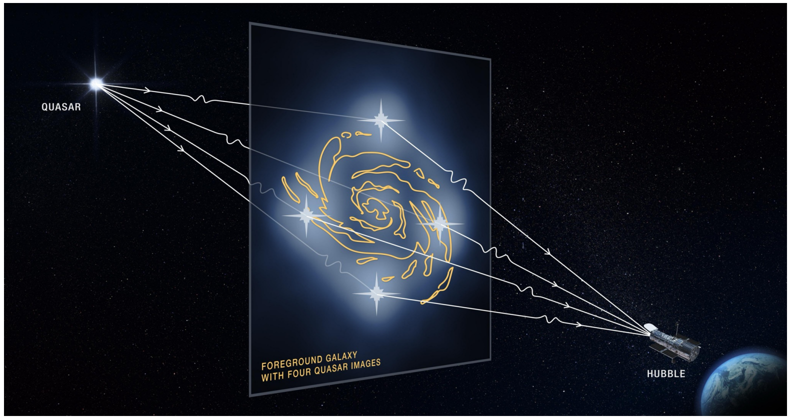


Lensing of gravitational waves:

Fundamental physics, astrophysics, cosmology

Chris Van Den Broeck

Universiteit Utrecht

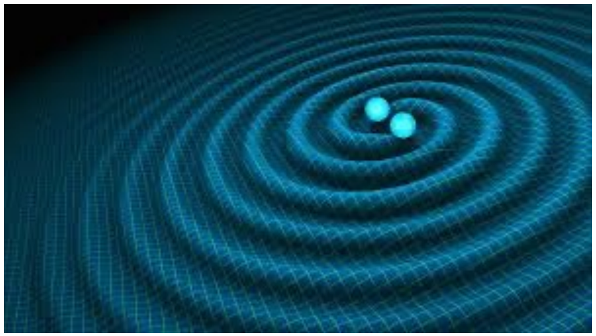
Nikhef

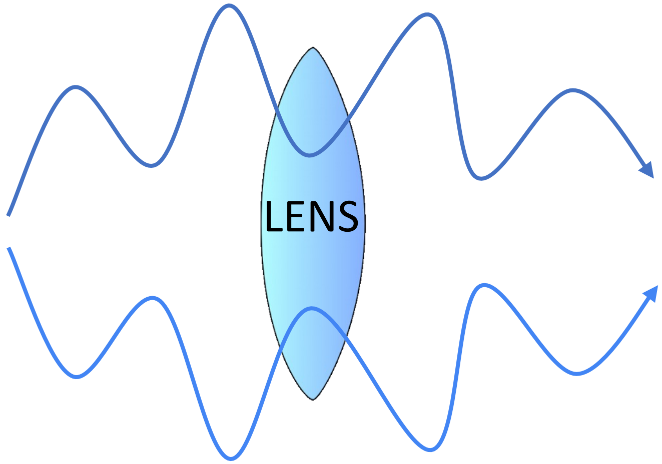
64th Cracow School of Theoretical Physics
Zakopane, Poland, 15-23 June 2024

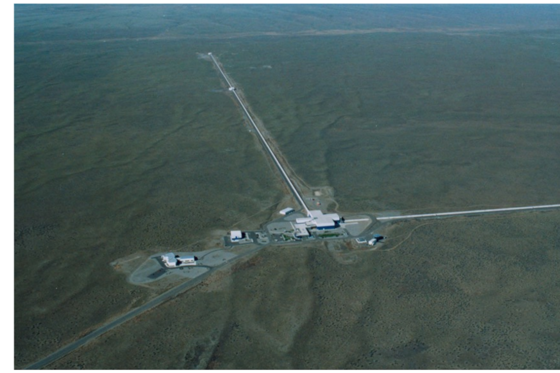
Gravitational lensing

Lensing of gravitational waves

- Same principle as for light: waves deflected by massive object on their path







Different lens properties
→ Different effect on the gravitational waves

Lensing of gravitational waves

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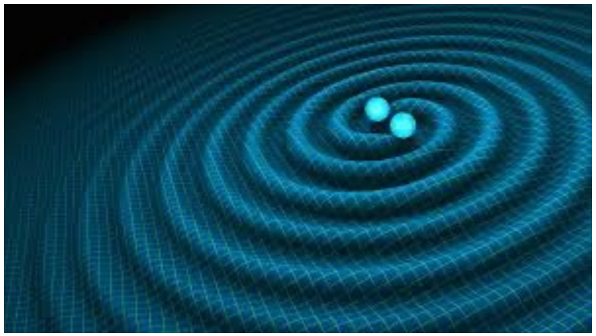
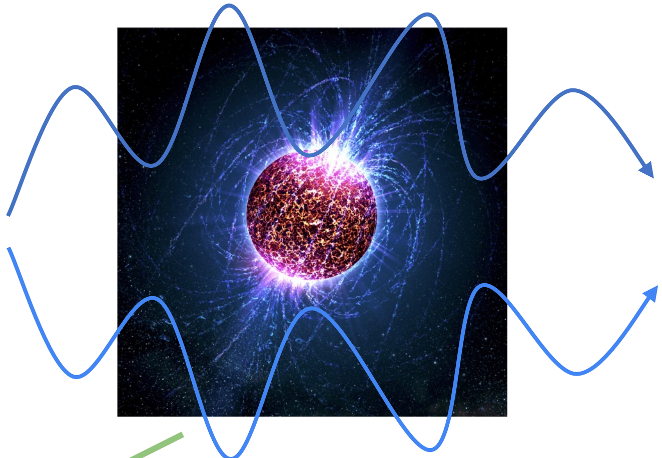
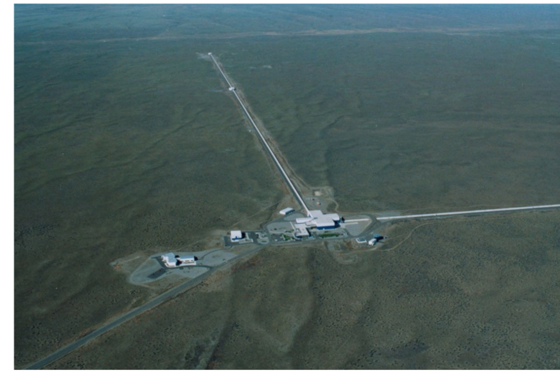
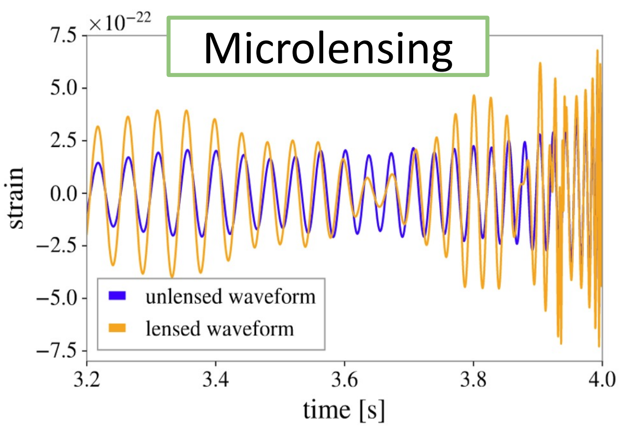






