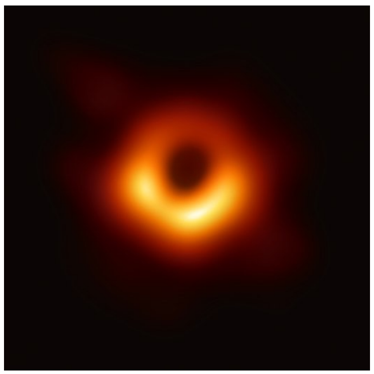
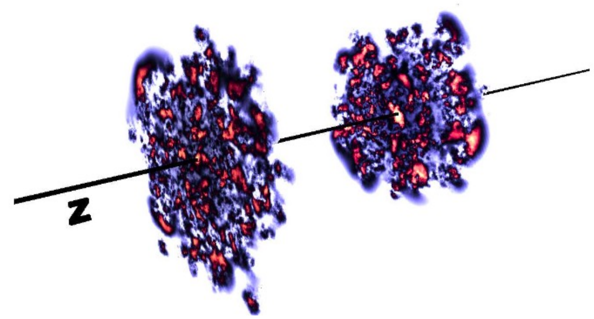
Universal features of 2→ N scattering in QCD and gravity from shockwave collisions-I



10^9 km ⟷ 10^{-19} km



Collisions of Color Glass Condensate gluon states in nuclei, arXiv:1206.6805

$M_{BH}=(6.5 \pm 0.2_{stat} \pm 0.7_{sys}) \times 10^9\, M_\odot$
at center of Messier 87
Event Horizon Telescope image of photon ring

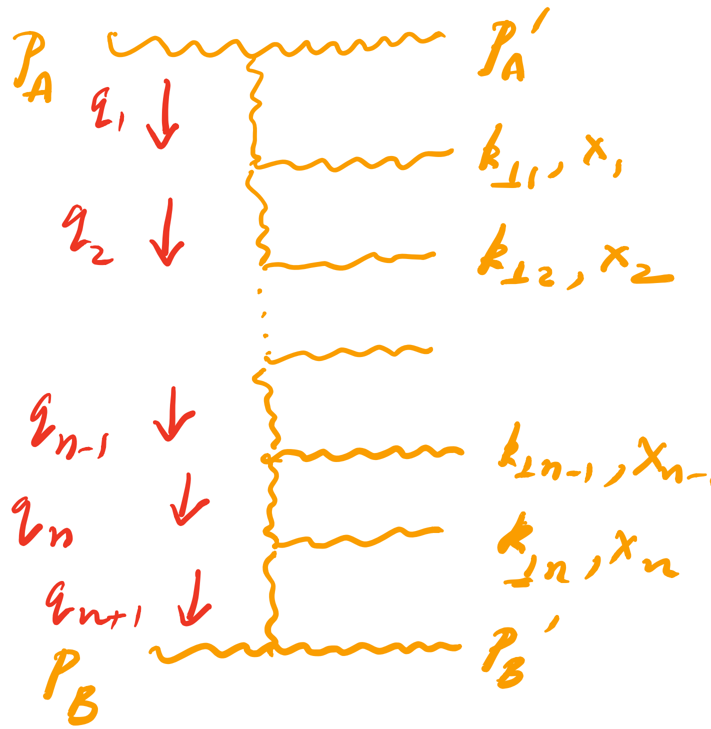
Raju Venugopalan
Brookhaven National Laboratory
CFNS, Stony Brook University

Zakopane Summer School, June 15-22, 2024

Based in part on recent work (arXiv:2311.03463, 2312.03507, 2312.11652 and 2406.10483) with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

S-matrix picture of 2 → N scattering in GR at trans-Planckian energies

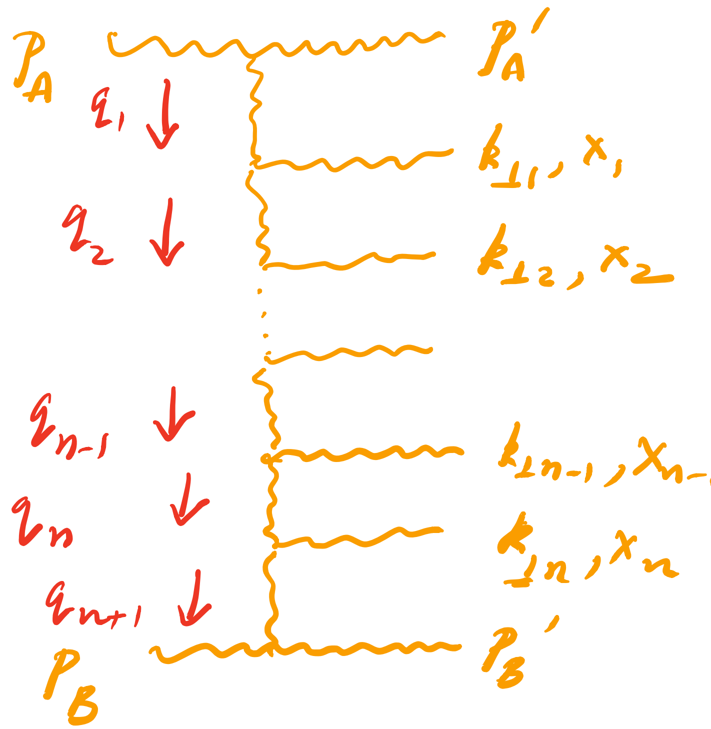
t'Hooft, Gross-Mende, Verlinde[2] ,...



What is the role of "wee*" gravitons in trans-Planckian scattering in gravity?

S-matrix picture of 2 → N scattering in GR at trans-Planckian energies

t'Hooft, Gross-Mende, Verlinde[2] ,…

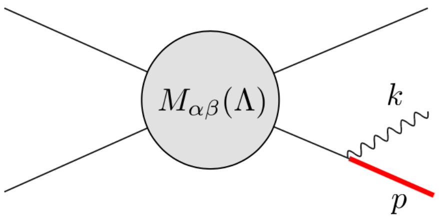


*Wee partons - a term coined by Richard Feynman to describe quarks and gluons carrying a very small fraction ($x \ll 1$) of a high energy hadron's momentum

Wee gravitons are not soft gravitons

Soft theorem's: exponentiation of clouds of soft photons and gravitons

Low's soft photon theorem:



$$\lim_{k\to 0} M^{\mu}_{\alpha\beta}(k,\Lambda) = \lim_{k\to 0}\left\{\frac{ep^{\mu}}{(p+k)^2 - m^2 - i\epsilon}\right\} M_{\alpha\beta}(\Lambda) = \lim_{k\to 0}\left\{\frac{ep^{\mu}}{p\cdot k - i\epsilon}\right\} M_{\alpha\beta}(\Lambda)$$

Because $p^2 - m^2 = 0$

Soft photon exponentiation $$\mathcal{S}^{(2)}_{fi} \simeq \exp\left\{-i\sum_{n,m=1}^{2} S^{nm}_{\mathrm{IR}}\right\}\mathcal{S}^{(2)}_{fi,\mathrm{UV}}$$

$$S^{nm}_{\mathrm{IR}} \simeq \frac{g^2}{8\pi^2}\sum_{n',m'}\eta^{n'}\eta^{m'}\gamma^{n'm'}\coth\gamma^{n'm'}\log\frac{\Lambda}{\lambda} - i\frac{g^2}{8\pi}\sum_{n',m'}{}'\,\eta^{n'}\eta^{m'}\coth\gamma^{n'm'}\log\frac{\Lambda}{\lambda}$$

with $\eta' = \pm 1$ and cusp angle defined in terms of scalar product of velocities $$\cosh\gamma^{nm} := \frac{\beta^n\cdot\beta^m}{\sqrt{(\beta^n)^2(\beta^m)^2}}$$

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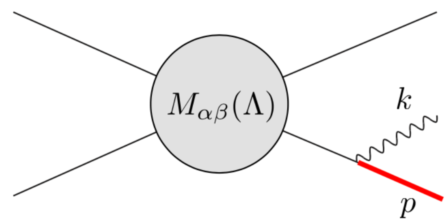
In gravity, Low's theorem gives



$$\lim_{k\to 0} M^{\mu\nu}_{\alpha\beta} = \lim_{k\to 0}\sqrt{8\pi G}\,\frac{p^\mu p^\nu}{p\cdot k - i\epsilon}\; M_{\alpha\beta}(\Lambda)$$

