

Odderon and Bartels–Kwieciński–Praszałowicz equation

Anna Staśto
Penn State University

Special session dedicated to Professor Michał Praszałowicz on his birthday

Outline

- Regge limit : Odderon as $C=-1$ amplitude
- High energy limit in QCD
 - BFKL equation for Pomeron
 - **Bartels-Kwieciński-Praszałowicz equation for Odderon**
- Solutions, beyond lowest order, unitarity corrections
- Experiments: past, present, future

Sources: apart from original papers, I learned a lot from an excellent review (2003) by Carlo Ewerz, talks by Sanjin Benić, Tamás Csörgő, Leszek Motyka, Yuri Kovchegov and Christophe Royon (<https://www.youtube.com/watch?v=yHBO3zcB3V4>) and discussions with Valery Khoze.

What ChatGPT says about Odderon?

ChatGPT 4o ▾

What is an Odderon ?



An Odderon is a theoretical concept in particle physics. It represents a specific type of exchange between particles in high-energy collisions, distinct from the more commonly discussed Pomeron. Here's a detailed breakdown:



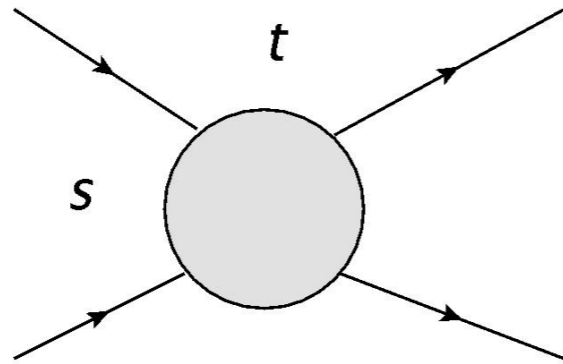
1. **Quantum Chromodynamics (QCD):** Odderons arise from QCD, the theory describing the interactions between quarks and gluons, the fundamental particles making up protons and neutrons.
2. **Charge-Parity (CP) Symmetry:** The Odderon is associated with the violation of charge-parity symmetry. This means it involves exchanges that have odd charge parity, unlike the Pomeron, which has even charge parity.
3. **High-Energy Physics:** Odderons are predicted to play a role in the scattering processes at very high energies, such as those investigated in particle accelerators like the Large Hadron Collider (LHC).
4. **Experimental Evidence:** While Odderons were first theorized in the 1970s, finding clear experimental evidence has been challenging. However, in recent years, there have been hints and potential signals of Odderon effects in experiments involving proton-proton and proton-antiproton collisions.

In summary, the Odderon is a theoretical construct in particle physics that represents a specific type of exchange process, differing from the more established Pomeron, and it is significant in the study of high-energy particle interactions and CP symmetry.

S-matrix and Regge limit

Pre QCD...

$$A(s, t)$$



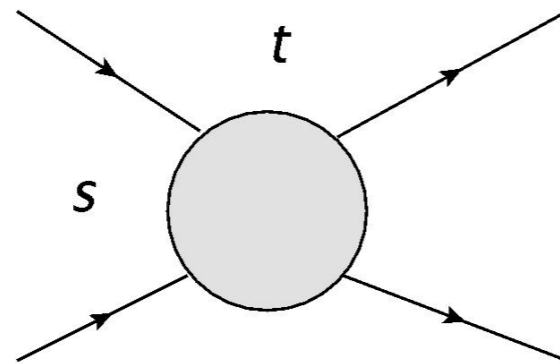
Properties of S matrix:

- Lorentz invariance
- unitarity
- analyticity

S-matrix and Regge limit

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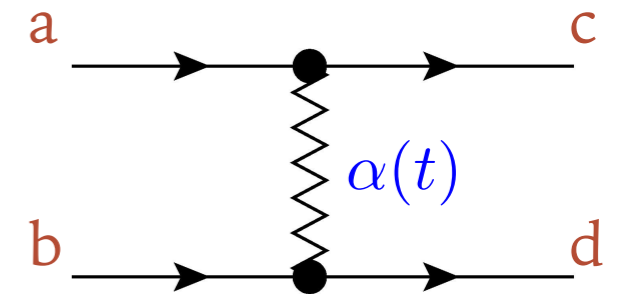
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Regge limit: $s \rightarrow \infty$ $s \gg |t|$

$$\mathcal{A}^{(\eta)}(s, t) = \beta(t)\Gamma(-\alpha(t))(1 + \eta e^{-i\pi\alpha(t)}) \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

Signature: $\eta = \pm 1$

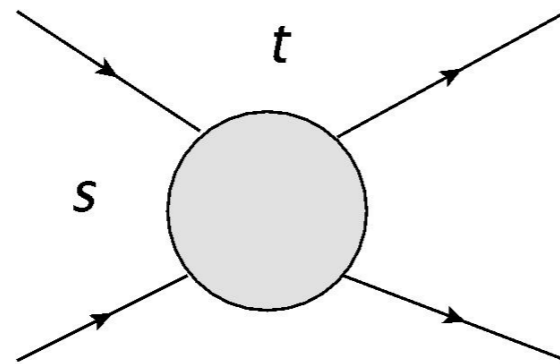


Energy dependence controlled by exchange of the **Regge trajectory**: $\alpha(t) = \alpha(0) + \alpha' t$

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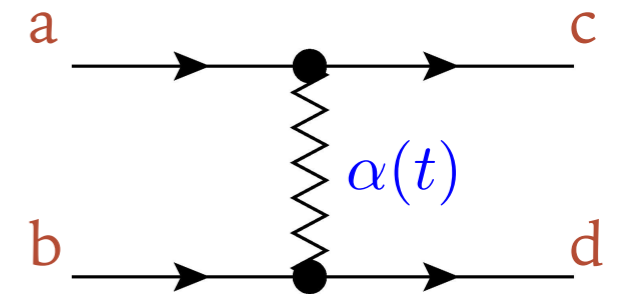
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Energy dependence controlled by exchange of the **Regge trajectory**: $\alpha(t) = \alpha(0) + \alpha' t$

Consider elastic scattering: $ab \rightarrow ab$

From optical theorem $\sigma_{\text{tot}} = s^{-1} \text{Im} A_{\text{el}}(s, 0) \sim s^{\alpha(0)-1}$

Intercept $\alpha(0)$ of Regge trajectory determines the behavior of the **total cross section**

Pomeron

- Reggeon with **even signature**, and intercept $\alpha_{\mathbb{P}}(0) > 1$
- Corresponds to the exchange of the **vacuum** quantum numbers.
- Amplitude (predominantly) **imaginary**
- Dominates the cross section at asymptotically **high energies**

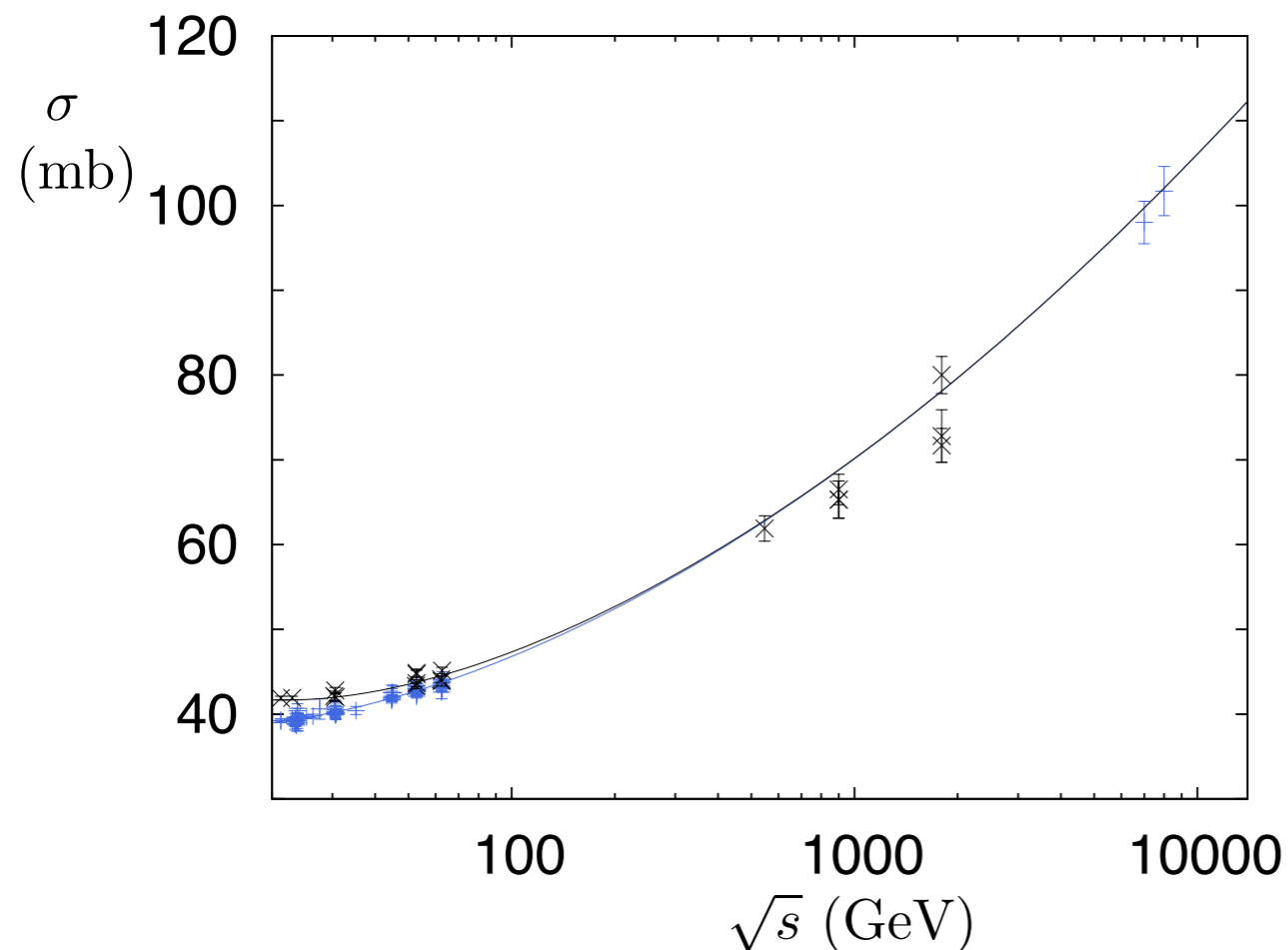
*Okun, Pomeranchuk;
Foldy, Peierls*

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Okun, Pomeranchuk;
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Donnachie, Landshoff



Soft Pomeron

$$\alpha_P(t) = 1.11 + 0.165 \text{GeV}^{-2} t$$

(2013 parameters of fit to data including LHC)

