A study of gauge dependence in polarized W boson production at the LHC

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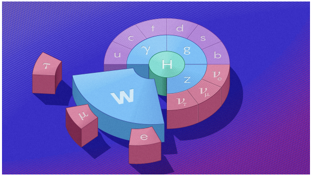
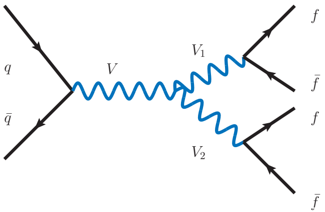
Focus: The W boson

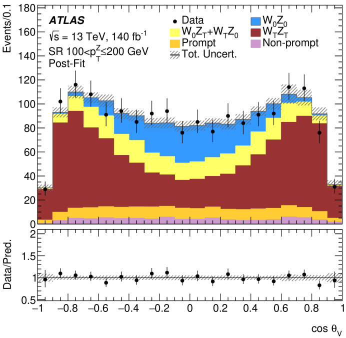


- Mediator of weak interactions (chiral nature)
- Massive gauge boson having spin ±1 (mass $\approx$ 80 GeV)
- Unlike photon and gluon (on-shell), it has three polarization states
- On-shell W: two transverse ($\lambda = \pm1$), one longitudinal ($\lambda = 0$)
- Off-shell W: scalar polarization (extra polarization state)

Big Motivation

- Helicity polarization of gauge bosons directly influences the kinematics and angular distributions of their decay products

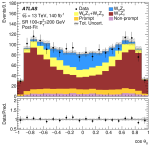


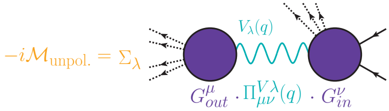


Source: ATLAS Collaboration, arXiv:2402.16365.

- Study of helicity polarized vector boson production is an important probe to understand the gauge theory and EWSB

Motivation





- Assumption in theory prediction: Interference between different helicity polarized states is negligible

- Amplitude involving unpolarized weak bosons (λ = boson helicity),

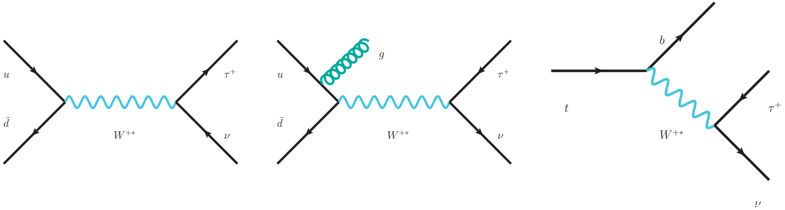
$$\mathcal{M}_{\text{full}} = \sum_{\lambda=0,\pm1,S} \mathcal{M}_\lambda = \sum_{\lambda=0,\pm1,S} J^\mu_{\text{in}}\, \Pi_\lambda J^\nu_{\text{out}} \tag{1}$$

$$|\mathcal{M}_{\text{full}}|^2 = \sum_{\lambda=0,\pm1,s} \underbrace{|\mathcal{M}_\lambda|^2}_{\text{polarized MEs}} + \underbrace{\sum_{\lambda\neq\lambda'} \mathcal{M}_\lambda \mathcal{M}'_\lambda}_{\text{interference terms}} \tag{2}$$

- In general, interference is not guaranteed to be negligible

Goal

- Our main goal:
  1. Investigate how large the interference term is in the theoretical prediction of the total and differential cross sections
  2. Identify when interference effects can be considered negligible
- Selected processes for investigation:
  1. Example 1: $u\bar{d} \to W^* \to \tau\nu$
  2. Example 2: $u\bar{d} \to W^* g \to \tau\nu g$
  3. Example 3: $t \to W^* b \to \tau\nu b$



Recap of Gauge Symmetry

- Lagrangian of photon field:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad \text{where} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

- Gauge transformation:

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \alpha(x) \tag{4}$$

- Gauge symmetry:

$$F'_{\mu\nu} = \partial_\mu (A_\nu + \partial_\nu \alpha) - \partial_\nu (A_\mu + \partial_\mu \alpha) = F_{\mu\nu} \tag{5}$$

