Domain Walls
and their
Gravitational Waves I

Alexander Vikman

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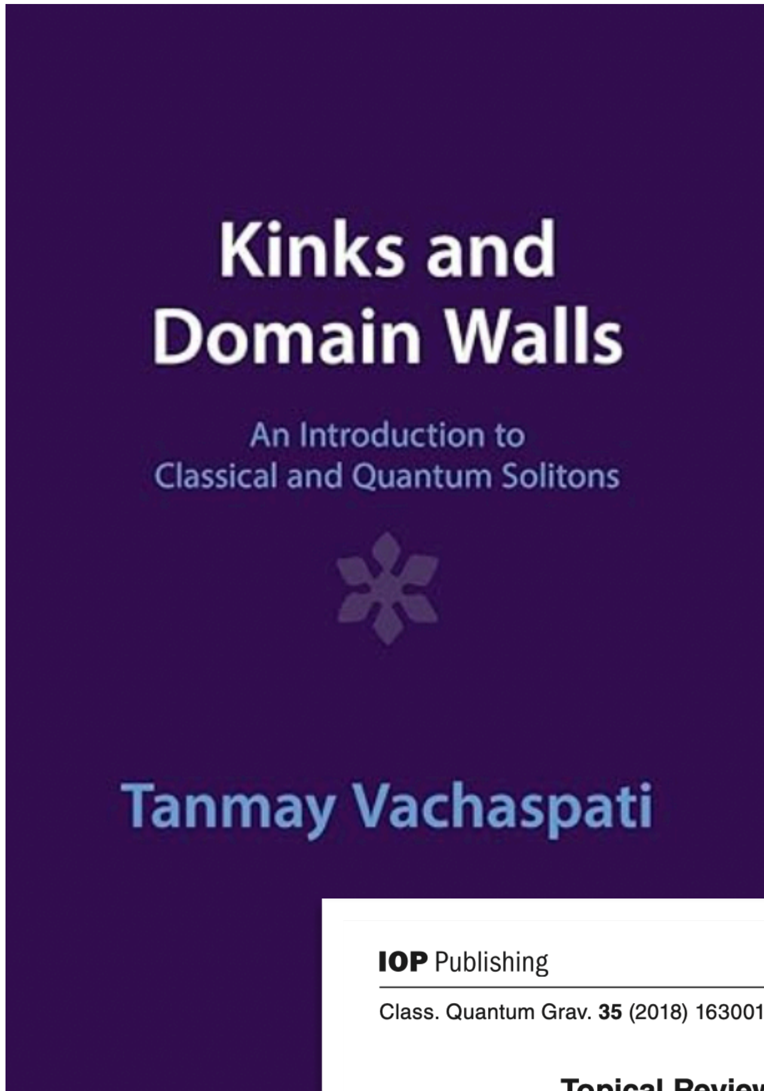