Low mass lens



Seo et al., arXiv:2110.03308

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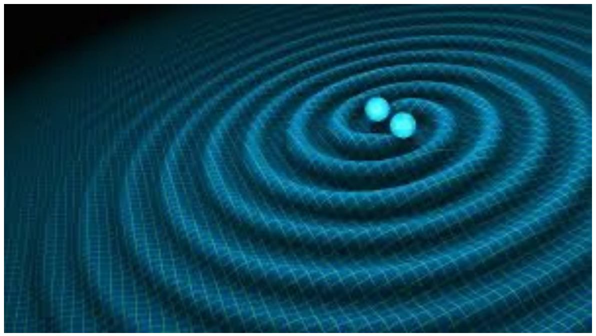
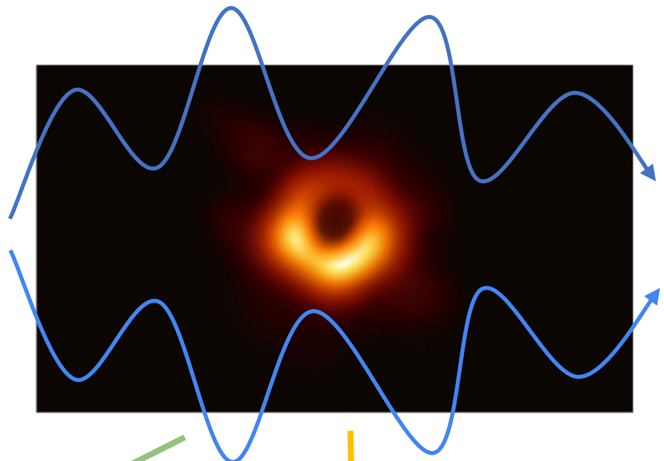
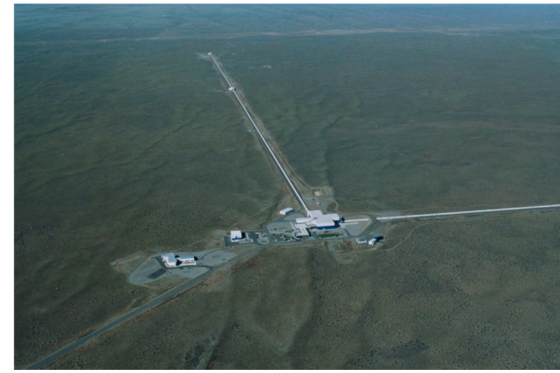
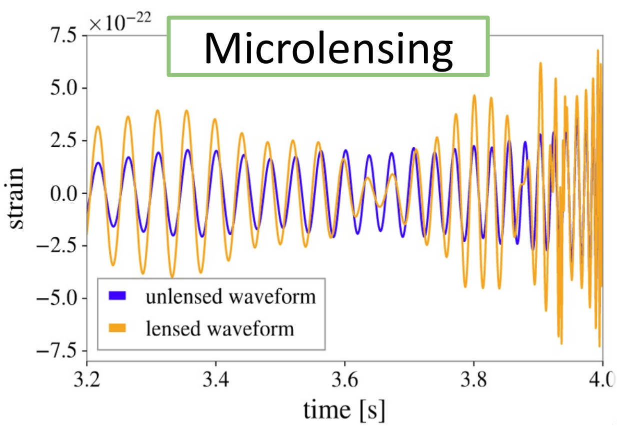


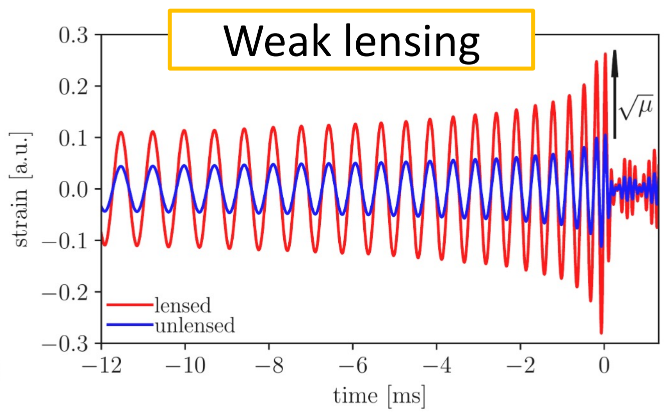




Low mass lens

Medium mass lens



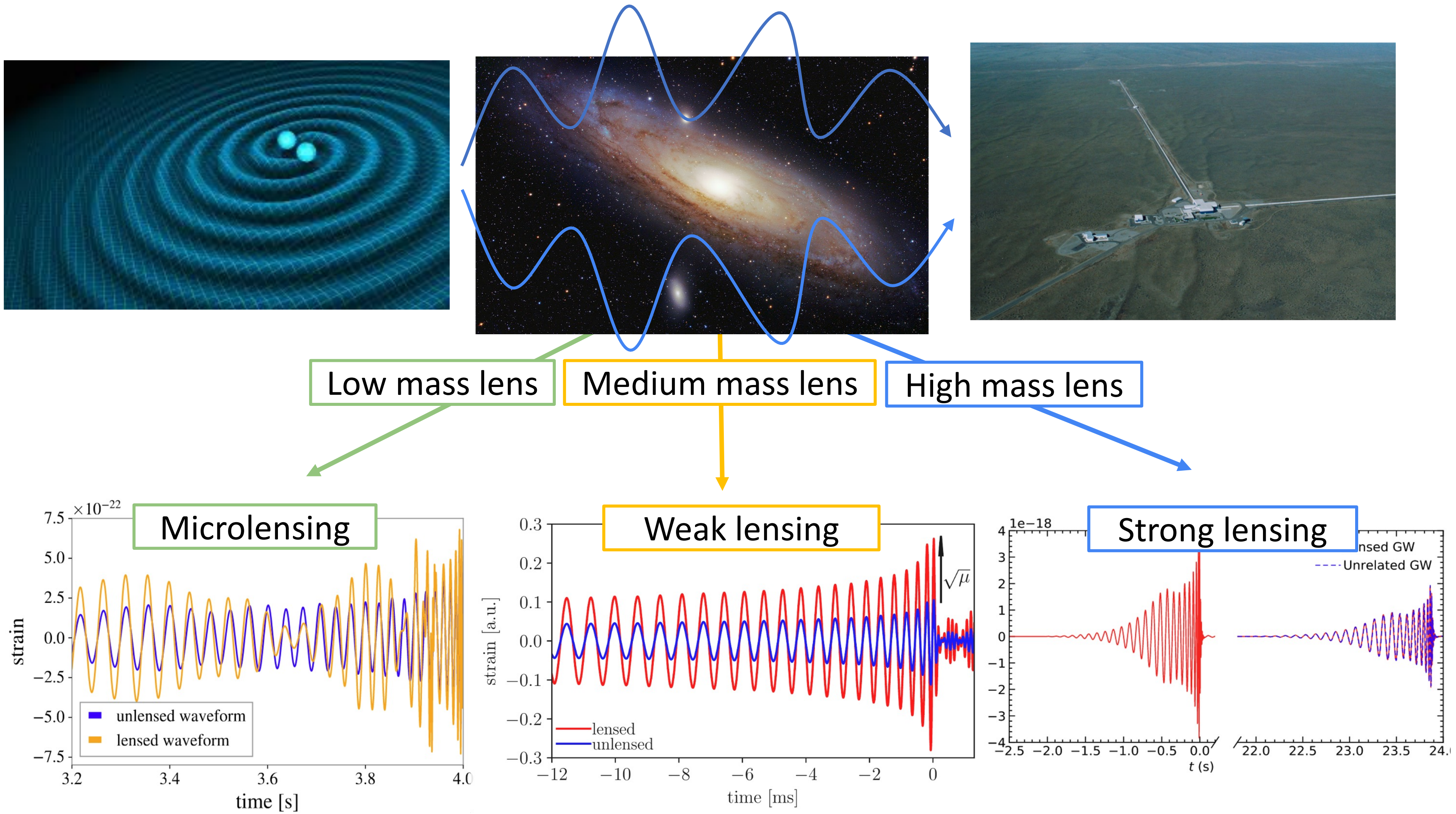


Seo et al., arXiv:2110.03308

Pang et al., MNRAS (2020)