Weinberg's soft graviton theorem:

$$\mathcal{S}^{(2)}_{fi} \simeq \exp\left\{-i\sum_{n,m=1}^{2} S^{nm}_{\rm IR}\right\}\mathcal{S}^{(2)}_{fi,{\rm UV}}$$

$$S^{nm}_{\rm IR} \simeq \frac{GM^2}{2\pi}\gamma\coth\gamma\,\frac{1+\tanh^2\gamma}{\sqrt{(1-\tanh^2\gamma}}\,{\rm Ln}\left(\frac{\Lambda}{\lambda}\right)$$

with $\eta' = \pm 1$

and cusp angle defined in terms of scalar product of velocities

$$\cosh\gamma^{nm} := \frac{\beta^n\cdot\beta^m}{\sqrt{(\beta^n)^2(\beta^m)^2}}$$

Wee gravitons are part of the "hard" matrix element with $k_\perp \geq \Lambda$

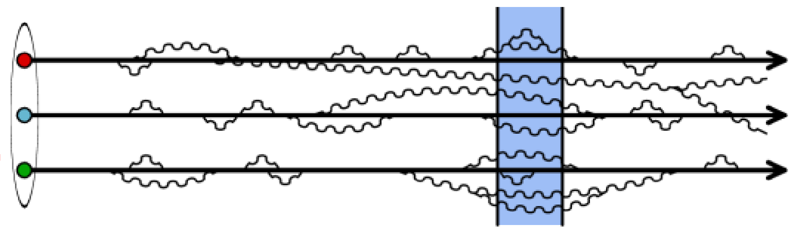
Gluon bremsstrahlung: ` ` Herculean task…might even not be possible”

After magisterially disposing of the problem of IR divergences of soft photons and gravitons, Weinberg notes…

Weinberg, Phys. Rev. 140 (1965) B516

Weinberg

But these remarks do not apply to theories involving charged massless particles. In such theories (including the Yang-Mills theory) a soft photon emitted from an external line can itself emit a pair of soft charged massless particles, which themselves emit soft photons, and so on, building up a cascade of soft massless particles each of which contributes an infrared divergence. The elimination of such complicated interlocking infrared divergences would certainly be a Herculean task, and might even not be possible.



We may be thankful that the zero charge of soft photons and the zero gravitational mass of soft gravitons saves the real world from this mess. Perhaps it would not be too much to suggest that it is the infrared divergences that prohibit the existence of Yang-Mills quanta or other charged massless particles.

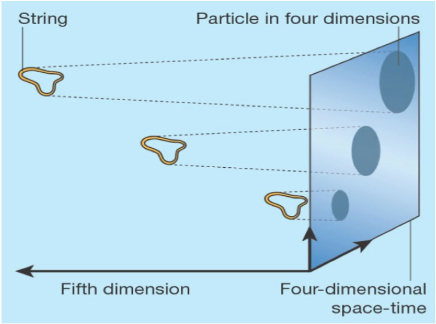
8 years prior to asymptotic freedom!

Gauge-Gravity correspondence

The classic one is the AdS/CFT correspondence

Clean dictionary to derive results in a strongly coupled gauge theory some of which may be universal (a candidate being the KSS bound on $\frac{\eta}{s}$)

Our universe is de Sitter and N=4 SUSY YM is not QCD



J.Maldacena, Nature 2003

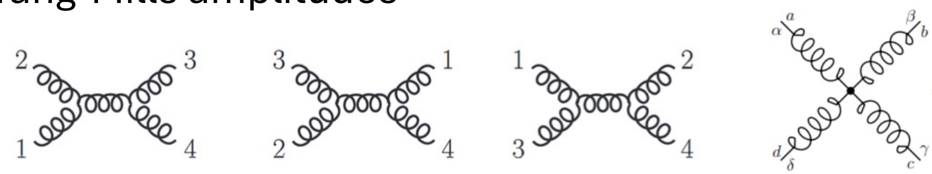
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy” limit between Einstein & Yang-Mills amplitudes

$$M_4^{\text{tree}}(1,2,3,4) = \left(\frac{\kappa}{2}\right)^2 s\, A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)$$

$$\kappa = 32\,\pi^2\, G_N$$



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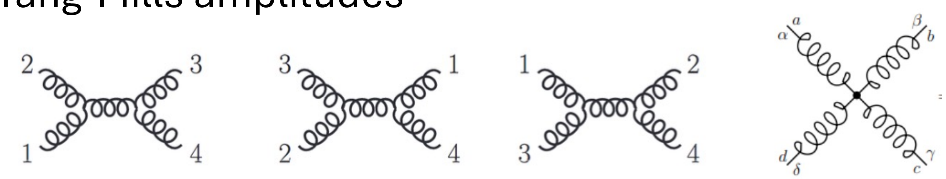


Remarkable "BCJ" color-kinematics duality

Bern, Carrasco,Johansson,arXiv:0805.3993

Tree level $gg \to gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\rm tree} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

with the s channel color factor $c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic factor $n_s = -\frac{1}{2}\Big\{\Big[(\epsilon_1.\epsilon_2)p_1^\mu + 2(\epsilon_1.p_2)\epsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\epsilon_3.\epsilon_4)p_3^\mu + 2(\epsilon_3.p_4)\epsilon_4^\mu - (3\leftrightarrow 4)\Big] + s\Big[(\epsilon_1.\epsilon_3)(\epsilon_2.\epsilon_4) - (\epsilon_1.\epsilon_4)(\epsilon_2.\epsilon_3)\Big]\Big\}$

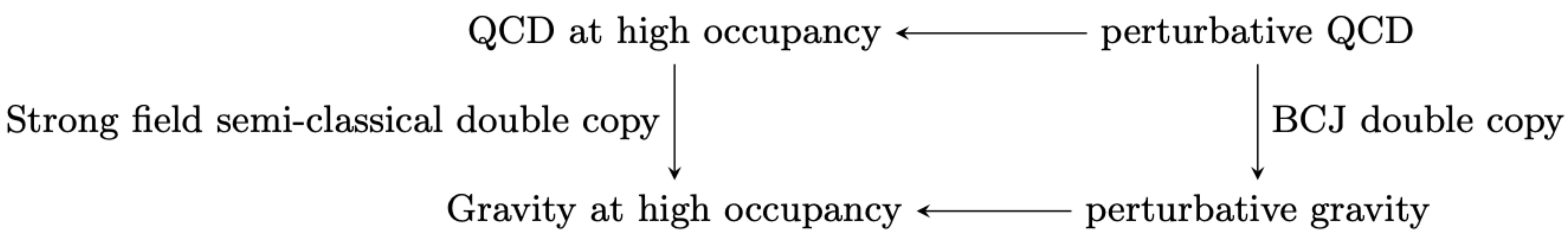
Tree level gravity amplitude obtained by replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\rm tree}|_{c_i\to n_i, g\to\kappa/2} = i\mathcal{M}_4^{\rm tree} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

Significant on-going work on extension to loop amplitudes

Review: Bern et al., arXiv: 1909.01358

Double Copy: gluon → gravitational radiation in shockwave collisions



Bern, Carrasco, Johannson,
arXiv: 1004.0476

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

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The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys*. 36 (1995) 6377; 3721 cites !**

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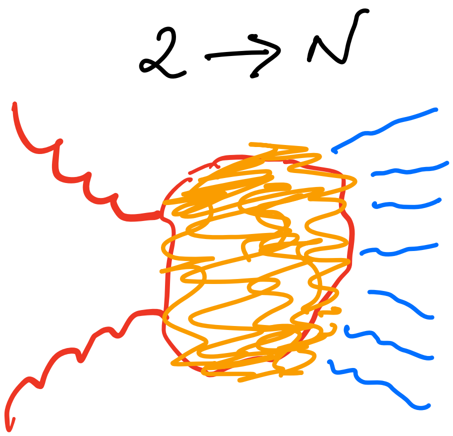
30+ years of work by ACV et al. exploring gravitational shockwave collisions I
n this 2-D EFT framework

2→ N graviton scattering: black hole quantum portrait

Dvali, Gomez, arXiv:1203.6575
Dvali, Gomez, arXiv:1112.3359
Dvali,Guidice,Gomez,Kehagis, arXiv:1010.1415



Estimate likelihood of forming a semi-classical metastable lump

$$P_{2\to N} \sim e^{S} \alpha_s^{N} N! \quad \Rightarrow \quad \text{If } N \sim \frac{1}{\alpha_s}$$

