65th Cracow School of Theoretical Physics, June 14-21, 2025

When Physics meets Quantum Information

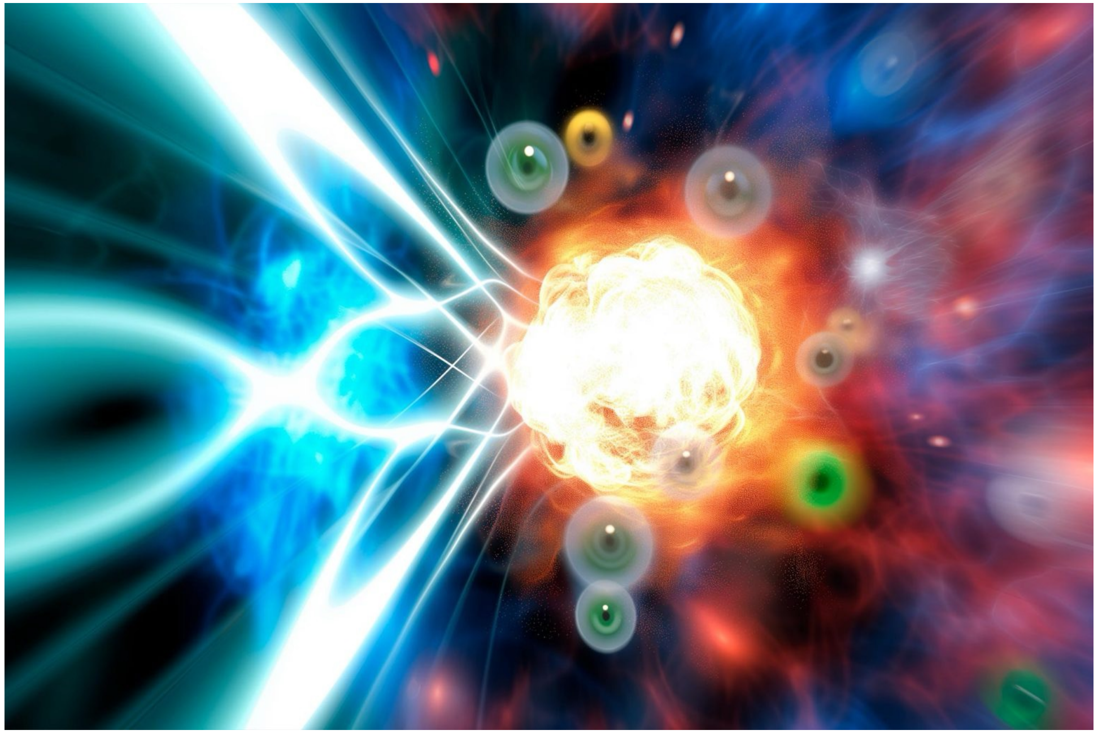


Image:
"Maximal entanglement inside the proton"
SciTechDaily

Dmitri Kharzeev

U.S. DEPARTMENT OF ENERGY
Office of Science

Center for Nuclear Theory
Stony Brook University

C2QA Co-design Center for Quantum Advantage

Brookhaven National Laboratory

ChatGPT on QIS@Zakopane:

🧭 Traditional Cracow/Zakopane Schools

- Since **1961**, the **Cracow School of Theoretical Physics** in Zakopane has primarily focused on particle physics, gravitation, cosmology, and general relativity th-www.if.uj.edu.pl +7.

- If you're specifically interested, the **2025 programme** explicitly includes **quantum information** in its topics—definitely worth checking out link.aps.org +1.

Outline

- Classical and quantum states
- Quantum entanglement: from paradox to technology
- Entanglement, thermalization and quantum AI
- Real-time quantum simulations of a QFT: thermalization through a maximal entanglement
- Evidence for maximal entanglement in nuclear/high energy physics

Statistics of classical states

Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung, respective den Sätzen über das Wärmegleichgewicht.

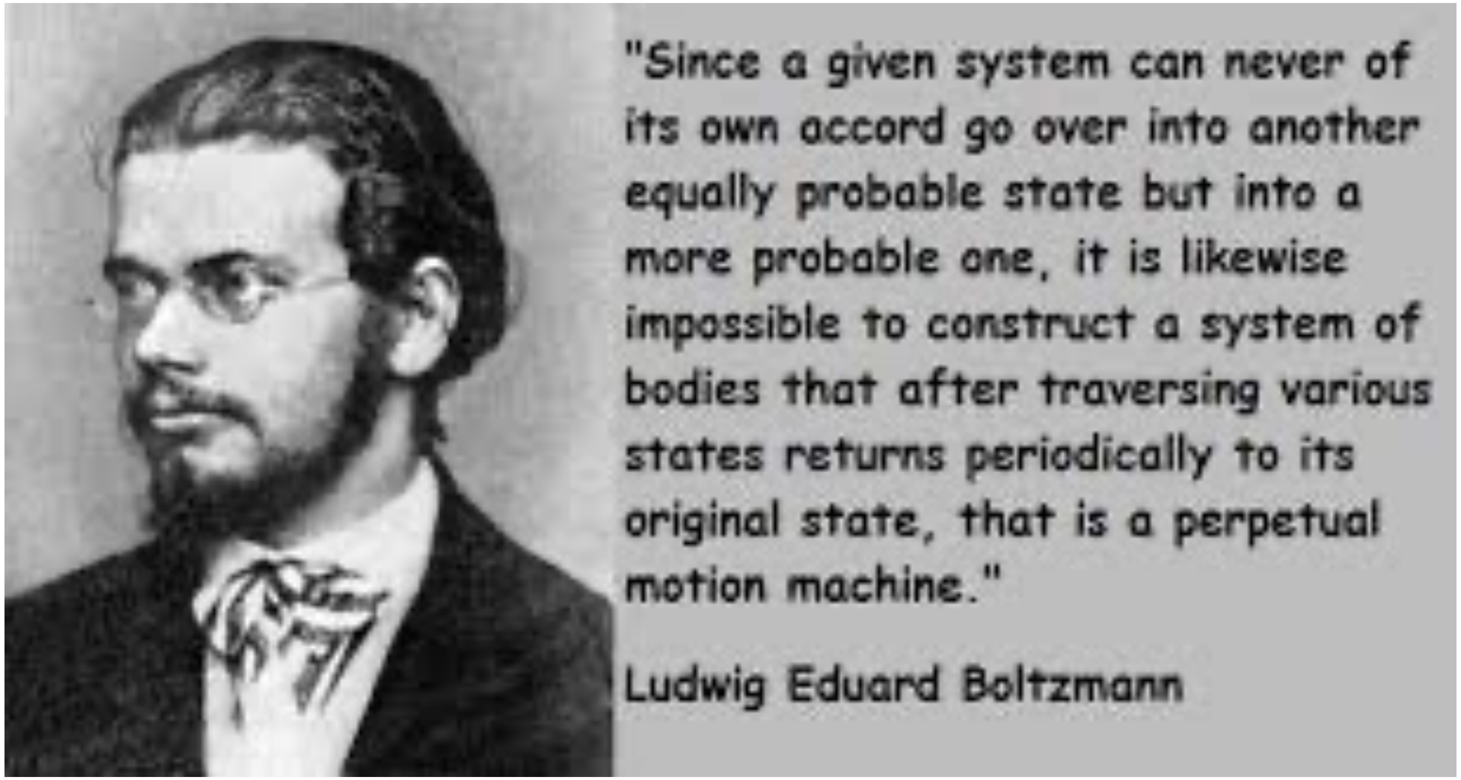
Von dem c. M. **Ludwig Boltzmann** in Graz.

Eine Beziehung des zweiten Hauptsatzes zur Wahrscheinlichkeitsrechnung zeigte sich zuerst, als ich nachwies, dass ein analytischer Beweis desselben auf keiner anderen Grundlage

mit folgenden Worten: „Es ist klar, dass jede einzelne gleichförmige Zustandsvertheilung, welche bei einem bestimmten

Anfangszustande nach Verlauf einer bestimmten Zeit entsteht, ebenso unwahrscheinlich ist, wie eine einzelne noch so ungleichförmige Zustandsvertheilung, gerade so wie im Lottospiele jede einzelne Quinterne ebenso unwahrscheinlich ist, wie die Quinterne

"It is clear that in thermal equilibrium all possible states of the system … are equally probable"



In classical statistical mechanics,
the system is driven to the most probable state with the largest entropy – equilibrium.

What if the system is quantum? Do quantum subsystems evolve to the state with the largest entanglement entropy (maximally entangled states)?

Quantum entanglement

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

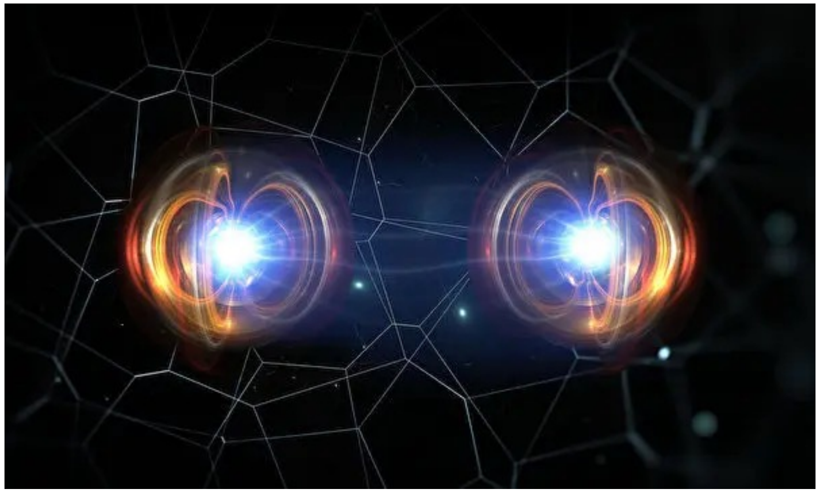
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Consider two separated quantum particles that had previously interacted

Measurement of momentum of one particle

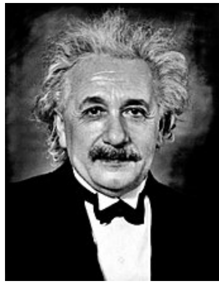


prevents the knowledge of another particle's coordinate!

``I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from ***spooky action at a distance****"*

A. Einstein, letter to M. Born, 1947

Quantum entanglement



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

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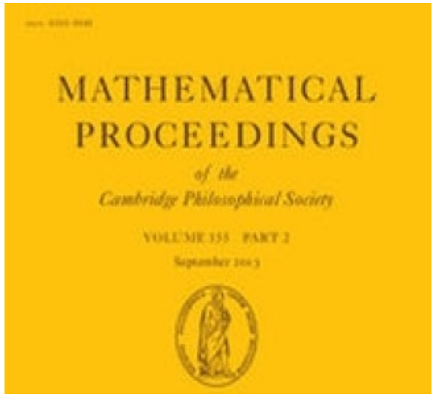
OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*
(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS



By E. SCHRÖDINGER

[Communicated by Mr M. Born]

[*Received* 14 August, *read* 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one privilege: to restrict the experiments to one only of the two systems. After re-establishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement*. Its sinister importance is due to its being involved in every measuring process and therefore forming the basis of the quantum theory of

Describing entanglement: the density matrix

EPR state (2 qubits):

$$\frac{|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B}{\sqrt{2}}$$

The corresponding density matrix:

$$\rho_{\mathrm{AB}}\left(\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\right) = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\frac{\langle 00| \pm \langle 11|}{\sqrt{2}} = \frac{|00\rangle\langle 00| \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

If the state of B is unknown, A is described by the reduced density matrix:

$$\rho_A = \mathrm{tr}_B(\rho_{AB}) = \frac{|0\rangle\langle 0|\ \langle 0|0\rangle \pm \cancel{|0\rangle\langle 1|\ \langle 0|1\rangle} \pm \cancel{|1\rangle\langle 0|\ \langle 1|0\rangle} + |1\rangle\langle 1|\ \langle 1|1\rangle}{2}$$

$$= \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathrm{I}}{2}.$$

Mixed state! $\rho_A \neq \rho_A^2$

Das Dämpfungsproblem in der Wellenmechanik.

Von **L. Landau** in Leningrad.

(Eingegangen am 27. Juli 1927.)

Es wird eine Formel für die wellenmechanische Behandlung der Dämpfung aufgestellt. Mit ihrer Hilfe werden einige diesbezügliche Fragen untersucht; auch die Kohärenzerscheinungen finden ihre Aufklärung. Ein Ausdruck für spontane Emission wird ermittelt, und die Intensitätsfrage der Spektrallinien auf diese Weise gelöst.

§ 1. Gekoppelte Systeme in der Wellenmechanik. In der Wellenmechanik kann ein System nicht eindeutig definiert werden; wir haben es immer mit einer Wahrscheinlichkeitsgesamtheit zu tun (statistische Auffassung)[1]. Ist das System mit einem anderen gekoppelt, so tritt in seinem Verhalten eine doppelte Unbestimmtheit auf.

Der Zustand des ersten Systems sei charakterisiert durch die Größen a_n in

$$\psi = \sum a_n \psi_n; \tag{1}$$

für das zweite gelte

$$\psi' = \sum b_r \psi'_r. \tag{2}$$

Die Schrödingersche Funktion für beide Systeme zusammen ist dann:

$$\Psi = \psi \psi' = \sum_n \sum_r a_n b_r \psi_n \psi'_r = \sum_n \sum_r c_{nr} \psi_n \psi'_r, \tag{3a}$$

wo

$$c_{nr} = a_n b_r. \tag{3b}$$

Tritt eine Kopplung auf, so wird c_{nr} Funktion der Zeit und kann nicht mehr der Gleichung (3b) gemäß zerlegt werden. Es können also a_n und b_r hier nicht mehr einzeln angewandt werden.

Quantifying entanglement: von Neumann entropy

$$\rho = \sum_n p_n |n\rangle\langle n|$$

Entanglement entropy:

$$S = -\mathrm{tr}\rho \ln \rho = -\mathrm{p_n} \ln \mathrm{p_n}$$

Pure states:

$$S = 0$$

e.g.

$$p_0 = 1,\ p_{n\neq 0} = 0$$

Mixed states:

$$S \neq 0$$

e.g. for EPR $\rho_A = \frac{\mathbf{I}}{2}$

$$p_0 = p_1 = \frac{1}{2} \to S = \ln 2$$

Maximally entangled states

Consider the entanglement entropy

$$S = -\mathrm{tr}\rho \ln \rho = -\sum_n p_n \ln p_n$$

for the case of N states with equal probabilities

$$p_n = 1/N$$

Then $$S = -N\frac{1}{N}\ln(1/N) = \ln N$$

This looks like the Boltzmann formula!

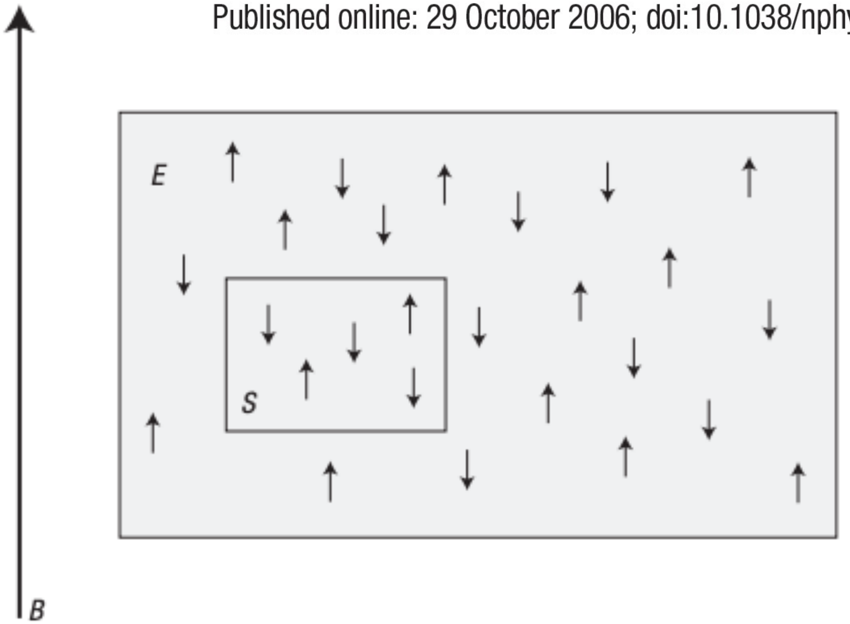
Maximal entanglement and thermalization

Entanglement and the foundations of statistical mechanics

SANDU POPESCU[1,2], ANTHONY J. SHORT[1]* AND ANDREAS WINTER[3]
[1]H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, UK
[2]Hewlett-Packard Laboratories, Stoke Gifford, Bristol BS12 6QZ, UK
[3]Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK
*e-mail: tony.short@bristol.ac.uk

nature physics
Published online: 29 October 2006; doi:10.1038/nphys444

Statistical mechanics is one of the most successful areas of physics. Yet, almost 150 years since its inception, its foundations and basic postulates are still the subject of debate. Here we suggest that the main postulate of statistical mechanics, the equal *a priori* probability postulate, should be abandoned as misleading and unnecessary. We argue that it should be replaced by a general canonical principle, whose physical content is fundamentally different from the postulate it replaces: it refers to individual states, rather than to ensemble or time averages. Furthermore, whereas the original postulate is an unprovable assumption, the principle we propose is mathematically proven. The key element in this proof is the quantum entanglement between the system and its environment. Our approach separates the issue of



In future work, we hope to go beyond the kinematic viewpoint presented here to address the dynamics of thermalization. In

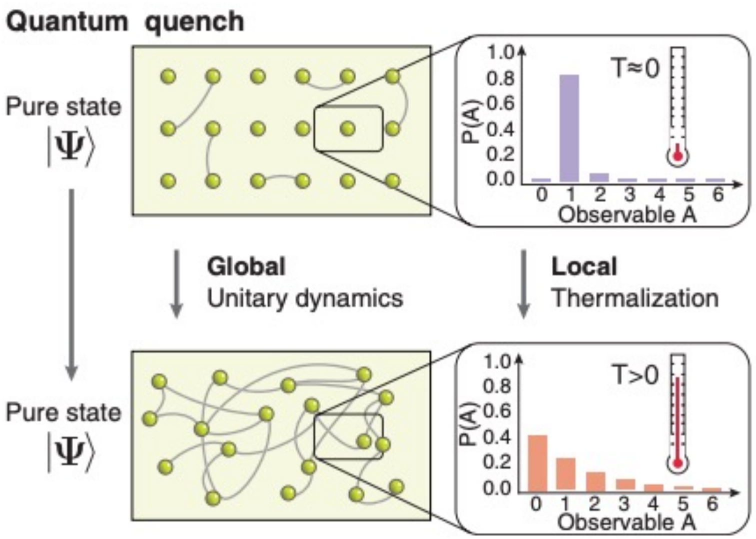
Maximal entanglement and thermalization

STATISTICAL PHYSICS

Quantum thermalization through entanglement in an isolated many-body system

Adam M. Kaufman, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Robert Schittko, Philipp M. Preiss, Markus Greiner*

Statistical mechanics relies on the maximization of entropy in a system at thermal equilibrium. However, an isolated quantum many-body system initialized in a pure state remains pure during Schrödinger evolution, and in this sense it has static, zero entropy. We experimentally studied the emergence of statistical mechanics in a quantum state and observed the fundamental role of quantum entanglement in facilitating this emergence.



