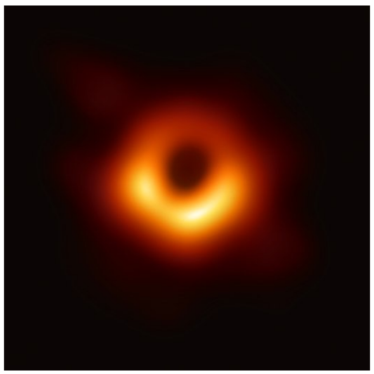
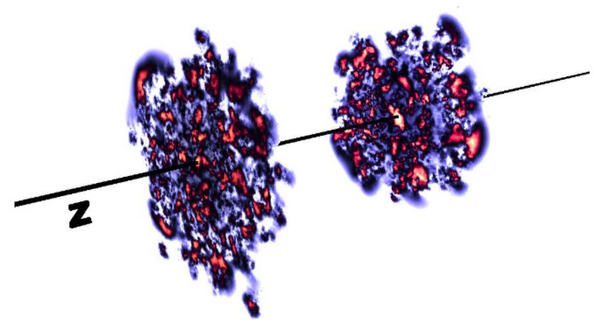
Universal features of 2→ N scattering in QCD and gravity from shockwave collisions-II



10^9 km ⟷ 10^{-19} km



Collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

$M_{BH}=(6.5 \pm 0.2_{stat} \pm 0.7_{sys}) \times 10^9\, M_\odot$
at center of Messier 87
Event Horizon Telescope image of photon ring

Raju Venugopalan

Brookhaven National Laboratory

CFNS, Stony Brook University

Work with Himanshu Raj: arXiv:2311.03463, 2312.03507, 2312.11652 and 2406.10483

Zakopane Summer School, June 15-22, 2024

Double Copy: gluon → gravitational radiation in shockwave collisions

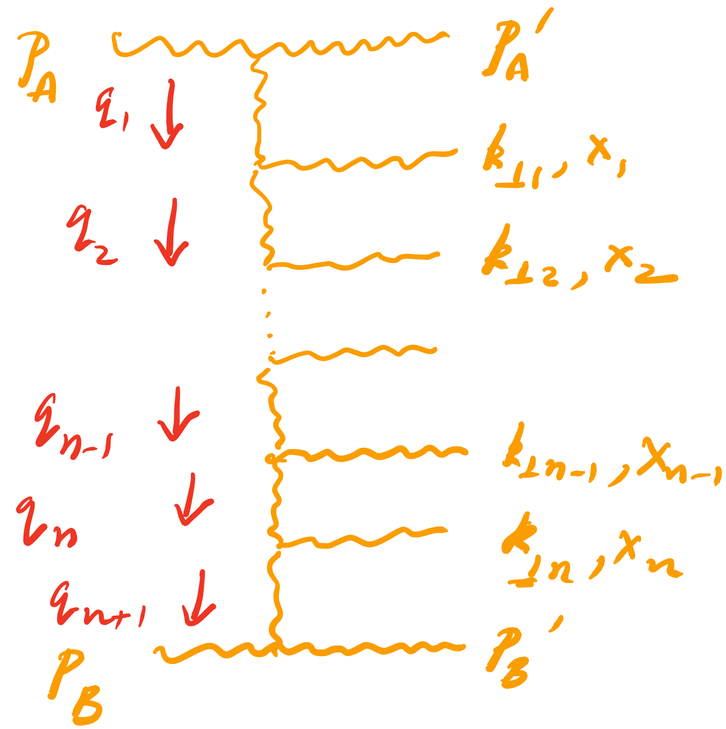
$$
\begin{tikzcd}
\text{QCD at high occupancy} \arrow[d, "\text{Strong field semi-classical double copy}"'] & \text{perturbative QCD} \arrow[l] \arrow[d, "\text{BCJ double copy}"] \\
\text{Gravity at high occupancy} & \text{perturbative gravity} \arrow[l]
\end{tikzcd}
$$

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

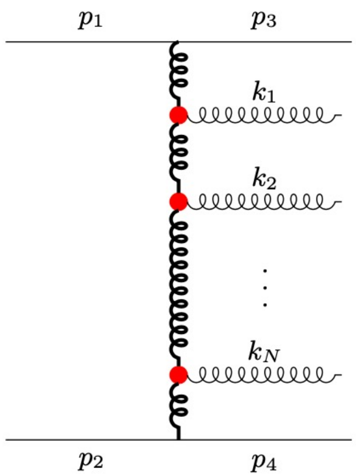
S-matrix picture of 2 → N scattering in GR at trans-Planckian energies

t'Hooft, Gross-Mende, Verlinde[2] ,...



What is the role of "wee*" gravitons in trans-Planckian scattering in gravity?

BFKL: 2→ N QCD amplitudes in Regge asymptotics*



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(``slow") but hard in transverse momentum – weak coupling Regge regime

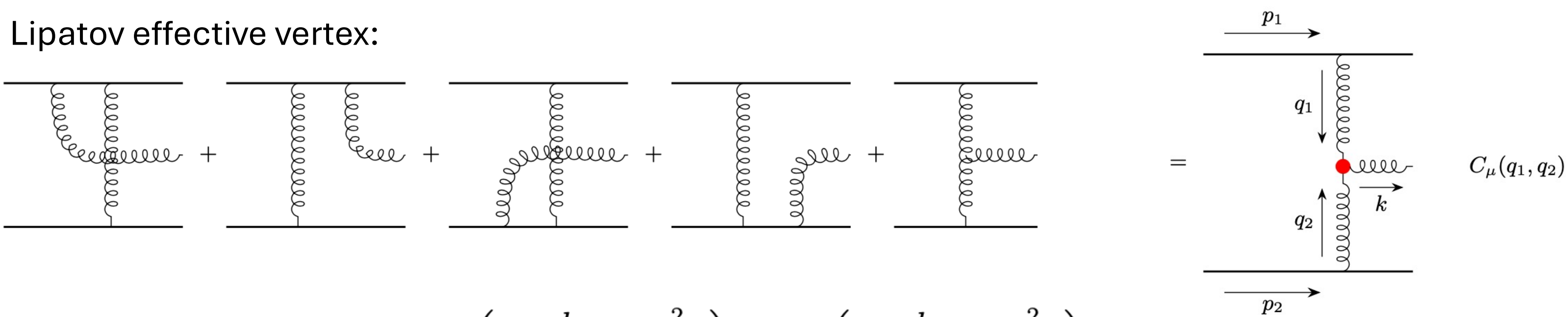
RG description rapidity of evolution given by the BFKL Hamiltonian
Very rapid growth of the amplitude with energy