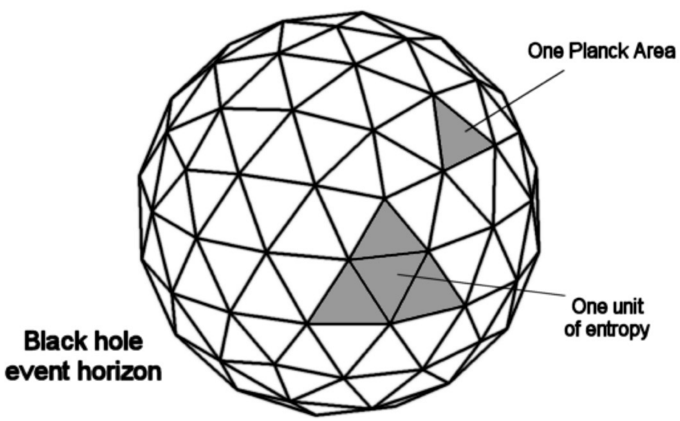
$$P_{2\to N} \sim e^{S} \alpha_s^{N} \left(\frac{1}{\alpha_s}\right)^{N} e^{-1/\alpha_s}$$

Exponentially suppressed…unless $S = \frac{1}{\alpha_S} = N$

$$\Rightarrow P_{2\to N} \sim O(1)$$

BHNP: quantum information perspective a la Dvali

BHNP saturates Bekenstein entropy bound $S \leq 2\pi\, E\, R/\hbar$



(for a nice discussion of many subtleties, see Bousso, arxlv:1810.01880)

Define $E = N\, Q_S$ as energy in critically packed volume = R_S^3 of quanta ("qubits") saturating unitarity (maximal information) and $Q_S = 1/R_S$

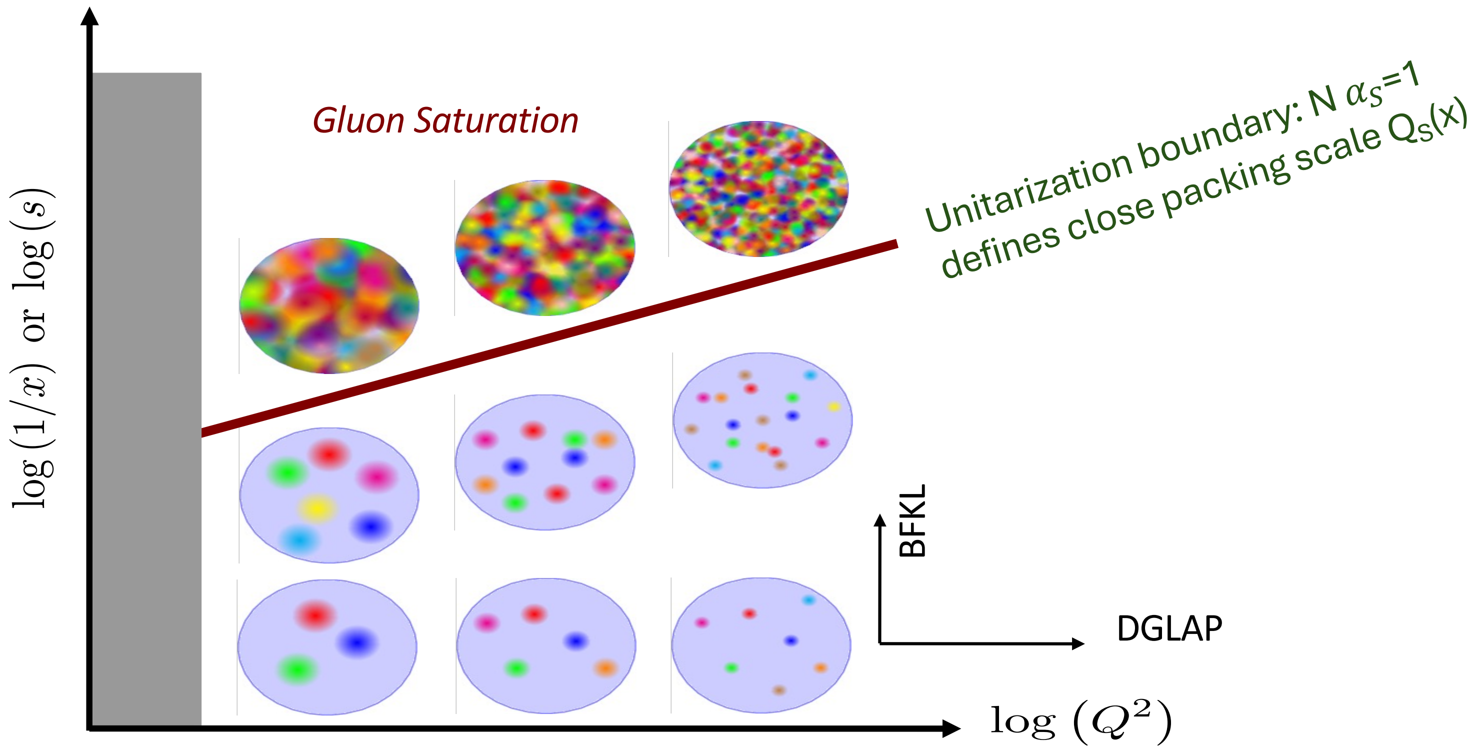
Then, $S \leq 2\pi\, N\, Q_S\, R_S$ is saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$

$$S_{Bek} = \frac{1}{\alpha_{gr}} = \frac{R_S^2}{L_P^2} = \frac{Area}{4G} = S_{BH}$$

Famous Bekenstein-Hawking area law

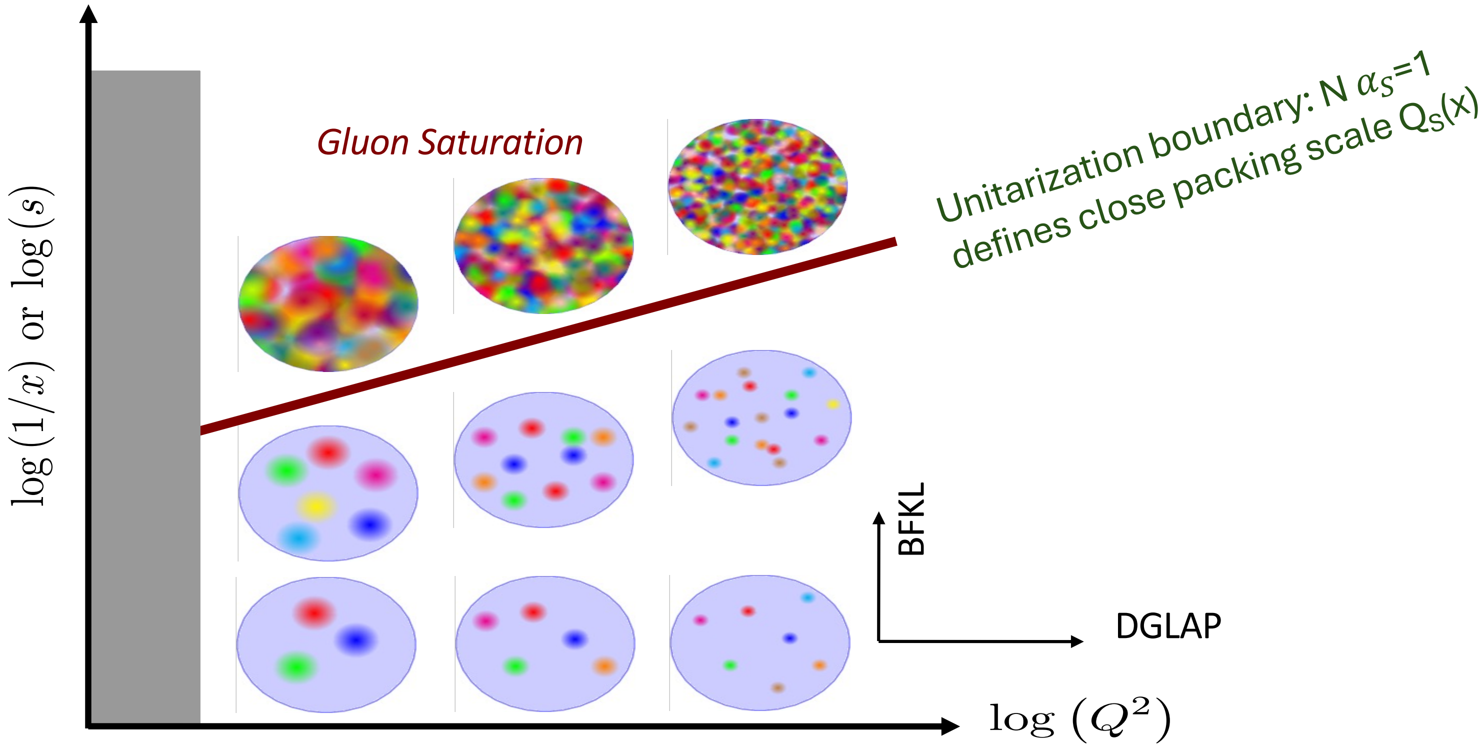
- Interpretation of M_{planck} as Goldstone scale representing breaking of Poincaré invariance by macroscopic quantum state