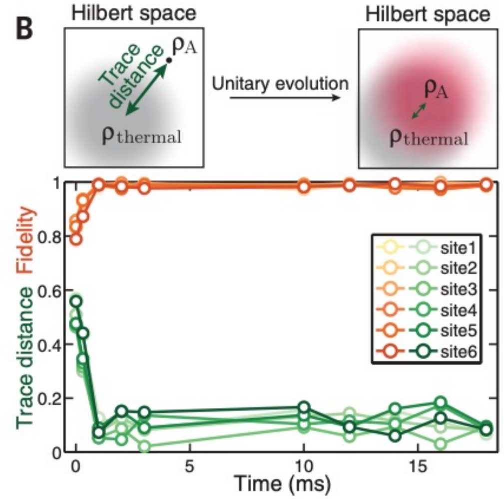
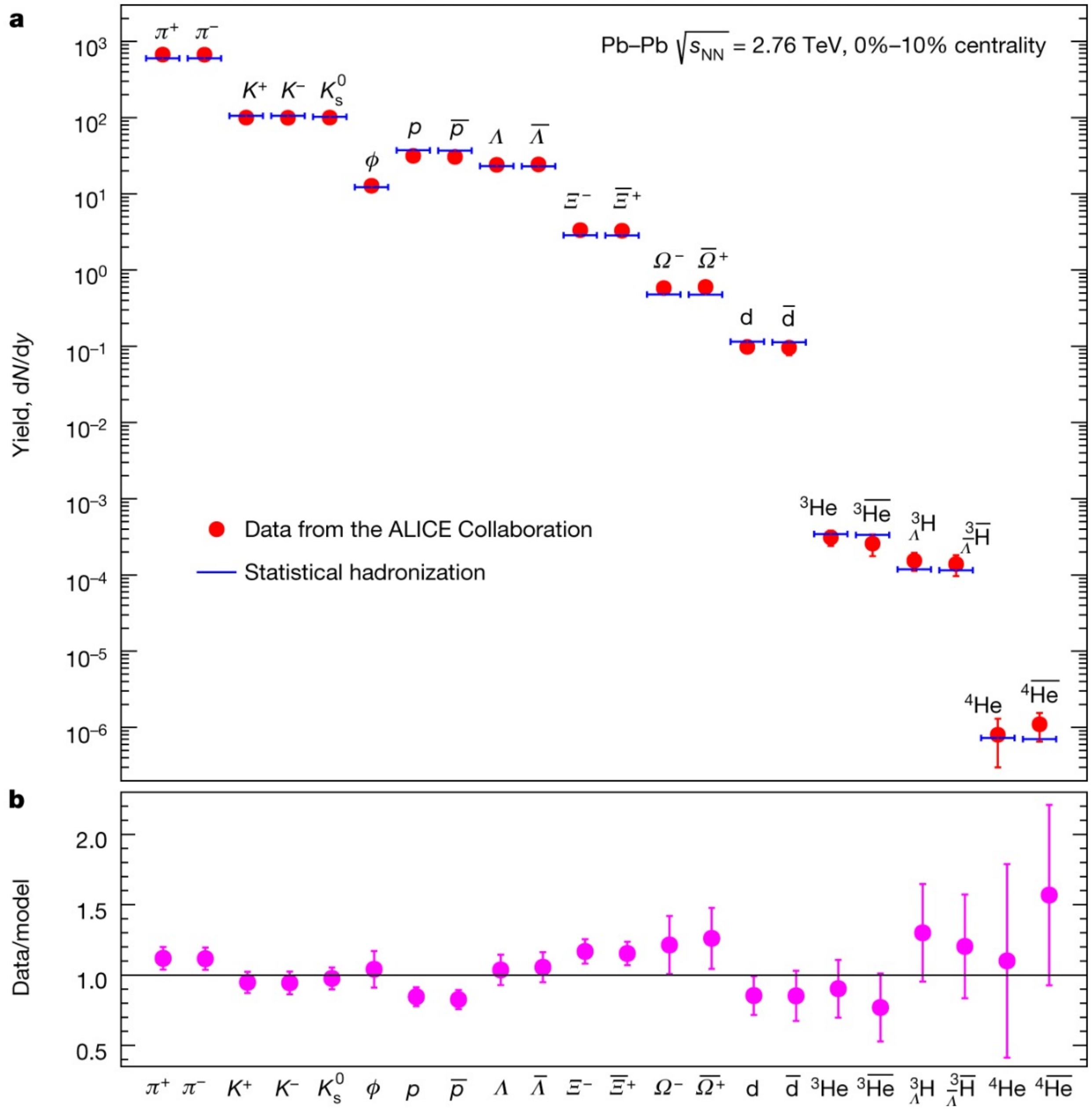


Fig. 5. Observation of local thermalization.

In nuclear and high energy physics, there is a long-standing puzzle of "early thermalization"

There is an ample evidence from experiments at RHIC, LHC and elsewhere that high energy heavy ion (and even pp and e^+e^- collisions) lead to some kind of fast thermalization:

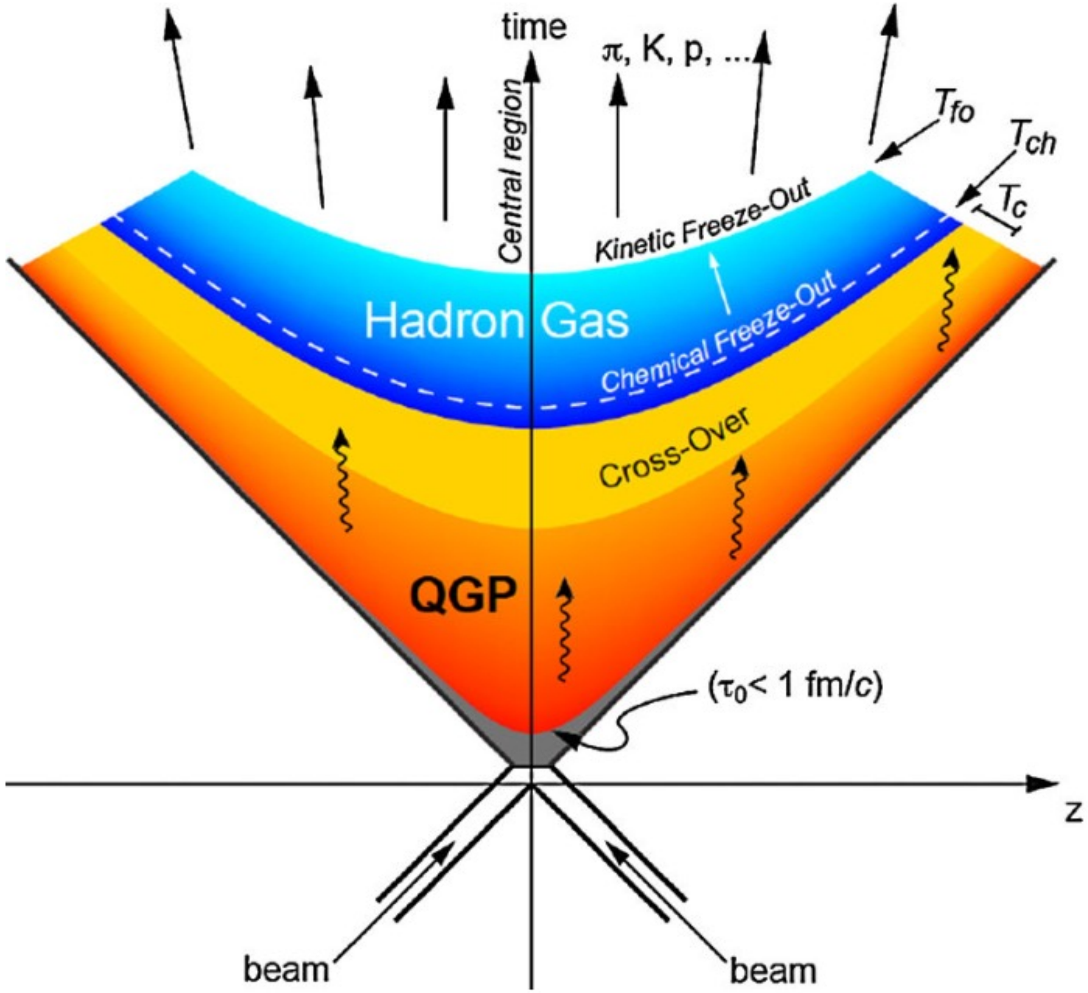
- Hadron abundances look thermal
- Hydrodynamics describes remarkably well the momentum spectra and azimuthal correlations of produced hadrons, assuming that the initial conditions are provided at a very early time $\tau \sim 0.5$ fm



A.Andronic, P.Braun-Munzinger, K.Redlich, J. Stachel, Nature 561 (2018) 321

What is the mechanism of this thermalization?

How can it happen so fast in a rapidly expanding system?



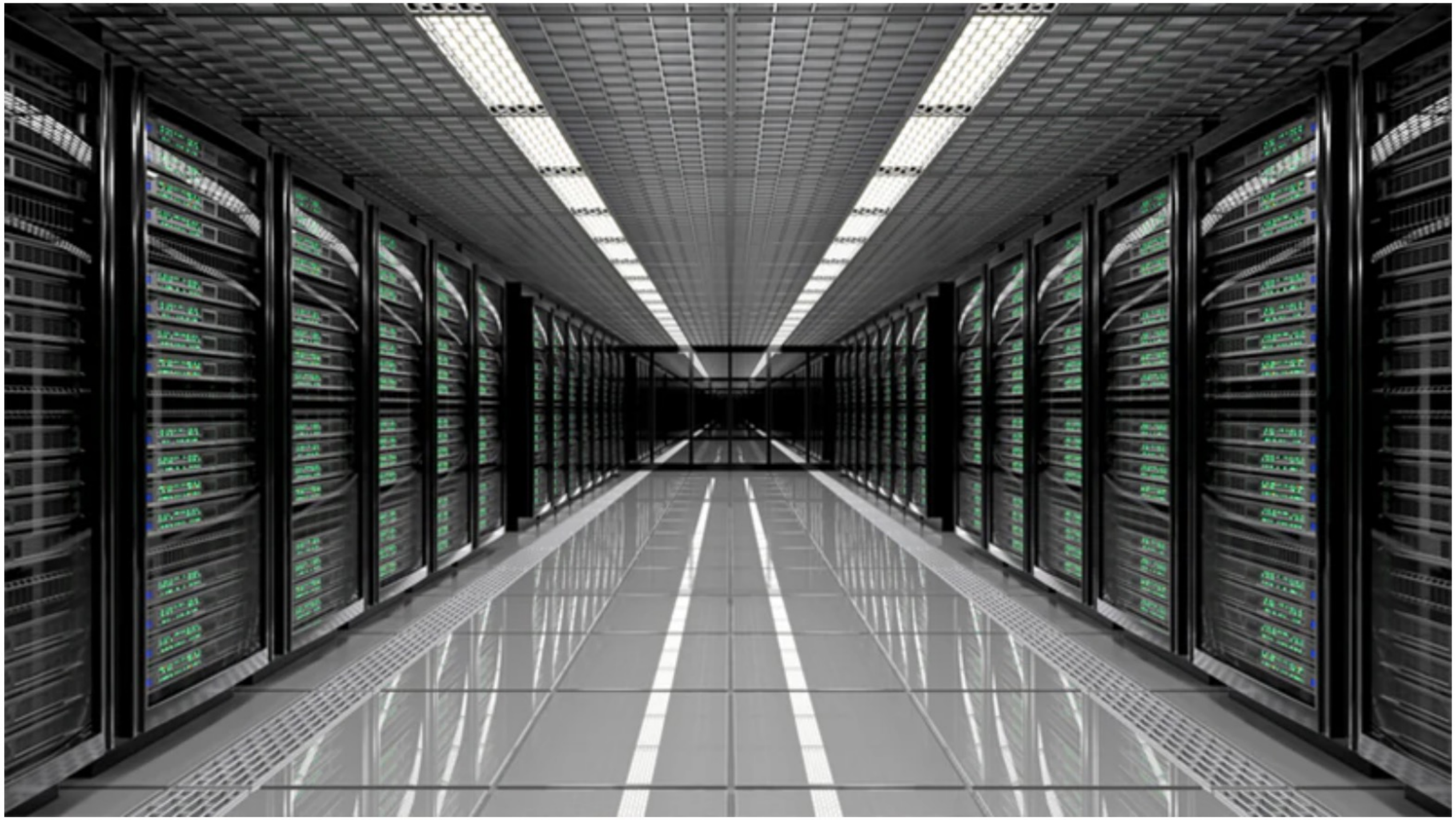
Is the mechanism of rapid thermalization, with its possible link to entanglement, a "million-dollar question"?

ARTIFICIAL INTELLIGENCE

Will the $1 trillion of generative AI investment pay off?

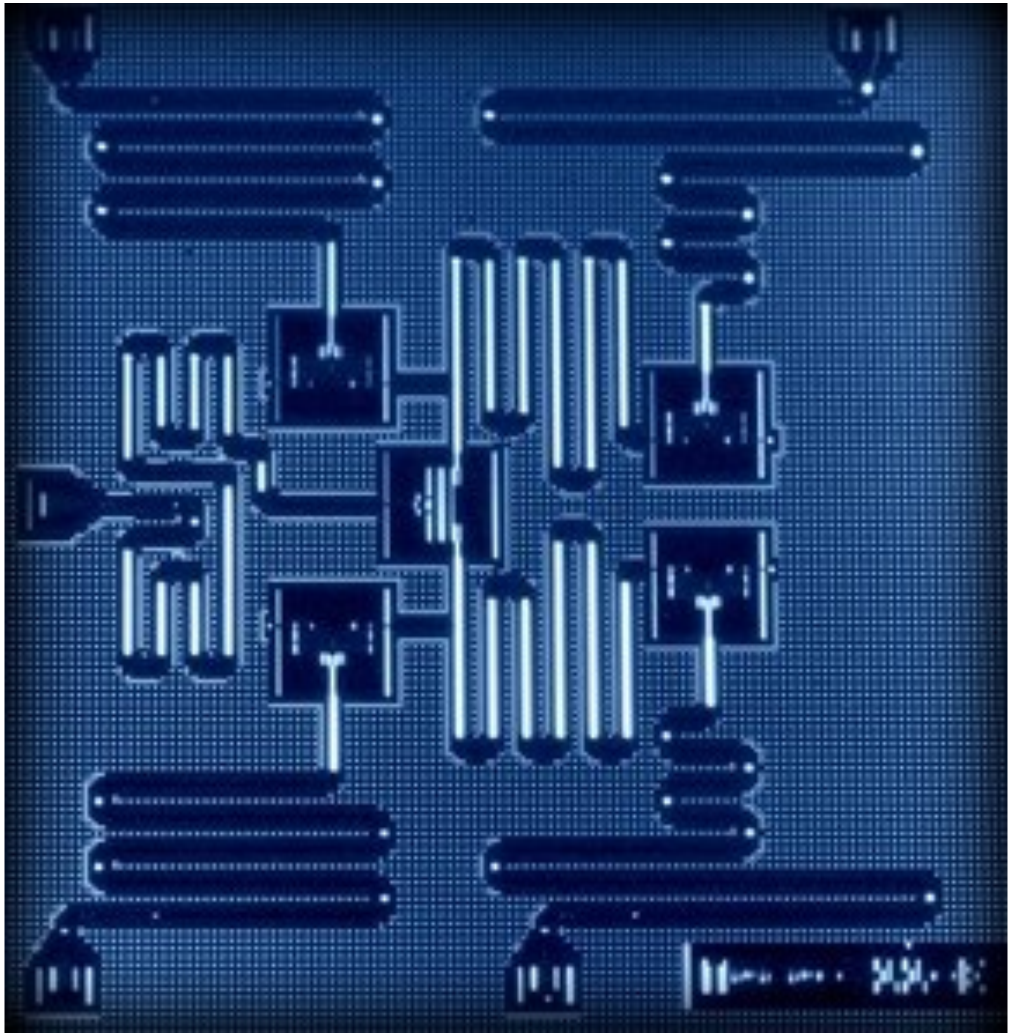
Goldman Sachs

August 5, 2024 Share

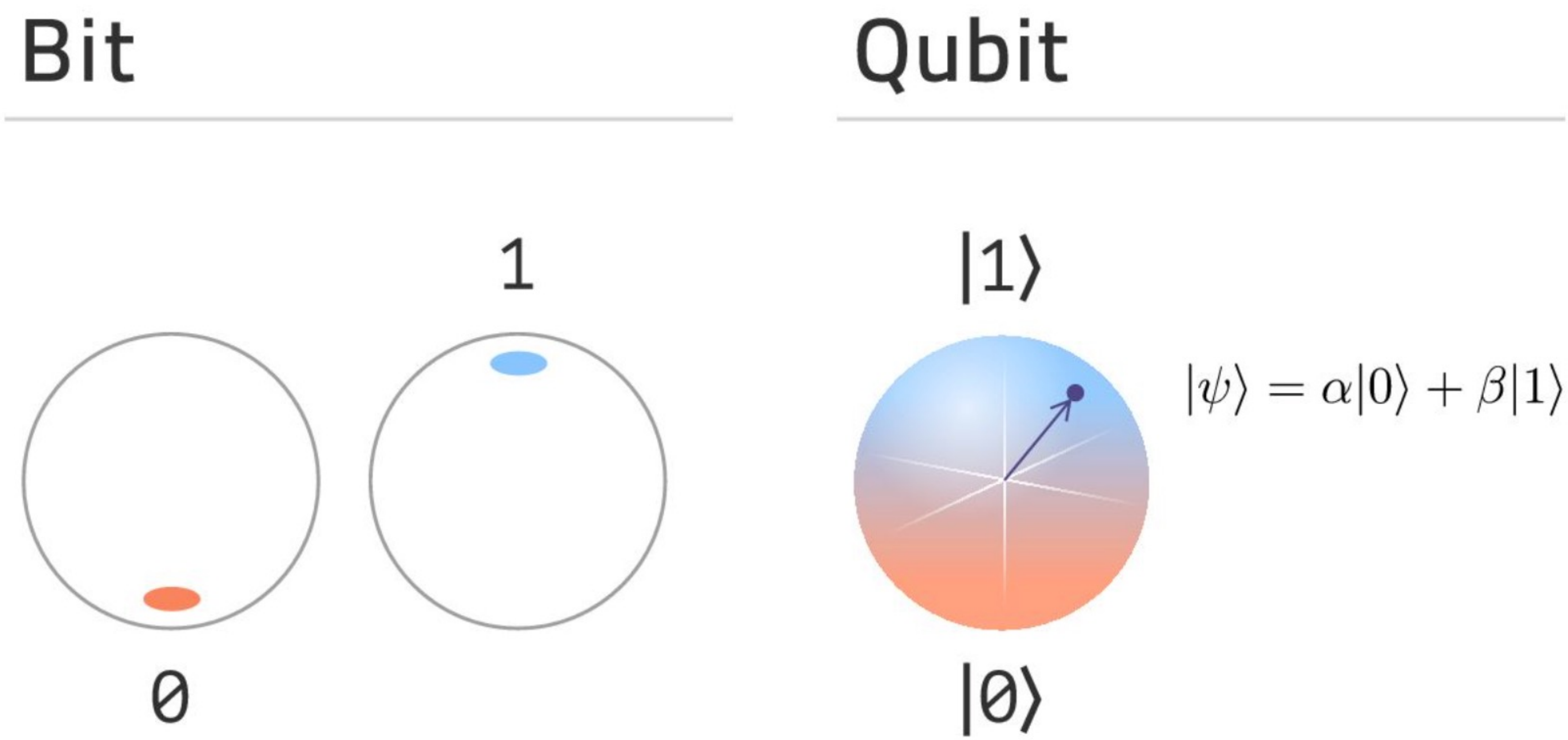


Quantum AI/ML

Entanglement at work: quantum computing



IBM five qubit processor credit: IBM-Q

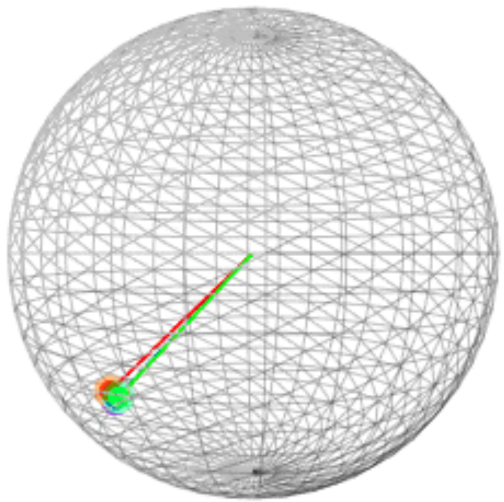


While a classical computer is made of bits, a quantum computer is made of quantum bits, or qubits for short. In contrast to classical bits, that can only be 0 or 1, qubits can exist in a superposition of these states.

Image: SINTEF

Why quantum computing?