A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

* Asymptotics is the calculus of approximations. It is used to solve hard problems that cannot be solved exactly and to provide simpler forms of complicated results

BFKL: Building blocks

Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu \, C^\mu$ =0

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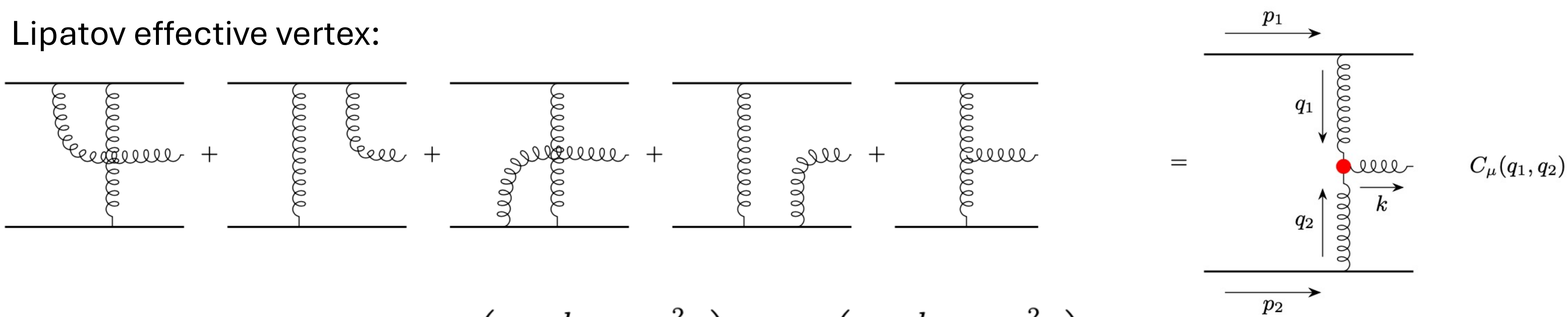

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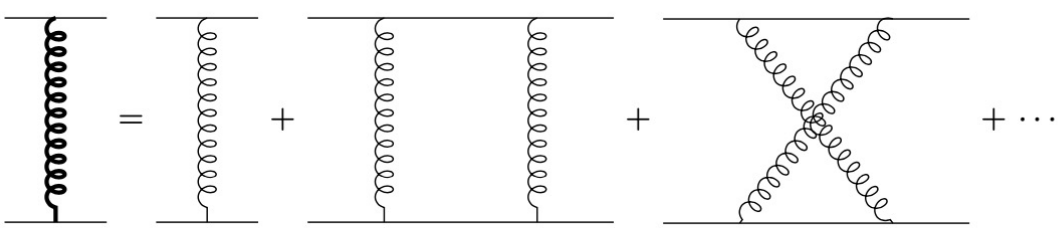
Reggeized gluon:

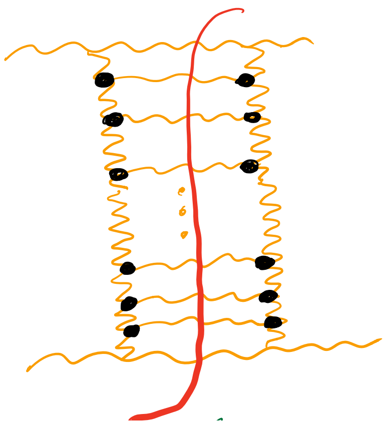


$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c\, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q} - \boldsymbol{k})^2}\ , \qquad t = -\boldsymbol{q}^2$$

$2 \to \mathrm{N} + 2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



$$\mathrm{Im}\, A(s,t) \propto \sum_{n=0}^{\infty} (\alpha_s C_T)^{n+2}$$

C_T is color factor

$$\times \int \prod_{i=1}^{n} \frac{dy_i}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{j\perp}}{(2\pi)^2}$$

Phase space factors

$$\times\, 2is \prod_{\ell=1}^{n+1} \frac{1}{t_\ell t_\ell'} e^{(y_{\ell-1}-y_\ell)[\alpha(t_\ell)+\alpha(t_\ell')]}$$

Reggeized propagators on both sides of cut

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

Product of Lipatov vertices

$$\sigma_{tot} = 2\,\mathrm{Im}\, A(s, t=0)$$

$$= s^{\lambda} \text{ with } \lambda = \frac{4\alpha_s N_c \ln 2}{\pi}$$

$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

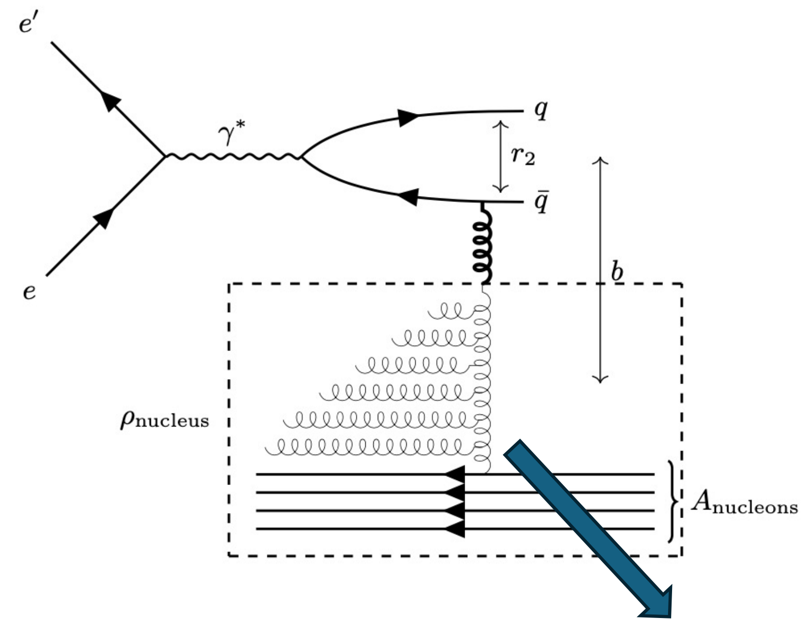
Real and virtual corrections combine to cancel infrared divergence !