BHNP-like idea realized quantitatively in Regge asymptotics of QCD



Classicalization and unitarization of $2 \to N$ cross-section occurs when $S_{CGC} = 1/\alpha_S$:
saturated semi-classical ”lump” is a Color Glass Condensate

BHNP-like idea realized quantitatively in Regge asymptotics of QCD



Condensate breaks Poincare invariance + global sub-group of $SU(3)_{color}$ $S_{CGC} = 1/\alpha_S = N = f_G^2 * \text{Area}$

CGC satisfies the Bekenstein-Hawking area law in units of a Goldstone scale $f_G^2 = N\, Q_S^2$

Dvali, RV, arXiv:2106.11989 (PRD, 2022)

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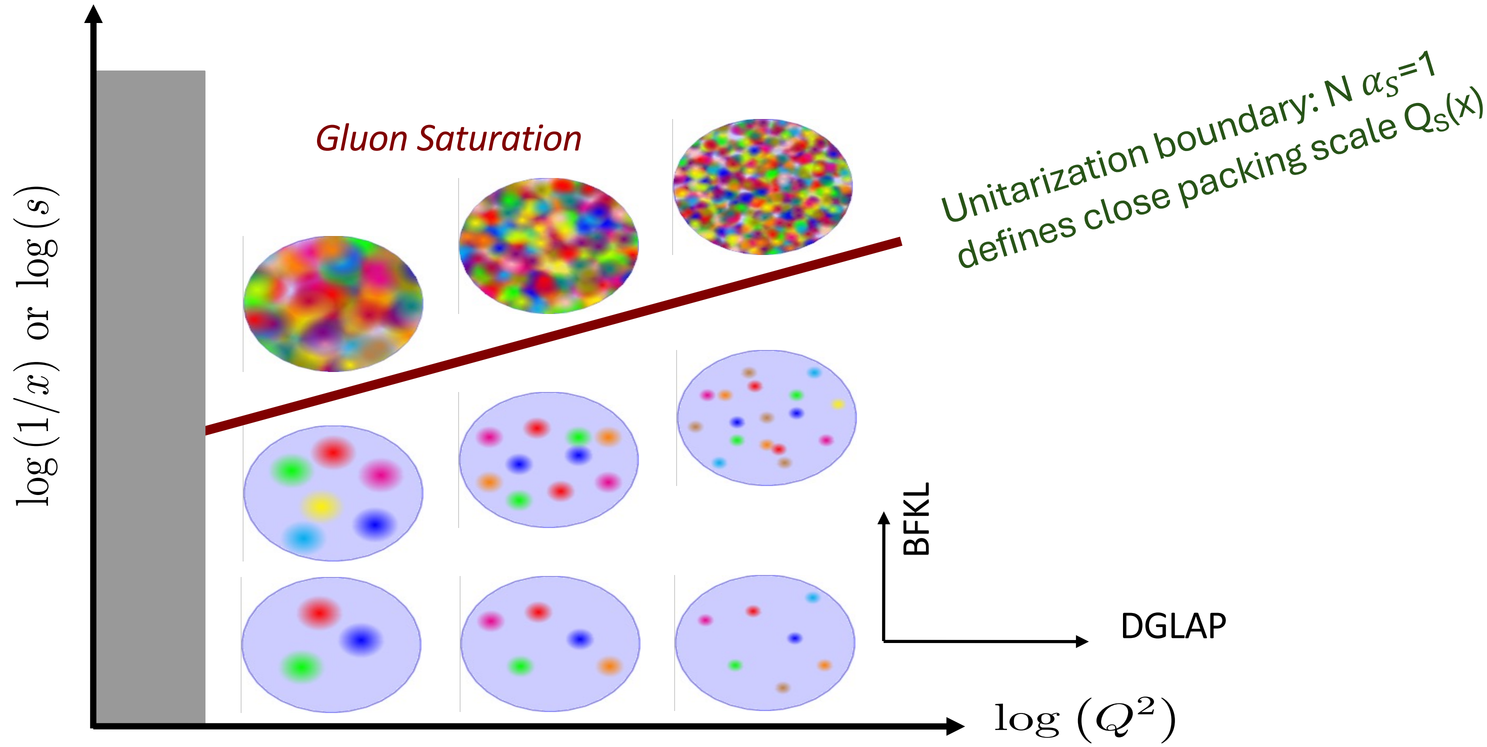


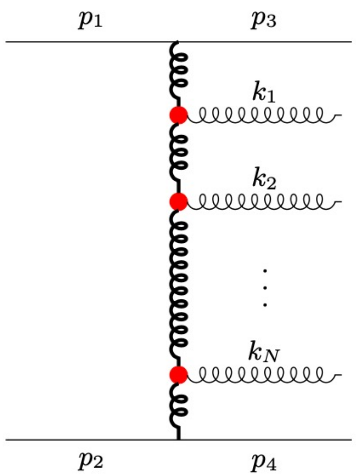
CGC a "classical" saddle point up to 1/N corrections: a leaky condensate, decays on time scale $\tau \sim \frac{1}{\alpha_S}\frac{1}{Q_S}$.

Consistent with bottom-up thermalization scenario of the Quark-Gluon Plasma (QGP)

R. Baier, A.H Mueller, D. Schiff,D. Son, hep-ph/0009237

Multiparticle production and gluon saturation in QCD:
from amplitudes to gluon shockwave collisions

BFKL: 2→ N QCD amplitudes in Regge asymptotics*



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(` `slow") but hard in transverse momentum – weak coupling Regge regime

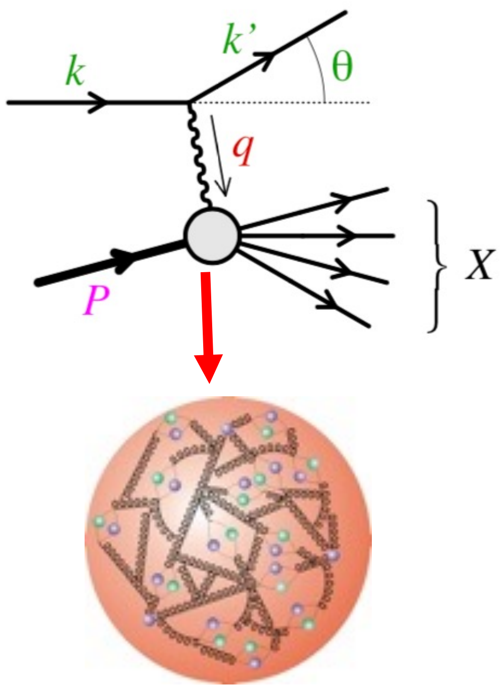
RG description rapidity of evolution given by the BFKL Hamiltonian
Very rapid growth of the amplitude with energy

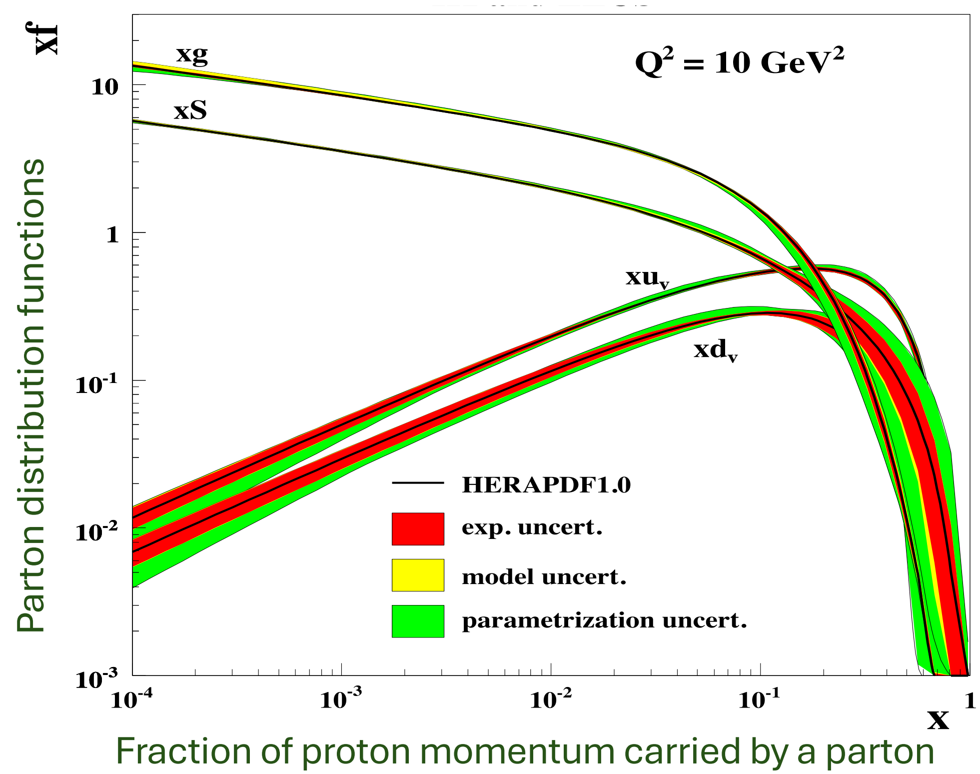
A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