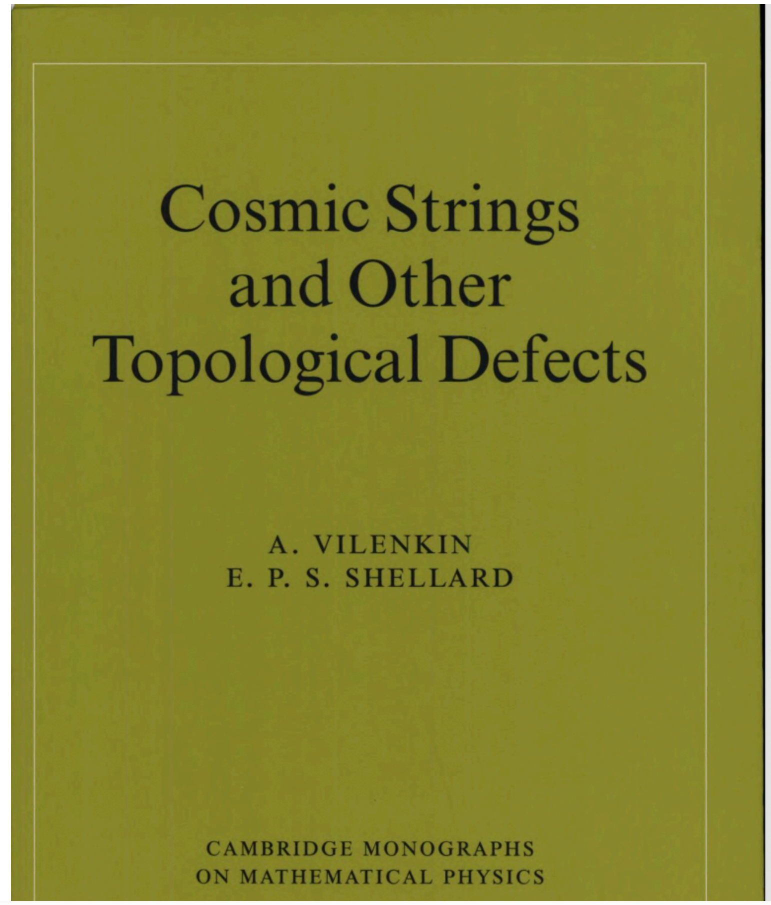
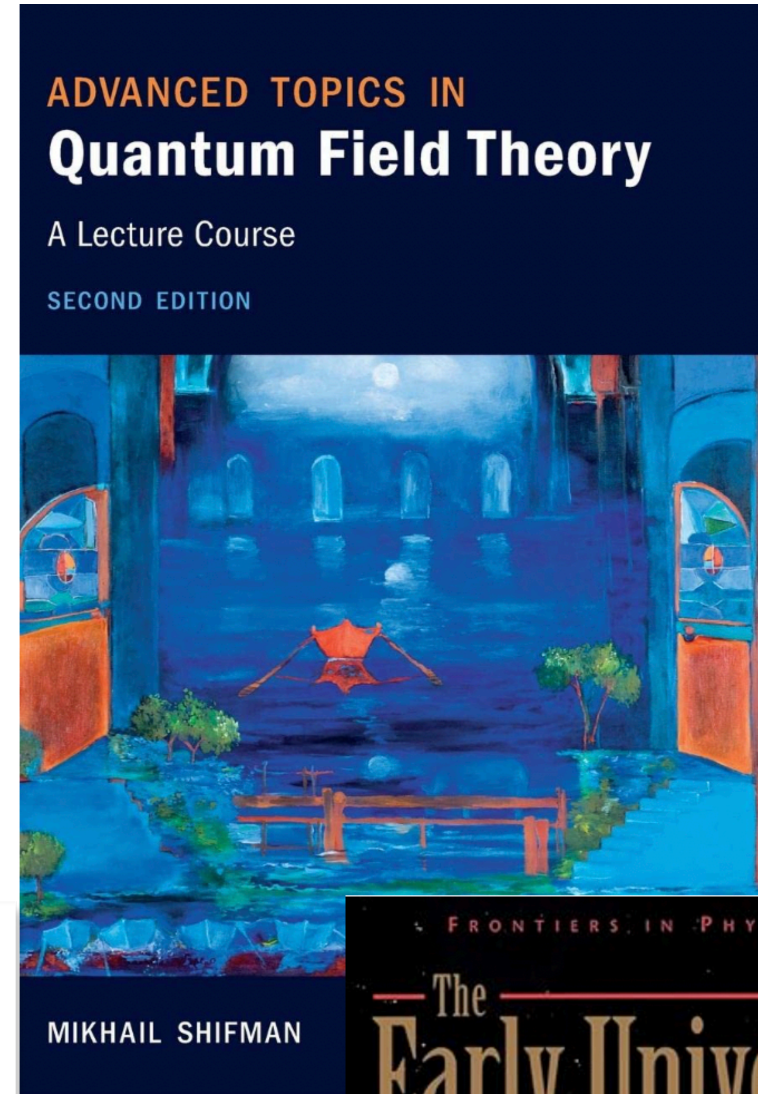
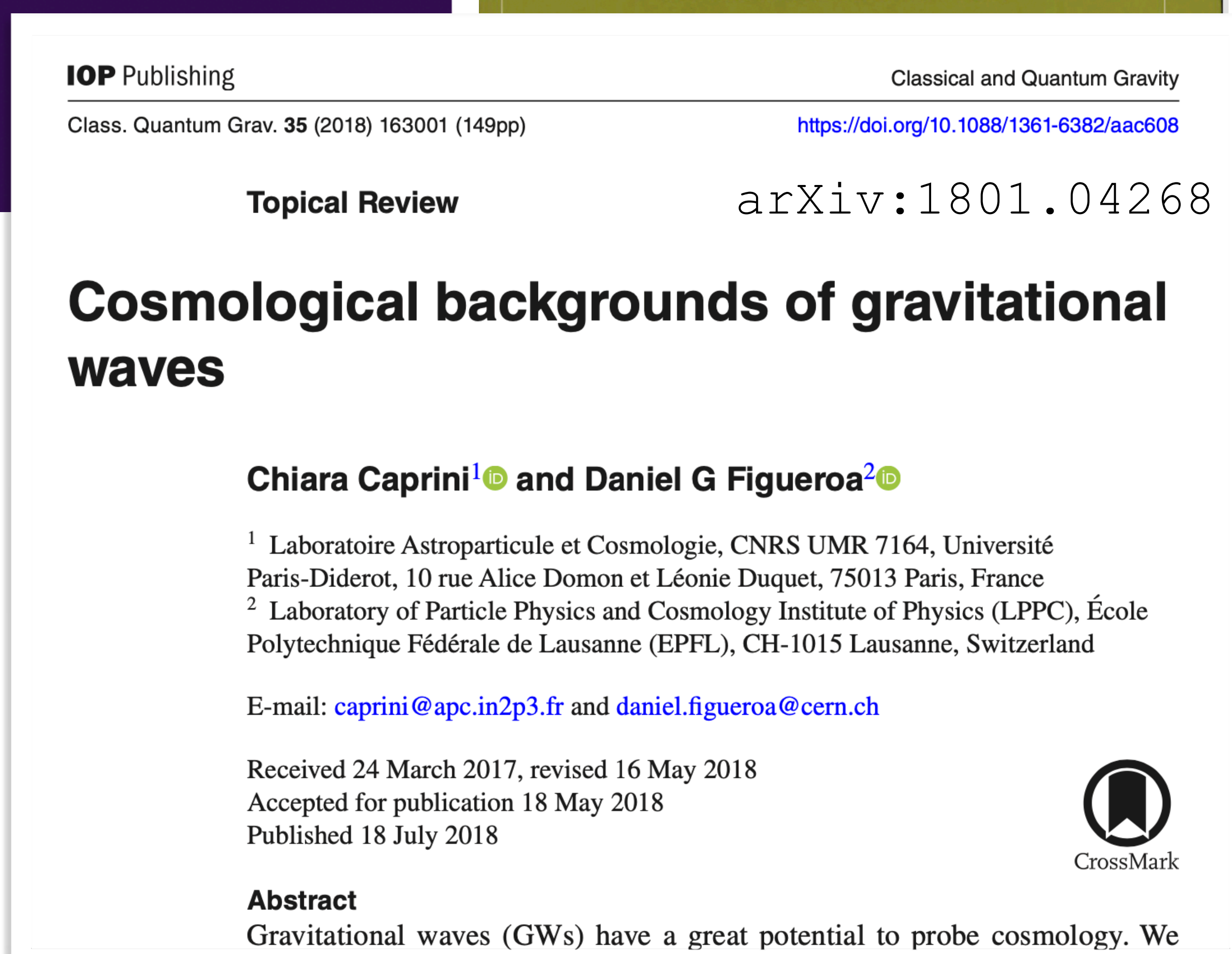
Plan