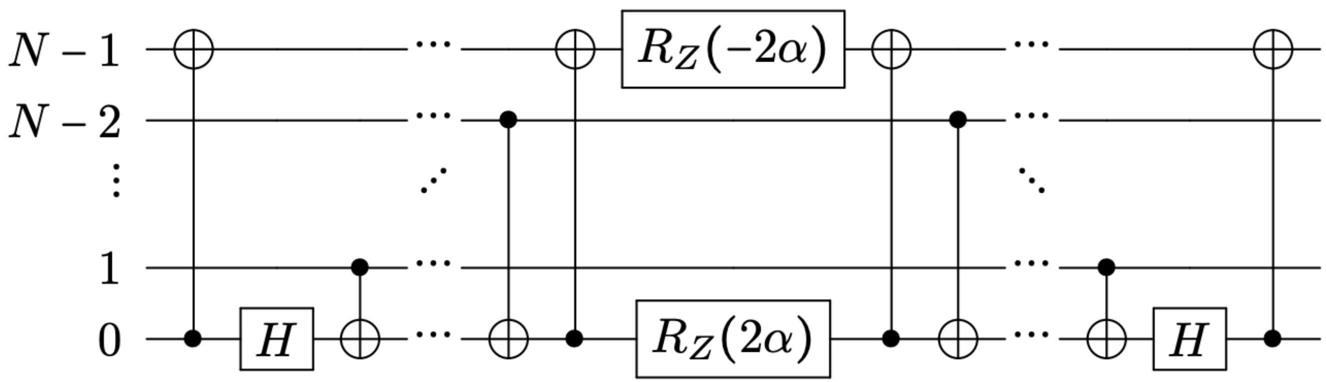
Quantum computing allows manipulation of entire vectors in Hilbert space, offering exponential speed-up:



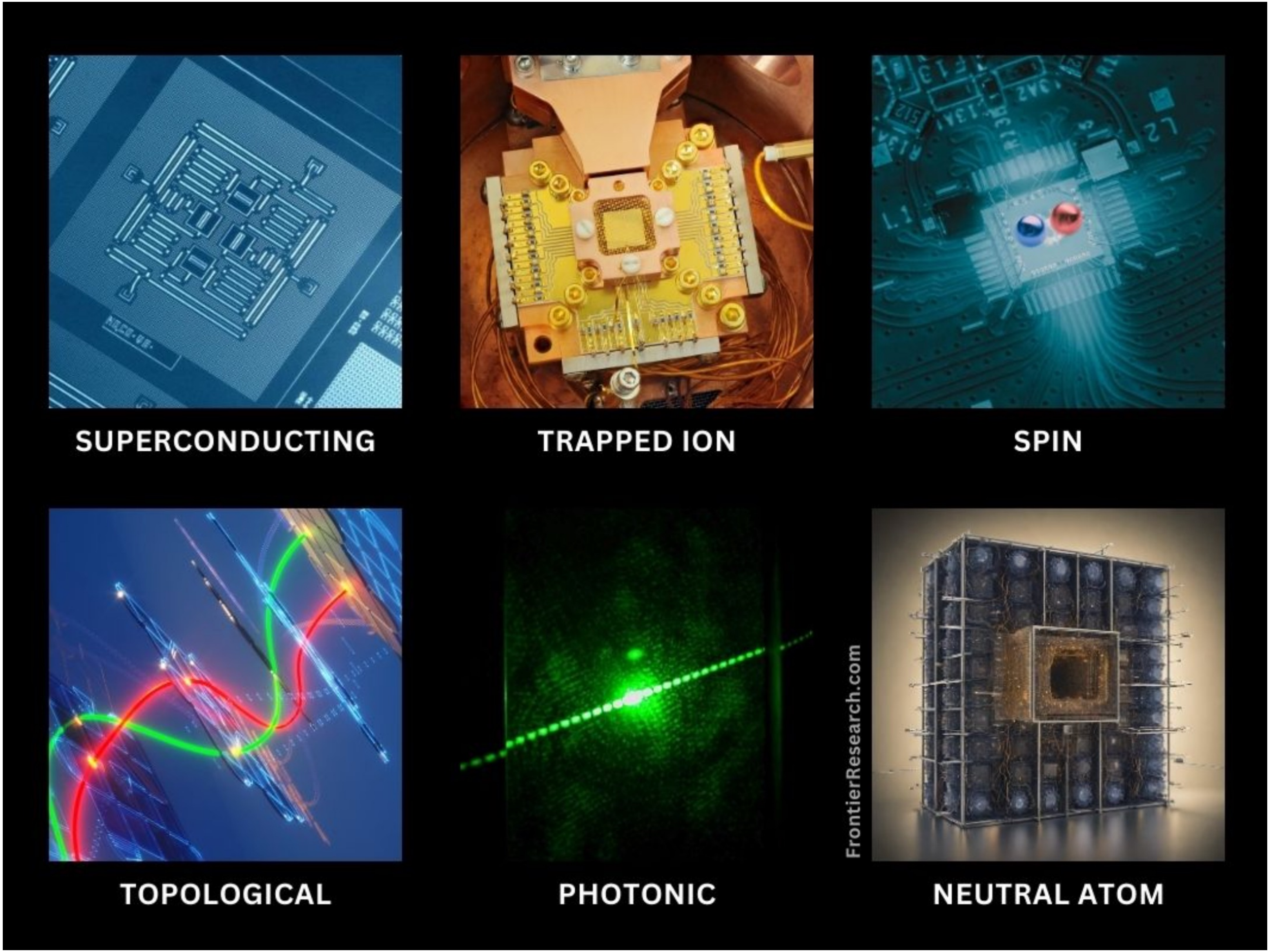
N bits – 2^N possible states; probability vector

Quantum state vector in 2^N dimensional Hilbert space

Promise of **Quantum advantage**



Circuit from:
DK, Y. Kikuchi,
Phys.Rev.Res.2(2020)


SUPERCONDUCTING
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IBM Quantum

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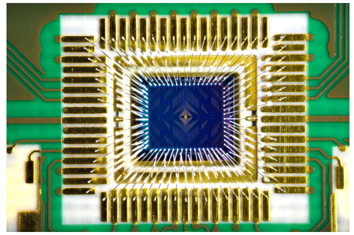
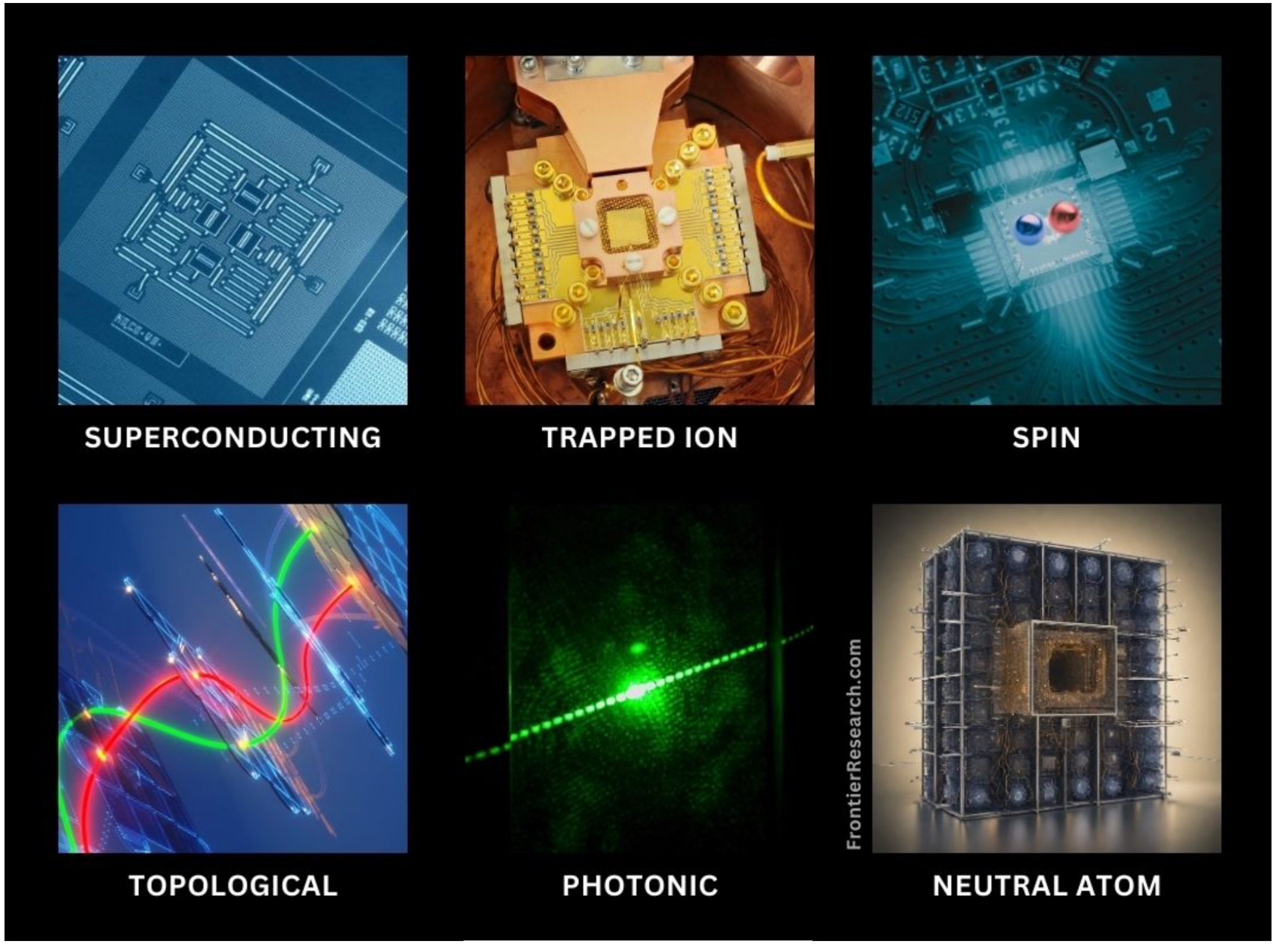


Google

Quantum AI

D:Wave
The Quantum Computing Company™

rigetti



Microsoft

QCi
QUANTUM COMPUTING INC

QUANTINUUM

Also: **Quantum-inspired classical computing** (quantum simulations on classical hardware, hybrid quantum-classical computing, …)

A recent example:

Efficient charge-preserving excited state preparation with variational quantum algorithms

Zohim Chandani[1], Kazuki Ikeda[2,3,4], Zhong-Bo Kang[5,6,7], Dmitri E. Kharzeev[3,4,8], Alexander McCaskey[9], Andrea Palermo[3], C.R. Ramakrishnan[10], Pooja Rao[11], Ranjani G. Sundaram[10], and Kwangmin Yu[12]

NVIDIA

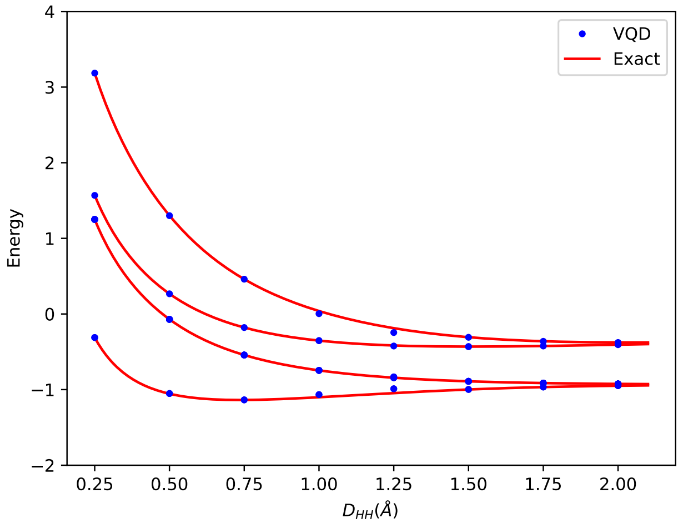


Figure 3: The bond-length dependence of the spectrum (in Hartree) of H_2 in the charge 0 sector, where all energy spectra carry charge 0 in the STO-3G basis. The red curves and the blue dots are obtained by exact diagonalization and VQD, respectively.

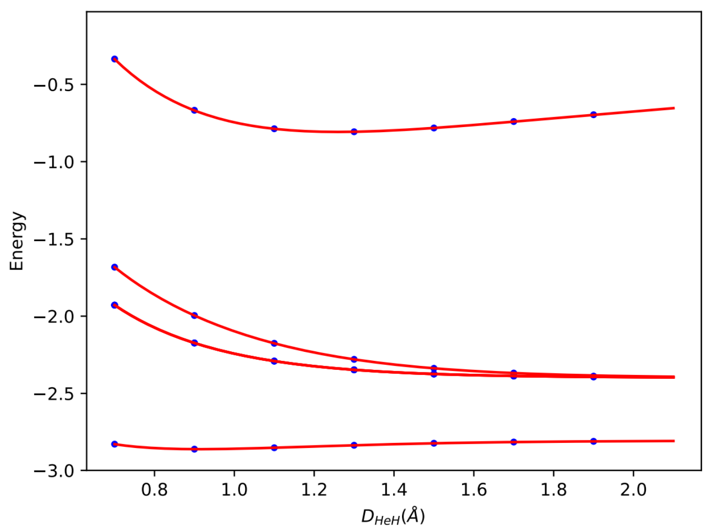
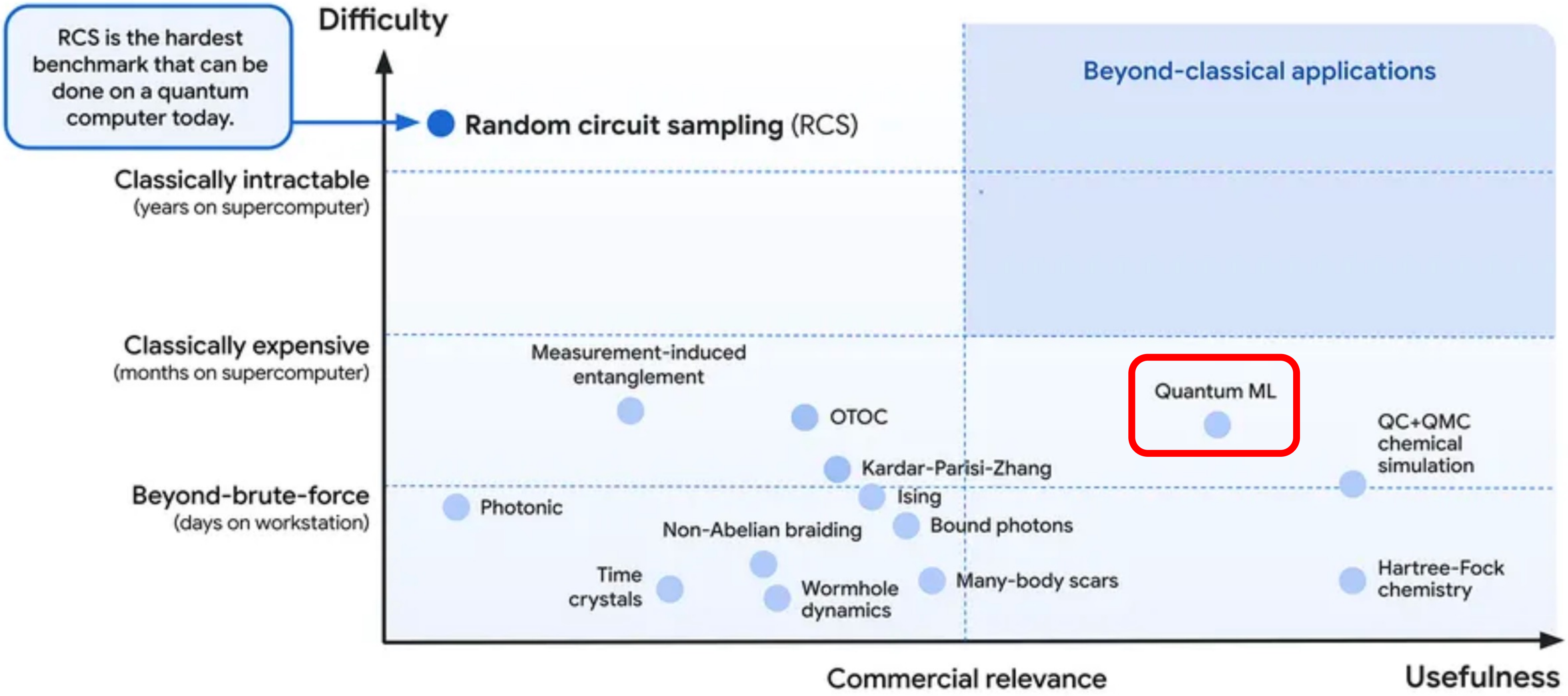
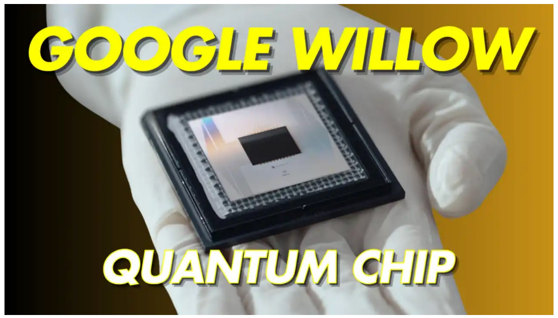


Figure 5: The bond-length dependence of the spectra (in Hartree) of HeH^+ in the charge 1 sector. All charge +1 spectra in the STO-3G basis are shown here. The red curves and the blue dots are obtained by exact diagonalization and CPVQD, respectively.





Estimated time on Willow vs classical supercomputer	5 minutes vs. 10^{25} years

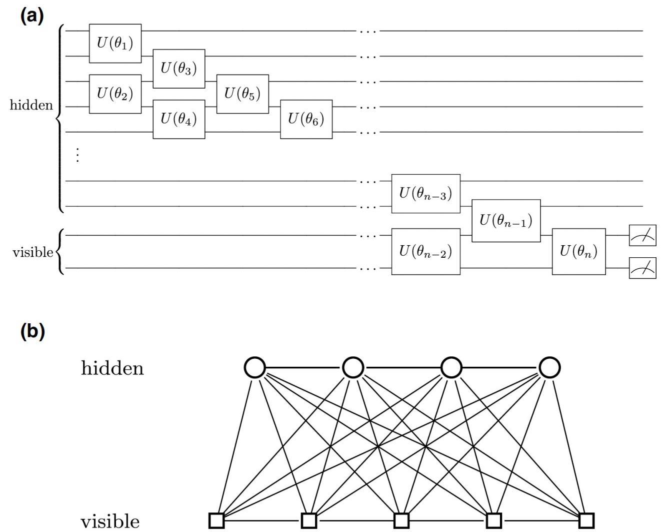
An obstacle to quantum deep neural networks:

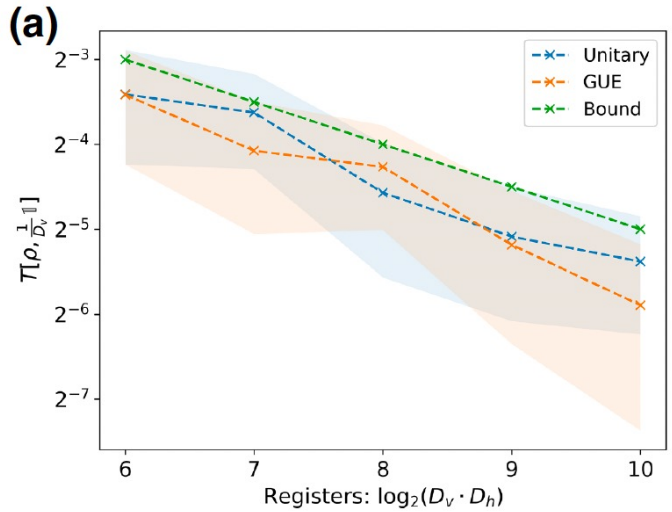
PRX QUANTUM **2,** 040316 (2021)

Entanglement-Induced Barren Plateaus

Carlos Ortiz Marrero,[1,*] Mária Kieferová,[2,†] and Nathan Wiebe[3,4,‡]

[1] *Data Sciences and Analytics Group, Pacific Northwest National Laboratory, Richland, Washington 99354, USA*
[2] *Centre for Quantum Computation and Communication Technology, Centre for Quantum Software and Information, University of Technology Sydney, New South Wales 2007, Australia*
[3] *Department of Computer Science, University of Toronto, Ontario M5S 1A1, Canada*
[4] *High Performance Computing Group, Pacific Northwest National Laboratory, Richland, Washington 99354, USA*



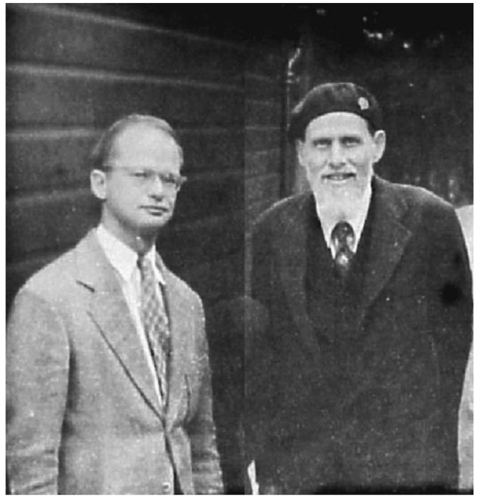


Neural networks

A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH and WALTER H. PITTS

Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying certain conditions, one can find a net behaving in the fashion it describes. It is shown that many particular choices among possible neurophysiological assumptions are equivalent, in the sense that for every net behaving under one assumption, there exists another net which behaves under the other and gives the same results, although perhaps not in the same time. Various applications of the calculus are discussed.



