64. Cracow School of Theoretical Physics

From the UltraViolet to the InfraRed:
a panorama of modern gravitational physics

June 15–23, 2024
Zakopane, Tatra Mountains, Poland

Topics include:
- Quantum/semiclassical gravity
- Amplitudes, soft theorems
- Black holes
- Gravitational waves, observation and theory
- Future detectors
- Modified gravity
- Cosmology
- Dark Matter, Dark Energy
- Mathematical aspects of GR

Domain Walls
and their
Gravitational Waves II

Alexander Vikman

20.06.2024

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MŠMT MINISTRY OF EDUCATION, YOUTH AND SPORTS
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PHYSICAL REVIEW D **105,** 063530 (2022)

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Beyond freeze-in: Dark matter via inverse phase transition and gravitational wave signal

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Gravitational shine of dark domain walls

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NANOGrav spectral index $\gamma = 3$ from melting domain walls

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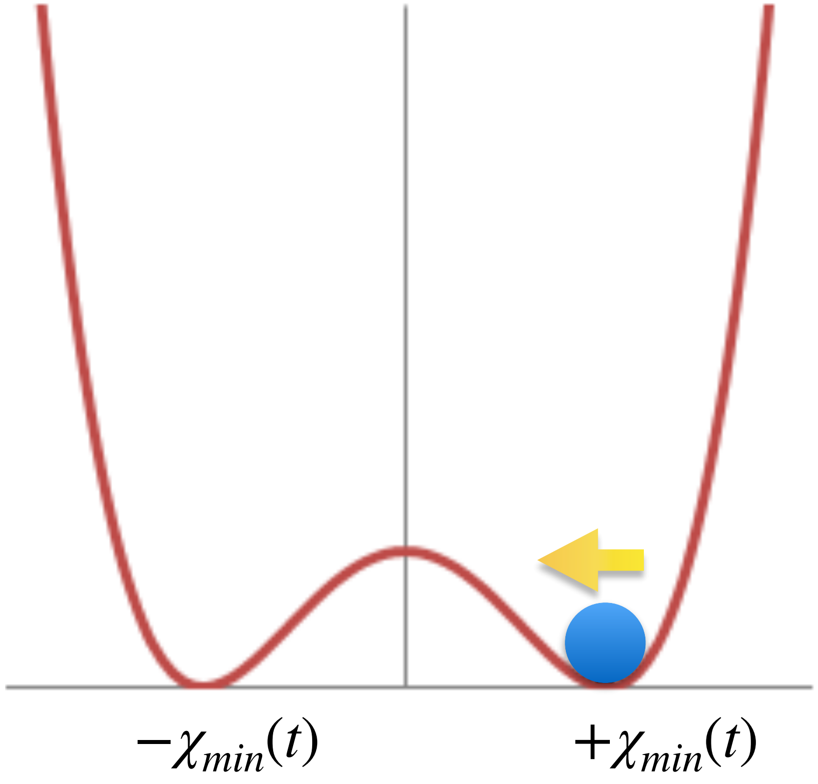
Inverse Phase Transition At Meltdown

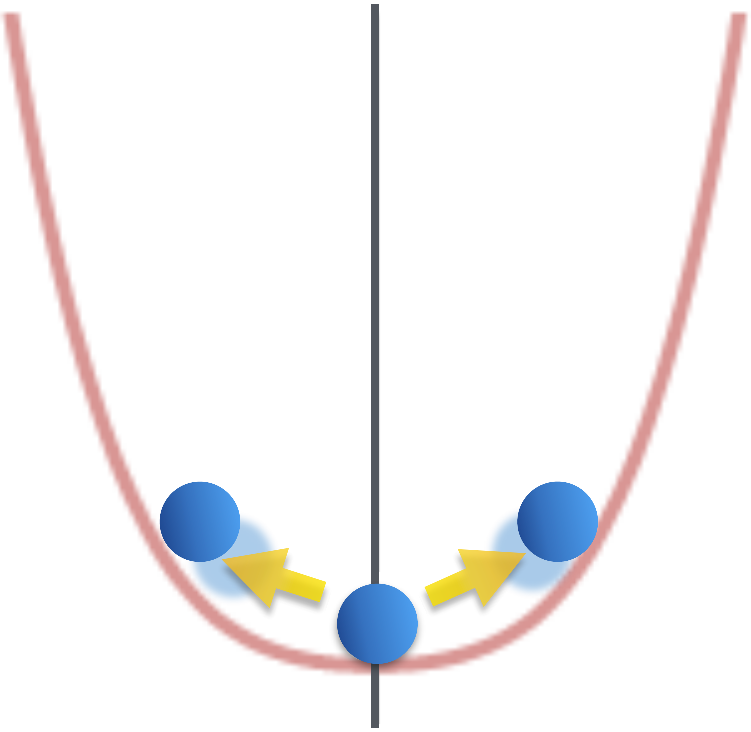
Early Universe

spontaneously Broken Phase



Late Universe

oscillations around restored symmetric vacuum



for Hubble parameter

$$H < M$$

Z_2 -symmetric DM scalar field χ coupled to ϕ - a multiplet of N *thermal* degrees of freedom

portal coupling

$$V = \frac{1}{2}\left(M^2 - g^2\phi^\dagger\phi\right)\cdot\chi^2 + \frac{\lambda}{4}\chi^4$$

$$g^2\langle\phi^\dagger\phi\rangle \simeq \frac{Ng^2T^2}{12}$$

potential bounded from below

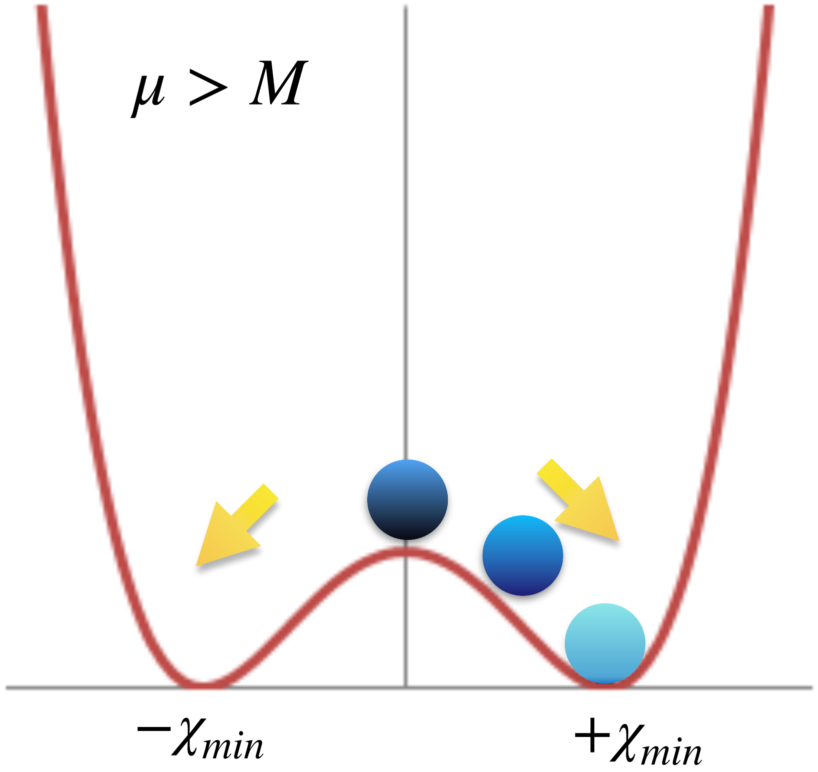
$$\beta = \frac{\lambda}{g^4} \geq \frac{1}{\lambda_\phi} \geq 1$$

potential bounded from below

weak coupling

Direct Phase Transition

Early universe spontaneously Broken Phase



Domain Walls!