- Simplest Domain Walls
- Cosmology of Domain Walls
- Melting Domain Walls and creation of ultralight **DM**





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Topical Review arXiv:1801.04268

Cosmological backgrounds of gravitational waves

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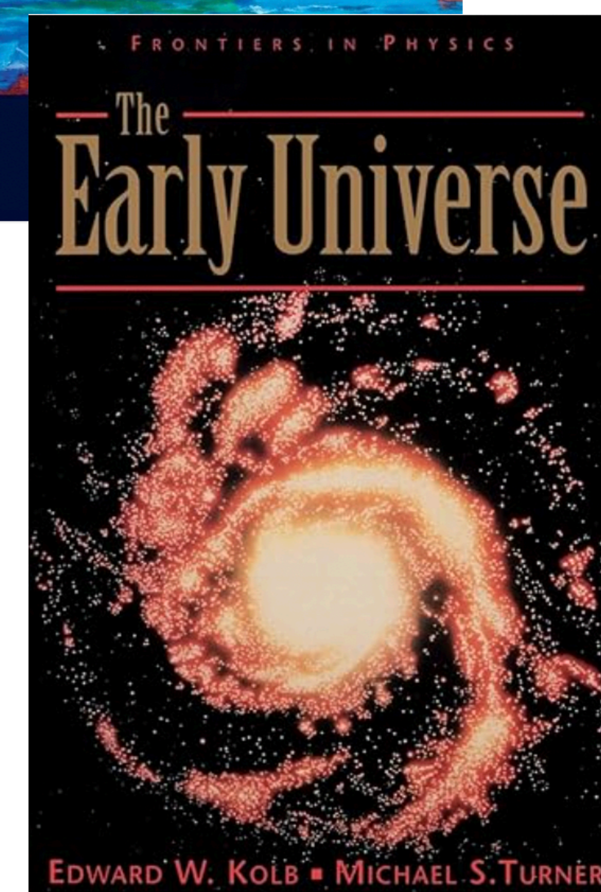
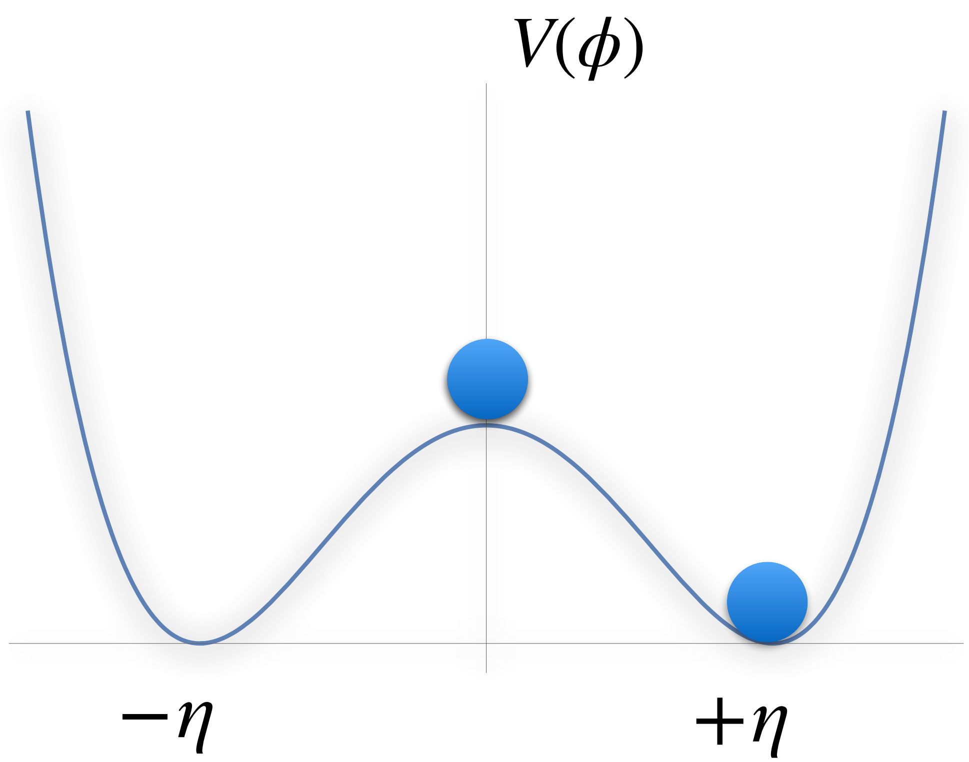
Abstract
Gravitational waves (GWs) have a great potential to probe cosmology. We



Spontaneous Breaking of Discrete Symmetry

Z_2 -symmetric scalar field



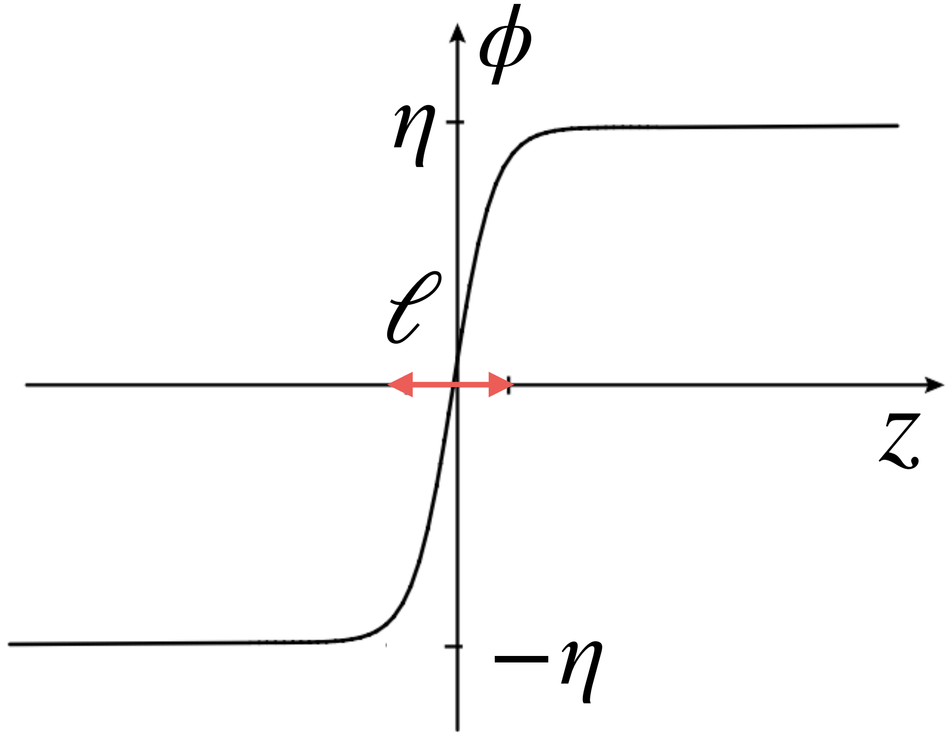
$$V = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$