McCulloch (right) and Pitts (left) in 1949

Active neuron: $\sigma_i = +1$

Inactive neuron: $\sigma_i = -1$

Activation threshold: θ

Weights (synapses): J_{ij}

$$\sigma_{\mathrm{i}}(t+1) = \mathrm{sgn}\left[\sum_{\mathrm{j}\neq\mathrm{i}} J_{\mathrm{ij}}\sigma_{\mathrm{j}}(t) - \theta_{\mathrm{i}}\right]$$

Bulletin of Mathematical Biophysics, Vol. 5, 1943, p. 115-133

Review: arXiv:2412.18030

Neural networks

Hopfield 1982:

Nobel prize in Physics, 2024,
with G. Hinton ("Boltzmann machines")

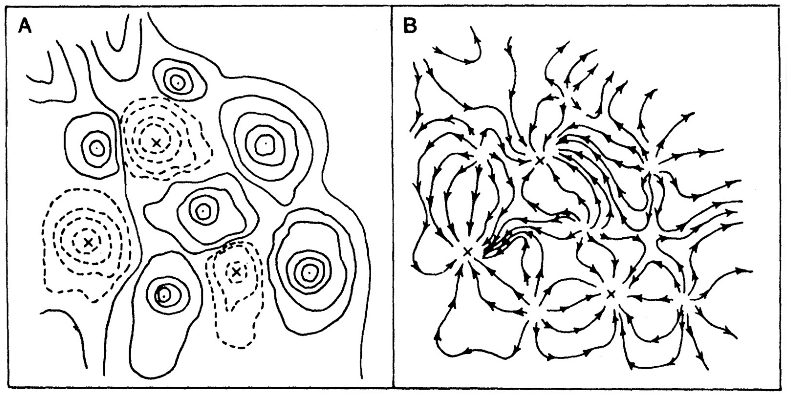
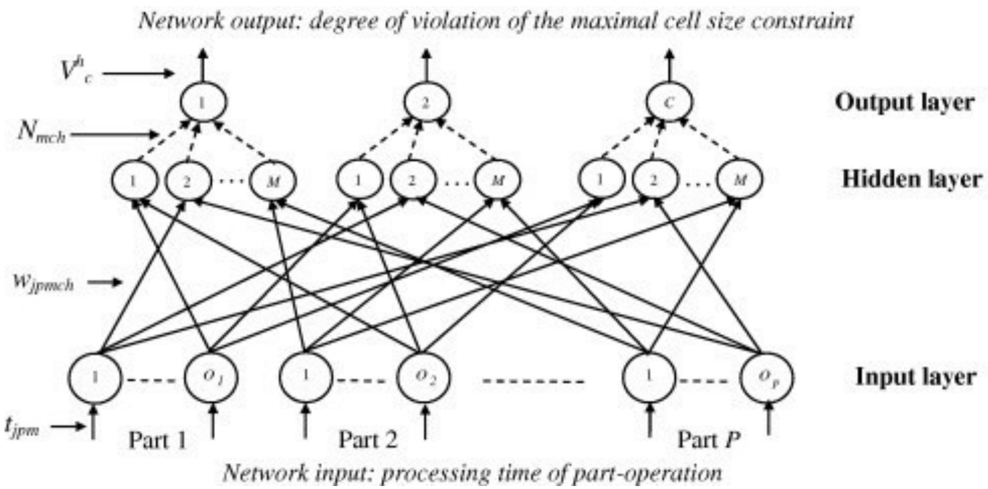


FIG. 2: Energy landscape and trajectories in a model of neural networks [39]. (A) Solid contours are above a mean level and dashed contours below, with X marking fixed points at the bottoms of energy valleys. (B) Corresponding dynamics, shown as a flow field.

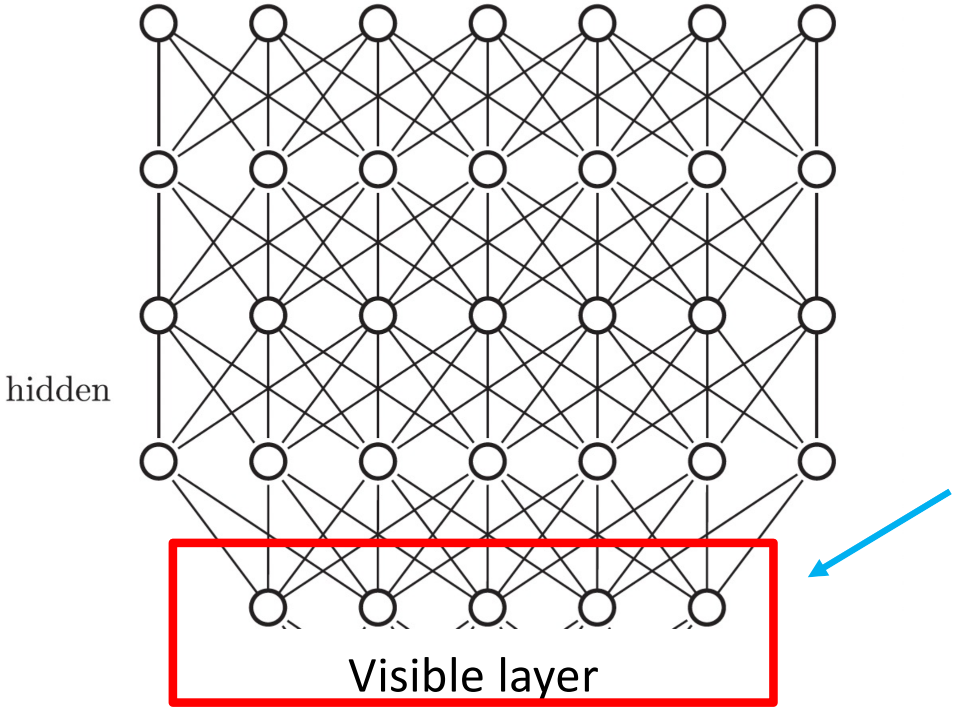
The network is sliding down on a landscape, which is an effective energy function.

"Coming to a rest at the minimum of the energy is a computation, analogous to recalling a memory."



Review: W. Bialek, arXiv:2412.18030

An obstacle to quantum deep neural networks:



Entangled pure state; entanglement entropy scales with the volume

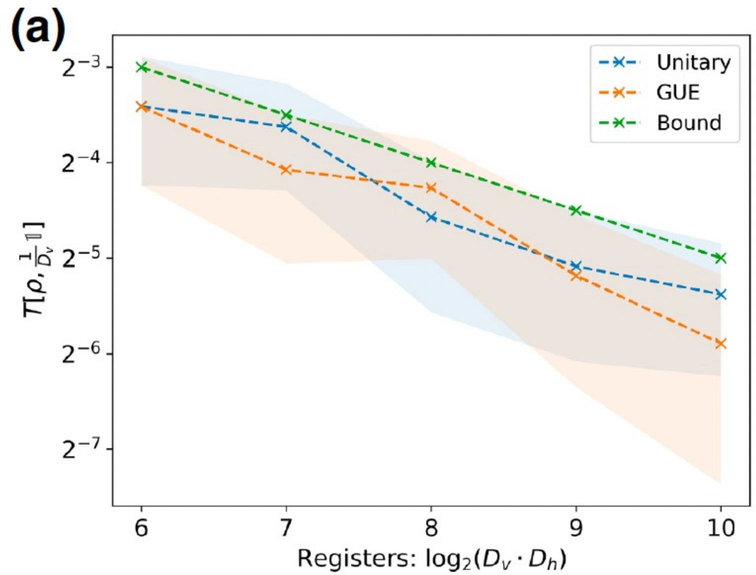
A small sub-system (the visible layer) is maximally entangled - **completely random output!**

Proposition 2. *Let $U \in \mathbb{C}^{D_v D_h \times D_v D_h}$ be drawn from a unitary 2-design and let $H = U^\dagger S U$ for some diagonal matrix $S \in \mathbb{C}^{D \times D}$, where $D = D_v D_h$. If either $\rho = U|0\rangle\langle 0| U^\dagger$ (unitary network) or $\rho = e^{-H}/Tr(e^{-H})$ (Boltzmann machine), then for any bounded operator $O_{obj} \in \mathbb{C}^{D_v \times D_v}$ acting on the visible subspace, we have that*

$$\left| Tr\left[(O_{obj} \otimes I_h)(\rho - I/D)\right]\right| \in O\left(\|O_{obj}\|_\infty \sqrt{\frac{D_v}{D_h}}\right),$$

Entanglement-Induced Barren Plateaus

Carlos Ortiz Marrero,[1,*] Mária Kieferová,[2,†] and Nathan Wiebe[3,4,‡]



What is the real-time dynamics of thermalization?
Is it accompanied by entanglement entropy (EE) production?
Can it be slowed down?

To answer these questions, we need a **real-time quantum simulation** in a field theory that is simple enough to solve numerically and still describes physical systems of interest.

Schwinger model is similar to QCD in a number of ways:
confinement, chiral condensate, anomaly, …

Perform a real-time quantum simulation of e^+e^- annihilation in massive Schwinger model, with the goal of understanding the possible **link between entanglement and thermalization**

Center for Nuclear Theory
Stony Brook University

The team:

C^{2}QA
Co-design Center for Quantum Advantage


David Frenklakh (SBU->BNL)


Adrien Florio (SBU->BNL)


Kazuki Ikeda (SBU->UMass)


Shuzhe Shi (SBU->Tsinghua)


Eliana Marroquin (SBU)


Vladimir Korepin (SBU)


Kwangmin Yu (BNL)


Sebastian Grieninger (SBU)


Andrea Palermo (SBU)

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio[1,*], David Frenklakh[2,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[4,ǁ], Shuzhe Shi[5,2,¶], and Kwangmin Yu[6,**]

Phys. Rev. Lett. **131**, 021902 – **Published 13 July, 2023**

U.S. Department of ENERGY Office of Science

Science Highlights

Quantum real-time evolution of entanglement and hadronization in jet production: Lessons from the massive Schwinger model

Adrien Florio[1,2,*], David Frenklakh[3,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[2,4,ǁ], Shuzhe Shi[5,3,¶], and Kwangmin Yu[6,**]

Phys. Rev. D **110**, 094029 – **Published 15 November, 2024**

Thermalization from quantum entanglement in the fragmentation of jets

Adrien Florio,[1,2,*] David Frenklakh,[1,†] Sebastian Grieninger,[3,2,‡]
Dmitri E. Kharzeev,[3,4,2,§] Andrea Palermo,[3,¶] and Shuzhe Shi[5,3,**]

To appear this week

Related work

PHYSICAL REVIEW D **105,** 114027 (2022)

Entanglement entropy and flow in two-dimensional QCD: Parton and string duality

Yizhuang Liu*
Institute of Theoretical Physics, Jagiellonian University, 30-348 Kraków, Poland

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Institute of Theoretical Physics and Mark Kac Complex Systems Research Center, Jagiellonian University, 30-348 Kraków, Poland

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(Received 3 April 2022; accepted 24 May 2022; published 17 June 2022)

PHYSICAL REVIEW D **105,** 114028 (2022)

Rapidity evolution of the entanglement entropy in quarkonium: Parton and string duality

Yizhuang Liu*
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Institute of Theoretical Physics and Mark Kac Center for Complex Systems Research, Jagiellonian University, 30-348 Kraków, Poland

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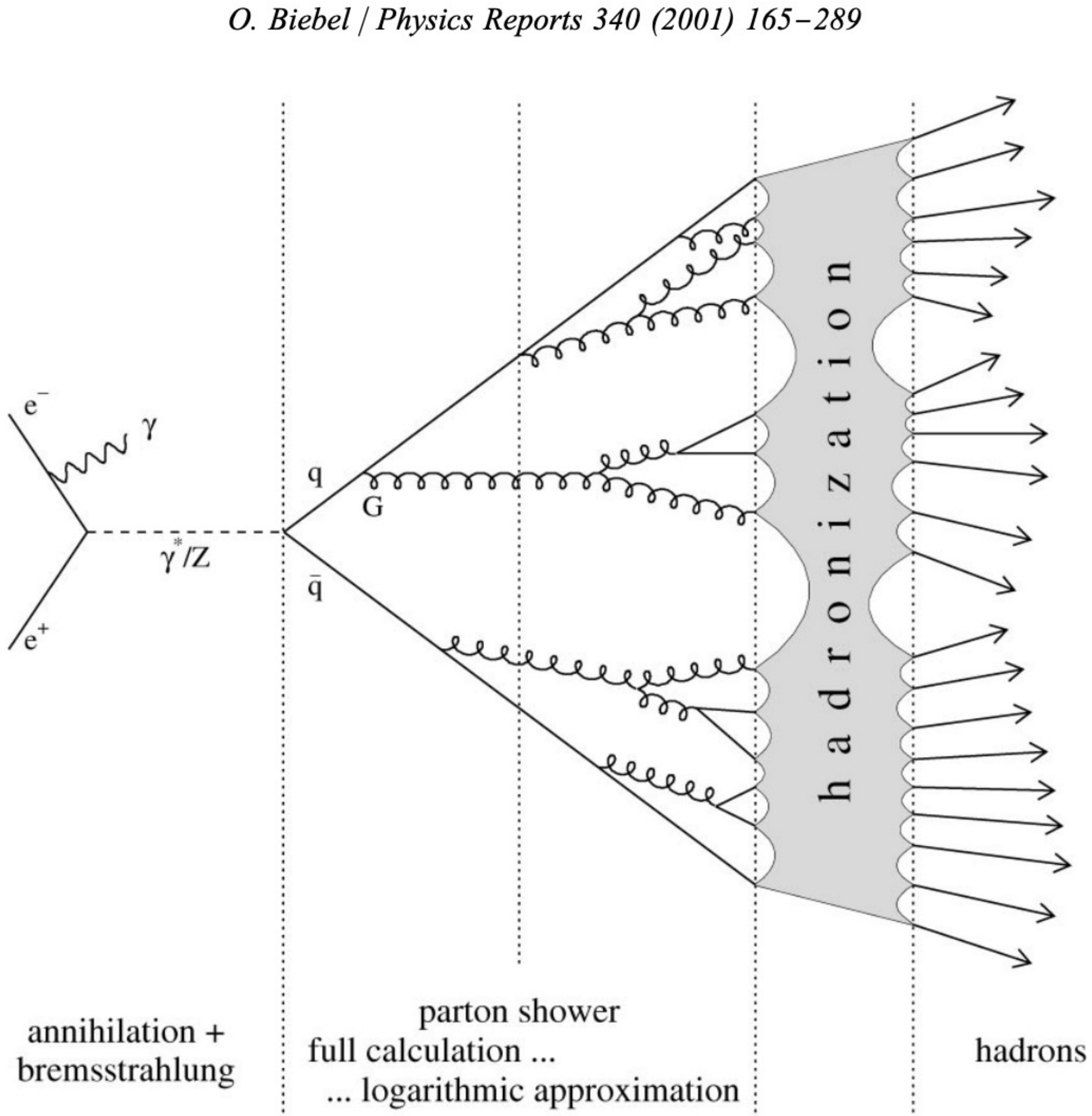
Universality and emergent effective fluid from jets and string breaking in the massive Schwinger model using tensor networks

Romuald A. Janik,[1,*] Maciej A. Nowak,[1,†] Marek M. Rams,[1,‡] and Ismail Zahed[2,§]
[1] *Institute of Theoretical Physics and Mark Kac Center for Complex Systems Research, Jagiellonian University, 30-348 Kraków, Poland*
[2] *Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794–3800, USA*
(Dated: February 19, 2025)

The setup


O. Biebel / Physics Reports 340 (2001) 165–289
e⁻
γ
q
G
γ*/Z
q̄
e⁺
h a d r o n i z a t i o n
annihilation +
bremsstrahlung
parton shower
full calculation ...
... logarithmic approximation
hadrons

Schwinger model: QED in (1+1) dimensions

$$S = \int \mathrm{d}^2x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\gamma^\mu D_\mu - m)\psi \right]$$

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Gauge Invariance and Mass. II*

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received July 2, 1962)

The possibility that a vector gauge field can imply a nonzero mass particle is illustrated by the exact solution of a one-dimensional model.

J. Schwinger
1965 Nobel prize with R. Feynman and S. Tomonaga for QED

it is plausible that some other types of excitation will then be located at fairly small fractions of m_0. Thus, one could anticipate that the known spin-0 bosons, for example, are secondary dynamical manifestations of strongly coupled primary fermion fields and vector gauge fields. This line of thought emphasizes that the question "Which particles are fundamental?" is incorrectly formulated. One should ask "What are the fundamental fields?"

ACKNOWLEDGMENTS

I have had the benefit of conversations on this and related topics with Kenneth Johnson and Charles Sommerfield.

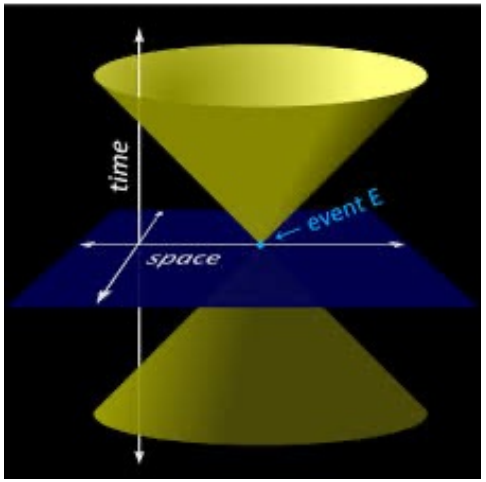
PHYSICAL REVIEW D VOLUME 10, NUMBER 2 15 JULY 1974

Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡
Tel Aviv University, Ramat-Aviv, Tel Aviv, Israel
(Received 29 June 1973; revised manuscript received 4 October 1973)

This paper is addressed to the question of why isolated quark partons are not seen. It is argued that in vector gauge theories it is possible to have the short-distance and light-cone behavior of quark fields without real quark production in deep-inelastic reactions. The physical mechanism involved is the flow of vacuum-polarization currents which neutralize any outgoing quarks. Our ideas are inspired by arguments due to Schwinger and an intuitive picture of Bjorken. Two-dimensional (1 space, 1 time) vector gauge field theories provide exactly soluble examples of this phenomenon. The resulting picture of deep-inelastic final states predicts jets of hadrons and logarithmically rising multiplicities as conjectured by Bjorken and Feynman.

Massless Schwinger model coupled to external sources:

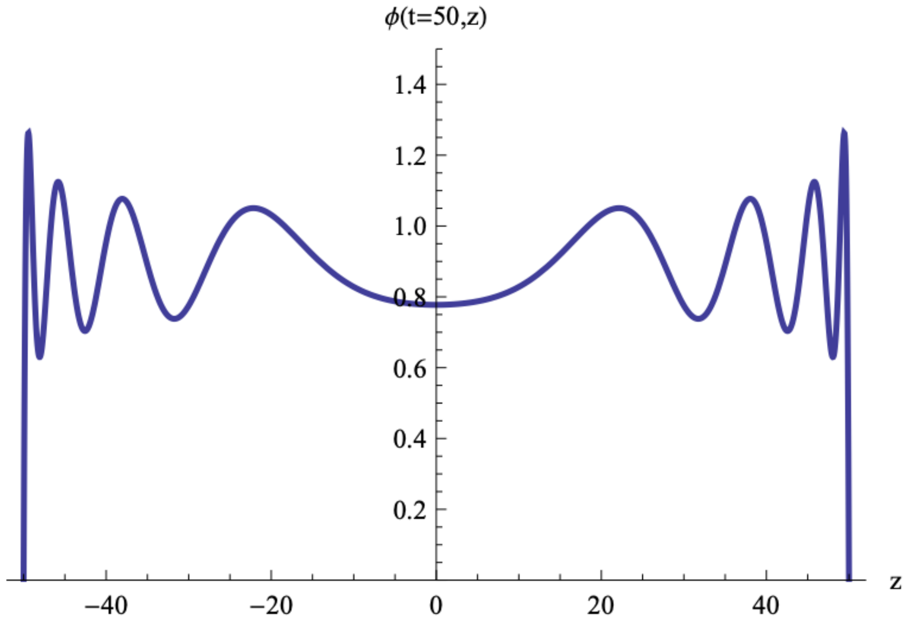


$$j_0^{\text{ext}} = g\delta(z - t), \quad j_1^{\text{ext}} = g\delta(z - t) \qquad \text{for } z > 0,$$

$$j_0^{\text{ext}} = -g\delta(z + t), \quad j_1^{\text{ext}} = g\delta(z + t) \qquad \text{for } z < 0,$$

In the massless case, can be solved exactly:

$$\phi(x) = \theta(t^2 - z^2)[1 - J_0(m\sqrt{t^2 - z^2})]$$



DK, F. Loshaj
Phys Rev D87 (2013) 7, 077501

String breaking due to production of quark-antiquark pairs;
the produced mesons form a rapidity plateau

To address thermalization, one needs to consider interacting mesons – this leads to the massive Schwinger model.

Non-integrable, no analytical solutions can be found – use digital quantum simulations!

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio, David Frenklakh, Kazuki Ikeda, Dmitri Kharzeev, Vladimir Korepin, Shuzhe Shi, and Kwangmin Yu
Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

Schwinger model: QED in (1+1) dimensions

The Hamiltonian:

$$H = \int \mathrm{d}x \left[\frac{1}{2} \left(\Pi + \frac{g\theta}{2\pi} \right)^2 + \bar{\psi}(\mathrm{i}\gamma_1 D_1 + m)\psi \right]$$

Gauge fixing, temporal gauge: $A_0 = 0.$

Generalized momentum: $\Pi = \dot{A}_1 - \frac{g\theta}{2\pi}$
(θ angle as background
electric field)

The staggered lattice Hamiltonian:

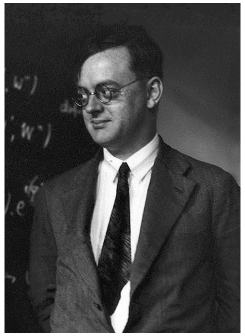
$$H_S^L = -\frac{i}{2a} \sum_{n=1}^{N-1} \left(\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n \right) + m \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n$$

$$+ \frac{ag^2}{2} \sum_{n=1}^{N-1} \left(L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t) \right)^2$$

Schwinger model: QED in (1+1) dimensions

Jordan-Wigner transformation:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\prod_{i=0}^{n-1}(-\mathrm{i}Z_i), \qquad \chi_n^\dagger = \frac{X_n + \mathrm{i}Y_n}{2}\prod_{i=0}^{n-1}\mathrm{i}Z_i,$$



Dirac	staggered	spin
$\bar\psi\psi$	$\frac{(-1)^n}{a}\chi_n^\dagger\chi_n$	$\frac{(-1)^n}{2a}Z_n$
$\bar\psi\gamma_0\psi$	$\frac{1}{a}\chi_n^\dagger\chi_n$	$\frac{1}{2a}Z_n$
$\bar\psi\gamma_1\psi$	$\frac{1}{2a}[\chi_n^\dagger\chi_{n+1} + \chi_{n+1}^\dagger\chi_n]$	$\frac{1}{4a}[X_nY_{n+1} - X_nY_{n+1}]$
$\bar\psi\gamma_5\psi$	$\frac{(-1)^n}{2a}[\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n]$	$-\frac{\mathrm{i}(-1)^n}{4a}[X_nX_{n+1} + Y_nY_{n+1}]$
$\bar\psi\gamma_1\partial_1\psi$	$-\frac{1}{2a^2}[\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n]$	$-\frac{\mathrm{i}}{4a^2}[X_nX_{n+1} + Y_nY_{n+1}]$

From:
DK and Y. Kikuchi,
Phys.Rev.Res. 2 (2020)
2, 023342

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio, David Frenklakh, Kazuki Ikeda, Dmitri Kharzeev, Vladimir Korepin, Shuzhe Shi, and Kwangmin Yu
Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

The form of Hamiltonian used in simulations:

$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}[L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t)]^2.$$

all-to-all qubit connectivity!

