Far from equilibrium phenomena: from RHIC and LHC theory to cold atom experiments (II)







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2307.07545 with Mazeliauskas and Preis [PRL], 2502.01622 with De Lescluze and 2504.18754 with Berges, Denicol and Preis

Lecture I: summary

Lecture I summary: nonthermal fixed points

Nonthermal fixed point: self-similar evolution in time at weak coupling

$$f(t,p) \approx A(t) \times f_s(B(t)p)$$
 with $A(t) = (t/t_{ref})^{\alpha}$ and $B(t) = (t/t_{ref})^{\beta}$

Relevant for QCD thermalization and for cold atomic gases

Significant simplification in the dynamics, as it reduces to a rescaling

Different systems might exhibit the same exponents (\longrightarrow universality)

Nonthermal fixed points act as attractors for a large class of initial states

Lecture I: slide 19/22

Lecture I summary: quasinormal modes

Quasinormal modes: frequency eigenmodes of systems with dissipation

Explain the attractive nature of (near-)equilibrium states:



I 103.3452 with Janik & Witaszczyk

Lecture 1: slide 20/22



2203.16549 with Du, Schlichting & Svensson

Lecture I summary: hydrodynamics

Framework for describing relaxation of spatial inhomogeneities

Macroscopic construction based on effective field theory principles

Microscopic input: equation of state and transport coefficient

Quasinormal mode manifestation: sound waves, shear mode

Working horse for simulating nuclear collisions:

QCD dynamics — relativistic fluid mechanics

Lecture 1: slide 21/22

What will happen now?

The goal of these lectures

two key theoretical mechanisms underlying state of the art understanding of thermalization in QCD are **nonthermal fixed points** and **attraction to equilibrium**

> novel phenomena in table top experiments with cold atomic gases far from equilibrium

Lecture II teaser

First take on quasinormal modes of nonthermal fixed points:



Lecture I: slide 22/22

Setup

Kinetic theory setup

We will be using (relativistic) isotropic kinetic theory

 $\partial_t f(t,p) = C[f](t,p)$

Example of C[f]: the Fokker-Planck QCD collision kernel see e.g. 1402.5049 by Blaizot, Wu & Yan

$$C_{FP}[f](t,p) \sim \mathcal{L}\left[I_a \frac{1}{p^2} \partial_p (p^2 \partial_p f) + I_b \frac{1}{p^2} \partial_p (p^2 f(1+f))\right]$$

$$\mathcal{L} = \int \frac{dq}{q} = \log \frac{\sqrt{\langle p^2 \rangle}}{m_D}, \quad \mathscr{L} = \text{const},$$
where
$$I_a = N_c \int \frac{d^3 p}{(2\pi)^3} f(1+f),$$

$$I_b = 2N_c \int \frac{d^3 p}{(2\pi)^3} \frac{f}{p}.$$

This collision kernel leads to a nonthermal fixed point with $\alpha = -\frac{4}{7}$ and $\beta = -\frac{1}{7}$

Result I: extending nonthermal fixed points to earlier times

Prescaling 1810.10554 by Berges and Mazeliauskas

Scaling with slowly varying in time exponents

$$f(t,p) \approx A(t) \times f_s(B(t)p)$$
 with $A(t) = (t/t_{ref})^{\alpha(t)}$ and $B(t) = (t/t_{ref})^{\beta(t)}$

might precede the exact scaling with α, β constant

Important for theory and experiment, since makes scaling visible much earlier

Nonthermal attractor



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The origin of (pre)scaling 2307.07545 with Mazeliauskas and Preis

Overoccupation simplifies C[f(t, p)] as the highest power of f dominates

$$C[f(t,p)] \sim \int dP_1 \dots (\dots) f^{\kappa} \text{ e.g. } \kappa_{2\leftrightarrow 2} = 3$$

This might make it possible to factor out *t*-dependence for $A(t)f_s(B(t)p)$:

$$C[f(t,p)] \equiv A(t)^{\mu_{\alpha}} B(t)^{\mu_{\beta}} C[f_{s}(\bar{p} \equiv B(t)p)]$$

Overoccupation leads to most of the energy (or particle number) in the relevant range of momenta

Imposing conservation of energy (or particle number) leads then to

$$A(t) \sim B(t)^{\sigma}$$
 with $\sigma = 4$ (or $\sigma = 3$)

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$$\mathcal{L} = \int_{q}^{dq} \log \sqrt{\langle p^{2} \rangle}, \quad \mathcal{L} = \text{const,}$$
where $I_{a} = N_{c}\int \frac{d^{3}p}{(2\pi)^{3}}f(1+f),$

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$$\mathcal{K} = \mathbf{3}$$

This collision kernel leads to a direct cascade in the UV

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Overoccupation leads to most of the energy (or particle number) in the relevant range of momenta, but the simplified kernel alters conservation properties

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Prescaling as dimensionality reduction 2307.07545 with Mazeliauskas and Preis