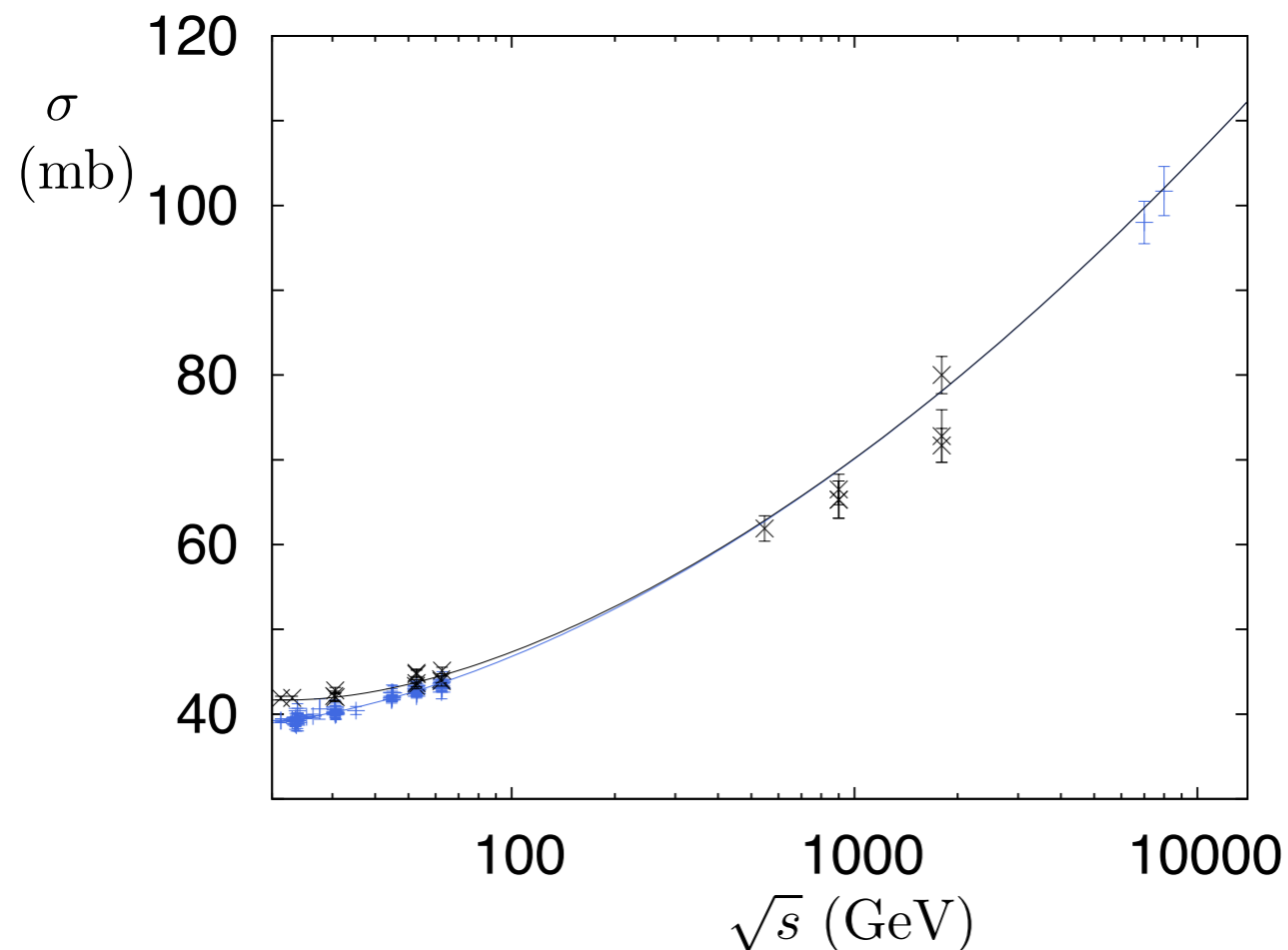
$$\sigma_{\text{tot}} \sim s^{\alpha_P(0) - 1}$$

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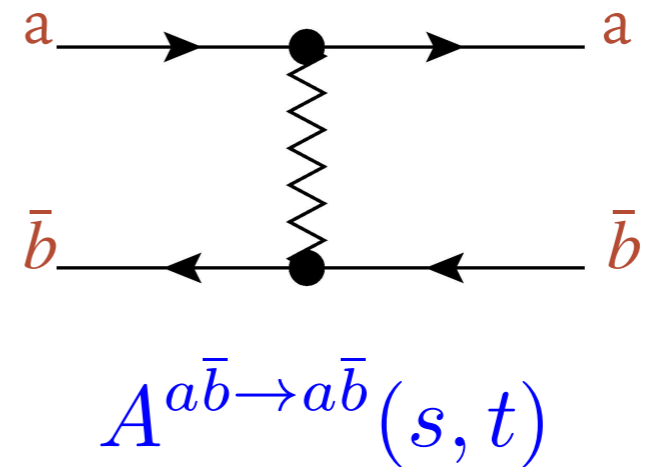
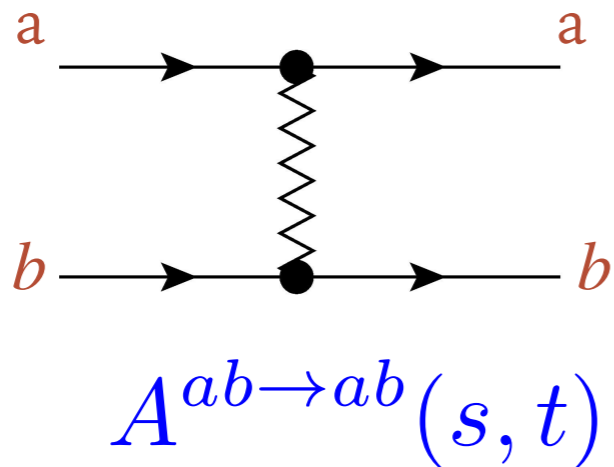
However, soft pomeron power behavior is potentially in conflict with **Froissart** bound from **unitarity**:

$$\sigma^{\text{tot}}(s) \leq C \log^2(s/s_0)$$

Need to include **multiple** exchanges

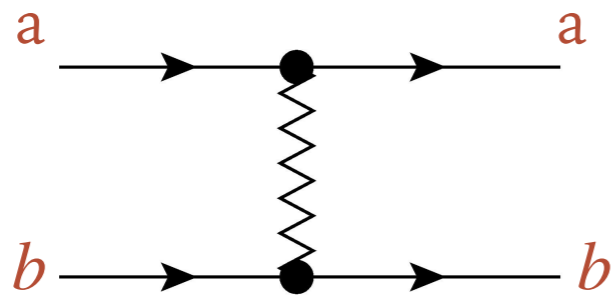
Crossing and the Odderon

What happens when **particle** is replaced by **antiparticle** ?

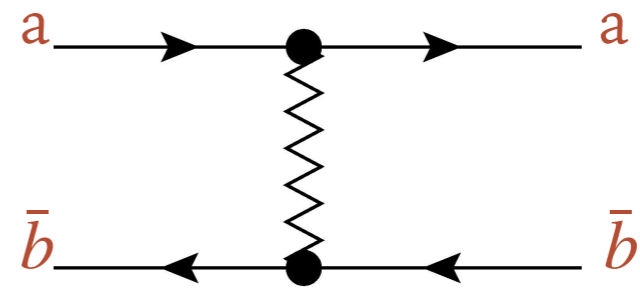


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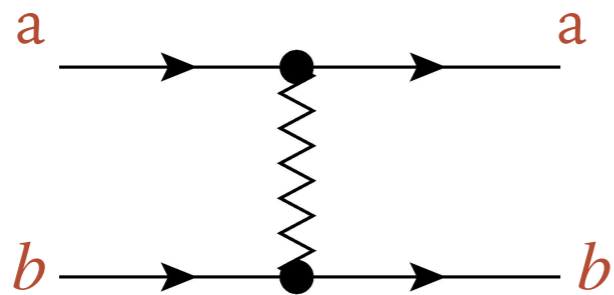


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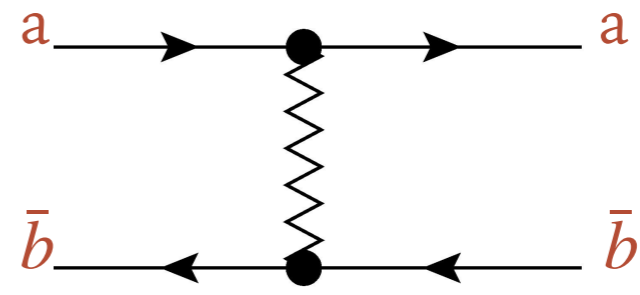
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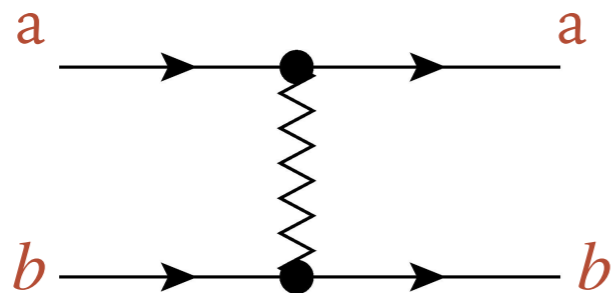
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Define two amplitudes: **even** and **odd** under crossing

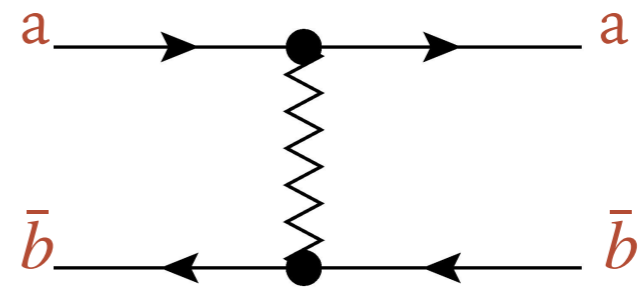
$$A_{\pm}(s, t) \equiv \frac{1}{2} (A^{ab \rightarrow ab}(s, t) \pm A^{a\bar{b} \rightarrow a\bar{b}}(s, t))$$

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What happens when **particle** is replaced by **antiparticle** ?



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Even: same for particle-particle and particle-antiparticle scattering

At high energy dominated by the **Pomeron**: **C = +1**

Odd: changes sign for particle-particle and particle-antiparticle scattering

At high energy dominated by another contribution, the **Odderon**: **C = -1**

Odderon in Regge theory

(General) **Pomeranchuk** theorem

$$\lim_{s \rightarrow \infty} \frac{\sigma_{\text{tot}}^{\bar{p}p}}{\sigma_{\text{tot}}^{pp}} \rightarrow 1$$

Difference does not have to vanish

$$\lim_{s \rightarrow \infty} |\Delta\sigma| = |\sigma_{\text{tot}}^{\bar{p}p} - \sigma_{\text{tot}}^{pp}| \neq 0$$

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LETTERE AL NUOVO CIMENTO

VOL. 8, N. 7

13 Ottobre 1973

A Possible Interpretation of pp Rising Total Cross-Sections.

L. ŁUKASZUK (*) and B. NICOLESCU

*Division de Physique Théorique (**), Institut de Physique Nucléaire (***)
and Laboratoire de Physique Théorique et Hautes Energies (***) - Paris*

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IL NUOVO CIMENTO VOL. 30 A, N. 3 1 Dicembre 1975

Non-Regge and Hyper-Regge Effects in Pion-Nucleon Charge Exchange Scattering at High Energies.

D. JOYNSON (*), E. LEADER (**), and B. NICOLESCU
*Division de Physique Théorique (**), Institut de Physique Nucléaire (***) - Paris
 Laboratoire de Physique Théorique des Particules Élémentaires - Paris (***)*

C. LOPEZ (*,*)
Laboratoire de Physique Théorique et Hautes Energies - Paris (,*)*

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Odderon name introduced by **Joynson, Leader, Nicolescu, Lopez 1975**

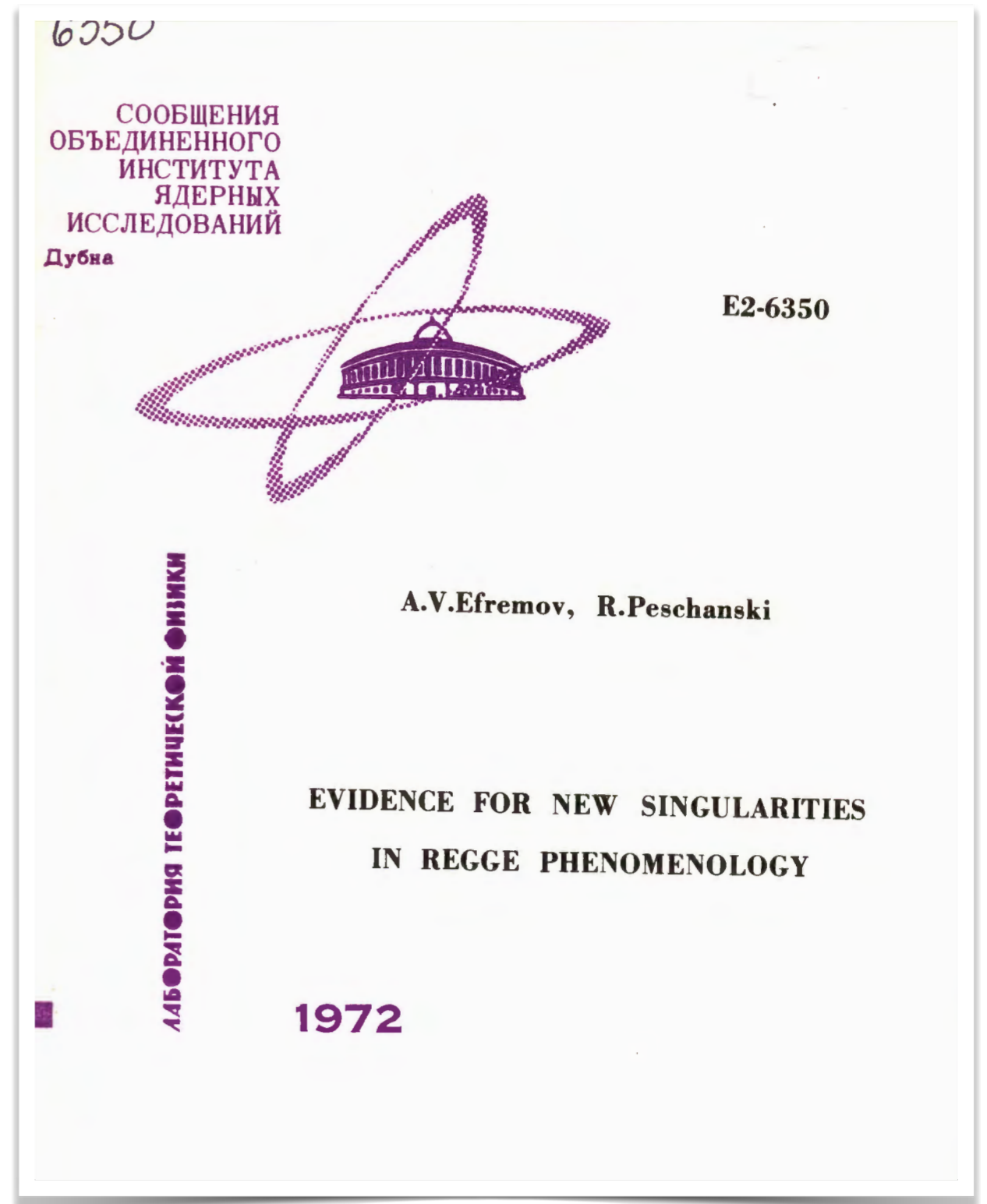
It would also be possible to interpret the new term as arising from an *odd signed* analogue of the pomeron, *i.e.* a Regge pole, which we shall refer to as the « odderon » and which has $\alpha(0) = 1$. This term would be purely real at $t = 0$ and would thus only play a rôle in $d\sigma/dt|_{t=0}$.