Strongly violates Froissart bound

Resummed NLO BFKL : $\lambda \approx 0.3$

s-channel picture of classicalization and unitarization of cross-sections

Dipole scattering off an
overoccuped hadron/nucleus configuration



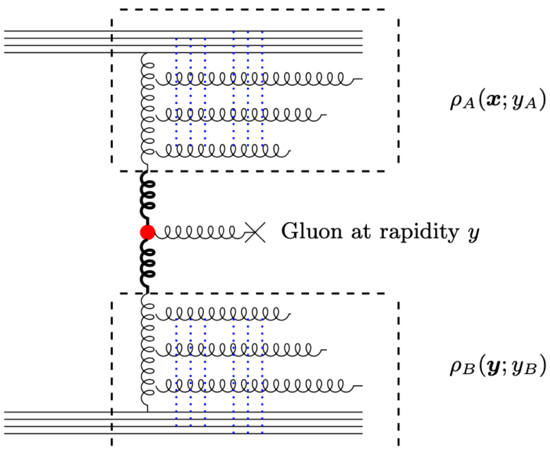
CGC EFT:
Powerful functional RG describes nonlinear (multi-Pomeron) evolution with rapidity
– at NLLx accuracy for multiple final states

Dense close-packed ($1/Q_S$) classical lump- gluon "shockwave"

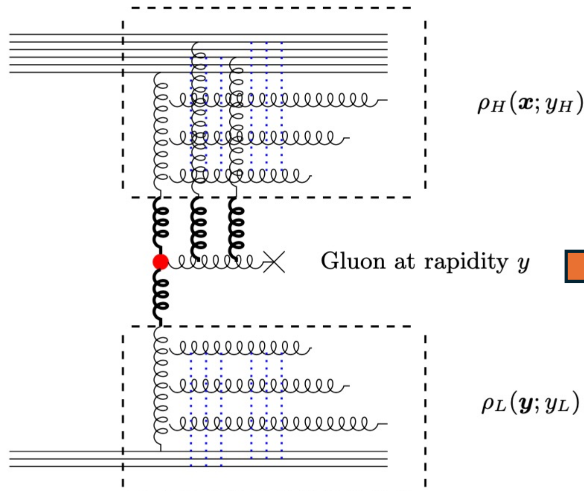
Reggeized gluon as field sourced
by lump's color charge density

CGC review: Gelis,Iancu,Jalilian-Marian, RV:arXiv 1002.0333

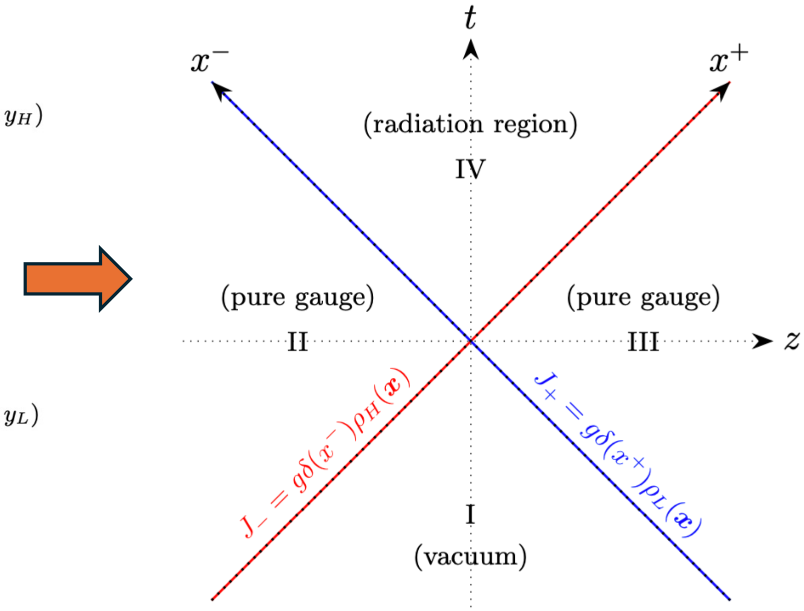
Gluon shockwave collisions: Lipatov vertex and reggeization



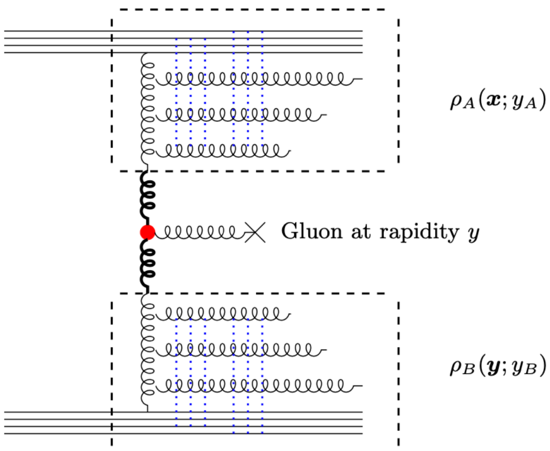
"dilute-dilute" collisions
($\frac{\rho_A}{\Delta_\perp^2}, \frac{\rho_B}{\Delta_\perp^2} \ll 1$)

