Far from equilibrium phenomena:
from RHIC and LHC theory to cold atom experiments (I)

Michal P. Heller

GHENT UNIVERSITY

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

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2307.07545 with Mazeliauskas and Preis [PRL], **2502.01622** with De Lescluze and **2504.18754** with Berges, Denicol and Preis

Happy B-day to the Cracow School

* 1961: first human in space, 1st school

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- 2025: 65th school

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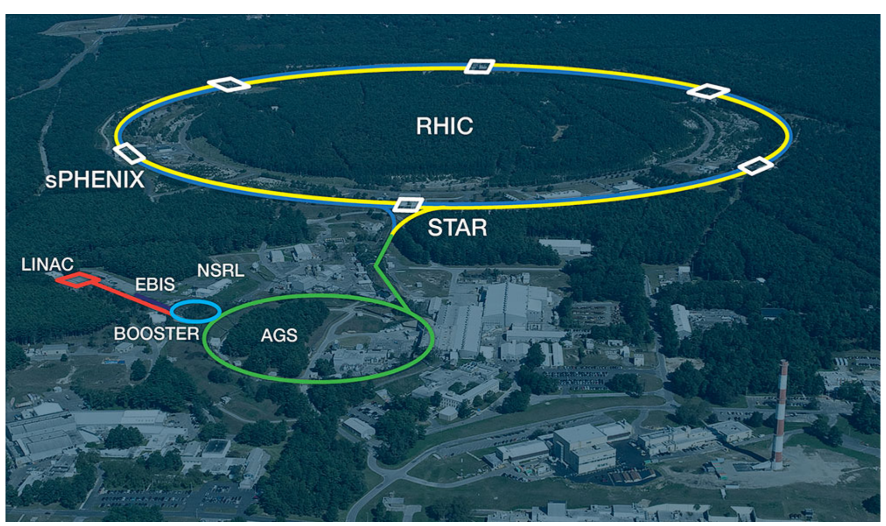
- 2016: 56th school
- 2025: 65th school

What these lectures will be about?

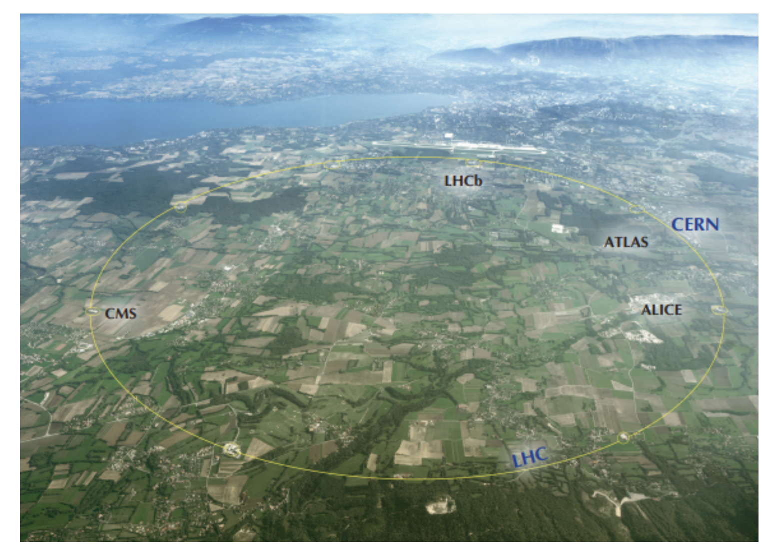
Beginnings

1974: "… it would be interesting to explore new phenomena by distributing a high amount of energy or high nuclear density over relatively large volume…"

Tsung-Dao Lee

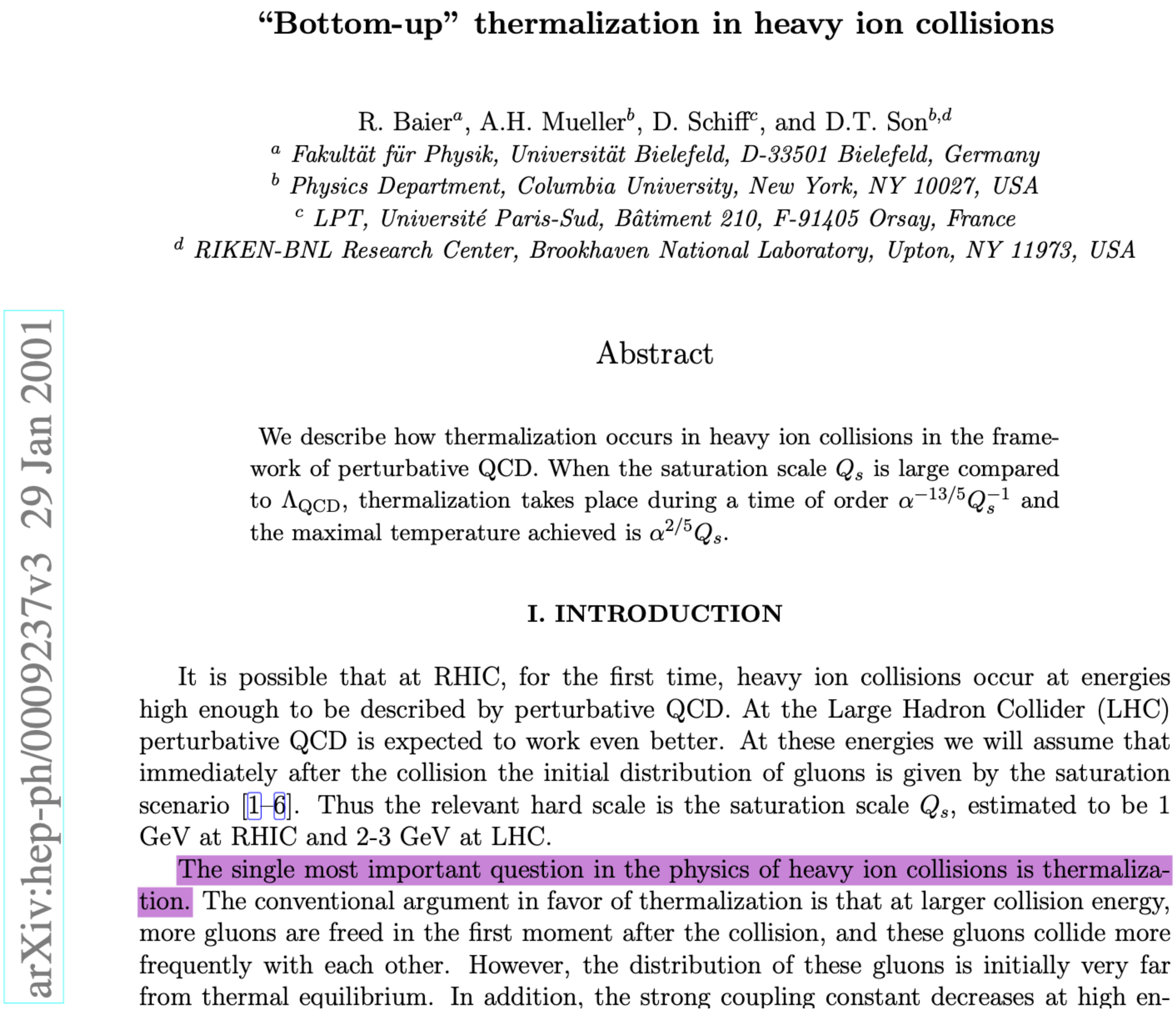
2000: 

source: BNL

2010: 

source: CERN

Early realization (BMSS)



"Bottom-up" thermalization in heavy ion collisions

R. Baier[a], A.H. Mueller[b], D. Schiff[c], and D.T. Son[b,d]
[a] *Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany*
[b] *Physics Department, Columbia University, New York, NY 10027, USA*
[c] *LPT, Université Paris-Sud, Bâtiment 210, F-91405 Orsay, France*
[d] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*

arXiv:hep-ph/0009237v3 29 Jan 2001

Abstract

We describe how thermalization occurs in heavy ion collisions in the framework of perturbative QCD. When the saturation scale Q_s is large compared to Λ_{QCD}, thermalization takes place during a time of order $\alpha^{-13/5}Q_s^{-1}$ and the maximal temperature achieved is $\alpha^{2/5}Q_s$.

I. INTRODUCTION

It is possible that at RHIC, for the first time, heavy ion collisions occur at energies high enough to be described by perturbative QCD. At the Large Hadron Collider (LHC) perturbative QCD is expected to work even better. At these energies we will assume that immediately after the collision the initial distribution of gluons is given by the saturation scenario [1–6]. Thus the relevant hard scale is the saturation scale Q_s, estimated to be 1 GeV at RHIC and 2-3 GeV at LHC.