Odderon in Regge theory

as pointed out by Tamás Csörgő

New contribution was also proposed
by *Efremov and Peschanski 1972*

- real with negative signature
- small $\alpha(0)$, α'



Pomeron and Odderon in QCD : lowest order

How Pomeron and Odderon can be realized in perturbative QCD ?

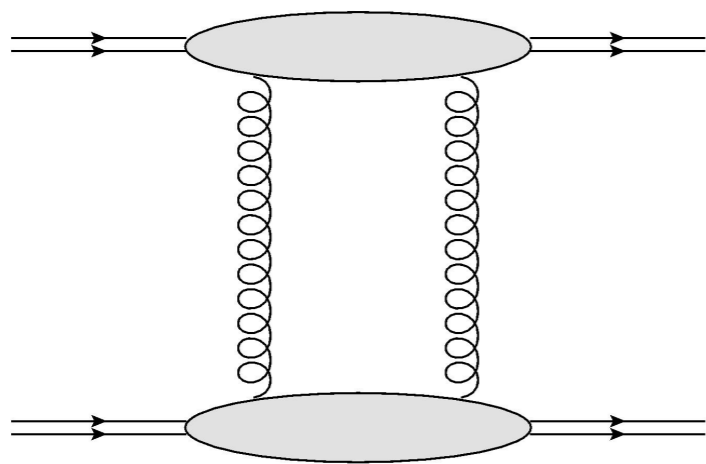
Charge conjugation: $\hat{A}_\mu(x) \rightarrow -\hat{A}_\mu^T(x)$ where $\hat{A}_\mu(x) = A_\mu^a(x) t^a$

Pomeron:

2 gluons, color singlet C= +1

$$\delta_{ab} A_\mu^a(x) A_\nu^b(y)$$

Low-Nussinov pomeron



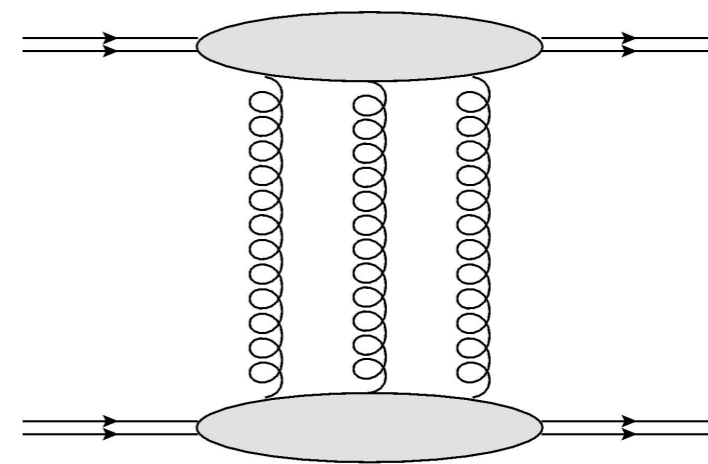
Odderon:

3 gluons, color singlet, C=-1

$$d_{abc} A_\mu^a(x) A_\nu^b(y) A_\lambda^c(z)$$

d_{abc} symmetric tensor

(structure with f_{abc} anti-symmetric tensor, carries positive C-parity)



Intercepts would be : $\alpha(0) - 1 = 0$ in both cases

BFKL Pomeron

There is no police in QCD, one can always emit another gluon...

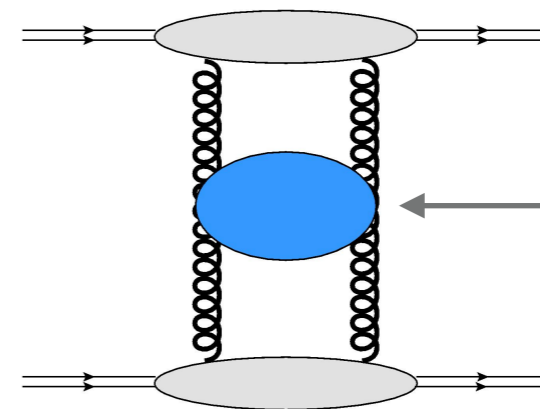
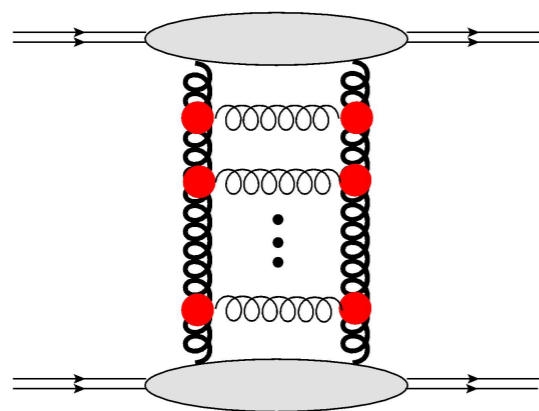
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Balitsky-Fadin-Kuraev-Lipatov **BFKL**: resummation of logarithms $\ln s$

Resummation of gluon emissions and exchanges: **evolution equation in energy** (or rapidity)



G is **Gluon Green's function**

$$\frac{\partial}{\partial Y} G(Y, \mathbf{k}, \mathbf{k}_0) = \mathcal{K} \otimes G(Y, \mathbf{k}, \mathbf{k}_0)$$

$Y = \ln \frac{s}{s_0}$: logarithm of energy

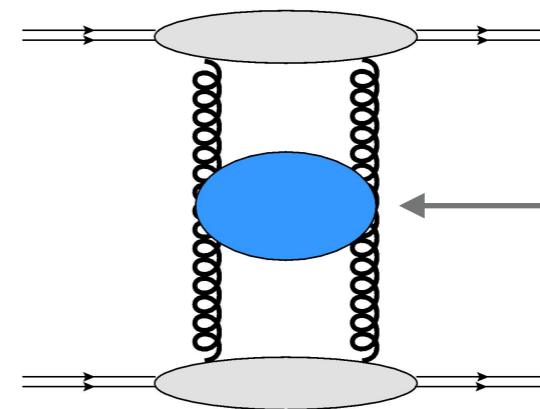
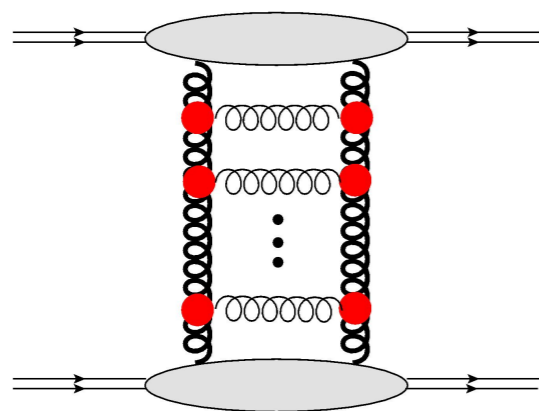
\mathbf{k} : transverse momentum

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$Y = \ln \frac{s}{s_0}$: logarithm of energy

\mathbf{k} : transverse momentum

Increase with energy: hard Pomeron

$$\omega_{IP} = \alpha_{IP} - 1 = \frac{\alpha_s N_c}{\pi} \chi(\gamma = 1/2) = 4 \ln 2 \frac{\alpha_s N_c}{\pi}$$

BKP Odderon

Bartels-Kwieciński-Praszałowicz equation: evolution equation for Odderon at high energy



Nuclear Physics B175 (1980) 365–401
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HIGH-ENERGY BEHAVIOUR IN A NON-ABELIAN GAUGE THEORY (II). First corrections to $T_{n \rightarrow m}$ beyond the leading $\ln s$ approximation

J. BARTELS

II. Institut für Theoretische Physik der Universität Hamburg, Germany

Received 26 March 1980

In this second part of our attempt to construct a unitary high-energy description of a spontaneously broken non-abelian gauge theory we calculate, for the $n \rightarrow m$ amplitude in the multi-Regge limit, the first corrections beyond the leading logarithmic approximation. The resulting amplitudes come in the form of the reggeon calculus where the number of reggeons in each t -channel is restricted to one or two. We then study the limit where the mass of the vector particle is taken to zero: for the $2 \rightarrow 2$ amplitude we show that this limit exists, not only for the approximation of the present paper but also for higher-order corrections.

Volume 94B, number 3

PHYSICS LETTERS

11 August 1980

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Institute of Nuclear Physics, Cracow, Poland

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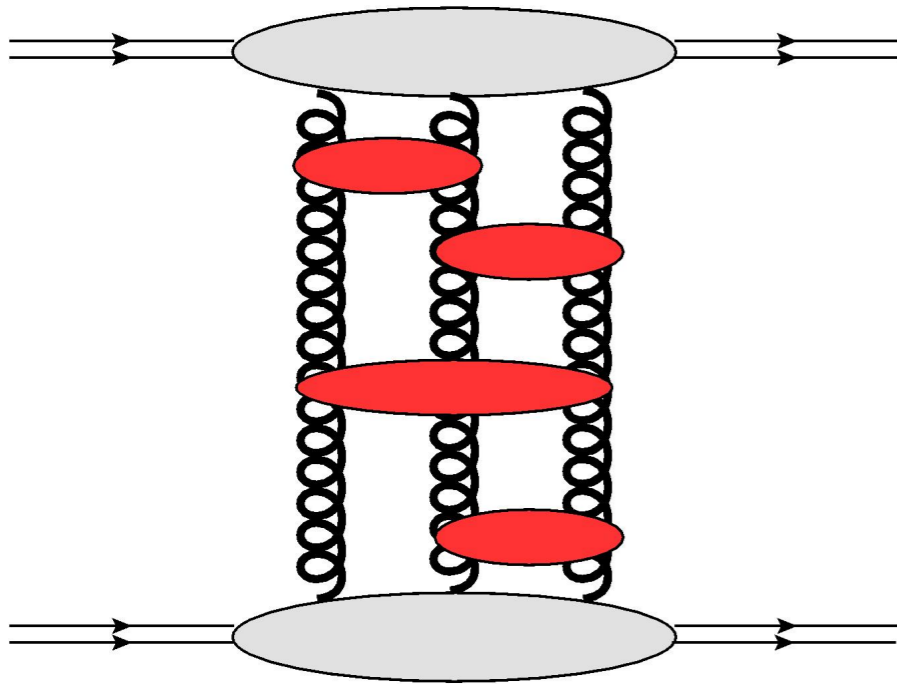
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Related work for n-gluons: [Jaroszewicz 1980](#)