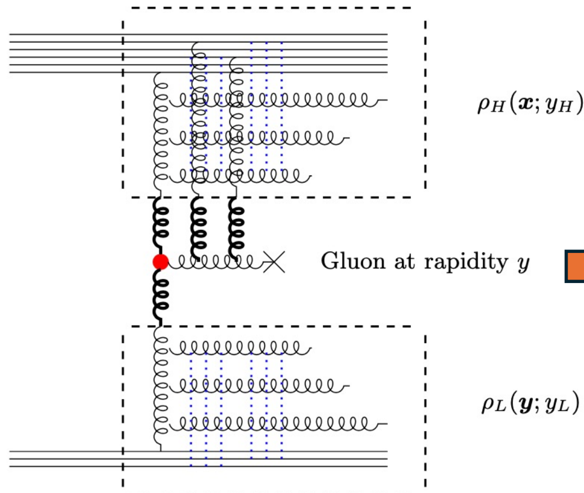


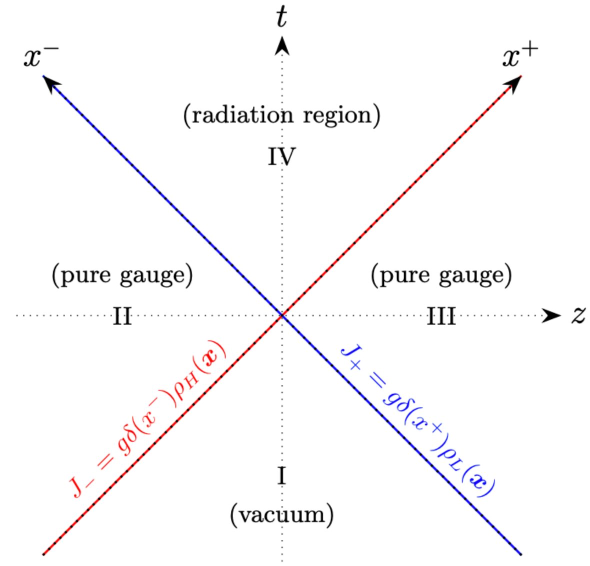
"dilute-dense" collisions
($\frac{\rho_L}{\Delta_\perp^2} << 1$, $\frac{\rho_H}{\Delta_\perp^2} \sim O(1)$)



Gluon shockwave collisions: Lipatov vertex and reggeization





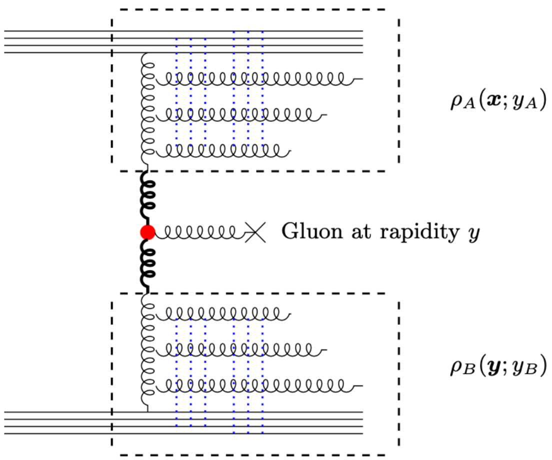


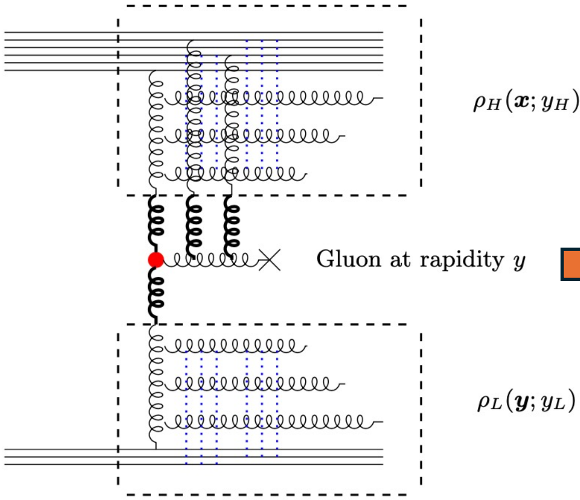
Weizsäcker-Williams gluon radiation field in light cone gauge

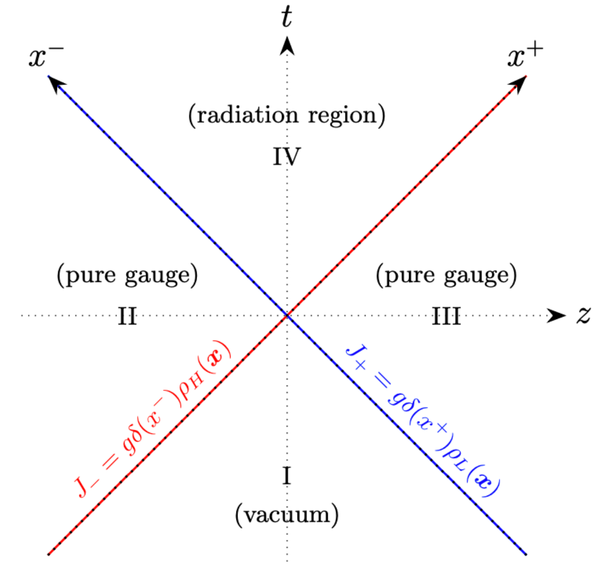
$$a_i(k) = -\frac{2ig}{k^2 + i\epsilon} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \left(q_{2i} - k_i \frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2} \right) \frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \left(U(\boldsymbol{k} + \boldsymbol{q}_2) - (2\pi)^2 \delta^2(\boldsymbol{k} + \boldsymbol{q}_2) \right)$$

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

Gluon shockwave collisions: Lipatov vertex and reggeization







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Lipatov vertex
in $A^- = 0$ gauge

$$U(x^-, \boldsymbol{x})\delta(x^+) = \exp\left(ig \int_{-\infty}^{x^-} dz^- \bar{A}_-(z^-, \boldsymbol{x}) \cdot T\right)$$

$$\bar{A}_\mu(x^-, \boldsymbol{x}) = -g\delta_{\mu -}\delta(x^-) \frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