All in all, overoccupation + conservation factorizes then the Boltzmann equation



Vast reduction of complexity in a sense of dimensionality reduction

Physics of prescaling 2307.07545 with Mazeliauskas and Preis

The solutions of

$$\frac{B(t)^{1-1/\beta}}{\partial_t B(t)} = \frac{t_{\text{ref}}}{\beta} = \frac{[\sigma + \bar{\mathbf{p}} \cdot \partial_{\bar{\mathbf{p}}}] f_S(\bar{\mathbf{p}})}{\mathcal{C}[f_S](\bar{\mathbf{p}})} \quad \text{where} \quad \frac{1}{\beta} \equiv (1 - \mu_\alpha)\sigma - \mu_\beta$$

predict the prescaling as

$$B(t) = \left(\frac{t - t_*}{t_{\text{ref}}}\right)^{\beta} \equiv t^{\beta(t)} \text{ with } \beta(t) = \beta \times \log_t \left(\frac{t - t_*}{t_{\text{ref}}}\right)$$

Prescaling is then just a scaling with an initial-condition dependent offset of time

$$B(t \gg t_*) \approx \left(\frac{t}{t_{\rm ref}}\right)^{\beta} \left(1 - \beta_{\infty} \frac{t_*}{t} + \ldots\right)$$

Experimental confirmation

2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ³⁹K in the lowest hyperfine state, confined in a square box trap of size $L = 50 \,\mu m$ [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions $(a \rightarrow 0)$ and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1-i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \rightarrow 30 a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k) dk$, where $\varepsilon = 2\pi \hbar^2 k^3 n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_{\rm B} = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].



Some perspectives from prescaling

Nonthermal fixed points = overoccupation + dimensionality reduction

(Pre)scaling is then a consequence of the equations of motion

Prescaling exhausts the dimensionally reduced ansatz $f(t,p) \approx A(t) \times f_s(B(t)p)$

Prescaling even at this level does not seem to have much to do with hydro

Result II: transient QNMs of nonthermal fixed points

Key idea behind 2502.01622 with De Lescluze

There is sense in which nonthermal fixed points are static: $f_s(\bar{p})$

As a result, the notion of quasinormal mode approach makes sense

Quasinormal modes of nonthermal fixed points 2502.01622 with De Lescluze

On a nonthermal fixed point:

directly before that:

$$f(t,p) \approx (t_{\text{uni}}/t_0)^{\alpha} f_{\text{scaling}} \left[(t_{\text{uni}}/t_0)^{\beta} p \right] \qquad (t_{\text{uni}}/t_0)^{-\alpha} f(t,p) \approx f_{\text{scaling}} \left[(t_{\text{uni}}/t_0)^{\beta} p \right]$$
with $t_{\text{uni}} \equiv t - t_*$

$$+ \sum_{\Omega} (t_{\text{uni}}/t_0)^{-i\Omega} \delta f_{\Omega} \left[(t_{\text{uni}}/t_0)^{\beta} p \right]$$



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Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta \langle T_{\mu\nu} \rangle \sim e^{-i\omega t + i \vec{q} \cdot \vec{x}}$



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Result III: hydrodynamics of nonthermal fixed points

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Hydrodynamics of nonthermal fixed points

2504.18754 with Berges, Denicol and Preis

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What we do in practice

2504.18754 with Berges, Denicol and Preis

$$f(t,p) = \left(t_{\text{uni}}/t_0\right)^{\alpha} f_{\text{scaling}} \left[\left(t_{\text{uni}}/t_0\right)^{\beta} p \right] + \delta f(x^{\mu},p)$$

Input:

with $\delta f(t,p) \sim p_{\mu} p_{\nu} \pi^{\mu\nu}(x^{\alpha})$

Method: take truncated low moments expansion of the Boltzmann equation that was used before to derive MIS/DNMR-type equations

Outcome
for 2-2 :
$$\tau_{\pi}(t)D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \dots$$
 with
scattering $\frac{\eta(t)}{\tau_{\pi}(t)} = \frac{4}{15} \times \text{(energy density)}$



Quasinormal modes of nonthermal fixed points 2502.01622 with De Lescluze

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Implications 2504.18754 with Berges, Denicol and Preis

Comparing the nonthermal fixed point liquid with near-equilibrium liquids requires going beyond the η/s paradigm, as this ratio is now time dependent



Using $\tau_{\pi}(t)D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + ...$ to model inhomogeneous nonthermal fixed point phenomena eying experimental confirmations in cold atomic gases

Extends to QCD kinetic theory ($\lambda = g^2 \times 3$)



2203.16549 with Du, Schlichting & Svensson

Lecture I: slide 18/22

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Summary

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Prescaling is the scaling with correctly accounted for the origin of time Consequence: nonthermal fixed points extend to earlier times 2307.07545 with Mazeliauskas and Preis

The emergence of nonthermal fixed points can be thought of as originating from the decay of transient quasinormal modes around them 2502.01622 with De Lescluze

Adding spatial momentum reveals hydrodynamic modes and opens a window on studying inhomogeneous nonthermal fixed points

2504.18754 with Berges, Denicol and Preis

Open problems (cold atoms / theory) experimentally confirmed in the Hadzibabic Lab Lorrectly account the Hadzibabic median setting? If confirmed in the nuclear collision setting? Prescaling is the scaling with correctly account Consequence: nonthermal fixed from the decay of transient qua experiment? less damped 2502.01622 with the prescaling one? 2502.01622 with the prescali Sum reveal is connections with a Hydro? Adding spatial momentum rever', 's and opens a window on studying inhomor with initial state fluctuations? with Berges, Denicol and Preis 15/15



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