Lensing of gravitational waves

- Same principle as for light: waves deflected by massive object on their path



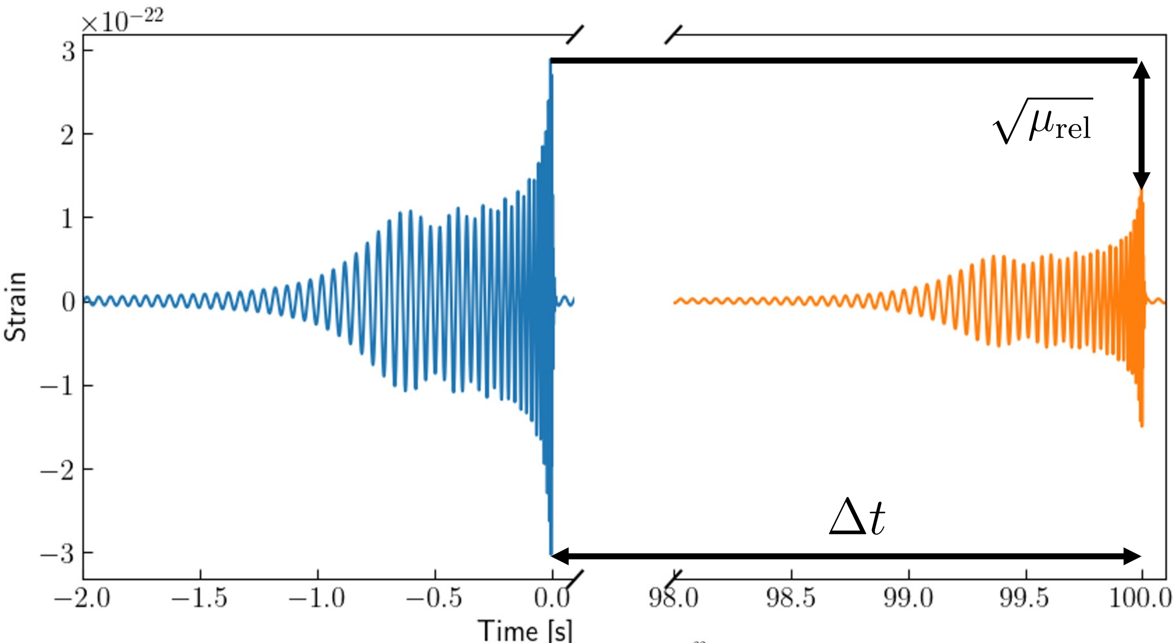
Seo et al., arXiv:2110.03308

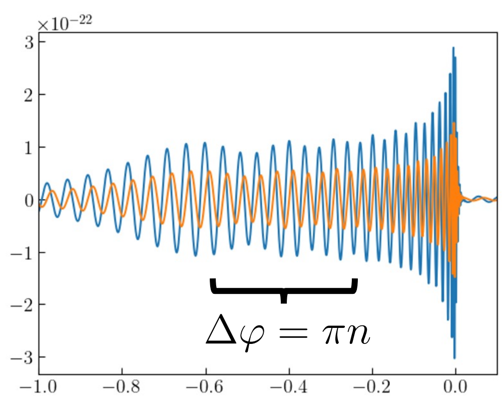
Pang et al., MNRAS (2020)

Wierda et al., ApJ (2021)

Strong lensing

- For strong lensing: $\lambda_{\mathrm{GW}} \ll R_{\mathrm{lens}}$, hence geometric optics approximation valid
  $\Longrightarrow$ The frequency evolution is unchanged
- Several images having taken different paths





- Relative magnification μ_{rel} : Different images undergo different (de)magnification
- Time delay Δt : Different images take different paths
  - Delays of minutes to months
  - Example: Milky Way as lens Schwarzschild time ~65 days
- Morse phase $\Delta\varphi = \pi n$: Each image undergoes a global phase shift, with $n \in \left\{0, \frac{1}{2}, 1\right\}$

Strong lensing: rates and searches

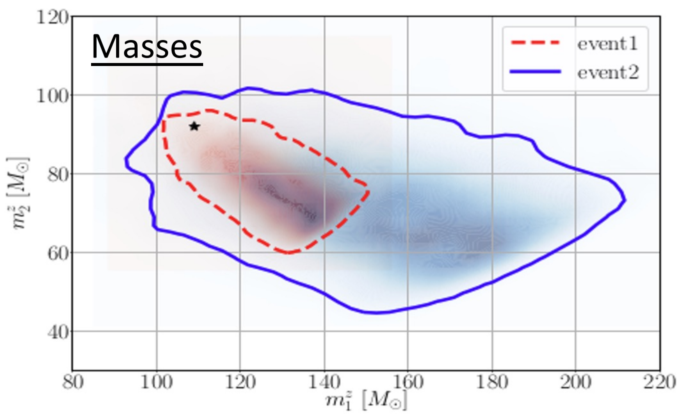
- Possibly ~1/year for Advanced LIGO, Virgo, KAGRA at design sensitivity

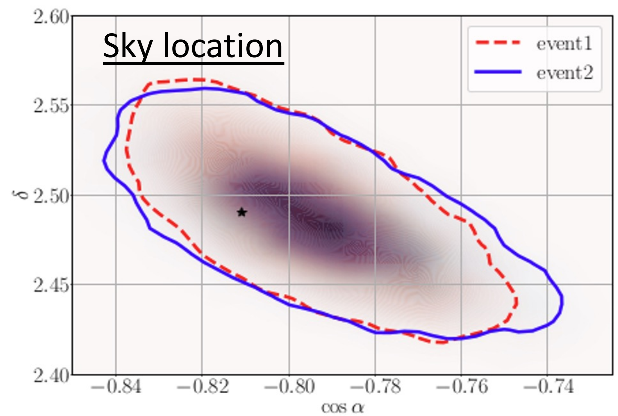
Observed rates		L	L/H	L/H/V/K	L/H/V/K (A+)	L/H/V/K (Voyager)
Lensed events:	total	$0.21^{+0.10}_{-0.07}$ yr^{-1}	$0.65^{+0.32}_{-0.22}$ yr^{-1}	$1.3^{+0.6}_{-0.4}$ yr^{-1}	$3.3^{+1.7}_{-1.1}$ yr^{-1}	$16.8^{+8.4}_{-5.6}$ yr^{-1}
	double	$0.17^{+0.08}_{-0.06}$ yr^{-1}	$0.50^{+0.25}_{-0.17}$ yr^{-1}	$0.92^{+0.46}_{-0.31}$ yr^{-1}	$2.5^{+1.2}_{-0.8}$ yr^{-1}	$13.1^{+6.5}_{-4.4}$ yr^{-1}
	triple	$0.032^{+0.016}_{-0.011}$ yr^{-1}	$0.11^{+0.06}_{-0.04}$ yr^{-1}	$0.23^{+0.12}_{-0.08}$ yr^{-1}	$0.55^{+0.28}_{-0.19}$ yr^{-1}	$2.0^{+1.0}_{-0.7}$ yr^{-1}
	quadruple	$0.011^{+0.005}_{-0.004}$ yr^{-1}	$0.038^{+0.019}_{-0.013}$ yr^{-1}	$0.12^{+0.06}_{-0.04}$ yr^{-1}	$0.30^{+0.15}_{-0.10}$ yr^{-1}	$1.6^{+0.8}_{-0.6}$ yr^{-1}
Unlensed events		370 yr^{-1}	1.1×10^3 yr^{-1}	1.9×10^3 yr^{-1}	5.8×10^3 yr^{-1}	31×10^3 yr^{-1}
Relative occurrence		1 : 1760	1 : 1650	1 : 1500	1 : 1740	1 : 1830

Wierda et al., ApJ (2021)

- How to search for strongly lensed events?
  - Frequency evolution determined by binary black hole masses and spins
    - Images have same frequency evolution:
      Posterior probability densities for e.g. masses should be consistent
  - Sky positions should be consistent