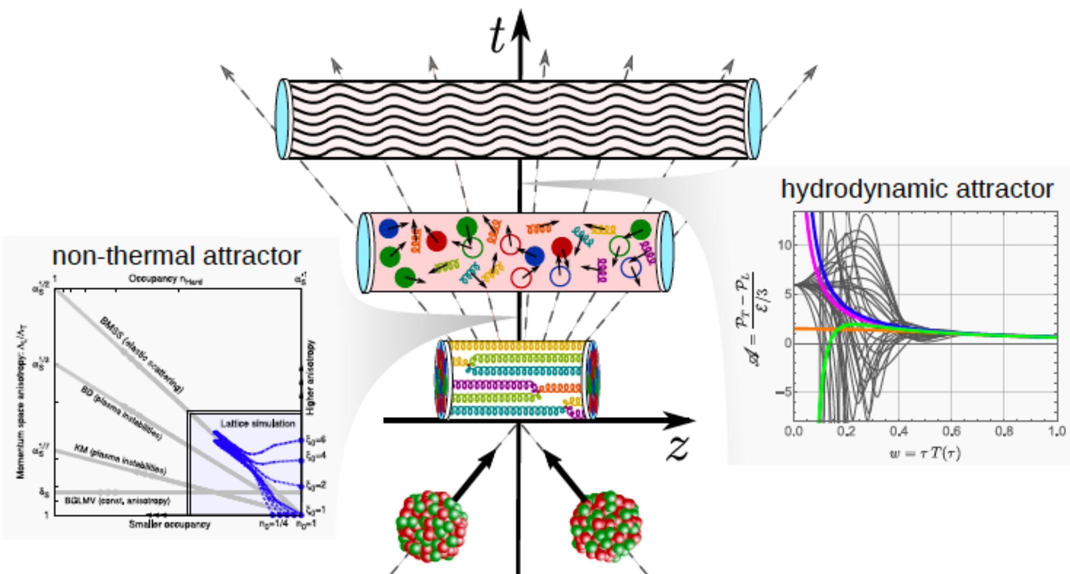
Ln (U) → *reggeized gluon*

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Dense-dense shockwave collisions: heavy-ion collisions

Quark-Gluon Plasma undergoing
hydrodynamic expansion



Collision of overoccupied Color Glass Condensate shockwaves

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

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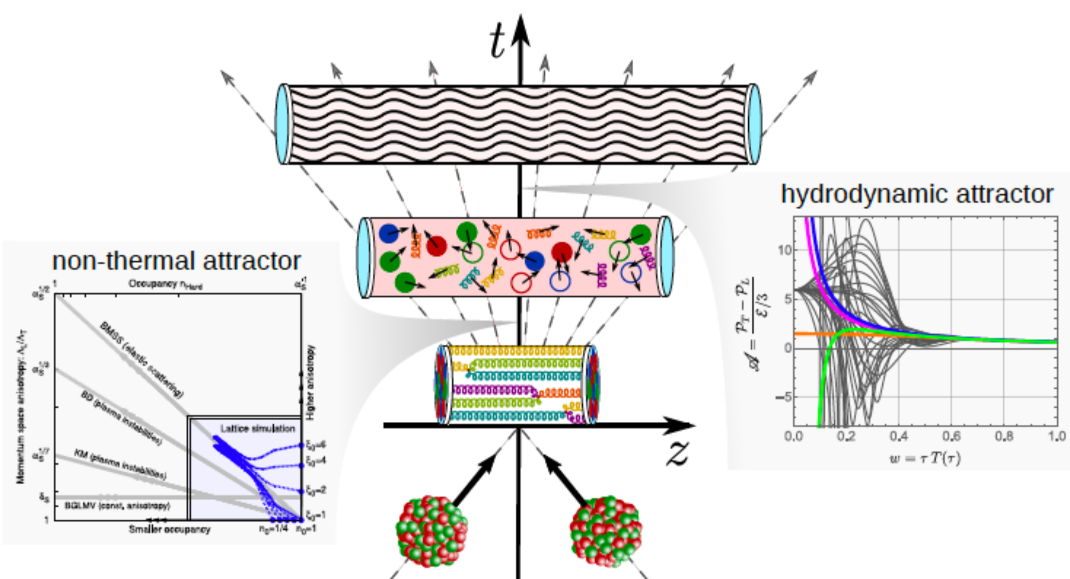

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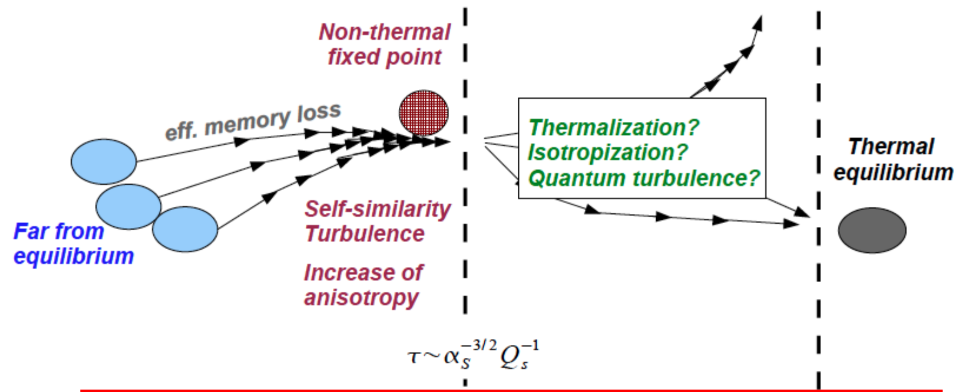

Thermal soft gluon bath for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

Very rapid thermalization
as $\alpha_S(Q_S) \rightarrow 0$ *and* $Q_S \rightarrow \infty$

