Introduction to Loop Quantum Black Hole Models

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Outline

A. Introduction of a Loop Quantum BH Model: Schw. interior as an example

—Canonical quantization of BH

—What is loop quantization

—Some results

B. Some recent results in LQGBH

—Spherically symmetric model

—Quantum Oppenheimer-Snyder model

—QG effects on BH image, et al

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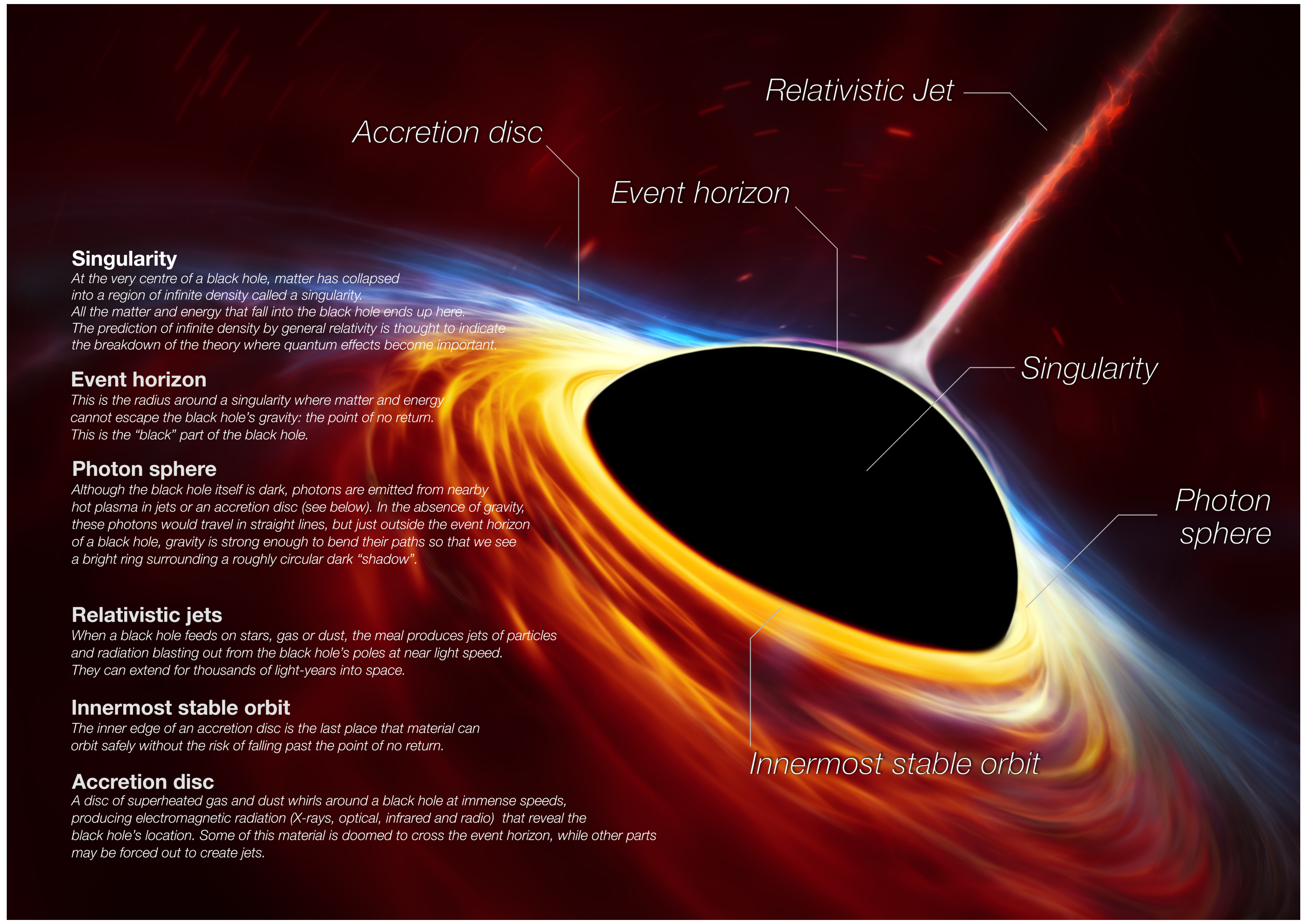
Motivation and background

The black holes of nature are the most perfect macroscopic objects there are in the Universe

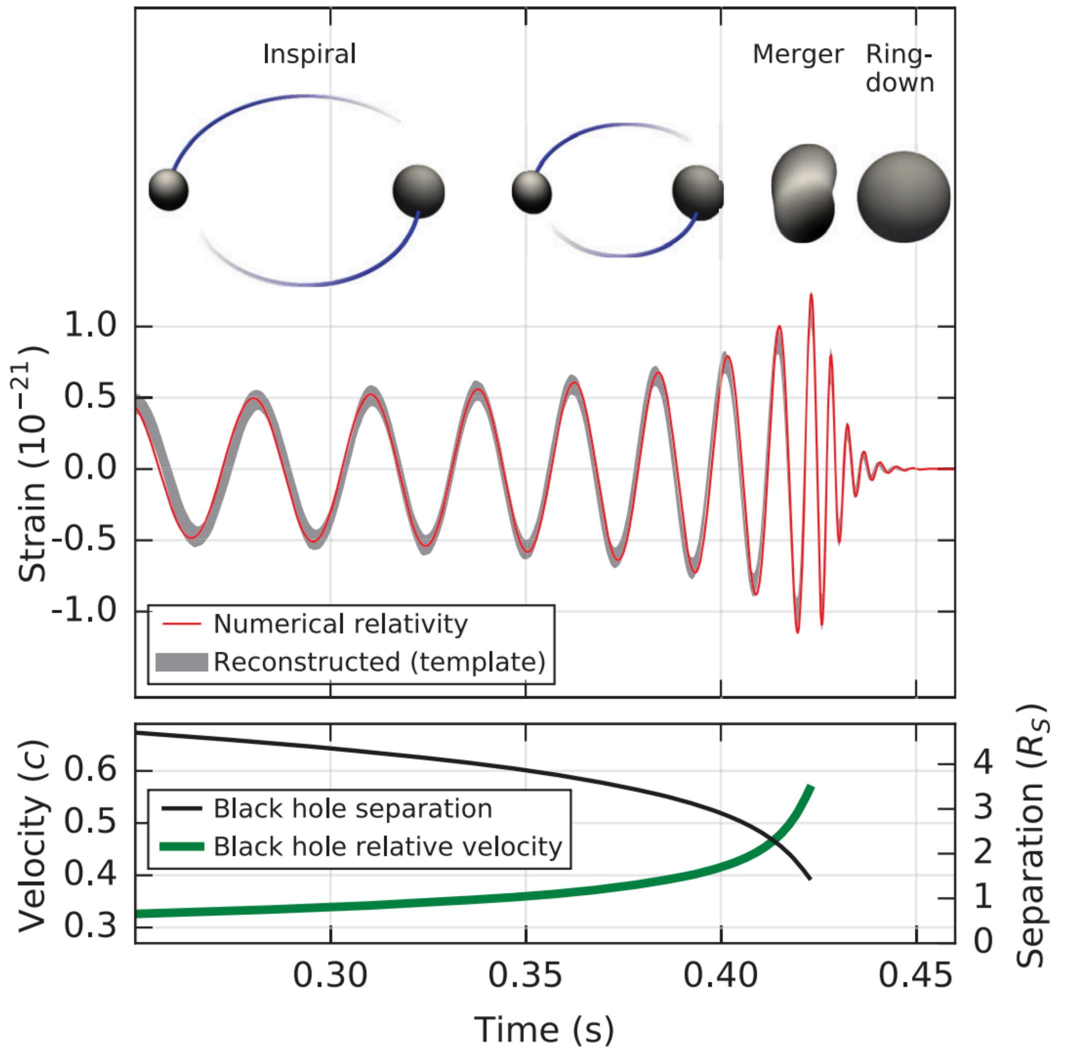
—Subrahmanyan Chandrasekhar



https://en.wikipedia.org/wiki/Black_hole

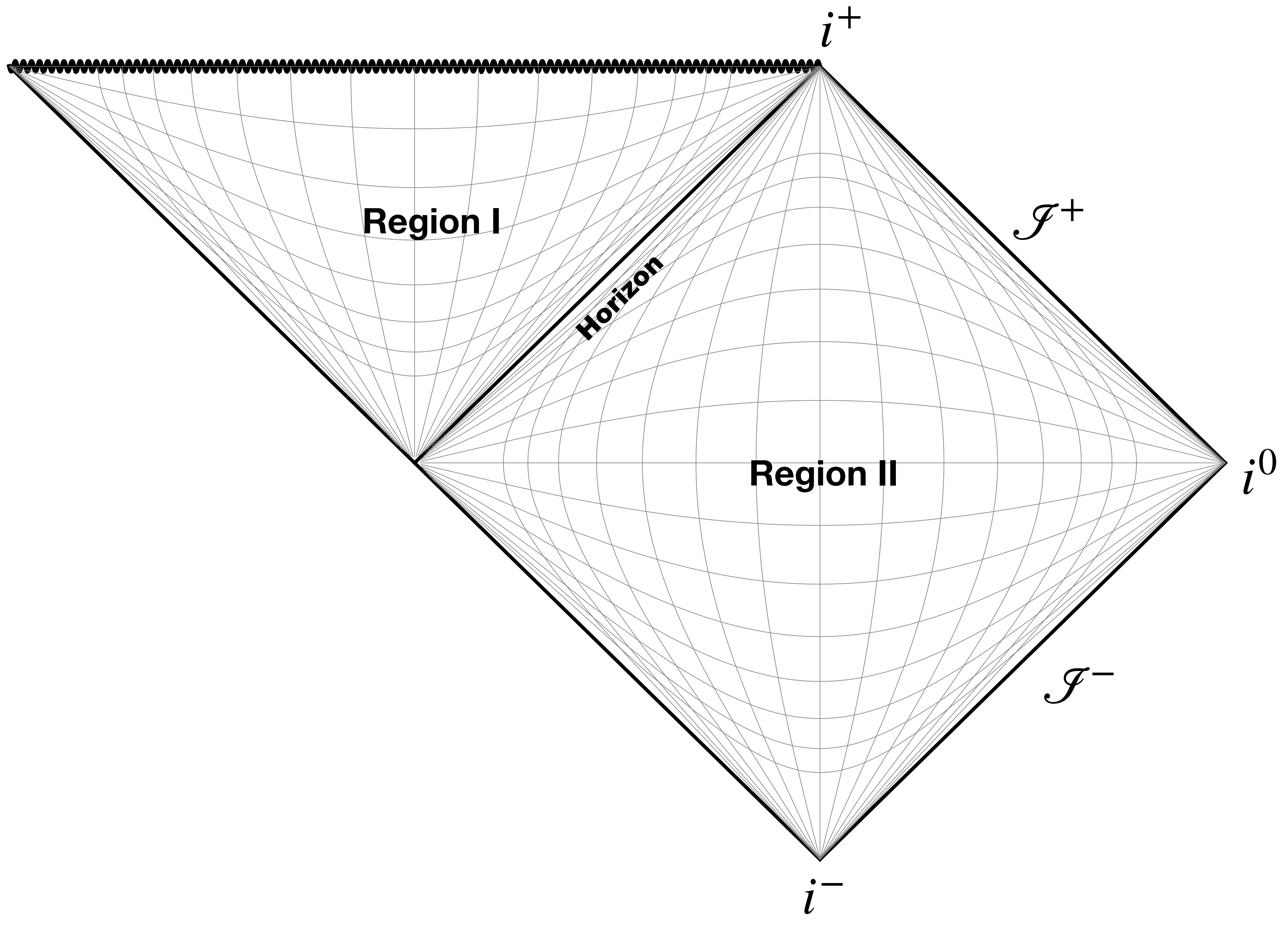
BHs have been observed due to the development of observational technique

Image of a BH at the core of M87
[https://eventhorizontelescope.org/]



Estimated gravitational-wave strain amplitude from GW150914 [PRL 116, 061102 (2016)]

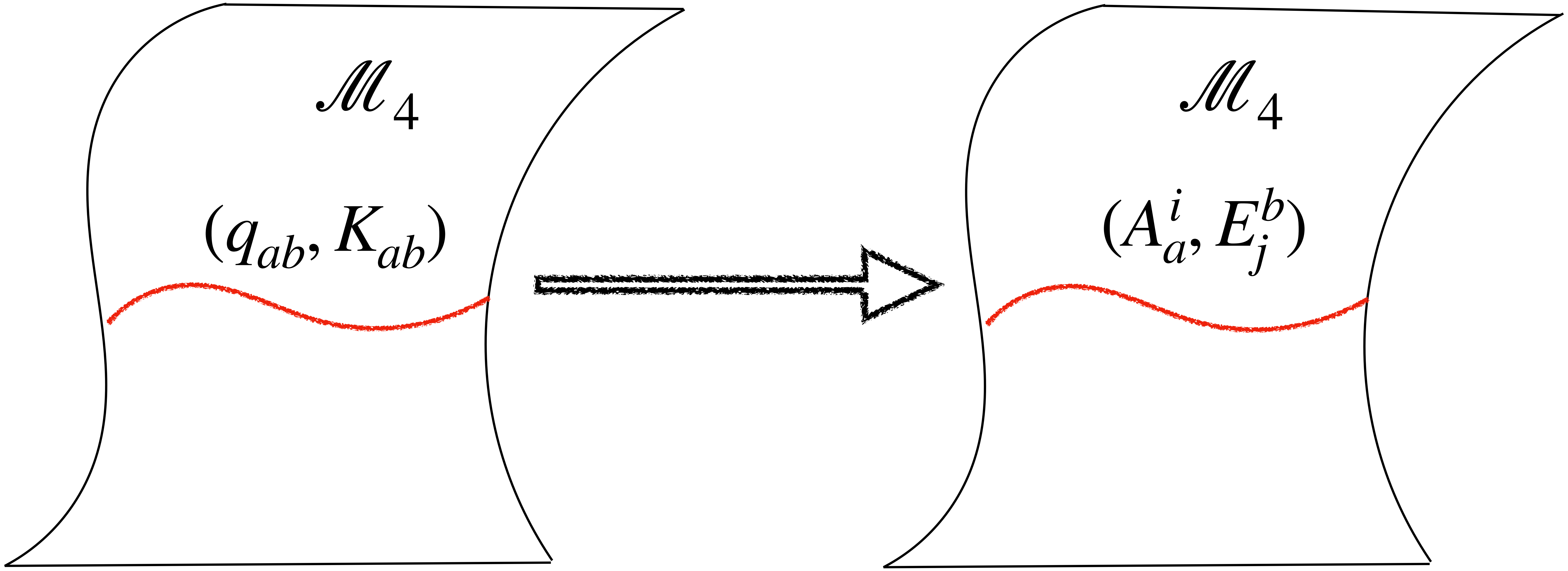
Motivation and background



Penrose Diagram of a Schwarzschild BH

- The existence of singularities in BHs motivate us to introduce QG in BH physics;
- In loop quantum gravity, our answer on quantum BH haven't formed a unique picture; There are, e.g., Ashtekar-Bojowald paradigm **[Ashtekar & Bojowald 05']**, the SF qBH model **[Rovelli, Harggard, Christodoulou, Speziale, Vilensky etc. 15', 16', Han, Qu, CZ 24' and so on]** and different loop quantum symmetry-reduced models **[Ashtekar, Bojowald, Bodendorfer, Boehmer, Chiou, Giesel, Gambini, Han, Husian, Li, Liu, Lewandowski, Modesto, Ma, Mehdi, ,Mena Marugan, Olmedo, Pullin, Singh, Vandersloot, Wang, Wilson-Ewing, Yang, Zhang and so on]**

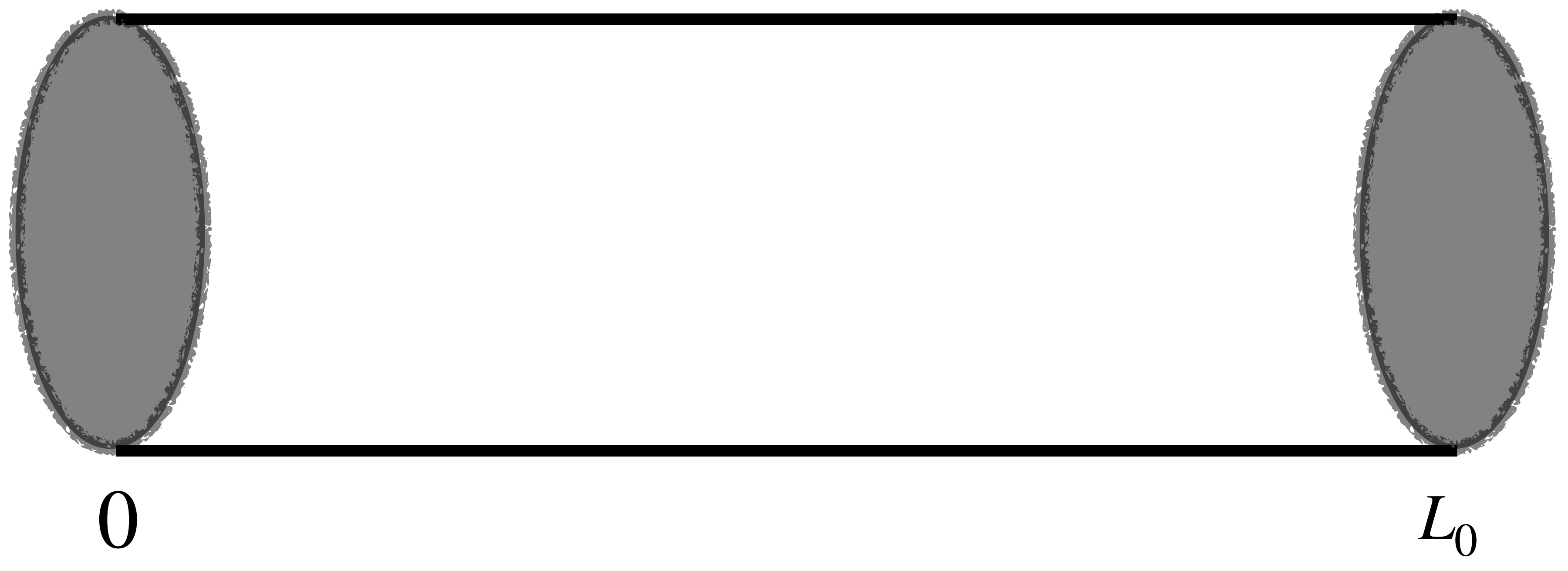
A few words on LQG full theory



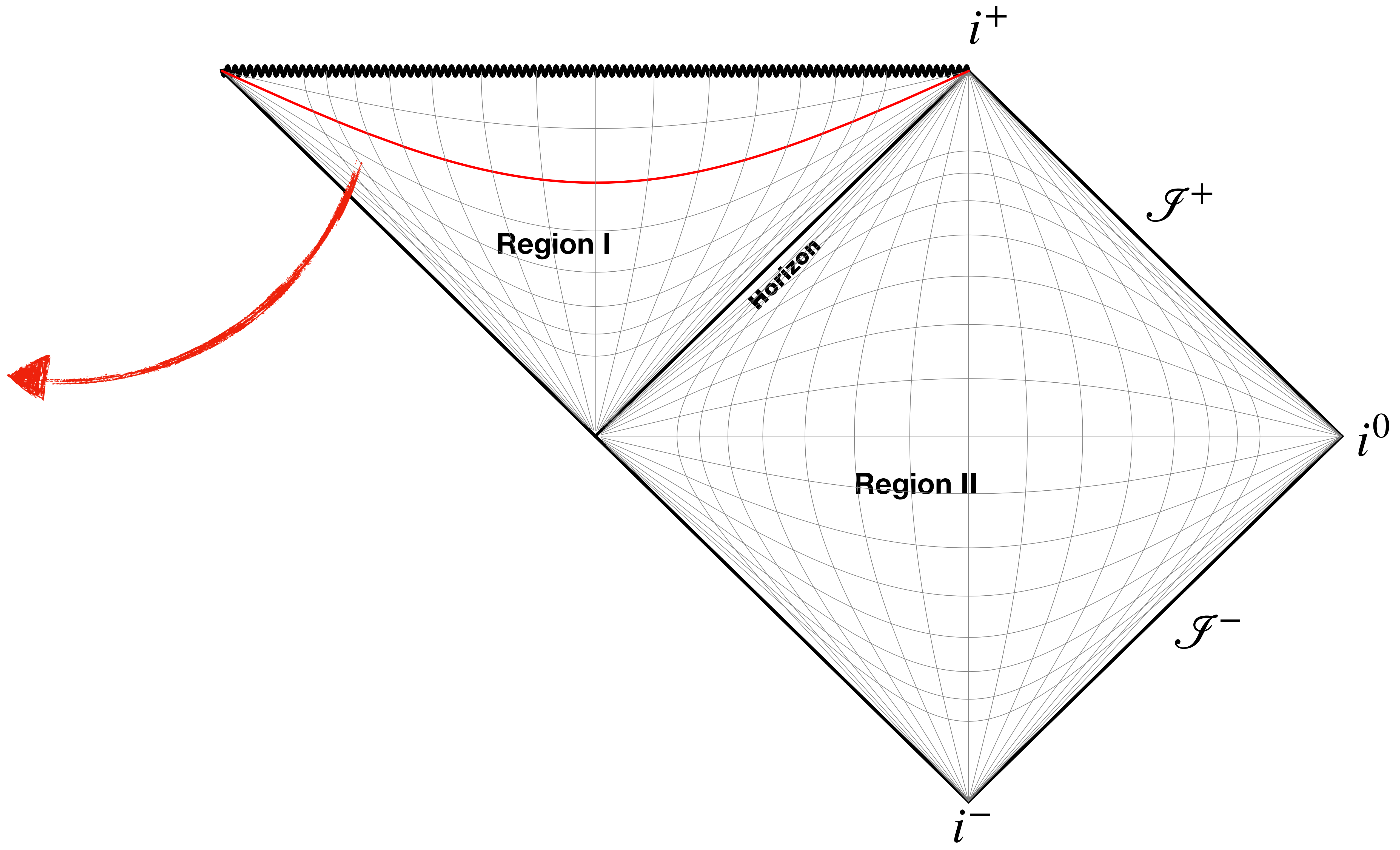
$$h_e = \mathscr{P}\exp\left(\int_e A_a dx^a\right) \qquad E_S = \int_S E^a \epsilon_{abc} dx^a dx^b$$

Background independent

quantum Schwarzschild BH: interior as an example



$\Sigma = \mathbb{R} \times S^2$, **homogeneity implies the symmetry group of** $\mathbb{T} \times \mathrm{SO}(3)$



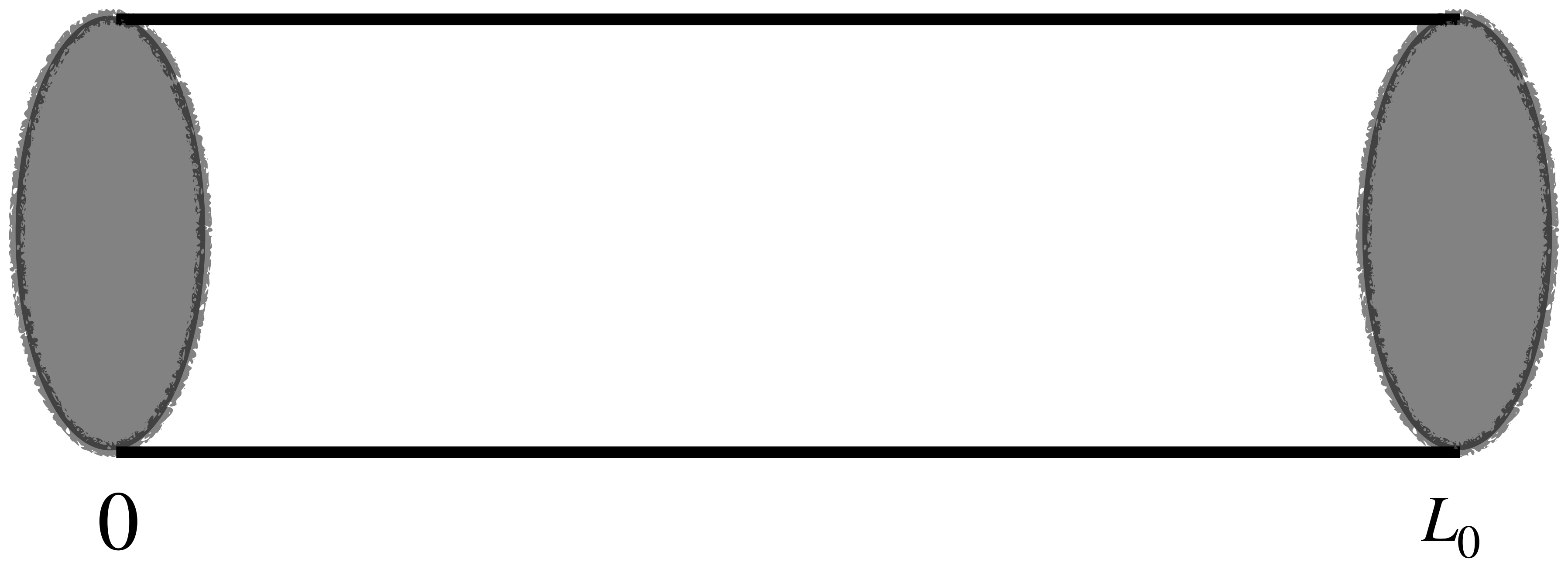
quantum Schwarzschild BH: interior as an example

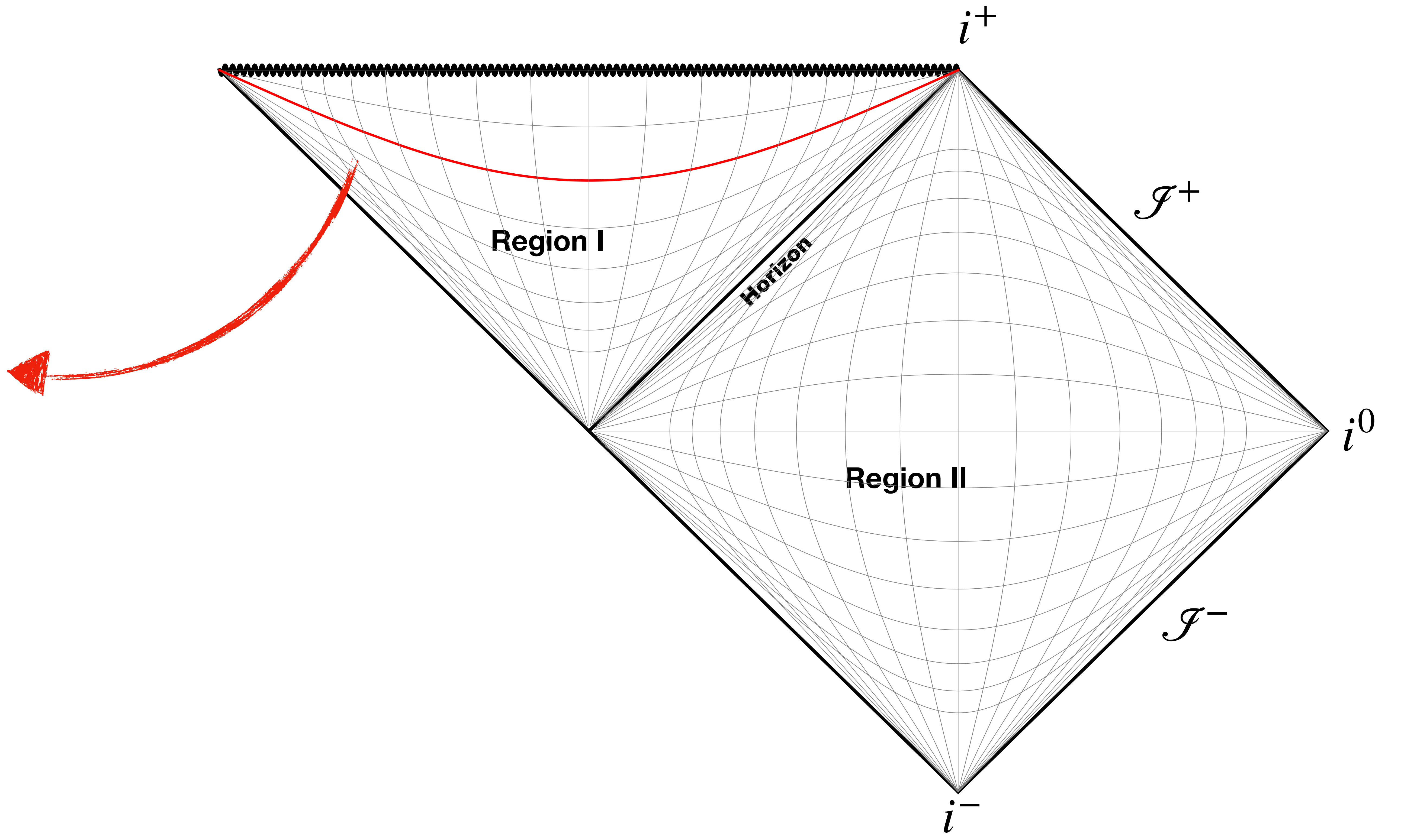
- $\mathscr{M}_4 = \mathbb{R} \times \mathbb{R} \times S^2 \ni (\tau, x, \theta, \phi)$
- **The metric with the symmetry $\mathbb{T} \times \mathrm{SO}(3)$:**

$$ds^2 = -N(\tau)^2 d\tau^2 + q_{xx}(\tau) dx^2 + q_{\theta\theta}(\tau) d\Omega^2$$

Einstein equation

$$ds^2 = -\left(\frac{2M}{\tau} - 1\right)^2 d\tau^2 + \left(\frac{2M}{\tau} - 1\right) dx^2 + \tau^2 d\Omega^2$$





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quantum Schwarzschild BH: interior as an example

We do canonical quantization for this system !