The single most important question in the physics of heavy ion collisions is thermalization. The conventional argument in favor of thermalization is that at larger collision energy, more gluons are freed in the first moment after the collision, and these gluons collide more frequently with each other. However, the distribution of these gluons is initially very far from thermal equilibrium. In addition, the strong coupling constant decreases at high en-

Continuous progress

arXiv:2005.12299v3 [hep-th] 17 Aug 2021

QCD thermalization: *Ab initio* approaches and interdisciplinary connections

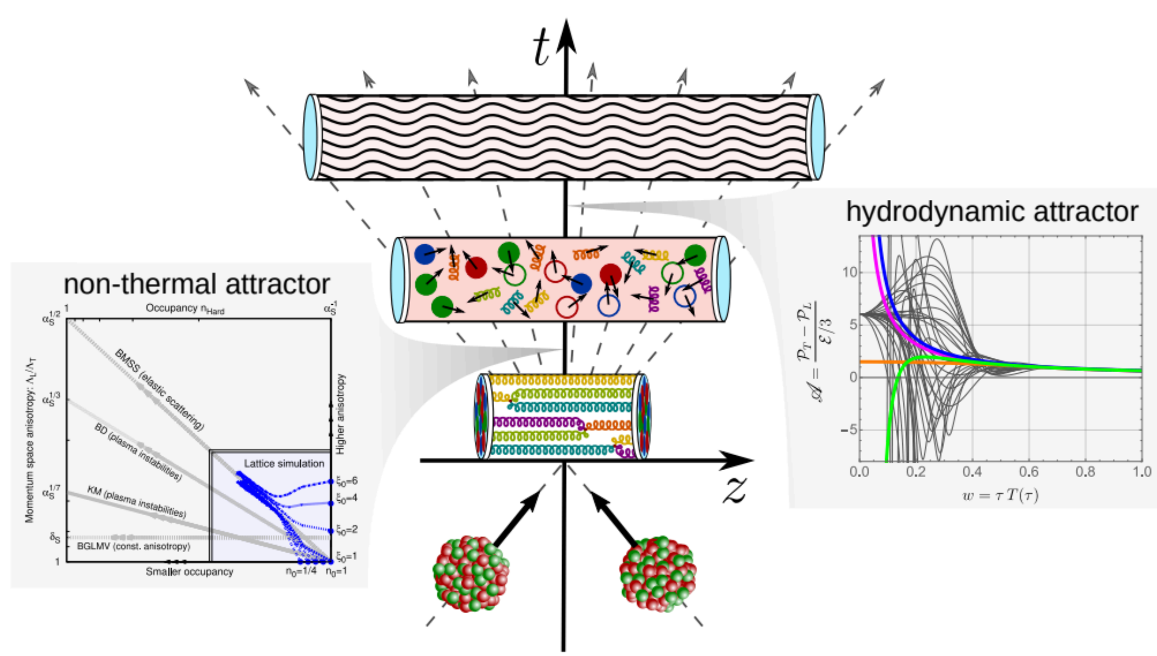
Jürgen Berges*
Institute for Theoretical Physics, Heidelberg University, Philosophenweg 16, D-69120 Heidelberg, Germany

Michal P. Heller†
Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, D-14476 Potsdam, Germany

Aleksas Mazeliauskas‡
Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland

Raju Venugopalan§
Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA

Heavy-ion collisions at BNL's Relativistic Heavy Ion Collider and CERN's Large Hadron Collider provide strong evidence for the formation of a quark-gluon plasma, with temperatures extracted from relativistic viscous hydrodynamic simulations shown to be well above the transition temperature from hadron matter. Outstanding problems in QCD include how the strongly correlated quark-gluon matter forms in a heavy-ion collision, its properties off-equilibrium, and the thermalization process in the plasma. We review here the theoretical progress in this field in weak coupling QCD effective field theories and in strong coupling holographic approaches based on gauge-gravity duality. We outline the interdisciplinary connections of different stages of the thermalization process to non-equilibrium dynamics in other systems across energy scales ranging from inflationary cosmology, to strong field QED, to ultracold atomic gases, with emphasis on the universal dynamics of non-thermal and hydrodynamic attractors. We survey measurements in heavy-ion collisions that are sensitive to the early non-equilibrium stages of the collision and discuss the potential for future measurements. We summarize the current state-of-the art in thermalization studies and identify promising avenues for further progress.



* berges@thphys.uni-heidelberg.de
† michal.p.heller@aei.mpg.de; *On leave from:* National Centre for Nuclear Research, Pasteura 7, Warsaw, PL-02093, Poland
‡ aleksas.mazeliauskas@cern.ch
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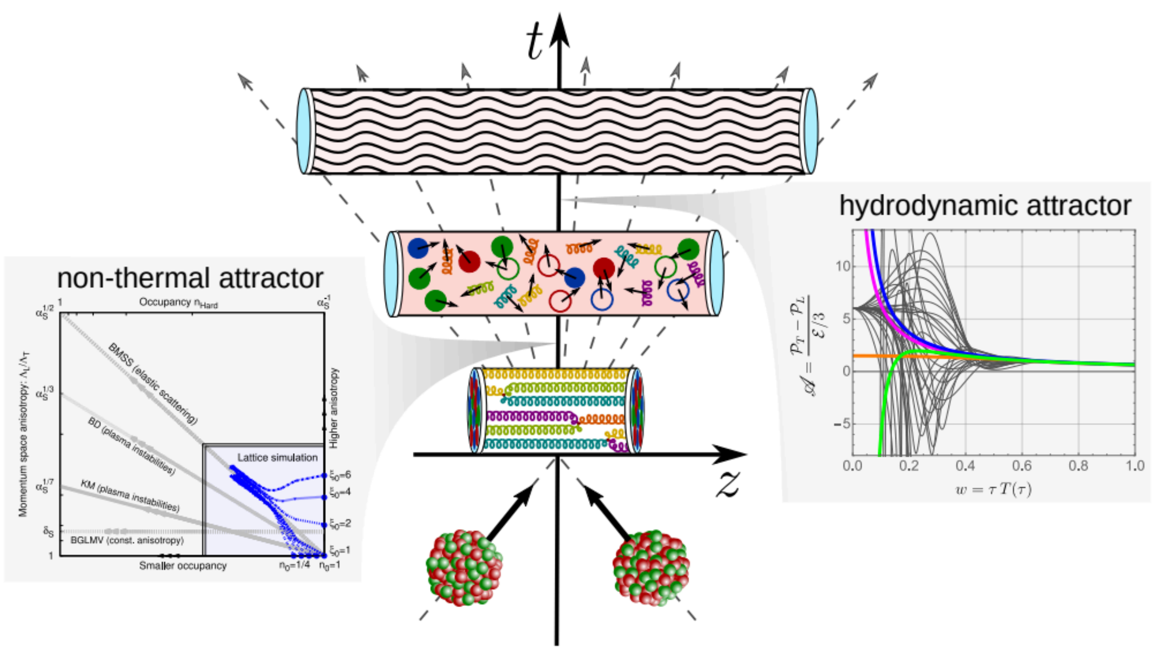

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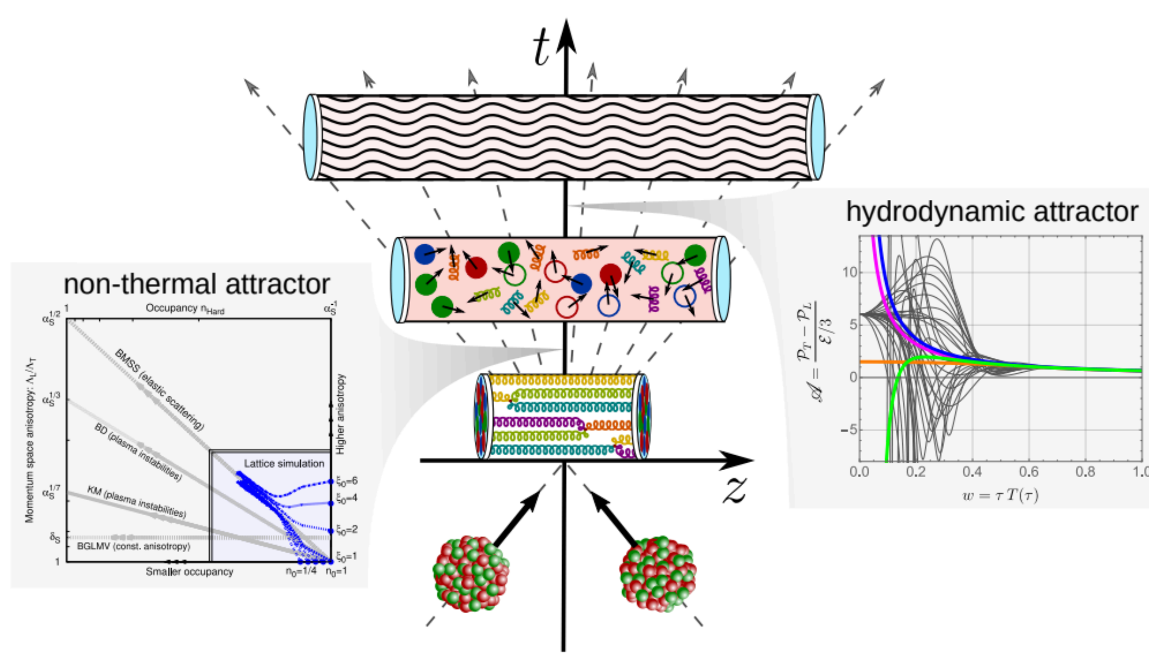