  Lagrangian is invariant under this transformation for different choices of α

- Gauge invariance: Redundancy of the sysmtem
  $\Longrightarrow$ each choice of $\alpha \longleftrightarrow$ same physical state

- Gauge fixing is needed to define a physically meaningful propagator
  $\Longrightarrow$ It introduces an unphysical parameter(s), e.g., ξ in R_ξ gauge

Propagators for W boson in R_ξ gauge

- Full (unpolarized) propagator: [2]

$$\Pi_{\mu\nu} = \frac{-i}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{1-\xi}{q^2 - m_W^2} q_\mu q_\nu\right) = \frac{i}{q^2 - m_W^2} \sum_{\lambda=0,\pm1,s} \eta_\lambda \epsilon_\mu \epsilon_\nu^* \tag{6}$$

- Transverse propagator sum ($\lambda = \pm 1$):

$$\Pi^t_{\mu\nu} = \frac{-i}{q^2 - m_W^2} \sum_{\lambda=\pm1} \eta_\lambda \epsilon_\mu \epsilon_\nu^* = \frac{i}{q^2 - m_W^2}(-g_{\mu\nu} - \Theta_{\mu\nu}) \tag{7}$$

- Longitudinal propagator ($\lambda = 0$):

$$\Pi^0_{\mu\nu} = \frac{i}{q^2 - m_W^2}\left(\Theta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \tag{8}$$

- Scalar propagator($\lambda = s$):

$$\Pi^s_{\mu\nu} = \frac{i}{q^2 - m_W^2}\left(\frac{q_\mu q_\nu}{q^2} + \frac{\xi - 1}{q^2 - \xi m_W^2} q_\mu q_\nu\right) \tag{9}$$

[2] $\eta_0 = \eta_{\pm1} = 1, \eta_s = -1$

Interference terms

- Full (unpolarized) matrix element squared,

$$|\mathcal{M}_{\text{full}}|^2 = \sum_{\lambda=0,\pm1,s} \underbrace{|\mathcal{M}_\lambda|^2}_{\text{polarized MEs}} + \underbrace{\sum_{\lambda\neq\lambda'} \mathcal{M}_\lambda \mathcal{M}'_\lambda}_{\text{interference terms}}$$

$$\text{Interference} = |\mathcal{M}_{\text{full}}|^2 - \sum_{\lambda=\pm1} |\mathcal{M}_\lambda|^2 - |\mathcal{M}_0|^2 - |\mathcal{M}_S|^2 \quad (10)$$

$$\begin{aligned} \text{Interference} \propto \quad & -\mathcal{O}\left[|\Theta_{\mu\nu}|^2\right] - \mathcal{O}\left[\text{Re}(g_{\mu\nu}, \Theta_{\mu\nu})\right] - \mathcal{O}\left[\text{Re}(q_\mu q_\nu, \Theta_{\mu\nu})\right] \\ & -\mathcal{O}\left[(q^2 - m^2)\text{Re}(q_\mu q_\nu, g_{\mu\nu})\right] \end{aligned} \quad (11)$$

- The term $\Theta_{\mu\nu}$ cancels out in the full propagator, but appears in the interference terms of the unpolarized matrix element squared

Interference terms (continued...)

- For W momentum $q^\mu = (E_V, |\vec{q}|\sin\theta_V\cos\phi_V, |\vec{q}|\sin\theta_V\sin\phi_V, |\vec{q}|\cos\theta_V)$, $\Theta_{\mu\nu}(\theta_V, \phi_V)$ is given by

$$\Theta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos^2\phi_V\ \sin^2\theta_V & \cos\phi_V\ \sin^2\theta_V\ \sin\phi_V & \cos\theta_V\ \cos\phi_V\ \sin\theta_V \\ 0 & \cos\phi_V\ \sin^2\theta_V\ \sin\phi_V & \sin^2\theta_V\ \sin^2\phi_V & \cos\theta_V\ \sin\phi_V\ \sin\theta_V \\ 0 & \cos\theta_V\ \cos\phi_V\ \sin\theta_V & \cos\theta_V\ \sin\phi_V\ \sin\theta_V & \cos^2\theta_V \end{bmatrix}$$