Avoid too much friction to start rolling

$$\mu \gtrsim H$$

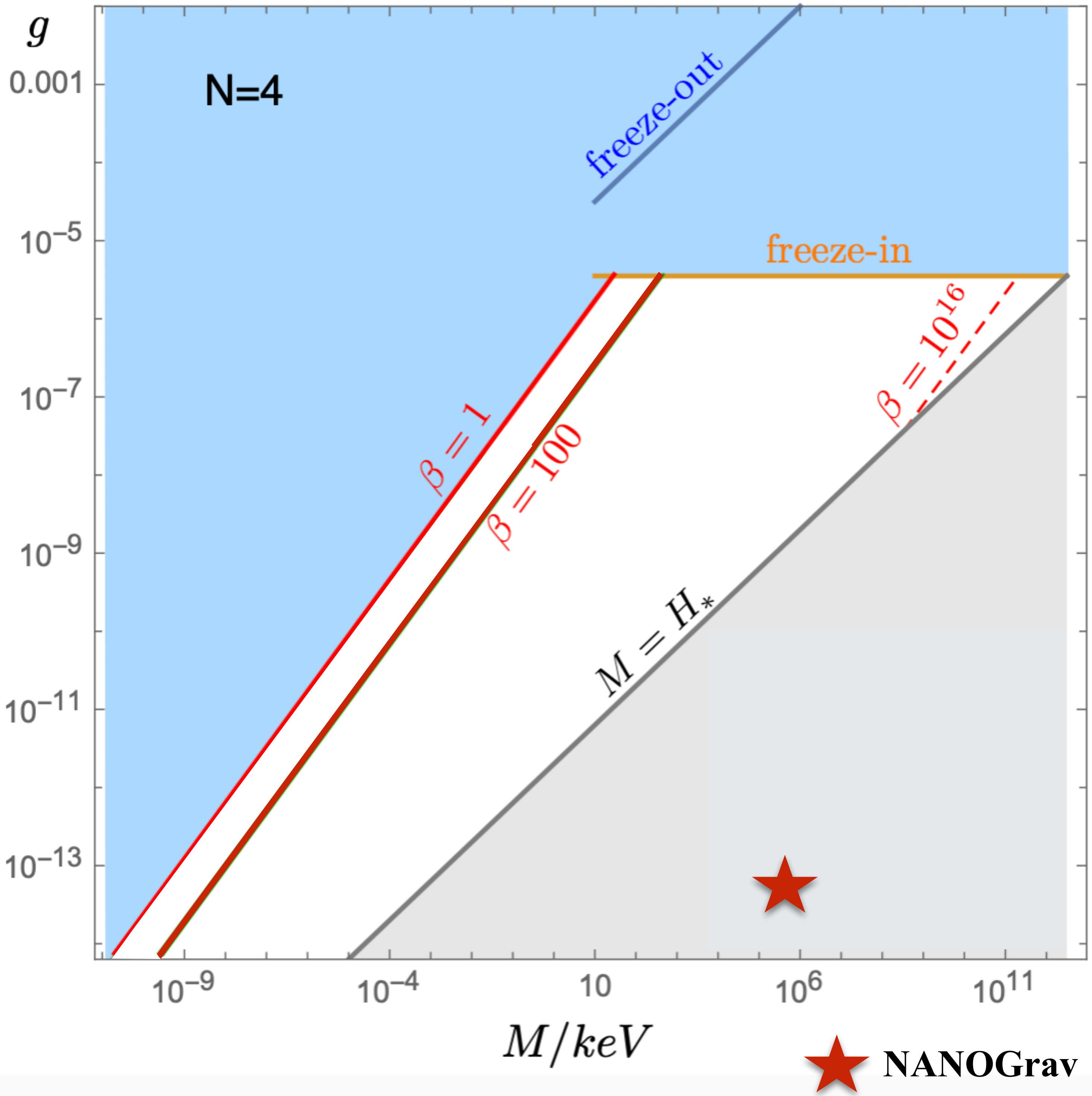
$$\sqrt{\frac{N}{12}}\, g T_i \simeq \sqrt{\frac{\pi^2 g_*}{90}}\,\frac{T_i^2}{M_{pl}}$$

$$T_i \simeq g\, M_{Pl}\sqrt{\frac{N}{g_*(T_i)}} \times \frac{1}{\sqrt{B}}$$

Correction taking into account time to get to the minimum

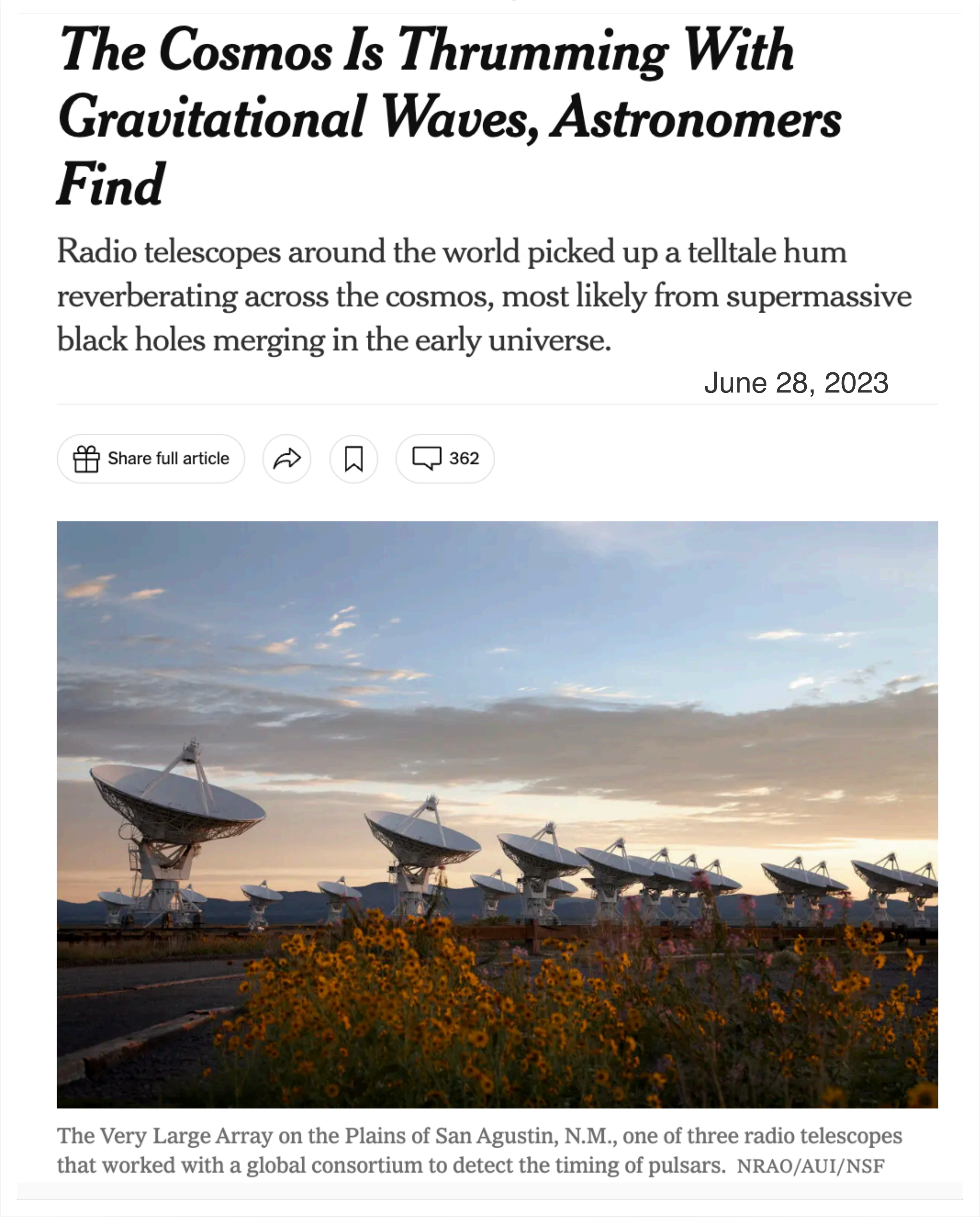
Allowed Parameter Space

$$M \simeq \; 10^{-13}\,\text{eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g_*(T_*)}{100}\right)^{2/5} \cdot \left(\frac{g}{10^{-18}}\right)^{7/5}$$



★ **NANOGrav**

The New York Times



The Cosmos Is Thrumming With Gravitational Waves, Astronomers Find

Radio telescopes around the world picked up a telltale hum reverberating across the cosmos, most likely from supermassive black holes merging in the early universe.

June 28, 2023

The Very Large Array on the Plains of San Agustin, N.M., one of three radio telescopes that worked with a global consortium to detect the timing of pulsars. NRAO/AUI/NSF