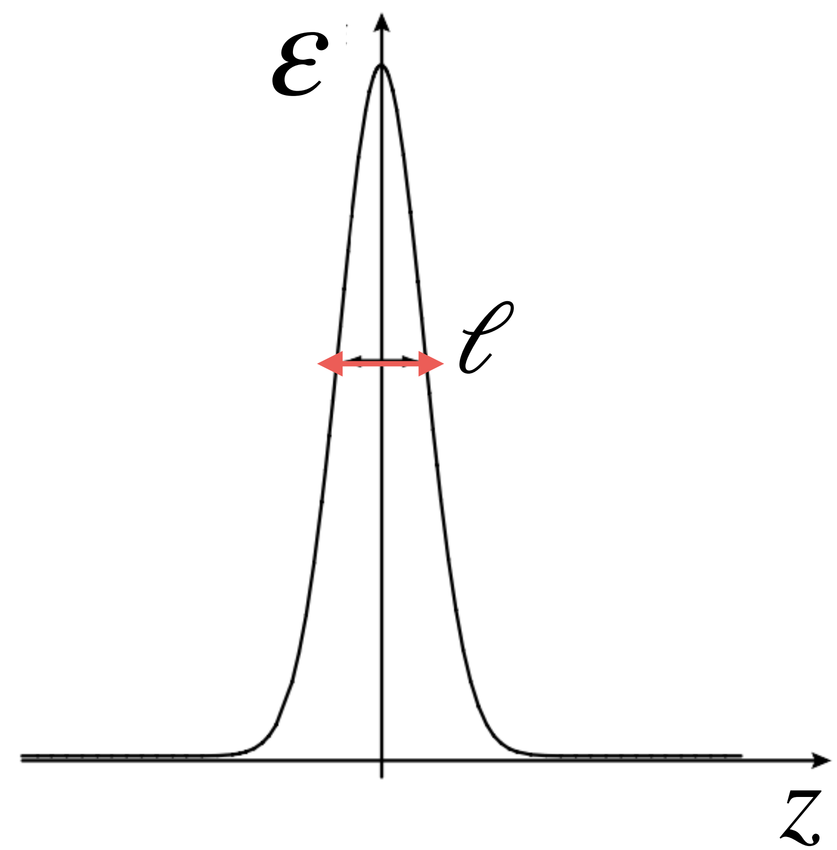
Minimising $$\sigma \simeq \varepsilon\ell \simeq \left(\frac{\eta}{\ell}\right)^2 \ell + \lambda\eta^4\ell$$

$$\ell \simeq \lambda^{-1/2}\eta^{-1} \qquad \sigma \simeq \lambda^{1/2}\eta^3$$

Kink

$$\phi(z) = \eta \tanh\left(z/\ell\right)$$





$$\ell = (\lambda/2)^{-1/2}\,\eta^{-1}$$

Energy-Momentum and Surface Tension

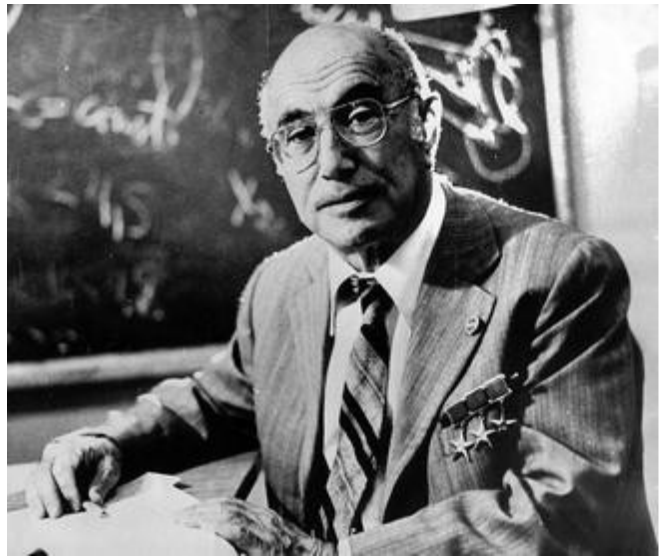
$$T^{\mu}_{\nu} = \partial^{\mu}\phi\,\partial_{\nu}\phi - \delta^{\mu}_{\nu}\left(\frac{1}{2}\left(\partial\phi\right)^{2} - V\left(\phi\right)\right)$$

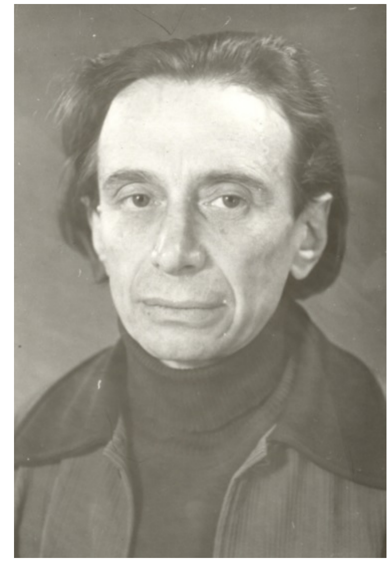
$$T^{\mu}_{\nu} = \frac{\lambda}{2}\eta^{4}\cosh^{-4}\left(\frac{z}{\ell}\right) diag\left(1,1,1,0\right)$$

$$\sigma_{wall} = \int dz T^{0}_{0} = \frac{2\sqrt{2\lambda}}{3}\eta^{3}(T)$$

Note that Great Wall

Even a Greater One!





Cosmological consequences of a spontaneous breakdown of a discrete symmetry

Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun'

Institute for Applied Mathematics, USSR Academy of Sciences

(Submitted January 31, 1974)

Zh. Eksp. Teor. Fiz. **67**, 3–11 (July 1974)

In theories involving spontaneous symmetry breakdown one may expect a domain structure of the vacuum. Such a structure does not exist near a cosmolgical singularity, when the temperature is above the Curie point, but this structure must appear later during the cosmological expansion and cooling down. We discuss the properties of the domain interfaces and of the space with domains in the large, the law of cosmological expansion in the presence of domains, and the influence of domains of the homogeneity of the Universe at a late stage.