Question 1: what are the canonical pairs?

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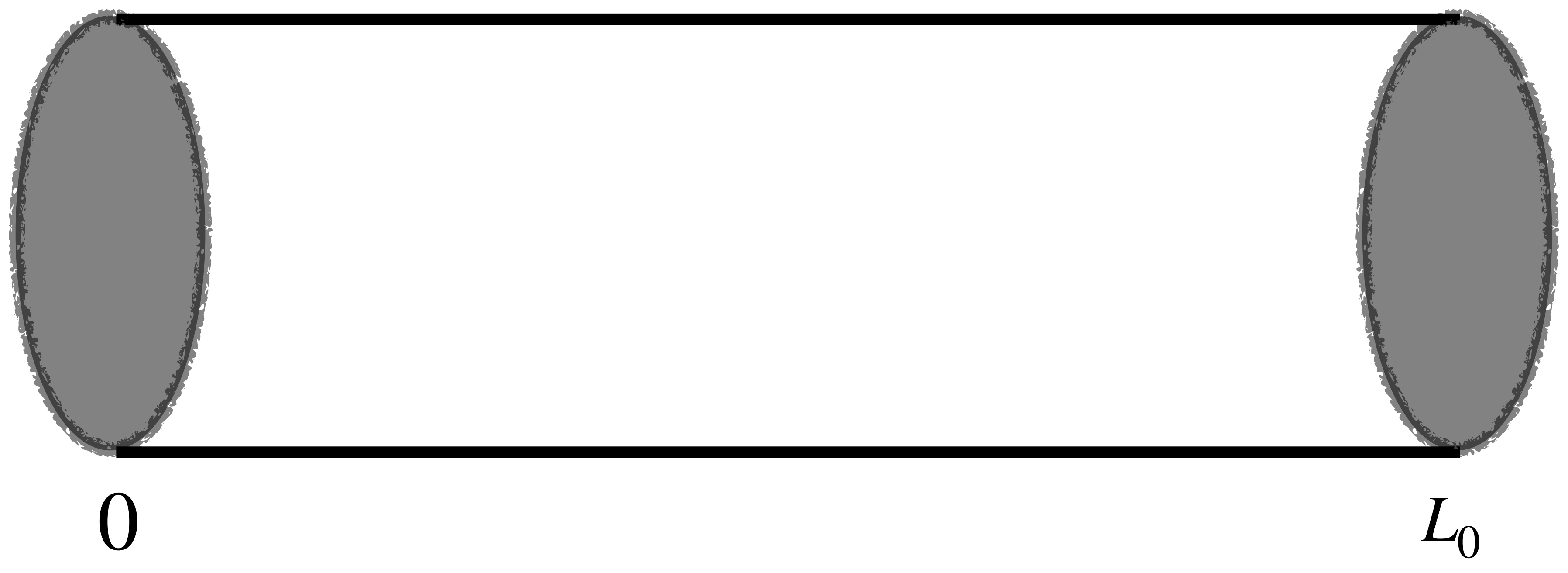

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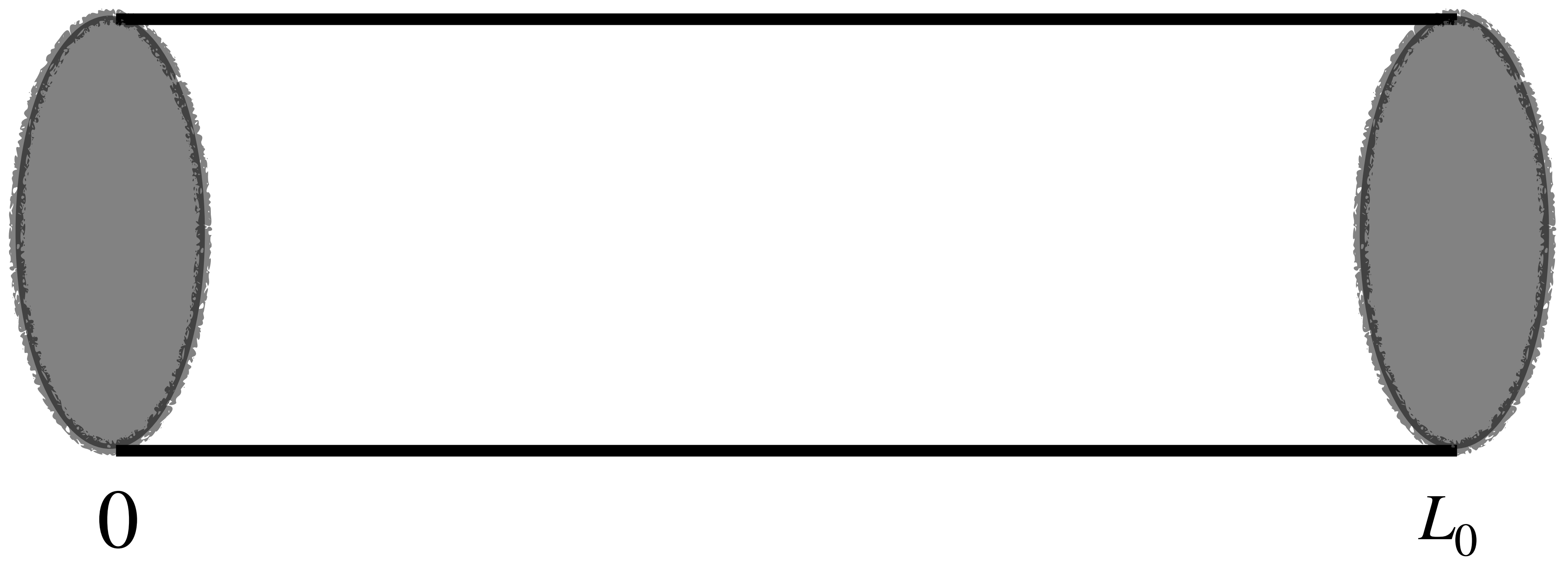

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Question 1: what are the canonical pairs?

The canonical pairs can be found via:

- $S = \int_{\mathbb{R}} d\tau \int_{[0,L_0]\times S^2} d^3x \sqrt{g} R[q]$
- $P^{xx} = \dfrac{\delta S}{\delta \dot{q}_{xx}}, \; P^{\theta\theta} = \dfrac{\delta S}{\delta \dot{q}_{\theta\theta}}$

quantum Schwarzschild BH: interior as an example

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However, in LQG, we prefer the Ashtekar variables (A_a^i, E_i^a), with

$$q^{ab} = \frac{E_i^a E_j^b \delta^{ij}}{\sqrt{\det(E)}},\ \{A_a^i(x), E_j^b(y)\} = 8\pi G\gamma\delta^3(x,y)$$

0 $\qquad$ L_0

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quantum Schwarzschild BH: interior as an example

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We thus need rewrite: $(q_{xx}, q_{\theta\theta}) \mapsto (\frac{p_b^2}{p_c}, p_c)$, so that E_i^a takes the simple form: $E_i^a \tau^i \partial_a = p_c \tau_3 \sin\theta \partial_x + \frac{p_b}{L_0}\tau_1 \sin\theta \partial_\theta + \frac{p_b}{L_0}\tau_2 \partial_\phi$.

quantum Schwarzschild BH: interior as an example

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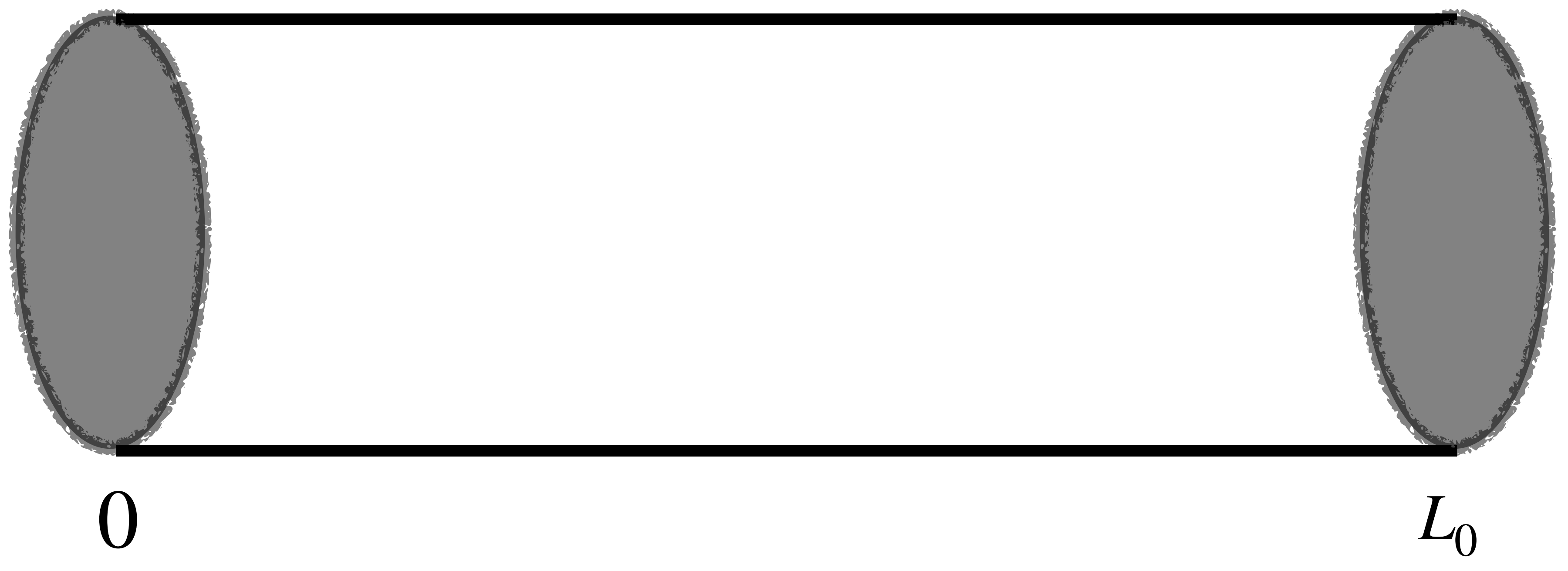

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Variables conjugate to p_b, p_c are denoted by b, c. The Ashtekar connection can be written in a, c as:

$$A_a^i \tau_i dx^a = \frac{c}{L_0} \tau_3 dx^2 + b\tau_1 d\theta + b\tau_2 \sin\theta d\phi + \tau_3 \cos\theta d\phi$$

$$\{b, p_b\} = G\gamma, \ \{c, p_c\} = 2G\gamma$$

quantum Schwarzschild BH: interior as an example

Question 2: How about the dynamics?

The dynamics is encoded in the Hamiltonian constraint H:

$$S = \int d\tau \left(\frac{1}{G\gamma}\dot{a}p_a + \frac{1}{2G\gamma}\dot{b}p_b - NH \right)$$

$$H = -\frac{1}{2G\gamma^2 p_b \sqrt{p_c}} \left(\gamma^2 p_b^2 + p_b^2 b^2 + 2bcp_b p_c \right)$$

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Question 3: How to reconstruct the metric?

- Choose a N
- $\dot{o} = \{o, NH\}, \forall o = b, c, p_b, p_c$ with initial data satisfying $H(b_o, c_o, p_b^{(o)}, p_c^{(o)}) = 0$
- $ds^2 = -N^2 d\tau^2 + \frac{p_b^2}{p_c} dx^2 + p_c d\Omega^2$ is independent of the choice of N
- ds^2 remains the same for initial data related with canonical transformation of H

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quantum Schwarzschild BH: interior as an example

- **Phase space containing canonical pairs** (b, p_b), (c, p_c)
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Canonical Quantization: $(b, p_b, c, p_c) \mapsto (\hat{b}, \hat{p}_b, \hat{c}, \hat{p}_c)$**, with** $[\hat{o}_1, \hat{o}_2] = i\hbar \widehat{\{o_1, o_2\}}$

Possible approach: **Schrodinger** **quantization,** $\mathscr{H} = L^2(\mathbb{R}^2),\ b, c$ **are multiplication operator,** $\hat{p}_b = -i\gamma G\hbar\partial_b,\ \hat{p}_c = -2i\gamma G\hbar\partial_c$

Loop quantum Schwarzschild BH: interior as an example

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Loop Approach: Inspired by full loop quantum gravity

- **Inner product:** $\langle f, g\rangle = \lim_{T\to\infty} \frac{1}{(2T)^2} \int_{-T}^{T}\int_{-T}^{T} \bar{g}(b,c) f(b,c)\, db dc$
- $\mathscr{H} = \{f, \|f\| < \infty\}$
- $\widehat{e^{i\lambda b}} f(b,c) = e^{i\lambda b} f(b,c), \quad \widehat{e^{i\lambda c}} f(b,c) = e^{i\lambda c} f(b,c)$
- $\hat{p}_b = -i\gamma\ell_p^2 \partial_b, \ \hat{p}_c = -2i\gamma\ell_p^2 \partial_c$

Loop quantum Schwarzschild BH: interior as an example

Inner product: $\langle f, g\rangle = \lim_{T\to\infty} \frac{1}{(2T)^2}\int_{-T}^{T}\int_{-T}^{T} \overline{g}(b,c)f(b,c)dbdc$

$$\langle e^{i\lambda b}, e^{i\lambda' b}\rangle = \lim_{T\to\infty} \frac{1}{(2T)^2}\int_{-T}^{T}\int_{-T}^{T} e^{i(\lambda'-\lambda)b}dbdc = \delta_{\lambda,\lambda'}$$

$|\lambda,\mu\rangle = e^{i\lambda b + i\mu c},\ \forall \lambda,\mu \in \mathbb{R}^2$ **forms the orthonormal basis of the Hilbert space. Our Hilbert space has uncountably many basis vectors. Non- separable Hilbert space.**

$$\widehat{e^{i\lambda_o b + i\mu_o c}}\,|\lambda,\mu\rangle = |\lambda+\lambda_o, \mu+\mu_o\rangle \qquad \hat{p}_b|\lambda,\mu\rangle = \gamma\ell_p^2\lambda|\lambda,\mu\rangle \qquad \hat{p}_b|\lambda,\mu\rangle = 2\gamma\ell_p^2\mu|\lambda,\mu\rangle$$

$\lim_{\lambda_o\to 0}\langle\lambda,\mu|\frac{\widehat{e^{i\lambda_o b}}-1}{\lambda_o}|\lambda,\mu\rangle = \lim_{\lambda_o\to 0}\frac{-1}{\lambda_o} = \infty$**, implies** $\hat{b} := \lim_{\lambda_o\to o}\frac{\widehat{e^{i\lambda b}}-1}{\lambda_o}$ **is not well-defined. The same for** $\hat{c}$

Question: How can we promote $H = -\frac{1}{2G\gamma^2 p_b\sqrt{p_c}}\left(\gamma^2 p_b^2 + p_b^2 b^2 + 2bcp_bp_c\right)$ **to an operator?**

Loop quantum Schwarzschild BH: interior as an example

$$\lim_{\lambda_o \to 0} \langle \lambda, \mu | \frac{\widehat{e^{i\lambda_o b}} - 1}{\lambda_o} | \lambda, \mu \rangle = \lim_{\lambda_o \to 0} \frac{-1}{\lambda_o} = \infty$$, implies $\hat{b} := \lim_{\lambda_o \to o} \frac{\widehat{e^{i\lambda b}} - 1}{\lambda_o}$ is not well-defined. The same for $\hat{c}$

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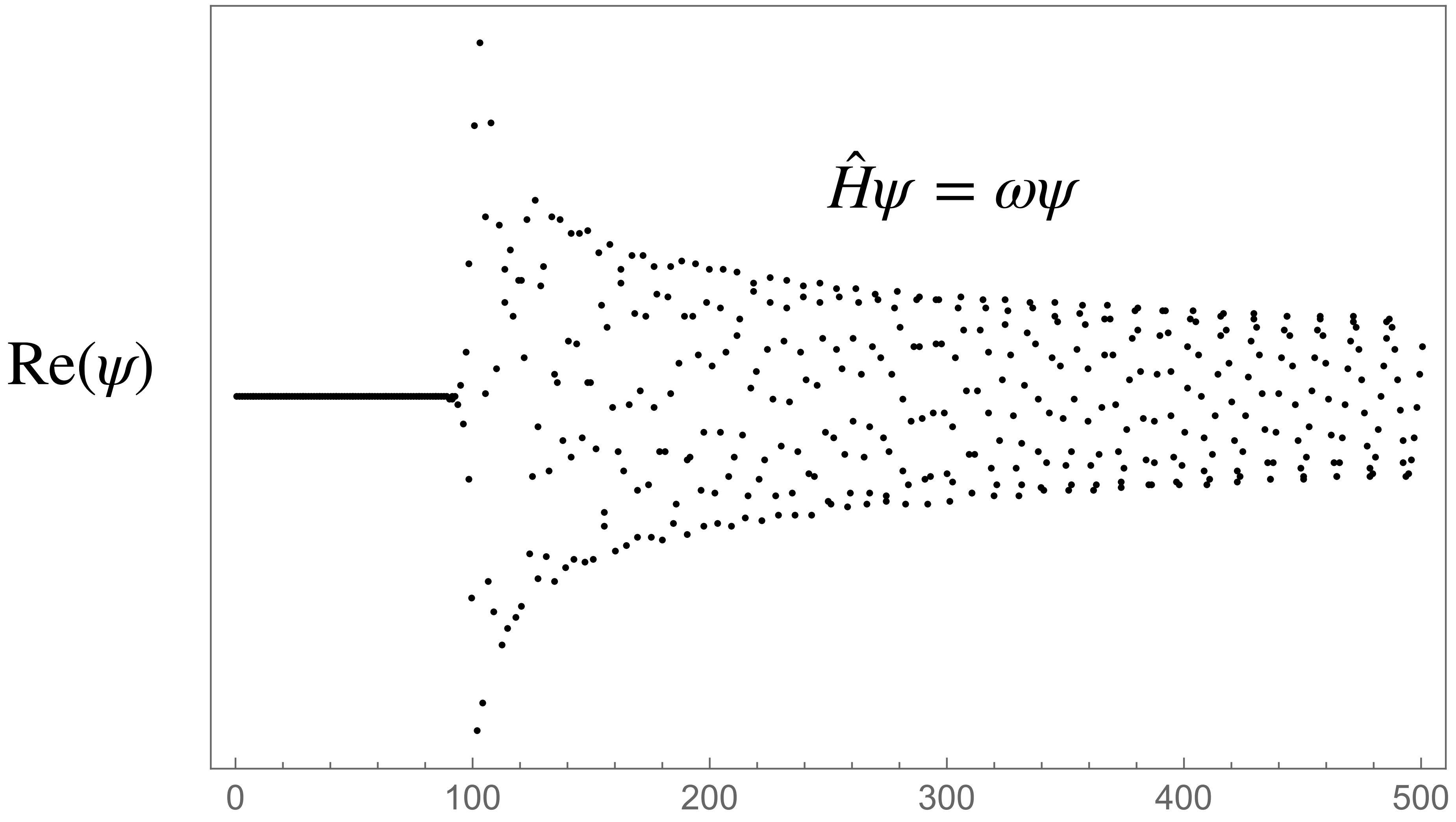
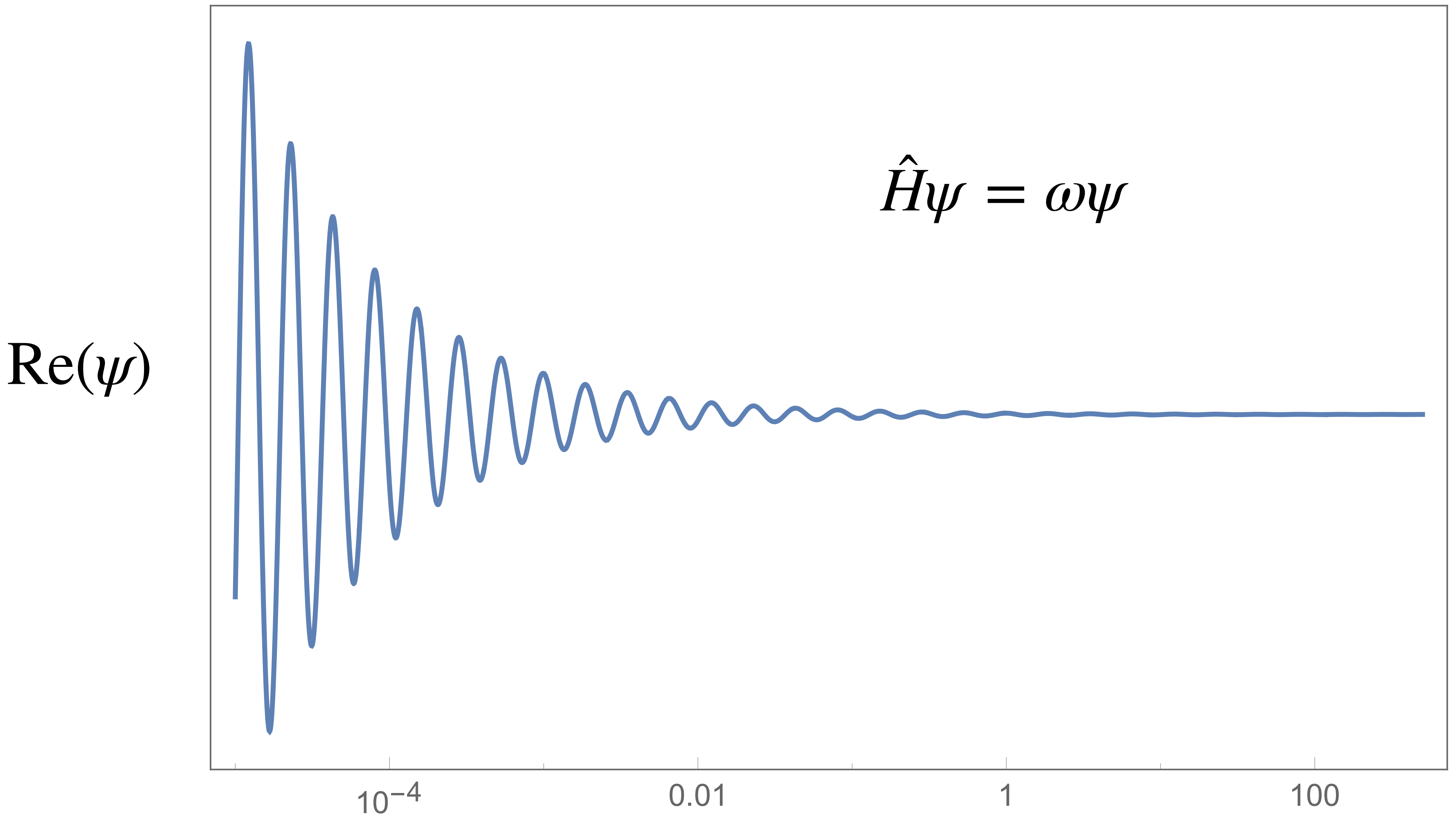
We need to do regularization for H**. Inspired by the full LQG, the regularization will replace** $b, c \to \frac{\sin(\lambda b)}{\lambda}, \frac{\sin(\mu c)}{\mu}$

