Observational Signatures of Quantum Gravity

Maulik Parikh — Arizona State University

Zakopane 2024

Near-Impossibility of Seeing Quantum Gravity at Accelerators

Dimensionless quantity involving Planck's constant, Newton's constant and energy:

$$\frac{\hbar c^5}{G_N E^2}$$

This is of order unity when

$$E \sim 10^{19} GeV$$

Near-Impossibility of Detecting Single Gravitons in a Gravitational Wave

Energy density of gravitational wave with amplitude h and angular frequency omega:

$$\rho = \frac{1}{4} h^2 \omega^2 M_P^2$$

Number density of gravitons:

$$n = \frac{\rho}{\hbar \omega}$$

Number in one cubic wavelength:

$$N = n \lambda^3$$

Number of gravitons in typical LIGO GW:

$$N \sim 10^{35}$$

F. Dyson (2013), "Is a graviton detectable?"

Why Quantum Gravity could be Observable: Lessons from EM

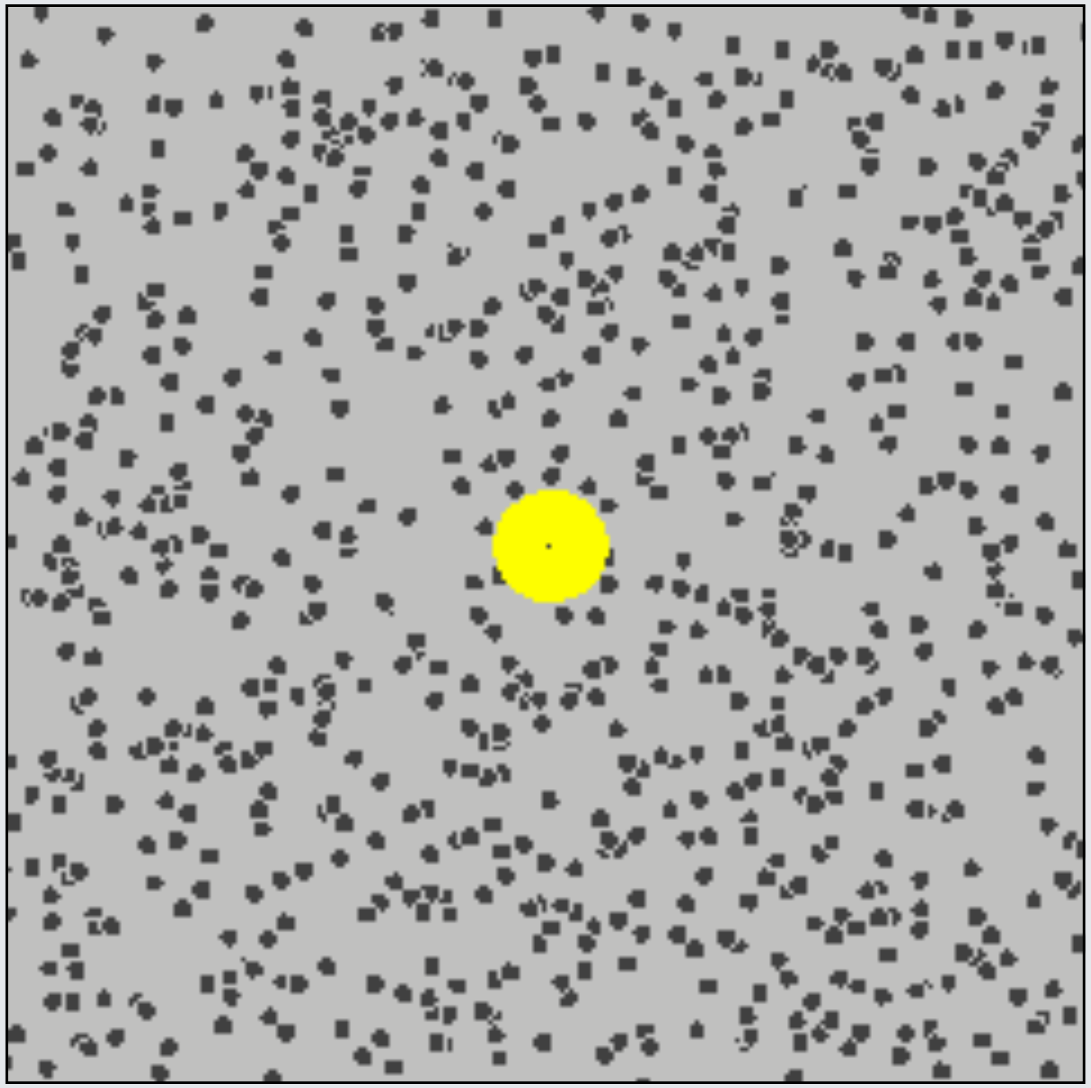
Many **observed phenomena** show that the **electromagnetic field** is **quantized:**

Photon anti-bunching, entangled photons, sub-Poissonian statistics,
Compton effect, Lamb shift, …

Most of these are ***tree-level*** effects in a **state** that has no classical counterpart

The same is true for gravity: there can be potentially observable effects
if the quantum state of the gravitational field is **not** a coherent state