Large CMB fluctuations

photon temperature due to redshift $T \propto \frac{1}{a}$

$$\frac{\delta T}{T} \propto \frac{\delta a}{a} \simeq \Phi$$

Poisson Equation $\Delta\Phi \sim G\,\sigma_{wall}\,\delta(z)$

$$\Phi \sim G\,\sigma_{wall} z$$

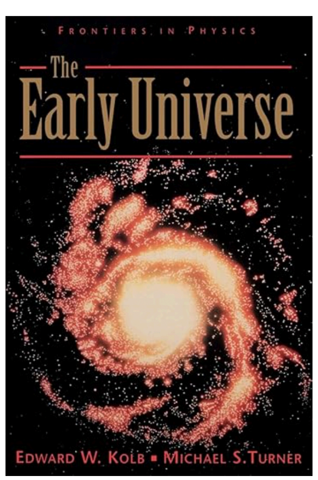
$$\frac{\delta T}{T} \simeq G\,\sigma_{wall} H_0^{-1} \simeq 10^{10}\ \lambda^{1/2} \left(\frac{\eta}{100\,GeV}\right)^3$$

Even Larger Mass

Mass inside the horizon H^{-1}

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\simeq 4 \times 10^{65}\ \lambda^{1/2} \left(\frac{\eta}{100\, GeV} \right)^3 \text{ grams}$$



"Apparently, domain walls are cosmological bad news…"

PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

Gauge and global symmetries at high temperature*

Steven Weinberg
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(Received 19 February 1974)

It is shown how finite-temperature effects in a renormalizable quantum field theory can restore a symmetry which is broken at zero temperature. In general, for both gauge symmetries and ordinary symmetries, such effects occur only through a temperature-dependent change in the effective bare mass of the scalar bosons. The change in the boson bare mass is calculated for general field theories, and the results are used to derive the critical temperatures for a few special cases, including gauge and nongauge theories. In one case, it is found that a symmetry which is unbroken at low temperature can be broken by raising the temperature above a critical value. An appendix presents a general operator formalism for dealing with higher-order effects, and it is observed that the one-loop diagrams of field theory simply represent the contribution of zero-point energies to the free energy density. The cosmological implications of this work are briefly discussed.

PHYSICAL REVIEW D VOLUME 23, NUMBER 4 15 FEBRUARY 1981

Gravitational field of vacuum domain walls and strings

Alexander Vilenkin
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(Received 10 October 1980)

The gravitational properties of vacuum domain walls and strings are studied in the linear approximation of general relativity. These properties are shown to be very different from those of regular massive planes and rods. It is argued that the domain walls are gravitationally unstable and collapse at a certain time $\sim t_c$ after their creation. If the vacuum walls ever existed, they must have disappeared at $t < t_c$.

Z_2 -symmetric DM scalar field χ coupled to ϕ - a multiplet of N *thermal* degrees of freedom

portal coupling

$$V = \frac{1}{2}\left(M^2 - g^2\phi^\dagger\phi\right)\cdot\chi^2 + \frac{\lambda}{4}\chi^4 + \frac{\lambda_\phi}{4}\left(\phi^\dagger\phi\right)^2$$

Tachyonic thermal mass

$$\mu^2 = g^2\langle\phi^\dagger\phi\rangle \simeq \frac{Ng^2T^2}{12}$$

increasing during preheating, then red-shifting

potential bounded from below

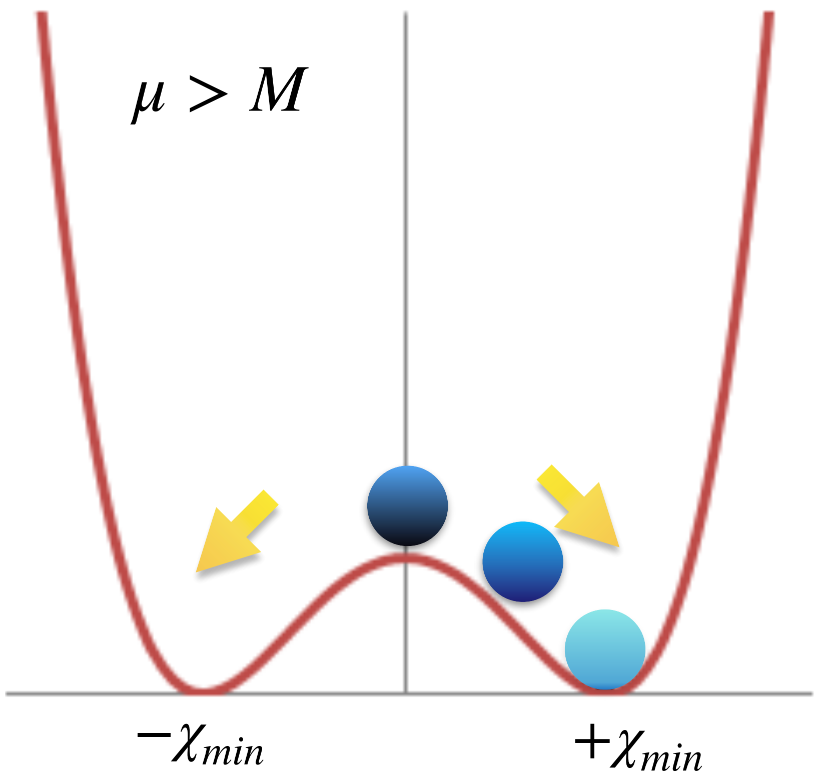
$$\beta = \frac{\lambda}{g^4} \geq \frac{1}{\lambda_\phi} \geq 1$$

potential bounded from below

weak coupling

Direct Phase Transition

Early universe spontaneously Broken Phase



Domain Walls!

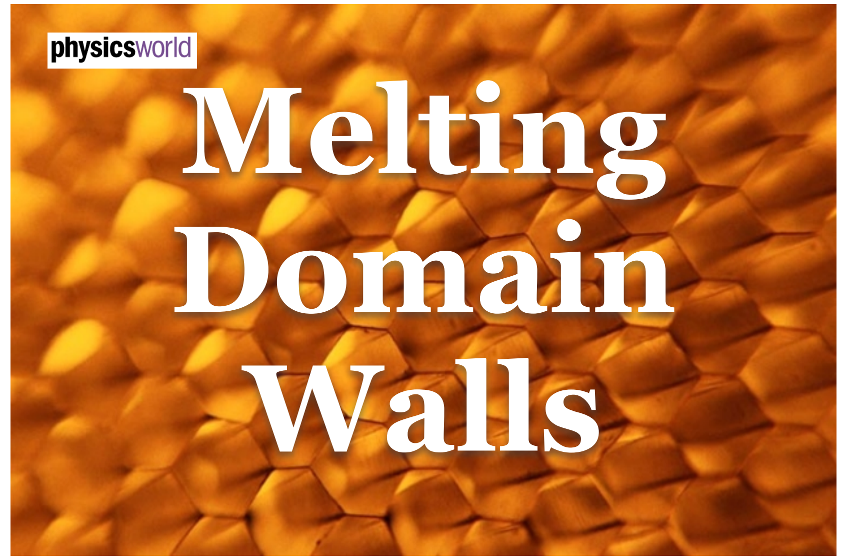
Avoid too much friction to start rolling