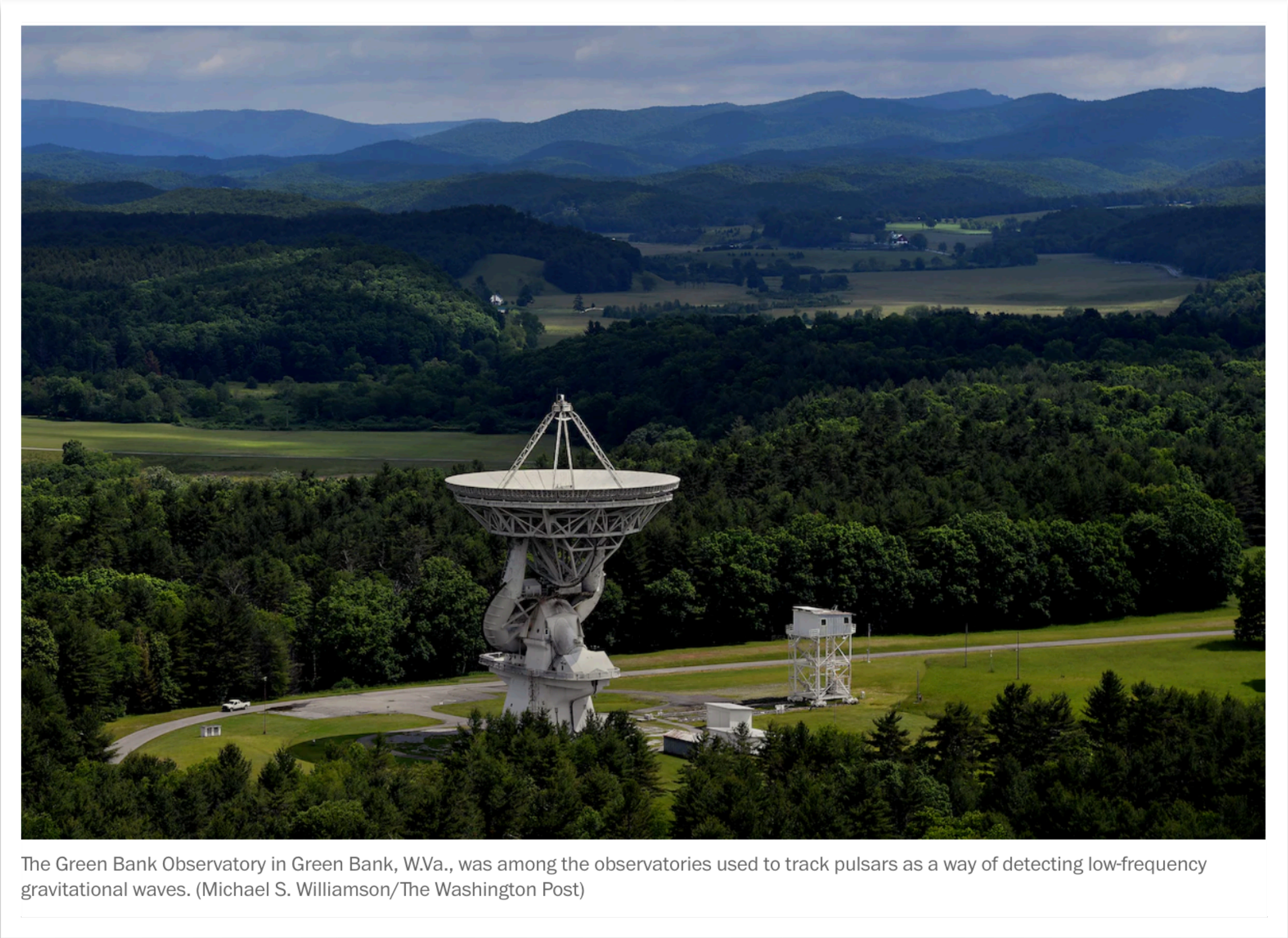
The Washington Post

In a major discovery, scientists say space-time churns like a choppy sea

The mind-bending finding suggests that everything around us is constantly being roiled by low-frequency gravitational waves

By Joel Achenbach and Victoria Jaggard

June 28, 2023 at 8:00 p.m. EDT

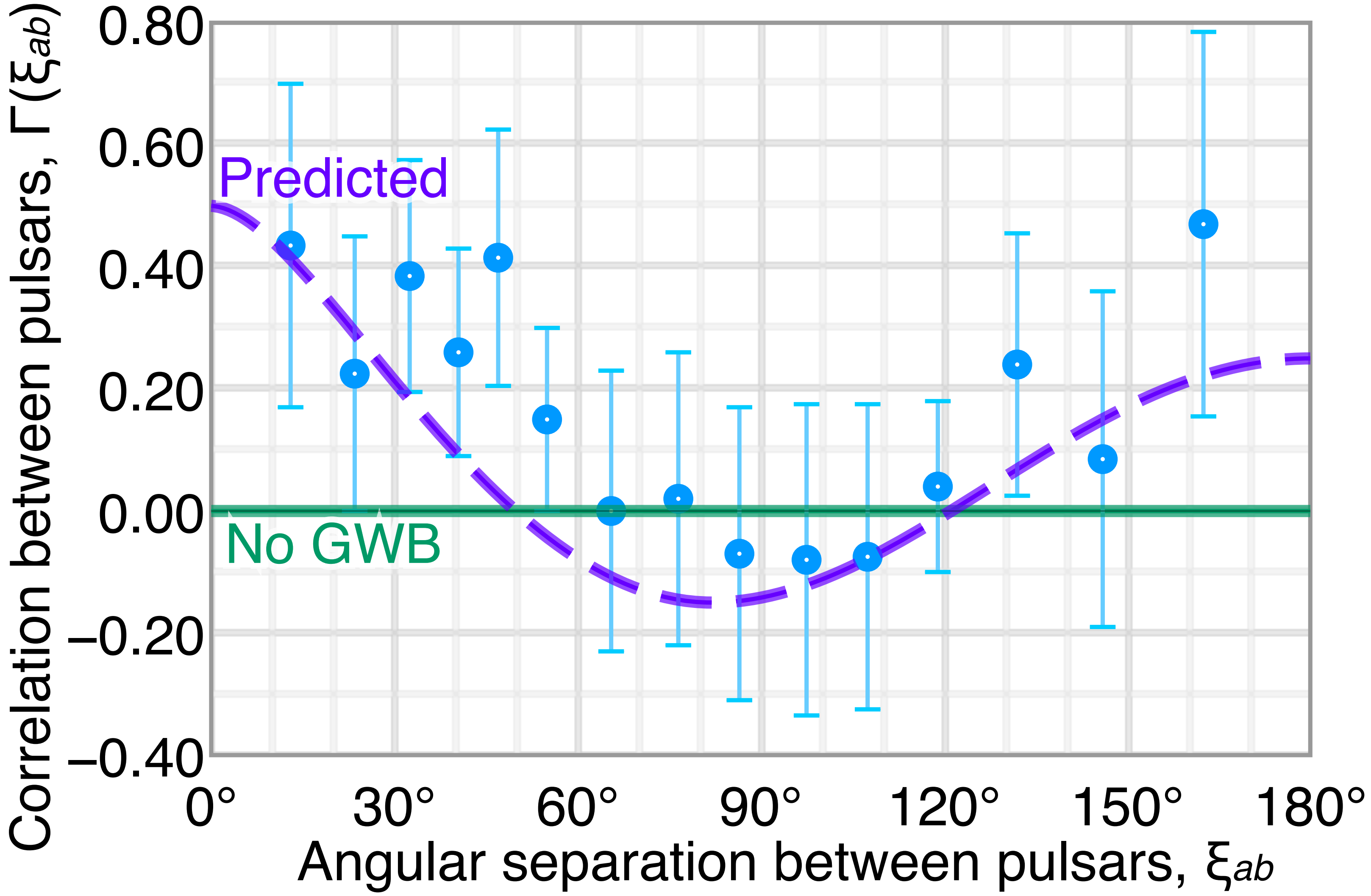


The Green Bank Observatory in Green Bank, W.Va., was among the observatories used to track pulsars as a way of detecting low-frequency gravitational waves. (Michael S. Williamson/The Washington Post)

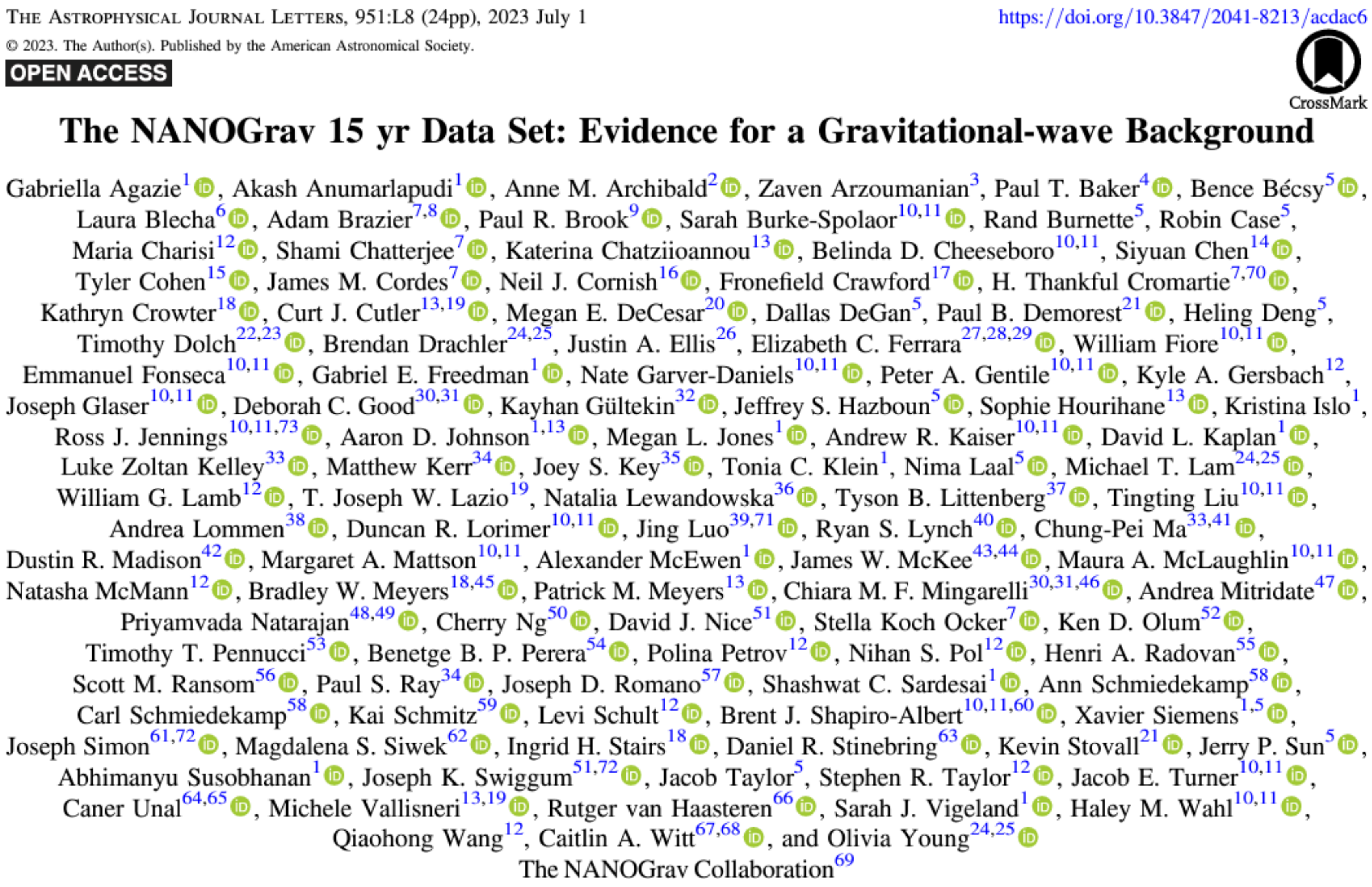
15 year of observations of 68 millisecond pulsars