Consequently, $\hat{b} | \lambda_o \rangle \mapsto \frac{\widehat{\sin(\lambda b)}}{\lambda} | \lambda_o \rangle = \frac{1}{2\lambda}(|\lambda_o + \lambda\rangle - |\lambda_o - \lambda\rangle)$

Difference operator is a key point for singularity resolution.

$$-i\frac{\partial}{\partial t}\psi = \hat{H}\psi, \quad \hat{H} = \sqrt{\hat{p}}\hat{x}\sqrt{\hat{p}}$$

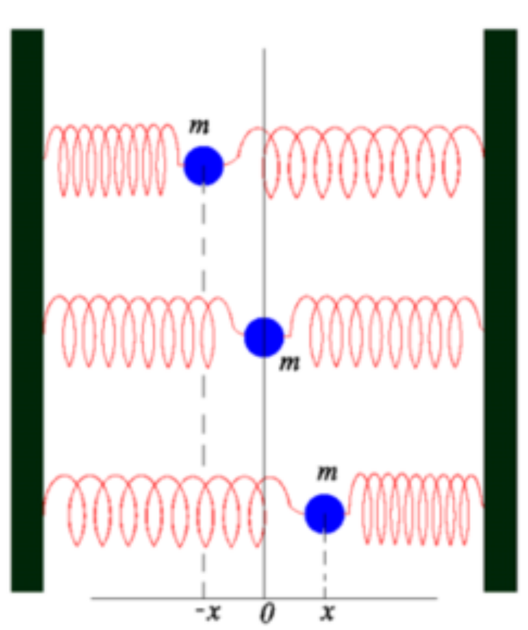


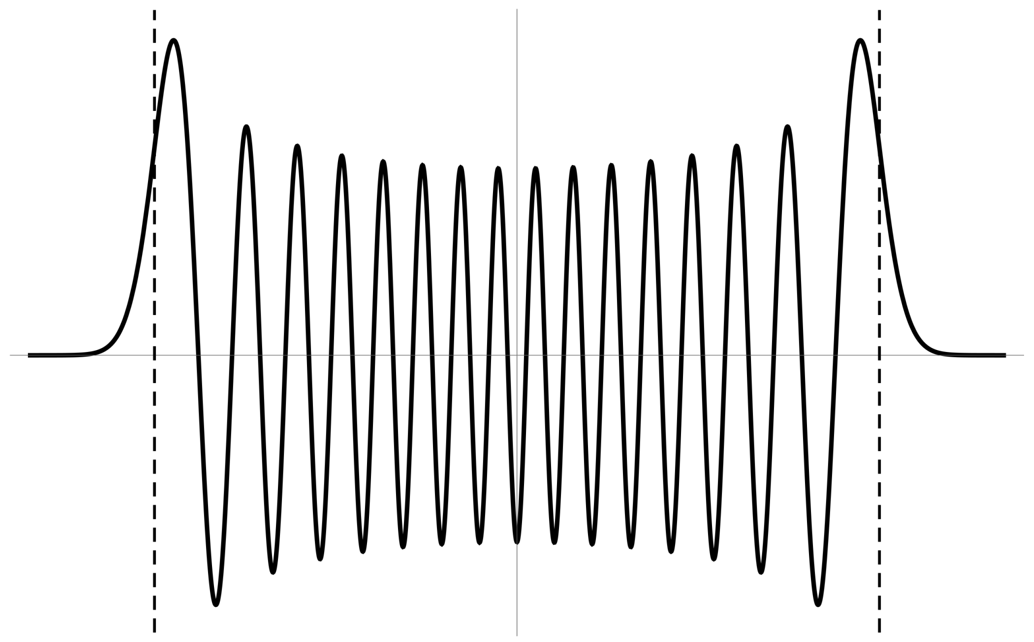


$$(\hat{x}\psi)(p) \to i(\psi(p+1) - \psi(p-1))$$

$$\hat{x}\psi \to i\frac{\mathrm{d}}{\mathrm{d}p}\psi$$

[CZ, Lewandowski, Ma 19']





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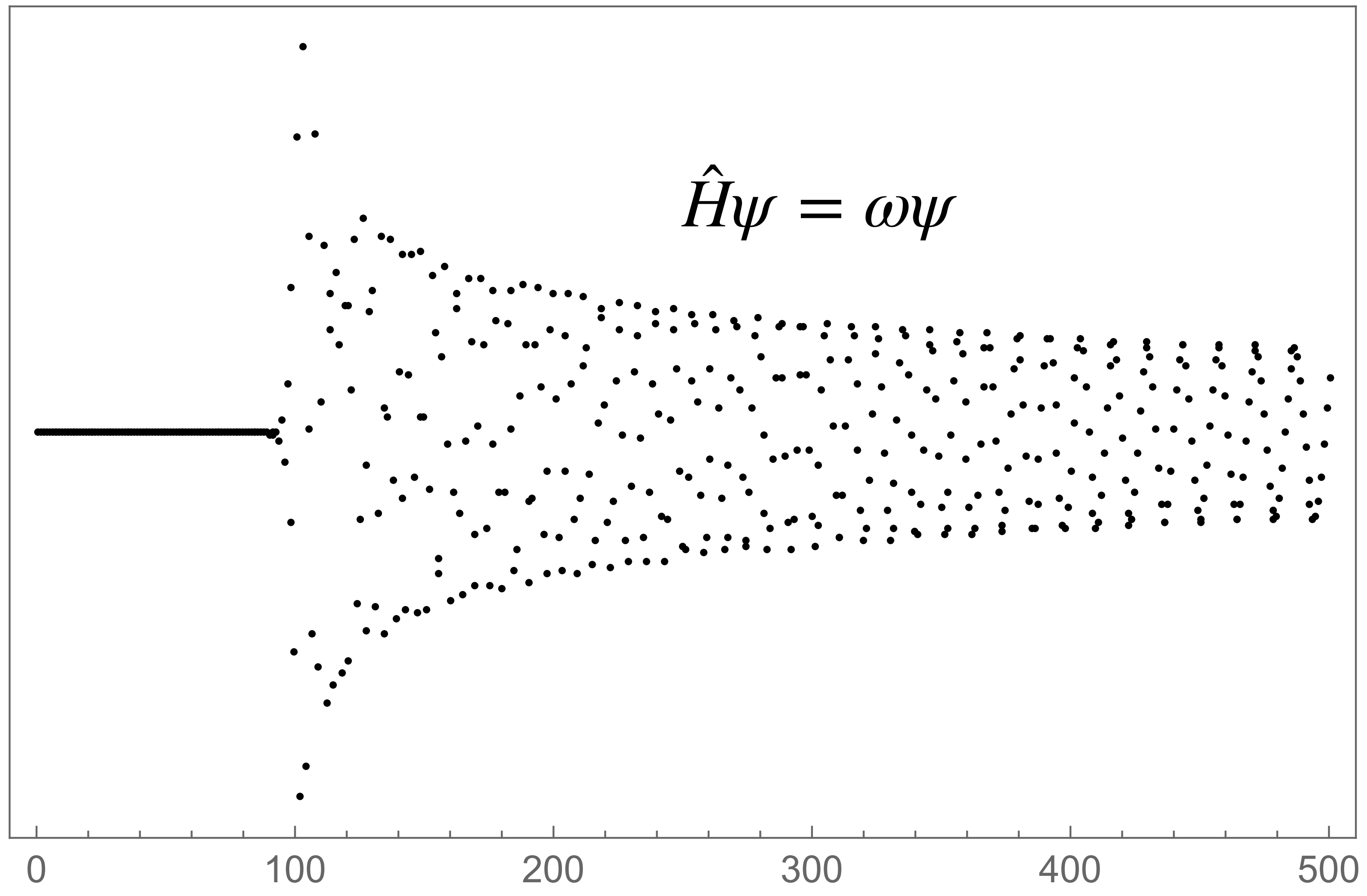
Re(ψ)



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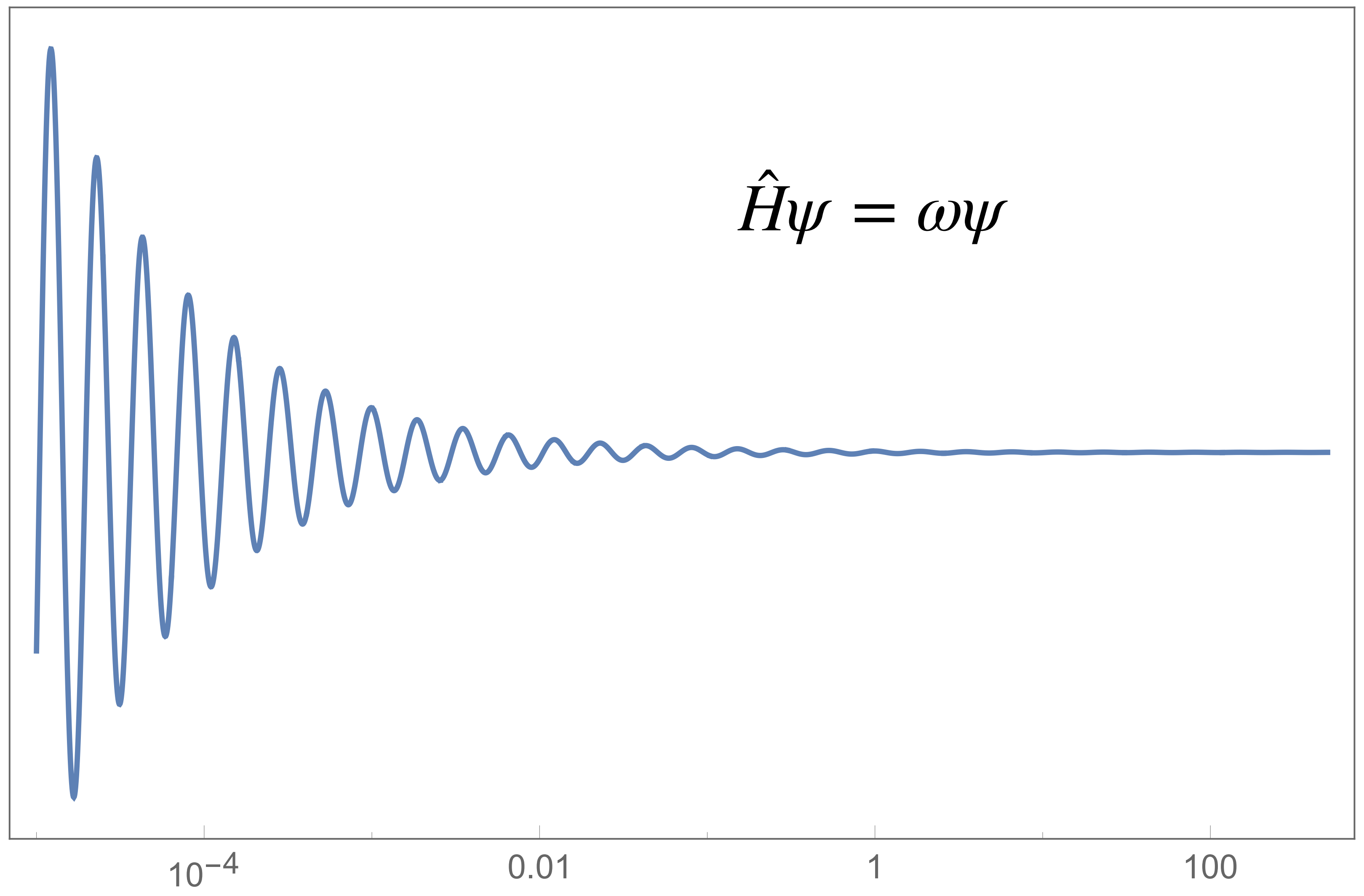

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[CZ, Lewandowski, Ma 19']

Loop quantum Schwarzschild BH: interior as an example

Choosing $N = -V = 2G\gamma^2 p_b \sqrt{p_c}$**, we have** $H[V] = 2p_b b c p_c + p_b^2 b^2 + \gamma^2 p_b^2$

Regularization leads to: $H[V]^{(\tilde{\delta}_b, \tilde{\delta}_c)} = 2p_b \frac{\sin(\tilde{\delta}_b b)}{\tilde{\delta}_b} p_c \frac{\sin(\tilde{\delta}_c c)}{\tilde{\delta}_c} + p_b^2 \frac{\sin^2(\tilde{\delta}_b b)}{\tilde{\delta}_b^2} + \gamma^2 p_b^2$

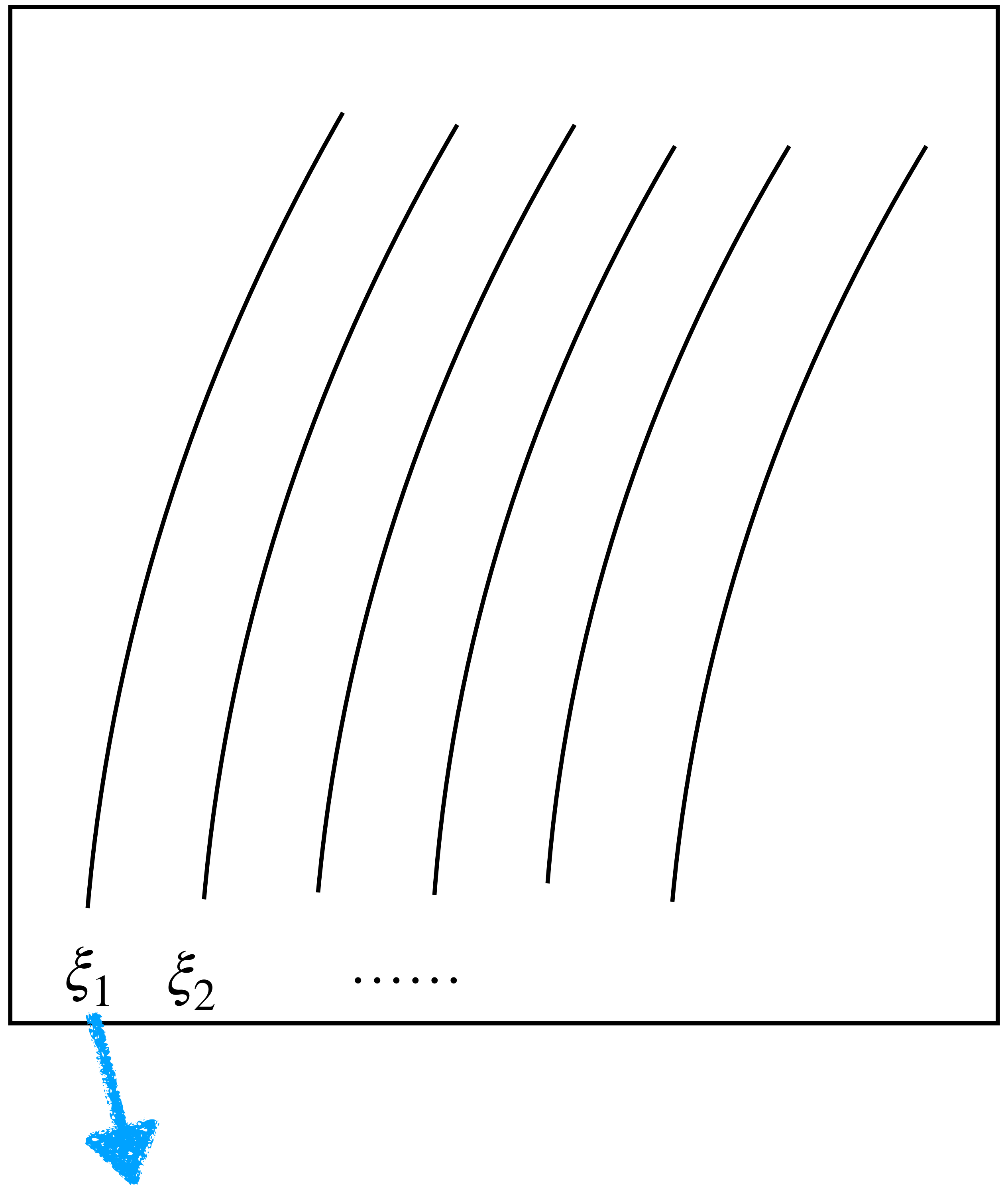
Classically, $H[V] = \lim_{\tilde{\delta}_b, \tilde{\delta}_c \to 0} H[V]^{(\tilde{\delta}_b, \tilde{\delta}_c)}$ **but in quantum theory,** $\widehat{H[V]} = \lim_{\tilde{\delta}_b, \tilde{\delta}_c \to \delta_b, \delta_c} \widehat{H[V]}^{(\tilde{\delta}_b, \tilde{\delta}_c)}$

Question: How to choose the parameters: $\delta_b,\ \delta_c$

Ambiguities arise due to various choices of δ_b, δ_c **:**

- μ_0**—scheme, constant** δ_b, δ_c **;** [Boehmer Vandersloot 07', Chiou 08']
- $\bar{\mu}$**—scheme,** δ_b, δ_c **being phase space function;** [Chiou 08']
- **New scheme,** δ_b, δ_c **being function of dynamical trajectories.** [Corichi, Singh 16', Ashtekar, Olmedo Singh 18']

(p_b, p_c) space



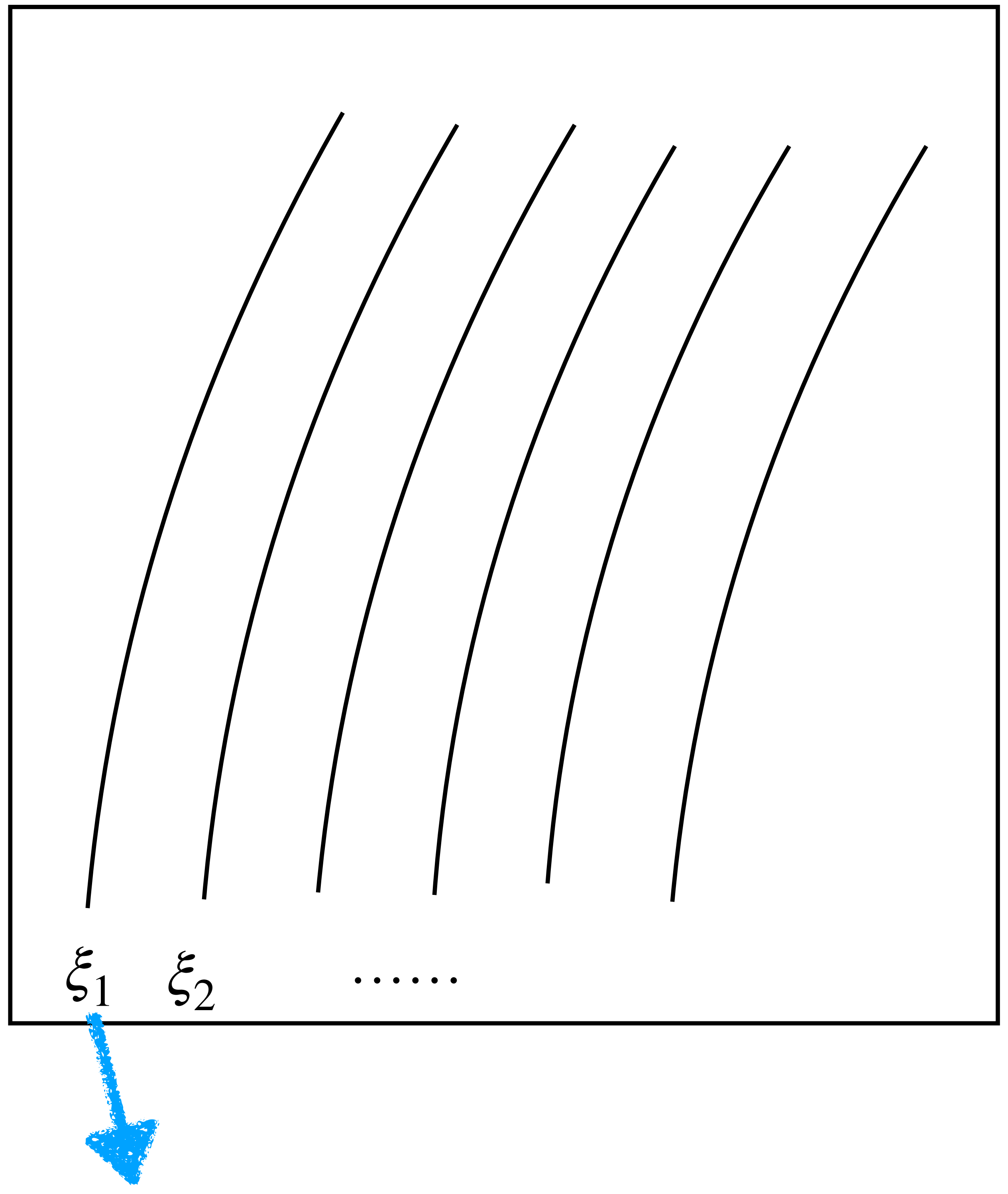
Labels of the dynamical trajectories

The basic idea:

- δ_b, δ_c have the physical interpretation of coordinate length of edges,
- the fundamental discreteness prevents the parameter from reaching 0.
- various lengths prevented by the discreteness leads to various schemes.

- μ_0—scheme, coordinate length is prevented $\Rightarrow$ constant δ_b, δ_c; [Boehmer and Vandersloot 07', Chiou 08']
- $\bar{\mu}$—scheme, physical length along the trajectory is presented $\Rightarrow$ $\delta_b(p_b, p_c), \delta_c(p_b, p_c)$; [Chiou 08']
- New scheme, the physical length at the bounce is not 0. [Ashtekar, Olmedo Singh 18']

(p_b, p_c) **space**



Labels of the dynamical trajectories

The basic idea:

- δ_b, δ_c **have the physical interpretation of coordinate length of edges,**
- **the fundamental discreteness prevents the parameter from reaching 0.**
- **various lengths prevented by the discreteness leads to various schemes.**

- μ_0**—scheme, coordinate length is prevented** $\Rightarrow$ **constant** δ_b, δ_c**;** [Boehmer and Vanderslhoot 07', Chiou 08']
- $\bar{\mu}$**—scheme, physical length along the trajectory is presented** $\Rightarrow$ $\delta_b(p_b, p_c), \delta_c(p_b, p_c)$**;** [Chiou 08']
- **New scheme, the physical length at the bounce is not 0.** [Ashtekar, Olmedo Singh 18']

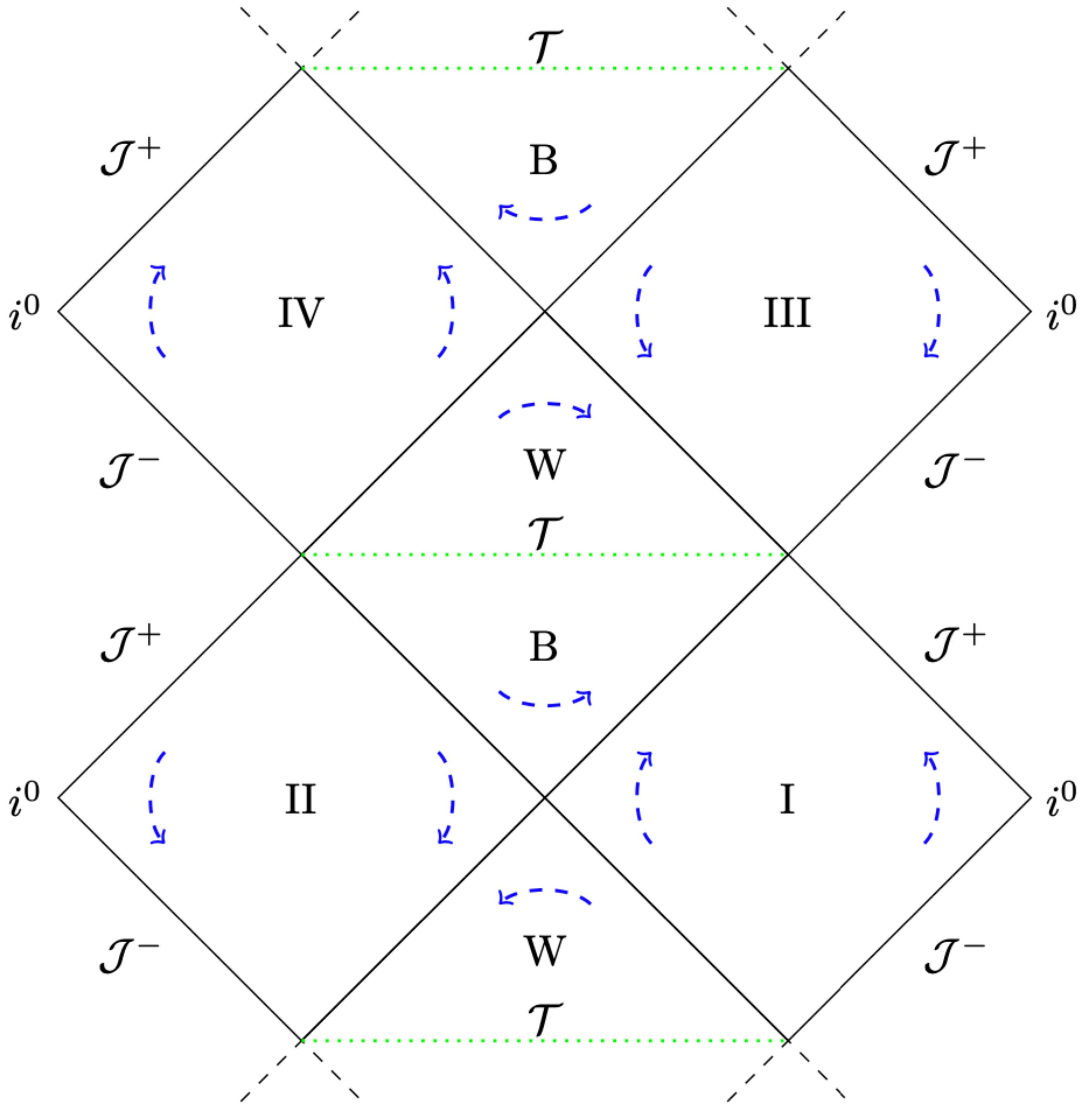
(Potential) limitation:

- μ_0**—scheme: the physical prediction depends on fiducial cell; bounce happens when curvature is small;**
- $\bar{\mu}$**—scheme: large departures from the classical theory very near the horizon; But the horizon is replace by singularity if matter is involved, then the large quantum correction is appropriate.**
- **New scheme: one actually needs to extend the phase space to include** δ_b, δ_c**.**

Loop quantum Schwarzschild BH: interior as an example

Some results:

- **Effective dynamics:** singularity resolution, BH-WH transition, etc.
  [Boehmer Vanderslhoot 07’, Chiou 08’, Corichi, Singh 16’, Ashtekar, Olmedo Singh 18’]
- **Quantum dynamics: discreteness of BH mass at the dynamical level;** [CZ, Ma, Song, Zhang 20’ & 21’]



[Ashtekar, Olmedo Singh 18’]

Loop quantum Schwarzschild BH: interior as an example

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- **Effective dynamics: singularity resolution, BH-WH transition, etc.** [Boehmer Vandersloot 07', Chiou 08', Corichi, Singh 16', Ashtekar, Olmedo Singh 18']
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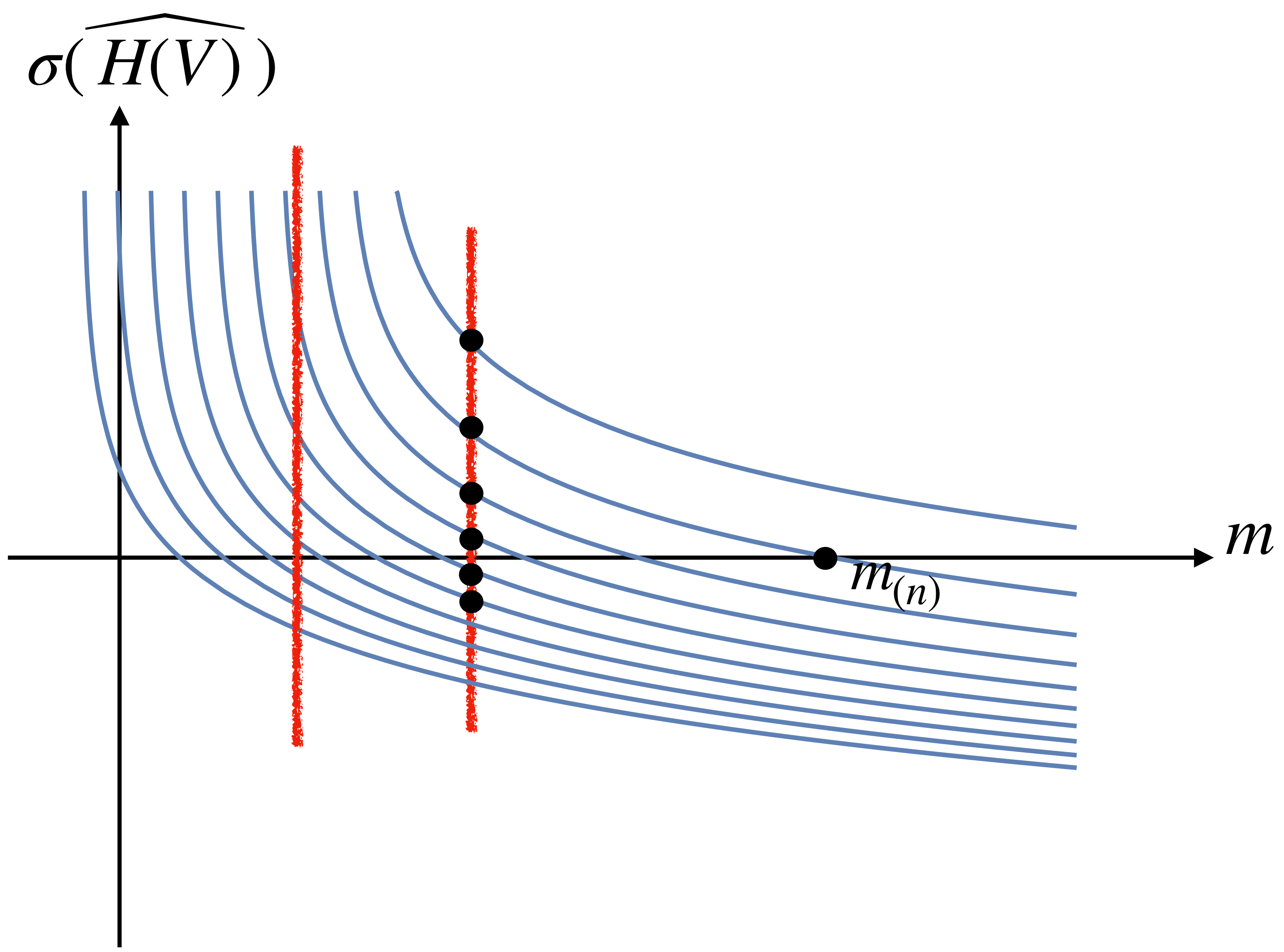


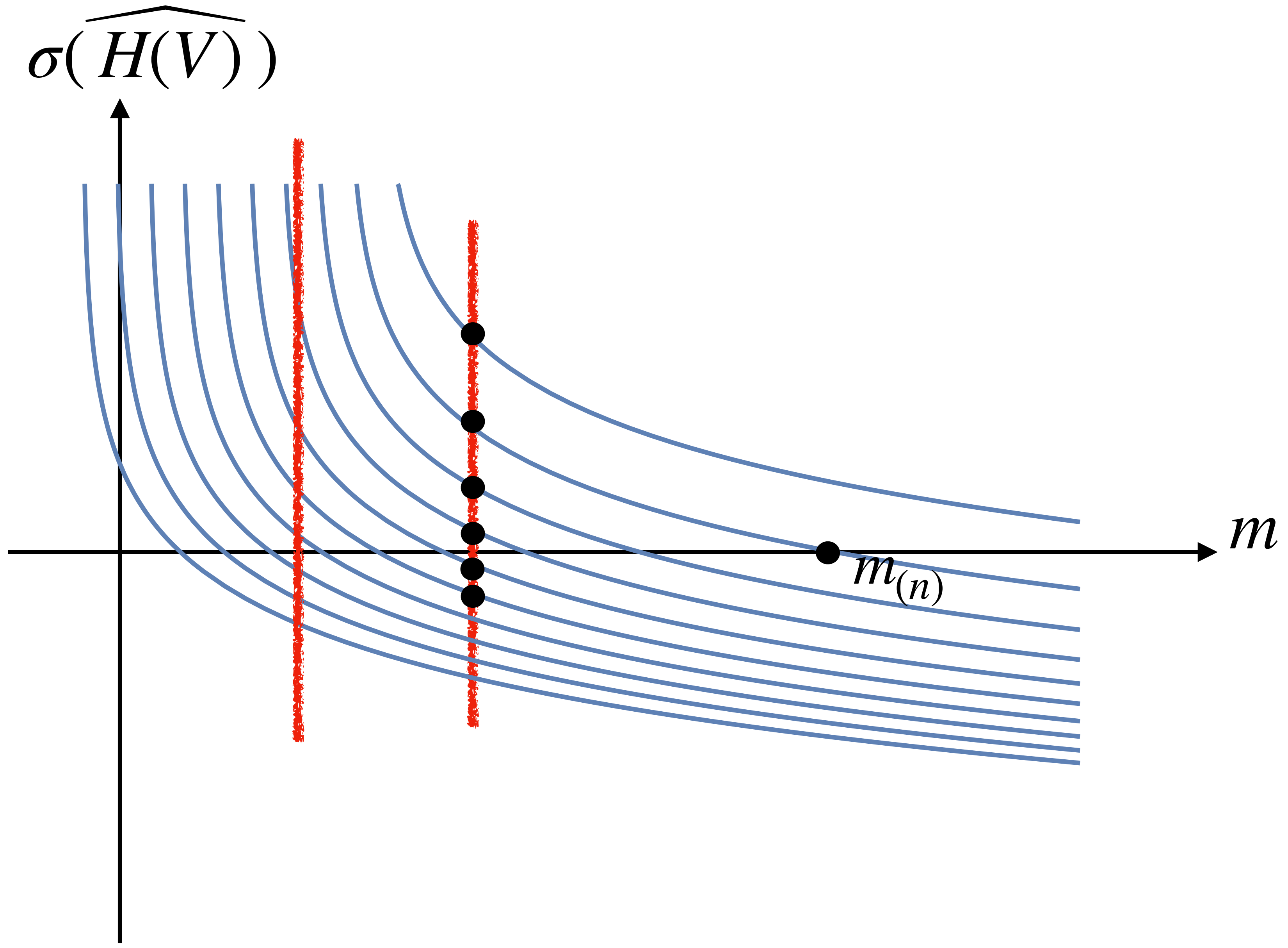
Constraint equation: $\widehat{H[V]} = 0$:

- **We find an operator $\hat{m}$ s.t $[\hat{m}, \widehat{H[V]}] = 0$;**
- **The Hilbert space is expanded by the common eigenstate $|m, h\rangle$;**
- **m is continuous but h is discrete; the range of h depend on m;**
- **Only for countably many values $m_{(n)}$, one can obtain $|m_{(n)}, h = 0\rangle$;**
- **The minimal value $m_{(0)}$ is not vanishing.**

Loop quantum Schwarzschild BH: interic

Some results:

- **Effective dynamics: singularity resolution, BH-WH transition, etc.**
  [Boehmer Vandersloot 07', Chiou 08', Corichi, Singh 16', Ashtekar, Olmedo Singh 18']
- **Quantum dynamics: discreteness of BH mass at the dynamical level**



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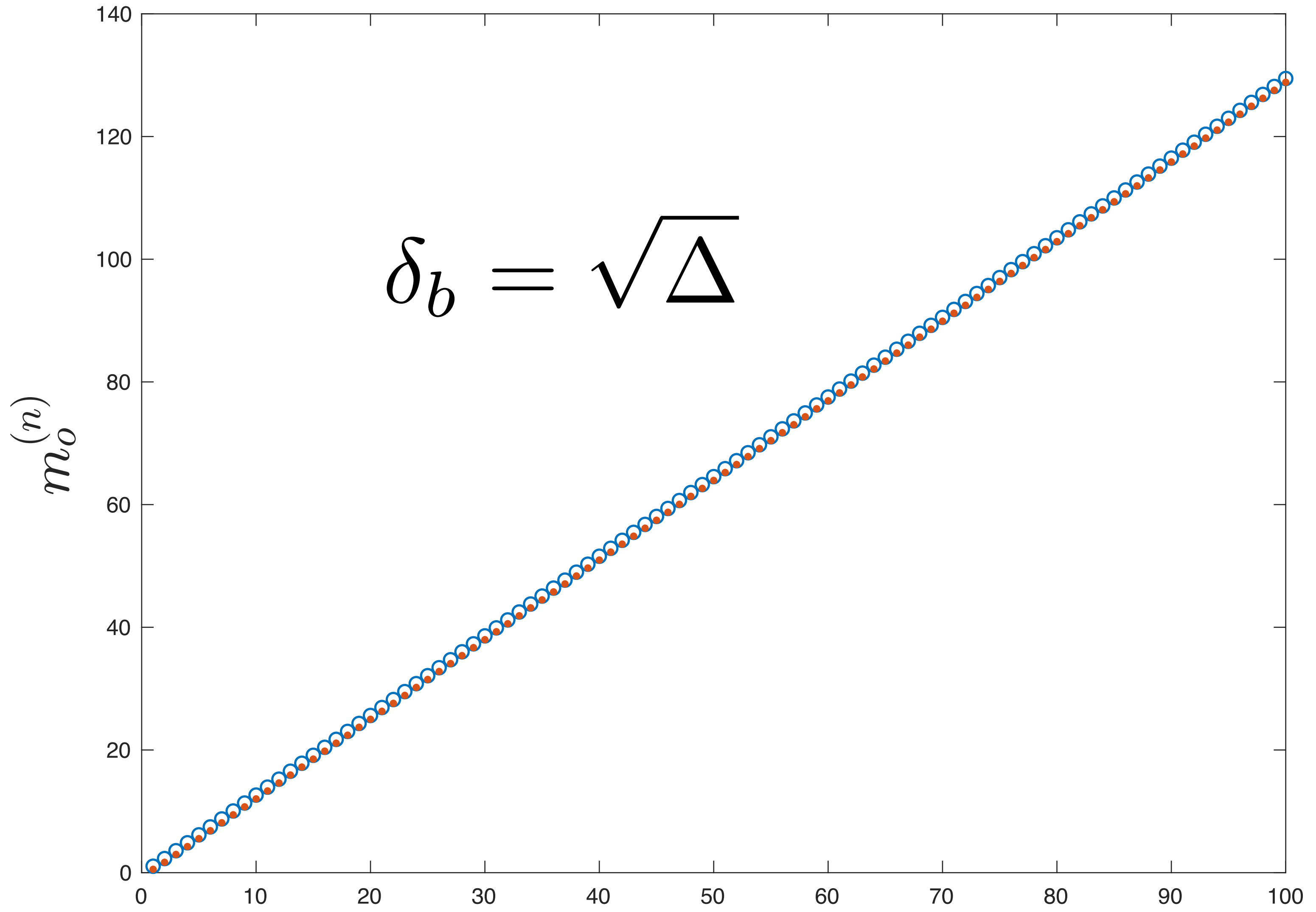
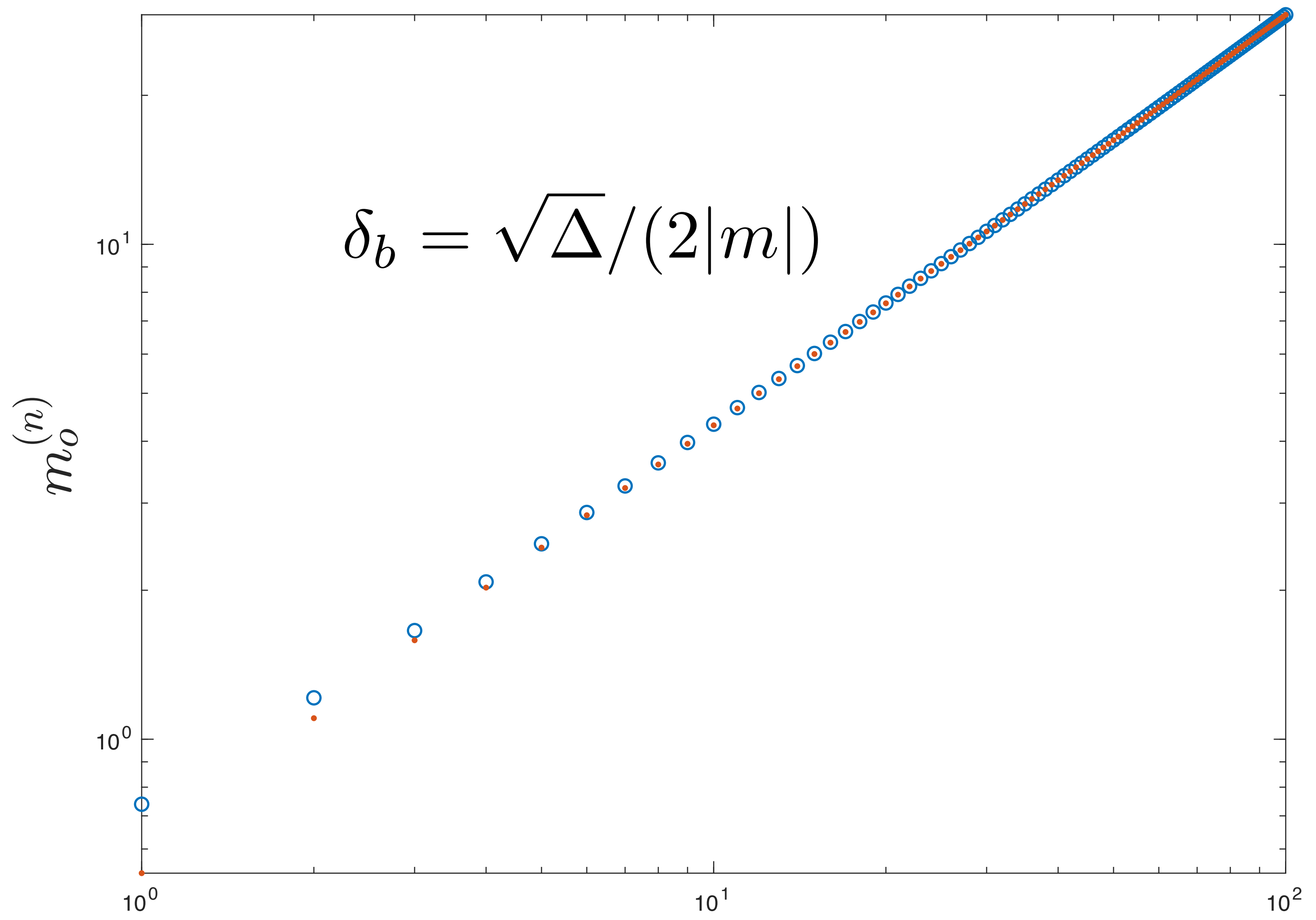




Summary

With the Schwarzschild interior as the example, we introduce:

- **Canonical quantization of a BH model**
- **Loop quantization**
- **Some recent results from LQGBH**

End of the First Lecture

Introduction to Loop Quantum Black Hole Models

Cong Zhang

Friedrich-Alexander-Universität Erlangen-Nürnberg

Review of the issue in the previous lecture

Choosing $N = -V = 2G\gamma^2 p_b \sqrt{p_c}$**, we have** $H[V] = 2p_b b c p_c + p_b^2 b^2 + \gamma^2 p_b^2$

Regularization leads to: $H[V]^{(\tilde{\delta}_b, \tilde{\delta}_c)} = 2p_b \dfrac{\sin(\tilde{\delta}_b b)}{\tilde{\delta}_b} p_c \dfrac{\sin(\tilde{\delta}_c c)}{\tilde{\delta}_c} + p_b^2 \dfrac{\sin^2(\tilde{\delta}_b b)}{\tilde{\delta}_b^2} + \gamma^2 p_b^2$

Classically, $H[V] = \lim_{\tilde{\delta}_b, \tilde{\delta}_c \to 0} H[V]^{(\tilde{\delta}_b, \tilde{\delta}_c)}$ **but in quantum theory,** $\widehat{H[V]} = \lim_{\tilde{\delta}_b, \tilde{\delta}_c \to \delta_b, \delta_c} \widehat{H[V]}^{(\tilde{\delta}_b, \tilde{\delta}_c)}$

Question: How to choose the parameters: $\delta_b,\ \delta_c$

Ambiguities arise due to various choices of δ_b, δ_c **:**

- μ_0**—scheme, constant** δ_b, δ_c **;** [Boehmer Vandersloot 07', Chiou 08']
- $\bar{\mu}$**—scheme,** δ_b, δ_c **being phase space function;** [Chiou 08']
- **New scheme,** δ_b, δ_c **being function of dynamical trajectories.** [Corichi, Singh 16', Ashtekar, Olmedo Singh 18']

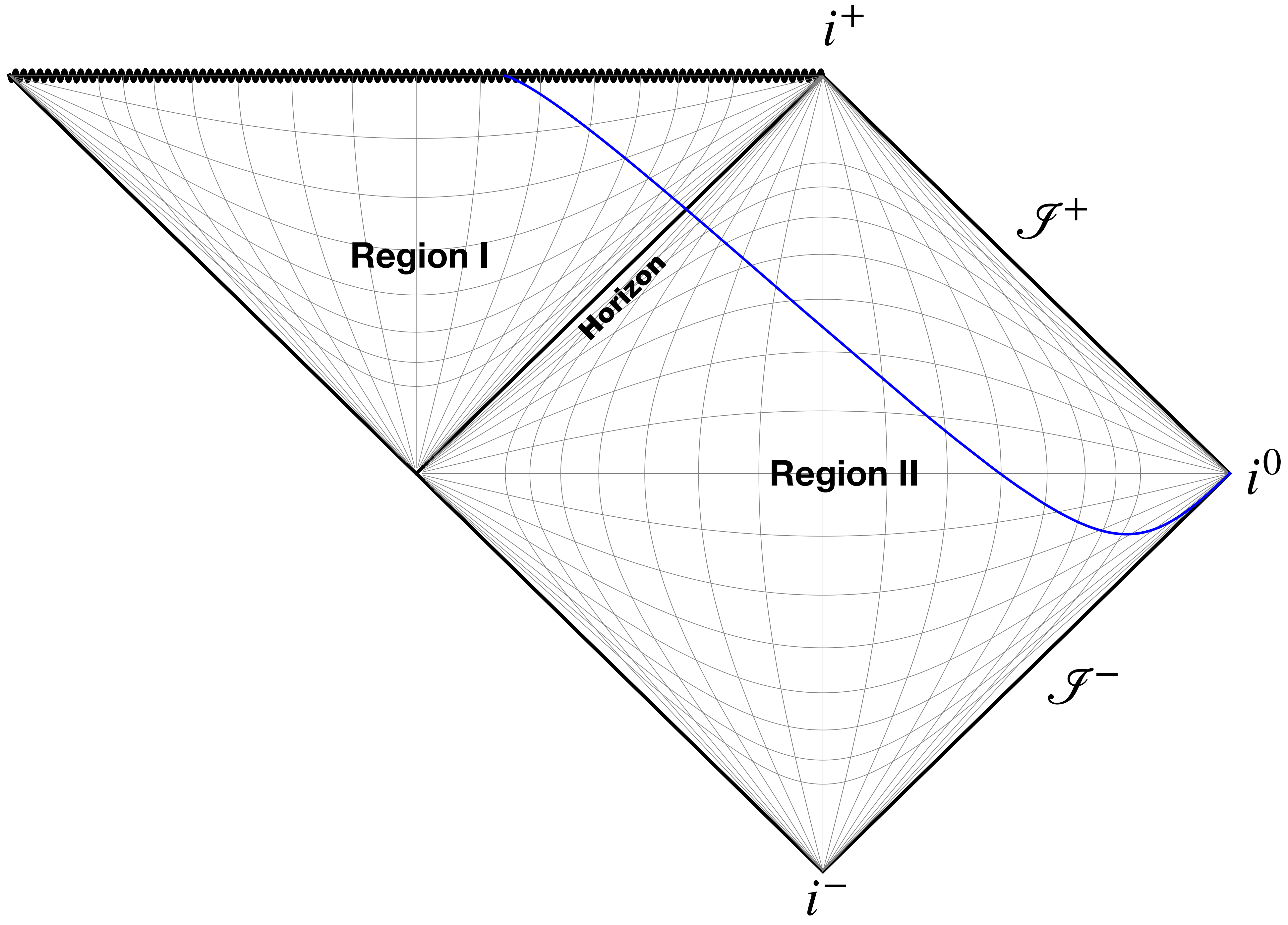
Outline

A. Introduction of a Loop Quantum BH Model: Schw. interior as an example
—Canonical quantization of BH
—What is loop quantization
—Some results

B. Some recent results in LQGBH
—Spherical symmetry model
—Quantum Oppenheimer-Snyder model
—QG effects on BH image, et al

Loop quantum Schwarzschild BH: spherically symmetric model

Spherically symmetric Model

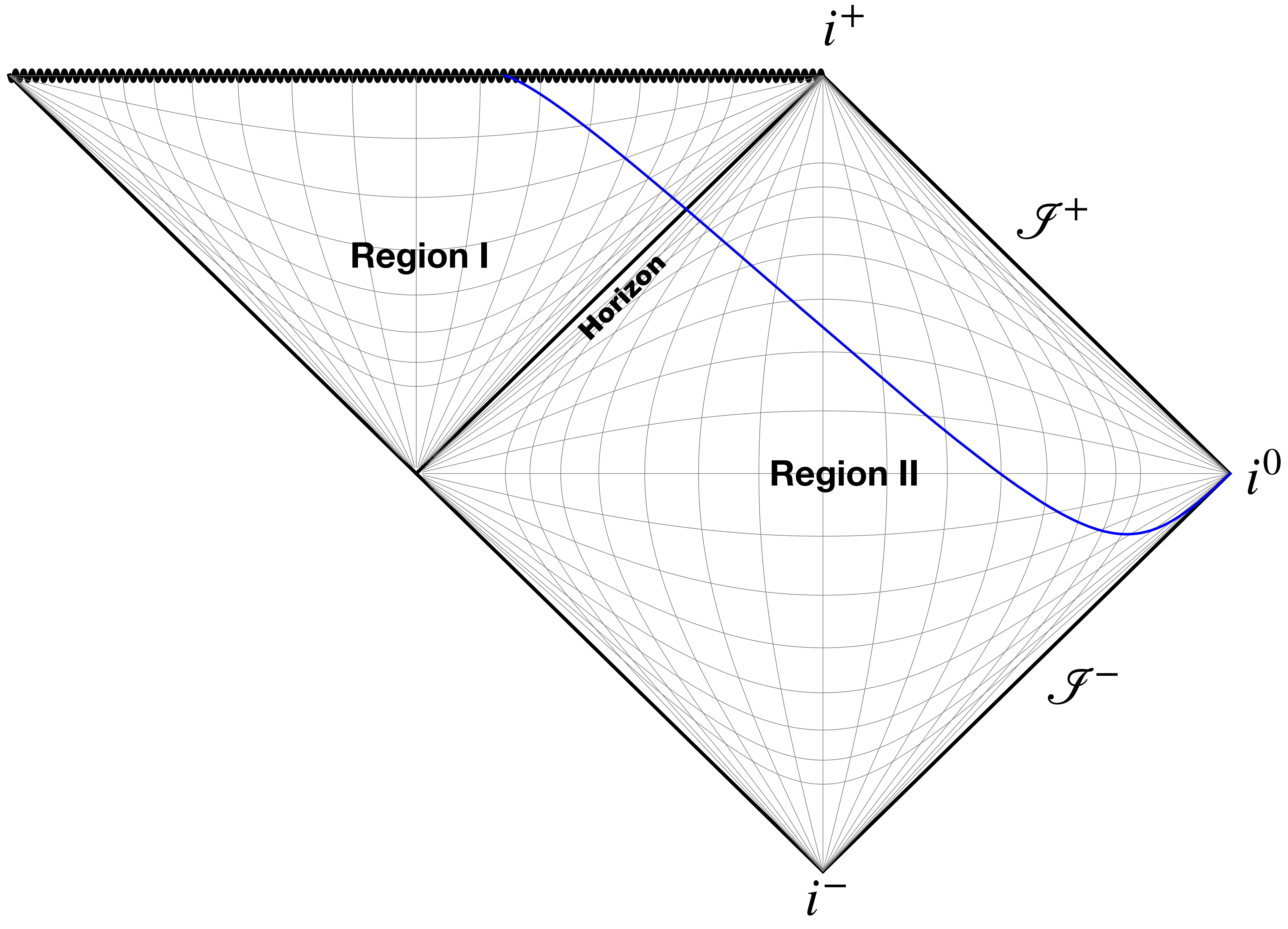


- $\mathbb{R} \times \mathrm{S}^2$ **with symmetry** $\mathrm{SO}(3)$ **:** $\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{(E^2)^2}{E^1}(\mathrm{d}x + N^x\mathrm{d}t)^2 + E^1\mathrm{d}\Omega^2$;
- **Quantization: promote** $E^1(x)$ **and** $E^2(x)$ **to operators acting on a Hilbert space.**

Loops quantization: choose the polymer Hilbert space as the home of the operators.