Warm-up: zero coupling, topological quench θ=θ(t)

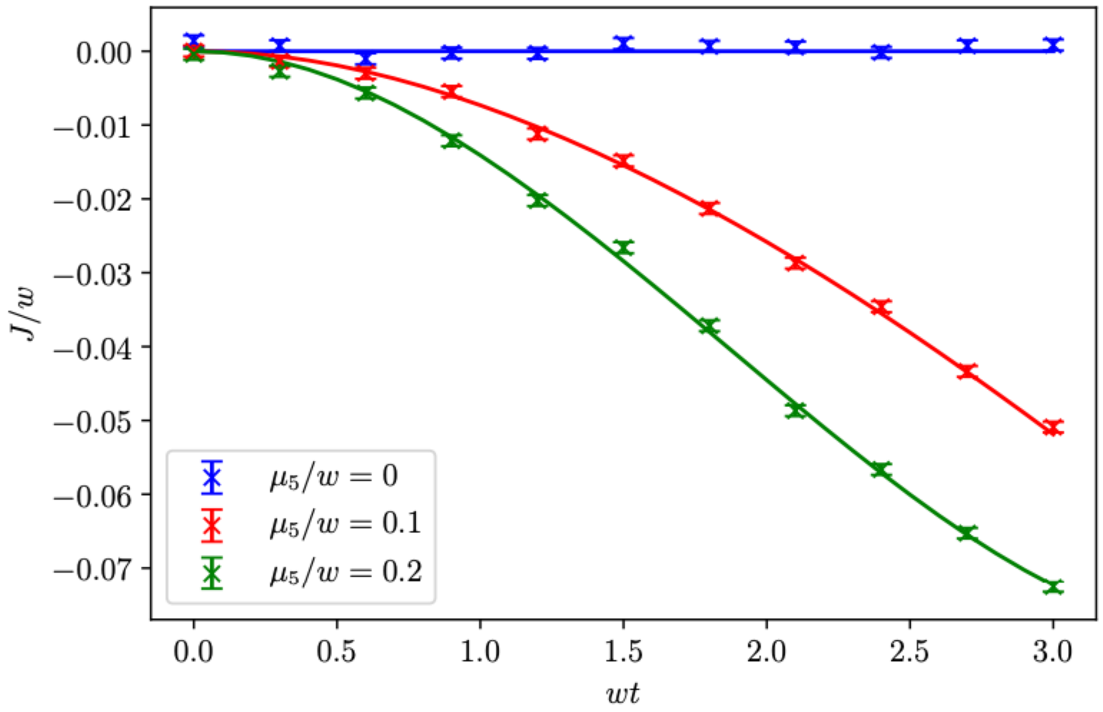
Real-time chiral dynamics from a digital quantum simulation

Dmitri E. Kharzeev[1,2,3,*] and Yuta Kikuchi[3,†]

Show more

Phys. Rev. Research **2**, 023342 – **Published 16 June, 2020**

Chiral magnetic current observed:

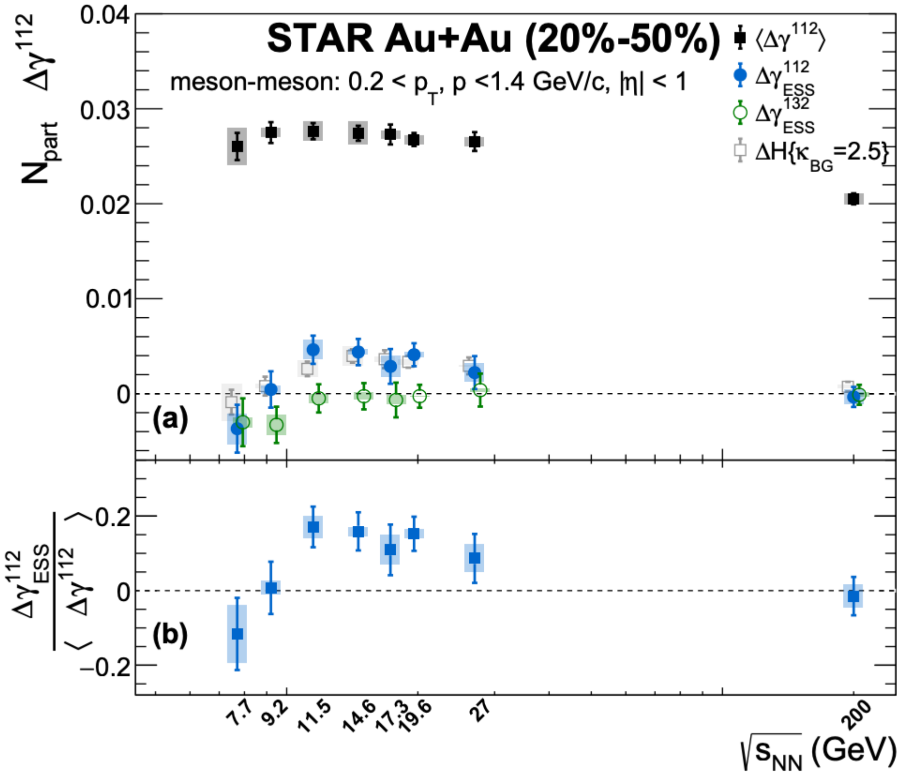


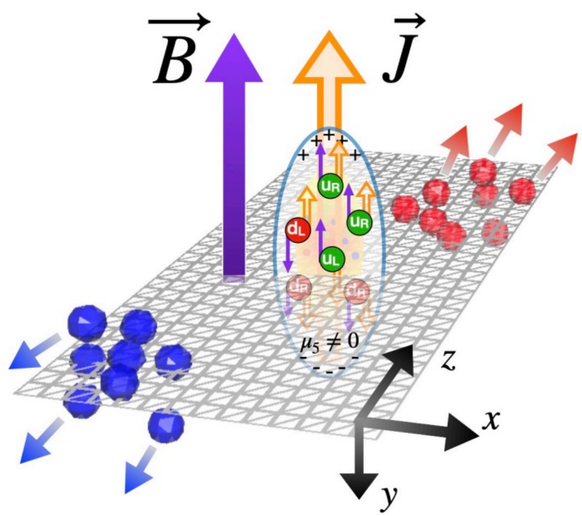
Very recent news: STAR result on CME in beam energy scan

Charge Separation Measurements in Au+Au collisions at $\sqrt{s_{NN}} = 7.7$–200 GeV in Search of the Chiral Magnetic Effect

The STAR Collaboration

arXiv:2506.00275





Charge Separation Measurements in Au+Au collisions at $\sqrt{s_{NN}}$ = 7.7–200 GeV in Search of the Chiral Magnetic Effect

The STAR Collaboration

arXiv:2506.00275

In summary, we have presented measurements of charge separation correlations along the magnetic field direction using Au+Au collisions at RHIC from $\sqrt{s_{NN}}$= 7.7 to 200 GeV energies, with the flow-related background effectively suppressed. We report a remaining charge separation signal in mid-central Au+Au collisions, positive finite with around 3σ significance at each of the center-of-mass energies of $\sqrt{s_{NN}}$= 11.5, 14.6, and 19.6 GeV. The results at $\sqrt{s_{NN}}$= 17.3 and 27 GeV also show positive values but with a lower significance of 1.3σ and 1.1σ. Below $\sqrt{s_{NN}}$= 10 GeV or at $\sqrt{s_{NN}}$= 200 GeV, the charge separation is consistent with zero. When the data between $\sqrt{s_{NN}}$= 10 and 20 GeV are combined, the significance rises to 5.5σ. The absence of a definitive CME signal from the top RHIC energy and the LHC energies [42, 77] can constrain the dynamical evolution of the magnetic field in the QGP phase in these collisions.

Why CME at BES?

One reason is the longer-living magnetic field.

But there may also be another, revealed through
the real-time simulations: enhancement of topological fluctuations
near the critical point:

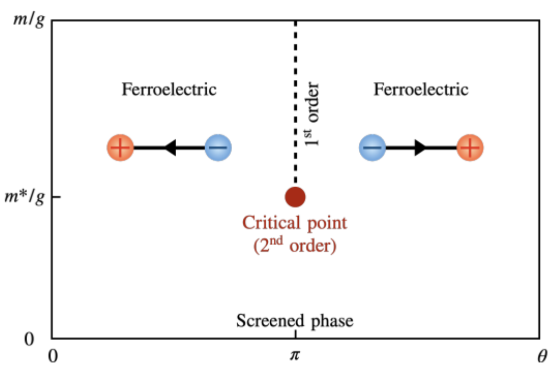
LETTER | OPEN ACCESS

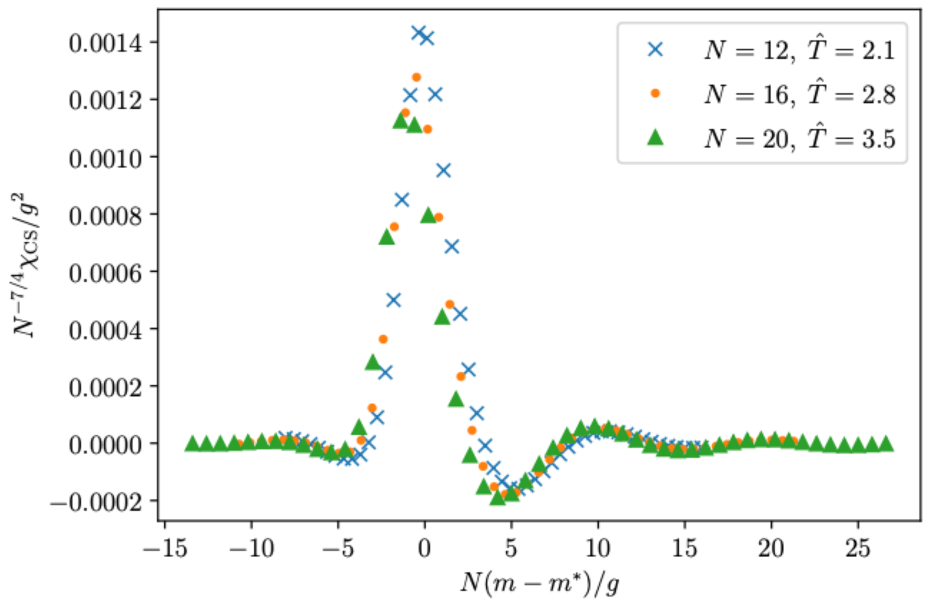
Real-time dynamics of Chern-Simons fluctuations near a critical point

Kazuki Ikeda[1,*], Dmitri E. Kharzeev[2,3,4,†], and Yuta Kikuchi[3,‡]

Show more

Phys. Rev. D **103**, L071502 – **Published 21 April, 2021**





Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

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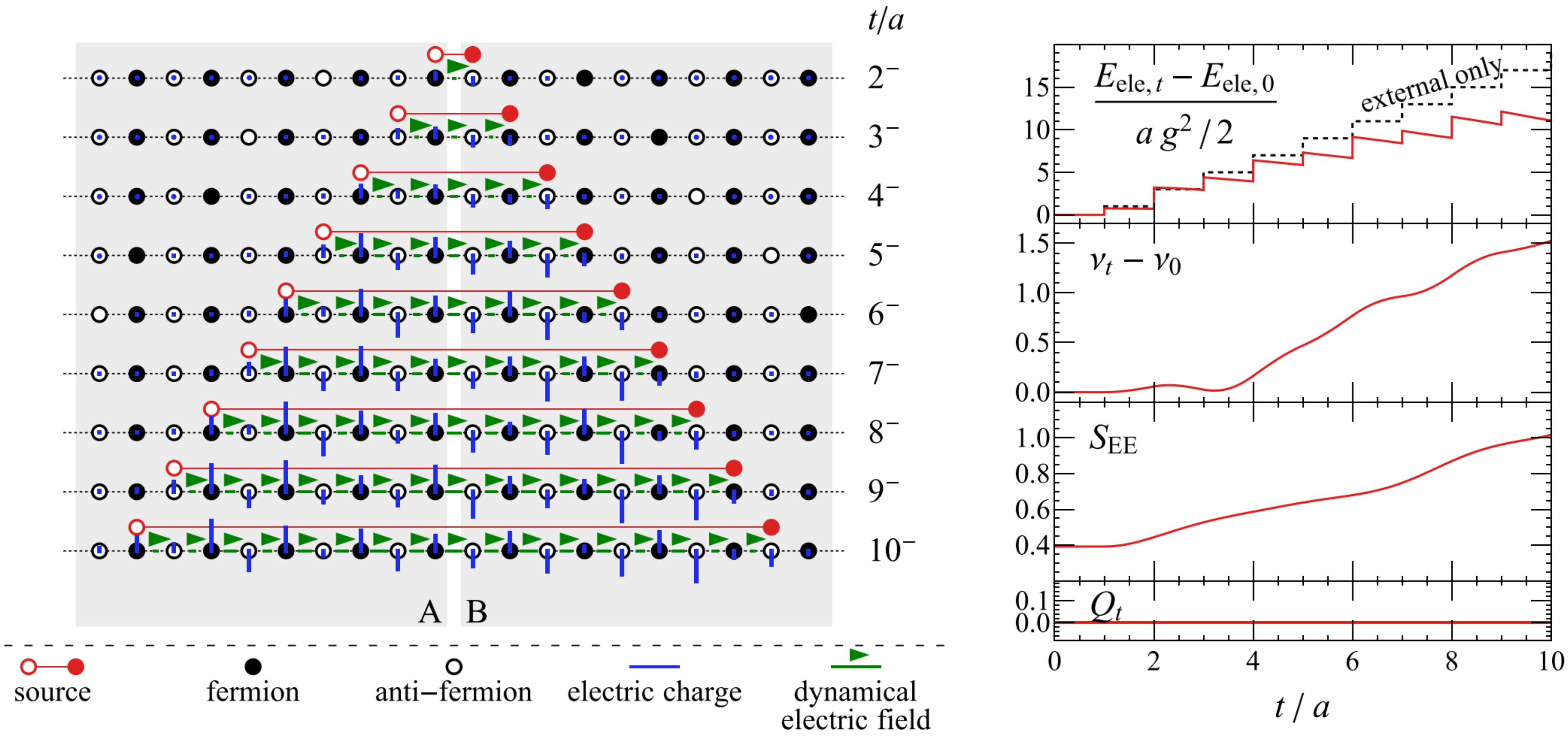
Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

PHYSICAL REVIEW LETTERS **131,** 021902 (2023)



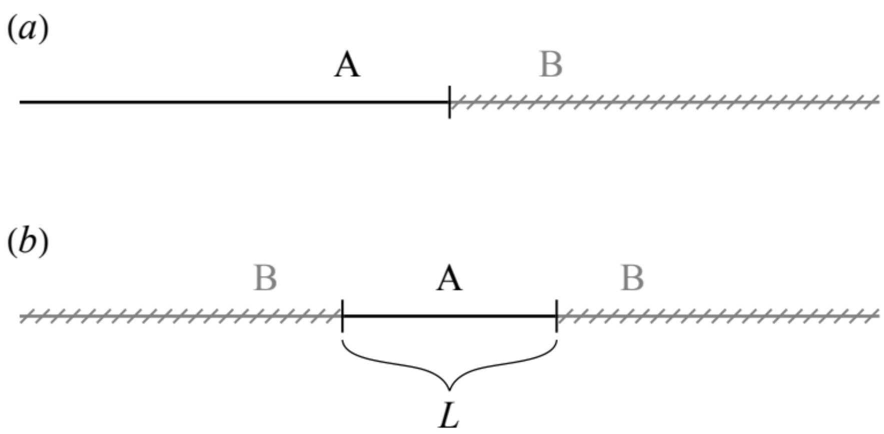
Screening of electric field, modification of the vacuum, growth of entanglement entropy!

What can we do to understand a possible approach to thermalization in our system?

Quantum simulation of entanglement and hadronization in jet production: lessons from the massive Schwinger model

Adrien Florio,[1,2,*] David Frenklakh,[3,†] Kazuki Ikeda,[2,3,‡] Dmitri Kharzeev,[1,2,3,§] Vladimir Korepin,[2,4,¶] Shuzhe Shi,[3,5,**] and Kwangmin Yu[6,††]

Let us start by examining the entanglement spectrum:



Entanglement among the quark and antiquark jet

Entanglement among the central region and the rest of the system

The entanglement spectrum

$$S_{EE}(t) = -\mathrm{Tr}_L[\rho_L(t)\ln\rho_L(t)] = -\sum_{i=1}^{2^{N/2}} \lambda_i \ln\lambda_i.$$

$$\rho_L(t) = \mathrm{Tr}_R\rho(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t)|\psi_i^L(t)\rangle\langle\psi_i^L(t)|,$$

At late times, a huge number of entanglement eigenstates start to contribute, with comparable eigenvalues – approach to the **maximal entanglement and thermalization**?

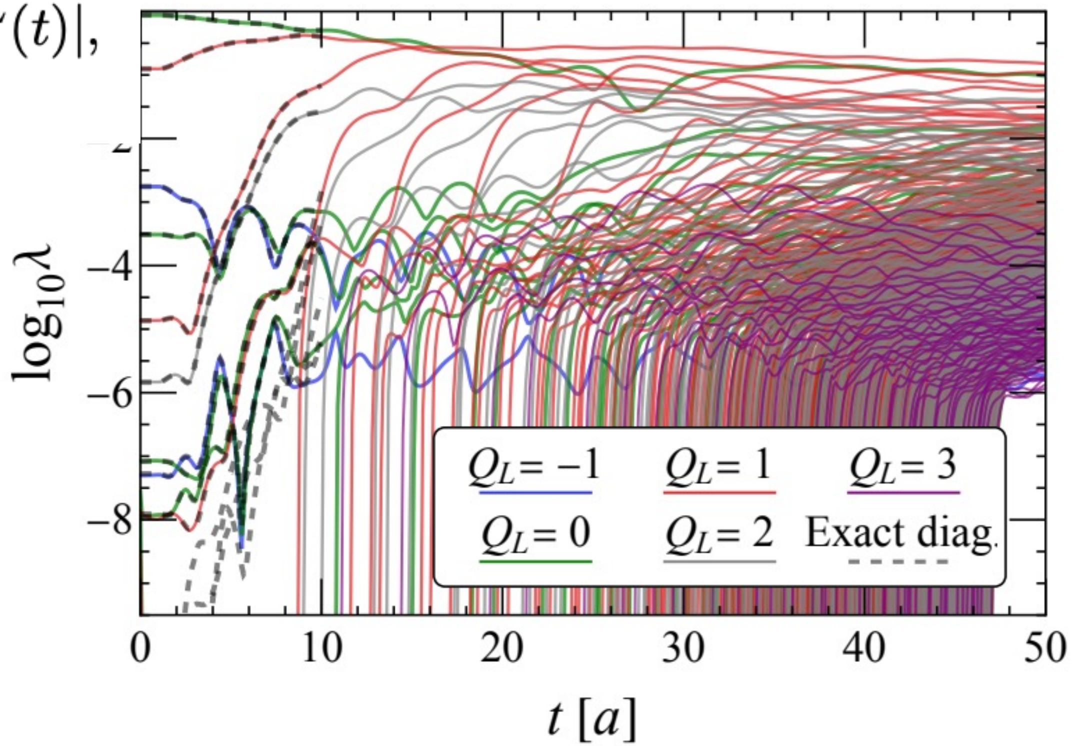


FIG. 2. Symmetry-resolved entanglement spectrum evolution for the lattice size $N = 100$, $m = 1/(4a), g = 1/(2a)$. For comparison the spectrum obtained with exact diagonalization for $N = 20$ at the same mass and coupling is shown as dashed curves.