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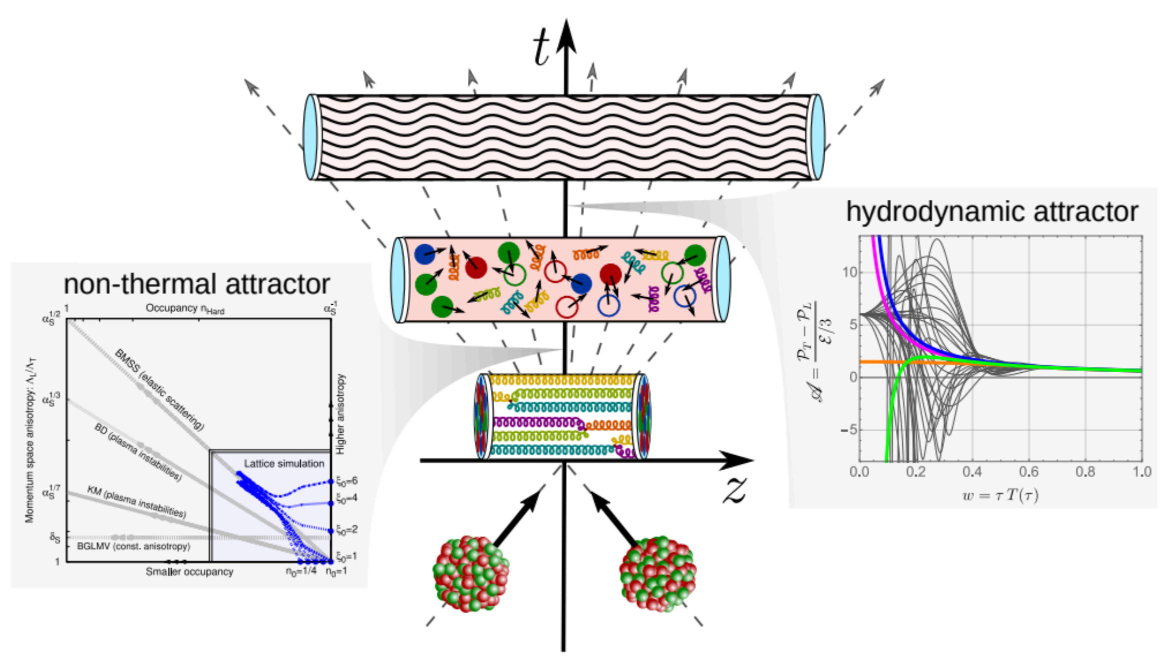

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Key take homes 25 years after BMSS

∞

models in 1+3D soluble with holography
(large number of degrees of freedom)

Patch-wise
theory understanding
in the coupling

models in 1+1D soluble with tensor networks
(includes gauge theories, potential for doing 1+2D)

0

strong classical fields, kinetic theory
(controllable ≈ QCD at energies beyond RHIC, LHC)

Fruitful connections to other systems and areas, in particular fluid mechanics, cold atomic gases and quantum information

Overlap with other lectures at this school

Dmitri KHARZEEV (Stony Brook University and BNL)
When physics meets quantum information (pdf)

Stanisław MRÓWCZYŃSKI (National Centre for Nuclear Research, Warszawa)
Glasma as a fluid

Continuous progress

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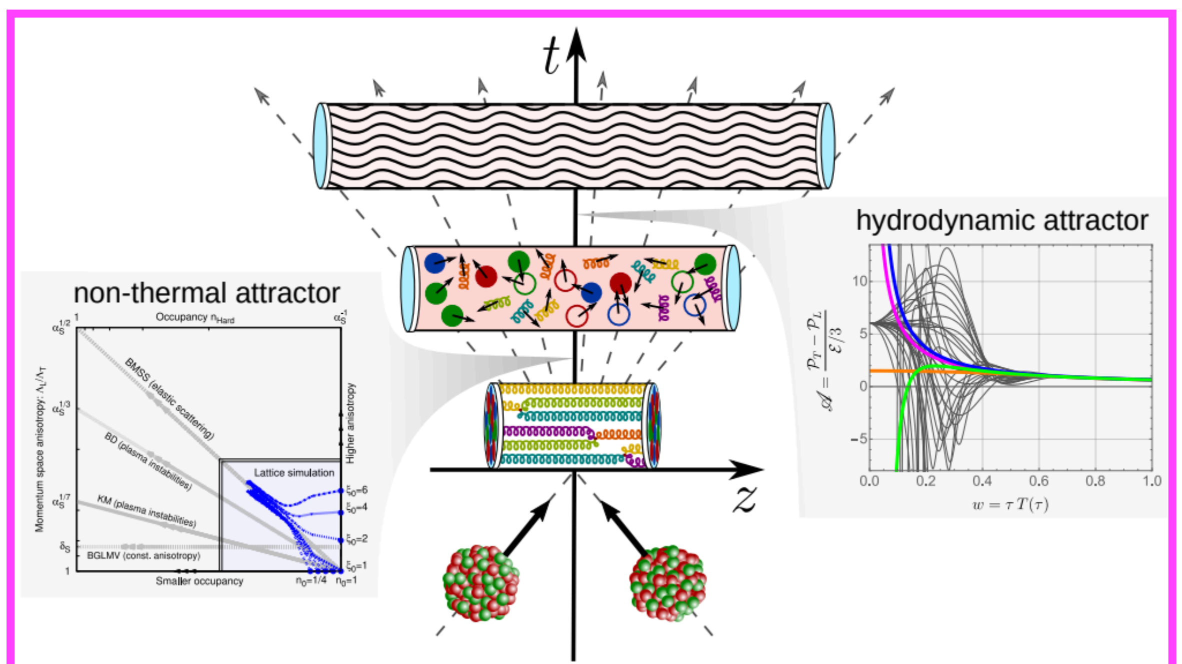

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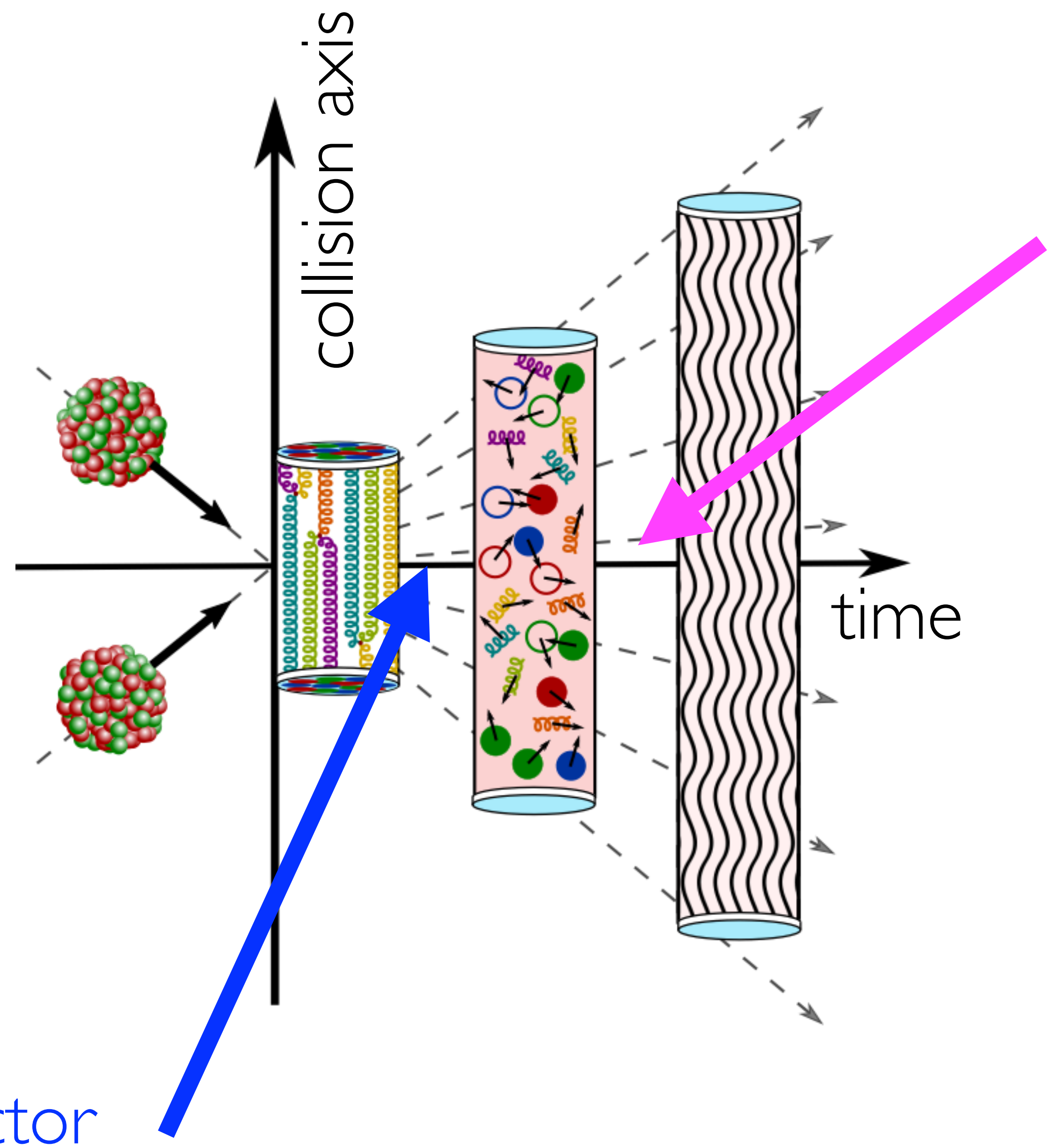
Seeking for a unifying idea