$$\mu \gtrsim H$$

$$\sqrt{\frac{N}{12}}gT_i \simeq \sqrt{\frac{\pi^2 g_*}{90}}\frac{T_i^2}{M_{pl}}$$

$$T_i \simeq g\, M_{Pl}\sqrt{\frac{N}{g_*(T_i)}} \times \frac{1}{\sqrt{B}}$$

Correction taking into account time to get to the minimum



$$V_{eff} \simeq \frac{\lambda \cdot \left(\chi^2 - \eta^2(T)\right)^2}{4} \qquad \eta^2(T) \approx \frac{Ng^2T^2}{12\lambda} = \mu^2/\lambda$$

Tension $\sigma_{wall} = \frac{2\sqrt{2\lambda}}{3}\eta^3(T)$ melting away as $\propto T^3$!

In the **scaling regime (Kibble 1976):** one domain wall per Hubble volume:

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{wall} \sim M_{wall}H^3 \sim \sigma_{wall}\,H \propto T^5$$

$$\frac{\rho_{wall}}{\rho_{rad}} \sim \frac{N^2}{30g_*(T)\beta} \cdot \frac{T}{T_i} < 1$$

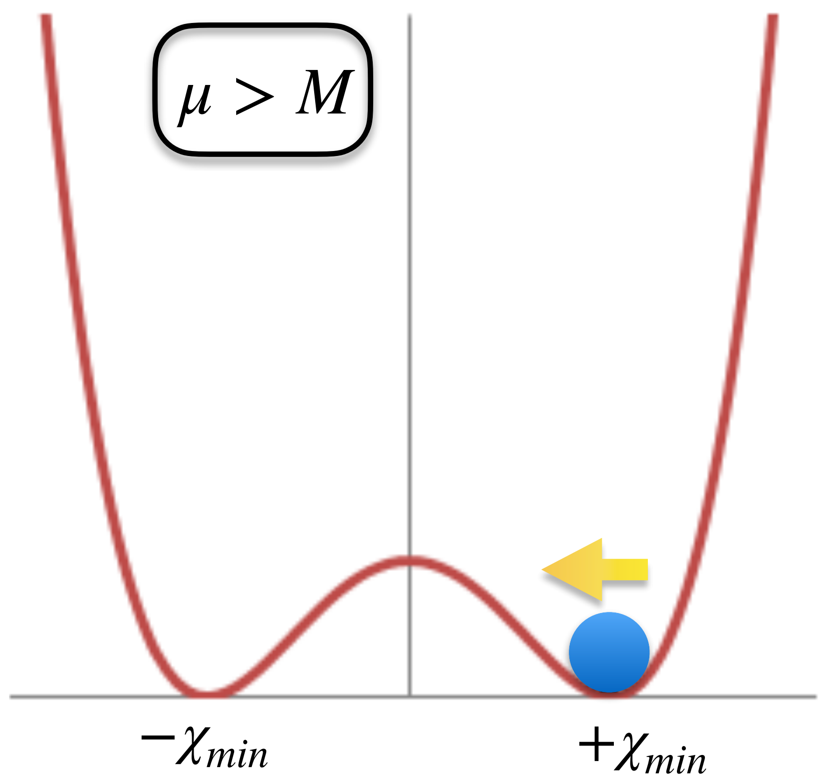
Usual Constant tension DW
$\rho_{wall} \propto T^2$

Inverse Phase Transition At Meltdown

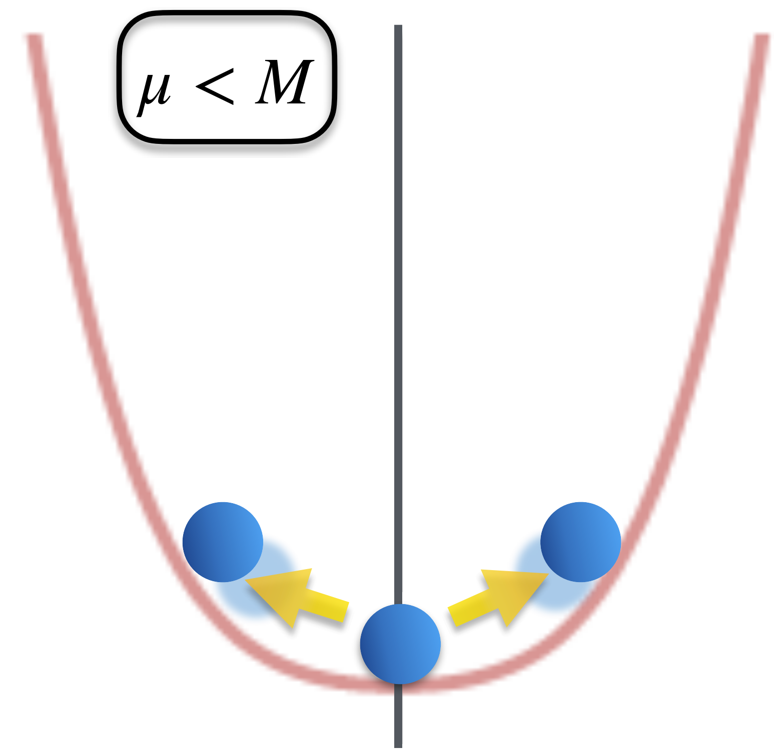
$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\left(M^2 - \mu^2(t,\mathbf{x})\right)\cdot\chi^2}{2} - \frac{\lambda\chi^4}{4}$$

Babichev, Gorbunov, Ramazanov(2020)