BKP Odderon

Kwieciński-Praszałowicz:

$$\begin{aligned} \omega T(\omega; \hat{k}_1, \hat{k}_2, \hat{k}_3) &= F_0(\hat{k}_1, \hat{k}_2, \hat{k}_3) + \int \prod_{j=1}^3 d^2 \hat{k}'_j \delta^{(2)}\left(\sum_{l=1}^3 \hat{k}'_l\right) \\ &\times \sum_{i=1}^3 K^{(i)}(\hat{k}_i; \hat{k}_{i+1}, \hat{k}_{i-1}, \hat{k}'_{i+1}, \hat{k}'_{i-1}) \delta^{(2)}(\hat{k}'_i - \hat{k}_i) G(\hat{k}'_{i+1}) G(\hat{k}'_{i-1}) [T(\omega; \hat{k}'_1, \hat{k}'_2, \hat{k}'_3) - T(\omega; \hat{k}_1, \hat{k}_2, \hat{k}_3)] \\ &\equiv F_0 + \sum_{i=1}^3 \tilde{K}^{(i)} T. \end{aligned}$$



Pairwise interactions between

$1 \leftrightarrow 2, 2 \leftrightarrow 3, 1 \leftrightarrow 3$ reggeized gluons

$$\frac{\partial}{\partial Y} \mathcal{O} = \mathcal{K}_{12} \otimes \mathcal{O} + \mathcal{K}_{23} \otimes \mathcal{O} + \mathcal{K}_{31} \otimes \mathcal{O}$$

Evolution equation of gluon Green's function for three reggeized gluons

Cancellation of infrared divergences between real and virtual terms

Solutions to the Odderon

Janik, Wosiek (JW) 1998

$$\alpha_{\mathbf{O}}(0) = 1 - 0.24717 \frac{\alpha_s N_c}{\pi}$$

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Bartels, Lipatov, Vacca (BLV) 1999

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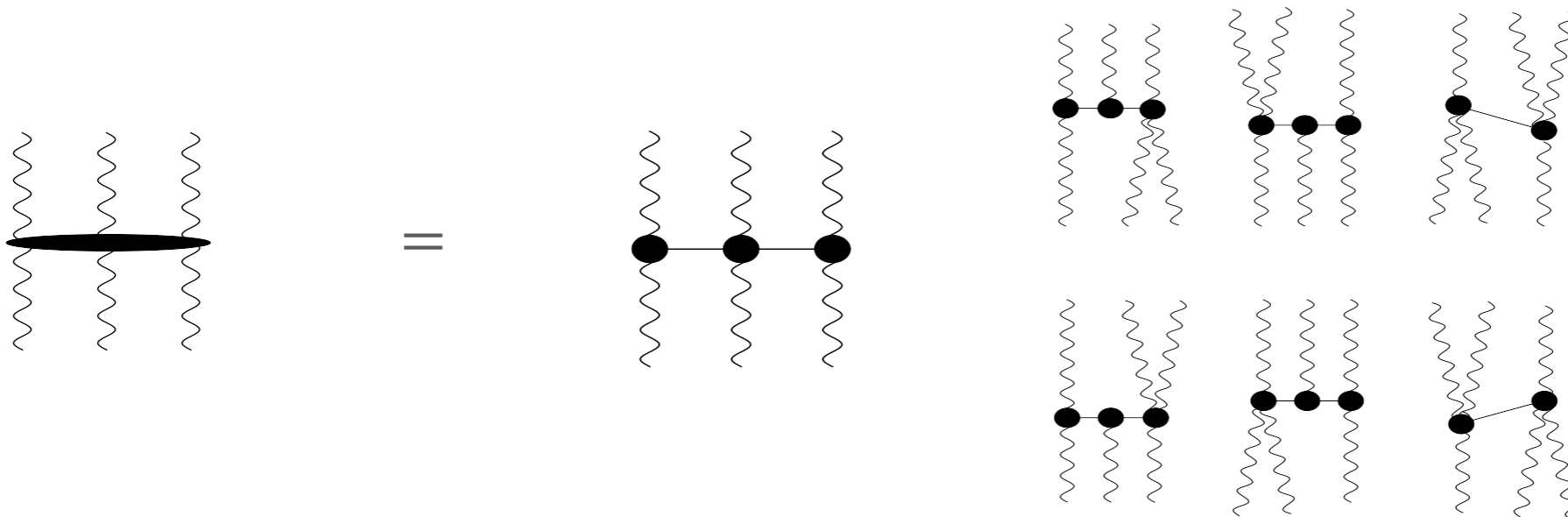
In coordinate space corresponds to the **same position** of **two gluons**

Described by the same **BFKL eigenvalue χ** but for odd value of **conformal spin n**

Odderon beyond lowest order

NLO corrections to the BKP equation *Bartels, Fadin, Lipatov, Vacca 2012*

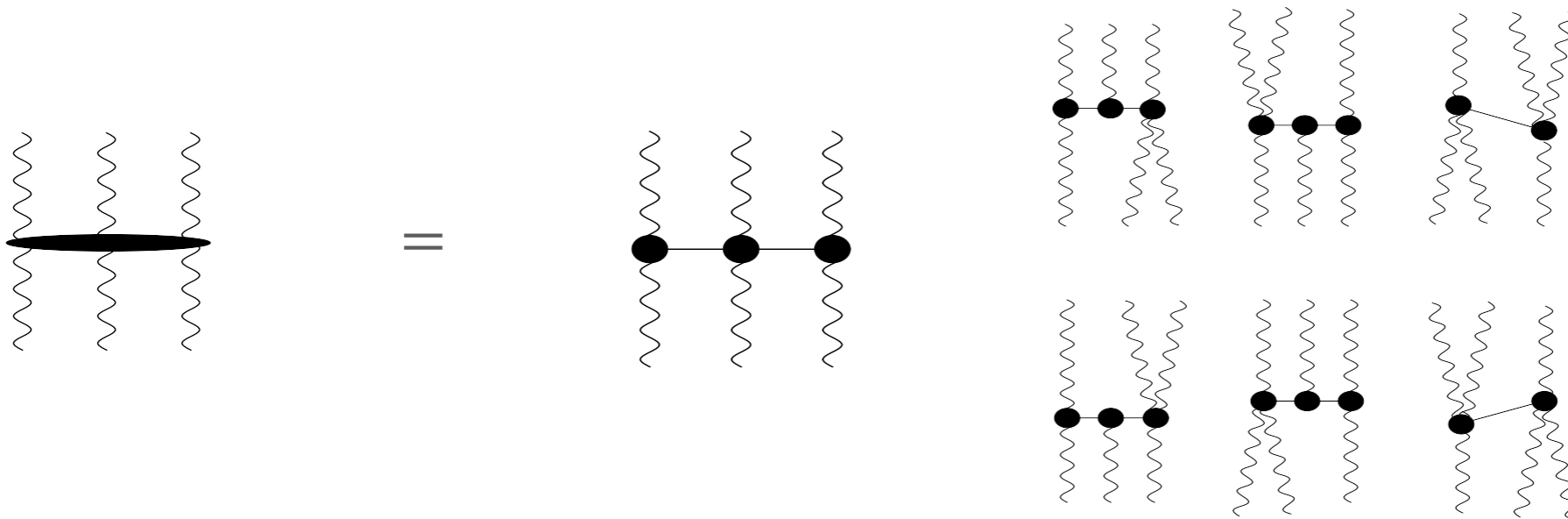
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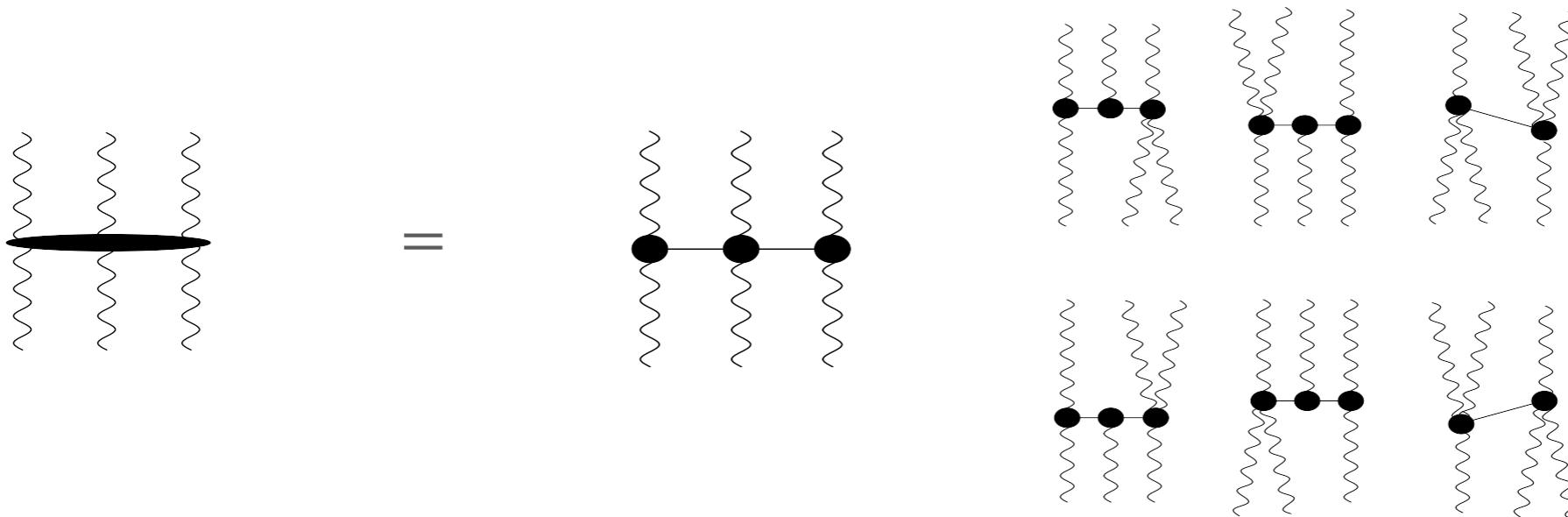


- Indications that the BLV Odderon intercept is 1 at this order, and perhaps beyond

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NLO BFKL in the octet representation and the connected 3-to-3 piece