For example, J0437–4715 has a period of 0.005757451936712637 s with an error of 1.7×10^{-17} s

Hellings–Downs curve



NANOGrav 15-year data set



The Astrophysical Journal Letters, 951:L8 (24pp), 2023 July 1 https://doi.org/10.3847/2041-8213/acdac6

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

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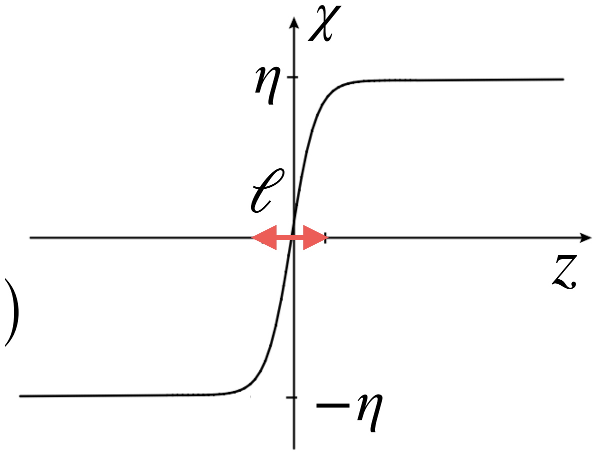
The NANOGrav Collaboration[69]

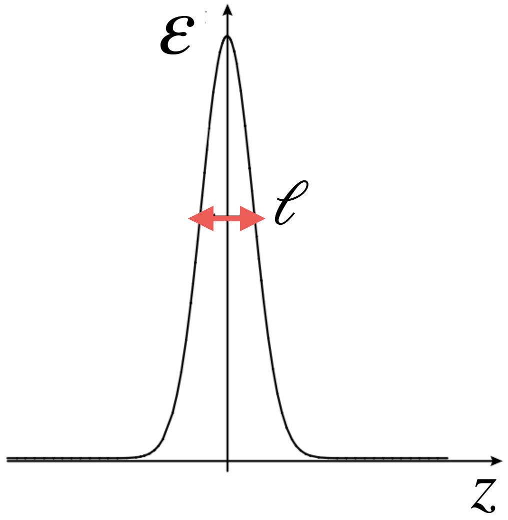
“The inferred gravitational-wave background amplitude and spectrum are consistent with astrophysical expectations for a signal from a population of supermassive black hole binaries, although more exotic cosmological and astrophysical sources cannot be excluded. The observation of Hellings–Downs correlations points to the gravitational-wave origin of this signal.”

Melting Domain Walls

$$\ell = (\lambda/2)^{-1/2}\,\eta^{-1}$$

$$\chi(z) = \eta \tanh\left(z/\ell\right)$$





$$V_{eff} \simeq \frac{\lambda \cdot \left(\chi^2 - \eta^2(T)\right)^2}{4}$$

$$\eta^2(T) \approx \frac{Ng^2T^2}{12\lambda}$$

Tension $\sigma_{wall} = \dfrac{2\sqrt{2\lambda}}{3}\,\eta^3(T)$ melting away as $\propto T^3$!

In the **scaling regime (Kibble 1976):** one domain wall per Hubble volume:

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{wall} \sim M_{wall}H^3 \sim \sigma_{wall}\,H \propto T^5$$

Usual Constant tension DW
$\rho_{wall} \propto T^2$

Gravitational Waves

Einstein's formula

$$P \sim \dddot{Q}_{ij}^2 / M_{Pl}^2$$

works well for domain wall network!!!

On the estimation of gravitational wave spectrum from cosmic domain walls

Takashi Hiramatsu (Kyoto U., Yukawa Inst., Kyoto), Masahiro Kawasaki (Tokyo U., ICRR and Tokyo U., IPMU), Ken'ichi Saikawa (Tokyo Inst. Tech.) (Sep 19, 2013)

Published in: *JCAP* 02 (2014) 031 • e-Print: 1309.5001 [astro-ph.CO]

Quadrupole Moment

$$|Q_{ij}| \sim M_{wall}/H^2$$

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{gw} \sim P \cdot t \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \propto T^6$$

If scaling regime attained almost instantaneously, the **peak frequency** is H_i!

$$f = H_i$$

Gravitational Waves Frequency

$$f_0 \simeq H_i \cdot \frac{a_i}{a_0} \propto T_i$$

$$f_0 \simeq \left(\frac{g_* \left(T_i\right)}{100} \right)^{1/6} \frac{T_0 T_i}{M_{pl}}$$

$$f_0 \simeq 6 \text{ nHz} \sqrt{\frac{N}{B}} \cdot \frac{g}{10^{-18}} \cdot \left(\frac{100}{g_*(T_i)} \right)^{1/3}$$


On the estimation of gravitational wave spectrum from cosmic domain walls #7

Takashi Hiramatsu (Kyoto U., Yukawa Inst., Kyoto), Masahiro Kawasaki (Tokyo U., ICRR and Tokyo U., IPMU), Ken'ichi Saikawa (Tokyo Inst. Tech.) (Sep 19, 2013)

Published in: *JCAP* 02 (2014) 031 • e-Print: 1309.5001 [astro-ph.CO]

Einstein formula estimation

$$\rho_{gw} \sim P \cdot t \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2}$$

Simulations

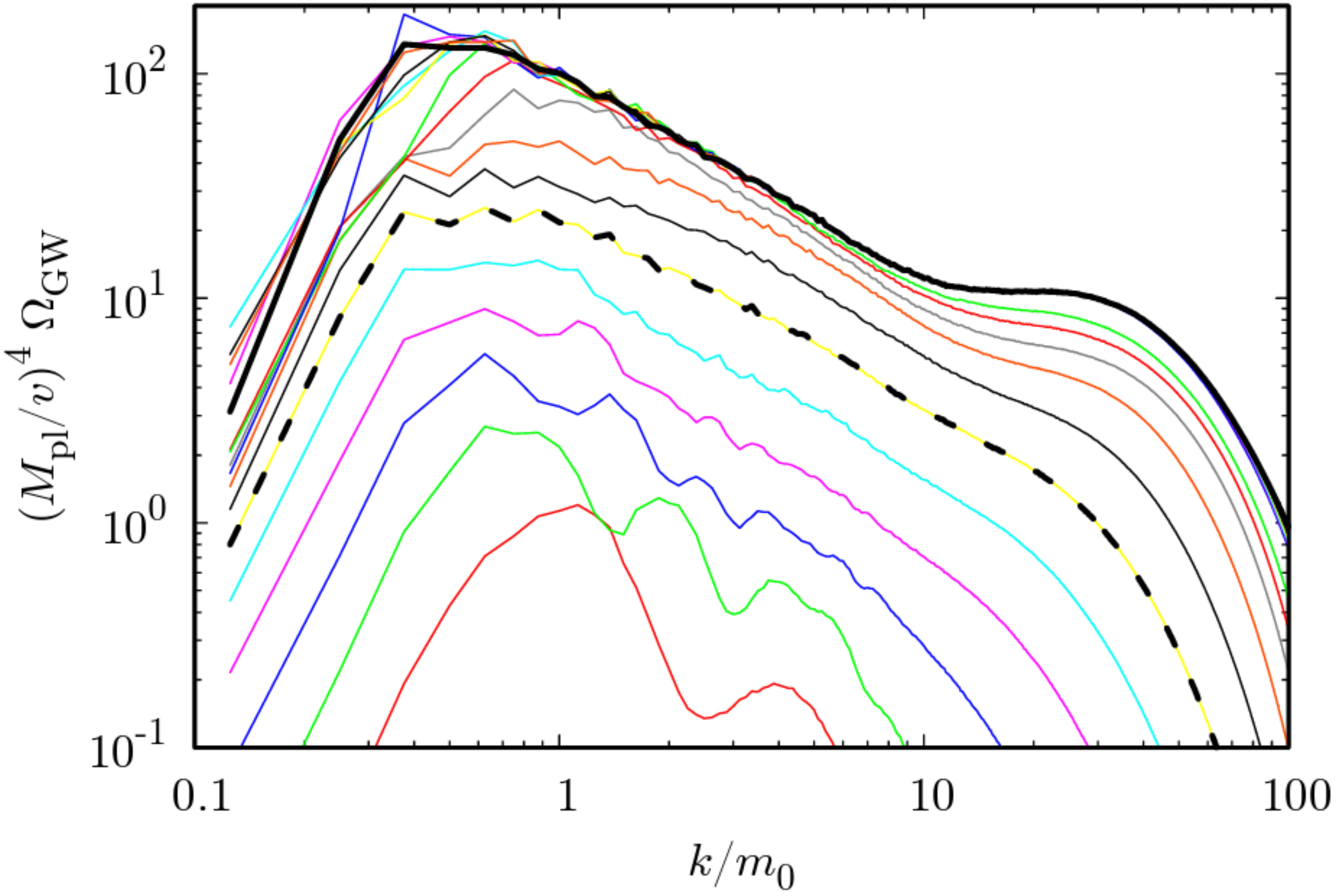
$$\frac{d\rho_{gw}}{d\ln f} \simeq \frac{\epsilon_{gw} A^2 \sigma_{wall}^2}{8\pi M_{Pl}^2} \qquad \propto T^6$$