Loop quantum Schwarzschild BH: spherically symmetric model

Spherically symmetric Model



$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{(E^2)^2}{E^1}(\mathrm{d}x + N^x\mathrm{d}t)^2 + E^1\mathrm{d}\Omega^2$$

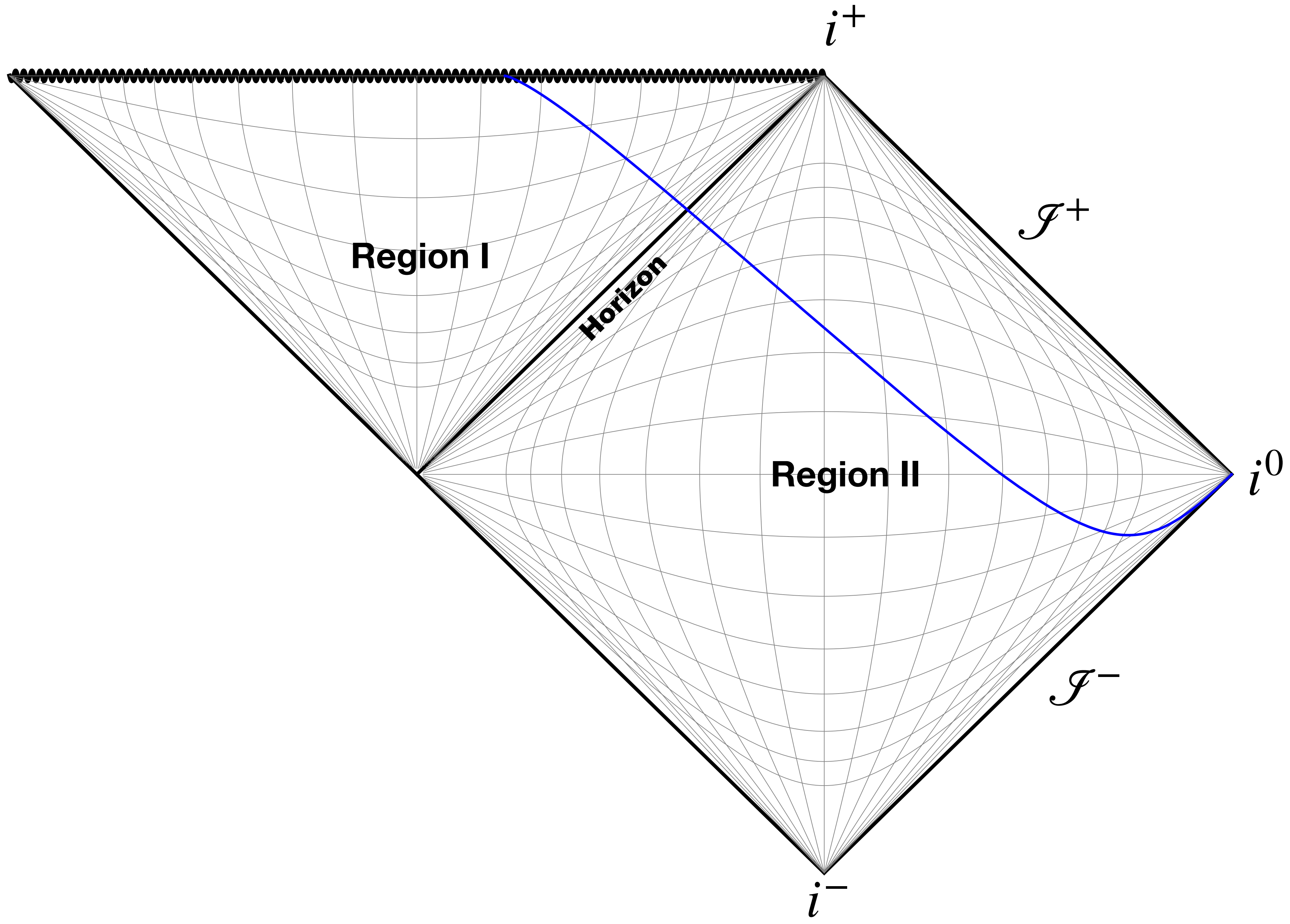
In the classical theory, we have the constraints algebra $H_x[N^x] + H[N]$ with:

$$H_x(x) = \frac{1}{2G}\left(2E^2(x)\partial_x K_2(x) - K_1(x)\partial_x E^1(x)\right)$$

$$H(x) = -\frac{1}{2G}\frac{1}{\sqrt{|E^1(x)|\,|E^2(x)|}}\Big\{[E^2(x)]^2 + [K_2(x)E^2(x)]^2 + 2K_1(x)E^1(x)K_2(x)E^2(x) - \frac{1}{4}[\partial_x E^1(x)]^2 - E^1(x)E^2(x)\partial_x\left[\frac{\partial_x E^1(x)}{E^2(x)}\right]\Big\}$$

Loop quantum Schwarzschild BH: spherically symmetric model

Spherically symmetric Model



$$ds^2 = -N^2 dt^2 + \frac{(E^2)^2}{E^1}(dx + N^x dt)^2 + E^1 d\Omega^2$$

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We can do loop regularization for:

1) $H(x)$ itself [Han, Liu 20', CZ 21']
2) $H(x) + N_o(x)H_x(x)$ [Gambini, Olmedo, Pullin 14' & 20]

Or another approach:

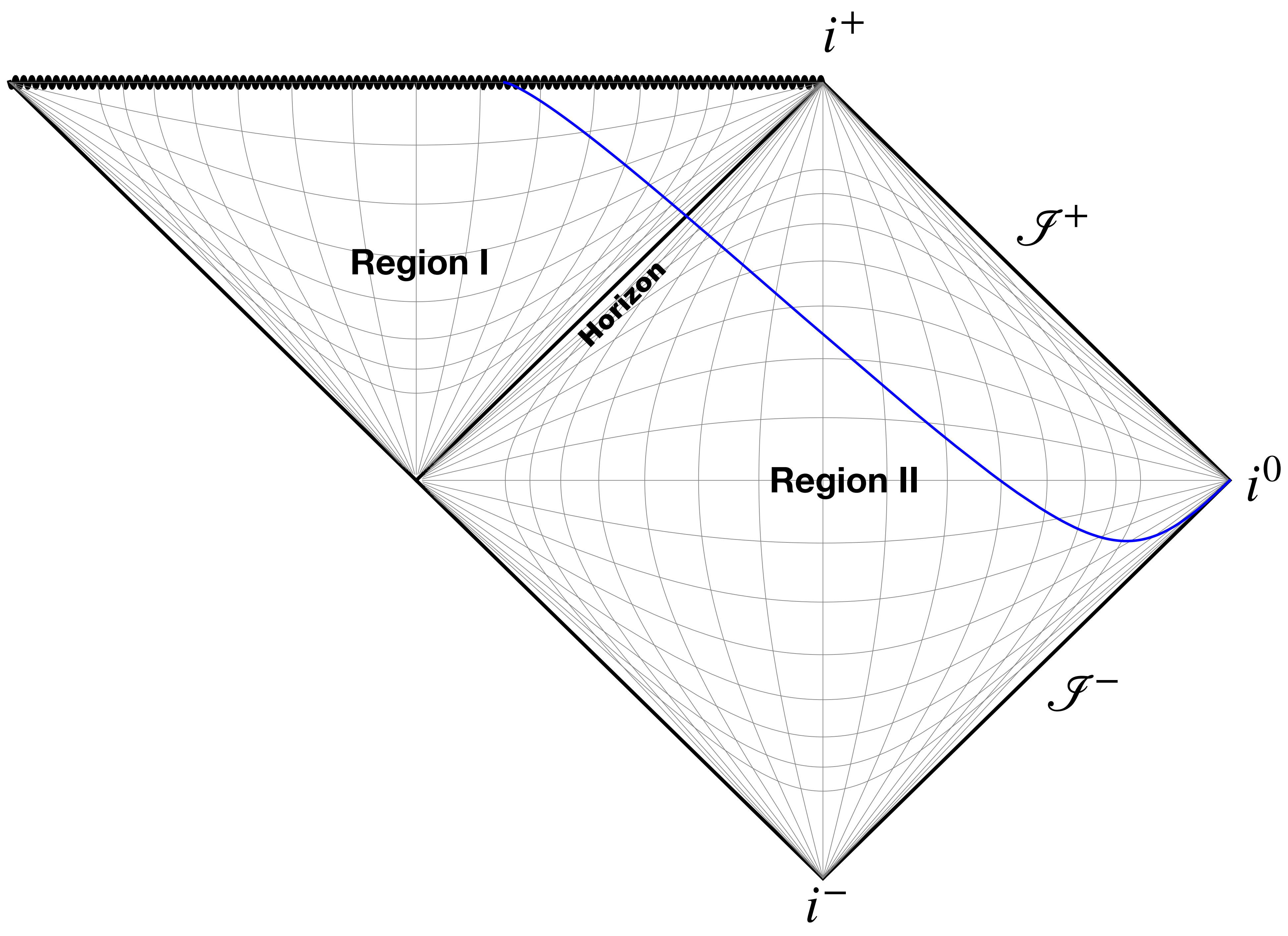
3) Choose $E^1(x) = x^2$ as the gauge solving the diff. constraint $H_x(x) = 0$ to get

$$H[N] = -\frac{1}{2G}\int dx \frac{N(x)}{|xE^2(x)|}\left\{[E^2(x)]^2\partial_x\left(x[K_2(x)]^2 + x - \frac{x^3}{[E^2(x)]^2}\right)\right\}.$$

Do loop regularization for this Gauge fixed Hamiltonian [Kelly, Santacruz, Wilson-Ewing 20'&22']

Loop quantum Schwarzschild BH: spherically symmetric model

Spherically symmetric Model



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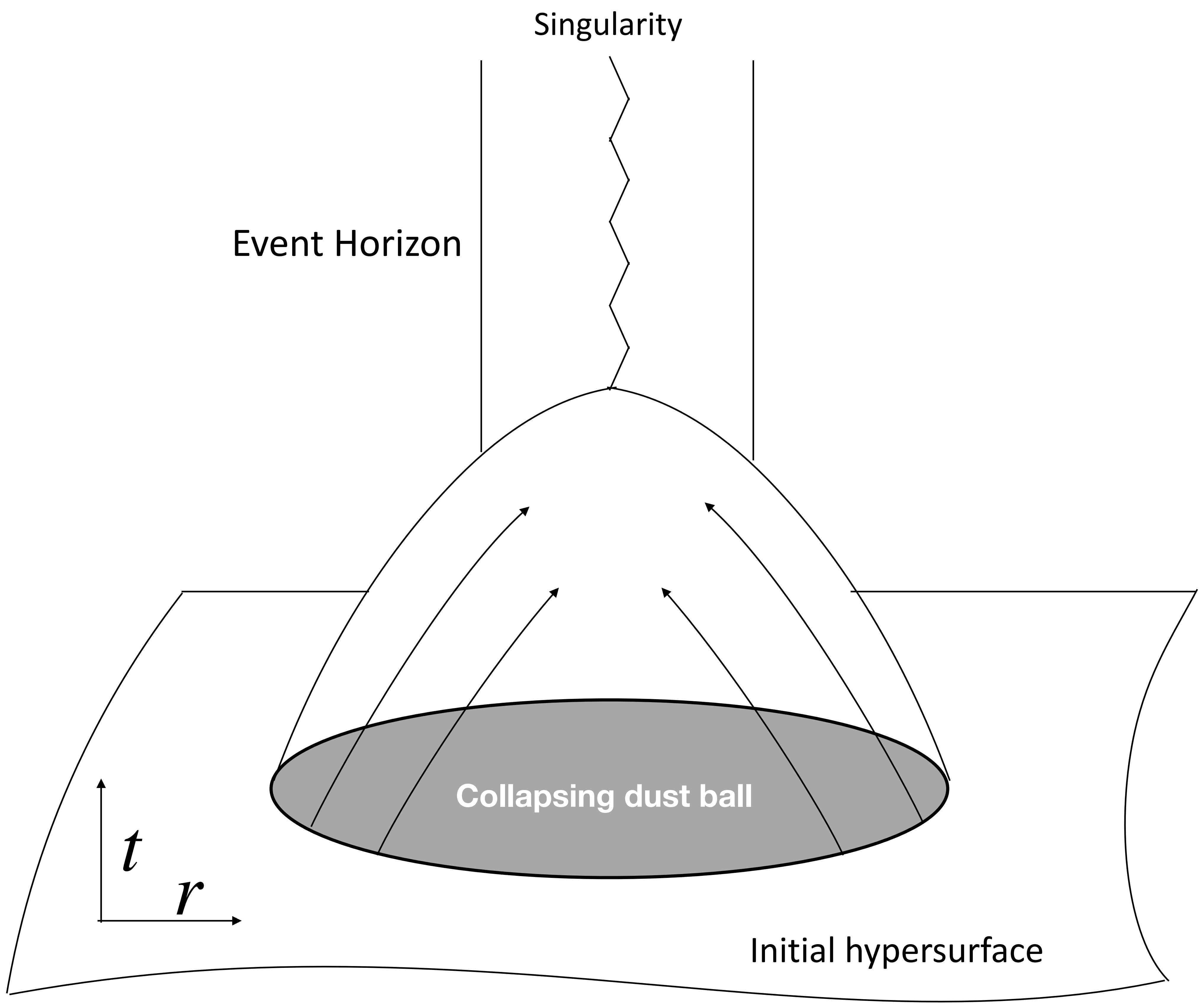
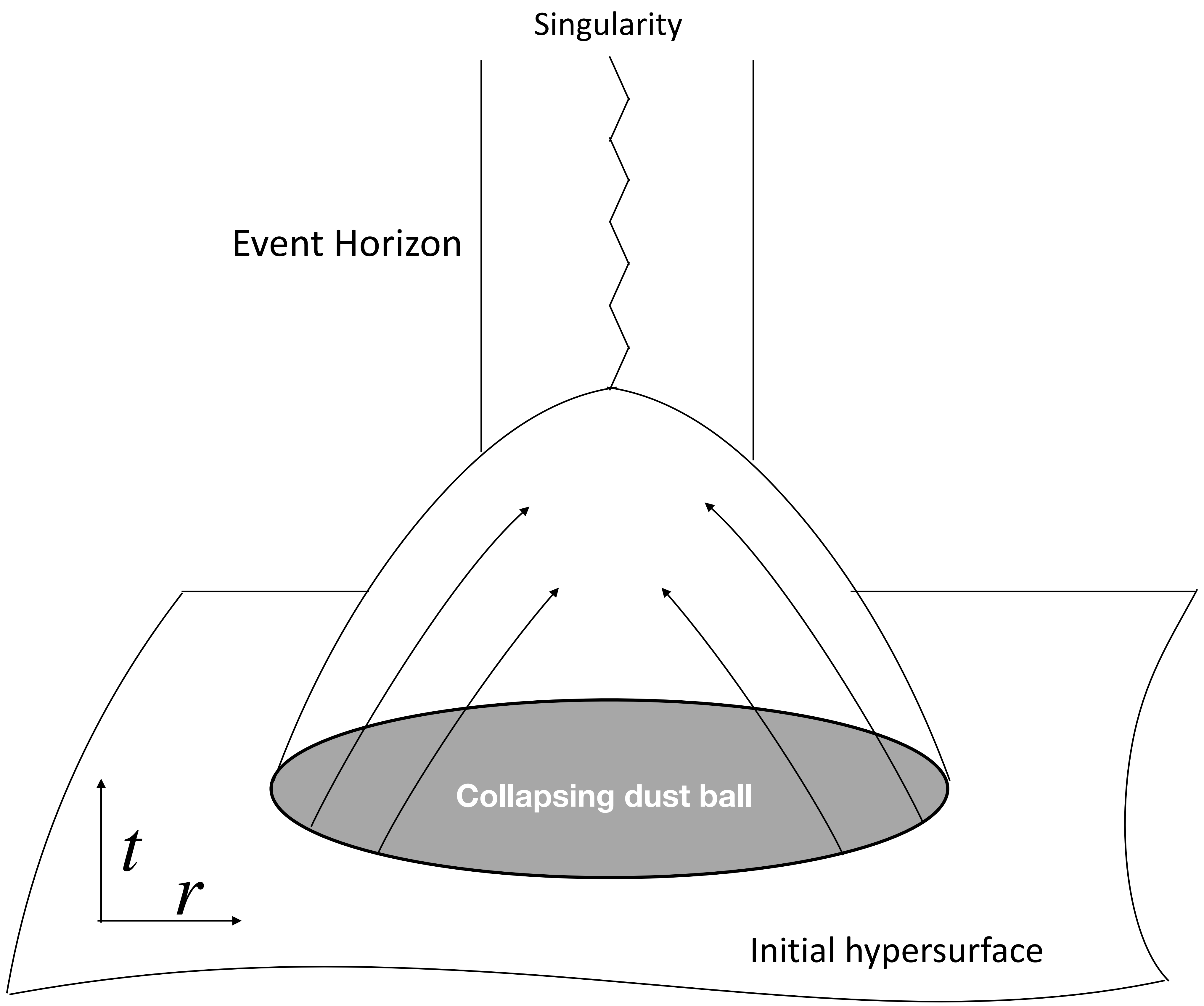
Do loop regularization for this Gauge fixed Hamiltonian [Kelly, Santacruz, Wilson-Ewing 20'&22']

Use the loop regularized Hamiltonian to solve the classical Hamilton equation. See [Giesel, Liu et. al. 23'] for mimetic gravity version of the approach 3).

Oppenheimer-Snyder model



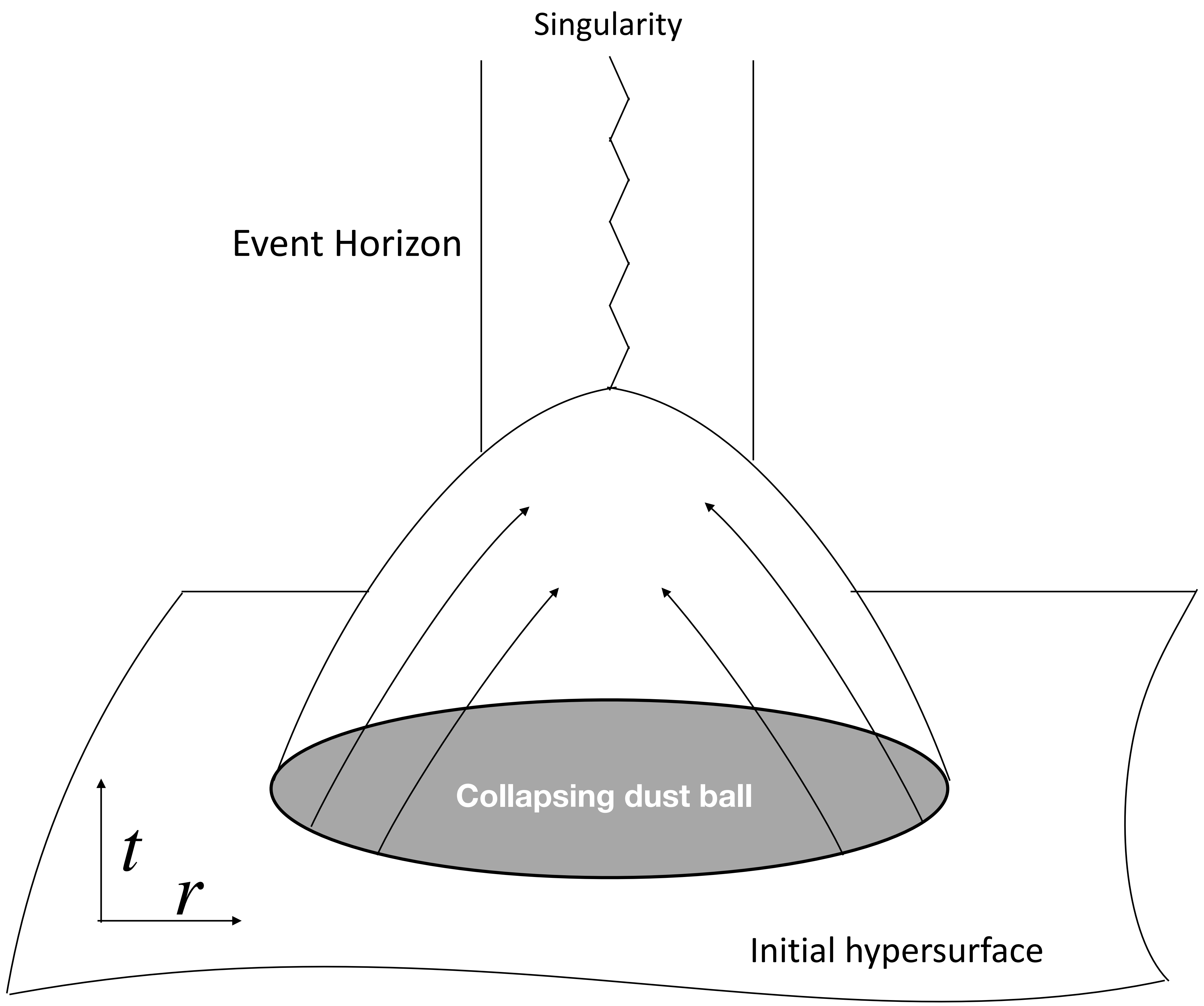
Oppenheimer-Snyder model



Some facts:

- **The dust ball takes the metric** $ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$ **;**
- $a(\tau)$ **is governed by:** $\mathbb{H}^2 = \dfrac{8\pi G}{3}\rho$ **and** $\partial_\tau(\rho a^3) = 0$ **;**
- **The Schwarzschild outside is the unique spherically symmetric and stationary metric that can be glued to the dust ball metric by the junction condition. This is the result without necessary to consider the EOM.**

Oppenheimer-Snyder model



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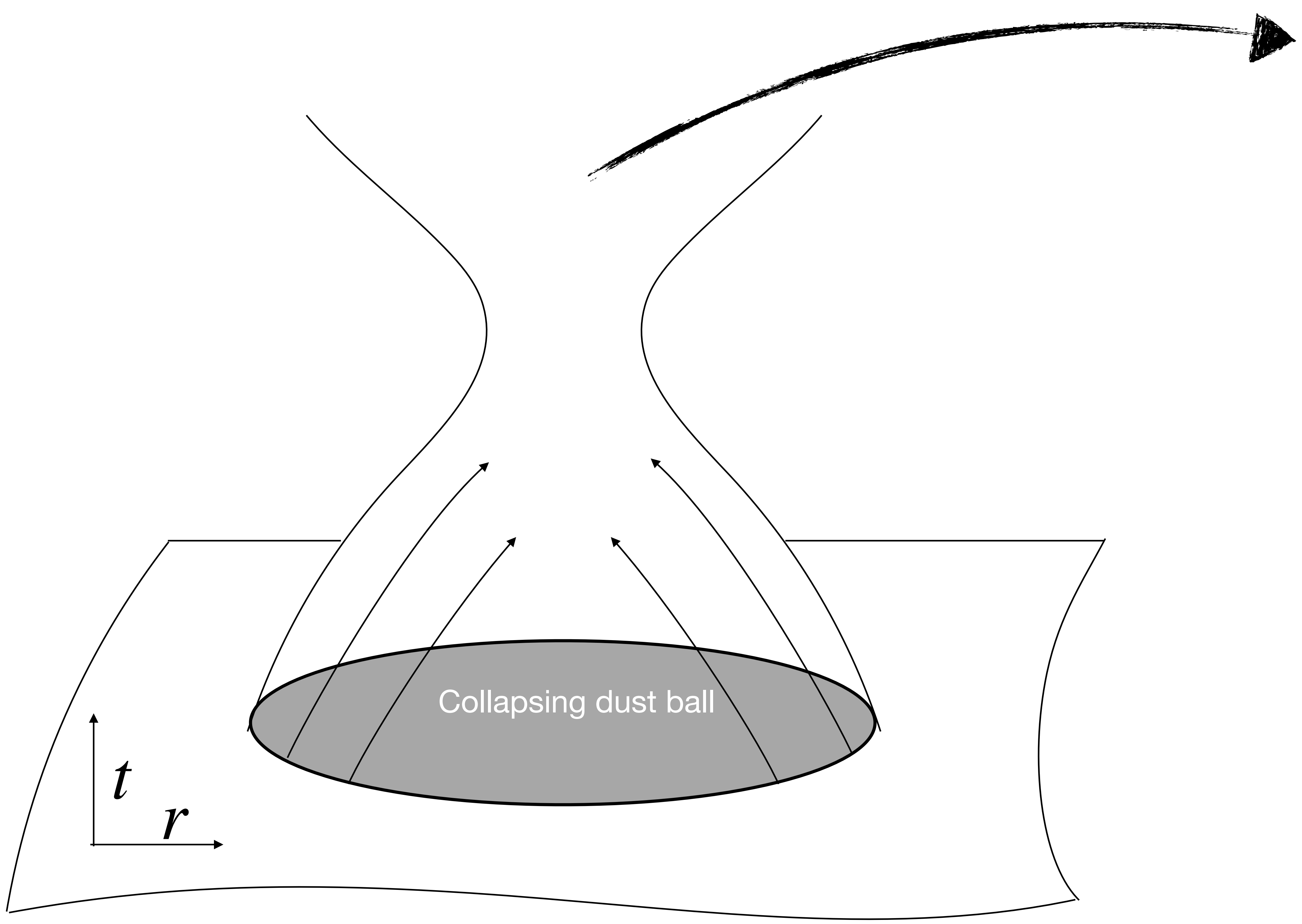
What will happen if the dust ball is a LQC one?