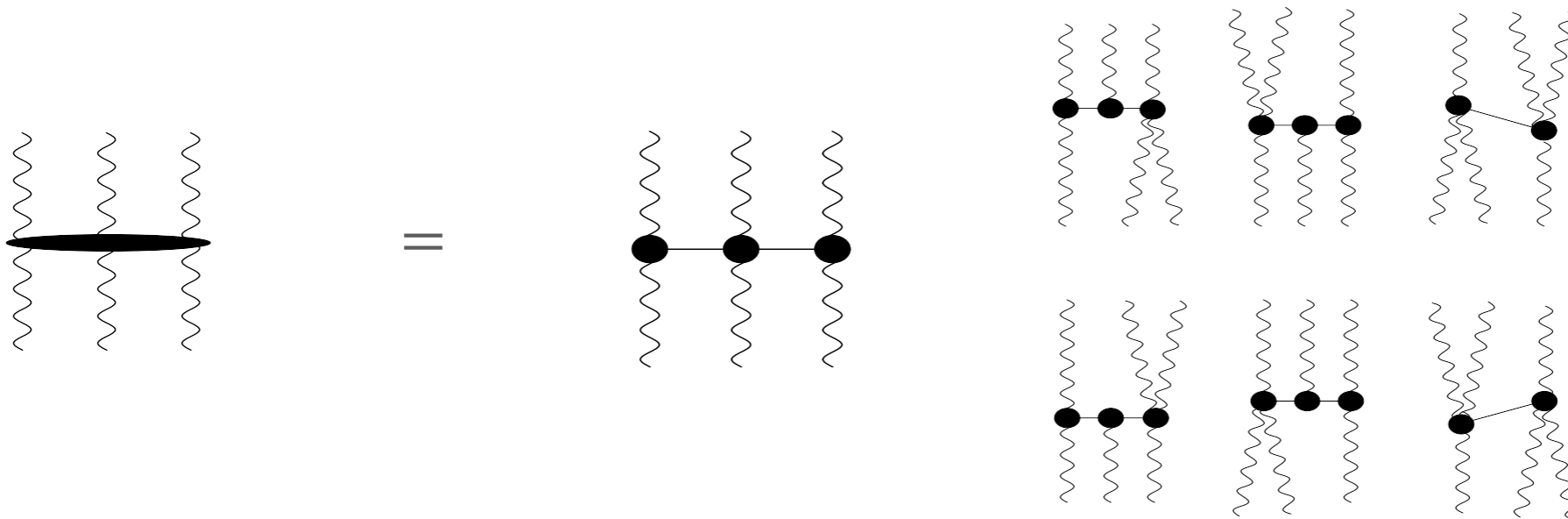


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Odderon beyond lowest order

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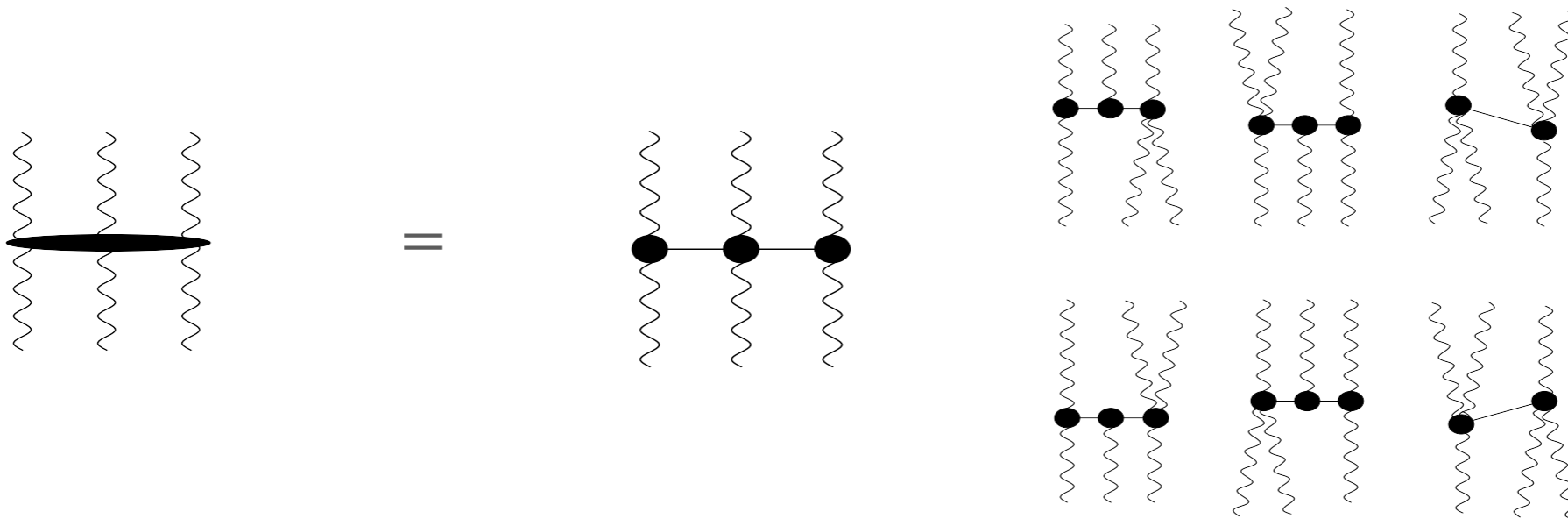


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 - The **ω expansion** for the high energy resummation suggests Odderon intercept could stay equal to 1 beyond the NLO order. *AS 2009*

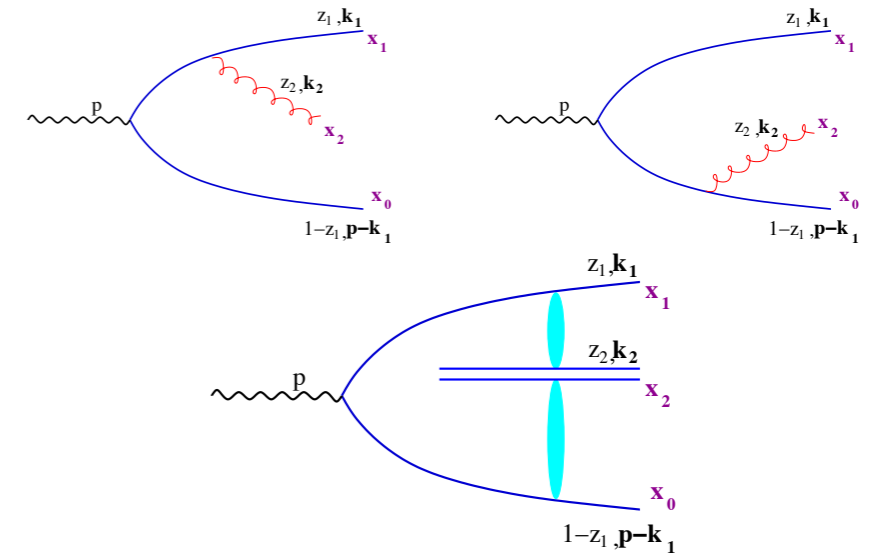
Odderon in dipole picture and unitarity corrections

But: Odderon can be modified by unitarity corrections

Can be computed using **dipole** evolution

Start from a quark-antiquark pair: a **dipole**
Emission of gluons : **splitting** of dipoles

Mueller 1994



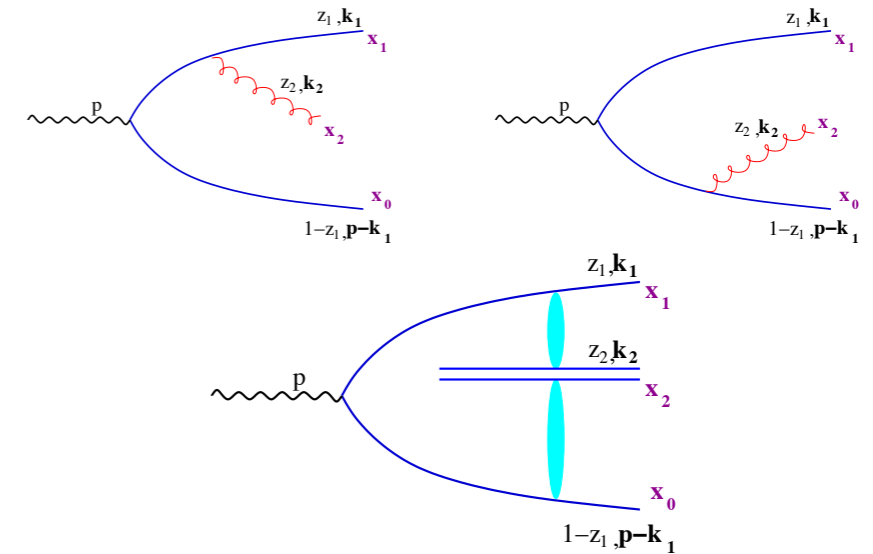
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BFKL equation in dipole framework

$$\frac{\partial N(\mathbf{x}, \mathbf{y}; Y)}{\partial Y} = \int d^2 z \mathcal{K}(\mathbf{x}, \mathbf{y}, \mathbf{z}) [N(\mathbf{x}, \mathbf{z}; Y) + N(\mathbf{z}, \mathbf{y}; Y) - N(\mathbf{x}, \mathbf{y}; Y)]$$

$N(\mathbf{x}, \mathbf{y}; Y)$ dipole amplitude (encodes interaction with the target)

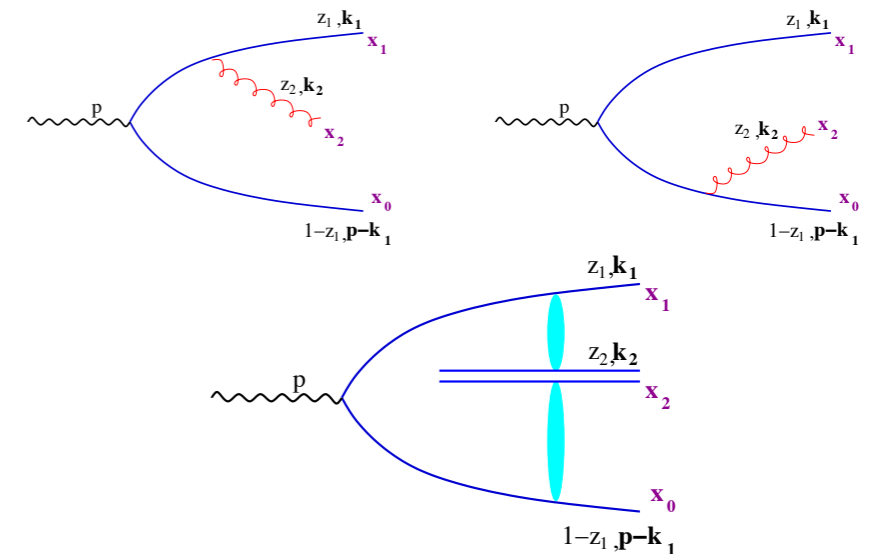
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BLV Odderon evolution equation in dipole framework is the same:

$$\frac{\partial O(\mathbf{x}, \mathbf{y}; Y)}{\partial Y} = \int d^2 z \mathcal{K}(\mathbf{x}, \mathbf{y}, \mathbf{z}) [O(\mathbf{x}, \mathbf{z}; Y) + O(\mathbf{z}, \mathbf{y}; Y) - O(\mathbf{x}, \mathbf{y}; Y)]$$

Initial condition for Odderon equation needs to be antisymmetric: $O(\mathbf{x}, \mathbf{y}; Y_0) = -O(\mathbf{y}, \mathbf{x}; Y_0)$

Odderon in dipole picture and unitarity corrections

Nonlinear evolution of the Odderon

Kovchegov, Szymanowski, Wallon 2003

Also derived within CGC/JIMWLK:

Hatta, Iancu, Itakura, McLerran 2005
Kovner, Lublinsky 2005

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Coupled to the Pomeron equation (**Balitsky-Kovchegov** equation with Odderon term)

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Solution:

- Pomeron is **leading** and unitarizes
- Odderon will be **suppressed**, absorbed by Pomeron

Motyka 2005

Experiments: elastic scattering at ISR

**Is there an experimental
evidence for Odderon ?**

Experiments: elastic scattering at ISR

Is there an experimental evidence for Odderon ?

VOLUME 54, NUMBER 20

PHYSICAL REVIEW LETTERS

20 MAY 1985

Measurement of $\bar{p}p$ and pp Elastic Scattering in the Dip Region at $\sqrt{s} = 53$ GeV

A. Breakstone,^{(1),(a)} H. B. Crawley,⁽¹⁾ G. M. Dallavalle,⁽⁵⁾ K. Doroba,⁽⁶⁾ D. Drijard,⁽³⁾ F. Fabbri,⁽³⁾ A. Firestone,⁽¹⁾ H. G. Fischer,⁽³⁾ H. Frehse,^{(3),(b)} W. Geist,^{(3),(c)} G. Giacomelli,⁽²⁾ R. Gokheli,⁽⁶⁾ M. Gorbics,⁽¹⁾ P. Hanke,⁽⁵⁾ M. Heiden,^{(3),(c)} W. Herr,⁽⁵⁾ E. E. Kluge,⁽⁵⁾ J. W. Lamsa,⁽¹⁾ T. Lohse,⁽⁴⁾ W. T. Meyer,⁽¹⁾ G. Mornacchi,⁽³⁾ T. Nakada,^{(5),(d)} M. Panter,⁽³⁾ A. Putzer,⁽⁵⁾ K. Rauschnabel,⁽⁴⁾ F. Rimondi,⁽²⁾ G. P. Siroli,⁽²⁾ R. Sosnowski,⁽⁶⁾ M. Szczekowski,⁽³⁾ O. Ullaland,⁽³⁾ and D. Wegener⁽⁴⁾

⁽¹⁾ Ames Laboratory and Iowa State University, Ames, Iowa 50011

⁽²⁾ Dipartimento di Fisica dell'Università and Istituto Nazionale di Fisica Nucleare, Bologna, Italy