* Asymptotics is the calculus of approximations. It is used to solve hard problems that cannot be solved exactly and to provide simpler forms of complicated results

The proton as a complex many-body system

Deeply Inelastic Scattering (DIS)



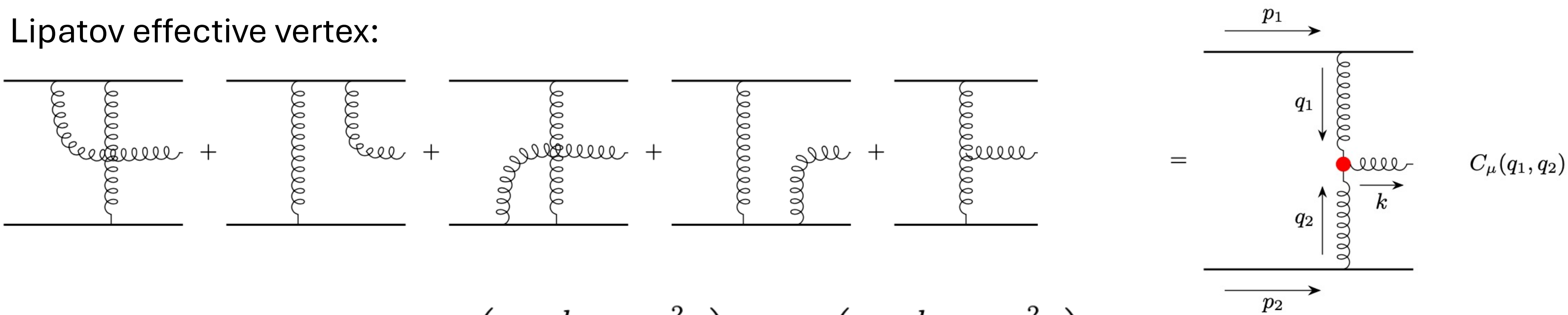


A key lesson from the HERA DIS collider:

Gluons and sea quarks dominate the proton wave-function at high energies

BFKL: Building blocks

Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu \, C^\mu$ =0

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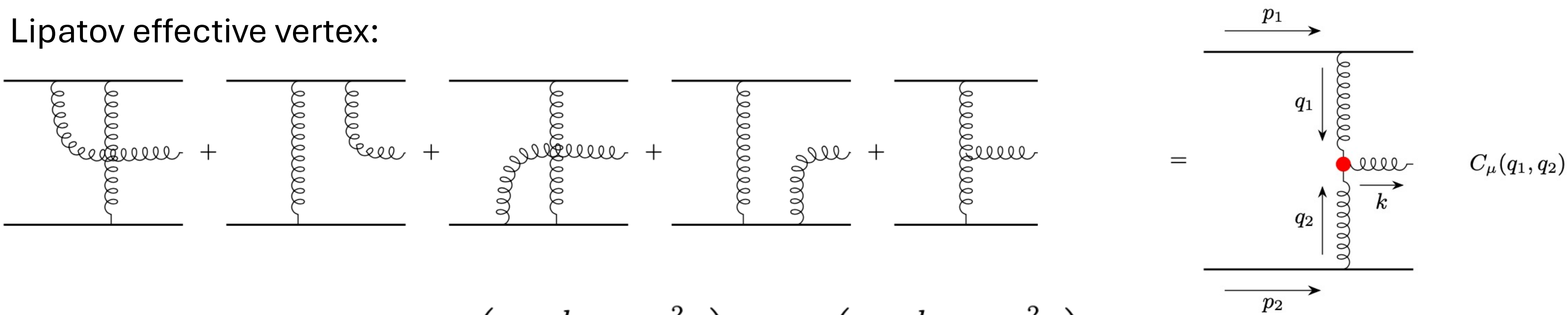

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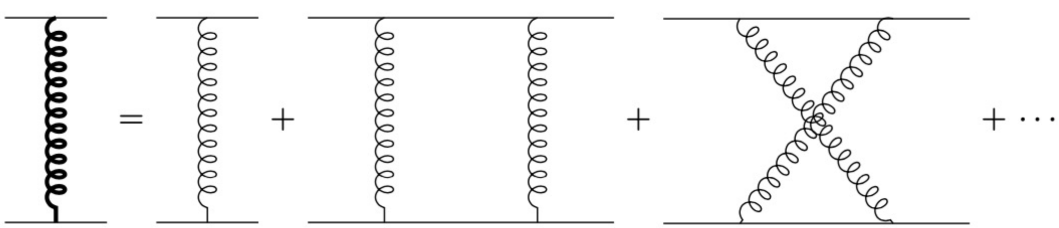
Gauge covariant, satisfies $k_\mu \, C^\mu = 0$

Reggeized gluon:

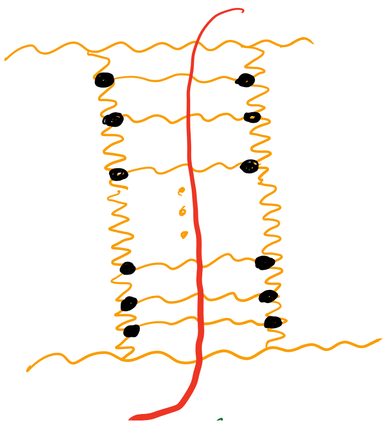


$$\frac{1}{t_i} \rightarrow \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c \, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q} - \boldsymbol{k})^2} \,, \qquad t = -\boldsymbol{q}^2$$

$2 \rightarrow \mathrm{N} + 2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



$$\mathrm{Im}\, A(s,t) \propto \sum_{n=0}^{\infty} (\alpha_s C_T)^{n+2}$$

C_T is color factor

$$\times \int \prod_{i=1}^{n} \frac{dy_i}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{jL}}{(2\pi)^2}$$

Phase space factors

$$\times\ 2is \prod_{l=1}^{n+1} \frac{1}{t_l t_l'}\, e^{(y_{l-1}-y_l)[\alpha(t_l)+\alpha(t_l')]}$$

Reggeized propagators on both sides of cut

$$\times \prod_{m=1}^{n} (C_\mu C^\mu)[q_m, q_{m+1}]$$

Product of Lipatov vertices

$$\sigma_{tot} = 2\, \mathrm{Im}\, A(s, t=0)$$

$$= s^{\lambda} \text{ with } \lambda = \frac{4\alpha_s N_c \ln 2}{\pi}$$

$$\simeq 0.5 \text{ for } \alpha_s = 0.2$$

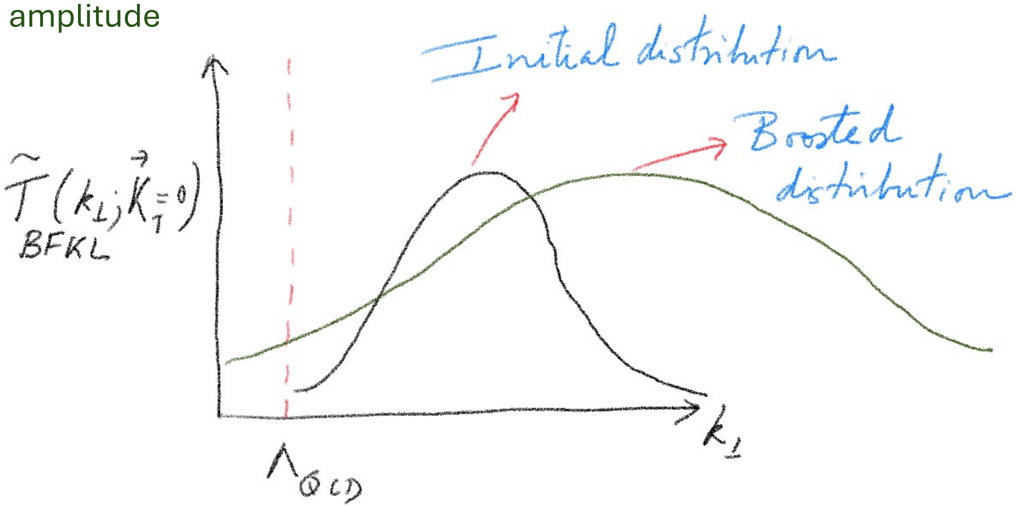
Real and virtual corrections combine to cancel infrared divergence !