2005.12299 with Berges, Mazeliauskas and Venugopalan

hydrodynamic attractor



nonthermal attractor
(aka nonthermal fixed point)

The goal of these lectures

two key theoretical mechanisms underlying
state of the art understanding of thermalization in QCD are
nonthermal fixed points and **attraction to equilibrium**

↓

novel phenomena in table top experiments
with cold atomic gases far from equilibrium

Far from equilibrium ingredient: nonthermal fixed points

Nonthermal attractor

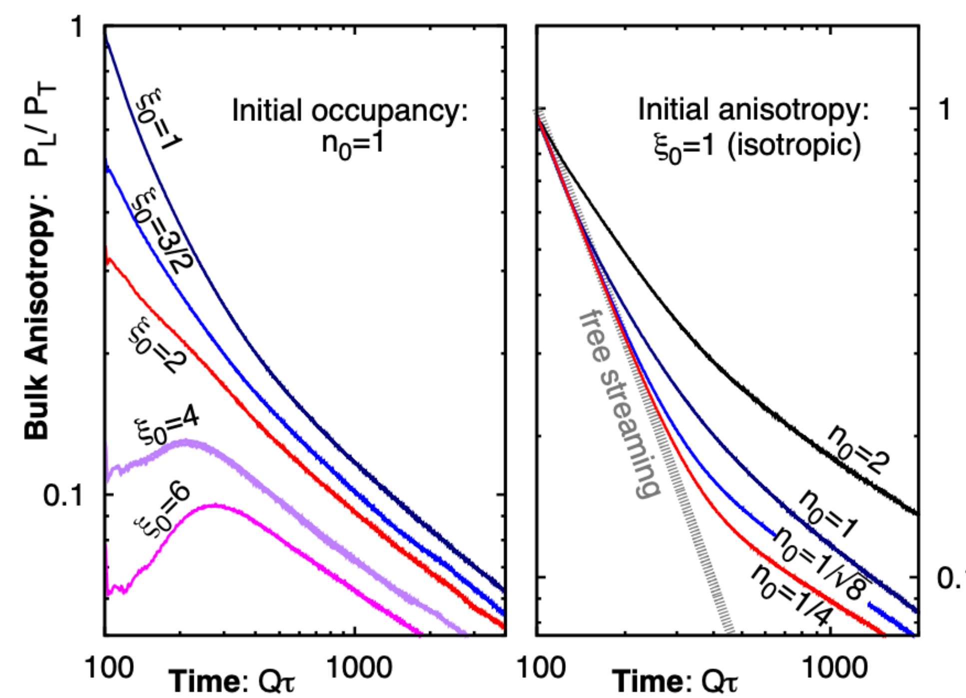
1303.5650 and **1311.3005** by Berges, Boguslavski, Schlichting and Venugopalan

After a collision at weak coupling: overoccupied gluons (strong classical fields):

$$f(p_T, p_z, \tau_0) = \frac{n_0}{8\pi\alpha_S}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right) \quad \text{with} \quad \alpha_S = 10^{-5}$$

Ab initio simulations revealed significant insensitivity to plasma instabilities and free streaming, consistent with elastic scattering dominance among hard particles in the kinetic theory regime $Q\tau \sim log^2(1/\alpha_S) = O(10^2)$: stage I of the "bottom up" thermalization

hep-ph/0009237 by Baier, Mueller, Schiff and Son

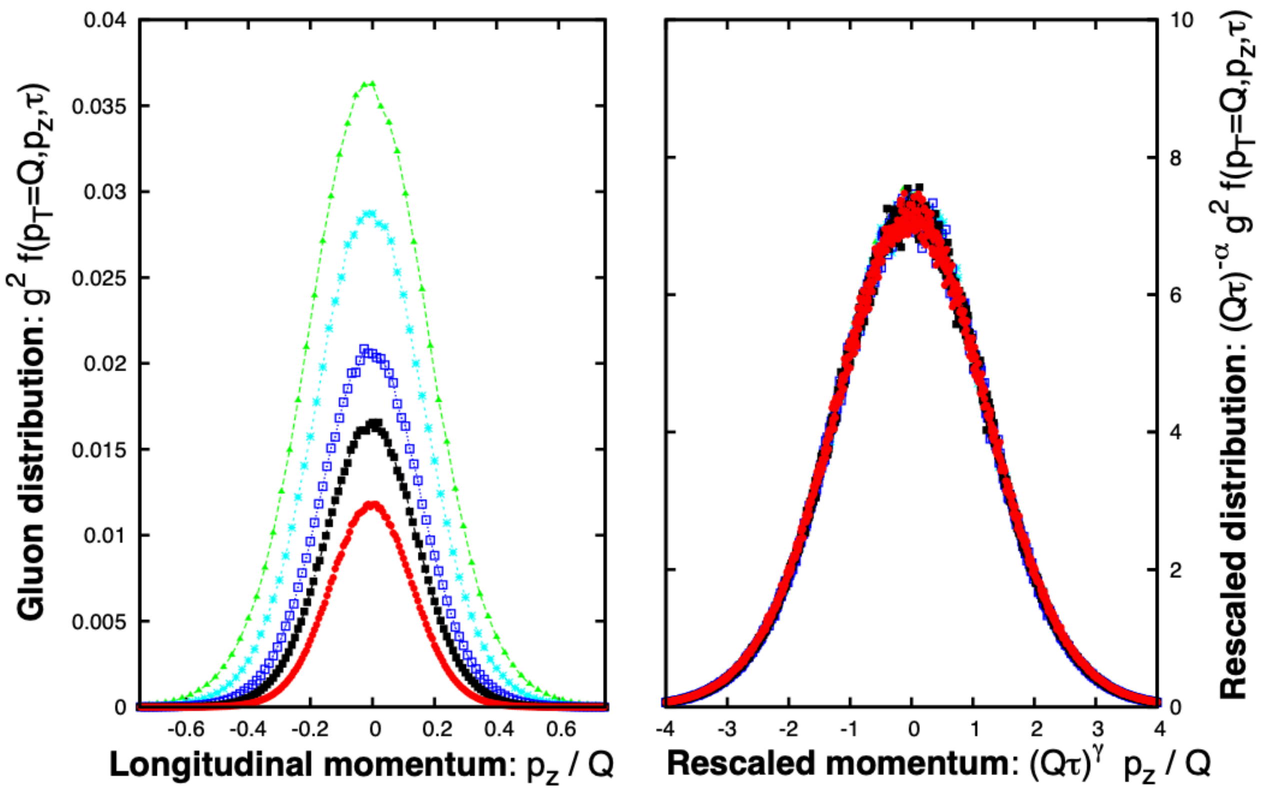


Overoccupation is expected then to drive the system to a self similar regime

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S\left((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z\right) \quad \text{with } \alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$$

Nonthermal attractor

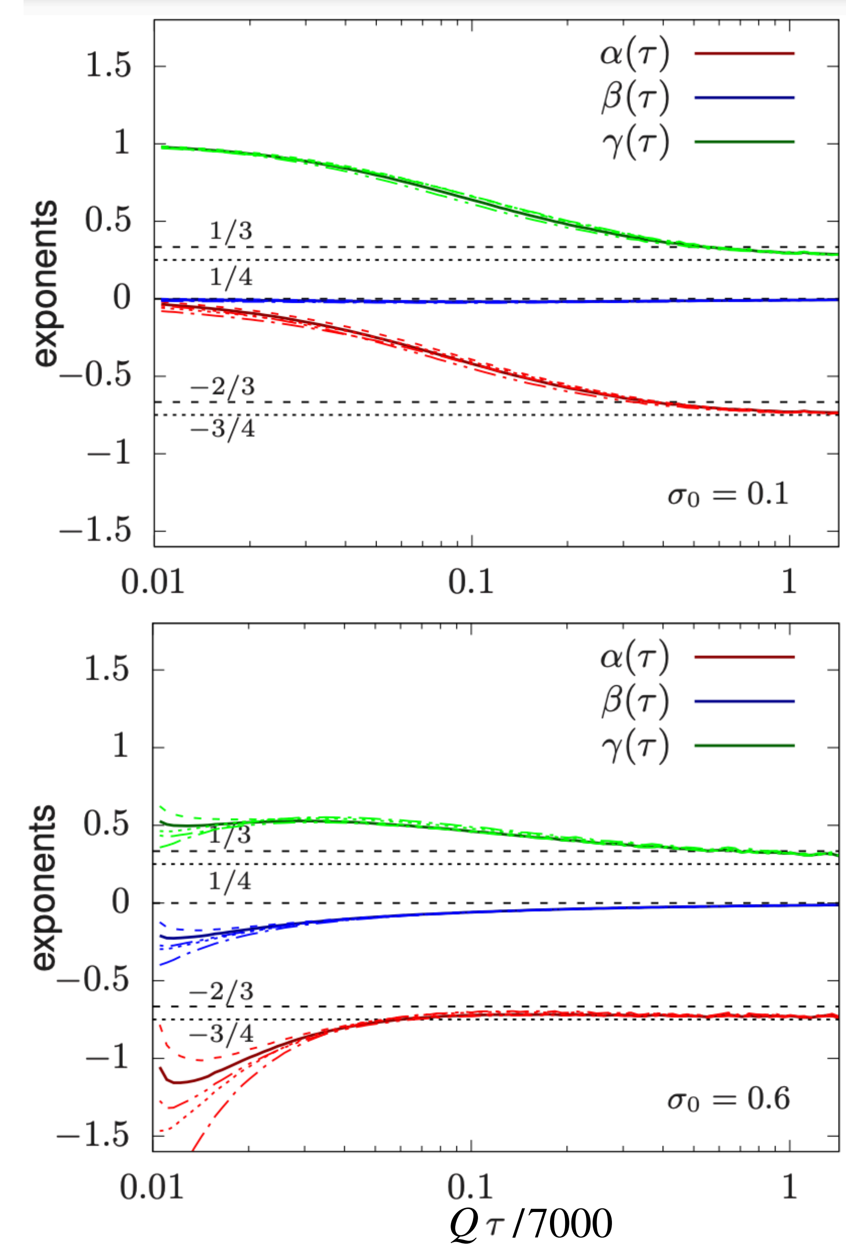
$$f(p_T, p_z, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_z\Big)$$ with $\alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$



