^2\mathcal{F}_K^{(2)}(z) + \dots \\ &= 2 + \frac{3}{K}\left[\frac{a}{z} + \frac{a^2}{z}\log z + \dots\right] \\ &= 2 + \frac{3a}{K} z^{-1+a}\end{aligned}$$

Agrees with the leading OPE contribution at small angles $z = \sin^2(\chi/2) \ll 1$ [Kologlu et al]

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle \sim 1 + \frac{\langle \mathbb{O}_3^+\rangle_H}{z^{1-\gamma_3^+/2}}\,, \qquad \langle \mathbb{O}_3^+\rangle_H = O(1/K)$$

$\mathbb{O}_3^+$ is the spin three light-ray operator of positive signature with anomalous dimension γ_3^+

Two regimes at small z / large K:

For fixed z and $K \to \infty$: $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle = 1 + O(1/K)$ *Flat regime*

For $z \to 0$ and fixed K : $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle \sim \dfrac{1}{z^{1-\gamma_3^+/2}}$ *OPE regime*

The transition happens for $K \sim 1/z^{1-\gamma_3^+/2}$

Conclusions

- The energy correlations in multi-particle states exhibit two characteristic regimes:

$$\mathbf{Flat}: \quad \text{EEC} \sim \text{const}, \qquad\qquad \mathbf{OPE}: \quad \text{EEC} \sim 1/\chi^{2-\gamma}$$

- The transition between the two regimes is controlled by the particle multiplicity K and the dynamics of the theory

$$\text{EEC} \sim 1 + \frac{C}{K\chi^{2-\gamma}}$$

- An analogous transition was observed in QCD in the measurement of the angular energy distribution of particles belonging to the same energetic jet

- Energy correlations are powerful tools for analyzing the internal structure of jets, studying an interplay between perturbative and nonperturbative effects

- A lot of interesting application in high-energy physics, gravitational waves physics and cosmology. Stay tuned!