Haris et al., arXiv:1807.07062

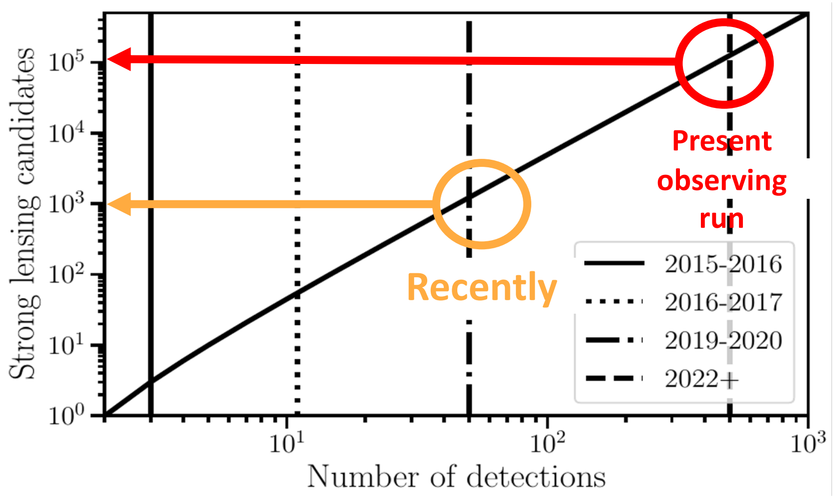


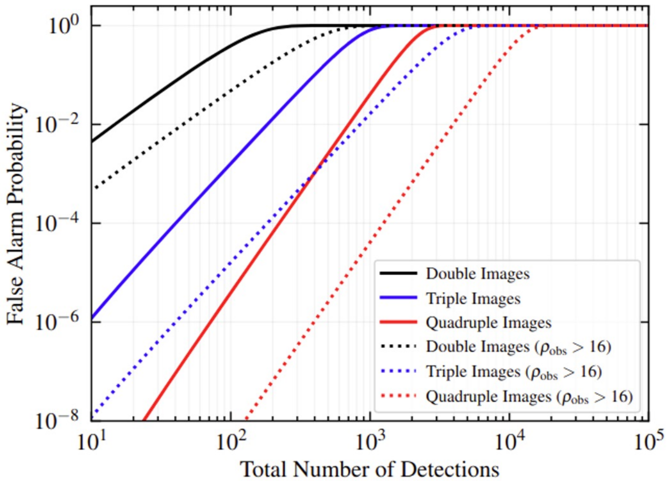


A needle in a haystack

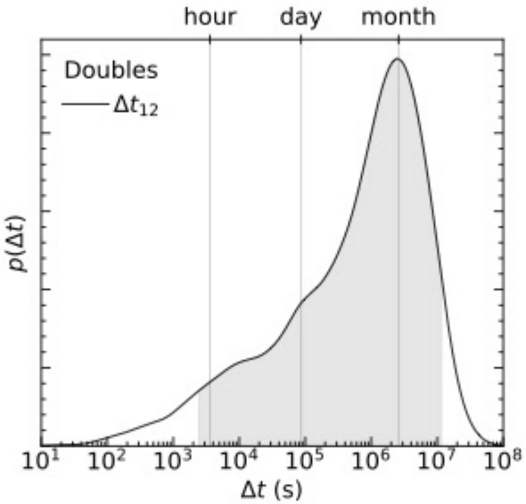
- To find a strongly lensed event, need to compare all pairs of detections
  - If *N* detections, false alarm probability grows as N^2

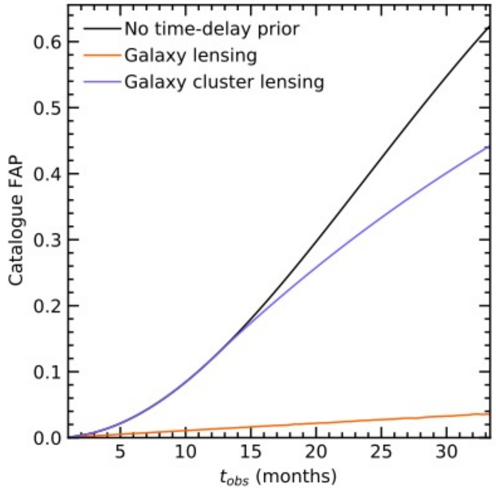
Çalişkan et al, PRD (2023)





  - Models predict distributions for time delays Δt and relative magnifications $\mu_{\rm rel}$
    - Folding these in makes the false alarm probability grow as *N*

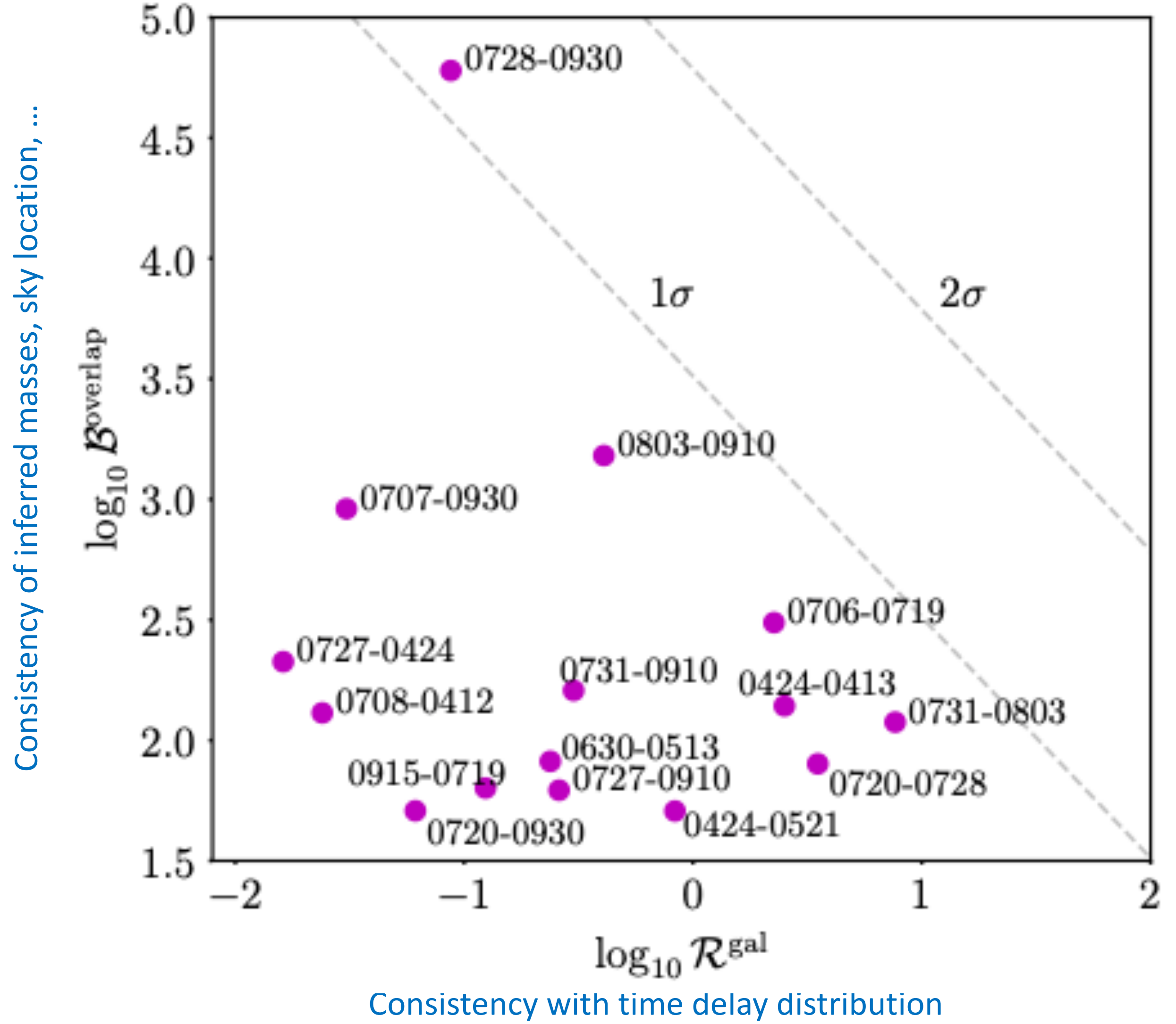




Wierda et al., ApJ (2021)

So far nothing found…

- A first search for strongly lensed events in LIGO-Virgo data:

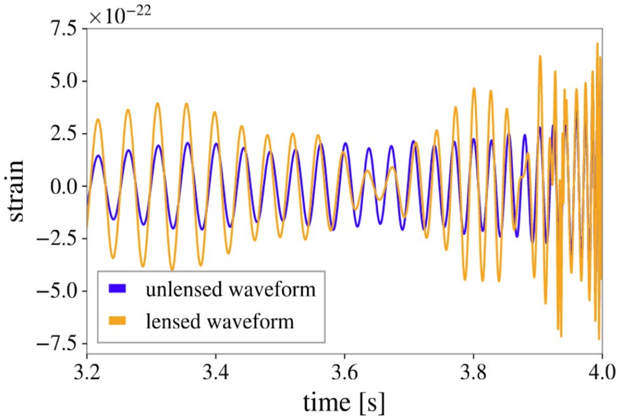


Abbott et al., ApJ (2021)

What about microlensing?