Tests of maximal entanglement

Renyi entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}.$$

"Entangleness"

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}.$$

$$\mathcal{E}[\mathrm{MES}] = 1.$$

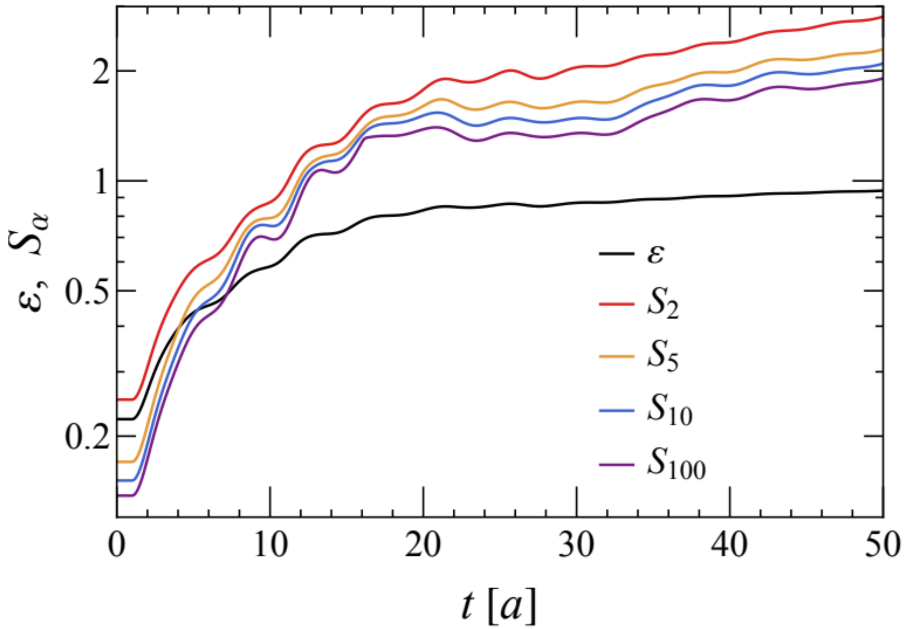
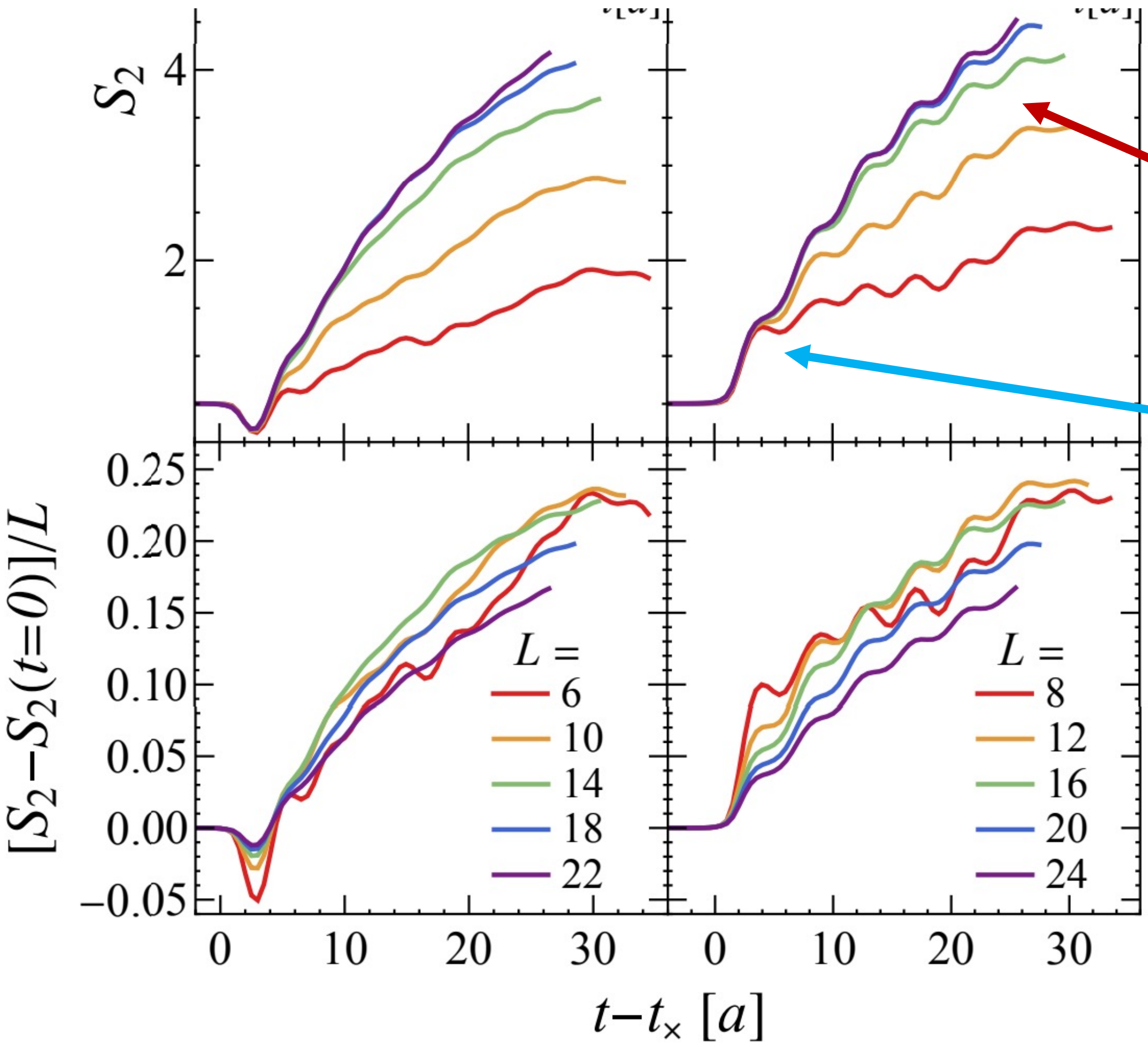


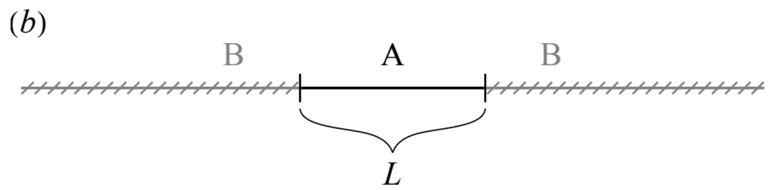
FIG. 3. Entangleness (black) and Rényi entropy with $\alpha = 2$ (red), 5 (gold), 10 (blue), and 100 (purple).

Approach to
maximal entanglement!
(in a subspace of
the full Hilbert space)

Transition from area to volume law scaling of entanglement

2nd Renyi entropy:



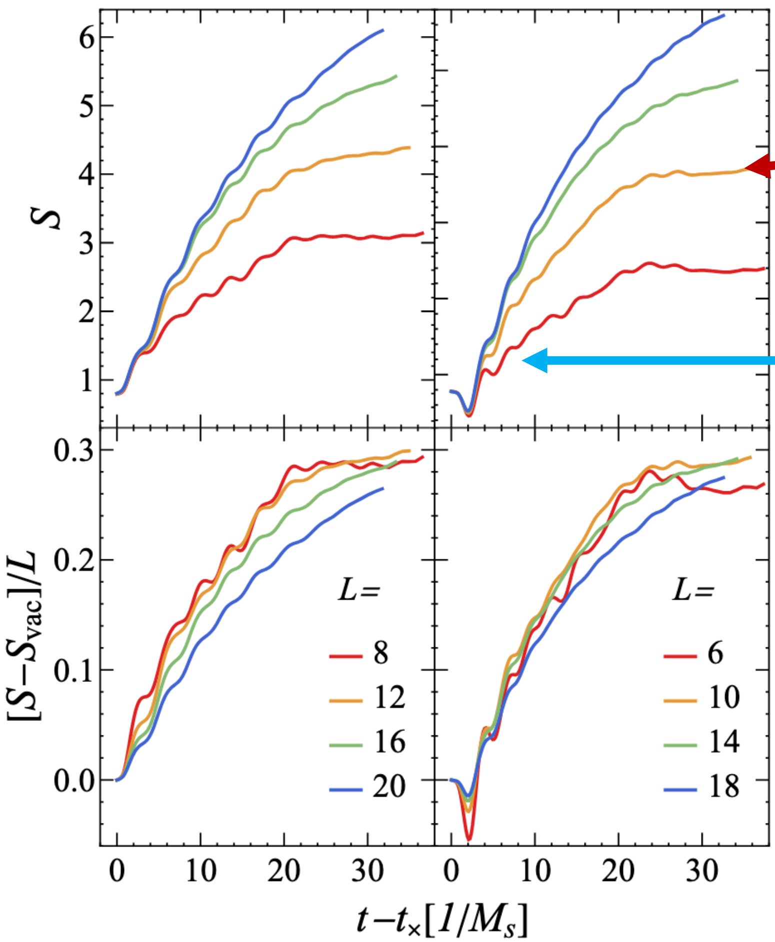


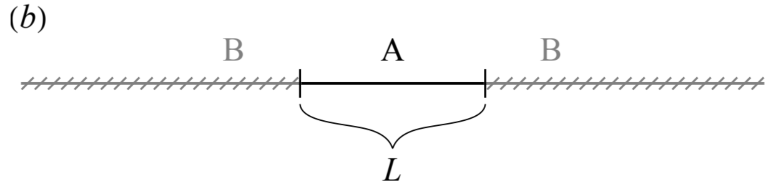
Volume law (~ L)

Area law
(L independent)

Transition from area to volume law scaling of entanglement

von Neumann entropy:





Volume law (~ L)

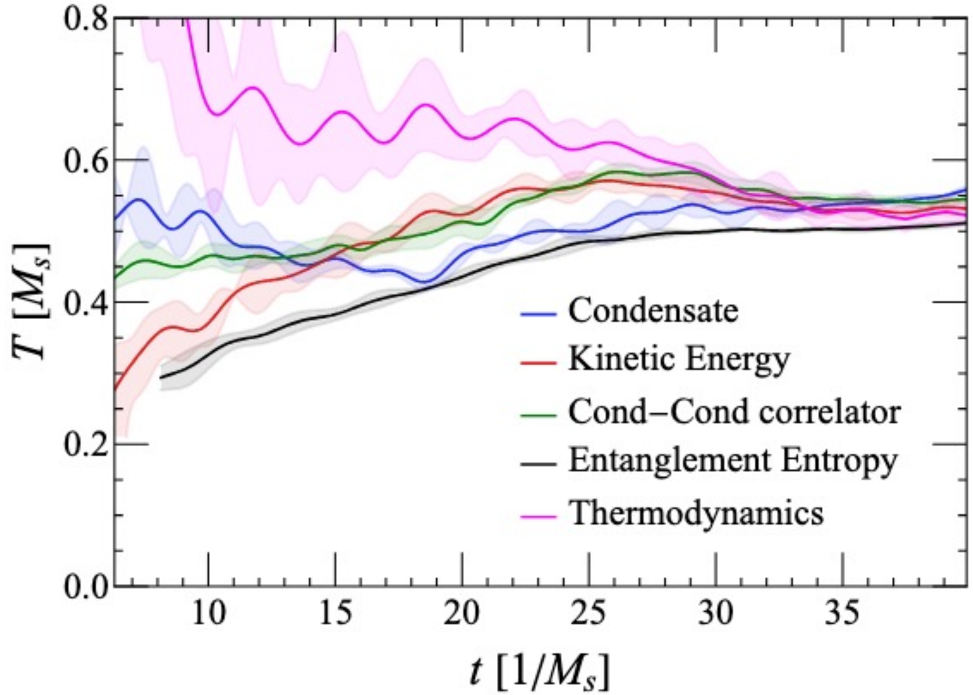
Area law
(L independent)

Thermalization from quantum entanglement in the fragmentation of jets

Adrien Florio,[1,2,*] David Frenklakh,[1,†] Sebastian Grieninger,[3,2,‡]
Dmitri E. Kharzeev,[3,4,2,§] Andrea Palermo,[3,¶] and Shuzhe Shi[5,3,**]

arXiv:2506.14983

Expectation values of local operators approach the thermal ones



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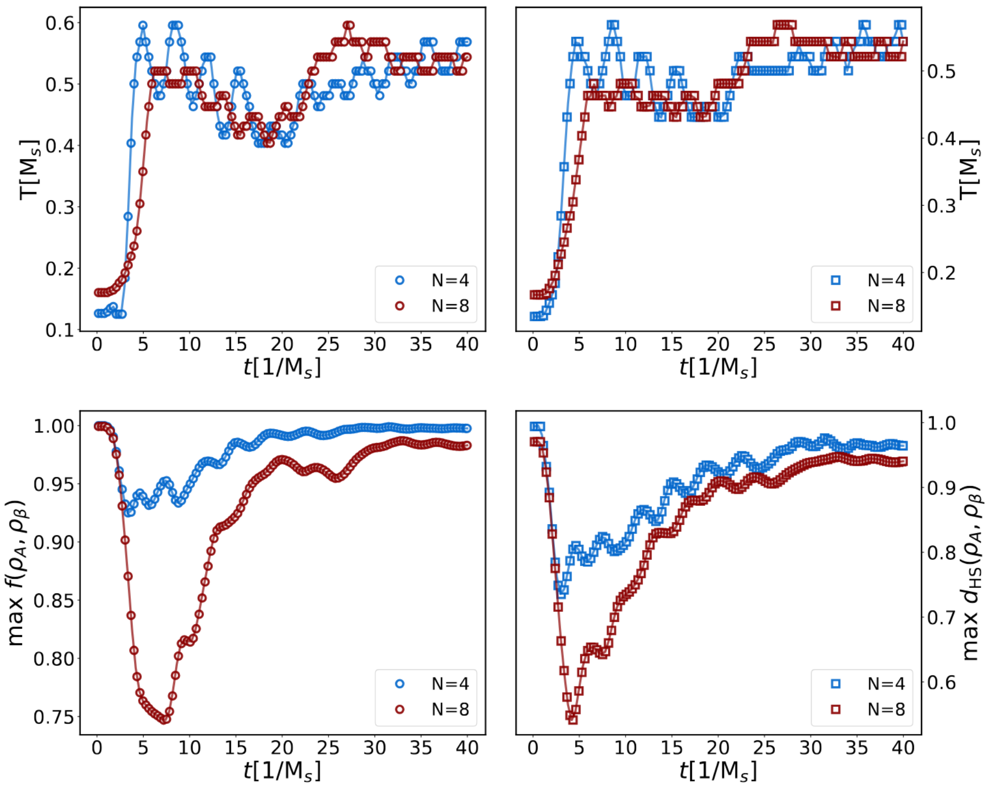
arXiv:2506.14983

Overlap with thermal density matrix approach the thermal ones



Overlap:

$$f(\rho_A, \rho_\beta) \equiv \frac{\mathrm{Tr}(\rho_A\, \rho_\beta)}{\sqrt{\mathrm{Tr}\, \rho_A^2\, \mathrm{Tr}\, \rho_\beta^2}}$$

Hilbert-Schmidt distance:

$$D_{\mathrm{HS}}(\rho_A, \rho_\beta) = \sqrt{\mathrm{Tr}[(\rho_A - \rho_\beta)^2]}.$$

$$d_{\mathrm{HS}}(\rho_A, \rho_\beta) = 1 - D_{\mathrm{HS}}(\rho_A, \rho_\beta)$$

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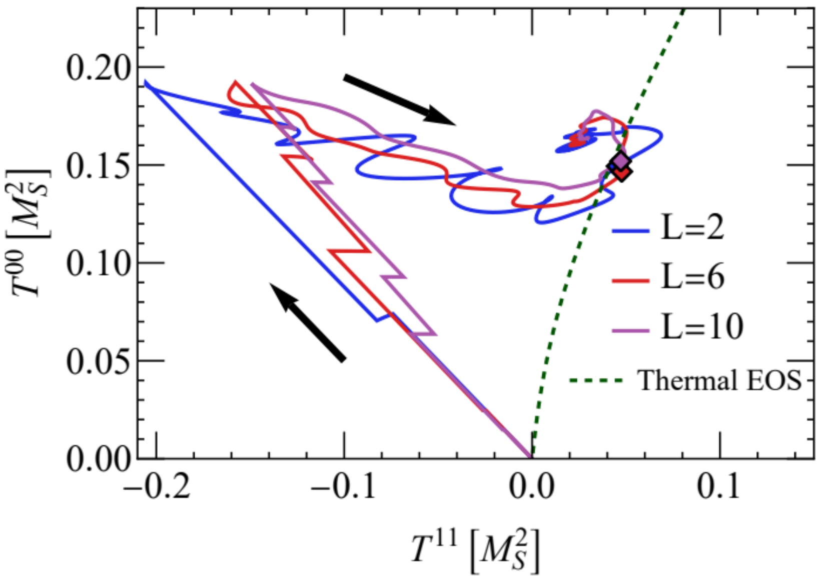
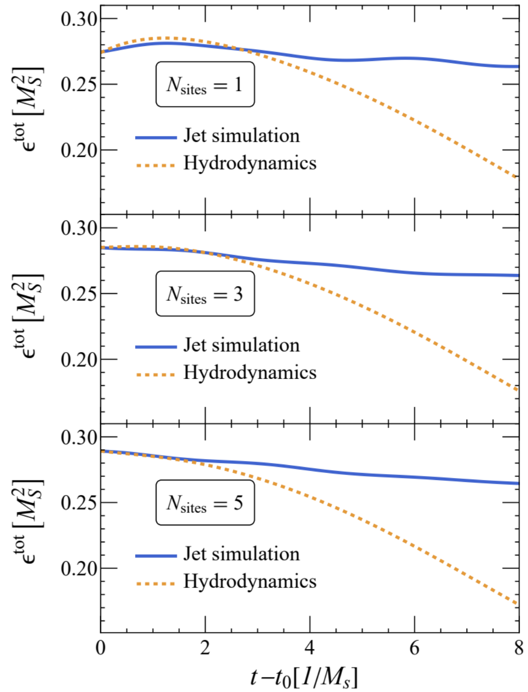
The onset of hydrodynamics

Negative pressure at early times:



Hydro works once the pressure is positive and before “freeze-out”:



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Dmitri E. Kharzeev,[3,4,2,§] Andrea Palermo,[3,¶] and Shuzhe Shi[5,3,**]

arXiv:2506.14983

Related work

PHYSICAL REVIEW D **105,** 114027 (2022)

Entanglement entropy and flow in two-dimensional QCD: Parton and string duality

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(Received 3 April 2022; accepted 24 May 2022; published 17 June 2022)

PHYSICAL REVIEW D **105,** 114028 (2022)

Rapidity evolution of the entanglement entropy in quarkonium: Parton and string duality

Yizhuang Liu[*]
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Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794–3800, USA

(Received 3 April 2022; accepted 24 May 2022; published 17 June 2022)

Universality and emergent effective fluid from jets and string breaking in the massive Schwinger model using tensor networks

Romuald A. Janik,[1, *] Maciej A. Nowak,[1, †] Marek M. Rams,[1, ‡] and Ismail Zahed[2, §]
[1] *Institute of Theoretical Physics and Mark Kac Center for Complex Systems Research, Jagiellonian University, 30-348 Kraków, Poland*
[2] *Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794–3800, USA*

(Dated: February 19, 2025)

The physical meaning of Schmidt states

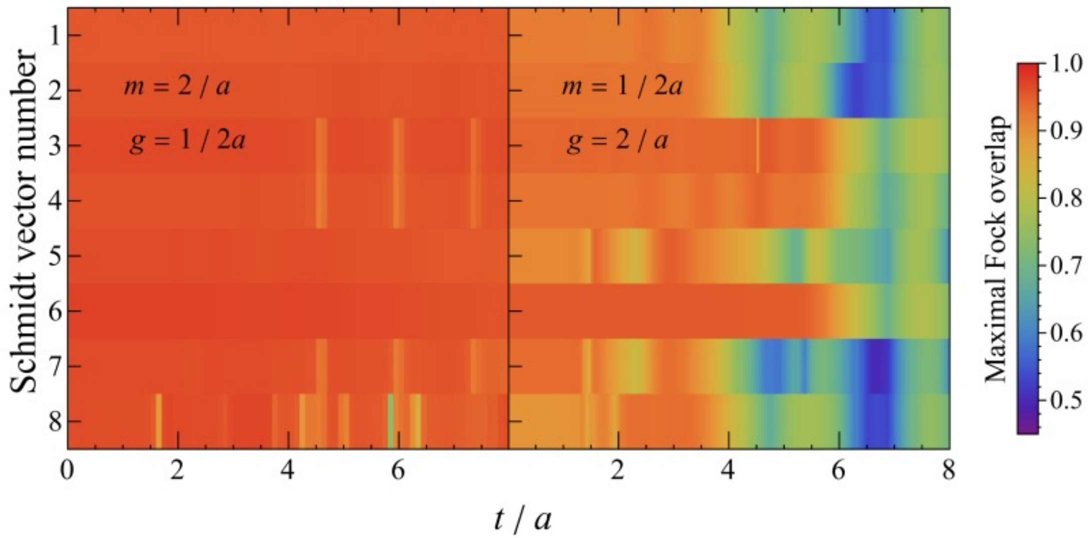

FIG. 5. Maximal overlap of each Schmidt vector with any Fock state. Comparison between $m = 2/a, g = 1/(2a)$ on the left panel and $m = 1/(2a), g = 2/a$ on the right panel is shown. In both cases, $N = 16$. To study continuous evolution, we choose to consider the 8 leading Schmidt vectors in the vacuum state at $t = 0$ and follow their evolution. Because of the level crossing in Schmidt spectrum, at later times these vectors are not necessarily the 8 leading Schmidt vectors.