Strongly violates Froissart bound

Resummed NLO BFKL : $\lambda \approx 0.3$

BFKL: infrared diffusion and gluon saturation

BFKL forward amplitude



For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLL BFKL does not cure infrared diffusion

Gluon saturation cures infrared diffusion

BFKL: infrared diffusion and gluon saturation

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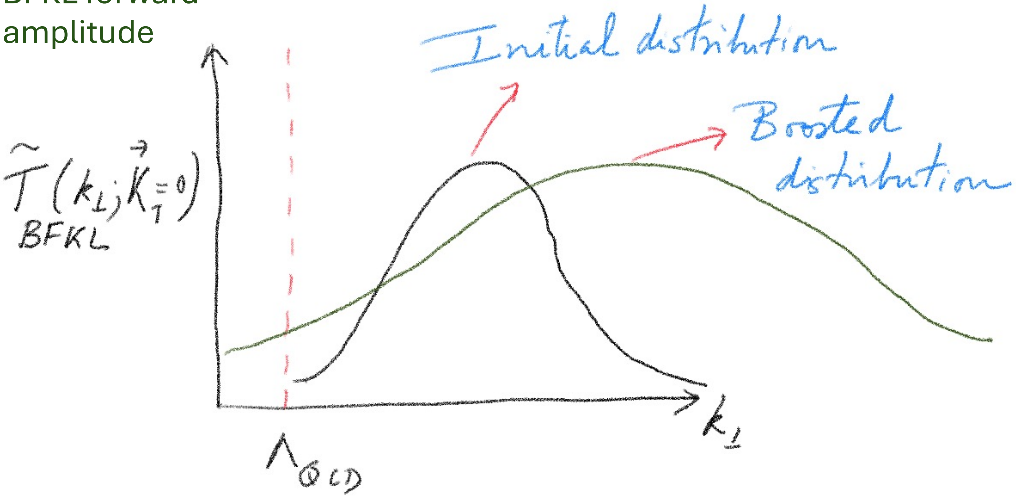

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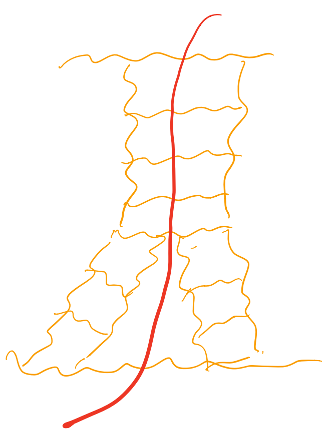
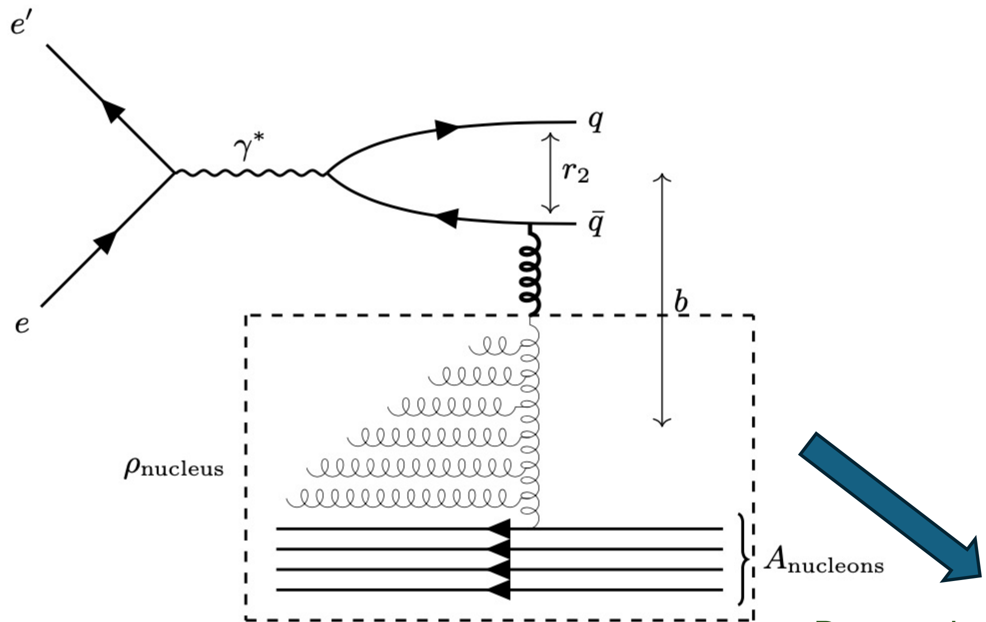


+ other higher twist cuts of O(1) when gluon occupancy $N \equiv \frac{xG_A(x, Q_S^2)}{2(N_c^2 - 1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$

Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Gluon saturation: classicalization and unitarization of cross-sections



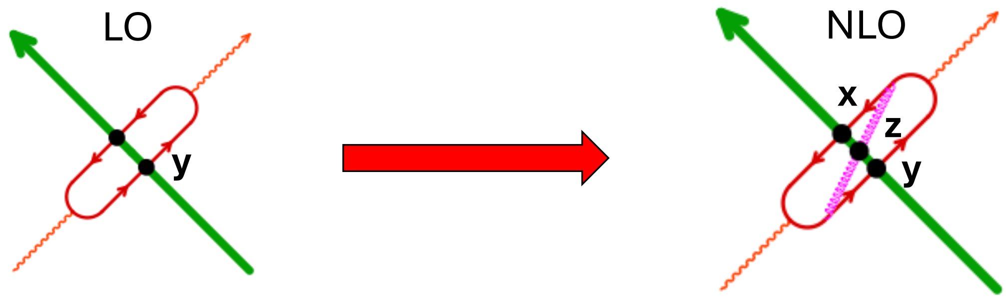
Dense close-packed ($1/Q_S$) classical lump- gluon "shockwave"

Reggeized gluon as field sourced by lump's color charge density

CGC EFT:
Powerful functional RG describes nonlinear (multi-Pomeron) evolution with rapidity
– at NLLx accuracy for multiple final states

CGC review: Gelis,Iancu,Jalilian-Marian, RV:arXiv 1002.0333

DIS: dipole evolution in gluon shockwave background



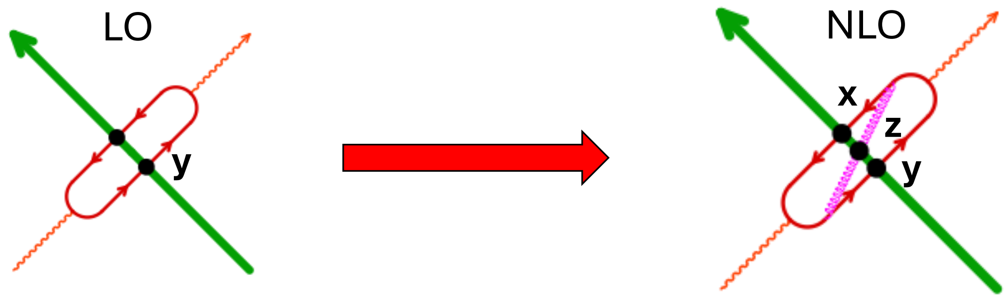
Path ordered 2-D Wilson lines
-describe transition between two pure gauges on either side of shockwave: “color memory”

“B-JIMWLK” RG eqns. for Wilson-line correlators: Eg. 2-point ”dipole”

$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Y = Ln(1/x)

DIS: dipole evolution in gluon shockwave background



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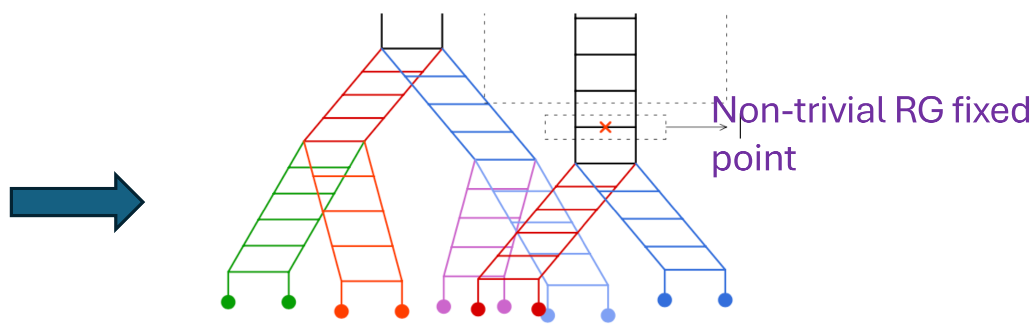
$Y = \mathrm{Ln}(1/x)$

Closed form expression for A >>1, Nc→ ∞:
non-linear Balitsky-Kovchegov (BK) eqn.