- Frequency-dependent magnification:

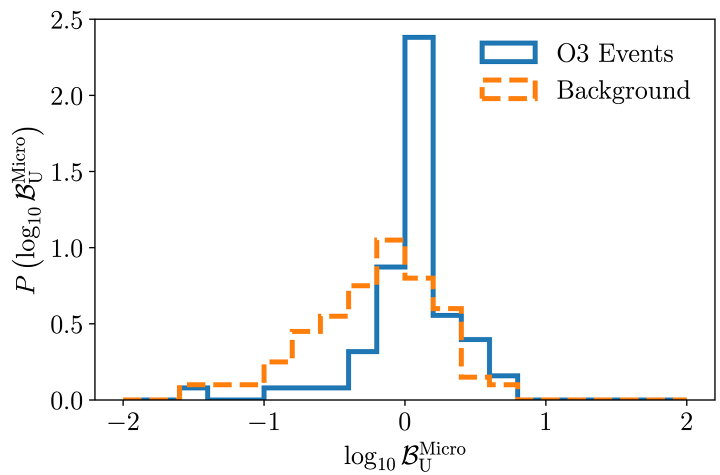
$$h^{\mathrm{Micro}}(f;\theta, M_{\mathrm{L}}^{z}, y) = \underbrace{h^{\mathrm{U}}(f;\theta)}_{\text{unlensed wave}} \underbrace{F(f; M_{\mathrm{L}}^{z}, y)}_{\text{magnification factor}}$$



Seo et al., arXiv:2110.03308

- M^{z}_{L} = lens mass (point mass for simplicity)
- y = dimensionless impact parameter

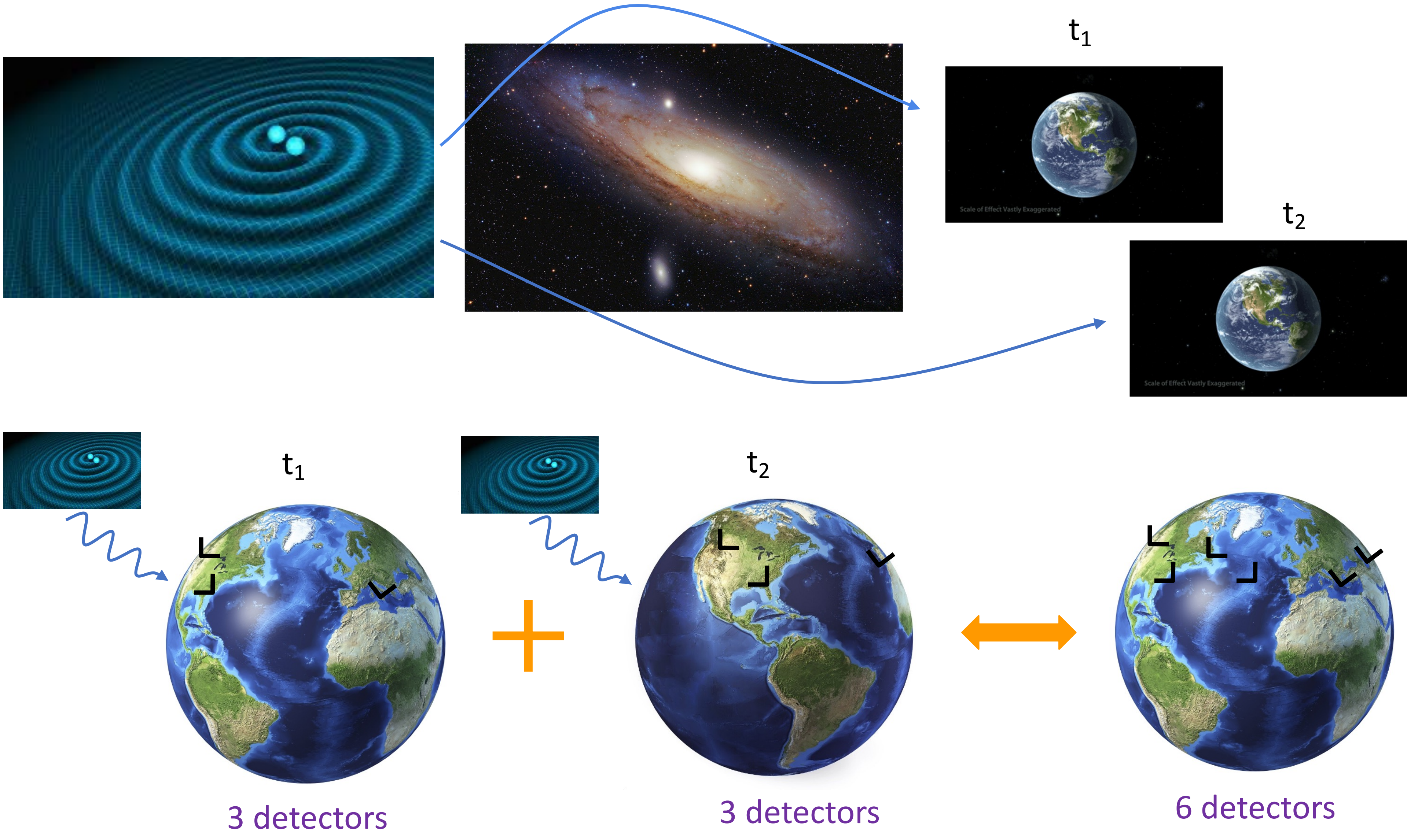
- Bayesian evidences:



Abbott et al., arXiv:2304.08393

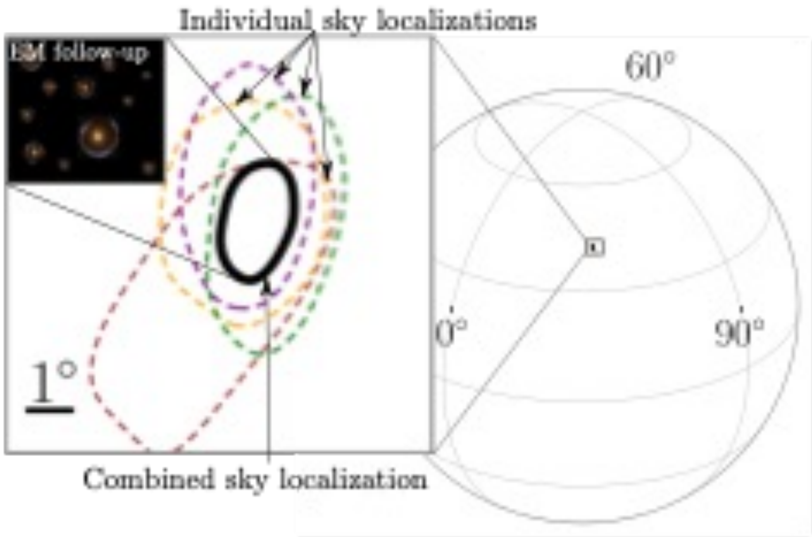
Why are (strongly) lensed gravitational waves interesting?