⁽³⁾ CERN, European Organization for Nuclear Research, Geneva, Switzerland

⁽⁴⁾ Institut für Physik der Universität, Dortmund, Germany

⁽⁵⁾ Institut für Hochenergiephysik der Universität, Heidelberg, Germany

⁽⁶⁾ University and Institute for Nuclear Studies, Warsaw, Poland

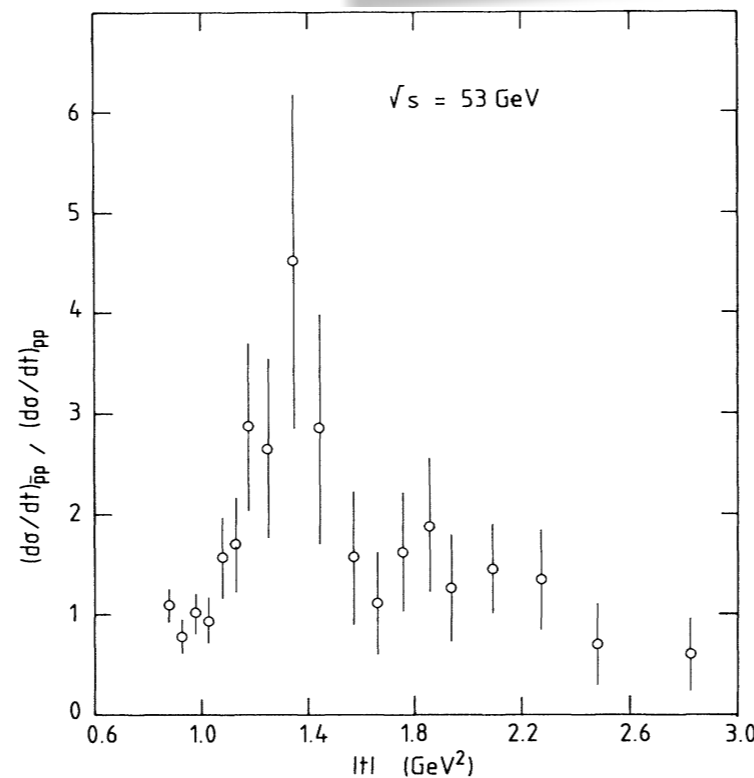
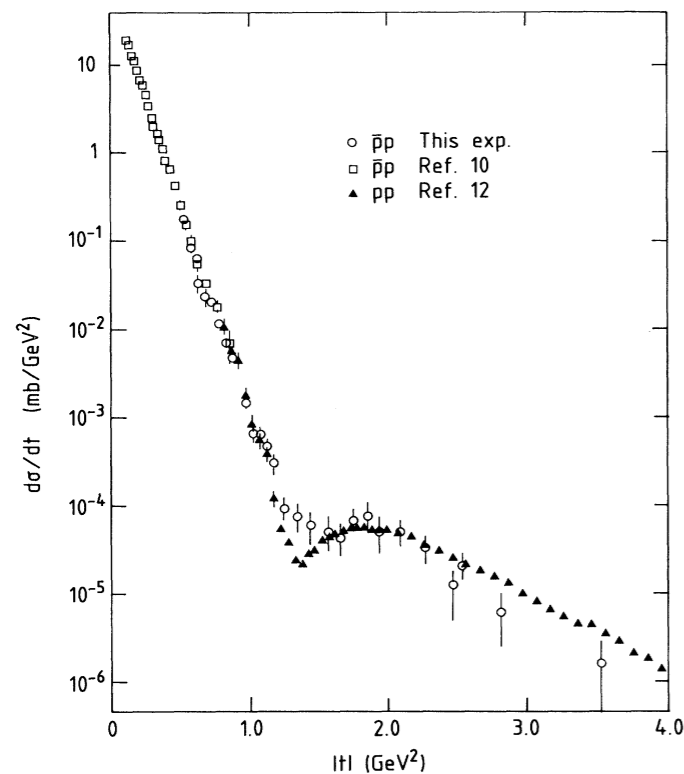
(Received 1 February 1985)

We have measured the differential cross section for $\bar{p}p$ and pp elastic scattering at $\sqrt{s} = 53$ GeV in the interval $0.5 < |t| < 4.0$ (GeV/c)² at the CERN intersecting storage rings using the split-field magnet detector. The shape of the differential cross section differs significantly between $\bar{p}p$ and pp scattering in the region $1.1 < |t| < 1.5$ (GeV/c)², with $\bar{p}p$ data showing a less pronounced dip structure than pp data.

Experiments: elastic scattering at ISR

Is there an experimental evidence for Odderon ?

$$\frac{d\sigma_{el}}{dt}$$



In particular, a difference between the $\bar{p}p$ and pp cross sections in the dip region [four-momentum transfer squared around 1.4 (GeV/c)^2] may indicate the presence of terms in the elastic scattering amplitude which are odd under crossing symmetry. The recent mea-

Based on a simple χ^2 calculation, the hypothesis that the $\bar{p}p$ and pp cross sections are equal (i.e., the ratio is 1) in the region $1.1 < |t| < 1.5 \text{ (GeV/c)}^2$ can be excluded to the 99.9% confidence level ($\chi^2/\text{d.o.f.} = 21/5$).

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ISR data and fits with Odderon

Donnachie, Landshoff 1984

Fit included:

Reggeon

Pomeron,

double Pomeron,

triple Pomeron,

Reggeon-Pomeron,

Pomeron and 2 gluons

Odderon

Prediction of no dip in $p\bar{p}$

Nuclear Physics B231 (1984) 189-204
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pp AND $p\bar{p}$ ELASTIC SCATTERING

A. DONNACHIE

Department of Theoretical Physics, University of Manchester, UK

P.V. LANDSHOFF

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK

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We present an analysis of pp and $p\bar{p}$ elastic scattering in terms of various exchanges. Three-gluon exchange dominates at large t , and single-pomeron exchange at small t . The dip seen in high-energy pp scattering is provided by the interference of both of these with double-pomeron exchange. We predict that this dip will not be found in high-energy $p\bar{p}$ scattering. The dip that is seen in low-energy $p\bar{p}$ scattering is the result of the additional presence of reggeon-pomeron exchange.

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- But: Odderon one of many contributions,
- Different models quite different parameters, eg. *Gauron, Nicolescu, Leader*
- Only one measurement, no s dependence
- Could not draw ultimate conclusions

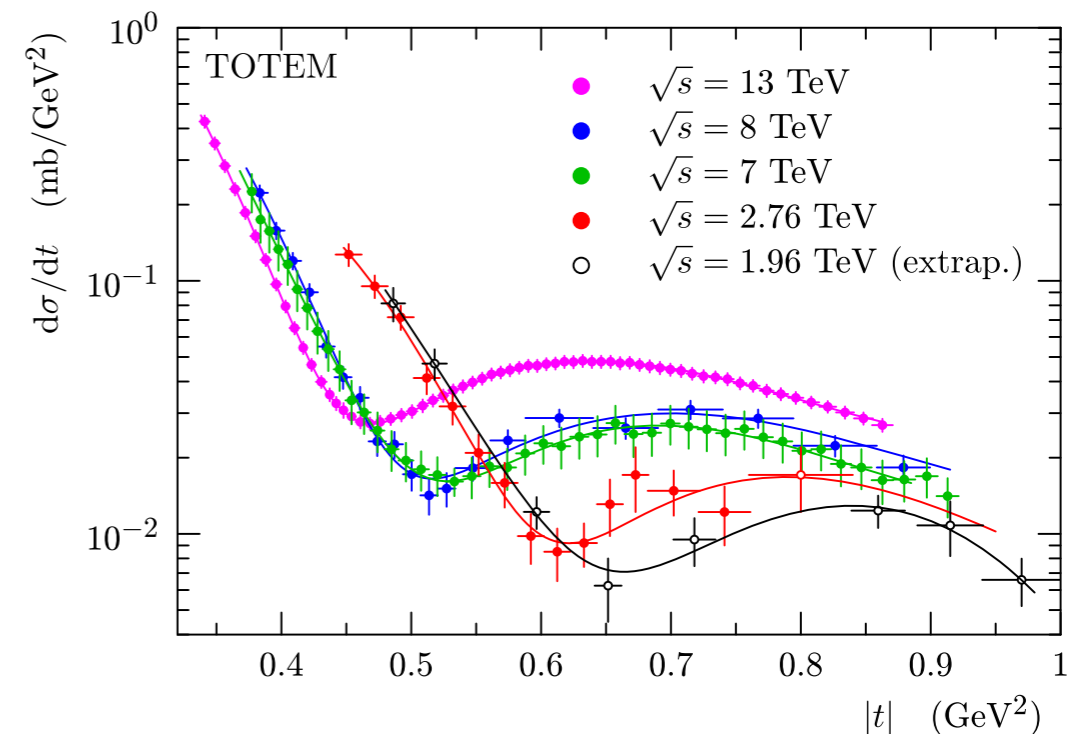
TOTEM-D0

PHYSICAL REVIEW LETTERS **127**, 062003 (2021)

Odderon Exchange from Elastic Scattering Differences between pp and $p\bar{p}$ Data at 1.96 TeV and from pp Forward Scattering Measurements

TOTEM & D0 collaborations

Idea by TOTEM & D0: take **LHC pp data** at 2.76, 7, 8, 13 TeV measured by TOTEM and **extrapolate down in energy** to 1.96 TeV to compare with **D0 data from $p\bar{p}$** Tevatron



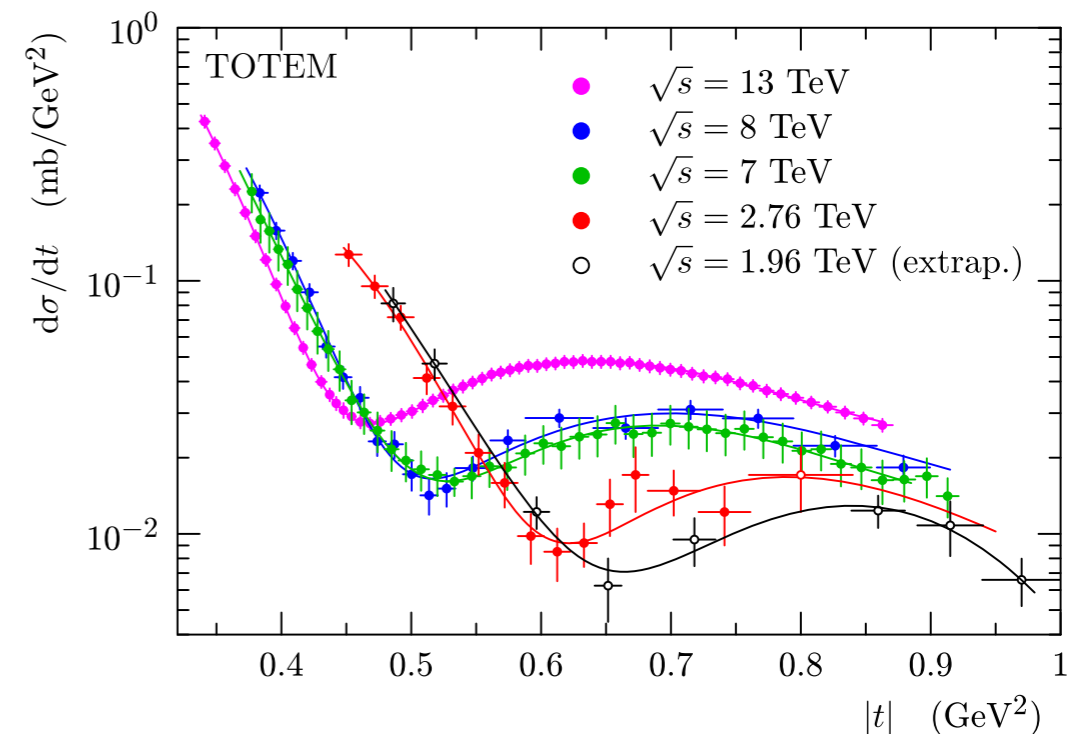
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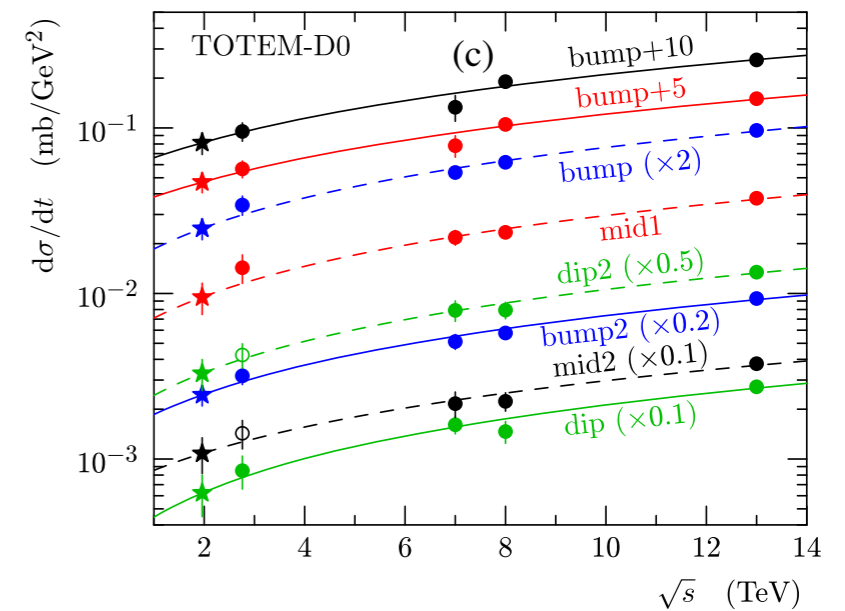
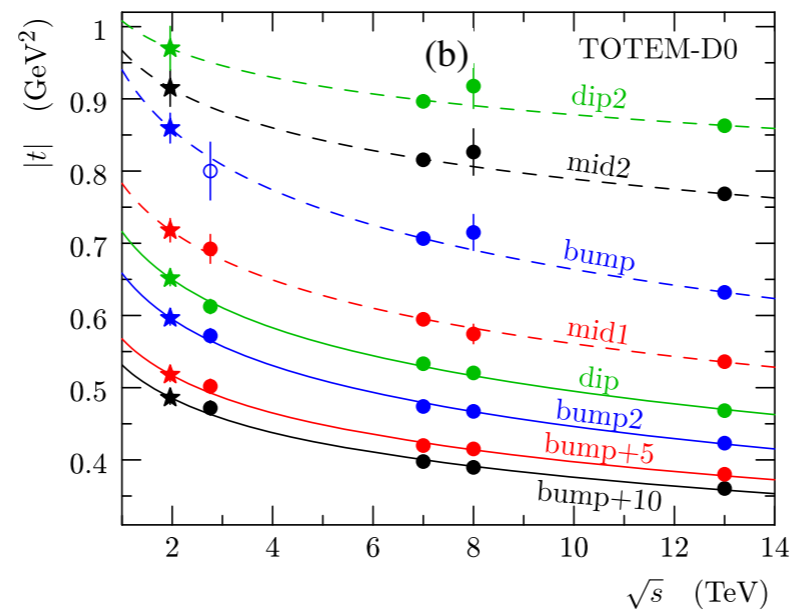
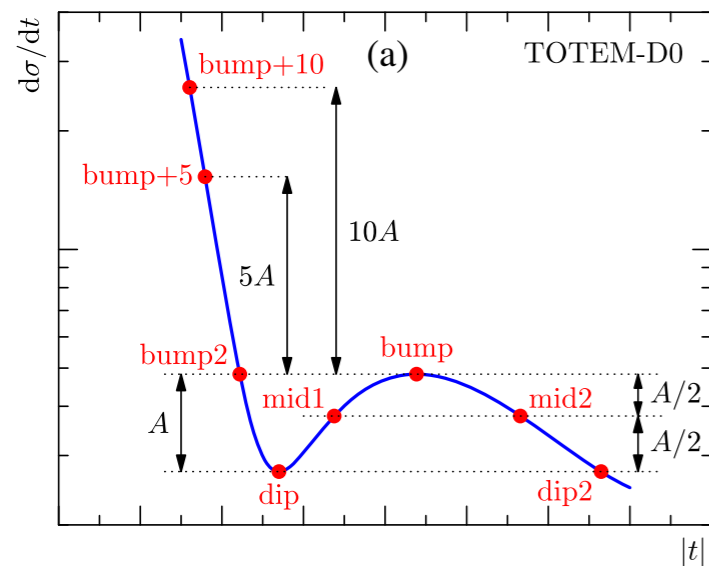