Classical statistical simulations of SU(2) gluons

1303.5650 and 1311.3005

by Berges, Boguslavski, Schlichting and Venugopalan



$g = 10^{-3}$ EKT with 2 massless quarks

1810.10554

by Berges and Mazeliauskas

Simplification in this talk: isotropy

No expansion: $ds^2 = -dt^2 + d\vec{x}^2$ and time dependence on t

Distribution function: $f(t, p \equiv |\vec{p}\,|)$

Nonthermal fixed point has only two scaling exponents:

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Isotropic nonthermal fixed points

e.g. **1810.08143** by Schmied, Mikheev and Gasenzer
or **2005.12299** with Berges, Mazeliauskas, Venugopalan

Ingredients:

weak coupling + overoccupied initial states $f(t=0,p) \gg f_{eq}(p)\Big|_{\text{same energy density}}$

Outcome:

simulations + cold atom experiments show prolonged self-similar time evolution

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Observation: the choice of the origin of time

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Special points for scaling phenomena

- $t = 0$
- $t \to \infty$

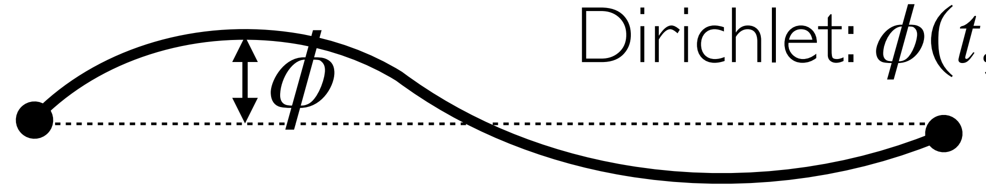
However, t used above has an origin chosen in a rather arbitrary way.

Near-equilibrium ingredient: quasinormal modes

Normal modes

Wave equation in a cavity $\begin{array}{l}\Box_{\text{cavity}}\,\phi = 0\\ \phi\big|_{\text{bdries}} = 0\end{array}$ $\longrightarrow$ spectrum of normal modes e.g.

Dirichlet: $\phi(t, L) = 0$



Dirichlet: $\phi(t,0) = 0$

$$\Big\downarrow \; -\partial_t^2\phi + \partial_x^2\phi = 0$$

$$\phi_{\text{NM}} \sim e^{-i\omega_{\text{NM}}t}\sin\left(n\pi x/L\right) \quad \text{with} \quad \omega_{\text{NM}} = \pm\, n\pi/L$$

If excited, normal modes just oscillate (possibly interfering with each other)

Quasinormal modes e.g. **0905.2975** by Berti, Cardoso and Starinets

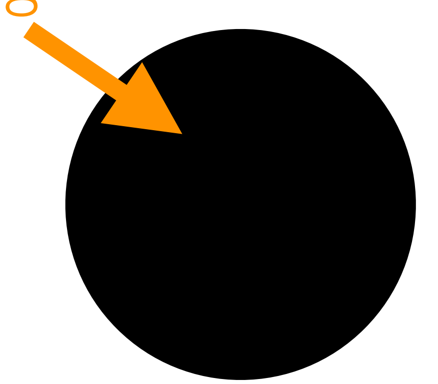
Option I: the equation breaks time symmetry explicitly, e.g.

$$\partial_t \phi - D\partial_x^2 \phi = 0 \quad \text{on a line} \quad \phi_{\rm QNM} \sim e^{-i\omega_{\rm QNM} t} e^{ikx} \quad \text{with} \quad \omega_{\rm QNM} = -iDk^2 \in \mathbb{C}$$

Option II: the boundary condition makes the problem non-Hermitian

ϕ purely ingoing at the horizon

black hole classically only absorbs : $\Box_{\text{black hole}}\,\phi = 0$ +



$\longrightarrow$ a tower of $\omega_{\rm QNM} \in \mathbb{C}$

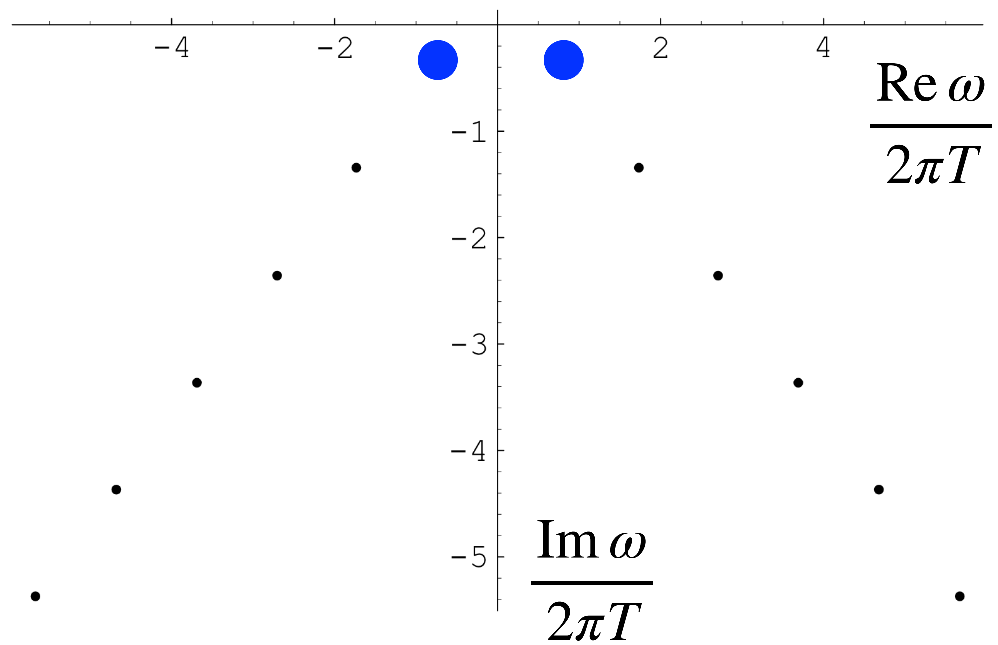
If excited, quasinormal modes decay in time (and often also oscillate)

Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\,t + i\,\vec{q}\cdot\vec{x}}$

plot for $q = 2\pi T$



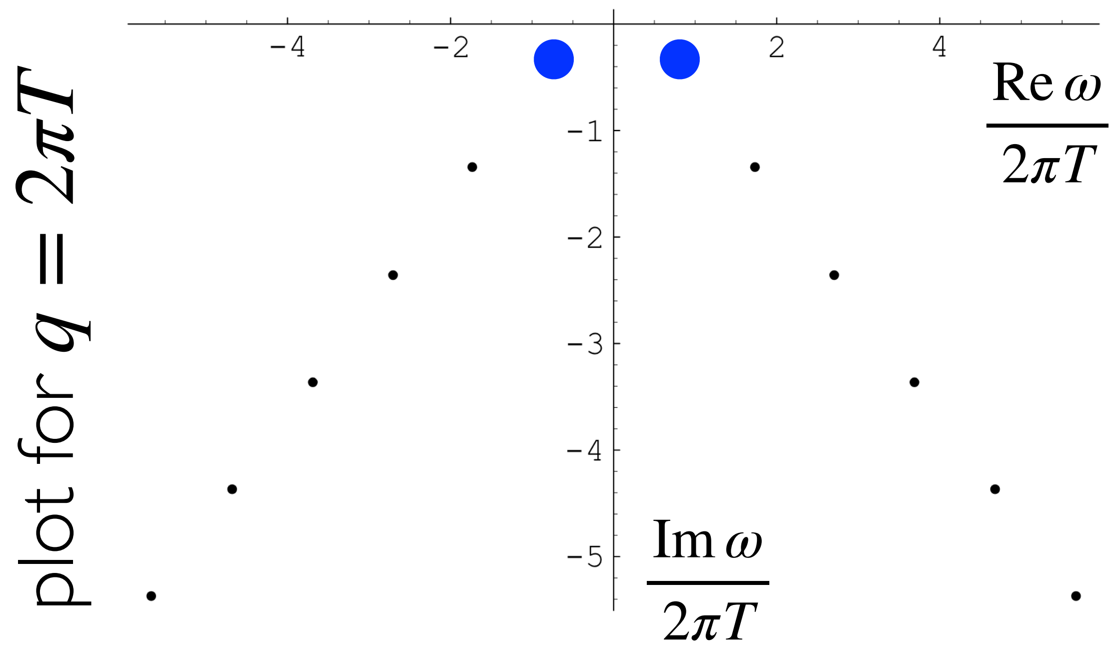
Consequences for thermalization → lots of short-lived excitations

