Holographic study of confining QCD-like theory on non-SUSY D2 brane and partial deconfinement

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Outline

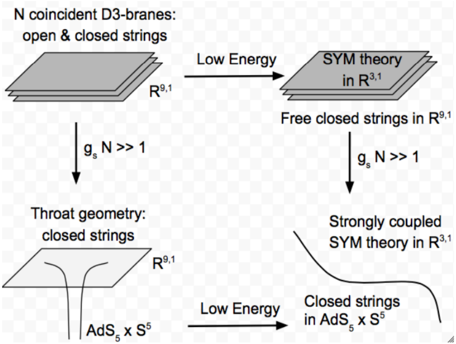
AdS/CFT holography

- **Strongly coupled d-dimensional Conformal Field Theory** $\Longleftrightarrow$ **Weakly coupled d+1 dimensional gravity theory in AdS spacetime**.

- **Non-perturbative Regime:** $g_{YM}^2 \sim g_s \to 0, N \to \infty$, $g_{YM}^2 N$=Large, finite.

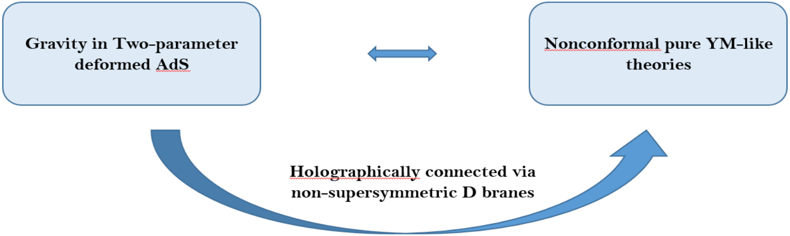
  [**Maldacena,'98; Witten, '98**]

- **AdS/CFT from D branes :**



[Aharony et al., '99]

Gauge-Gravity Duality and Non-SUSY D branes

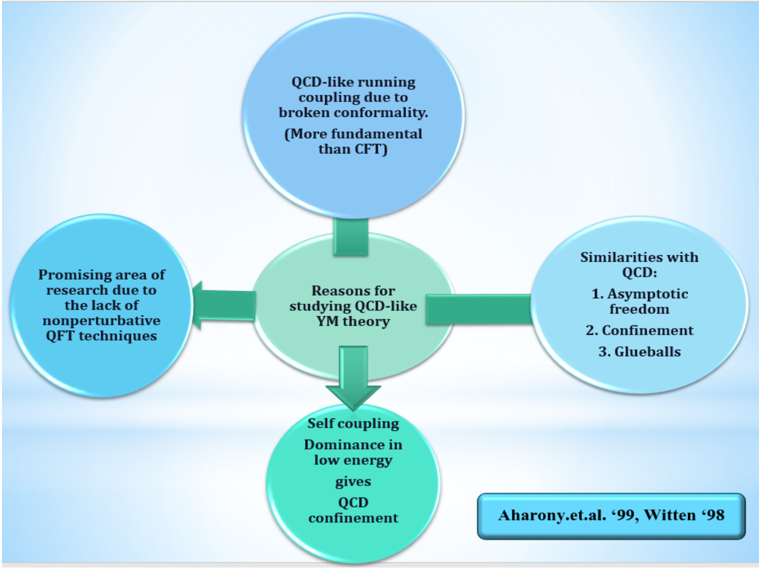


Nonconformal Dp Branes:

- Relaxation in supersymmetry
- More parameters than usual D branes.
- Nonconformal YM theory in the worldvolume
- Asymptotically merges to usual D branes.
- Delivers a platform to study QCD confinement

[Lu, Roy, '04]

Why pure Yang-Mills theory??



Previous studies and Goal of our work

*Confinement of 3+1D pure YM-like theories with dimensionless coupling are achieved by using low energy decoupling limit of nonsupersymmetric D3 branes.

*A top-down holographic model (similar to Witten-Sakai-Sugimoto model) serves such purpose by assuming the gauge theory as pure YM-type and henceforth confirming it by studying different low-energy confining properties.

*Such study does not require setting any cutoff along the energy scale as it is required while probine confinement with AdS/CFT holography.