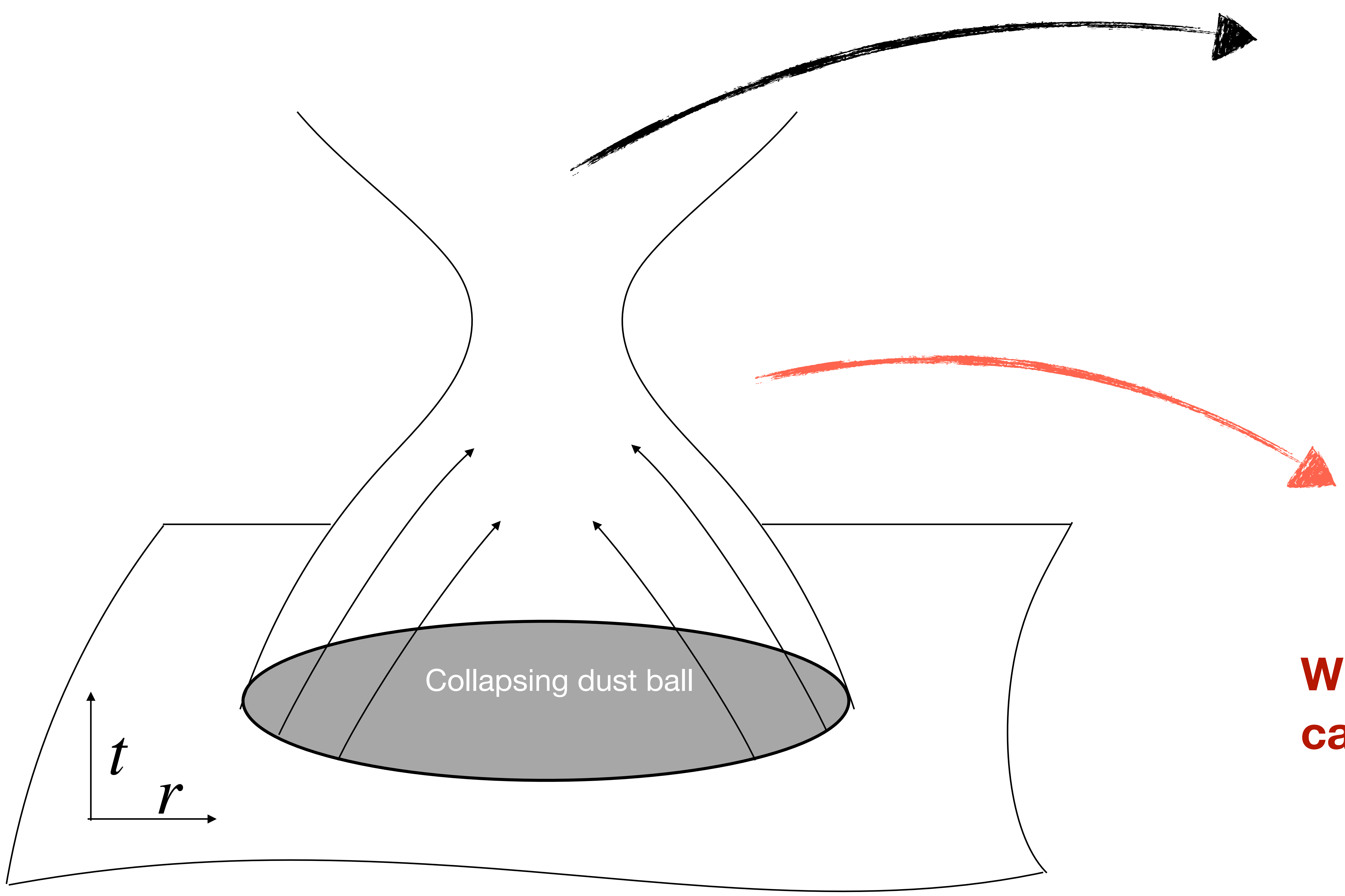
Quantum Oppenheimer-Snyder model



$$ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$$

$$\mathbb{H}^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$ **and** $$\partial_\tau(\rho a^3) = 0$$

Quantum Oppenheimer-Snyder model



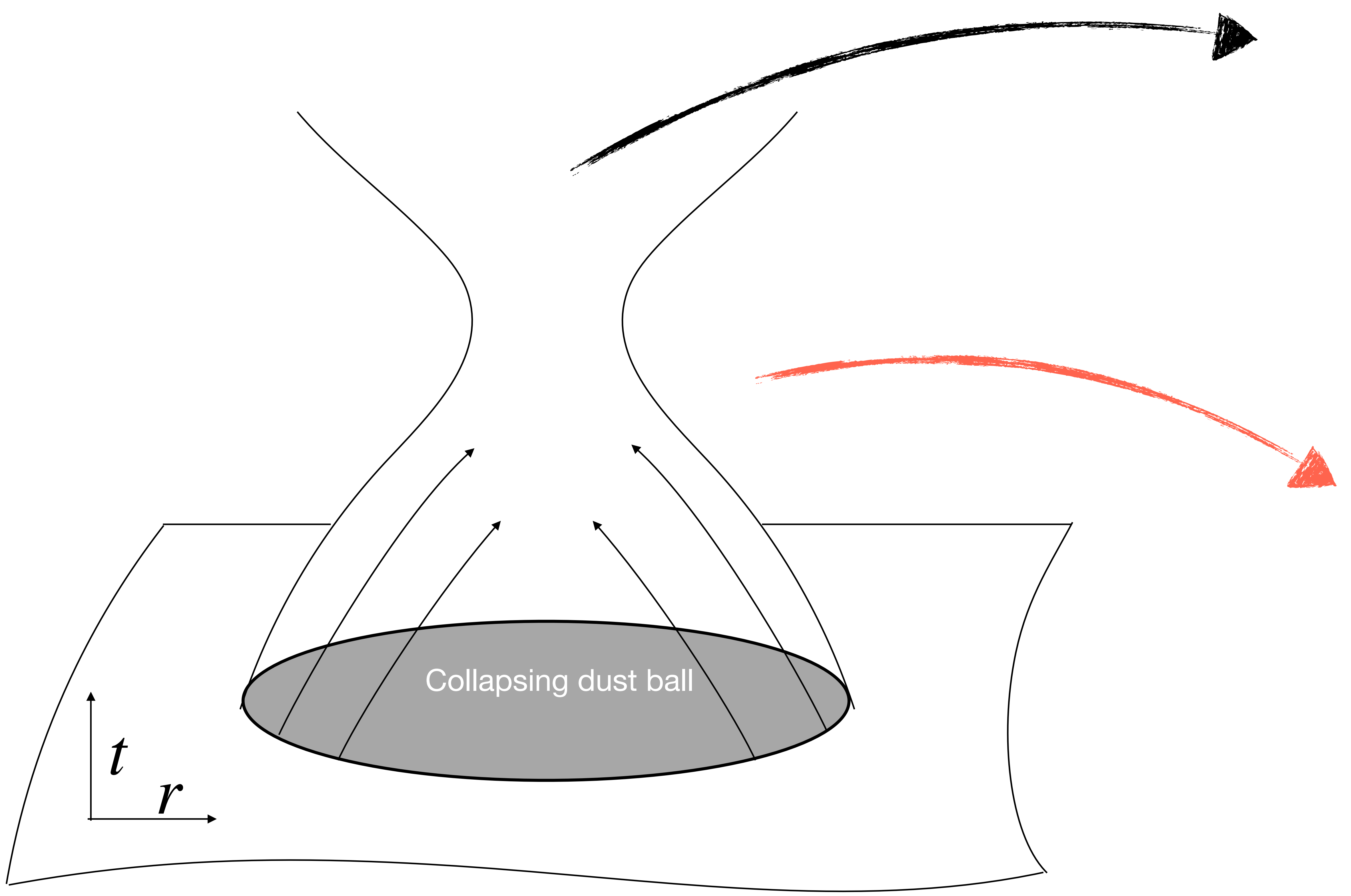
$$ds^2 = -d\tau^2 + a(\tau)^2 ds_E^2$$

$$\mathbb{H}^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \text{ and } \partial_\tau(\rho a^3) = 0$$

$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

What is the expression for $f(r)$ and $g(r)$ so that the outside can be glued with the inside by the junction condition?

Quantum Oppenheimer-Snyder model



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$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = g(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$

$$\alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$

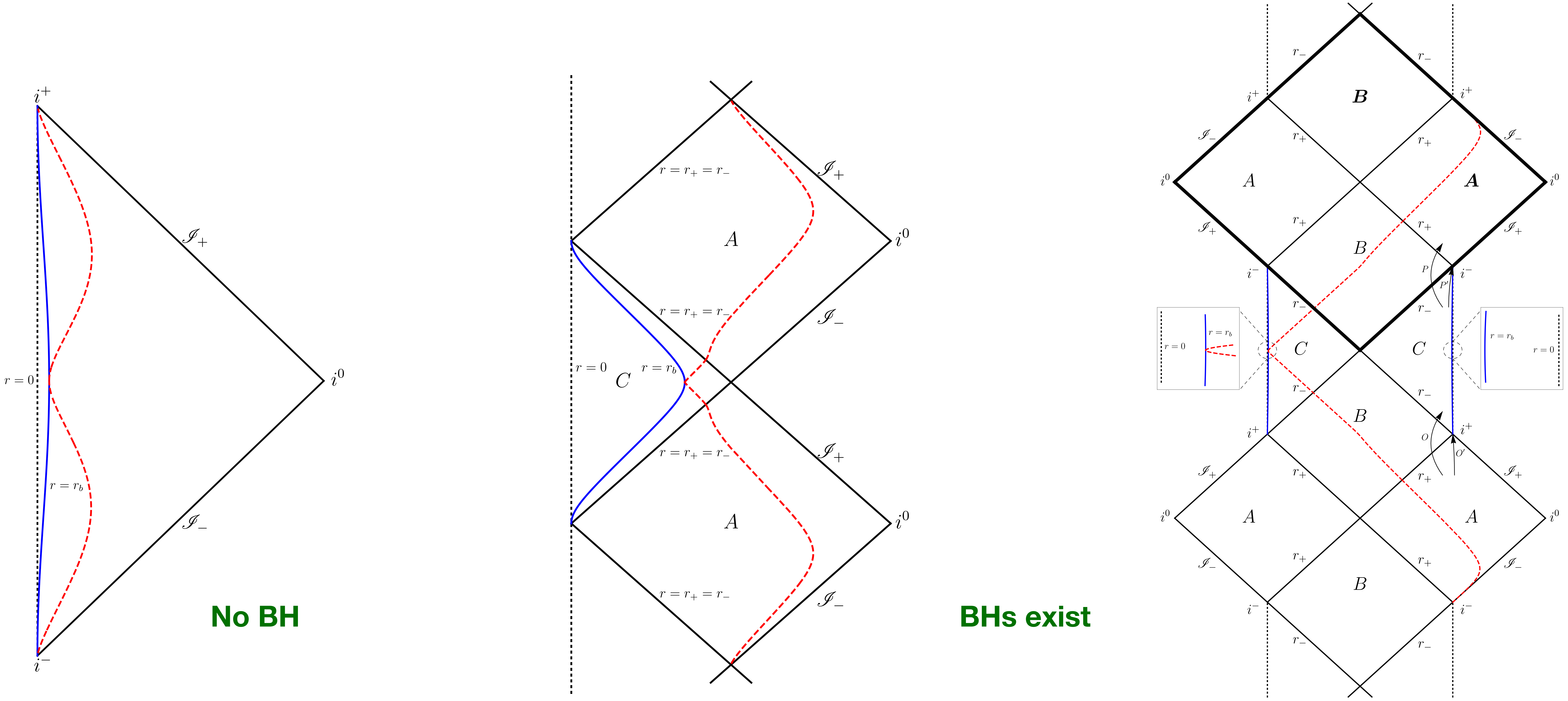
[Lewandowski, Ma, Yang, CZ 23']

Quantum Oppenheimer-Snyder model

- **The outside metric is uniquely determined by the modified Friedemann equation** [see Luca Cafaro, Jerzy Lewandowski 24’ and Luca’s talk]

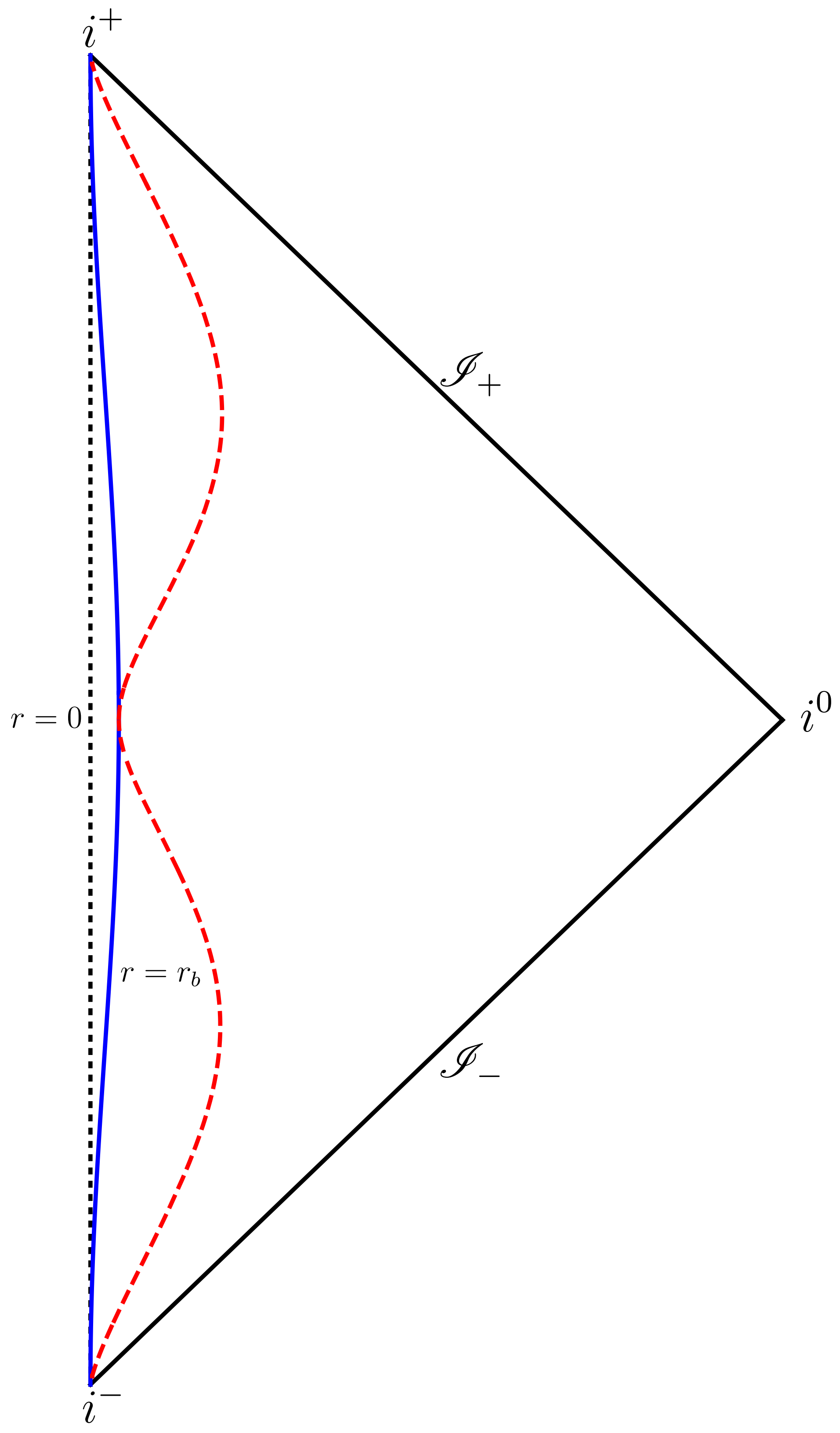
  for $\mathbb{H}^2 = \frac{8\pi G}{3}\rho X(\rho)$**, we get** $f(r) = g(r) = 1 - 2GMr^{-1}X(3M/(4\pi r^3))$
- **The same metric is obtained by other people from various approaches** [e.g., Marto, Tavakoli & Moniz 15’, Kelly, Santacruz & Wilson-Ewing 20’, Bobula & Powłowski 23’, and Giesel, Liu, Rullit, Singh & Weigl 23’]
- **The Penrose diagram of the maximally extended spacetime is studied as follows:**

Quantum Oppenheimer-Snyder model

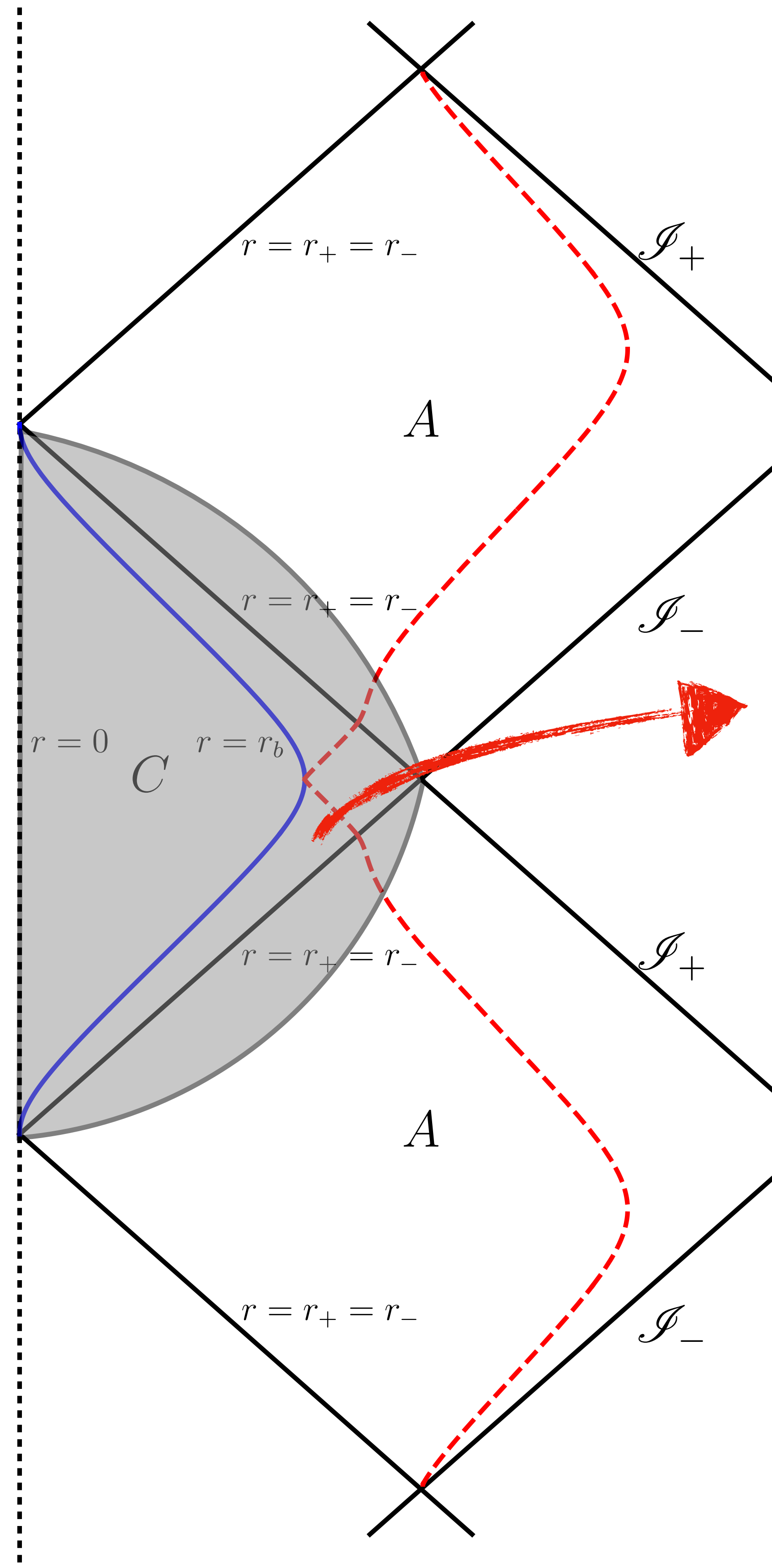


$M < M_{\text{min}}$ $M = M_{\text{min}}$ $M > M_{\text{min}}$

Quantum Oppenheimer-Snyder model



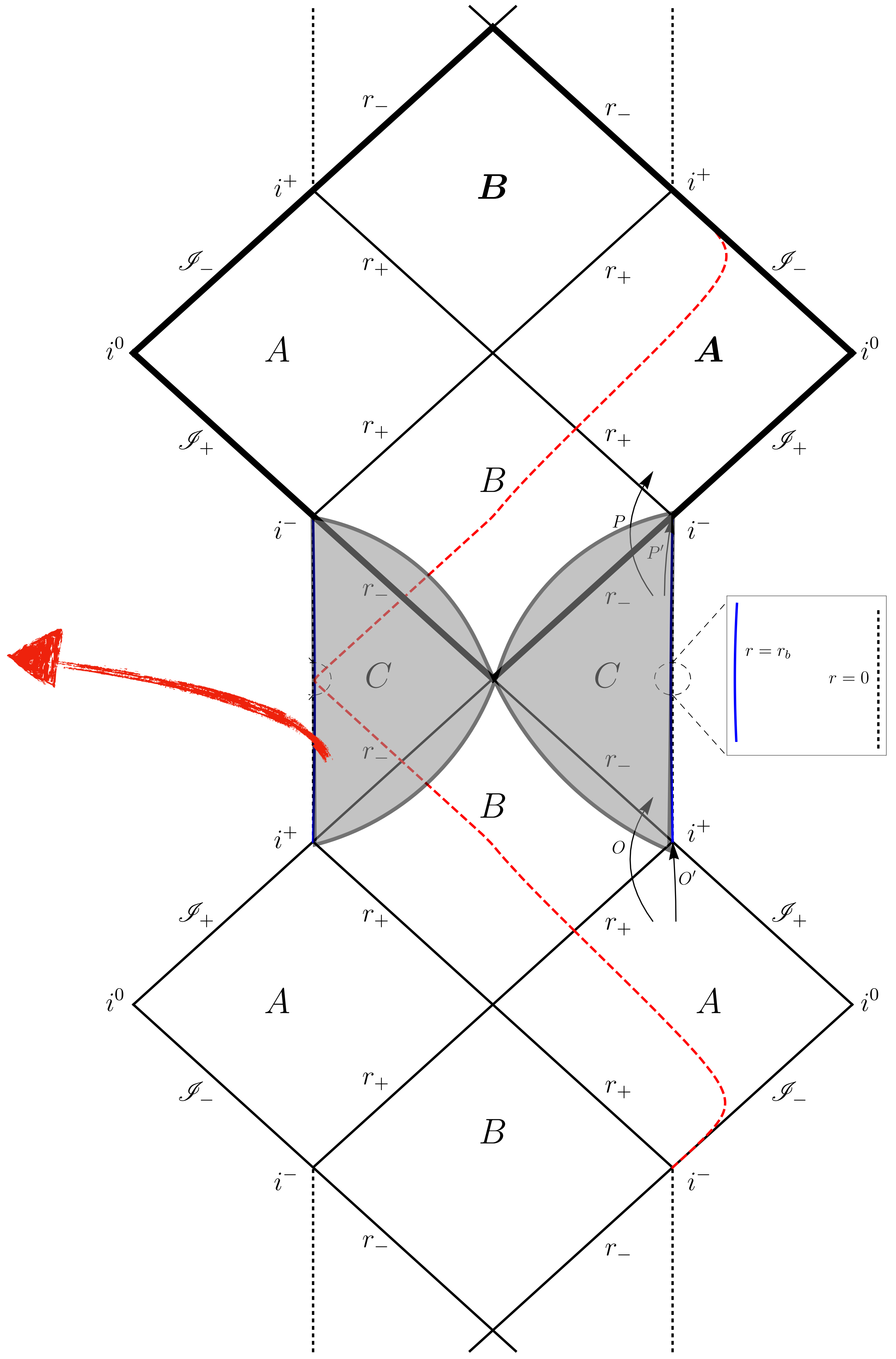
$M < M_{\min}$



$M = M_{\min}$

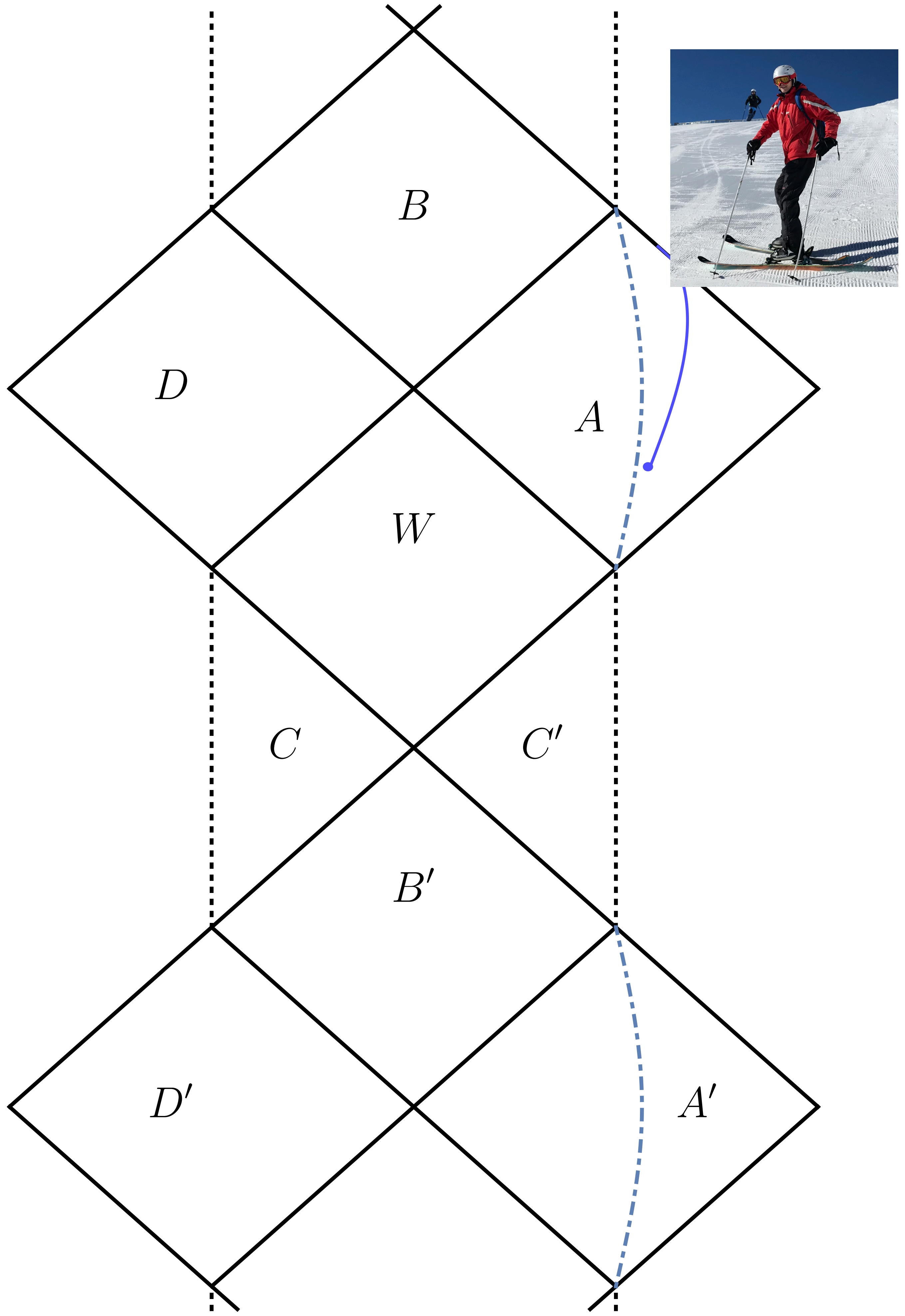
BH-WH transition, but singularity is replaced by a transition region