our case

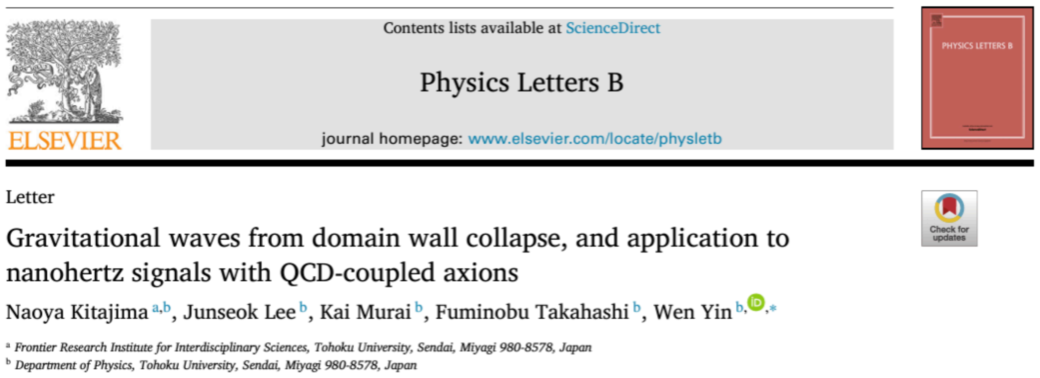
$$\epsilon_{gw} = 0.7 \pm 0.4 \qquad A = 0.8 \pm 0.1$$

$$\Omega_{gw}(f, t) = \frac{1}{\rho_{tot}(t)} \left(\frac{d\rho_{gw}}{d\ln f} \right) \qquad \propto T^2$$

our case



Phys. Lett. B 851 (2024) 138586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Letter

Gravitational waves from domain wall collapse, and application to nanohertz signals with QCD-coupled axions

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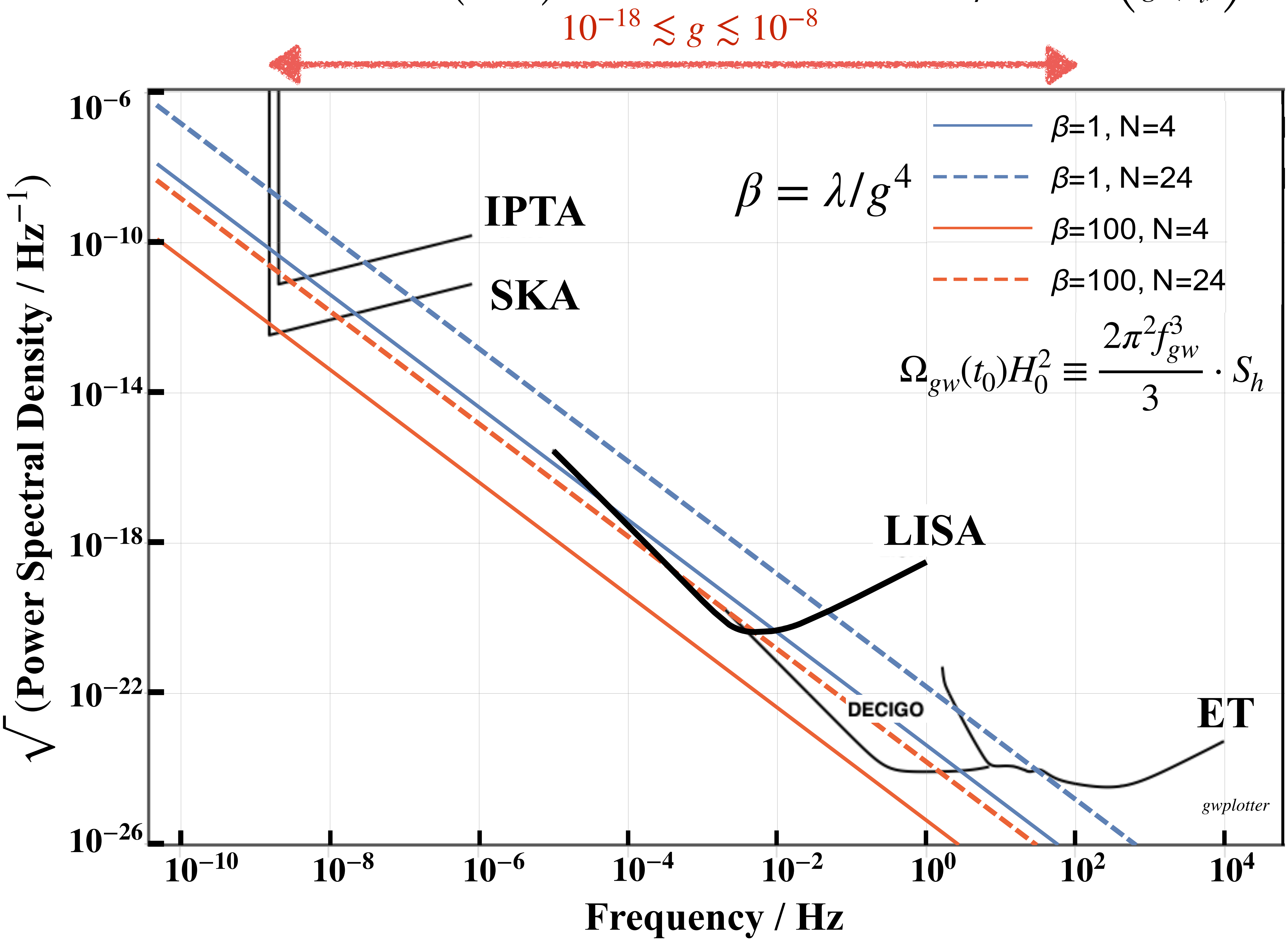
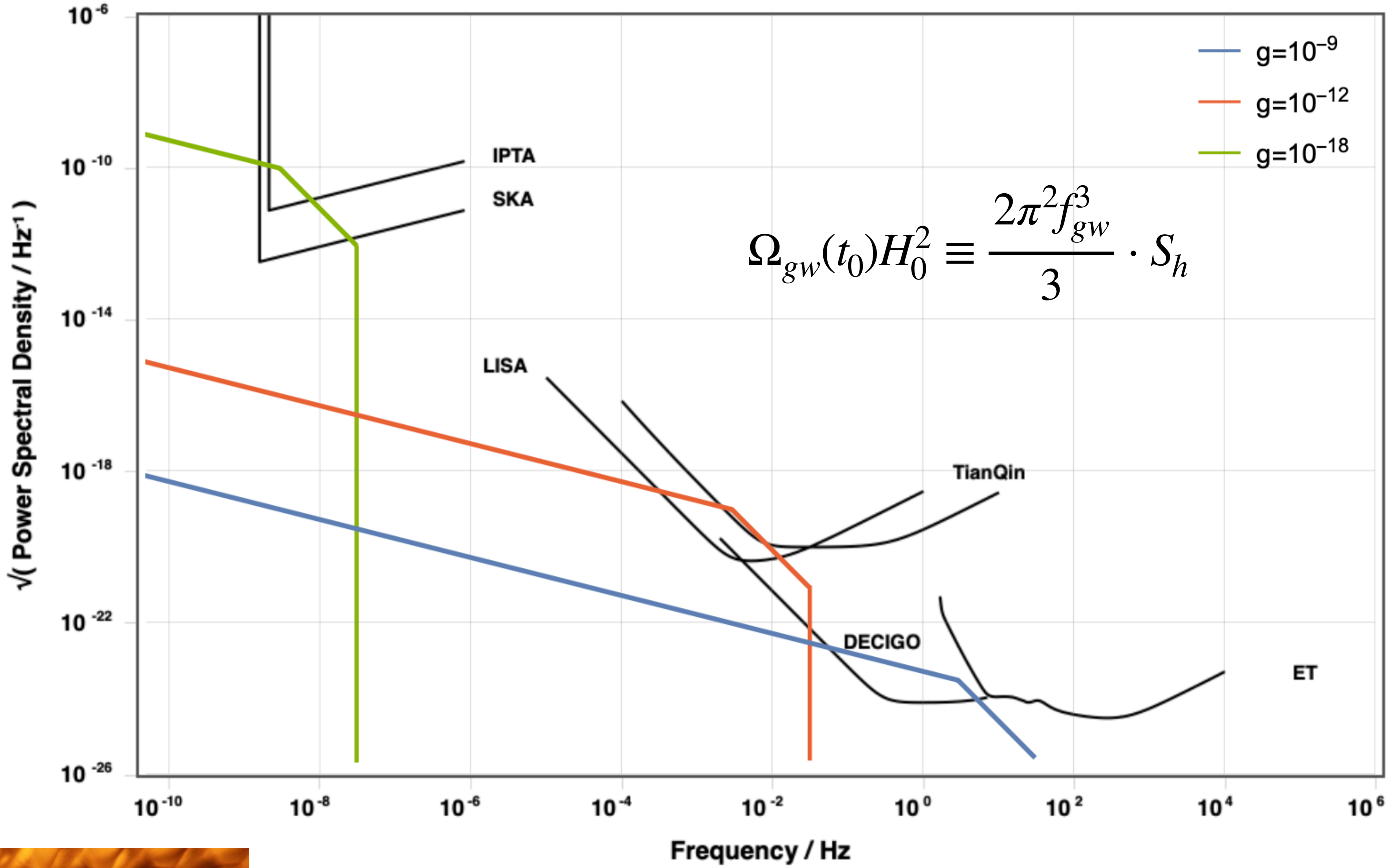
$$f_{gw} \equiv f_{gw}(t_0) \simeq 6\ \text{nHz} \cdot \sqrt{N} \cdot \frac{g}{10^{-18}} \cdot \left(\frac{100}{g_*(T_i)}\right)^{1/3} \qquad \Omega_{gw}h^2(t_0) \approx \frac{4 \cdot 10^{-14} \cdot N^4}{\beta^2} \cdot \left(\frac{100}{g_*(T_i)}\right)^{7/3}$$