Transition from "quark-antiquark" states at early times to "mesons" at late times –

Hadronization seen in real time!

Entanglement as a probe of hadronization

Jaydeep Datta,[1,*] Abhay Deshpande,[1,2,†] Dmitri E. Kharzeev,[3,4,‡] Charles Joseph Naïm,[1,§] and Zhoudunming Tu[5,¶]

[1] *Center for Nuclear Frontiers in Nuclear Science, Department of Physics and Astronomy, Stony Brook University, New York 11794-3800, USA*
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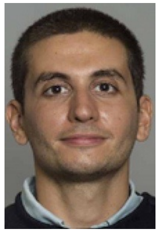
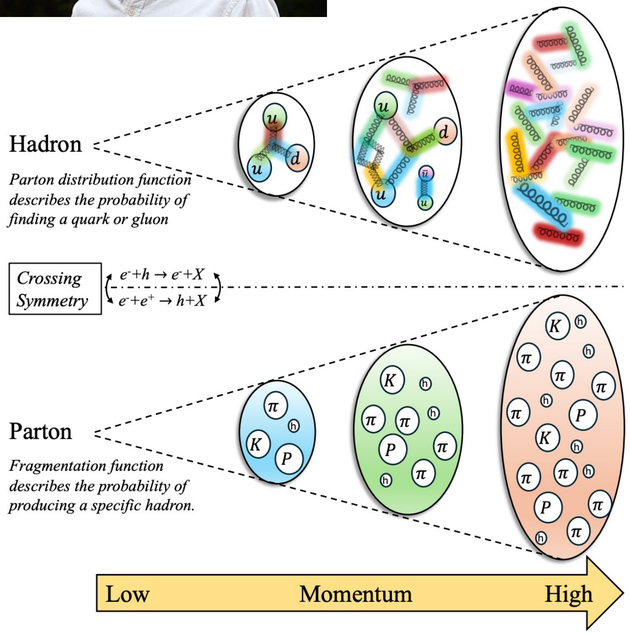
(Dated: October 30, 2024)



arXiv:2410.22331, Phys. Rev. Lett.(2025)



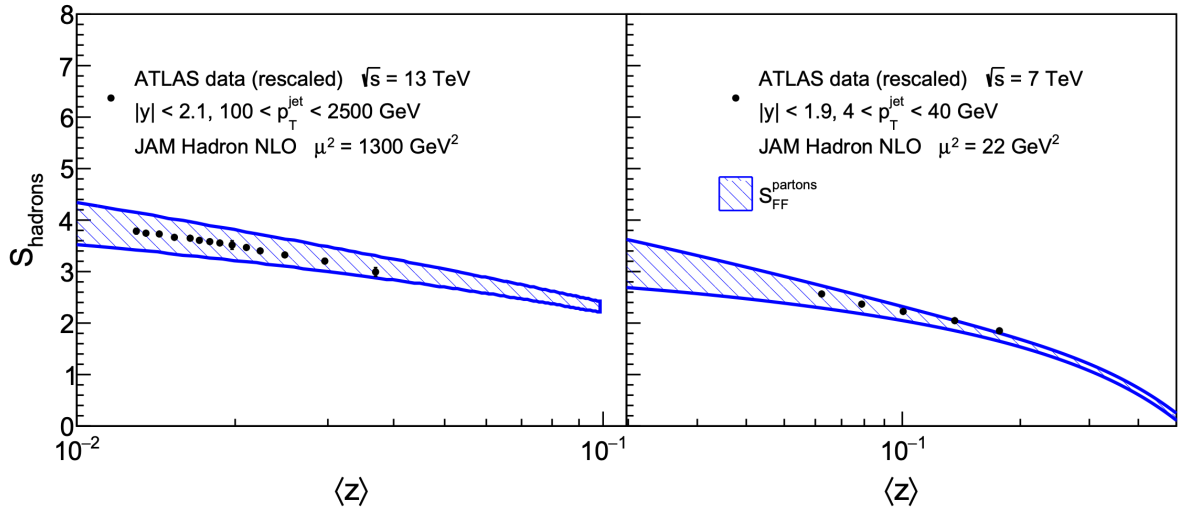
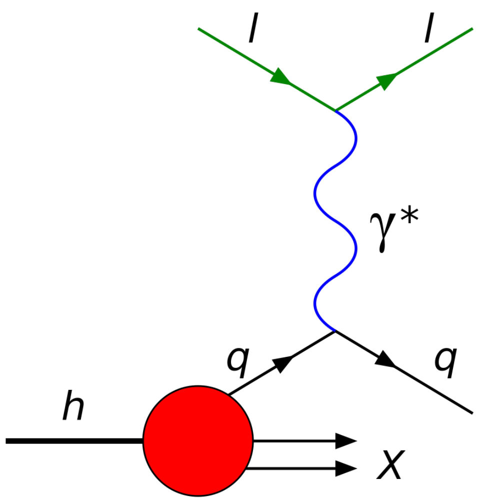


FIG. 3. The entropy $S_{\rm hadrons}$ as a function of $\langle z\rangle$ for $S_{\rm FF}^{\rm partons}$ — incorporating gluons, u-(anti)quarks, and d-(anti)quarks — is shown using JAM fragmentation functions at NLO for $\mu^2 = 1300$ GeV^2, compared with ATLAS data at $\sqrt{s} = 13$ TeV [45] (left). Additionally, the results at $\mu^2 = 22$ GeV^2 are compared with ATLAS data at $\sqrt{s} = 7$ TeV [43] (right). The uncertainties are calculated at the 1σ level. The total entropy $S_{\rm FF}^{\rm partons}$ is derived from the sum of the individual entropies of each parton, with each contribution normalized by the average fraction of jets produced by that parton from PYTHIA simulation.

Evidence for maximal entanglement from jet fragmentation

The puzzle of the parton model



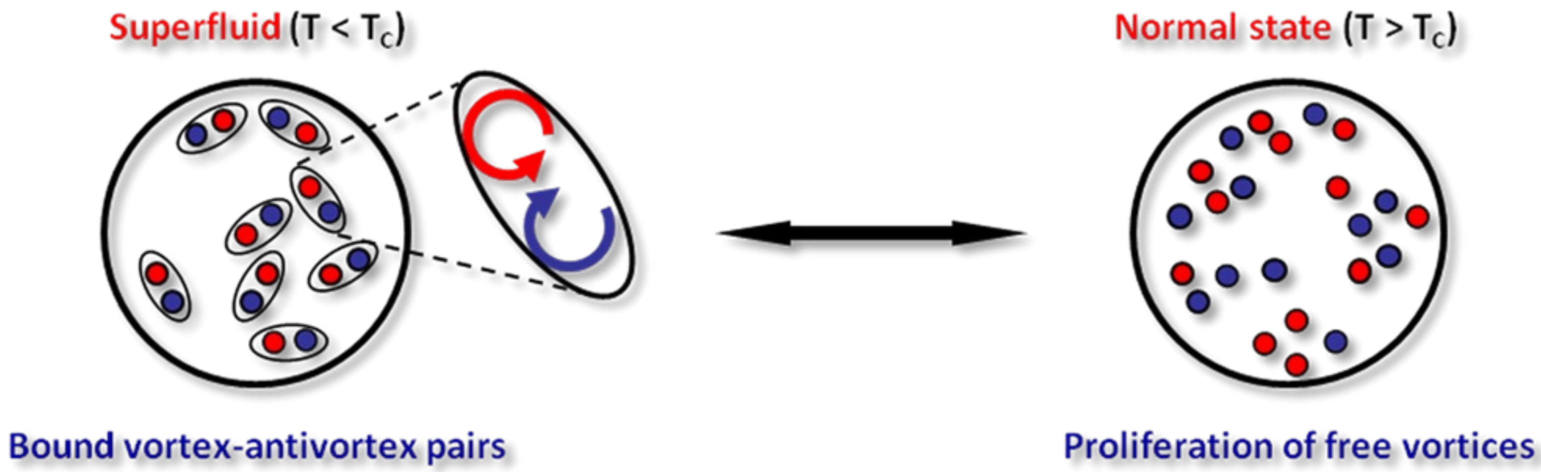
In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?

The puzzle of the parton model

In quantum mechanics, the proton is a pure state with zero entropy. Yet, a collection of free partons does possess entropy… Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems: BKT phase transition (Nobel prize 2016)

PHYSICAL REVIEW D **95**, 114008 (2017)

Deep inelastic scattering as a probe of entanglement

Dmitri E. Kharzeev[1,2,*] and Eugene M. Levin[3,4,†]

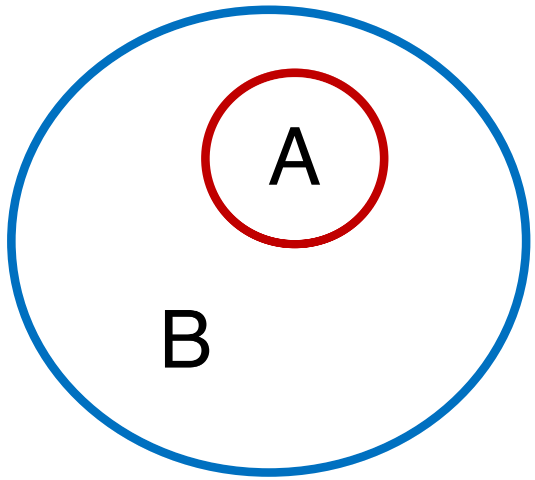
Our proposal: the key to solving this apparent paradox is entanglement.

DIS probes only a part of the proton's wave function (region A). We sum over unobserved region B; in quantum mechanics, this corresponds to accessing the density matrix of a mixed state

$$\hat{\rho}_A = \mathrm{tr}_{\mathrm{B}} \hat{\rho}$$

with a non-zero entanglement entropy

$$S_A = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}\right]$$



The quantum mechanics of partons and entanglement

What is “region B” in DIS? It may be the phase!

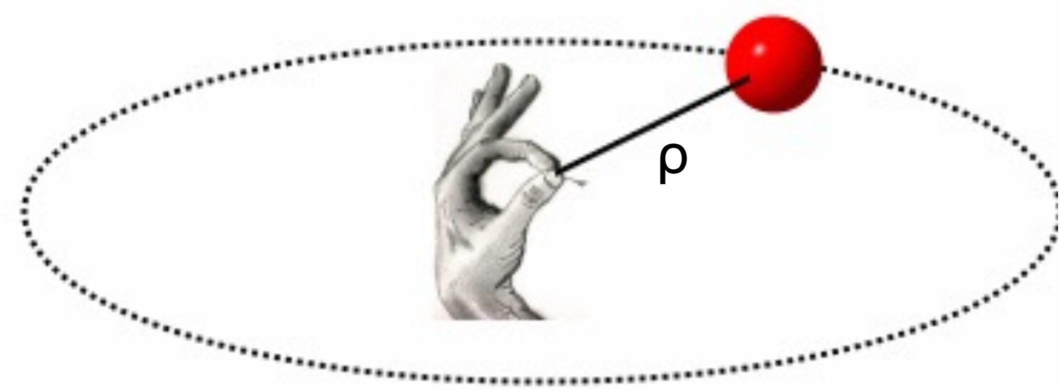
DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

DIS takes an instant snapshot of the proton’s wave function. This snapshot cannot measure the phase of the wave function.

Classical analogy:

$$z = \rho \ \exp(i\omega t)$$

Instant snapshot can measure the amplitude ρ, but not the angular velocity ω !



The quantum mechanics of partons and entanglement

A simple quantum mechanical model (proton rest frame):

DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

Expand the proton wave function in oscillator Fock states:

$$|n\rangle = \frac{1}{\sqrt{n!}} \prod_i^n a_i^\dagger |0\rangle,$$

$$|\Psi\rangle = \sum_n \alpha_n |n\rangle,$$

The density matrix:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \sum_{n,n'} \alpha_n\, \alpha_{n'}^* \,|n\rangle\langle n'|,$$

depends on time:

$$\hat{\rho}(t) = \sum_{n,n'} e^{i(n'-n)\omega t}\, \hat{\rho}(t=0).$$

But this time dependence cannot be measured by a light front – it crosses the hadron too fast, at time $t_{light} = R,$

Decoherence in high energy interactions

DK, Phil. Trans. Royal Soc (2022)

Therefore, the observed density matrix is a trace over an unobserved phase:

$$\hat{\rho}_{parton} = \mathrm{Tr}_{\varphi}\hat{\rho} = \sum_{n,n'} \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(n'-n)\varphi}\ \alpha_n\ \alpha^*_{n'} |n\rangle\langle n'| = \sum_n |\alpha_n|^2\ |n\rangle\langle n|.$$

U(1) Haar measure

“Haar scrambling” = decoherence
Y.Sekino, L.Susskind ‘08

After “Haar scrambling”, the density matrix becomes diagonal in parton basis (Schmidt basis) –

Probabilistic parton model!

This is a density matrix of a mixed state, with non-zero entanglement entropy!

The quantum mechanics of partons and entanglement

The parton model density matrix:

$$\hat{\rho}_{parton} = \sum_n p_n \, |n\rangle\langle n|$$

is mixed, with purity

$$\gamma_{parton} = \mathrm{Tr}(\rho^2_{parton}) = \sum_n p_n^2 < 1.$$

entanglement entropy

$$S_E = -\sum_n p_n \ln p_n$$

Parton model expressions for expectation values of operators:

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}(\hat{\mathcal{O}} \hat{\rho}_{parton}) = \sum_n p_n \langle n | \hat{\mathcal{O}} | n \rangle;$$

The quantum mechanics of partons and entanglement on the light cone

The density matrix on the light cone:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \sum_{n,n'}^{\infty} \int d\Gamma_n \, d\Gamma_{n'} \, \Psi^*_{n'}(x_{i'}, \vec{k}_{\perp i'}) \Psi_n(x_i, \vec{k}_{\perp i}) |n\rangle\langle n'|.$$

Haar scrambling: on the light cone, $t_i - z_i = x_i^- = 0,$
but t, z and $x^+ = z + t$ cannot be independently determined:

$$\int \frac{dx^+}{2\pi} e^{i(P_n^- - P_{n'}^-)x^+} = \delta(P_n^- - P_{n'}^-),$$

⬇

$$\hat{\rho}_{parton} = \mathrm{Tr}_{x^+} |\Psi\rangle\langle\Psi| = \sum_{n}^{\infty} \int d\Gamma_n \, |\Psi_n(x_i, \vec{k}_{\perp i})|^2 |n\rangle\langle n|,$$

Phase-occupation number uncertainty relation and parton model

$$\Delta\phi\Delta n \geq \tfrac{1}{2}\left|\langle\Psi|[\hat{\phi},\hat{n}]|\Psi\rangle\right|$$

High energies – phase cannot be measured, number is fixed:

parton model applies

Low energies - phase shifts can be measured, number is uncertain:

parton model does not apply

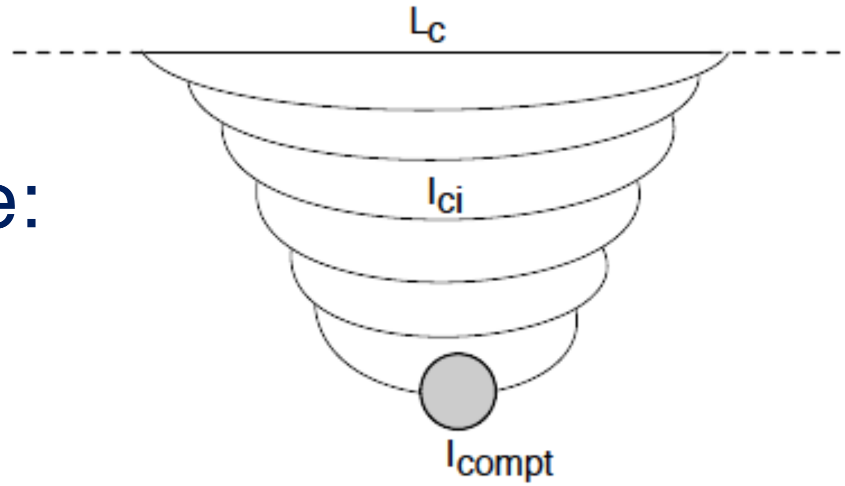
PHYSICAL REVIEW A VOLUME 48, NUMBER 4 OCTOBER 1993

Measurement of number-phase uncertainty relations of optical fields

D. T. Smithey, M. Beck, J. Cooper,* and M. G. Raymer

The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method

(A.H. Mueller '94; E. Levin, M. Lublinsky '04):

$$Z(Y,u) = \sum_n P_n(Y) u^n.$$

Solution:

$$P_n(Y) = e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

The resulting von Neumann entropy is

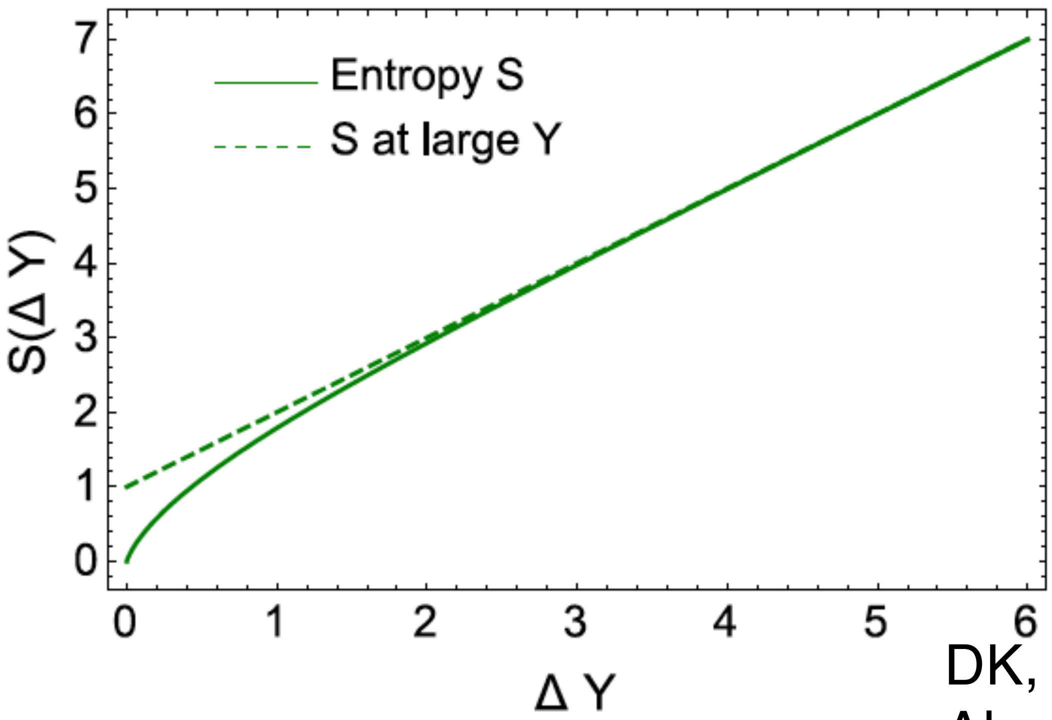
$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln\left(\frac{1}{1 - e^{-\Delta Y}}\right)$$

DK, E. Levin, arXiv:1702.03489; PRD

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \to \Delta Y$$

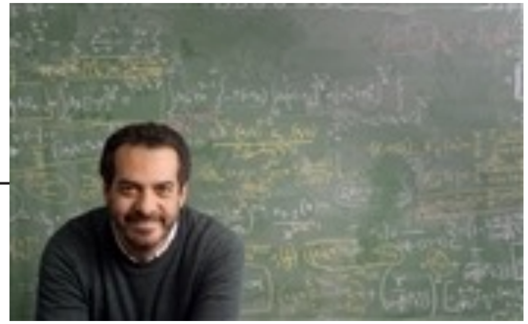


This "asymptotic" regime starts rather early, at

$$\Delta Y \simeq 2$$

DK, E. Levin, arXiv:1702.03489; PRD
Also: K. Kutak, arXiv:1103.3654

Linear dependence on rapidity is a consequence of (approximate) conformal invariance:



PHYSICAL REVIEW D **110,** 074008 (2024)

Universal rapidity scaling of entanglement entropy inside hadrons from conformal invariance

Umut Gürsoy,[1] Dmitri E. Kharzeev,[2,3] and Juan F. Pedraza[4]

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[2]*Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*

[3]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*

[4]*Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, Madrid 28049, Spain*

description. In this paper, we use an effective conformal field theoretic description of hadrons on the light cone to show that the linear dependence of the entanglement entropy on rapidity found in parton description is a general consequence of approximate conformal invariance and does not depend on the assumption of weak coupling. Our result also provides further evidence for a duality between the parton and string descriptions of hadrons.

$$S_A = \frac{c}{6}\Delta\eta + \ldots,$$

The entanglement entropy from QCD evolution

At large ΔY (x ~ 10^{-3}) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\Delta}$$

becomes very simple:

$$S = \ln[xG(x)]$$

DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

The entanglement entropy from QCD evolution

What is the physics behind this relation?