Consequences for thermalization → a few long-lived hydrodynamic modes

Ingredient III: (relativistic) hydrodynamics

Relativistic hydrodynamics e.g. Florkowski, Heller & Spaliński 1707.02282

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\,t + i\,\vec{q}\cdot\vec{x}}$



Relativistic Navier-Stokes equations dictate the properties of sound propagation:

$$\omega = \pm\, c_s\, q - i\frac{4}{3T}\frac{\eta}{s}q^2 + \ldots$$

(shear) viscosity describes the dominant dissipative effect

Shear viscosity across systems

The KSS bound conjecture

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

hep-th/0405231 by Kovtun, Son, Starinets

$$\frac{\eta}{s} \overset{?}{\geq} \mathcal{O}\left(\frac{1}{4\pi}\right)$$

0812.2521 by Buchel, Myers, Sinha

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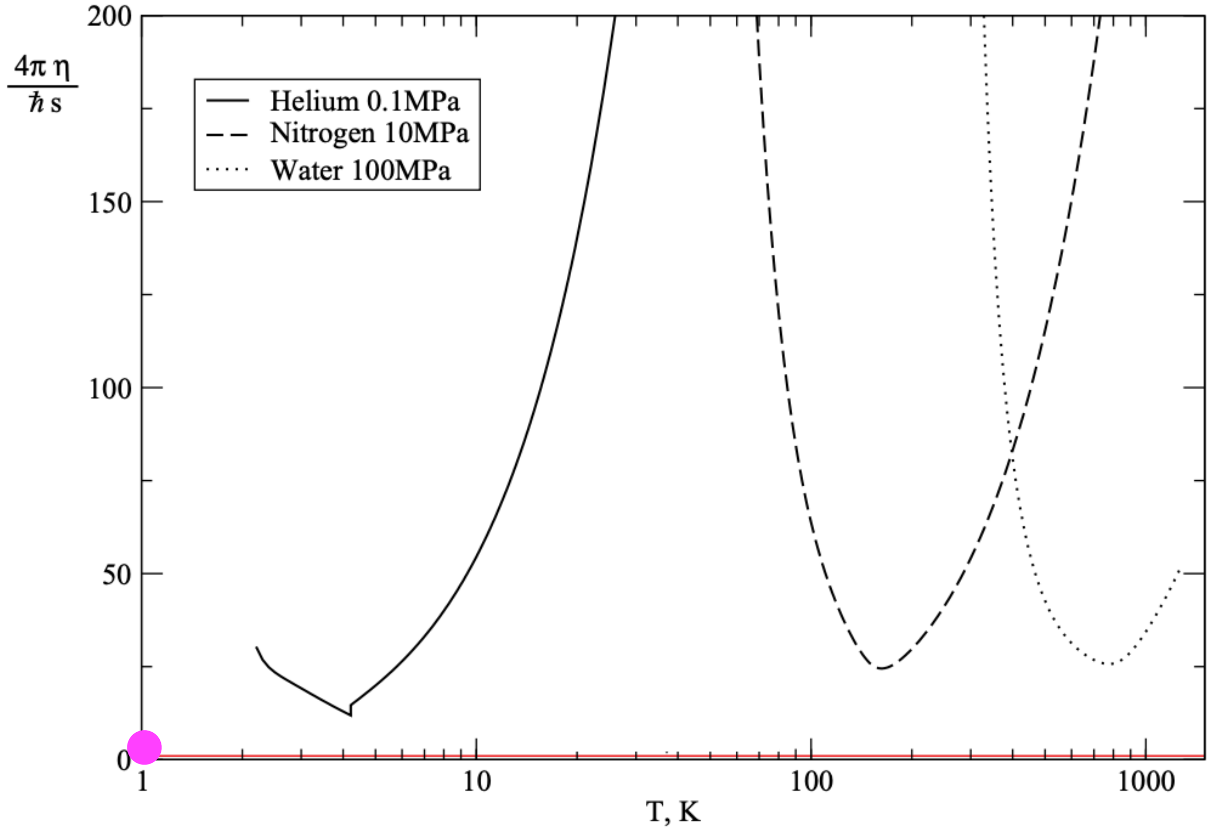
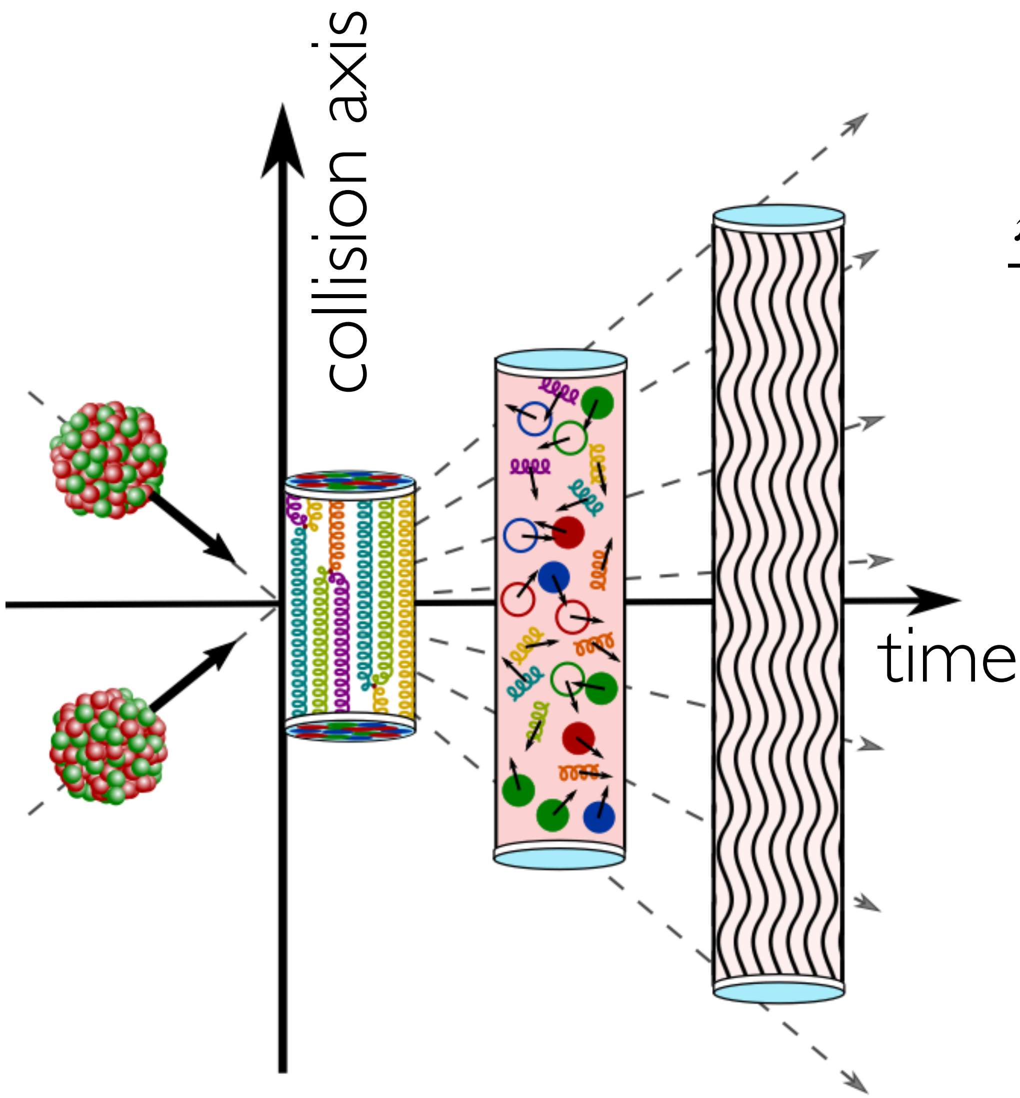
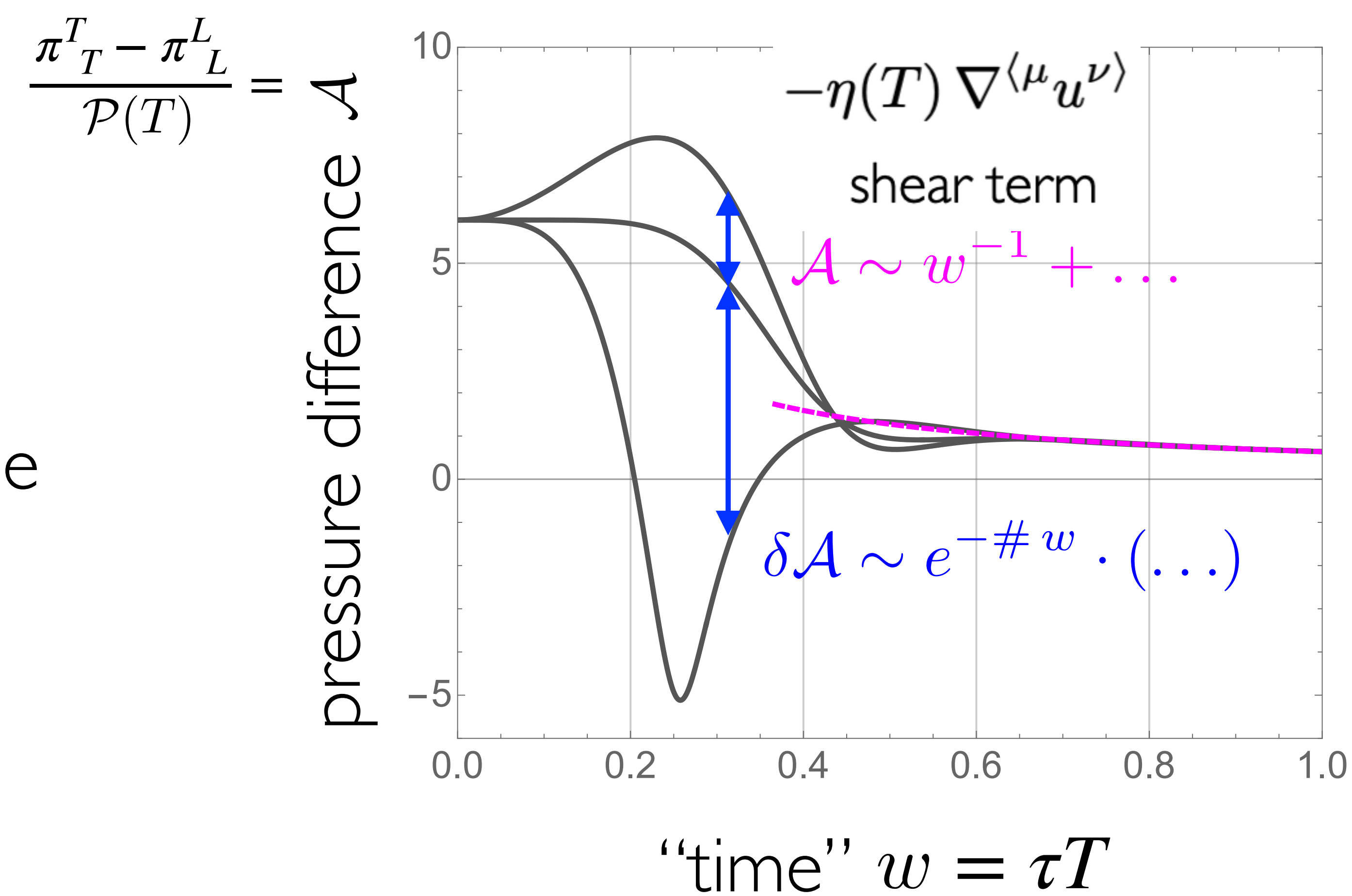