Early Universe
spontaneously Broken Phase



Late Universe
oscillations around restored symmetric vacuum



Tachyonic mass $\mu(t)$ slowly decreases / redshifts due to cosmological expansion

for Hubble parameter

$H < M$

scalar field traces vacuum $\chi_{min}(t) = \sqrt{\frac{\mu^2(t) - M^2}{\lambda}}$ as long as $\left|\frac{\dot{M}_{eff}}{M_{eff}^2}\right| \ll 1$

Tracing the vacuum

The minimum moves as $$\chi_{min}(t) = \sqrt{\frac{\mu^2(t) - M^2}{\lambda}}$$

In the minimum $$M_{eff}^2(t) = 2 \cdot (\mu^2(t) - M^2)$$

Adiabatically tracing the minimum $$\left| \frac{\dot{M}_{eff}}{M_{eff}^2} \right| \ll 1$$

Adiabaticity is definitely violated when $M_{eff} = 0$ i.e. when $\mu_* \simeq M$ **!**

At this point one cannot trace the minimum as $\dot{\chi}_{min} = \frac{\mu\dot{\mu}}{\sqrt{\lambda\left(\mu^2(t) - M^2\right)}}$ diverges**!**

Resulting Energy Density

adiabaticity is violated at t_*, *before* $\mu \simeq M$, if $M > H_*$ the field starts to oscillate with amplitude

$$\chi_* \simeq \frac{\left(2M^2\right)^{1/3}}{\sqrt{2\lambda}} \left|\frac{\dot{\mu}}{\mu}\right|_*^{1/3} \simeq \frac{(\kappa H_* M^2)^{1/3}}{\sqrt{2\lambda}}$$

Position of the minimum at t_* when

$$\left|\frac{\dot{M}_{eff}}{M_{eff}^2}\right| = 1$$

where we assume cosmological evolution

$$\frac{d\mu^2(t)}{dt} = -\kappa H \mu^2(t)$$

the field behaves as DM

$$\rho_\chi(t) = \frac{M^2 \chi_*^2}{2} \cdot \left(\frac{a_*}{a(t)}\right)^3 \simeq \frac{(\kappa \cdot M^5 \cdot H_*)^{2/3}}{4\lambda} \cdot \left(\frac{a_*}{a(t)}\right)^3$$

for the model of this talk $\kappa = 2$

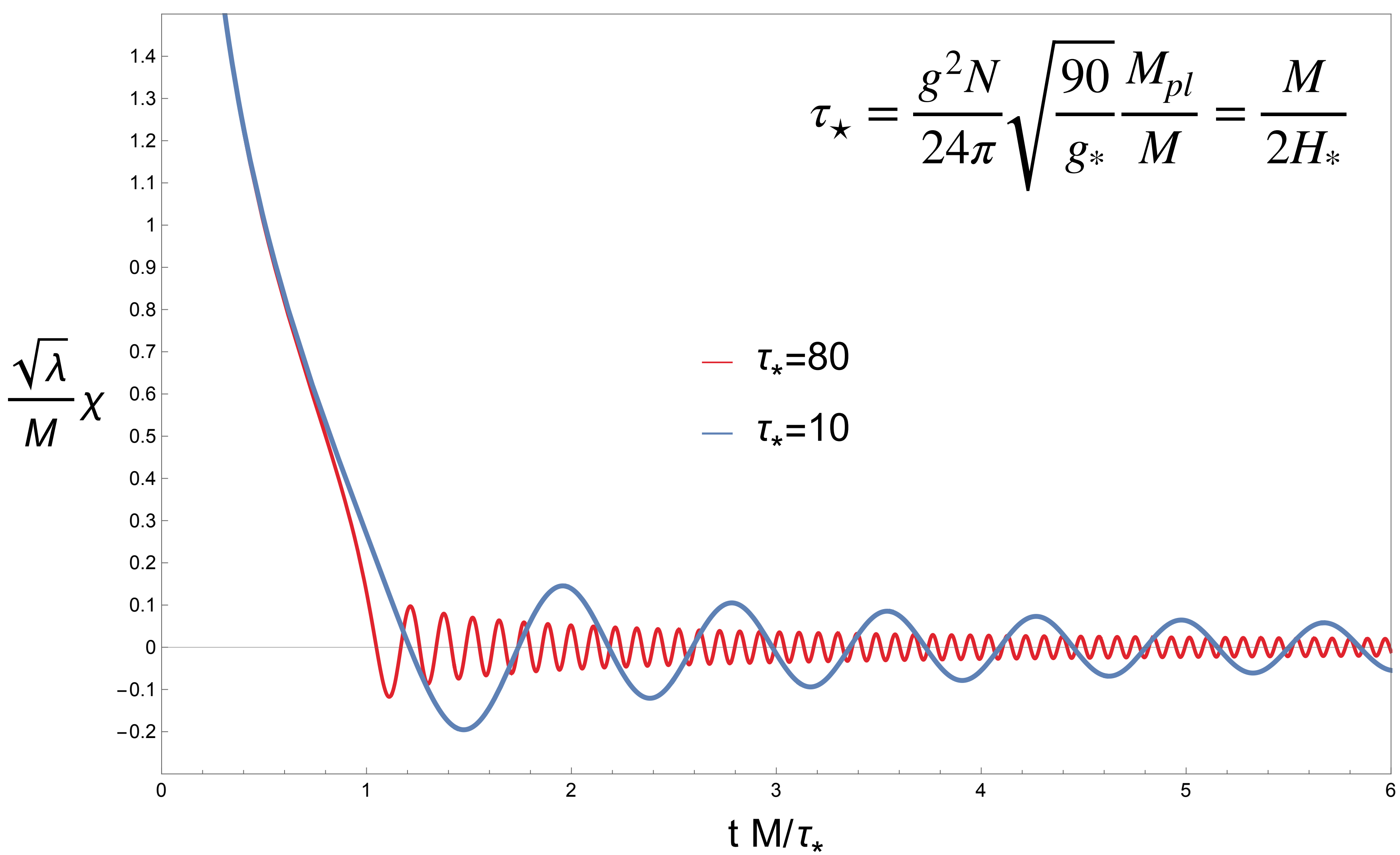
Dynamics only depends on one single free dimensionless parameter

$$\ddot{\chi} + 3H\dot{\chi} + \left(M^2 - \frac{g^2 N T^2}{12}\right)\chi + \lambda\chi^3 = 0 \quad \textit{with} \quad H = \frac{1}{2t} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{T^2}{M_{pl}}$$

$$\frac{1}{\tau_\star^2}\left(\bar{\chi}'' + \frac{3}{2}\frac{\bar{\chi}'}{\tau}\right) + \left(1 - \frac{1}{\tau}\right)\bar{\chi} + \bar{\chi}^3 = 0$$

cf. WKB

$$\tau_\star = \frac{g^2 N}{24\pi}\sqrt{\frac{90}{g_*}}\frac{M_{pl}}{M} = \frac{M}{2H_*}$$