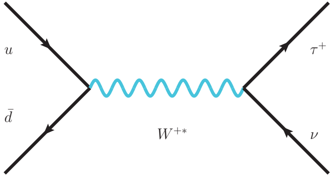
- Also $\Theta_{\mu\nu}$ has a nice structure (in R_ξ and axial gauges),

$$\Theta_{\mu\nu} = \frac{(n\cdot q)}{(n\cdot q)^2 - q^2 n^2}\left[-n_\mu q_\nu - q_\mu n_\nu + \frac{q^2}{(n\cdot q)} n_\mu n_\nu + \frac{n^2}{(n\cdot q)} q_\mu q_\nu\right] \tag{12}$$

where the choices of n are frame dependent,
$n = (1, \vec{0})$ (time-like) or
$n = (0, -\hat{q})$(space-like) or
$n = (1, -\hat{q})$(light-like)

- We are interested in knowing how this Θ interacts with vector currents

Example 1: $u\bar{d} \to W^{+*} \to \tau^+\nu_\tau$



- Polarized matrix element: $\mathcal{M}^\lambda = J^\mu_{ud}\Pi^\lambda_{\mu\nu}J^\nu_{\tau\nu}$
- Currents:

$$J^\mu_{ud} = \bar{v}_L(p_d)\left(-\frac{ig}{\sqrt{2}}\gamma^\mu P_L\right)v_L(p_u) \qquad J^\nu_{\tau\nu} = \bar{u}_L(p_\nu)\left(-\frac{ig}{\sqrt{2}}\gamma^\nu P_L\right)v_L(p_\tau) \tag{13}$$

- Rest frame of W^*: $|\vec{q}| = 0,\ \theta_V = \phi_V = 0 \implies q^\mu = (Q, \vec{0}), \hat{q} = (0, 0, 1)$
  Momentum conservation: $q^\mu = p^\mu_u + p^\mu_d = p^\mu_\tau + p^\mu_\nu$,
  Choosing n spacelike: $n^\mu = (0, -\hat{q}) = (0, 0, 0, -1)$

Example 1: $u\bar{d} \to W^{+*} \to \tau^+\nu_\tau$(continued...)

- For massless quarks, $J^\mu_{ud} = -\frac{ig}{\sqrt{2}}[\bar{v}_R(p_d)\gamma^\mu P_L u_L(p_u)] = (0, Q, -iQ, 0)$

  $\implies J^\mu_{ud} q_\mu = -\frac{ig}{\sqrt{2}}[\bar{v}_R(p_d)(m_d P_L + m_u P_R)u_L(p_u)] = 0$ [from Dirac eqn]

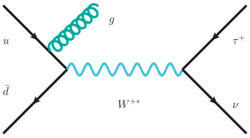
  $\implies J^\mu_{ud} n_\mu = -J^3_{ud} = 0$

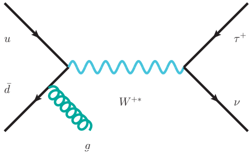
$$J^\mu_{ud}\Theta_{\mu\nu}J^\nu_{\tau\nu} \sim J^\mu_{ud}\left[-n_\mu q_\nu - q_\mu n_\nu + \frac{q^2}{(n\cdot q)}n_\mu n_\nu + \frac{n^2}{(n\cdot q)}q_\mu q_\nu\right]J^\nu_{\tau\nu} = 0 \tag{14}$$

- For two body scattering for massless incoming particles, the interference is zero

$$\text{Interference} \propto \; -\cancelto{0}{\mathcal{O}\left[\left|\Theta_{\mu\nu}\right|^2\right]} - \cancelto{0}{\mathcal{O}\left[\mathrm{Re}(g_{\mu\nu}, \Theta_{\mu\nu})\right]} - \cancelto{0}{\mathcal{O}\left[\mathrm{Re}(q_\mu q_\nu, \Theta_{\mu\nu})\right]}$$
$$-\cancelto{0}{\mathcal{O}\left[\mathrm{Re}(q_\mu q_\nu, g_{\mu\nu})\right]}$$