Our goal

Probing low-energy confinement in nonperturbative 2+1D pure YM-like theory with dimensionful coupling on the worldvolume of non-SUSY D2 brane via top-down holography

Non-SUSY Dp brane Solutions

Non-SUSY Dp brane

$$ds^2 = \frac{G(\rho)^{\frac{\delta}{4}}}{\sqrt{F(\rho)}}\left(-dt^2 + \sum_{i=1}^{p}(dx^i)^2\right) + F(\rho)^{\frac{1}{2}}G(\rho)^{\frac{1}{7-p}+\frac{\delta}{4}}\left(\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_{8-p}^2\right)$$

$$e^{2(\phi-\phi_0)} = F(\rho)^{\frac{3-p}{2}}G(\rho)^{\delta}, F(\rho) = G(\rho)^{\frac{\alpha}{2}}\cosh^2\theta - G(\rho)^{-\frac{\beta}{2}}\sinh^2\theta,\ G(\rho) = 1 + \frac{\rho_p^{7-p}}{\rho^{7-p}}$$

[J. X. Lu, S. Roy, '04]

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[J. X. Lu, S. Roy, '04]

- **The singularity**: $\rho = 0$ ($\rho \perp$ to worldvolume, Dp brane is sitting at singularity)
- $\rho_p \to$ mass parameter, $\delta \to$ Dilaton parameter.
- Dilaton $\phi = \phi(\rho) \implies$ **running effective coupling**

• **Asymptotic Limit**: $\rho_p \to 0$

Isotropic Non-SUSY D2 brane in decoupling limit

Metric and associated dilaton along with flux

Low Energy Decoupling Limit: $l_s \to 0, \ \rho = \alpha^{'} u, \ \rho_2 = \alpha^{'} u_2$

$$ds^2 = \alpha' \left[\sqrt{\frac{\gamma u_2^5}{L}} F(u)^{-\frac{1}{2}} G(u)^{\frac{\delta}{4}+\frac{\beta}{4}} \left(-dt^2 + \sum_{i=1}^{2} (dx^i)^2 \right) + \sqrt{\frac{L}{\gamma u_2^5}} F(u)^{\frac{1}{2}} G(u)^{\frac{1}{5}+\frac{\delta}{4}-\frac{\beta}{4}} \left(\frac{du^2}{G(u)} + u^2 d\Omega_6^2 \right) \right]$$

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$$e^{2\phi} = \frac{g_{\mathrm{YM}}^4}{4} \sqrt{\frac{L}{\gamma u_2^5}} F(u)^{\frac{1}{2}} G(u)^{\delta - \frac{\beta}{4}},$$

$G(u) = 1 + u_2^5/u^5$ and $F(u) = G(u)^\gamma - 1$, extra parameters: u_2

[**Nayek, Roy, ’16**]

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[**Nayek, Roy, '16**]

- $G(u) > 1,\ F(u) > 0$, for $0 < u < \infty$, $-\frac{8}{5} < \delta < \frac{8}{5}$
- $u \implies$ **Energy scale**, $u_2 \implies$ Constant point due to nonconformality.,
- Dilaton $\phi = \phi(u) \implies$ **running effective coupling**
- Asymptotic limit: $u_2 \to 0$

Effective Running Coupling

- **The effective gauge coupling**

$$\lambda^2 = (N_c e^{\phi})^{(4/5)} = \frac{1}{(6\pi^2)^{\frac{4}{5}}\gamma^{\frac{1}{5}}}\left(\frac{L}{u_2}\right) F(u)^{\frac{1}{5}} G(u)^{\frac{2}{5}\delta - \frac{\beta}{10}}$$

[Aharony, Gubser, Maldacena, Ooguri, Oz]

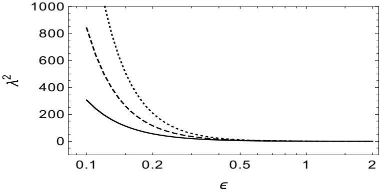

Figure: Dependence of λ^2 on the energy scale $u = \epsilon u_2$ for $\delta = 0.98$ (solid line), 1.28 (dashed line) and 1.58 (dotted line)

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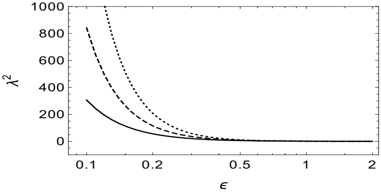

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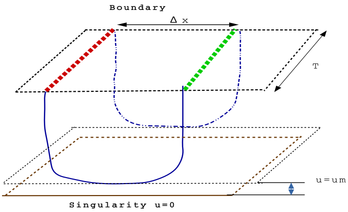
- **Holographic Approach is valid for** $\boxed{\frac{1}{\gamma^{1/5}} g_{\mathrm{YM}}^2 N_c^{\frac{1}{5}} \leq u_2 \leq g_{\mathrm{YM}}^2 N_c}$.