[in comparison with, e.g., AOS 18']



$M > M_{\min}$

Observational effects of quantum correction



[Yang, CZ, Ma 23']

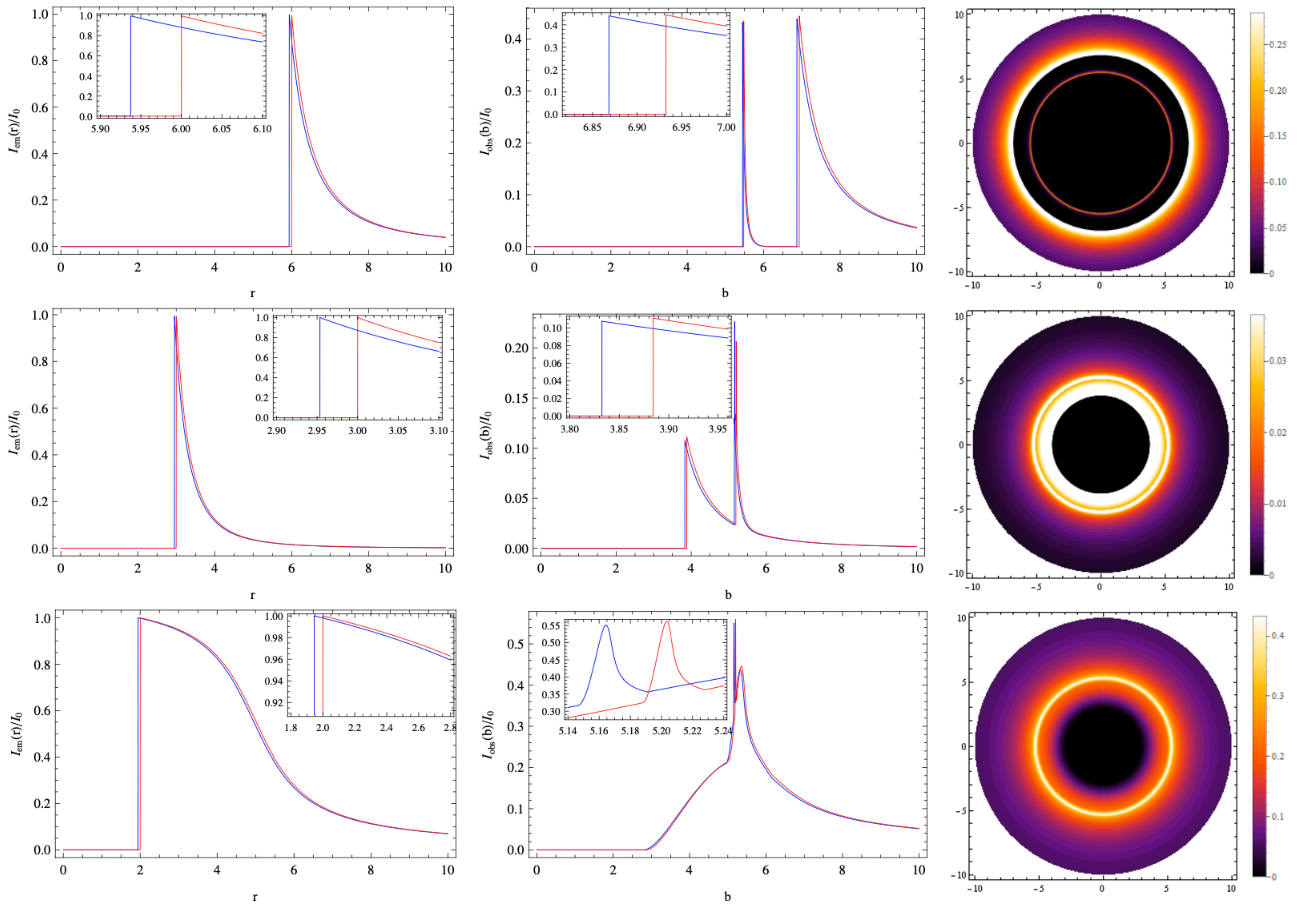
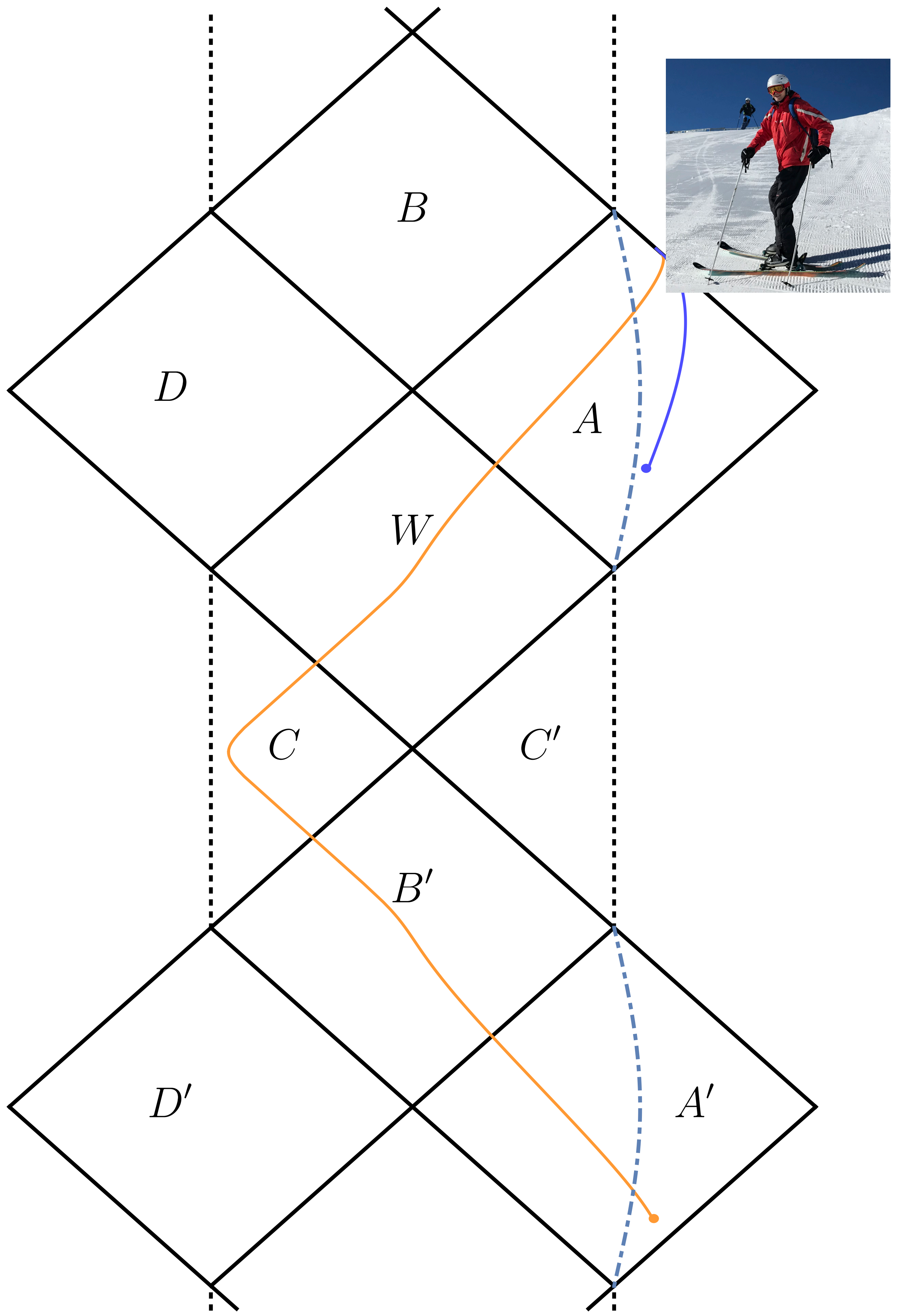
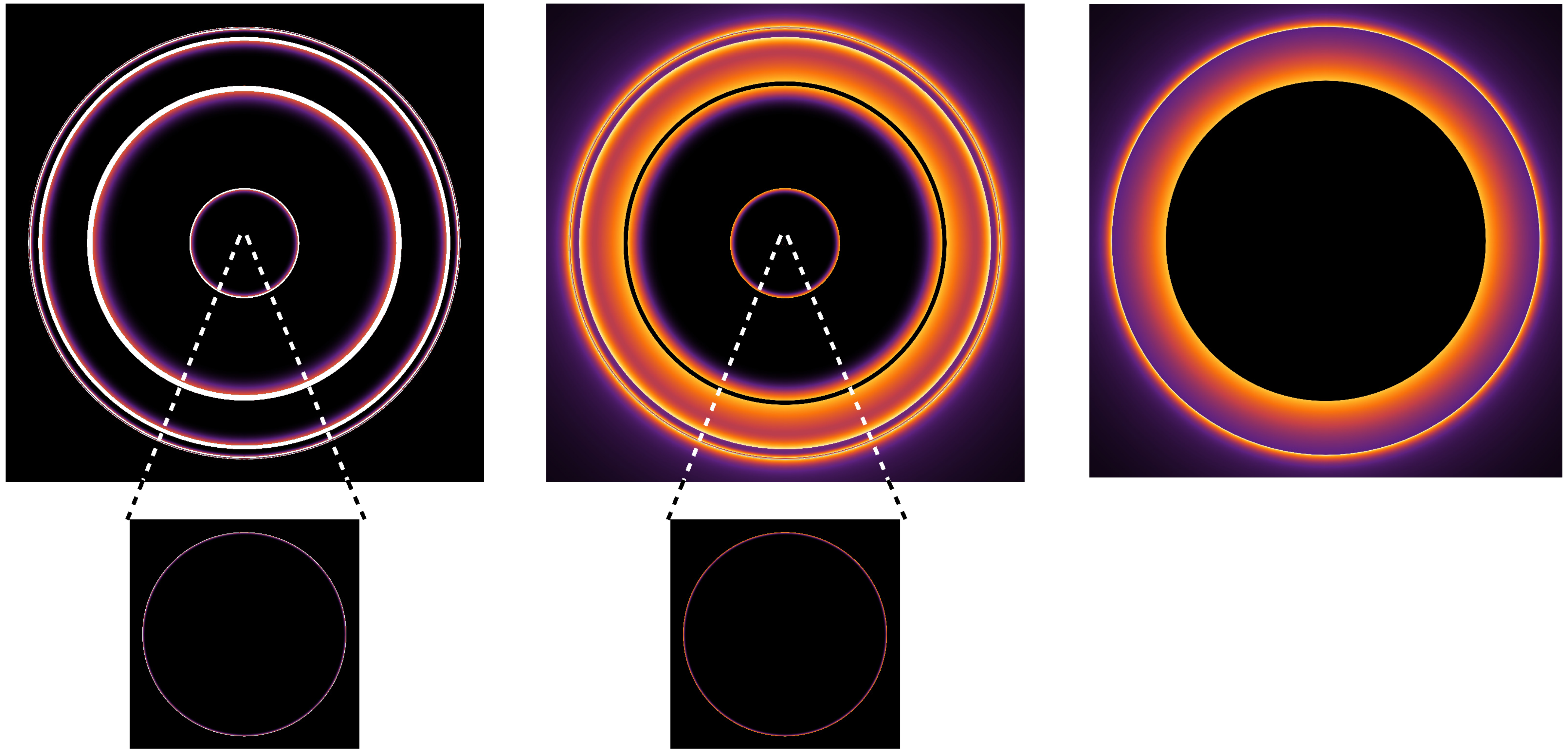


FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles. In each row, the first two panels show the emission intensity I_{em}/I_0 and observational intensity I_{obs}/I_0, normalized to the maximum value I_0, of a thin disk near the quantum-corrected BH (blue) compared to those of the Schwarzschild BH (red), and the third panel depicts the density plot of I_{obs}/I_0 of a thin disk near the quantum-corrected BH. The parameters are $R_s = 2$, $\gamma = 1$ and $\Delta = 0.1$.

Observational effects of quantum correction



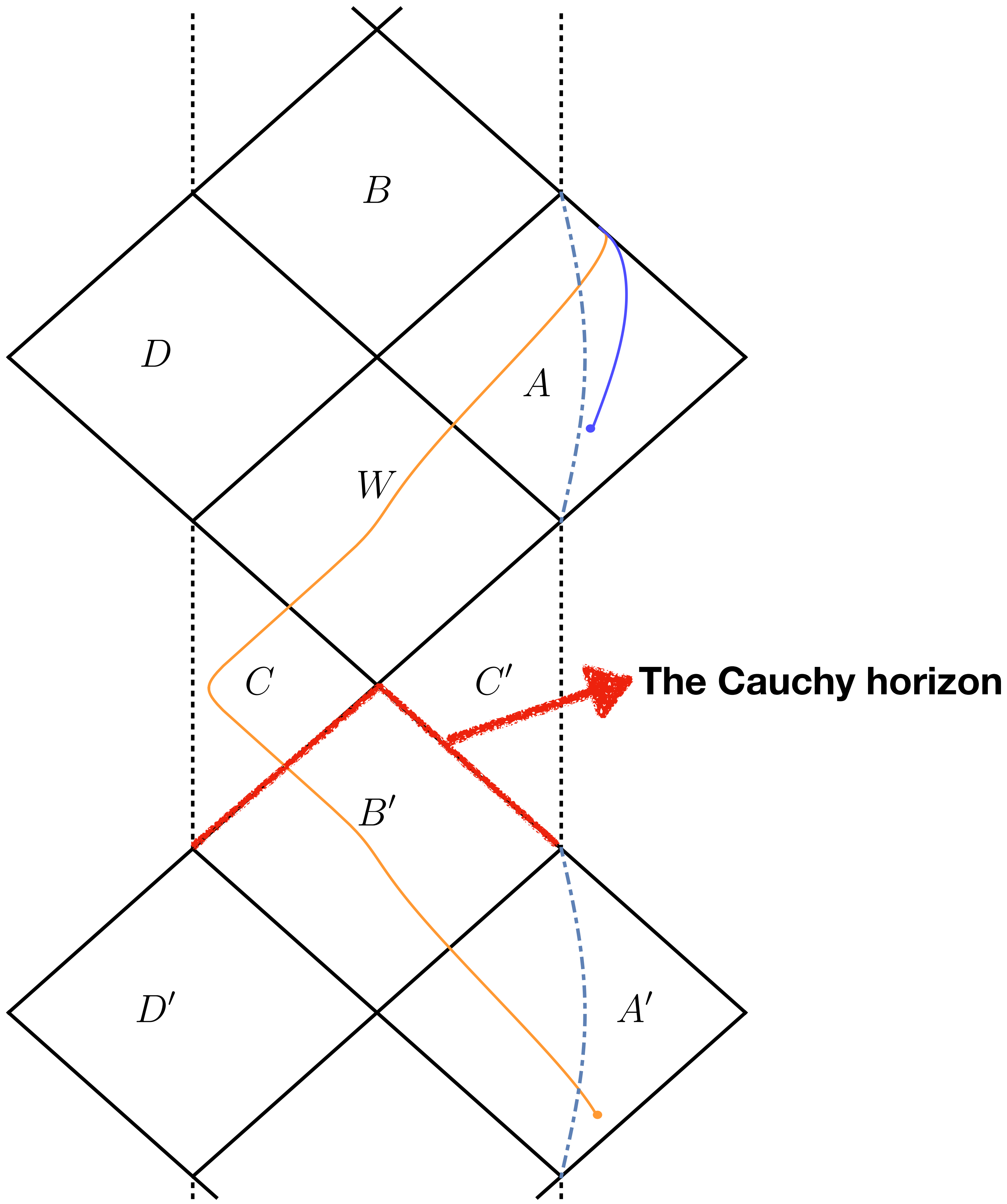
[CZ, Ma, Yang 23']



By measuring the position and width of the light rings, we could get the details of the quantum correction.

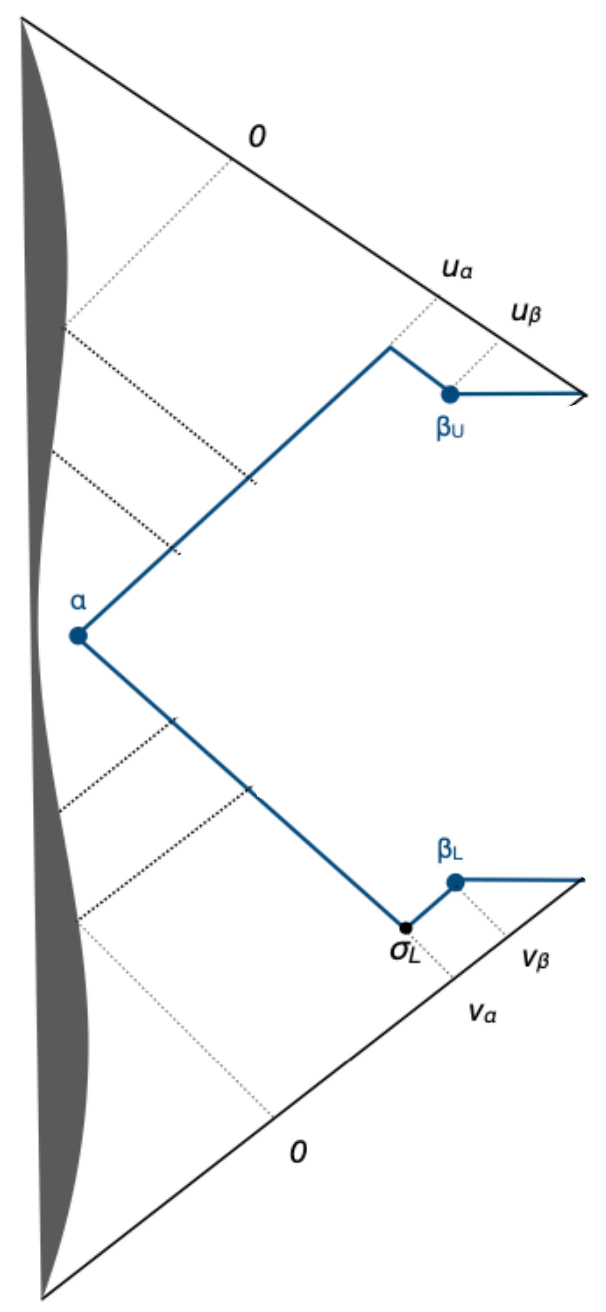
[see Cao, Li, Liu, Zhou 24' for similar work in regular BH]

BH model with spinfoam



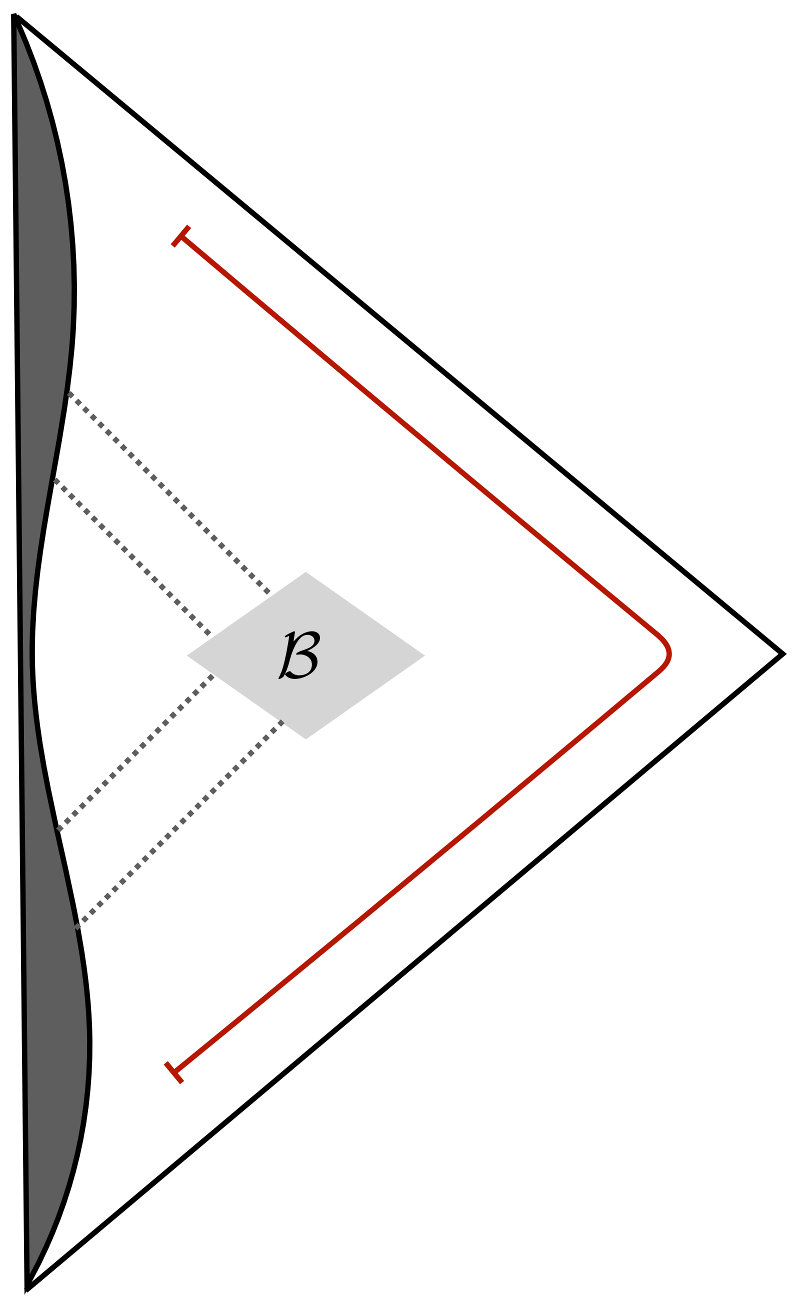
While the spacetime offers distinct advantages, it is not without debates: The existence of Cauchy horizon implies that the spacetime could be unstable under perturbation [Cao, Li et.al. 23’ and 24’, Shao, CZ, et.al. (2023)].