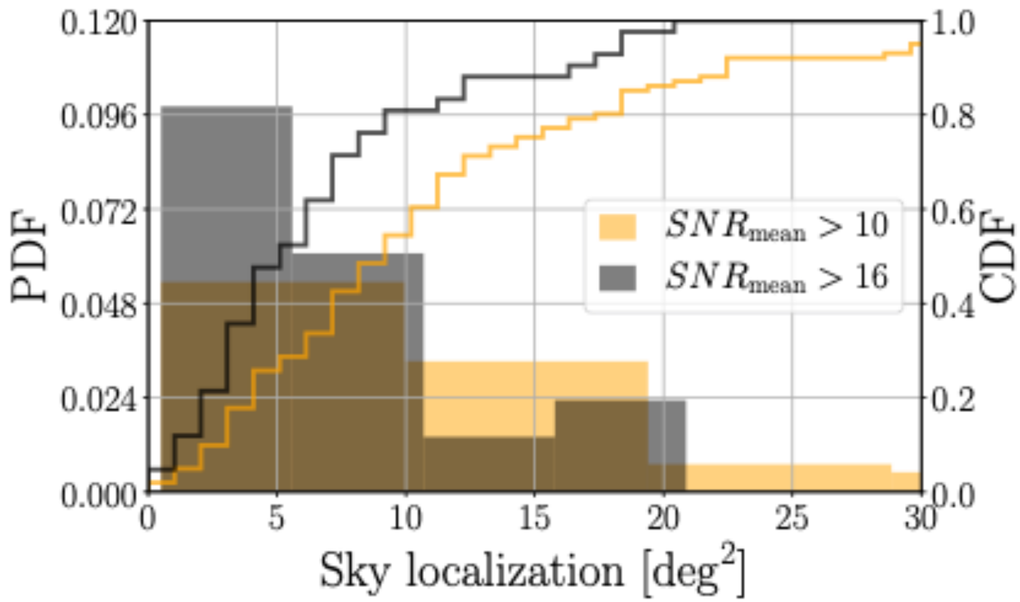
- Seeing 2 images with 3 detectors = seeing 1 signal with 6 virtual detectors



Why are (strongly) lensed gravitational waves interesting?

- Lensing allows for improved sky localization
- With **four images**:



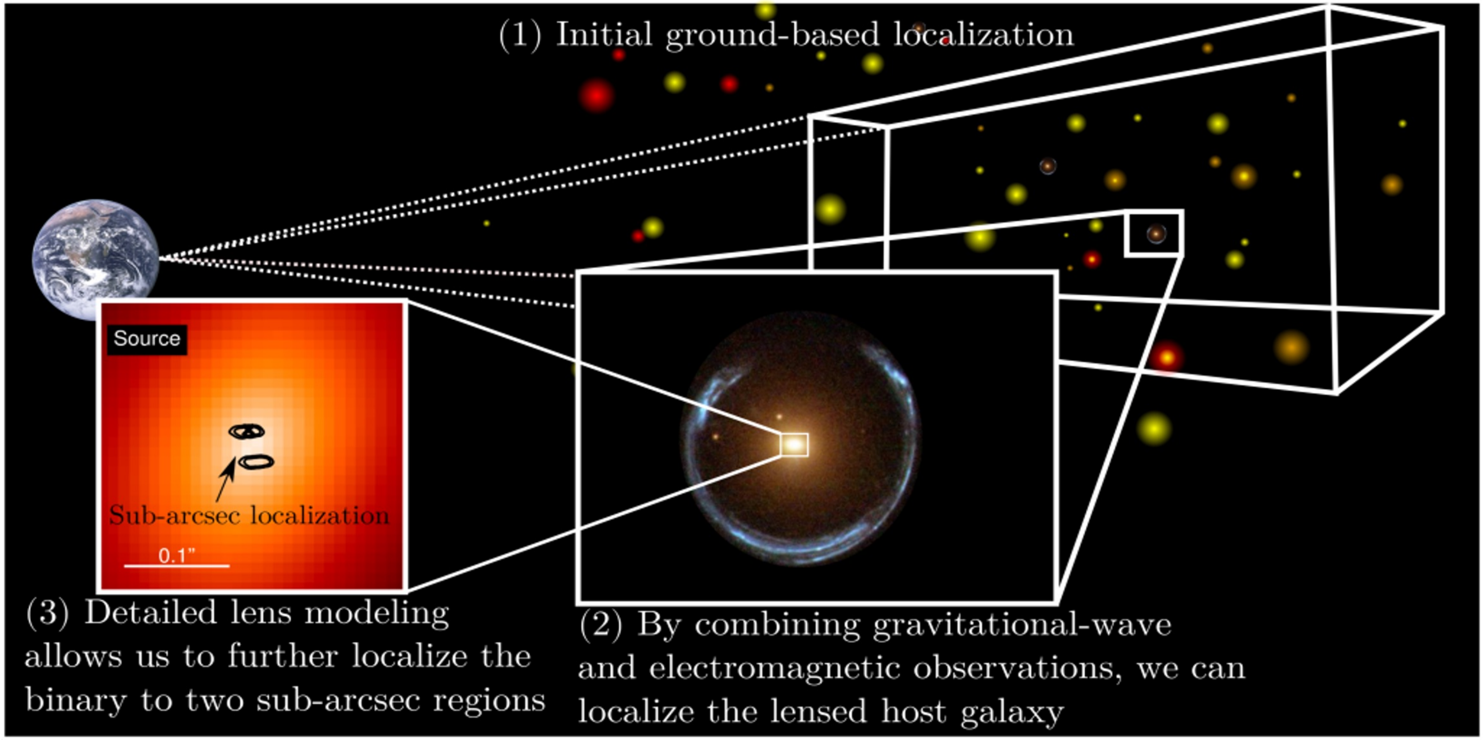


Hannuksela et al., MNRAS (2020)
Janquart et al., MNRAS (2021)

Why are (strongly) lensed gravitational waves interesting?

- Lensing allows for improved sky localization
- Sky error box must contain host galaxy, which will also be lensed
  - Electromagnetic telescopes
    + requiring consistency of lensed galaxy with lensed gravitational wave
    $\Longrightarrow$ Identification of the host galaxy

  A way to find the host galaxy of a binary black hole merger
  - Lens modeling: pin down location *inside* the galaxy with sub-arcsec precision

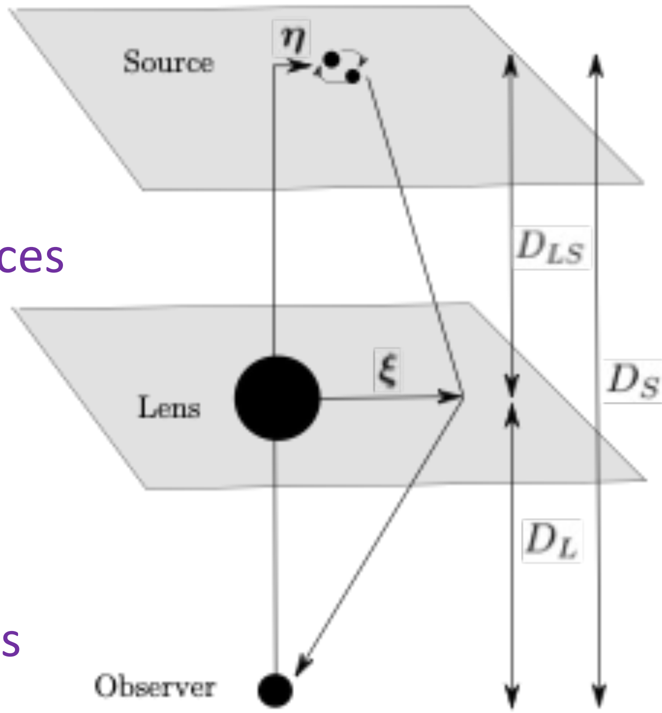


Hannuksela et al., MNRAS (2020)