Baier,Mueller,Schiff,Son,
hep-ph/0009237

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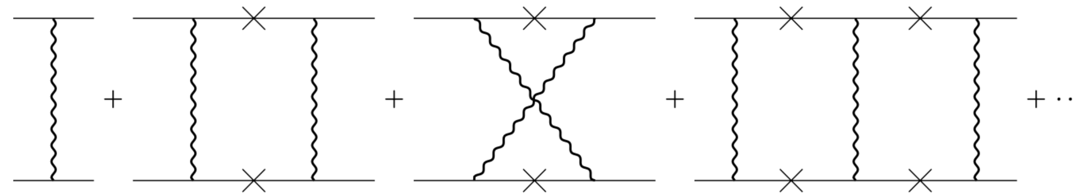
Multiparticle production and saturation in gravity: from amplitudes to shockwave collisions

An analogous program can be followed for 2→N scattering in gravity with remarkable quantitative double copy relations emerging at every step…

From QCD to gravity in Regge asymptotics: reggeization

In Einstein gravity, at large impact parameters,
the dominant contribution is eikonal scattering



$$i\mathcal{M}_{\mathrm{Eik}} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \qquad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2} \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

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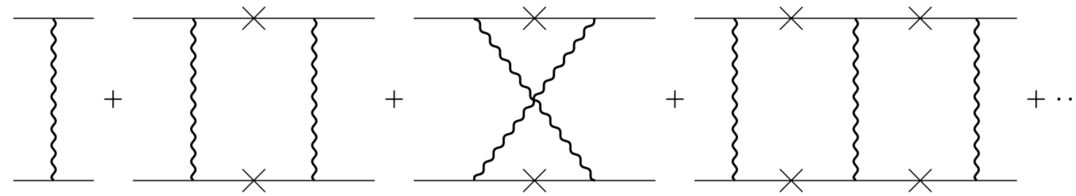

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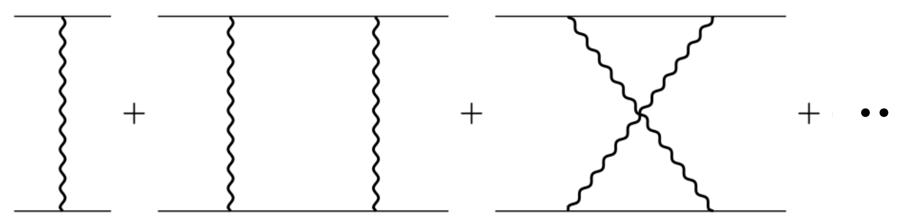
Genuine loop contributions formally suppressed by

$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(\underbrace{-i\pi s\log\left(\frac{-t}{\Lambda^2}\right)}_{\text{Eikonal}} + \underbrace{t\log\left(\frac{s}{-t}\right)\log\left(\frac{-t}{\Lambda^2}\right)}_{\text{Loop}}\right)$$

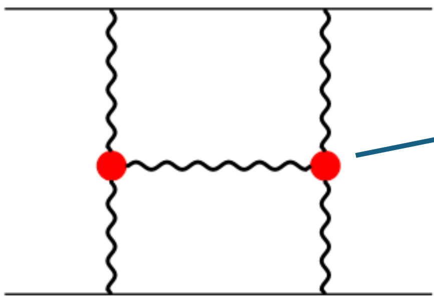


Graviton Regge trajectory $\alpha(t) = -\kappa^2 t\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2\left(\frac{1}{\boldsymbol{k}^2}+\frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2}\right)-\boldsymbol{q}^2\right], \qquad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitation Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \underbrace{\frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QCD Lipatov vertex}} - \underbrace{\frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)}_{\text{Double copy of QED Bremsstrahlung vertex}}$$

$$N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2\boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}\right)$$

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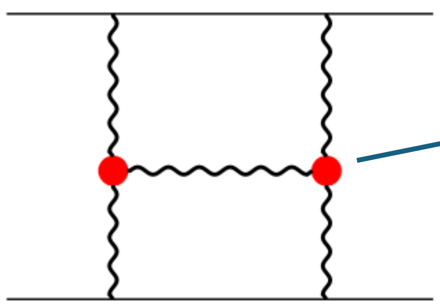

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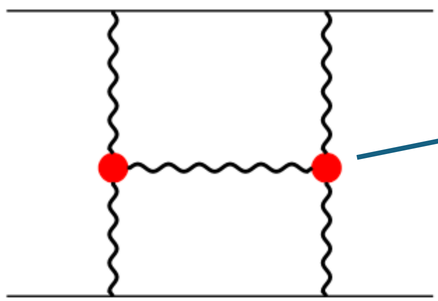
In amplitudes language, extra terms in double copy are imposed by so-called Steinmann relations
- required by unitarity to cancel spurious poles of energy variables ($s_1 = (k+p_1)^2$ and $s_2 = (k+p_2)^2$) in overlapping channels

Eg. Sabio Vera, Campillo, Vasquez-Mozo, 1112.4494

From QCD to gravity in Regge asymptotics: Lipatov vertex



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S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0+\delta_1+\delta_2+\cdots)}$$

$$\delta_0 = Gs\log\left(\frac{L}{b}\right), \quad \delta_1 = \frac{6G^2s}{\pi b^2}\log s, \quad \delta_2 = \frac{2G^3s^2}{b^2}\left[1+\frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2}+2\right)\right]$$

Leading Eikonal term (real)

Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$
- includes absorptive piece

$$\boxed{\delta_2 \gg \delta_1 \text{ for } R_S \gg l_P}$$

Shockwave collisions in general relativity

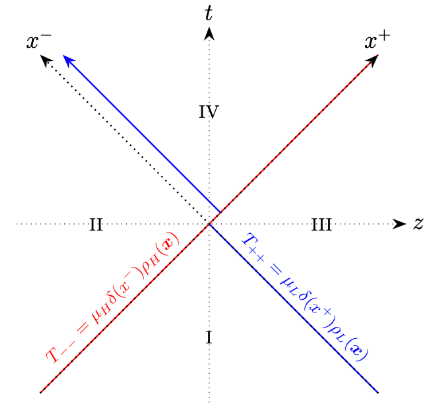
Aichelburg-Sexl shockwave metric of shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\left(dx^-\right)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi} \mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\ \ln \Lambda |\boldsymbol{x} - \boldsymbol{y}| \rho_H(\boldsymbol{y})$