Evolved shockwave scattering off dipole probe contains all-twist multi-pomeron "fan" diagrams

BFKL obtained as the leading twist result…



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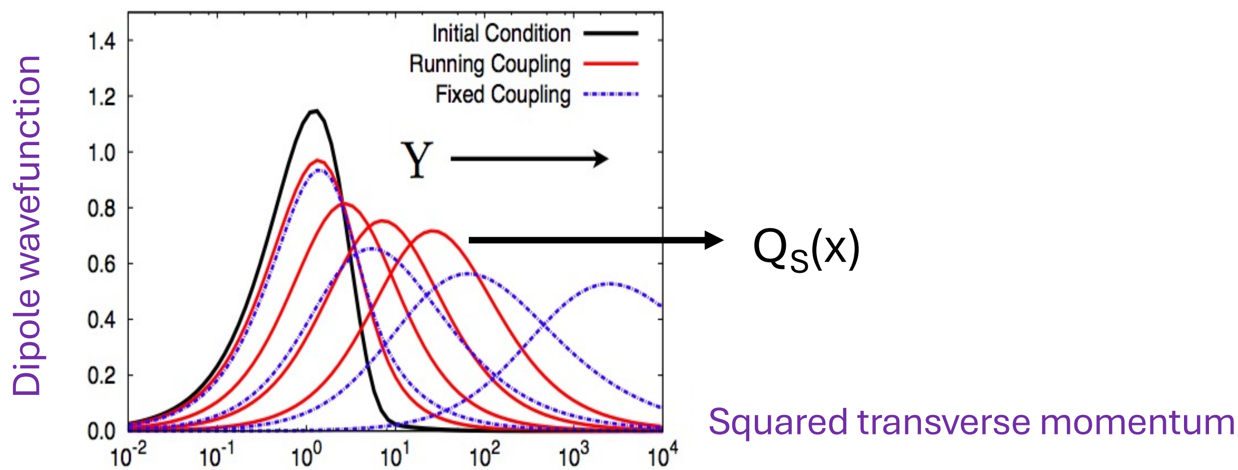
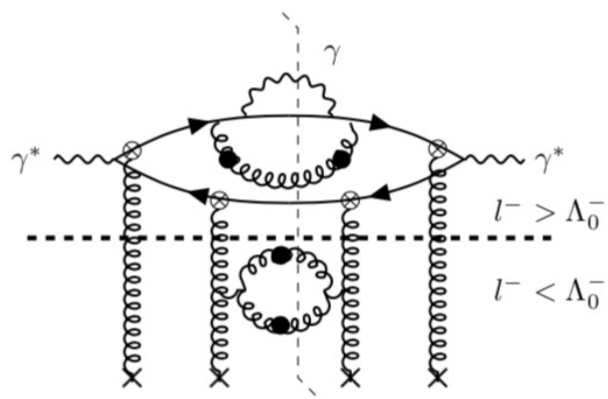




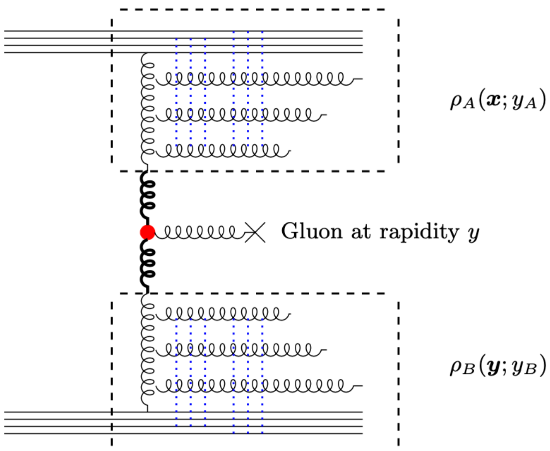
”impact” factors for multiple final states to NLO accuracy

Small x RG evolution of shockwave to NLLx accuracy

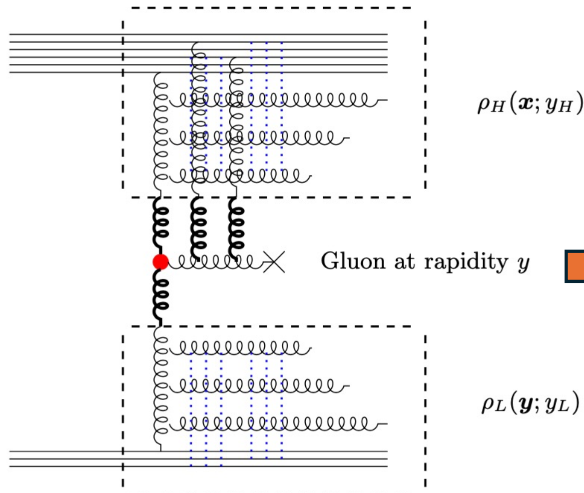
State-of-the-art:

Significant amount of phenomenology describing small x HERA, RHIC, LHC data
- precision tests at EIC in future

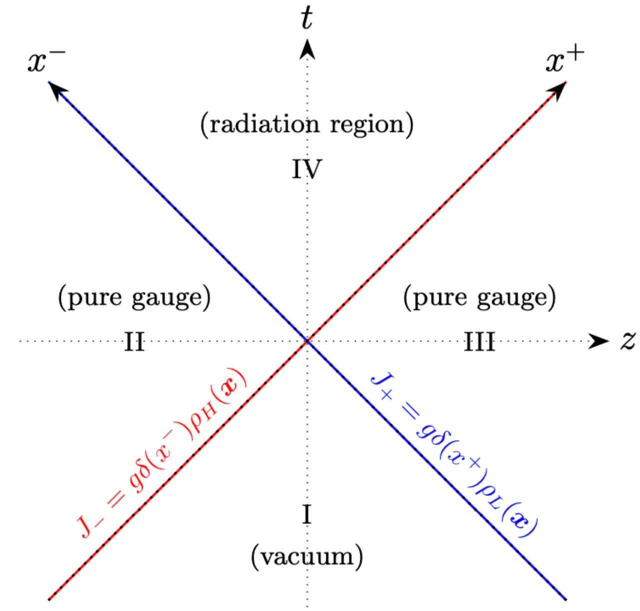
Gluon shockwave collisions: Lipatov vertex and reggeization



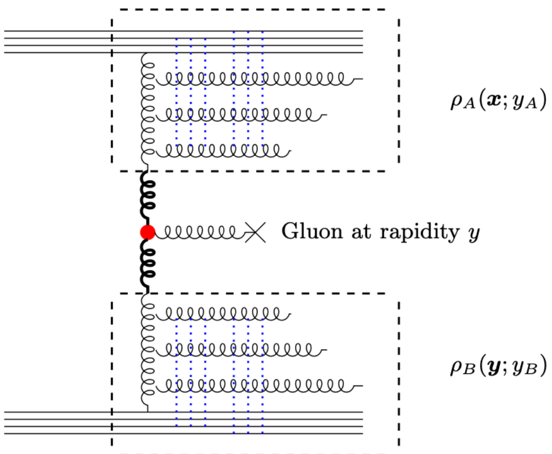
"dilute-dilute" collisions
($\frac{\rho_A}{\Delta_\perp^2}, \frac{\rho_B}{\Delta_\perp^2} \ll 1$)