Assume Whole DM is in χ

equality time $$\rho_\chi = \varepsilon_{rad}\left(T_{eq}\right) = \frac{\pi^2 g_*\left(T_{eq}\right)}{30} T_{eq}^4$$

from entropy conservation $$sa^3 = const$$ where $$s = \frac{2\pi^2 g_*(T)\,T^3}{45}$$

$$\left(\frac{a_*}{a_{eq}}\right)^3 = \frac{g_*\left(T_{eq}\right) T_{eq}^3}{g_*(T_*)\,T_*^3}$$

which one uses in

$$\rho_\chi = \frac{\left(4M^{10}H_*^2\right)^{1/3}}{4\lambda}\left(\frac{a_*}{a_{eq}}\right)^3 = \varepsilon_{rad}\left(T_{eq}\right)$$

to obtain M

Mass of DM

$$M = \frac{\lambda^{3/5}}{g}\sqrt{\frac{12}{N}}\left(\frac{\pi^4 g_*^2(T_*)}{75}\left(\frac{M_{pl}}{T_{eq}}\right)^2\right)^{1/5} T_{eq}$$

Recalling that $T_{eq} \simeq 0.8\,\mathrm{eV}$

$$M \simeq 10^{-13}\,\mathrm{eV}\,\frac{\beta^{3/5}}{\sqrt{N}}\cdot\left(\frac{g_*(T_*)}{100}\right)^{2/5}\cdot\left(\frac{g}{10^{-18}}\right)^{7/5}$$

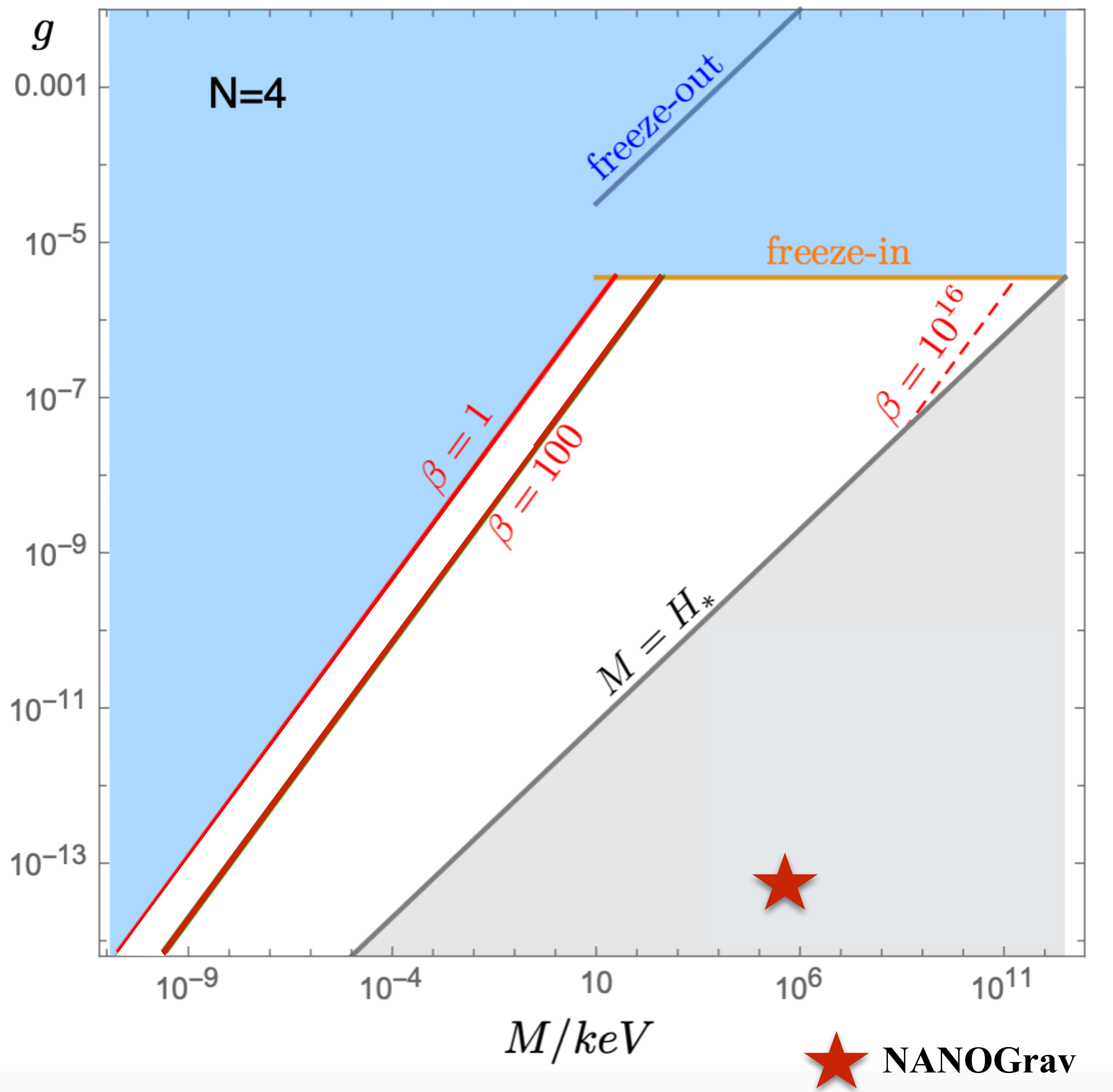
$$\beta = \frac{\lambda}{g^4} \geq \frac{1}{\lambda_\phi} \geq 1$$

potential bounded from below

weak coupling

Allowed Parameter Space

$$M \simeq 10^{-13}\,\text{eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g_*(T_*)}{100}\right)^{2/5} \cdot \left(\frac{g}{10^{-18}}\right)^{7/5}$$



Thanks a lot for attention!