Example 2: $u\bar{d} \to W^{+*}g \to \tau^+\nu g$





- Polarized matrix element:

$$\mathcal{M}^\lambda = \mathcal{M}_u^\lambda + \mathcal{M}_d^\lambda \qquad \mathcal{M}_u^\lambda = J_{ud}^\mu \Pi_{\mu\nu}^\lambda J_{\tau\nu}^\nu \tag{15}$$

- Currents:

$$J_{ud}^\mu = \bar{v}_R(p_d) \underbrace{\left(-\frac{ig}{\sqrt{2}}\gamma^\mu P_L\right)}_{\text{W-u-d vertex}} \underbrace{\left(\frac{i\not{p}_a}{p_a^2}\right)}_{\text{Fermion propagator}} \underbrace{\left(-ig\gamma^\rho T_{ij}^a\right)}_{\text{Quark-gluon vertex}} \epsilon_\rho^*(k) u_L(p_u) \tag{16}$$

$$J_{\tau\nu}^\nu = \bar{u}_L(p_\nu) \left(-\frac{ig}{\sqrt{2}}\gamma^\nu P_L\right) v_L(p_\tau) \tag{17}$$

- This scattering process is sensitive to the $\Theta_{\mu\nu}$ term: non-zero interference

Example 2: Plots of $|\mathcal{M}|^2$ vs q

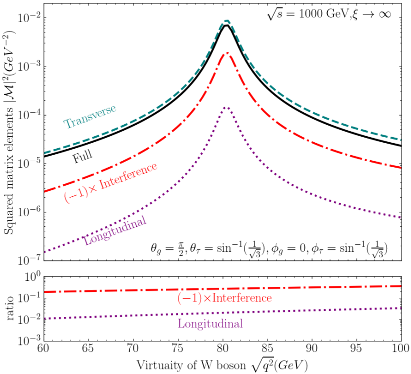


Figure: (a)

- For all polarization states, peak is at 80 GeV (as expected)

$$|\mathcal{M}|^2 \sim \frac{1}{(q^2 - m_W^2)^2 + (\Gamma_W m_W)^2}$$

- Interference terms

$$= |\mathcal{M}_{\text{Full}}|^2 - |\mathcal{M}_{\text{T}}|^2 - |\mathcal{M}_{\text{L}}|^2$$
$$\sim \mathcal{O}(|\Theta_{\mu\nu}|^2)$$
$$\sim \mathcal{O}(10\%) \text{ [From the plot (a)]}$$

- Scalar polarization contribution vanishes between two diagrams for each helicity configuration

Example 2:Plots of $|\mathcal{M}|^2$ vs q(continued...)

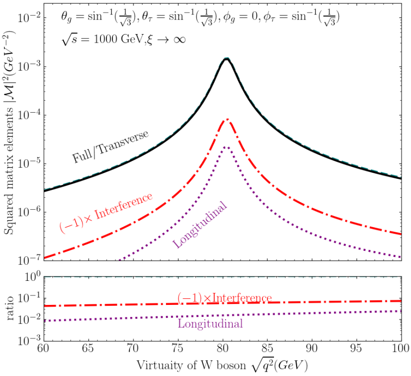


Figure: (b)

- Interference terms

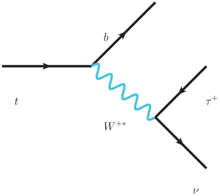
  $=|\mathcal{M}_{\mathsf{Full}}|^2 - |\mathcal{M}_{\mathsf{T}}|^2 - |\mathcal{M}_{\mathsf{L}}|^2$

  $\sim\mathcal{O}(|\Theta_{\mu\nu}|^2)$

  $\sim\mathcal{O}(5\%)$ [From the plot (b)]