$$\Omega_{gw}(t_0)H_0^2 \equiv \frac{2\pi^2 f_{gw}^3}{3} \cdot S_h$$

$\Omega_{gw}(IR) \sim f^2$ $\Omega_{gw}(UV) \sim f^{-1}$ Cutoff $\ell = (\lambda/2)^{-1/2}\eta^{-1}$

Usual Domain Walls $\Omega_{gw}(IR) \sim f^3$

More on f^2 in IR

Dimensional analysis
supported by simulation
for constant tension

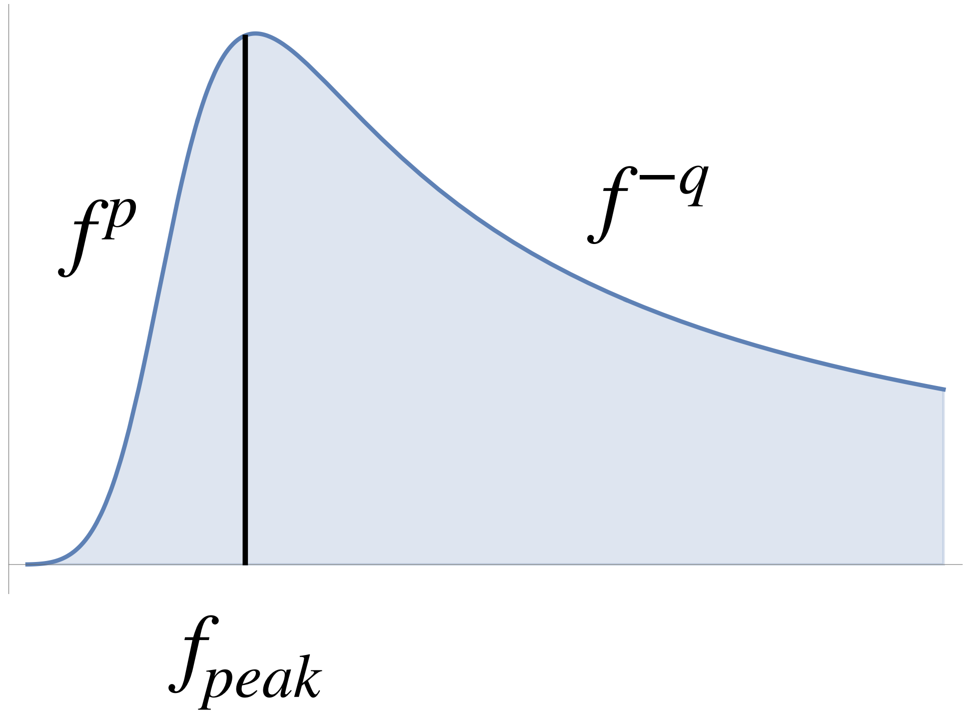
$$\Omega_{gw}\left(t_{now}\right)_{peak} \simeq A\left(\frac{f_{peak}}{F_{max}}\right)^2$$

energy is additive

Σ over t_{em} = Σ over f_{peak}

$$\delta\Omega_{gw}(f) = 2A\left(\frac{f_{peak}}{F_{max}^2}\right)\delta f_{peak}\left(\frac{f}{f_{peak}}\right)^p \frac{2}{1+\left(f/f_{peak}\right)^{p+q}}$$

for $f_{min} \ll f \ll F_{max}$



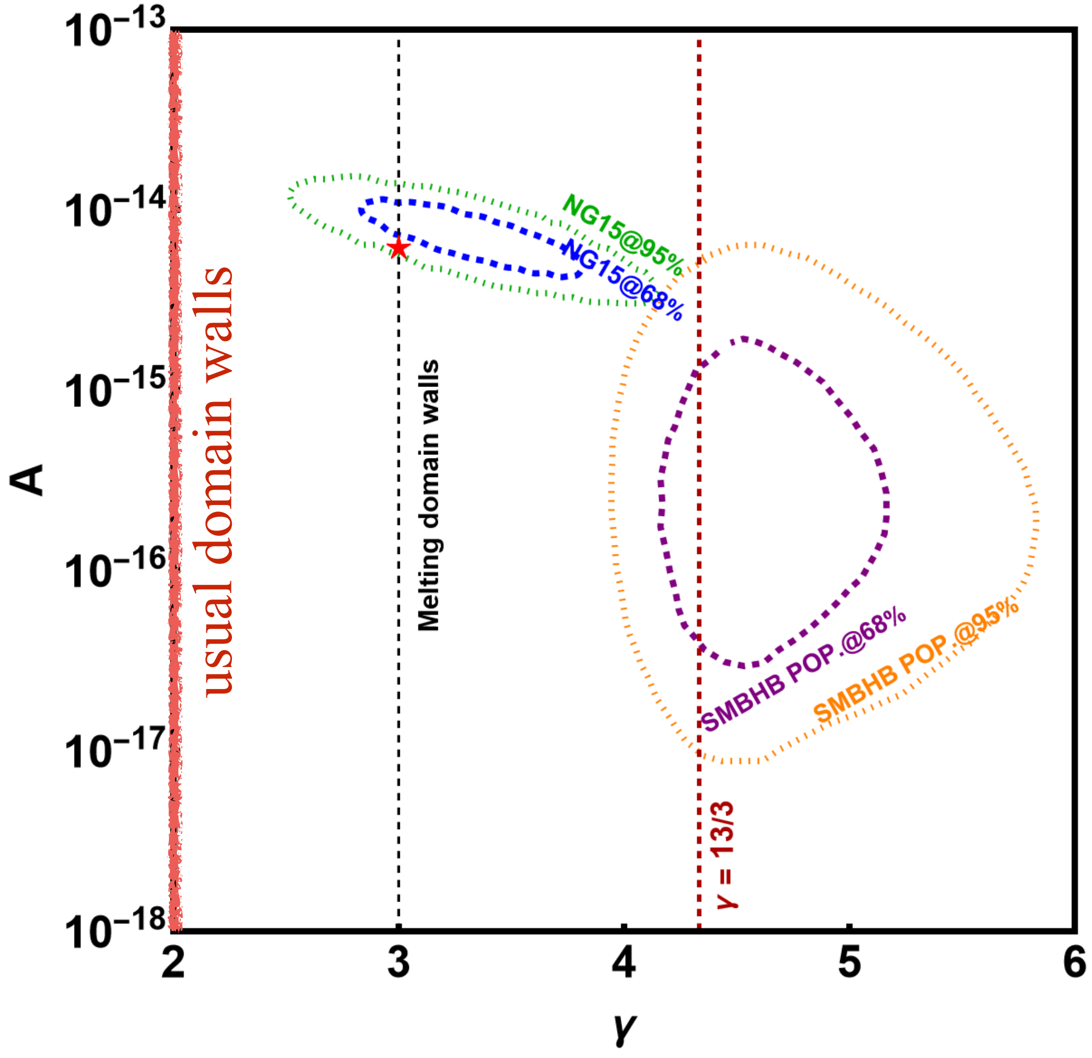
$$\Omega_{gw}(f) = \int_{f_{min}}^{F_{max}} \delta\Omega_{gw}(f) \propto \left(\frac{f}{F_{max}}\right)^2\left[1-\mathcal{O}\left(\frac{f}{F_{max}}\right)^n - \mathcal{O}\left(\frac{f_{min}}{f}\right)^m\right]$$