Note: scaling of elastic cross section

Baldenegro, Royon, AS 2022
Baldenegro, Praszalowicz, Royon, AS 2024

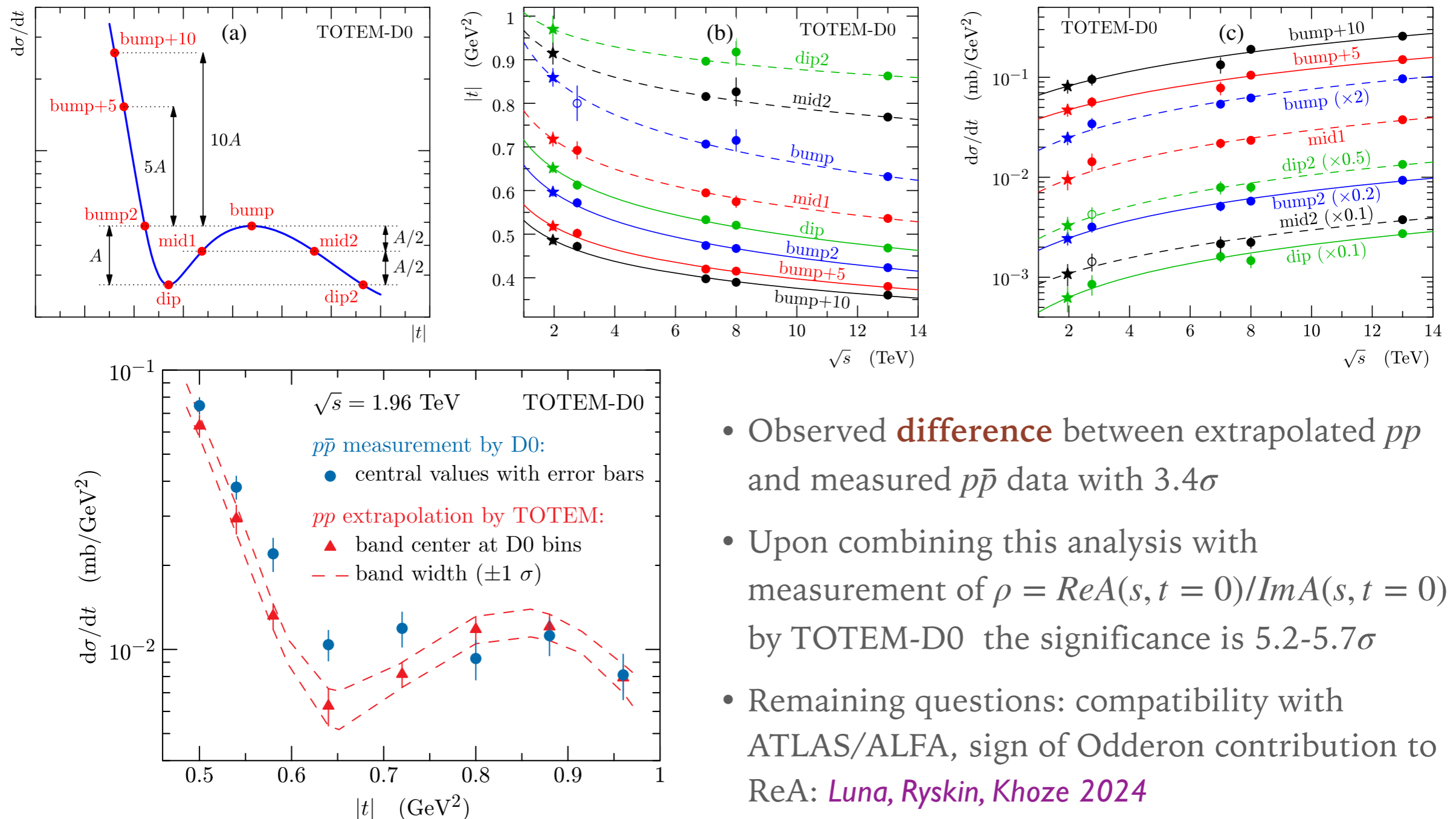
TOTEM-D0 analysis

- Selected points in the elastic cross section around **dip-bump** region
- Fit to these points and **extrapolate to 1.96 TeV**



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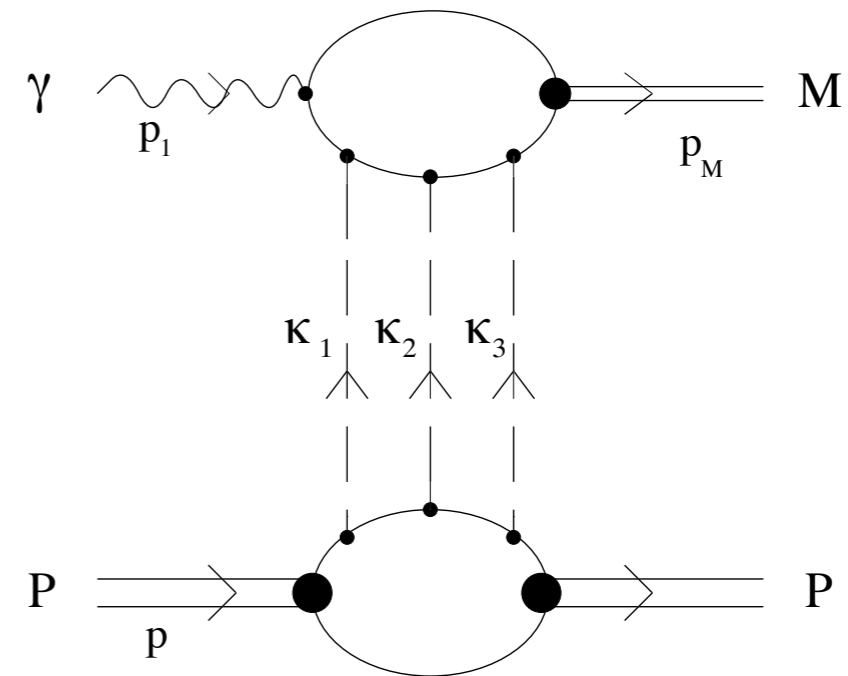
- Observed **difference** between extrapolated pp and measured $p\bar{p}$ data with 3.4σ
- Upon combining this analysis with measurement of $\rho = ReA(s, t = 0)/ImA(s, t = 0)$ by TOTEM-D0 the significance is $5.2-5.7\sigma$
- Remaining questions: compatibility with ATLAS/ALFA, sign of Odderon contribution to ReA : [Luna, Ryskin, Khoze 2024](#)

Odderon in ep collider

Schäfer, Mankiewicz, Nachtmann 1991

Testing Odderon in ep : **diffractive**
production of **$C=+1$ mesons**

Opposite C of photon and meson: this channel is sensitive to Odderon exchange



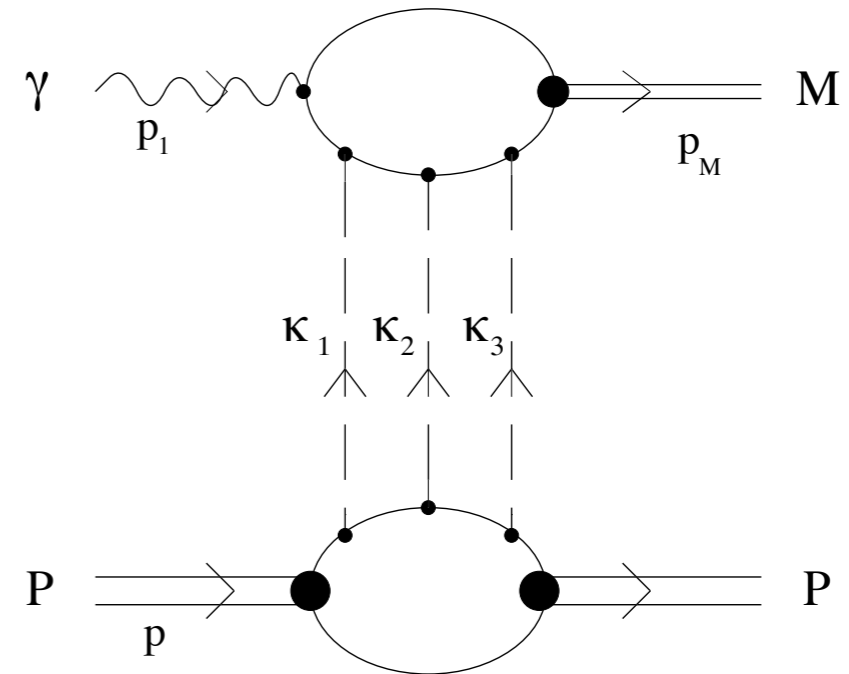
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$$M = \pi^0, a_2, \eta_c, \chi_c, \dots$$