Pinning down the sky location of the source

- Sky location of the images and the source:
  - $\boldsymbol{\eta}$ displacement of the source from line of sight
  - $\boldsymbol{\xi}_i$ positions of the images in the lens plane
  - $\boldsymbol{\eta} = D_S\boldsymbol{\beta}$ and $\boldsymbol{\xi}_i = D_L\boldsymbol{\theta}_i$

    where D_S , D_L , D_{LS} are angular diameter distances



- Fermat potential: $\phi(\boldsymbol{\theta},\boldsymbol{\beta}) = \frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta})$

  where deflection potential

$$\psi(\boldsymbol{x}) = \frac{1}{\pi}\int d^2\boldsymbol{x}'\,\kappa(\boldsymbol{x}')\,\ln|\boldsymbol{x}-\boldsymbol{x}'|$$

  with $\kappa(\boldsymbol{x})$ normalized surface mass density of the lens

- Image locations are extrema of the Fermat potential:

$$\boldsymbol{\nabla}_{\boldsymbol{\theta}}\left[\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta})\right] = 0$$

- Image time delays and magnifications:

$$t_{d,j} = \frac{D_L D_S}{D_{LS}}\frac{1+z_L}{c}\left[\frac{1}{2}(\boldsymbol{\theta}_i-\boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta}_i)\right]$$

$$\mu_j = \left[1/\det\left(\frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}}\right)\right]_{\boldsymbol{\theta}=\boldsymbol{\theta}_j}$$

Pinning down the sky location of the source

- Image locations are extrema of the Fermat potential:

$$\nabla_{\boldsymbol{\theta}} \left[\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta}) \right] = 0$$

"lens equation"

- Image time delays and magnifications:

$$t_{d,j} = \frac{D_L D_S}{D_{LS}} \frac{1 + z_L}{c} \left[\frac{1}{2}(\boldsymbol{\theta}_i - \boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta}_i) \right]$$

$$\mu_j = \left[1/\det \left(\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} \right) \right]_{\boldsymbol{\theta} = \boldsymbol{\theta}_j}$$

- From gravitational wave observations:
  - $\Delta t_{ij} = t_{d,i} - t_{d,j}$ differences in image arrival times (highly accurate)
  - $\mu_{ij} = \mu_i / \mu_j$ relative magnifications (less accurate)

- In the case of **four images:** $\boldsymbol{\beta}$, $\boldsymbol{\theta}_i$, $i = 1, \ldots, 4$ together **10 unknowns**
  - $\Delta t_{12}/\Delta t_{13}$, $\Delta t_{12}/\Delta t_{14}$: **2** observables that only depend on $\boldsymbol{\beta}$, $\boldsymbol{\theta}_i$
  - Lens equation: 4x2 = **8** constraints
  - Assume lens sufficiently well modeled, i.e. function $\psi(\boldsymbol{x})$ is known

$\Longrightarrow$ Solve for $\boldsymbol{\beta}$, $\boldsymbol{\theta}_i$

Two ways of measuring the Hubble constant

- Once $\boldsymbol{\beta}$, $\boldsymbol{\theta}_i$ are known, calculate magnifications:

$$\mu_j = \left[1/\det\left(\frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}}\right)\right]_{\boldsymbol{\theta}=\boldsymbol{\theta}_j}$$

  - In the image amplitudes: $d_L/\sqrt{\mu_i} \implies d_L$ luminosity distance to the source
  - Redshift of the host galaxy known from EM measurements
  - Fix cosmological parameters except for H_0

  $\implies$ Measurement of H_0

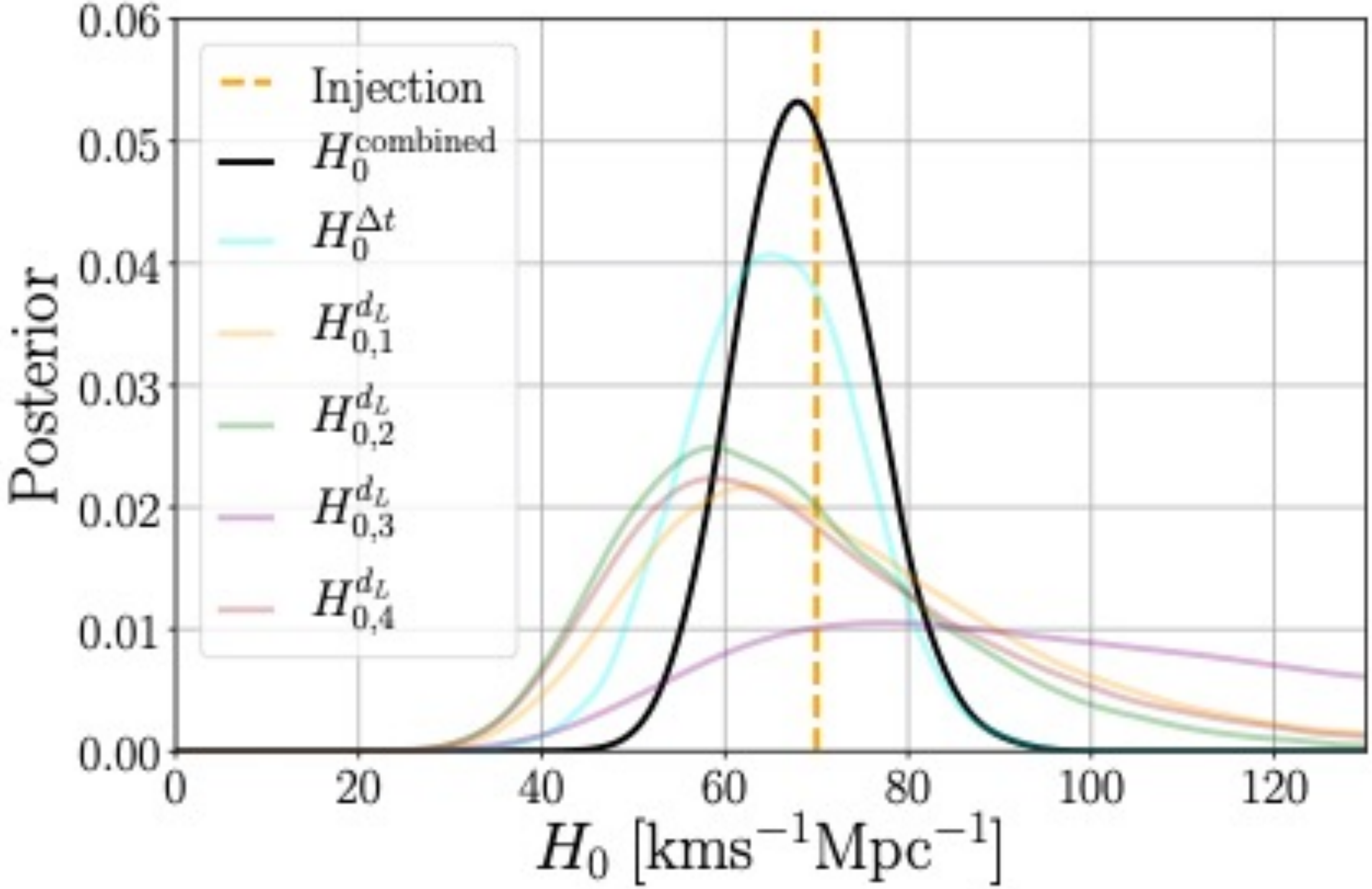
- Differences in arrival time:

$$\Delta t_{ij} = \frac{D_L(H_0; z_L) D_S(H_0; z_S)}{D_{LS}(H_0; z_L, z_S)} \frac{1+z_L}{c} \left[\phi(\boldsymbol{\theta}_i, \boldsymbol{\beta}) - \phi(\boldsymbol{\theta}_j, \boldsymbol{\beta})\right]$$