- Interference effect varies with phase space points

Example 3: Top quark decay $t \to bW^{+*} \to b\tau^+\nu$

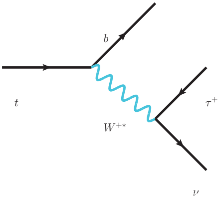


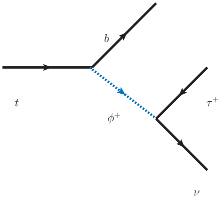
- This decay process is sensitive to scalar polarization as here top, bottom and tau are massive
- Total matrix element, $\mathcal{M}_{\text{full}} = \mathcal{M}_{\pm 1} + \mathcal{M}_0 + \mathcal{M}_S + \mathcal{M}_\phi$
- Matrix element for scalar polarization in R_ξ gauge:

$$\mathcal{M}^s = J^\mu_{tb} \Pi^s_{\mu\nu} J^\nu_{\tau\nu} = J^\mu_{tb} \left[\frac{i}{q^2 - m_W^2} \left(\frac{1}{q^2} + \frac{\xi - 1}{q^2 - \xi m_W^2} \right) q_\mu q_\nu \right] J^\nu_{\tau\nu}$$
$$= \mathcal{O}\left(\frac{1}{q^2}\right) \text{term} + \underbrace{\mathcal{O}(\xi)\ \text{term}}_{\mathcal{M}^{s\xi}} \qquad (18)$$

Example 3: Top quark decay (continued...)

- We need to consider the goldstone contribution to remove the gauge dependence in R_ξ gauge





$$
\begin{aligned}
(\mathcal{M}^{\phi^+} + \mathcal{M}^s) \sim \left(\frac{1}{q^2 - \xi m_W^2}\right)\left(\frac{1}{m_W^2} - \frac{\xi - 1}{q^2 - m_W^2}\right) - \frac{1}{q^2}\left(\frac{1}{q^2 - m_W^2}\right) \\
= \frac{1}{m_W^2 q^2} \neq 0
\end{aligned} \tag{19}
$$

- Scalar polarization has a survival physical contribution to the matrix element when W is off-shell

- ξ dependence is cancelled out between scalar polarization and goldstone boson

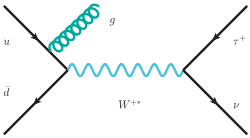
Conclusion and future outlook

1. Study of helicity polarized vector boson production is an important probe to understand the gauge theory and EWSB
2. Theory predictions for experiments assume that interference between helicity polarizations is negligible (not guaranteed)
3. Our main goal:
   - Investigate how large the interference term is in the theoretical prediction of the total cross section
   - Identify when interference effects can be considered negligible
4. In a simple two body scattering with massless incoming particles, the interference terms vanish
5. In $W + 1g$ production, interference effects are non-zero and can reach $\mathcal{O}(5\% - 10\%)$ at the squared matrix element level, depending on the phase-space point
6. Further investigation of interference effects in several other processes and their gauge dependence

Thank You

Back up slides

Why Scalar Polarization is zero?



Figure

- For internal u-diagrams:

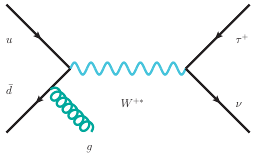
$$J^{\mu}_{u\bar{d}} = \bar{v}_R(p_d)\left(-\frac{ig}{\sqrt{2}}\gamma^{\mu}P_L\right)\left(\frac{i\not{p}_a}{p_a^2}\right)\left(-ig\gamma^{\rho}T^a_{ij}\right)\epsilon^*_{\rho}(k)u_L(p_u)$$

$$J^{\mu}_{u\bar{d}}q_{\mu} \sim -i\bar{v}_R(p_d)\gamma^{\mu}P_L\not{p}_a\gamma^{\rho}\epsilon^*_{\rho}(k)u_L(p_u)q_{\mu}\frac{1}{p_a^2}$$

$$= -i\bar{v}_R(p_d)\gamma^{\mu}(p_{a\mu}+p_{d\mu})P_L\not{p}_a\gamma^{\rho}\epsilon^*_{\rho}(k)u_L(p_u)\frac{1}{p_a^2} \quad [\bar{v}_R(p_d)\gamma^{\mu}p_{d\mu} = m_d\bar{v}_R]$$

$$= -i\bar{v}_R(p_d)\gamma^{\mu}p_{a\mu}P_L\not{p}_a\gamma^{\rho}\epsilon^*_{\rho}(k)u_L(p_u)\frac{1}{p_a^2}$$

$$= -i\bar{v}_R(p_d)P_R\gamma^{\rho}\epsilon^*_{\rho}(k)u_L(p_u) \qquad [\not{p}_a^2 = p_a^2 I]$$