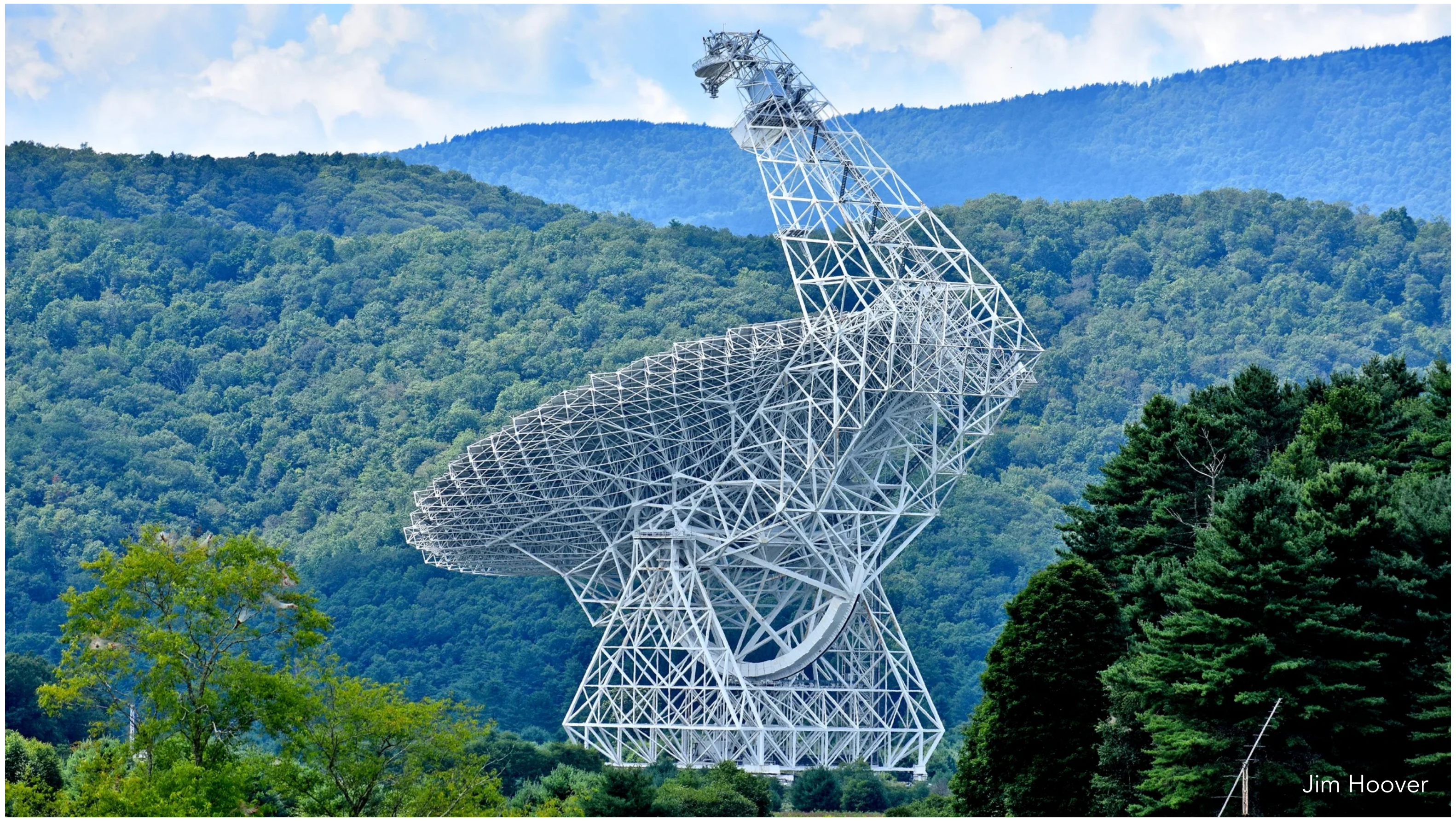
NANOGgrav

$$\Omega_{\rm GW}(f) = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma},$$

$$f_{yr} = 32\,\text{nHz}$$

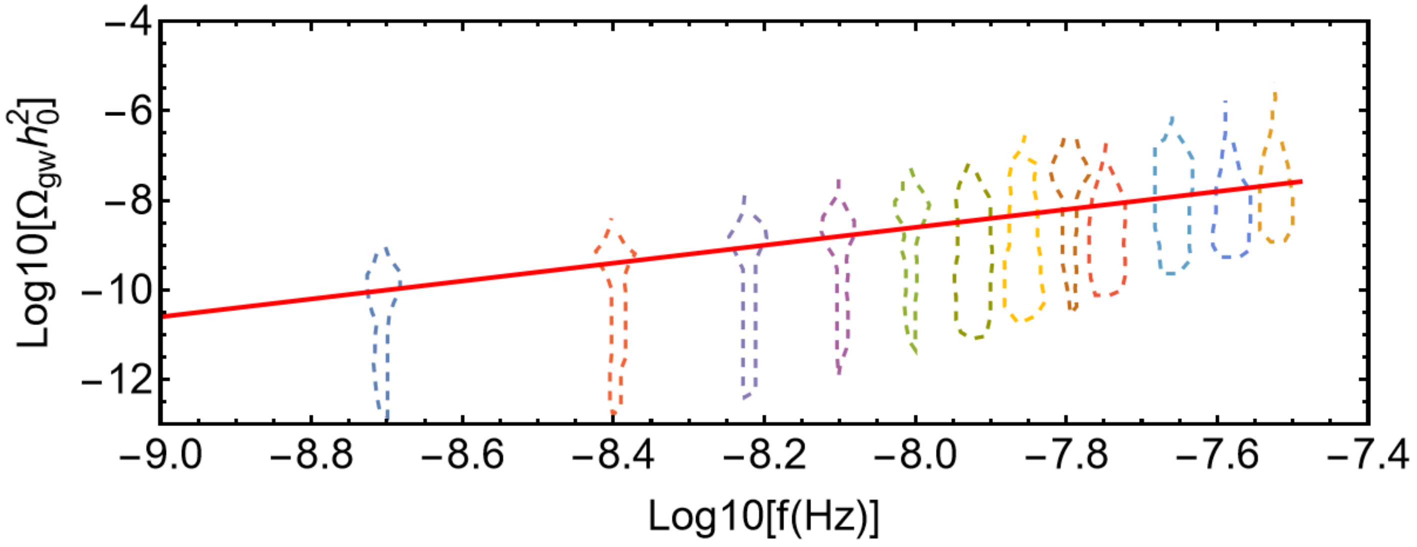
$$\Omega_{yr} = \frac{2\pi^2}{3H_0^2}A^2 f_{yr}^2$$



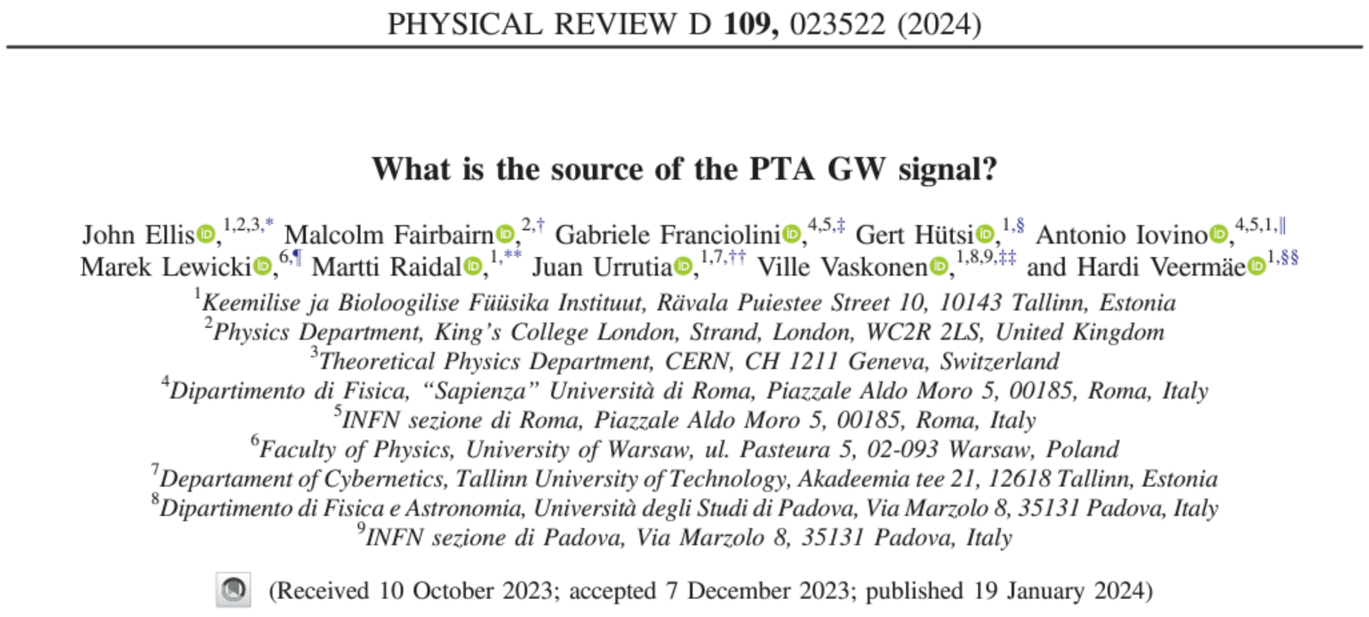


The 100-meter Green Bank Telescope, the world's largest fully steerable telescope and a core instrument for pulsar timing array experiment.

parameters $g = 10^{-18}$, $\beta = \lambda/g^4 = 1$, $N = 24$, $g_* = 75$



$$T_i \simeq 1.2\,\text{GeV}\left(\frac{f_0}{f_{yr}}\right)\cdot\left(\frac{100}{g_*(T_i)}\right)^{1/6}$$



PHYSICAL REVIEW D **109,** 023522 (2024)

What is the source of the PTA GW signal?