Soln of Einstein's eqns sourced by the EM tensor

$$T_{\mu\nu} = \delta_{\mu -}\delta_{\nu -}\mu_H \delta(x^-)\rho_H(\boldsymbol{x})$$



$\mu_H = m_H \gamma$ = fixed for $\gamma \to \infty$

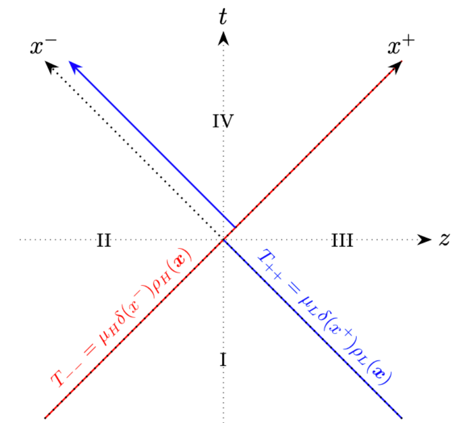
$\kappa^2 = 8\pi G$

Shockwave collisions in general relativity: single shock background

Aichelburg-Sexl shockwave metric of a shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\left(dx^-\right)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-)\dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi}\mu_H\delta(x^-)\displaystyle\int d^2\boldsymbol{y}\ \ln\Lambda|\boldsymbol{x}-\boldsymbol{y}|\rho_H(\boldsymbol{y})$



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

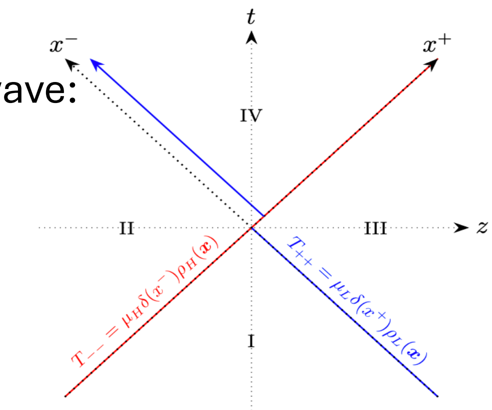
Fix light cone gauge $h_{\mu+}$=0. Find solution: $h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x})h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\dfrac{1}{2}\displaystyle\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\,\partial_+\right)$

Exactly analogous to the QCD case
with $A_- \to\ g_{--}$ and $T^a \to \partial_+$

Melville, Nachulich, Schnitzer, White,
arXiv:1306.6019

Shockwave collisions in general relativity: dilute-dilute approximation



Now consider the interaction of the “dilute” source ρ_L with the dense ρ_H shockwave:

$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H\delta\left(x^-\right)\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L\delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H\delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

We decompose the perturbation $h_{\mu\nu}$ into a term linear in ρ_L and one bi-linear in $\rho_L\rho_H$

Linearized Einstein’s equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2\tilde{h}_{ij} - \Box\tilde{h}_{ij} = \kappa^2\left[\left(2\partial_i\partial_j - \Box_\perp\delta_{ij}\right)\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}\left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right)\right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij}h \text{ where } h = \delta_{ij}h_{ij}$$

Shockwave collisions in general relativity: geodesics

Unlike the QCD case, the sub-Eikonal contributions T_{+i}, T_{ij} required for consistency of eqns of motion

Notot uniquely fixed by energy-momentum conservation--the dynamics of sources is needed.
In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-\bar{g}}} \int_{-\infty}^{\infty} d\lambda \; \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

The solution of the corresponding null geodesic equations $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \, , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

In the shockwave background, given by $X^- = \lambda \, , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

From the geodesic solutions, we can reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtain

Gravitational radiational field

$$\boxed{\tilde{h}_{ij}^{(2)}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

Gravitational Lipatov vertex

recovering Lipatov's result $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

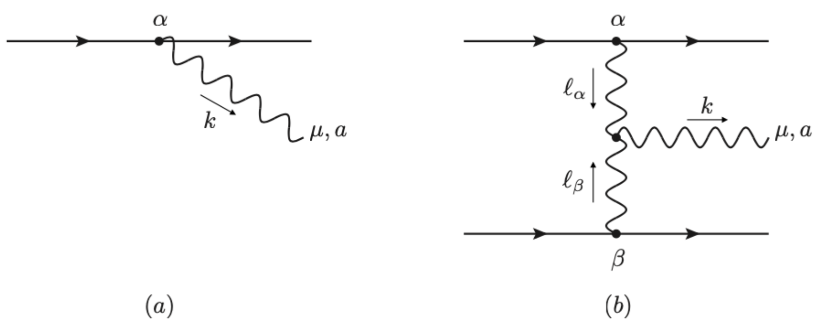
Compare to gauge theory radiation field

$$\boxed{a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

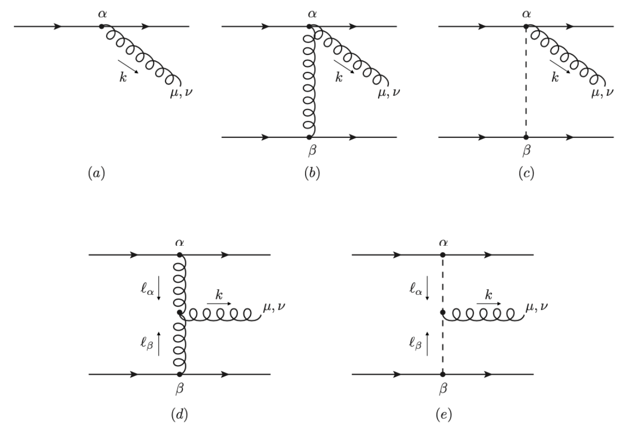
$$-if^{abc} T_b T_c C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \xleftrightarrow[\text{CK relation?}]{\text{Is there a}} s\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2)$$

H.Johansson, A.Sabio Vera, E.Serna Campillo, and M.Vaszquez-Mozo, JHEP10,215(2013),arXiv:1307.3106 [hep-th]