  - z_L , z_S known from electromagnetic measurements

  $\implies$ Measurement of H_0

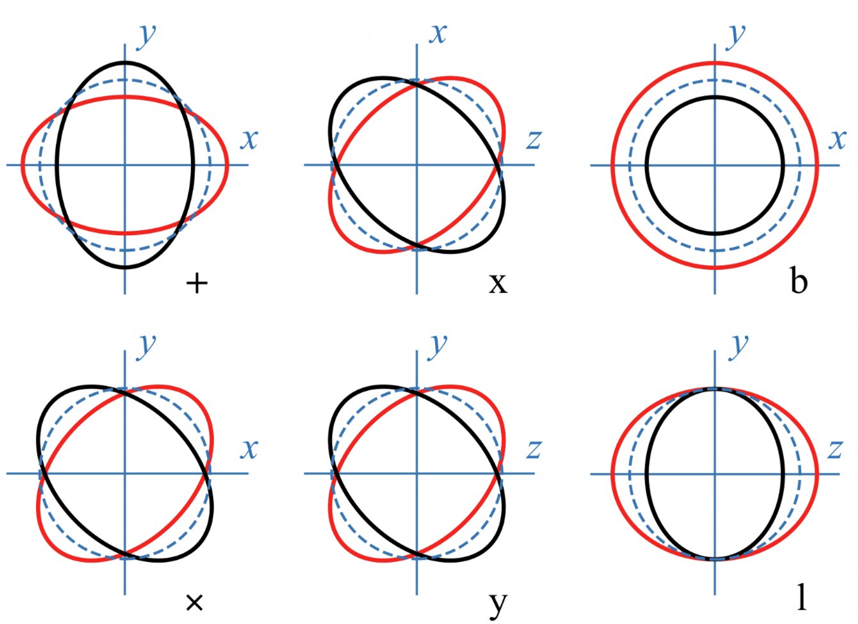
Two ways of measuring the Hubble constant

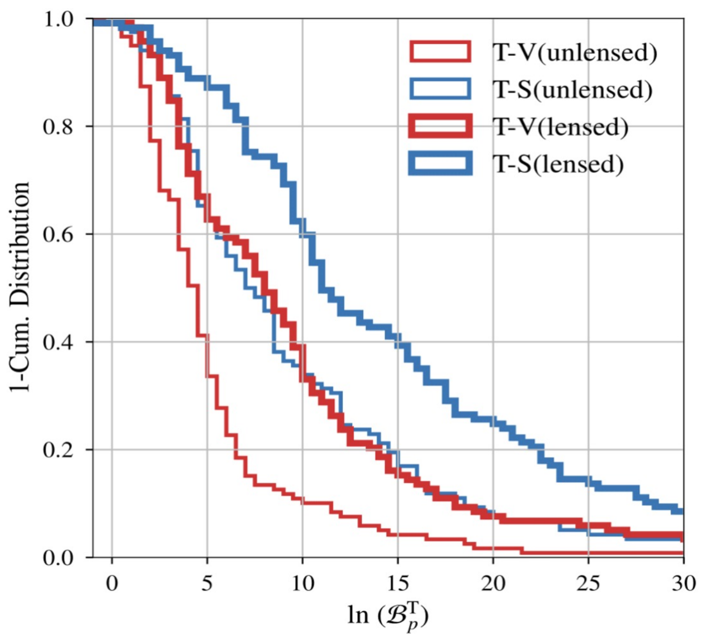


Hannuksela et al., MNRAS (2020)

Science with lensed gravitational waves

- Localization of binary black hole events
  - Link between black hole binaries and their host galaxies
- High-redshift Hubble constant measurements
- **Fundamental physics: How many polarizations?**





Goyal et al., PRD (2021)

Science with lensed gravitational waves

- Localization of binary black hole events
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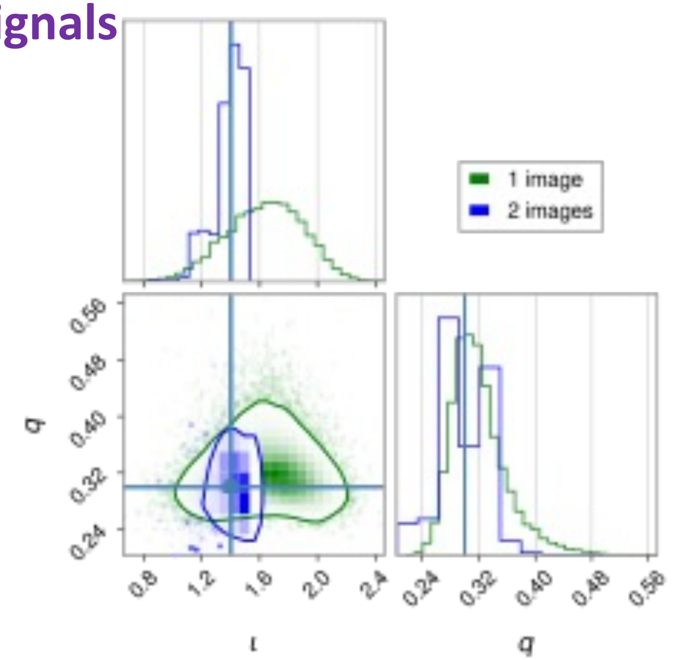
- Fundamental physics: How many polarizations?

- **Probing higher-order modes in gravitational wave signals**
  - Better constraints on higher-order mode content means better localization
  - Better understanding of the binary: Enhanced tests of the dynamics of GR



Janquart et al., ApJ Lett (2021)

Science with lensed gravitational waves

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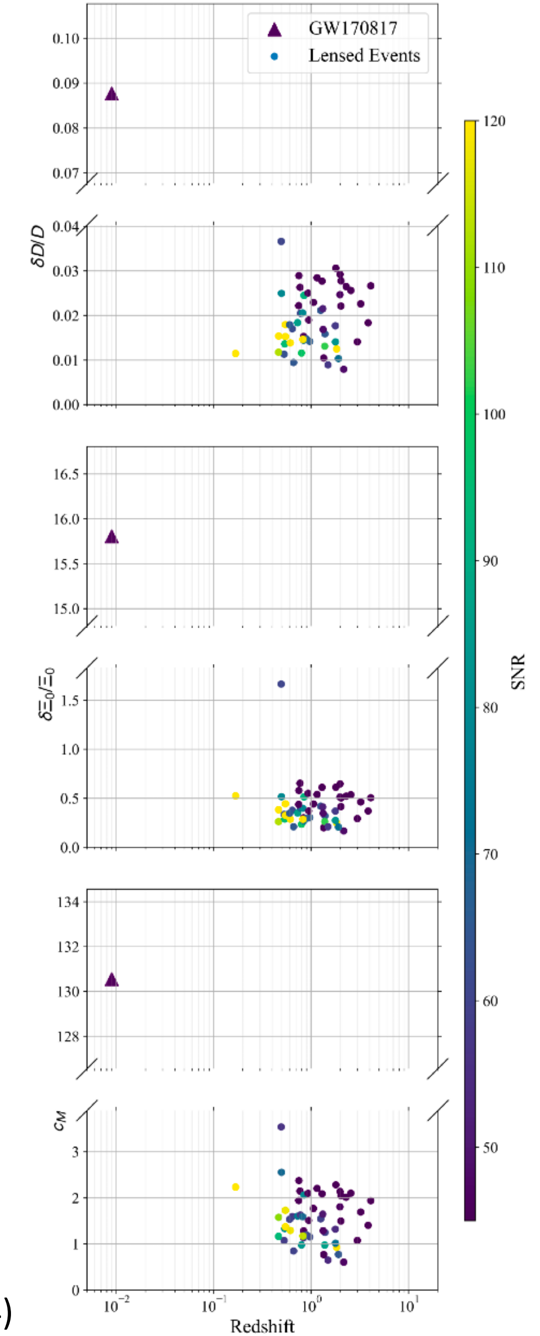
- High-redshift Hubble constant measurements
- Fundamental physics: How many polarizations?
- Probing higher-order modes in gravitational wave signals
  - Better constraints on higher-order mode content means better localization
  - Better understanding of the binary: Enhanced tests of the dynamics of GR
- **Alternative theories of gravity:**
  - Large extra dimensions
  - Theories with friction
  - Variable Planck mass

Compare luminosity distance measured from GW versus EM

Finke et al., PRD (2021)

Narola et al., PRD (2024)



Thank you for your attention!