Hydro and transient QNMs in action

heavy-ion collisions
at RHIC and LHC

behaviour in
of theoretical models
(here: holographic boost-invariant flow)



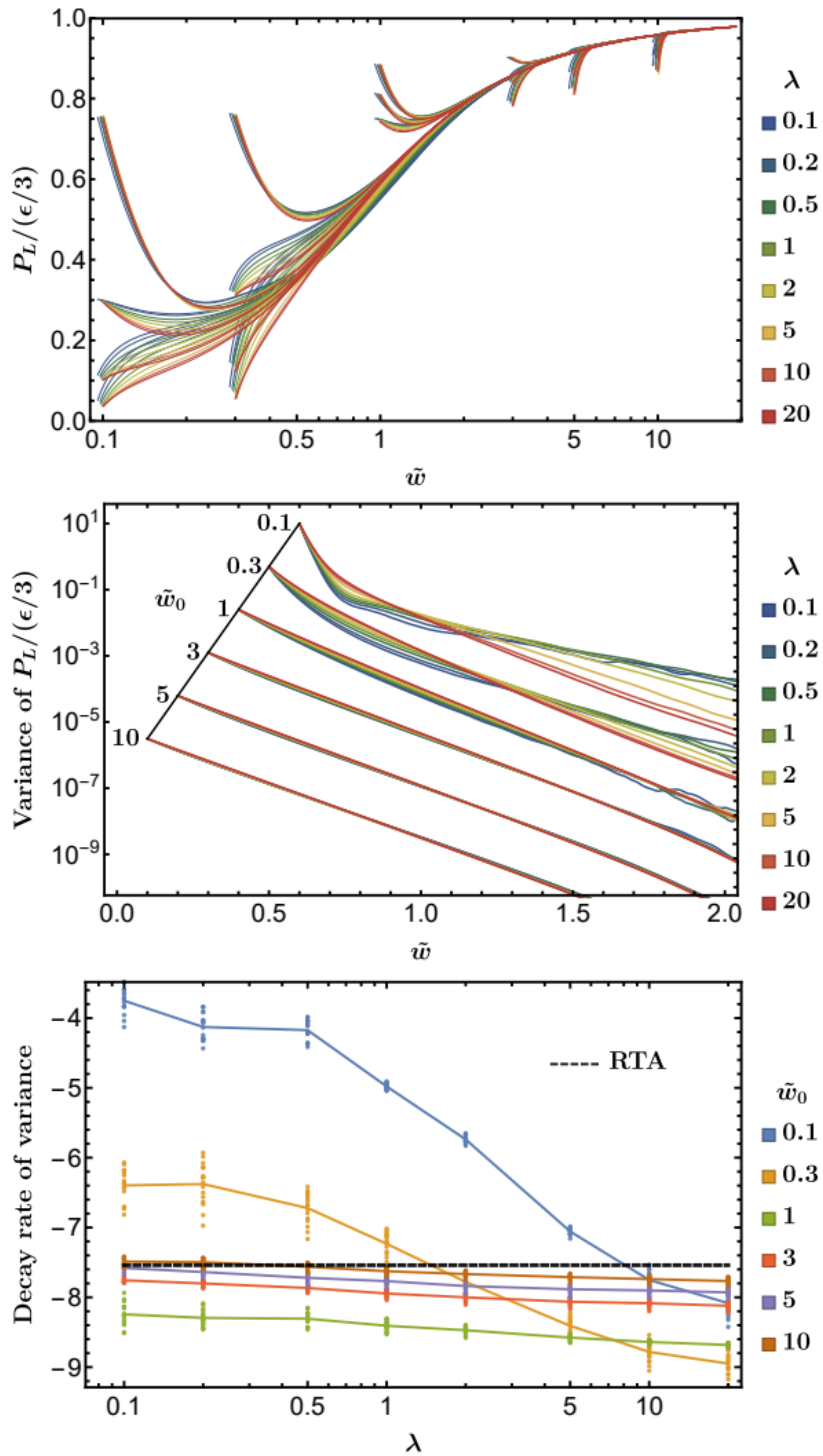


2005.12299
with Berges, Mazeliauskas & Venugopalan

1103.3452 with Janik & Witaszczyk

Extends to QCD kinetic theory ($\lambda = g^2 \times 3$)

2203.16549 with Du, Schlichting & Svensson



$$\delta\mathcal{A} \sim e^{-\#\, w} \cdot (\ldots)$$

$\# \sim (\eta/s)^{-1}$

Lecture 1: summary

Lecture I summary: nonthermal fixed points

Nonthermal fixed point: self-similar evolution in time at weak coupling

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Relevant for QCD thermalization and for cold atomic gases