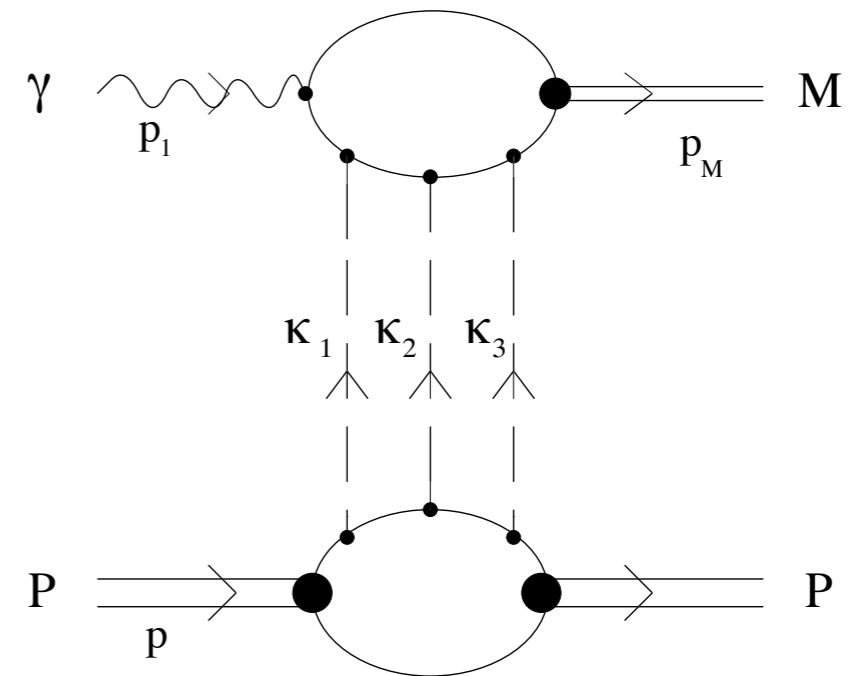


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Czyżewski, Kwieciński, Sadzikowski, Motyka 1996

η_c total cross section at $Q^2 = 0$: 11-45 pb

Engel, Ivanov, Kirschner, Szymanowski 1997

studied π^0, a_2, η_c

H1 measurement of exclusive π^0



Physics Letters B 544 (2002) 35–43

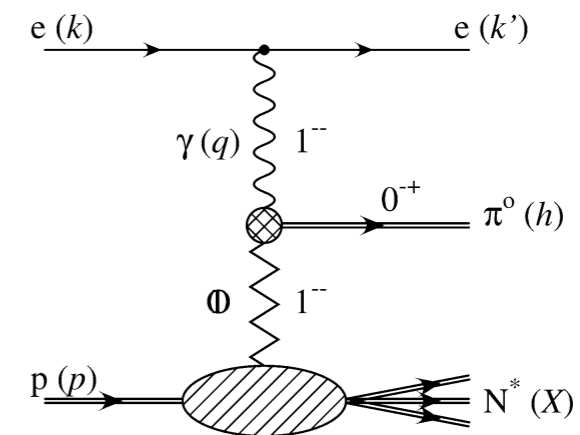
PHYSICS LETTERS B

www.elsevier.com/locate/npe

Search for odderon-induced contributions to exclusive π^0
photoproduction at HERA

H1 Collaboration

$$ep \rightarrow e\pi^0 N^*$$



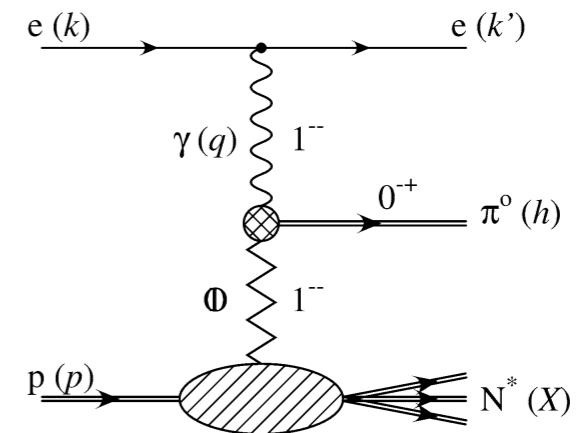
H1 measurement of exclusive π^0



Physics Letters B 544 (2002) 35–43

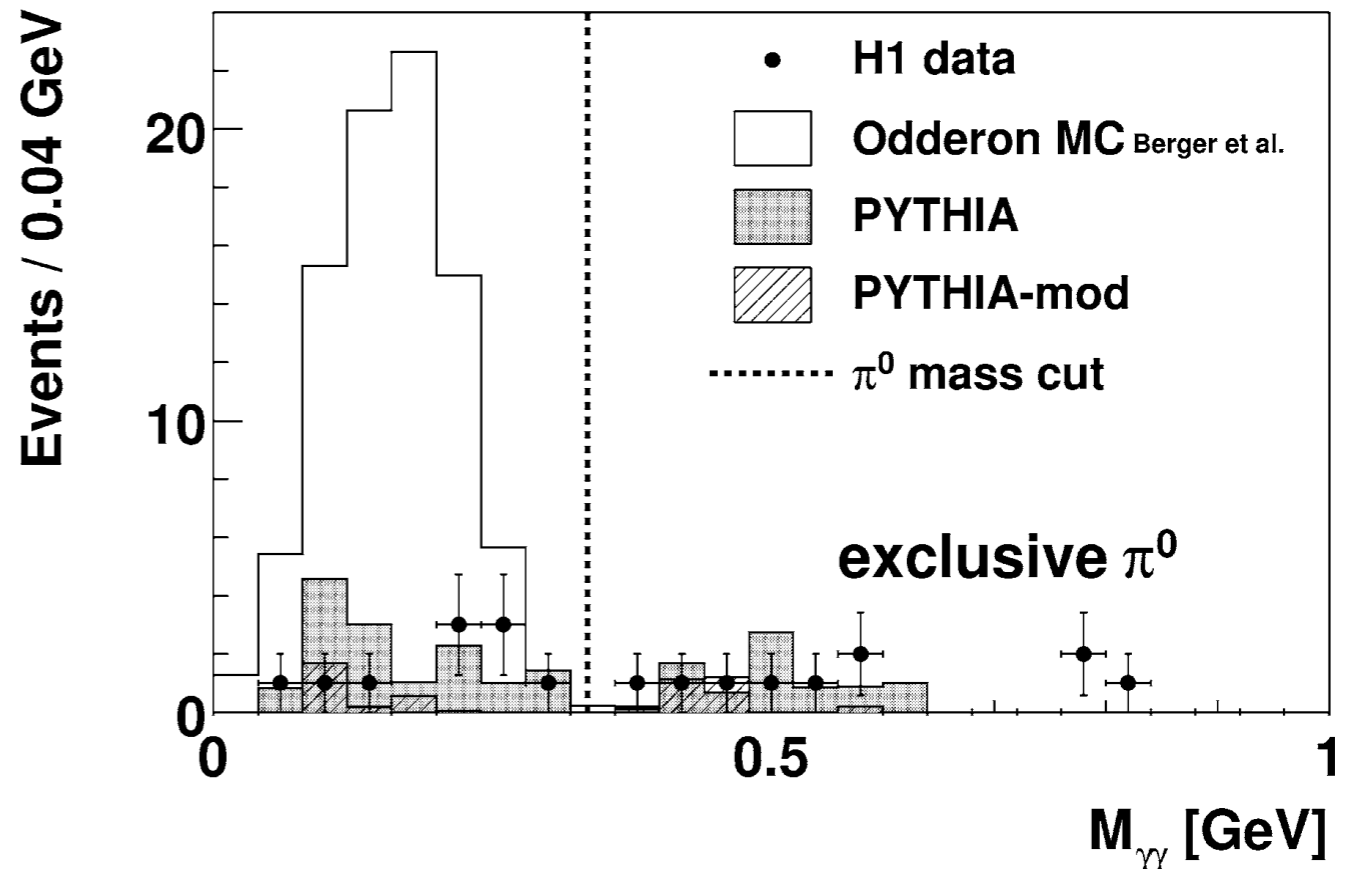
PHYSICS LETTERS B

www.elsevier.com/locate/npe



Search for odderon-induced contributions to exclusive π^0 photoproduction at HERA

H1 Collaboration



Data compatible with background

$$\sigma_{\gamma p \rightarrow \pi^0 N^*} (\gamma \mathbb{O} \text{ fusion}) < 49 \text{ nb}$$

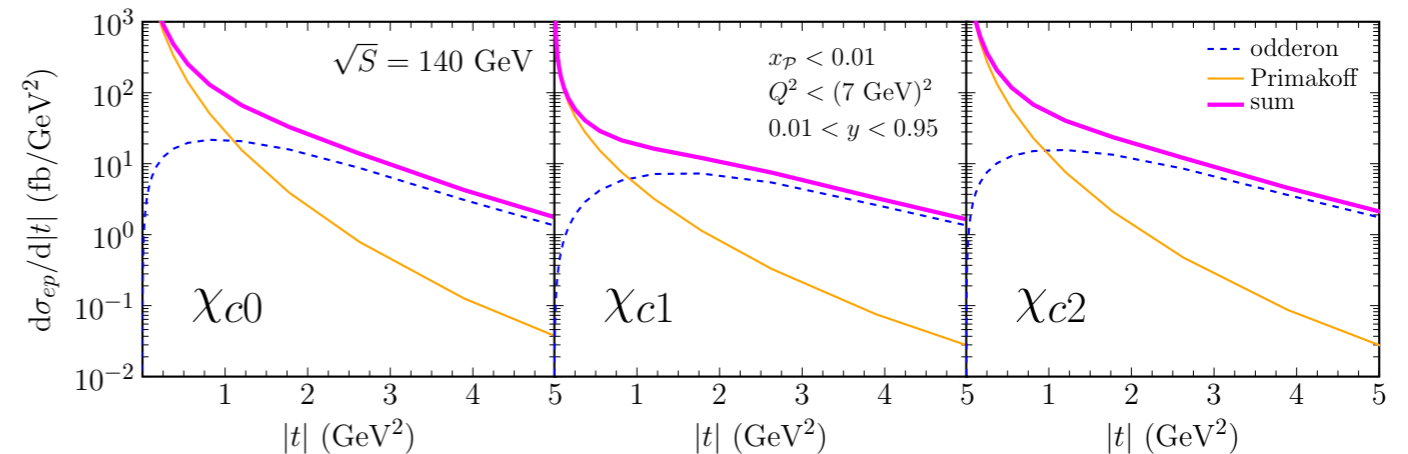
Prospects for Odderon searches: selected examples

ep collider

Electron Ion Collider EIC:

100-1000 higher luminosity than HERA

predictions for exclusive χ_c at EIC due to Odderon exchange

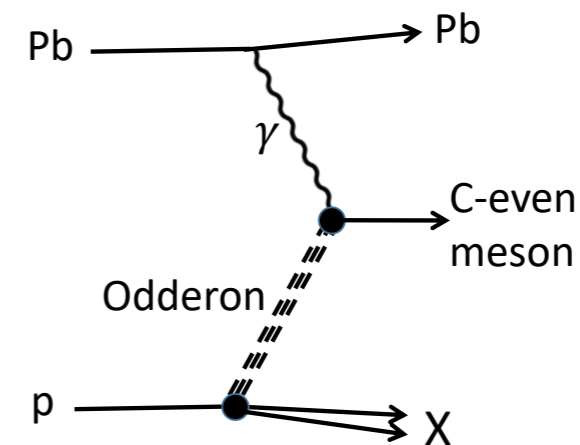
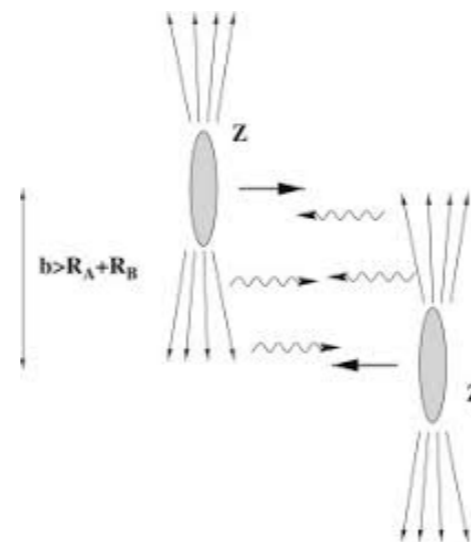


Benić, Dumitru, Kaushik, Motyka, Stebel 2024

pp collider: LHC

ultraperipheral collisions

LHC as **photon-photon** collider



Harland-Lang, Khoze, Martin, Ryskin 2018

Also : charge asymmetries in diffractive γp , Pomeron-Odderon in pp, Siverts function



Image by DALL.E