Classical color-kinematic duality



From Goldberger, Ridgway
arXiv:1611.03493



A color-kinematic duality exists but requires inclusion of sub-eikonal corrections to the Lipatov vertex

For this, require a detailed theory of sources: Yang-Mills+Wong equations for classical color sources c^a:

$$D_\mu F_a^{\mu\nu} = g J_a^\nu \qquad J_a^\mu(x) = \sum_{\alpha=1,2} \int d\tau\, c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d\left(x - x_\alpha(\tau)\right)$$

$$\frac{dc^a}{d\tau} = g f^{abc} v^\mu A_\mu^b(x(\tau)) c^c(\tau) \qquad \frac{dp^\mu}{d\tau} = g c^a F_{a\nu}^\mu v^\nu$$

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+c_1\cdot c_2\left\{\frac{q_1^2 c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2 c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\qquad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Classical color-kinematic replacement rule:

$$c_\alpha^a \to p_\alpha^\mu\,,$$

$$if^{a_1a_2a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2}+\eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3}+\eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right)$$

$$g \to \kappa\,,$$

Gluon 3-pt vertex with f^{abc} stripped off

Classical color-kinematic duality

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+\,c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\quad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Performing the substitution, one finds the result we obtained by direct computation!

$$A^{\mu\nu}(k) = \frac{\kappa^3 s}{2\,k^2}\int\frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\frac{1}{2}\left[C^\mu C^\nu - N^\mu N^\nu + \cancel{k^\mu\left(\frac{p_1^\nu}{p_1\cdot k}\boldsymbol{q}_1^2+\frac{p_2^\nu}{p_2\cdot k}\boldsymbol{q}_2^2\right)}\right]$$

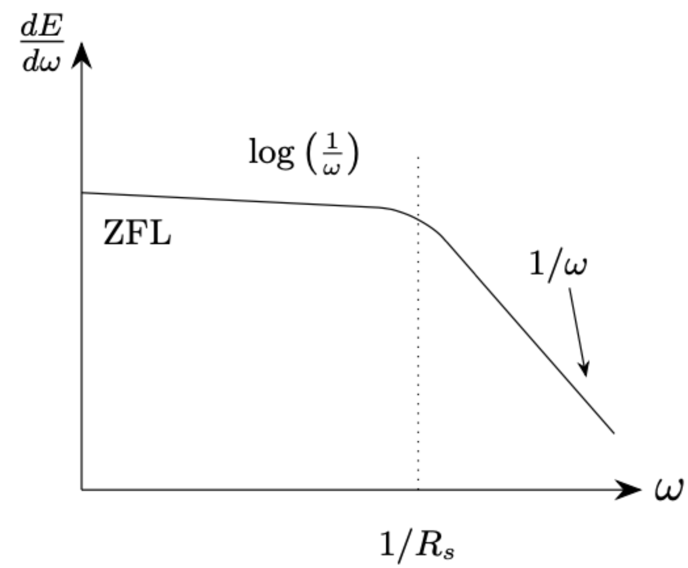
Unphysical – drops out when contracted with the gravitational polarization tensor

Next steps...

Compute WW spectrum in GR for dilute-dilute and dilute-dense cases

$$\frac{dE^{\mathrm{GW}}}{d\omega d\Omega} = \frac{1}{2\pi^2}\omega^2 \sum_{\lambda} \left| \mathscr{M}^{(\lambda)} \right|^2 \qquad \mathscr{M}^{(\lambda)} = k^2 \tilde{h}_{ij}^{(2)}(k)\varepsilon_{ij}^{(\lambda)}$$

Eg. Gruzinov, Veneziano, arXiv:1409.4555
Ciafaloni,Colferai,Coradeschi,Veneziano, arXiv:1512.00281



What is the shape of the spectrum as the compact objects get close. Solution of the Lipatov equation will tell us the spectrum as a function of impact parameter and rapidity

Amazingly, this has not been done...

Next steps...

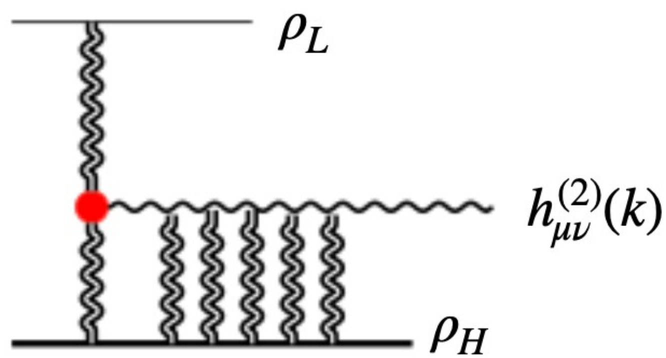
Compute shockwave propagators
(graviton-reggeized graviton-graviton)

Raj, RV, arXiv:2406.10483

Remarkably, they satisfy double-copy relations to the QCD shock wave propagators

Next steps…

Extend analysis to the dilute-dense case to compute coherent multi-gravi-reggeon contributions to the radiation spectrum

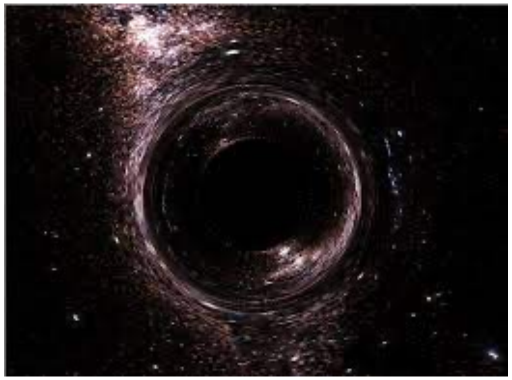


Next steps…

Just as in the QCD case, can we derive an RG ?
(ingredients are Wightman propagators and one loop corrections
to the radiation field in the shockwave background)

Far more difficult because one has to worry about significant modifications
to the geodesic trajectories – Raychaudhuri equations

Is there a non-trivial fixed point that corresponds to black-hole formation
at high occupancy ?



http://spiro.fisica.unipd.it/~antonell/schwarzschild/