$$S = \ln[xG(x)]$$

It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

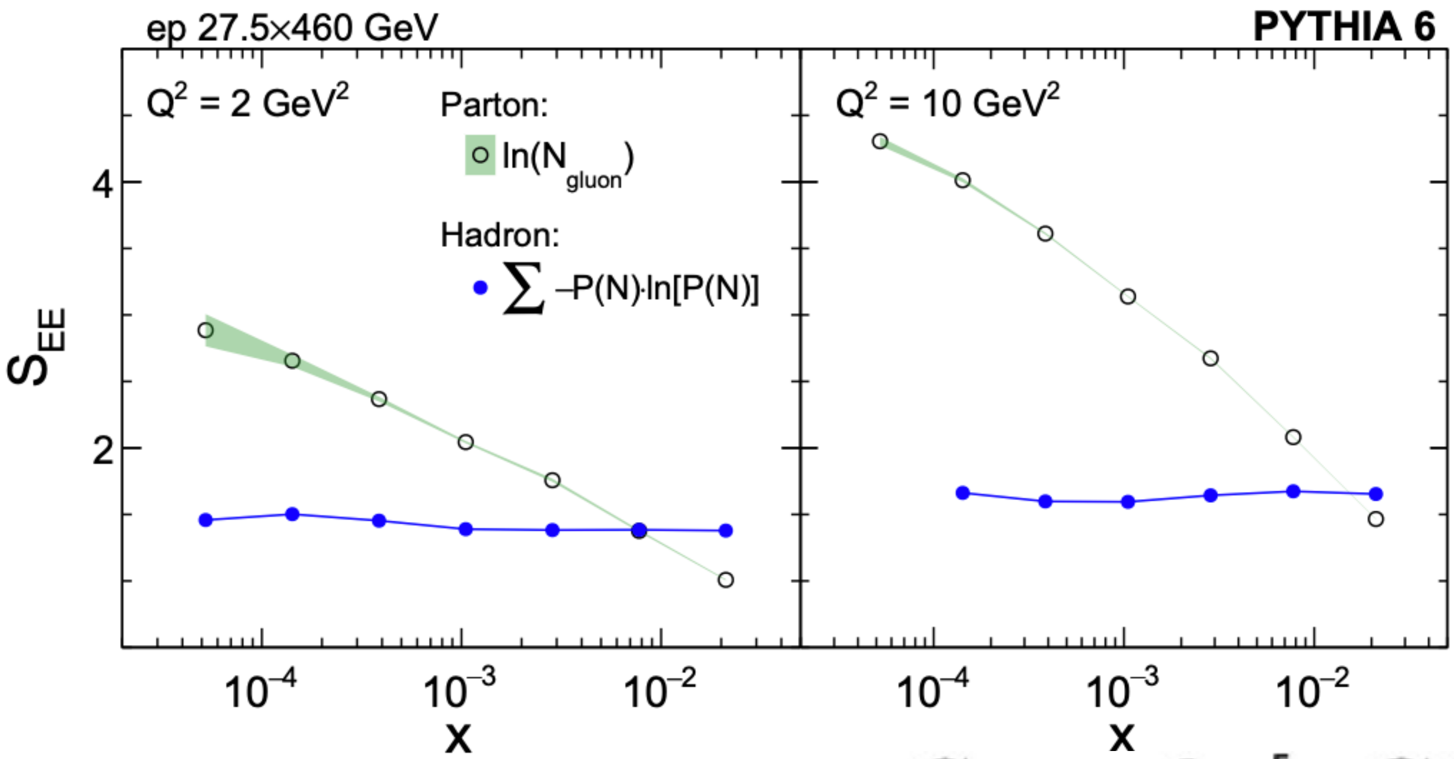
DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

Test of the entanglement at the LHC

PHYSICAL REVIEW LETTERS **124**, 062001 (2020)

Einstein-Podolsky-Rosen Paradox and Quantum Entanglement at Subnucleonic Scales

Zhoudunming Tu[1,*] Dmitri E. Kharzeev,[2,3] and Thomas Ullrich[1,4]



MC generator PYTHIA: $S = \ln[xG(x)]$

is not satisfied at small x (no entanglement)

Test of the entanglement at the LHC

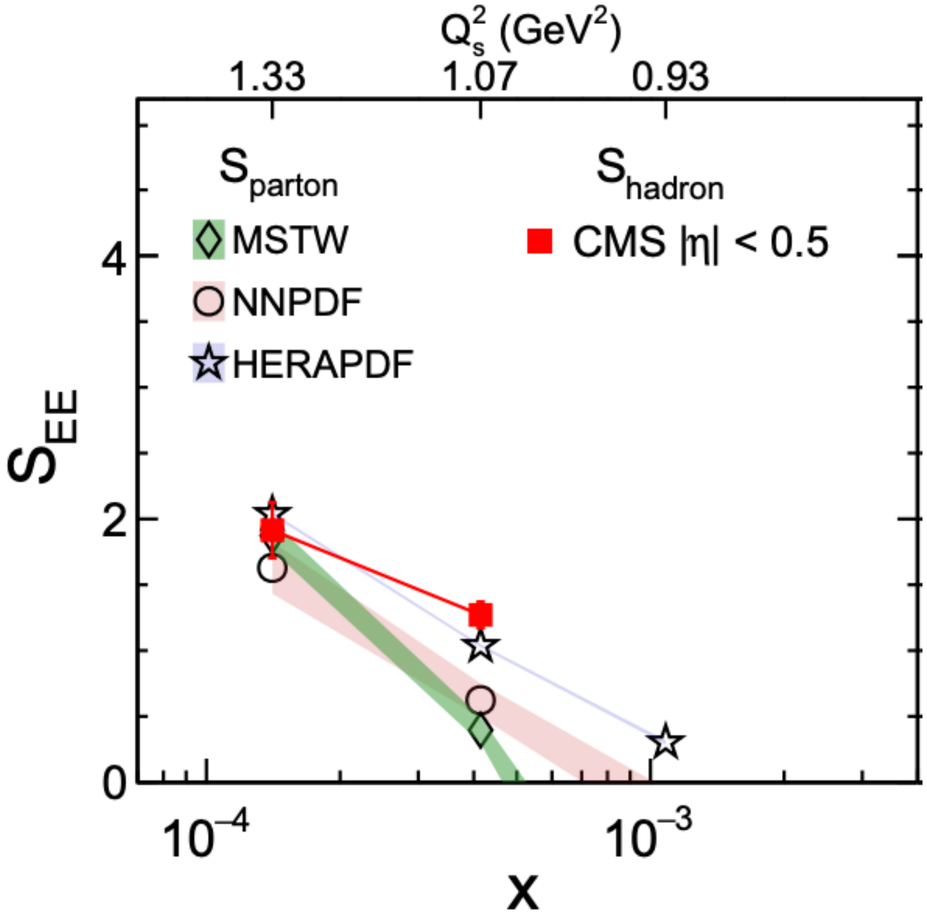
LHC data:

arXiv:1904.11974

$$S = \ln[xG(x)]$$

is satisfied at small x (**entanglement?!**)

K. Tu, DK, T. Ullrich,
arXiv:1904.11974;
PRL (2020)



Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

Martin Hentschinski[1] and Krzysztof Kutak[2]

[1]Departamento de Actuaria, Física y Matemáticas, Universidad de las Americas Puebla, San Andrés Cholula, 72820 Puebla, Mexico
[2] Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342, Kraków, Poland

December 14, 2021

Abstract

We investigate the proposal by Kharzeev and Levin of a maximally entangled proton wave function in Deep Inelastic Scattering at low x and the proposed relation between parton number and final state hadron multiplicity. Contrary to the original formulation we determine partonic entropy from the sum of gluon and quark distribution functions at low x, which we obtain from an unintegrated gluon distribution subject to next-to-leading order Balitsky-Fadin-Kuraev-Lipatov evolution. We find for this framework very good agreement with H1 data. We furthermore provide a comparison based on NNPDF parton distribution functions at both next-to-next-to-leading order and next-to-next-to-leading with small x resummation, where the latter provides an acceptable description of data.

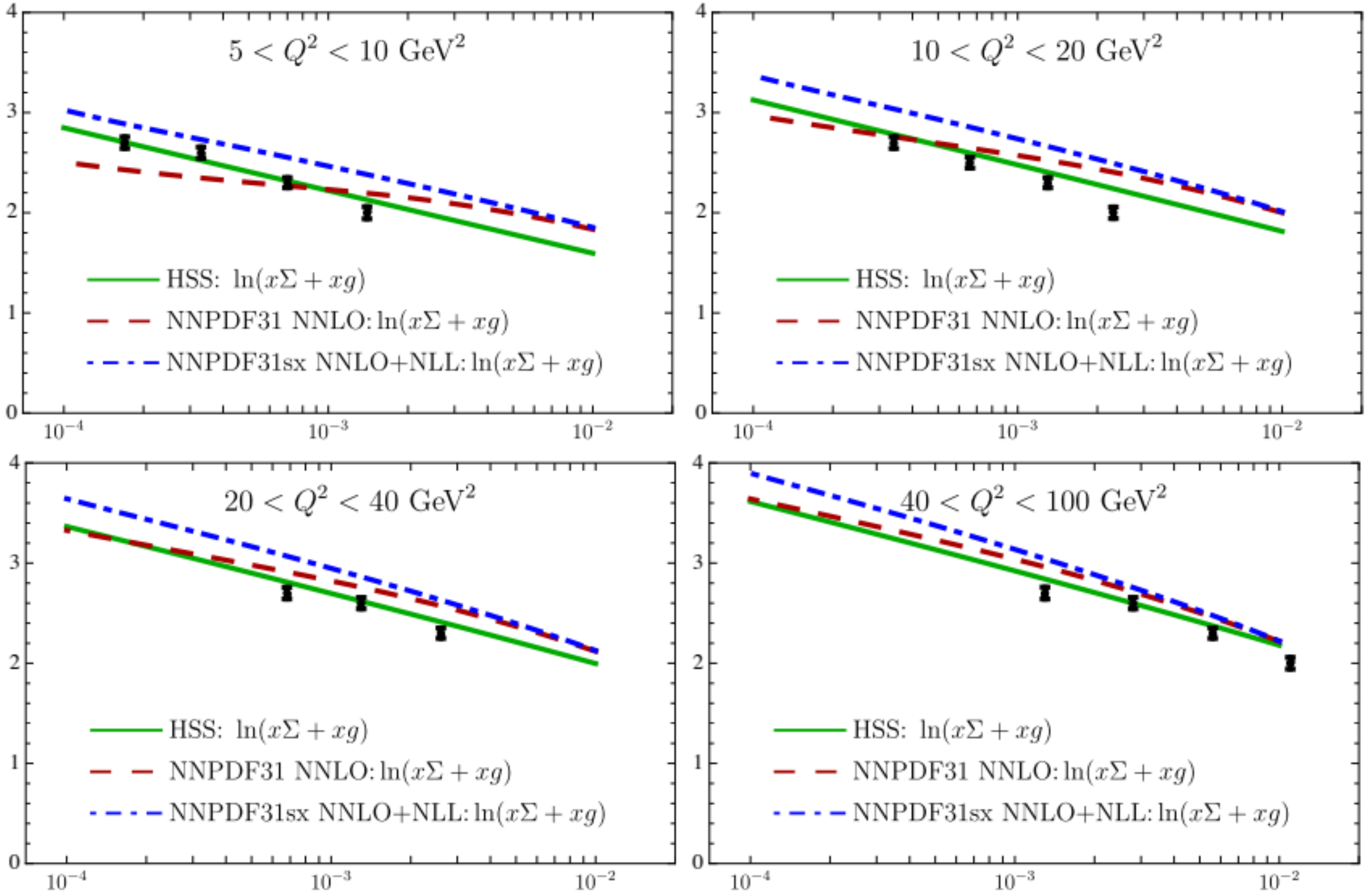


Figure 1: *Partonic entropy versus Bjorken x, as given by Eq. (1) and Eq. (2). We furter show results based on the gluon distribution only as well as a comparison to NNPDFs. Results are compared to the final state hadron entropy derived from the multiplicity distributions measured at H1 [19]*

Maximal entanglement: experimental tests at HERA and EIC

Probing the Onset of Maximal Entanglement inside the Proton in Diffractive Deep Inelastic Scattering

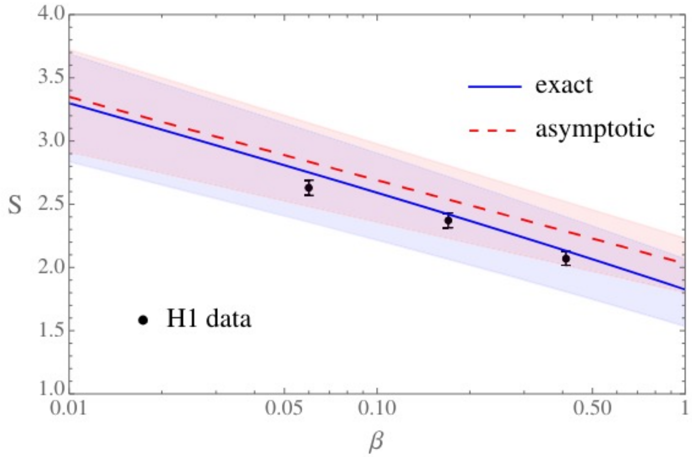
Martin Hentschinski, Dmitri E. Kharzeev, Krzysztof Kutak, and Zhoudunming Tu

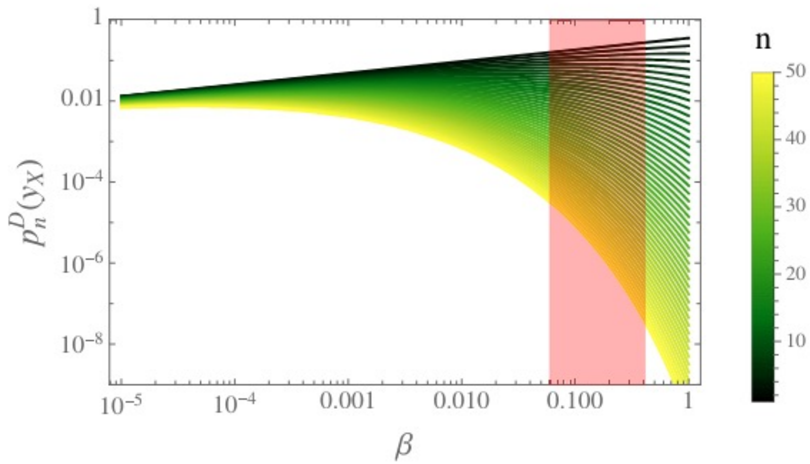
Phys. Rev. Lett. **131**, 241901 – Published 13 December 2023

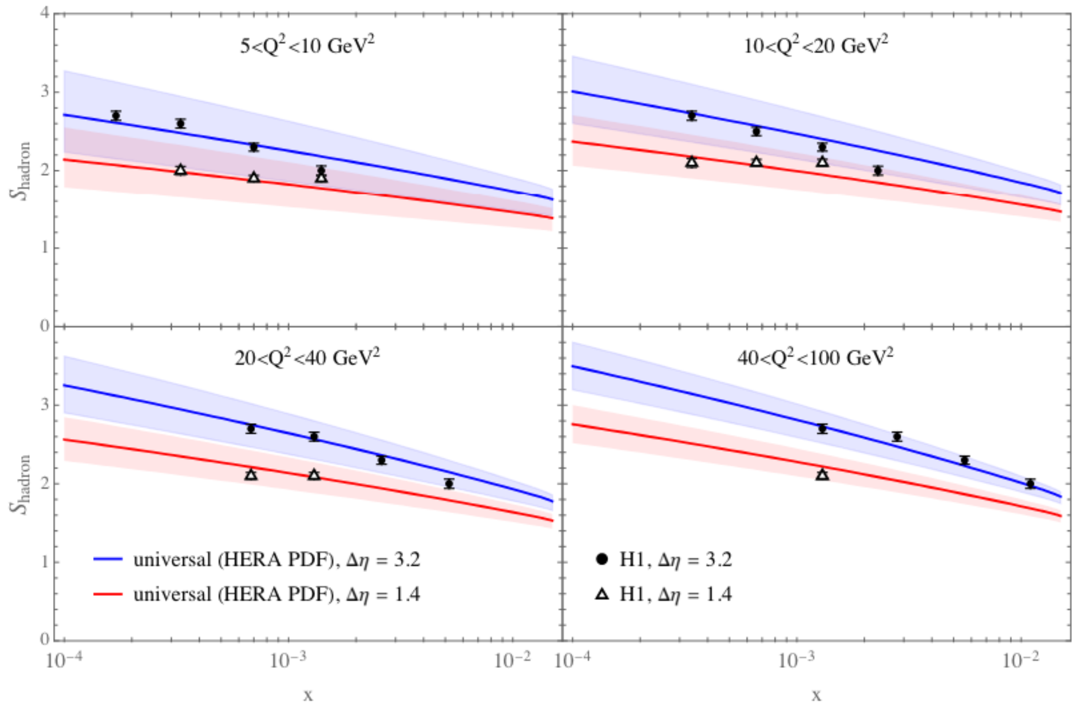
QCD evolution of entanglement entropy

Martin Hentschinski,[1,*] Dmitri E. Kharzeev,[2,3,†] Krzysztof Kutak,[4,‡] and Zhoudunming Tu[3,§]

arXiv:2408.01259, Rep.Prog.Phys.(2025)



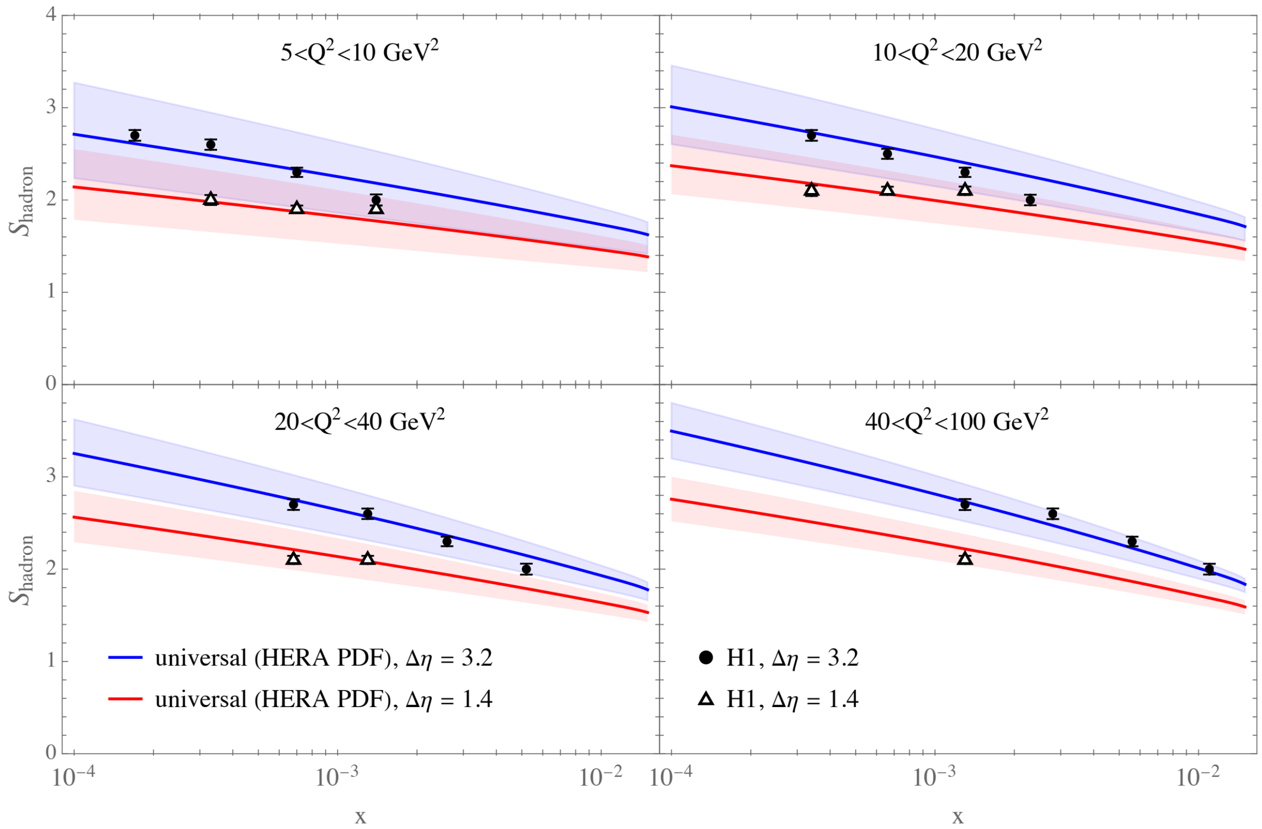




QCD evolution of entanglement entropy

Martin Hentschinski[1], **Dmitri E Kharzeev**[2,3], **Krzysztof Kutak**[4]
and Zhoudunming Tu[3,*]

arXiv: 2408.01259; Reports on Progress in Physics, 2024, in press



Maximal entanglement agrees with H1 measurements in different rapidity windows

Summary:

- Fundamental research in physics has been, and will continue to drive the quantum technology
- Deep connection emerges between entanglement and thermalization in statistical physics – fundamentally interesting, crucially important for technology (decoherence, quantum AI/ML)
- Studies of real-time behavior of quantum field theories relevant for nuclear/high energy/condensed matter physics are key to understanding thermalization, and ways to control it