Significant simplification in the dynamics, as it reduces to a rescaling

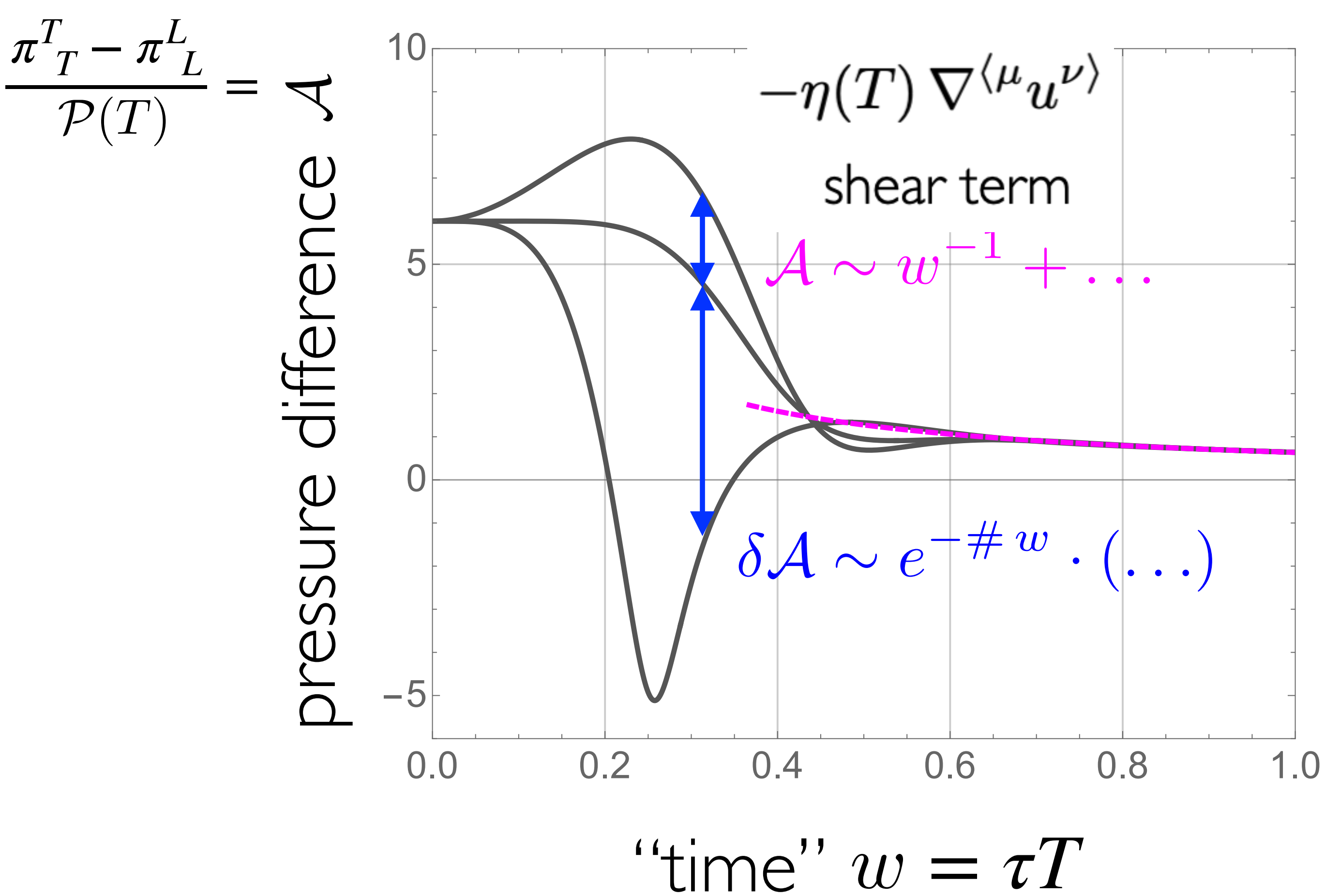
Different systems might exhibit the same exponents ($\longrightarrow$ universality)

Nonthermal fixed points act as attractors for a large class of initial states

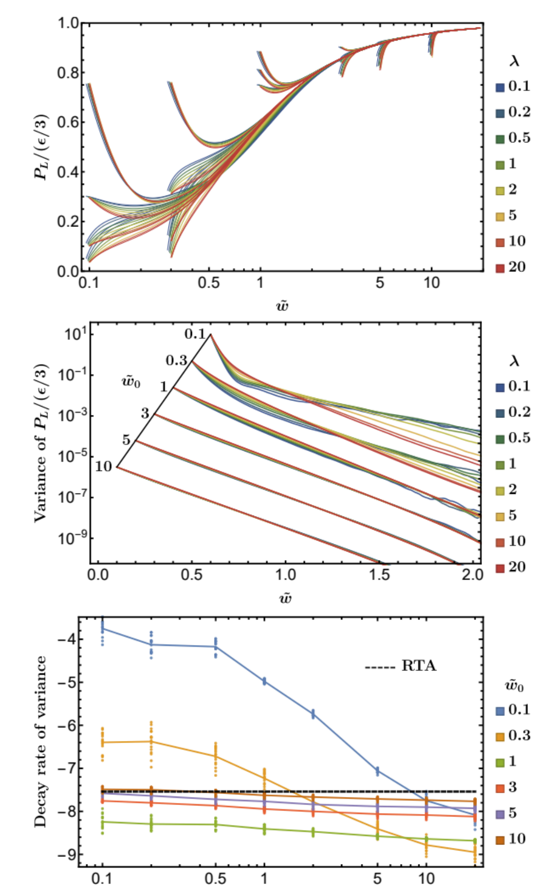
Lecture I summary: quasinormal modes

Quasinormal modes: frequency eigenmodes of systems with dissipation

Explain the attractive nature of (near-)equilibrium states:



1103.3452 with Janik & Witaszczyk



2203.16549 with Du, Schlichting & Svensson

Lecture I summary: hydrodynamics

Framework for describing relaxation of spatial inhomogeneities

Macroscopic construction based on effective field theory principles

Microscopic input: equation of state and transport coefficient

Quasinormal mode manifestation: sound waves, shear mode

Working horse for simulating nuclear collisions:

QCD dynamics $\longrightarrow$ relativistic fluid mechanics

Lecture II: teaser

The goal of these lectures

two key theoretical mechanisms underlying
state of the art understanding of thermalization in QCD are
nonthermal fixed points and **attraction to equilibrium**

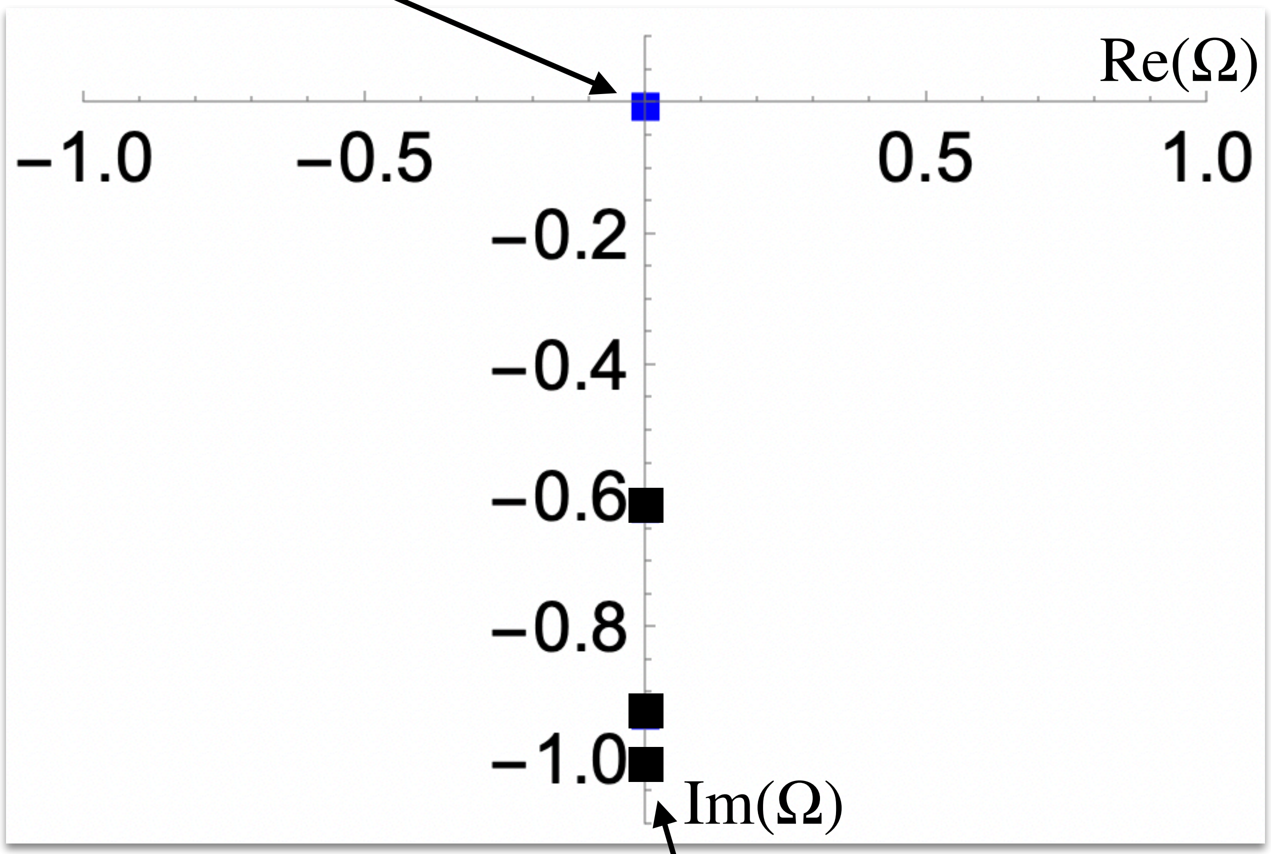
↓

novel phenomena in table top experiments
with cold atomic gases far from equilibrium

Lecture II teaser

First take on quasinormal modes of nonthermal fixed points:

hydrodynamics



experimentally confirmed