• $u_2 = g_{YM}^2 N_c \implies$ **entirely non-perturbative regime**.

Quark-Antiquark Potential



* $V \sim \sigma r$ in strong coupling regime, σ is QCD string tension.

$< W_F(\mathcal{C}) >= Exp[iV(x)\tau] = Exp[i\sigma A]$

Nambu-Goto action of the open string $\equiv$ Minimal Wilson loop area A.

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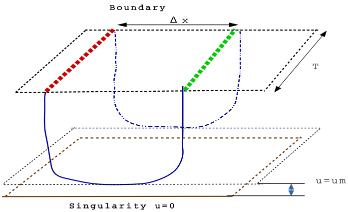

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- $V(x) = \frac{u_2^{\frac{5}{2}}}{2\pi\sqrt{L}} \left[\frac{5}{8}\delta^2 - \frac{3}{5}\right]^{\frac{1}{4}} \Sigma^{\frac{5\delta}{16\gamma}} \Delta x,$ for largest $q\bar{q}$ separation Δx.
- QCD string tension: $\frac{\sigma}{g_{\mathrm{YM}}^4 N_c^2}\Big|_{u_2 = g_{YM} N_c^2} = \frac{1}{2\sqrt{3}\pi^2} \left(\frac{5}{8}\delta^2 - \frac{3}{5}\right)^{\frac{1}{4}} \Sigma^{\frac{5\delta}{16\gamma}}$

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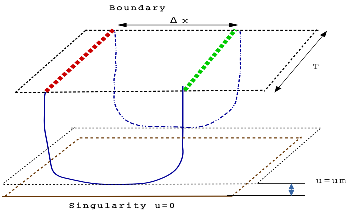

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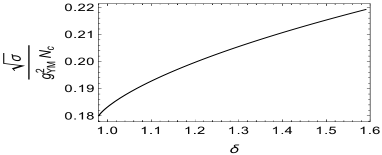

Strong confinement for higher δ in nonperturbative regime, Consistent with λ results.

Anistropic D2 brane and its decoupled geometry

- Anisotropy along the time direction
- $\delta_1 = \frac{4}{21}\left(8\delta \pm \sqrt{21 - 20\delta^2}\right)$

- δ: **Anisotropy parameter**, (plays crucial role in QCD phase transition)
- **Running couping:** $\lambda^2 = \frac{1}{(6\pi^2)^{\frac{4}{5}}}\left(\frac{L}{u}\right)\left(1 + \frac{u_2^5}{u^5}\right)^{-\frac{1}{10}+\frac{21}{40}\delta_1-\frac{4}{5}\delta}$,

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- **Confining potential**:

$$V = \frac{\Delta x}{2\pi}\sqrt{\frac{u_2^5}{8L}}\,(7\delta_1 - 8\delta - 4)^{\frac{1}{4} - \frac{7}{16}\delta_1 + \frac{1}{2}\delta} \times (7\delta_1 - 8\delta + 4)^{\frac{1}{4} + \frac{7}{16}\delta_1 - \frac{1}{2}\delta}$$

- **QCD string tension:**

$$\frac{\sqrt{\sigma}}{g_{YM}^2 N_c} = \frac{1}{2\pi}\left(\frac{1}{6}\right)^{\frac{1}{4}}\left[(7\delta_1 - 8\delta)^2 - 16\right]^{\frac{1}{8}} \times \left(\frac{7\delta_1 - 8\delta + 4}{7\delta_1 - 8\delta - 4}\right)^{\frac{7}{32}\delta_1 - \frac{1}{4}\delta}$$

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δ	0.0	-0.1	-0.2	-0.3	-0.4	-0.49
$(-\delta)^{1/5}$	0.0	0.63	0.72	0.78	0.83	0.86
$\frac{\sqrt{\sigma}}{g_{YM}^2 N_c}$	0.2010	0.1980	0.1942	0.1891	0.1824	0.1729

σ decreases with increasing $-\delta$
$\Longrightarrow$ σ increases with self-coupling.