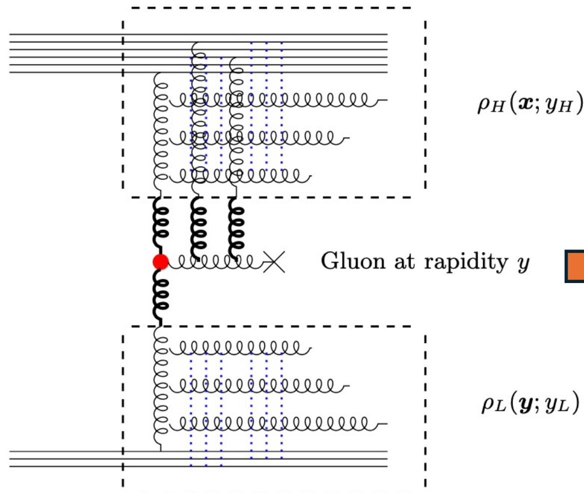


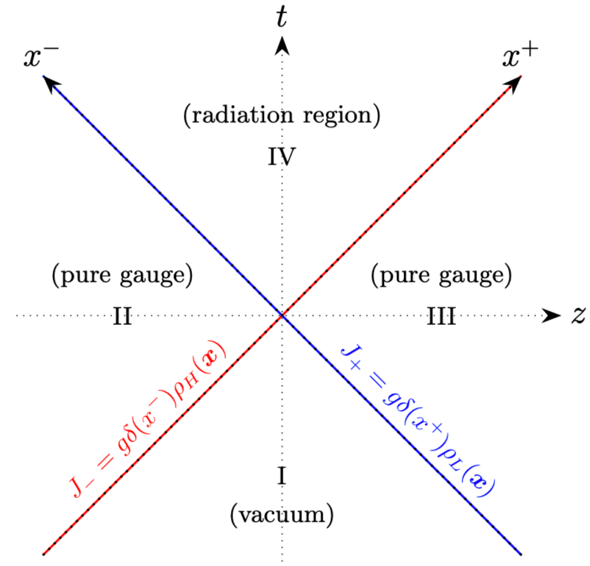
"dilute-dense" collisions
($\frac{\rho_L}{\Delta_\perp^2} << 1 , \frac{\rho_H}{\Delta_\perp^2} \sim O(1)$)



Gluon shockwave collisions: Lipatov vertex and reggeization







Weizsäcker-Williams gluon radiation field in light cone gauge

$$a_i(k) = -\frac{2ig}{k^2 + i\epsilon} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \left(q_{2i} - k_i \frac{\boldsymbol{q}_2^2}{\boldsymbol{k}^2}\right) \frac{\rho_L(\boldsymbol{q}_2)}{\boldsymbol{q}_2^2} \left(U(\boldsymbol{k}+\boldsymbol{q}_2) - (2\pi)^2 \delta^2(\boldsymbol{k}+\boldsymbol{q}_2)\right)$$

$$U(x^-, \boldsymbol{x})\delta\left(x^+\right) = \exp\left(ig \int_{-\infty}^{x^-} dz^- \bar{A}_-(z^-, \boldsymbol{x}) \cdot T\right)$$

$$\bar{A}_\mu(x^-, \boldsymbol{x}) = -g\delta_{\mu-}\delta\left(x^-\right) \frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

Lipatov vertex
in $A^- = 0$ gauge

reggeized gluons from
semi-classical source dists.

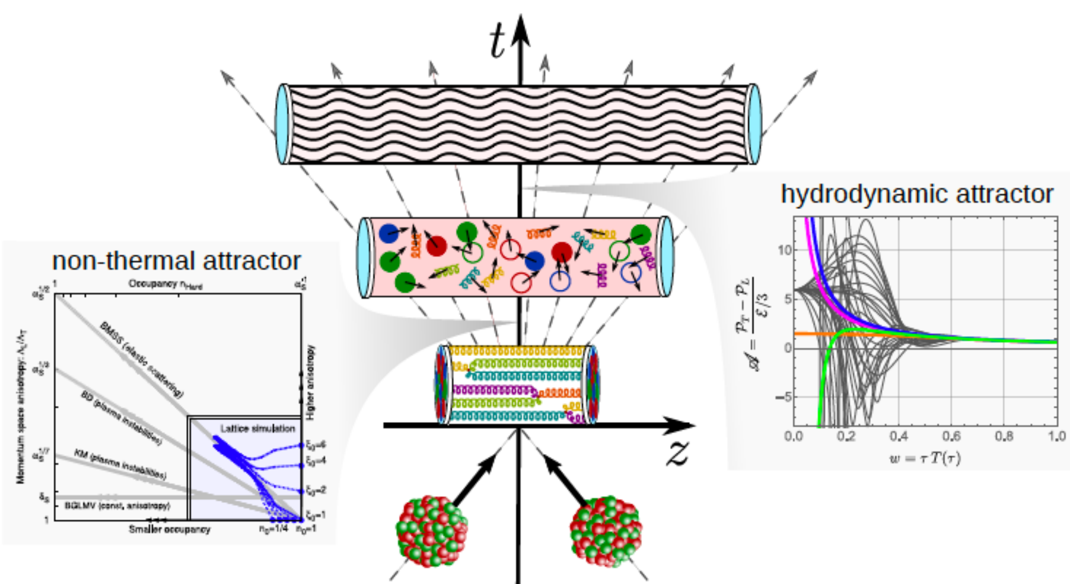
Ln (U) → *reggeized gluon*

Blaizot,Gelis,RV (2004)
Gelis-Mehtar-Tani (2005)

Jalilian-Marian,Jeon,RV (2000); Caron-Huot (2013)

Dense-dense shockwave collisions: heavy-ion collisions

Quark-Gluon Plasma undergoing
hydrodynamic expansion



Collision of overoccupied Color Glass Condensate shockwaves

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and RV
Rev. Mod. Phys. **93**, 035003 (2021)

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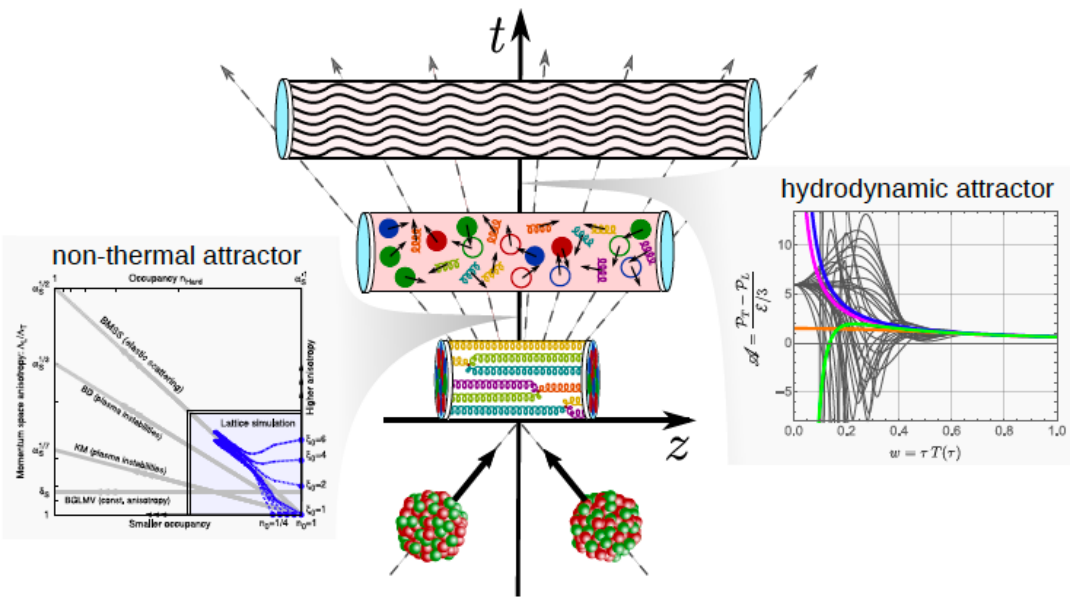

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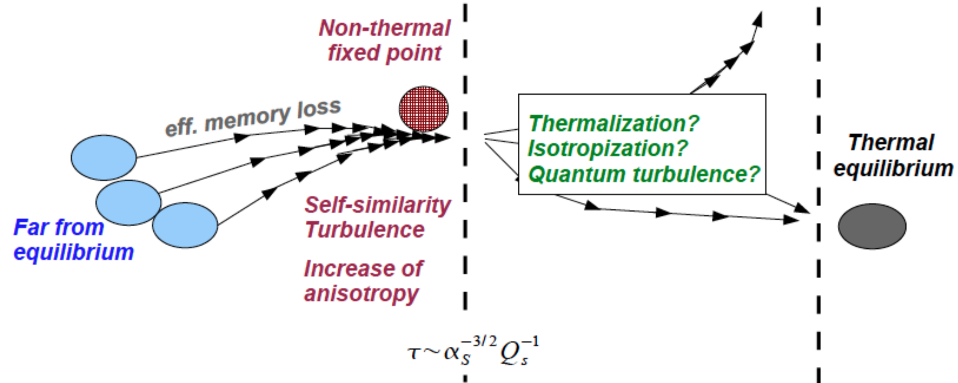

Thermal soft gluon bath for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature: $T_i = \alpha_S^{2/5} Q_S$

Very rapid thermalization
as $\alpha_S(Q_S) \rightarrow 0$ and $Q_S \rightarrow \infty$

Baier, Mueller, Schiff, Son,
hep-ph/0009237

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Multiparticle production and saturation in gravity: from amplitudes to shockwave collisions

An analogous program can be followed for 2→N scattering in gravity with remarkable quantitative double copy relations emerging at every step…