BH model with spinfoam



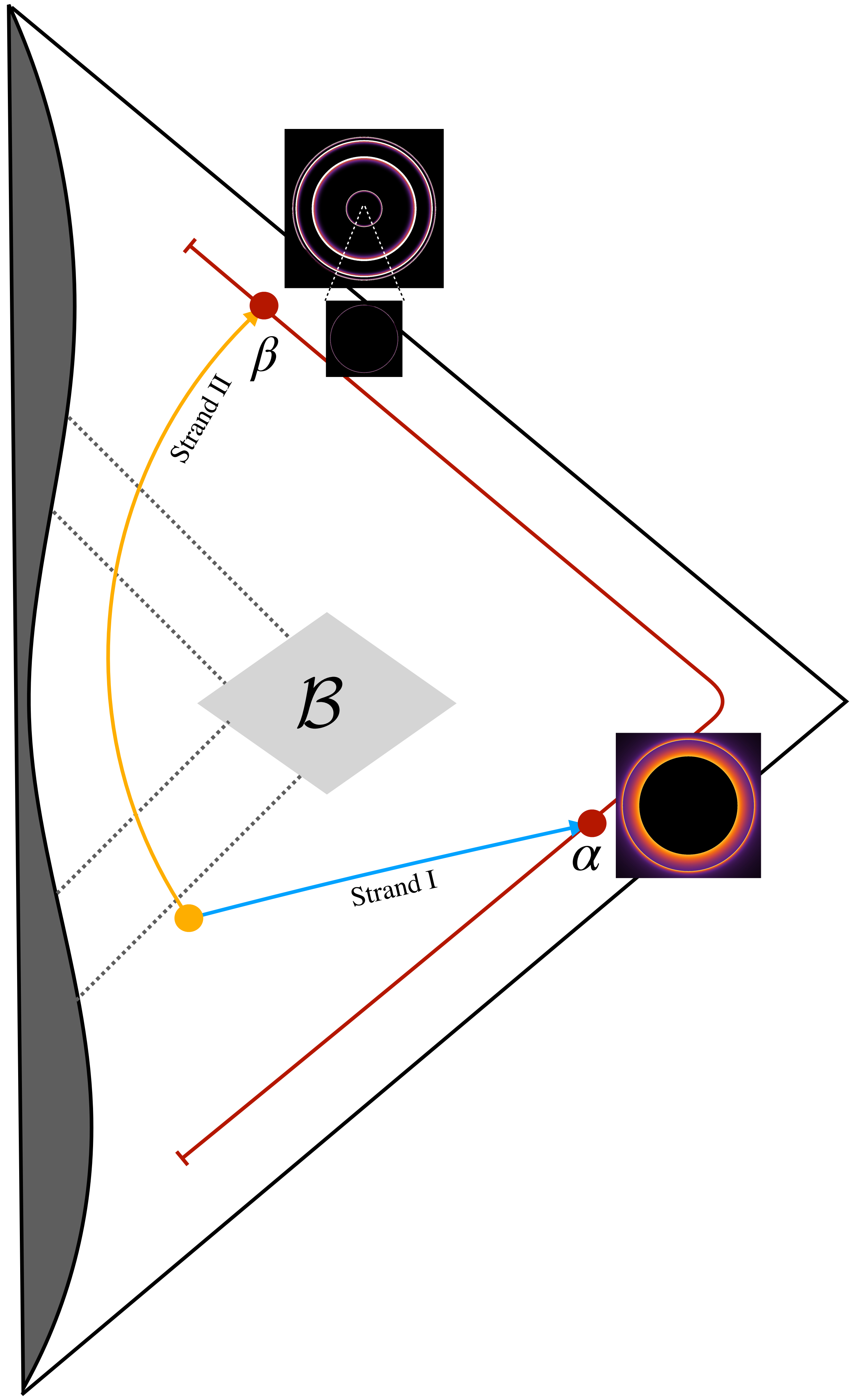
Consider the QG effect at the end of evaporation

[Han, Rovelli & Soltani 23']

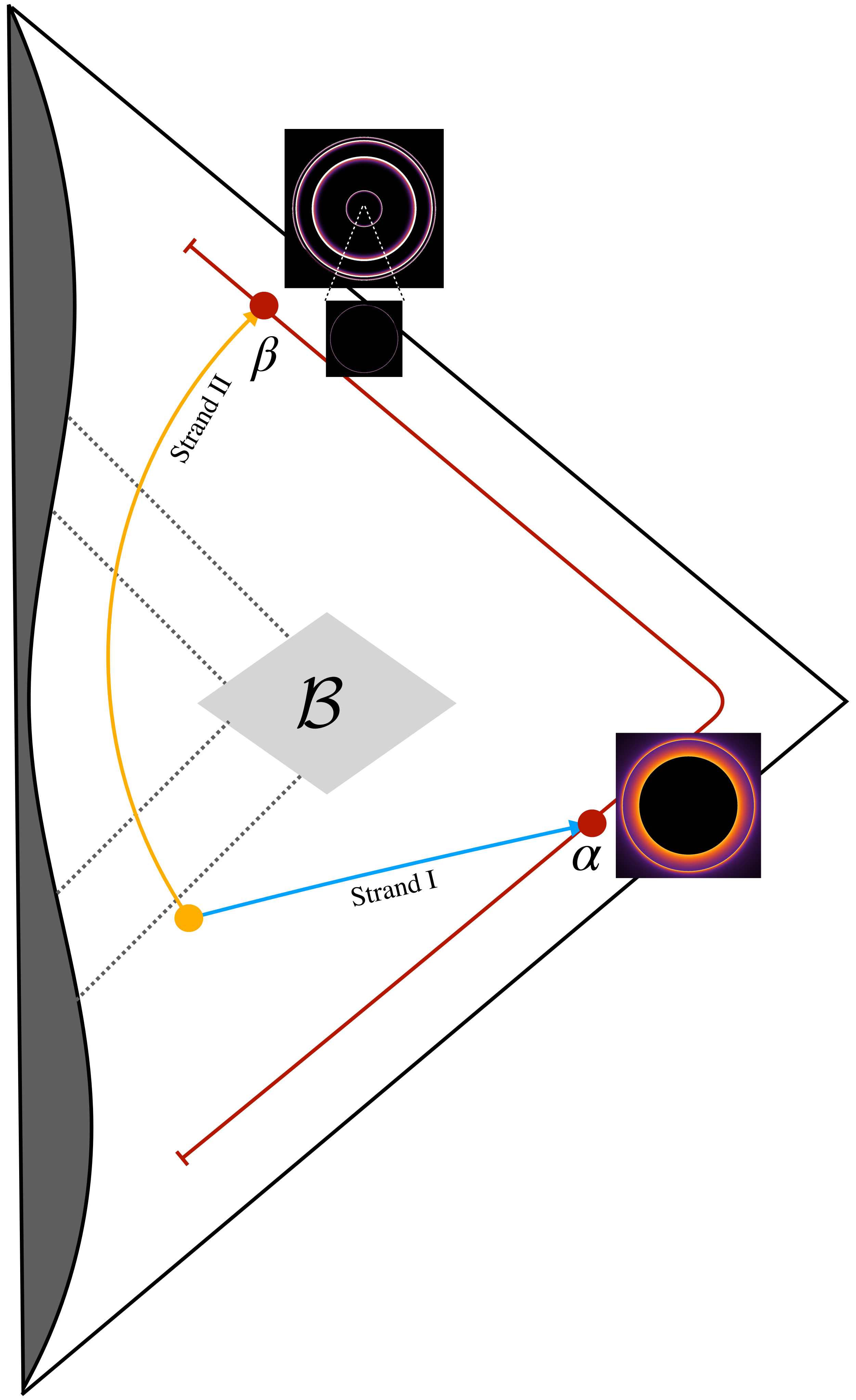


- The metric is locally the same as ours except for the B-Region in the new spacetime;
- No Cauchy horizon.

BH model with spinfoam

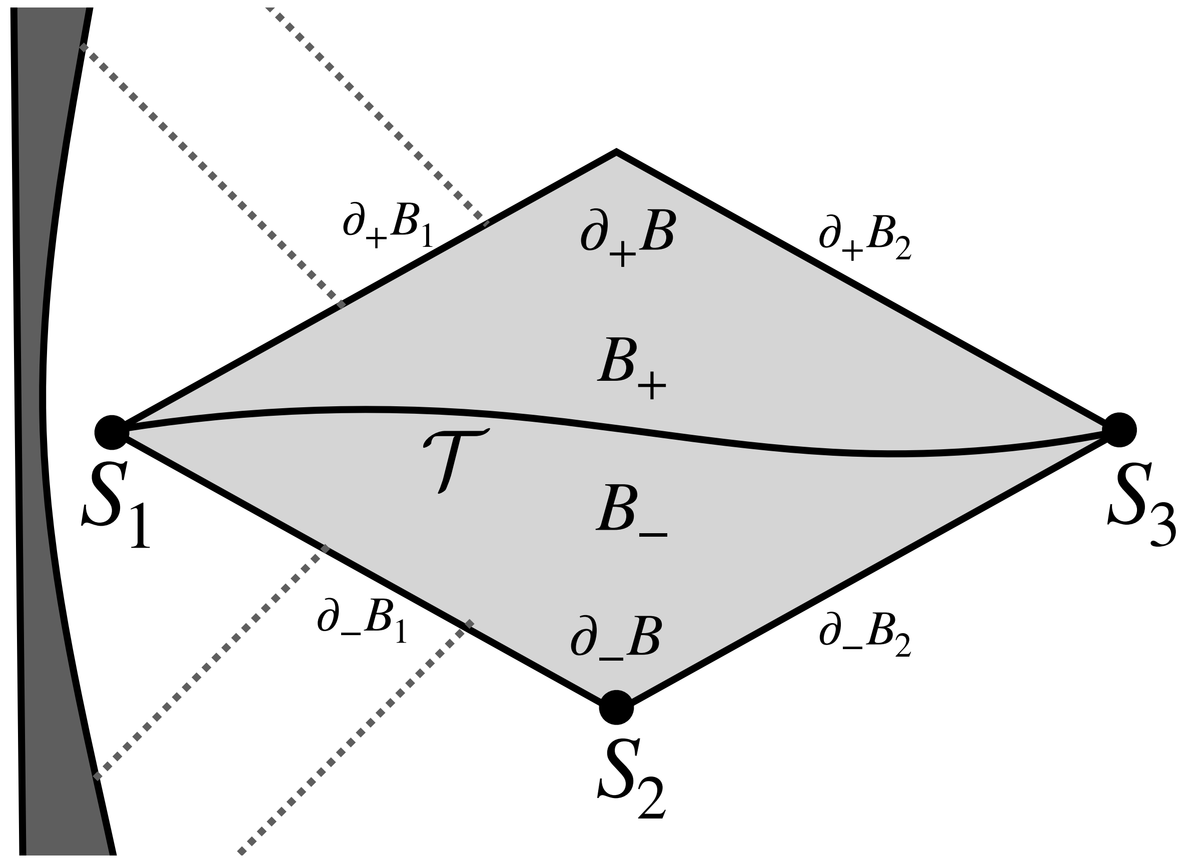


BH model with spinfoam



What is the dynamics in the B region?

BH model with spinfoam

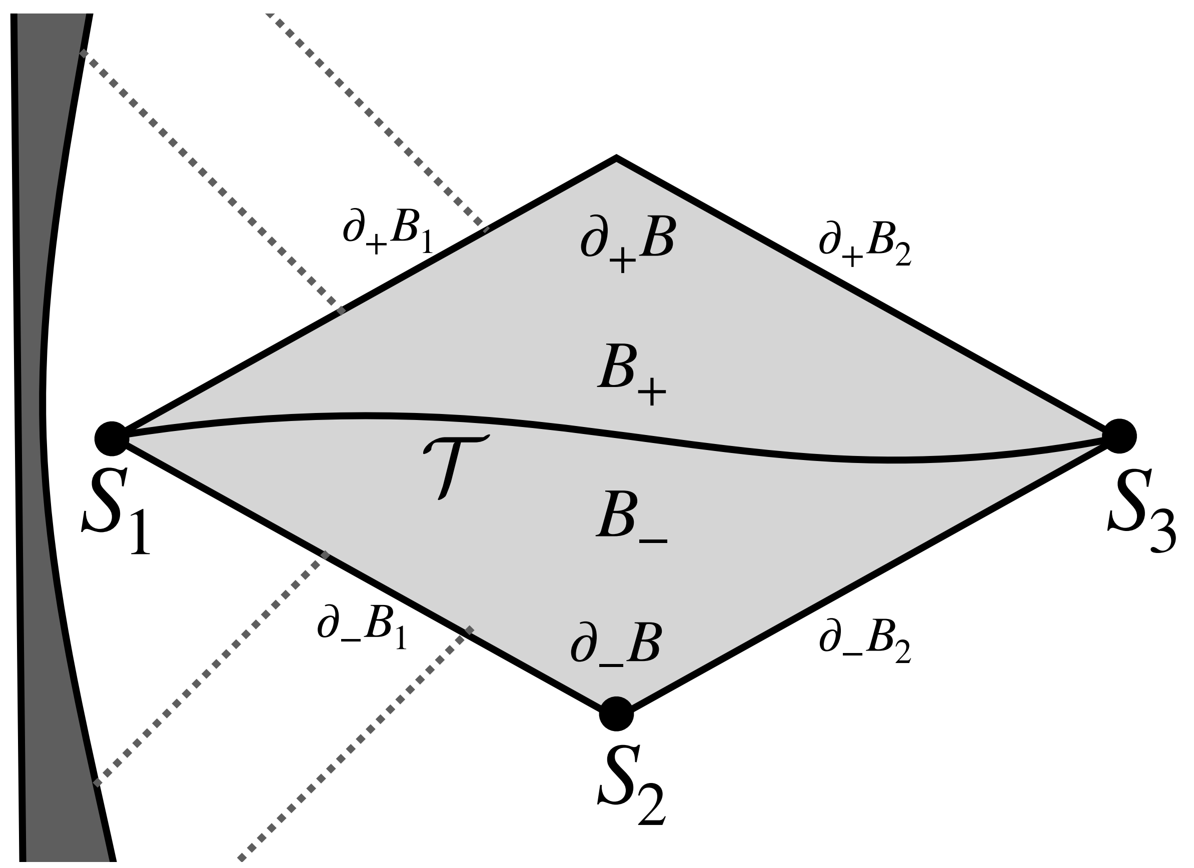


The dynamics of B region is governed by the spinfoam model **[Carlo's lecture]**

The SF amplitude can be numerical calculated with various algorithm **[Hongguang's lecture]:**
Small spin regime: e.g. Soltani, Rovelli & Martin-Dussaud 21', Donà & Frisoni 23'.
Large spin regime: Han, Liu & Qu 23'.

BH model with spinfoam

[Han, Qu & CZ 24']



- ∂B is located in the semiclassical region, so that the boundary state can be chosen as the coherent state "labelled" by (e^i_a, K^i_a) with spread t.
- We consider the amplitude as $t \to 0$, equivalent as $j \to \infty$;
- In LQG, $\pm e^i_a$ are regarded as different states due to the $\mathrm{SU}(2)$ gauge;
- $\pm e^i_a$ give the same 3-D metric q_{ab};
- The boundary state is proposed as the superposition

$$\left(\psi_{(e_-,K_-)} + \psi_{(-e_-,K_-)}\right) \otimes \left(\psi_{(e_+,K_+)} + \psi_{(-e_+,K_+)}\right)$$

$$A = A\left(\psi_{(K_+,e_+)} \otimes \psi_{(K_-,e_-)}\right) + A\left(\psi_{(K_+,-e_+)} \otimes \psi_{(K_-,e_-)}\right) + A\left(\psi_{(K_+,e_+)} \otimes \psi_{(K_-,-e_-)}\right) + A\left(\psi_{(K_+,-e_+)} \otimes \psi_{(K_-,-e_-)}\right)$$

- We consider a non-degenerate 2-complex containing 56 vertices in our work;
- The first two terms dominate the amplitude;
- The first two terms imply the transition $\pm e_- \to \pm e_+$ with $\det(e_+) = -\det(e_-)$;
- Tunneling between opposite orientations accompanying the BH-WH transition;
- The value of the effective action in the amplitude is computed with the results:

$$S^{(++)} = -0.0458193513442056, S^{(--)} = -0.0458193513442275,$$

where the parameter is chosen as $t = 1/246.34$, and $GM = 2 \times 10^5 \sqrt{\beta \kappa \hbar}, \quad \beta = \dfrac{1}{10}$.

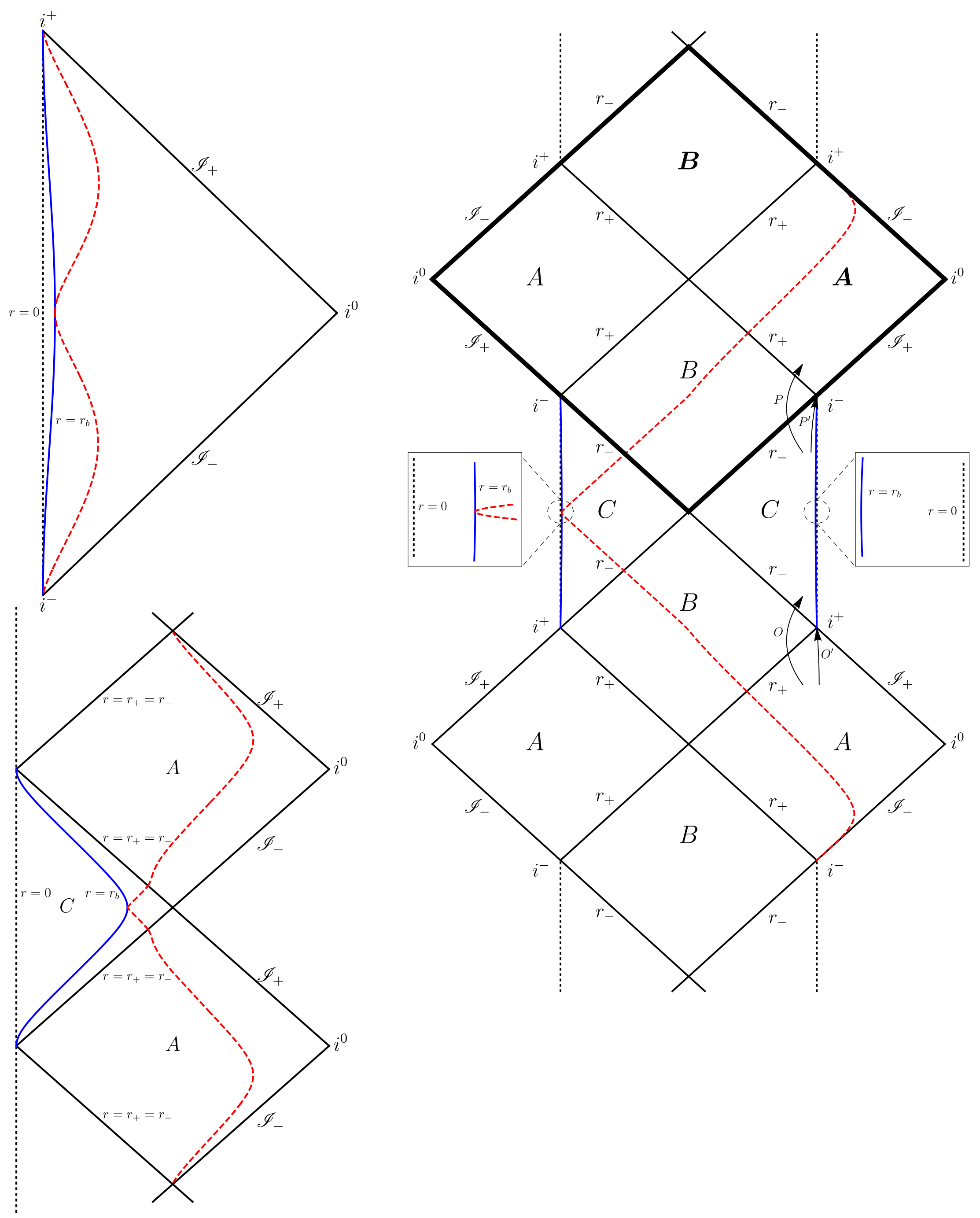
[https://github.com/czhangUW/BH2WHTranstionInSF]

Summary

We introduced our works related to the quantum OS model with the results:

$$ds^2 = -f(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = g(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$



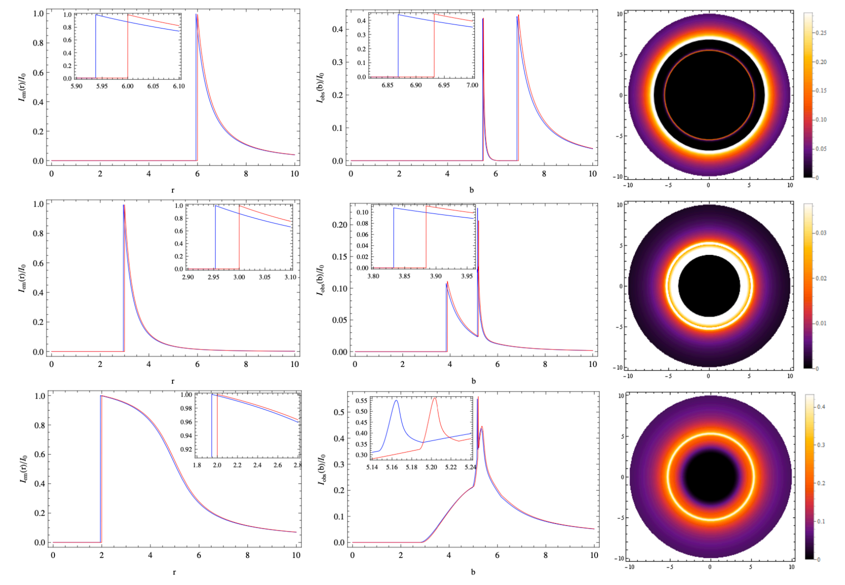
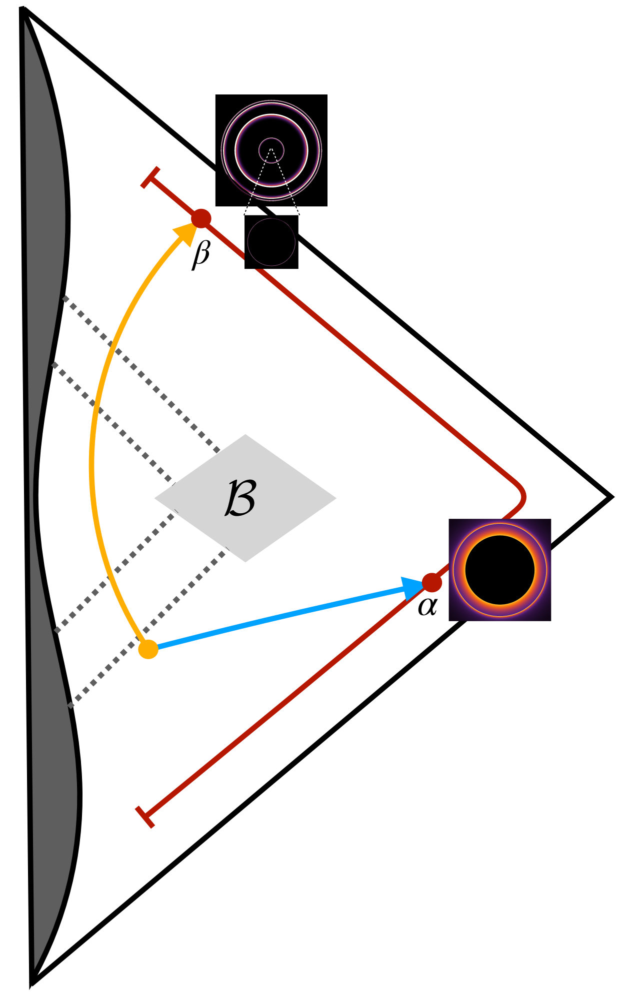
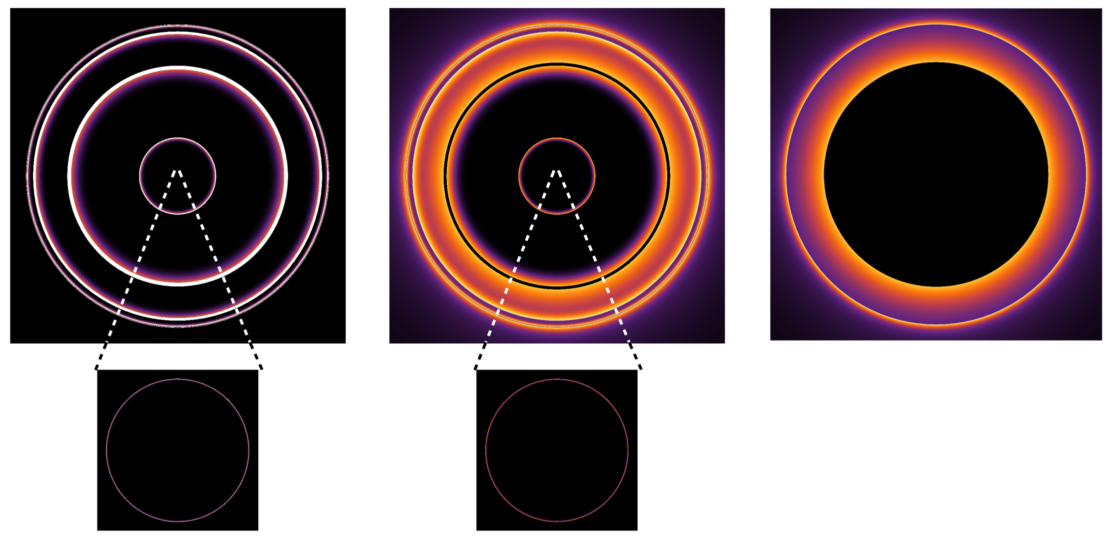
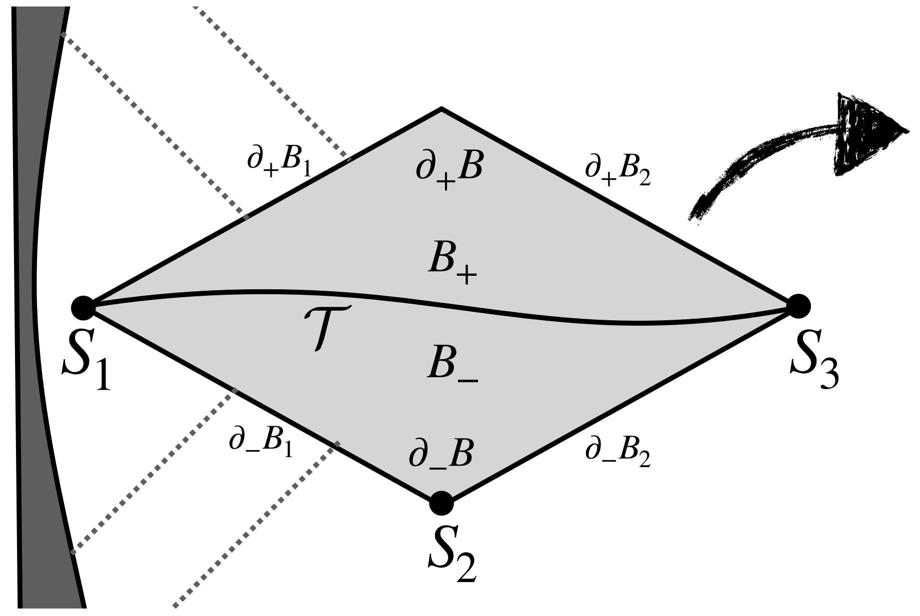


FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles. In each row, the first two panels show the emission intensity I_{em}/I_0 and observational intensity I_{obs}/I_0, normalized to the maximum value I_0, of a thin disk near the quantum-corrected BH (blue) compared to those of the Schwarzschild BH (red), and the third panel depicts the density plot of I_{obs}/I_0 of a thin disk near the quantum-corrected BH. The parameters are $R_s = 2$, $\gamma = 1$ and $\Delta = 0.1$.







The SF dynamics with the complex critical point method

Thank you for your attention !