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Marek Lewicki,[6,¶] Martti Raidal,[1,**] Juan Urrutia,[1,7,††] Ville Vaskonen,[1,8,9,‡‡] and Hardi Veermäe[1,§§]
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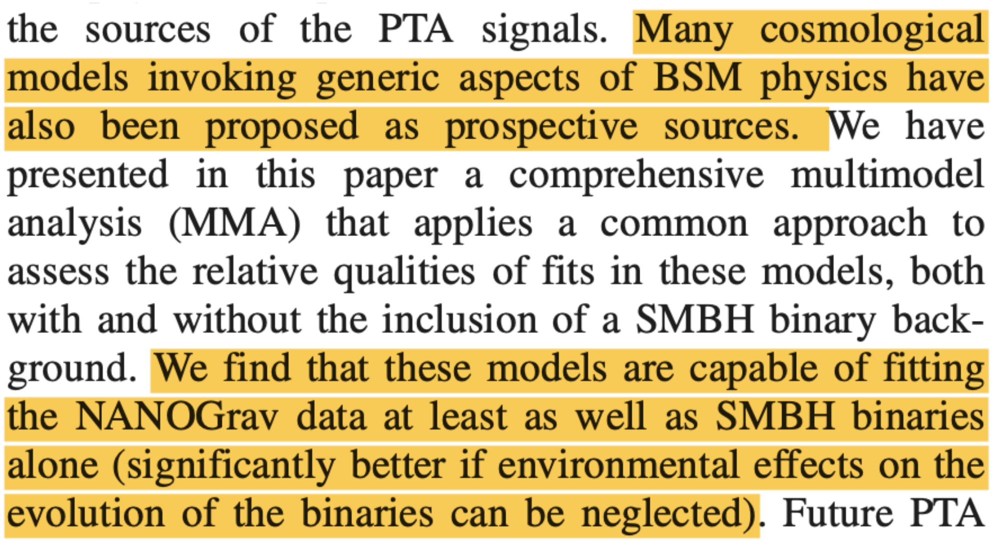
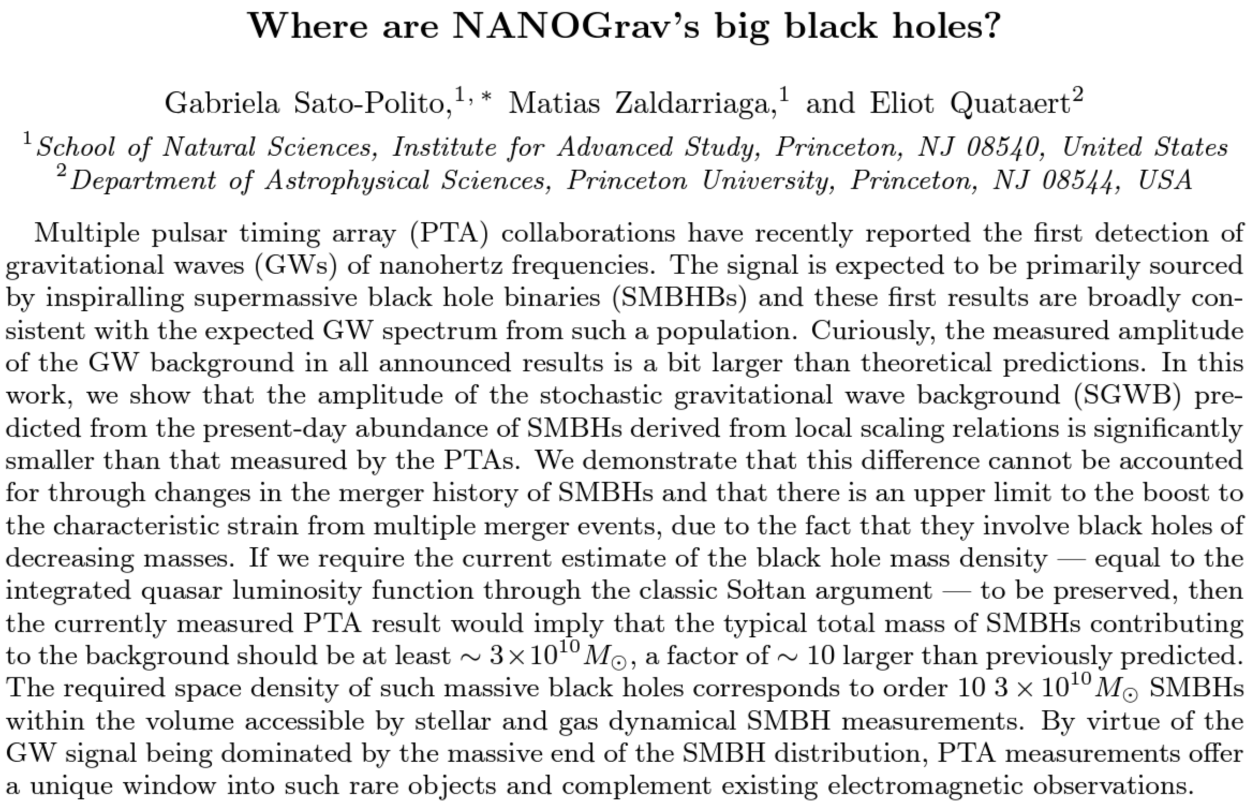


the sources of the PTA signals. Many cosmological models invoking generic aspects of BSM physics have also been proposed as prospective sources. We have presented in this paper a comprehensive multimodel analysis (MMA) that applies a common approach to assess the relative qualities of fits in these models, both with and without the inclusion of a SMBH binary background. We find that these models are capable of fitting the NANOGrav data at least as well as SMBH binaries alone (significantly better if environmental effects on the evolution of the binaries can be neglected). Future PTA



Where are NANOGrav's big black holes?

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Multiple pulsar timing array (PTA) collaborations have recently reported the first detection of gravitational waves (GWs) of nanohertz frequencies. The signal is expected to be primarily sourced by inspiralling supermassive black hole binaries (SMBHBs) and these first results are broadly consistent with the expected GW spectrum from such a population. Curiously, the measured amplitude of the GW background in all announced results is a bit larger than theoretical predictions. In this work, we show that the amplitude of the stochastic gravitational wave background (SGWB) predicted from the present-day abundance of SMBHs derived from local scaling relations is significantly smaller than that measured by the PTAs. We demonstrate that this difference cannot be accounted for through changes in the merger history of SMBHs and that there is an upper limit to the boost to the characteristic strain from multiple merger events, due to the fact that they involve black holes of decreasing masses. If we require the current estimate of the black hole mass density — equal to the integrated quasar luminosity function through the classic Sołtan argument — to be preserved, then the currently measured PTA result would imply that the typical total mass of SMBHs contributing to the background should be at least $\sim 3\times10^{10} M_\odot$, a factor of ~ 10 larger than previously predicted. The required space density of such massive black holes corresponds to order 10 $3\times10^{10} M_\odot$ SMBHs within the volume accessible by stellar and gas dynamical SMBH measurements. By virtue of the GW signal being dominated by the massive end of the SMBH distribution, PTA measurements offer a unique window into such rare objects and complement existing electromagnetic observations.

Superradiance: From NANOGrav to LIGO or LISA

DM from the inverse phase transition

$$M_\chi \simeq 10^{-12}\ \text{eV} \cdot B^{9/20} \cdot \left(\frac{g_*(T_{sym})}{100}\right)^{1/5} \cdot \left(\frac{g_*(T_i)}{100}\right)^{1/20} \cdot \left(\frac{m_\phi}{10\ \text{MeV}}\right)^{1/2} \times \left(\frac{f_{peak}}{30\ \text{nHz}}\right)^{6/5} \cdot \left(\frac{10^{-8}}{\Omega_{gw,peak} h_0^2}\right)^{3/20}$$

Superradiance for $M_{BH} \simeq 10^2 M_\odot$ → LIGO

DM from the direct phase transition

$$M_\chi \simeq 6.5 \cdot 10^{-17}\ \text{eV} \cdot \left(\frac{f_{peak}}{30\ \text{nHz}}\right) \cdot \left(\frac{g_*(T_i)}{100}\right)^{1/6} \cdot \sqrt{\frac{10^{-8}}{\Omega_{gw,peak} \cdot h_0^2}}$$

Superradiance for $M_{BH} \simeq 10^7 M_\odot$ → LISA

Thanks a lot for attention!