Figure

- For internal d-diagrams:

$$
\begin{aligned}
J^{\mu}_{u\bar{d}} =& \bar{v}_R(p_d)\left(-ig\gamma^{\rho}T^a_{ij}\right)\left(\frac{-i\not{p}_b}{p_b^2}\right)\left(-\frac{ig}{\sqrt{2}}\gamma^{\mu}P_L\right)\epsilon^*_{\rho}(k)u_L(p_u) \\
J^{\mu}_{u\bar{d}}q_{\mu} \sim& i\bar{v}_R(p_d)\gamma^{\rho}\epsilon^*_{\rho}(k)\not{p}_b\gamma^{\mu}P_L u_L(p_u)q_{\mu}\frac{1}{p_b^2} \\
=& i\bar{v}_R(p_d)\gamma^{\rho}\epsilon^*_{\rho}(k)\not{p}_b P_R\gamma^{\mu}(p_{b\mu}+p_{u\mu})u_L(p_u)\frac{1}{p_b^2} \\
=& i\bar{v}_R(p_d)P_R\gamma^{\rho}\epsilon^*_{\rho}(k)u_L(p_u) \qquad [\not{p}_b^2 = p_b^2\mathbb{I}]
\end{aligned}
$$

- For scalar polarization, currents for internal u and d diagrams are exactly equal and opposite so they cancel each other (specially for massless quarks)

Top quark decay (through goldstone boson)

- In R_ξ gauge, goldstone boson diagram will come into play
- Currents for goldstone-fermions vertices:

$$
\begin{aligned}
J^{\mu}_{tb\phi} =& \bar{u}_L(p_2)\left[\frac{ig}{\sqrt{2}}\left(\frac{m_b}{m_W}P_L - \frac{m_t}{m_W}P_R\right)\right]u_L(p_1) \\
J^{\nu}_{\tau\nu\phi} =& \bar{u}_L(p_3)\left[\frac{-ig}{\sqrt{2}}\frac{m_\tau}{m_W}\right]v_L(p_3)
\end{aligned}
$$

- Goldstone boson propagator:

$$
\Pi^{\phi^+}_{\mu\nu} = \frac{i}{q^2 - \xi m_W^2}
$$

Top quark decay (through goldstone boson)

- Gauge dependent term in scalar polarized matrix element:

$$\begin{aligned}\mathcal{M}_{s\xi} =& \frac{ig^2 m_\tau}{2}\left(\frac{1}{q^2 - m_W^2}\right)\left(\frac{1-\xi}{q^2 - \xi m_W^2}\right) \\ & \times [\bar{u}_L(p_2)(m_t P_R - m_b P_L)u_L(p_1)][\bar{u}_L(p_3)P_R v_L(p_4)]\end{aligned}$$

- Matrix element for goldstone diagram:

$$\begin{aligned}\mathcal{M}_{\phi^+} =& J^{\mu}_{tb\phi}\Pi^{\phi^+}_{\mu\nu}J^{\nu}_{\tau\nu\phi} \\ =& \frac{-ig^2 m_\tau}{2}\left(\frac{1}{m_W^2}\right)\left(\frac{1}{q^2 - \xi m_W^2}\right)[\bar{u}_L(p_2)(m_t P_R - m_b P_L)u_L(p_1)] \\ & \times [\bar{u}_L(p_3)P_R v_R(p_4)]\end{aligned}$$

Polarized cross section measurement template used in LHC

$$\mathcal{M}_{\lambda}^{\text{decay}} = \epsilon_{\lambda\mu} J^{\mu}, \qquad \mathcal{M}_{0}^{\text{decay}} = ig\sqrt{2}E\sin\theta, \qquad \mathcal{M}_{R/L}^{\text{decay}} = igE(1 \pm \cos\theta)e^{\pm i\phi} \tag{20}$$

At the LHC, polarization fractions are measured by template fitting the angular distribution of the decay products

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} \propto \frac{3}{8}(1 \pm \cos\theta)^2 f_R + \frac{3}{8}(1 \mp \cos\theta)^2 f_L + \frac{3}{4}\sin\theta^2 f_0 \tag{21}$$

$$f_L + f_R + f_0 = 1 \tag{22}$$