Hawking-Page Transition

* **Hawking-Page transition** : Thermal brane (without horizon)$\rightarrow$ Black brane(with horizon)

At $\delta = -1$, $\delta_1 = -\frac{4}{3}$ generates

The standard black D2 brane in the Einstein frame:

$$ds^2 = \bar{H}(r)^{\frac{3}{8}} \left(\frac{dr^2}{\tilde{f}(r)} + r^2 d\Omega_6^2 \right) + \bar{H}(r)^{-\frac{5}{8}} \left(-\tilde{f}(r) dt^2 + dx_1^2 + dx_2^2 \right)$$

$$\bar{H}(r) = 1 + \frac{\rho_2^5 \sinh^2\theta}{r^5}, \quad \tilde{f}(r) = 1 - \frac{\rho_2^5}{r^5}$$

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At $\delta = -1$, $\delta_1 = -\frac{12}{7}$ generates

$$ds^2 = \bar{H}(r)^{\frac{3}{8}} \tilde{f}(r)^{\frac{1}{40}} \left(\frac{dr^2}{\tilde{f}(r)} + r^2 d\Omega_6^2 \right) + \bar{H}(r)^{-\frac{5}{8}} \tilde{f}(r)^{-\frac{1}{24}} \left(-\tilde{f}(r) dt^2 + dx_1^2 + dx_2^2 \right)$$

$e^{2\phi} = g_s^2 \tilde{H}(r)^{\frac{1}{2}} \tilde{f}(r)^{-\frac{1}{2}}$, For, $\rho_2 \ll r \implies$ The metric reduces to that of black D2 brane

Light pseudoscalar glueball masses

- **Glueball (J^{PC})** : Bound state of self interacting gluons.
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- **For isotropic and anisotropic D2-brane** : $M_{0^{-+}} \approx n\sqrt{\sigma}$ for $n = 0, 1, 2$.

$$\sigma \propto \lambda \implies M_{0^{-+}} \propto \lambda$$ **(Strong confinement)**

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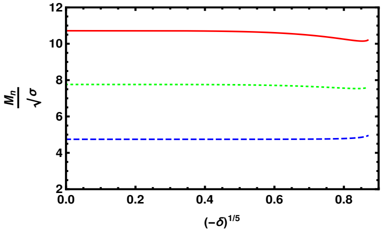
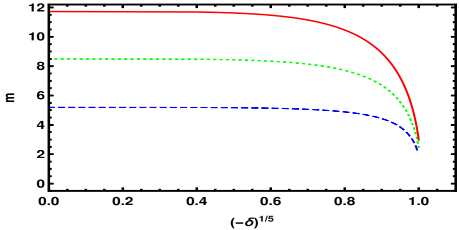


Increase in anisotropy $\implies$ Towards deconfinement

QCD phase transition : Partial deconfinement



- At $\delta = -1$, sharp fall in glueball mass as well as mass gap.
- Hawking Page transition $\equiv$ QCD Confinement-deconfinement phase transition.
- **Smooth crossover at HP transition point**
- Implication of **Partial Deconfinement**, i.e., the deconfinement of only some subgroup SU(M) ($M < N$) of SU(N) gauge theories.

[Hanada, Ishiki and Watanabe; 2018]

Influence of finite temperature for anisotropic case

- Empirical relation between T and δ: $\boxed{T = (-\delta)^{1/5}\, T_c}$
- At $\delta = -1$, $\boxed{\text{T=T}_c \text{ (Hawking-Page transition point)}}$

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Some observations regarding the variation in temperature:

- σ decreases slowly with T
- M_n is max at $T = 0$ and decreases slowly with T's
- Experiences a sharp fall at $T = T_c \rightarrow$ confinement-deconfinement transition.
- $M_{n_1} - M_{n_2} \propto T$, reduces abruptly at $T = T_c$.

Summary

- $2+1$ dimensional strongly coupled QCD-like theory is consistent as a worldvolume theory on both non-SUSY isotropic and anisotropic D2 brane.
- Effective coupling, string tension and glueball spectrum behave quite similarly as in confined QCD.
- With finite anisotropy, their variations quite consistently connect the HP transition in the bulk with QCD phase transition in the gauge theory at a certain δ value.
- Trivially small glueball mass at HP transition $\implies$ Partial deconfinement.

Future scope:

- Studying confined hadrons with a bottom-up construction of nonconformal gauge/gravity holography...(work in progress)
- Explicit analysis of partially deconfined phases using similar top-down holography for black D brane geometries...

Thank you for your attention