Why Quantum Gravity could be Observable: Lessons from Brownian Motion



Even if collisions with individual molecules cannot be detected, a **cumulative effect** can be apparent

Potential Loopholes

Non-classical states

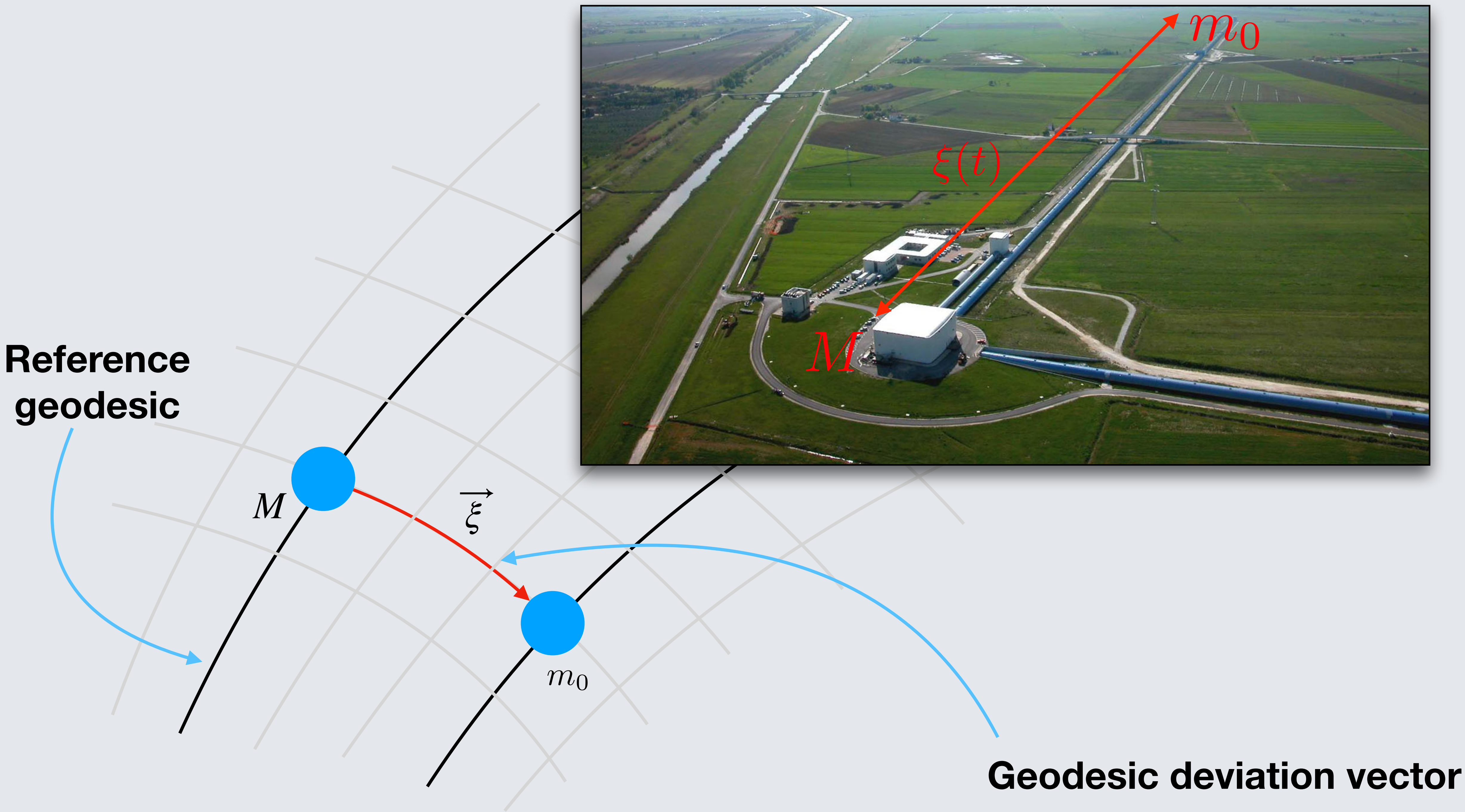
Cumulative effects

Breakdown of effective field theory e.g. non-locality

Vast number of states of quantum black holes

How We Probe Gravity

Geodesic Deviation



Geodesic Deviation Equation

$$\ddot{\xi}^{\mu} = -R^{\mu}_{0\nu 0}\xi^{\nu}$$

geodesic deviation

Riemann tensor

$$R_{i0j0}(t,0) = -\frac{1}{2}\ddot{h}_{ij}(t,0)$$

gravitational wave

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

This is the geodesic deviation equation in the presence of a **classical** gravitational wave

Quantum Geodesic Deviation Equation?

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

What is the generalization of this equation when the spacetime metric is treated as a **quantum** field?

"The Noise of Gravitons," arXiv:2005.07211

The Noise of Gravitons

Maulik Parikh*×1, Frank Wilczek*†2, and George Zahariade×3

*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA
×Beyond: Center for Fundamental Concepts in Science, Arizona State University, Tempe, Arizona 85287, USA
†Department of Physics, Stockholm University, Stockholm SE-106 91, Sweden
†Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
†Wilczek Quantum Center, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

Abstract

We show that when the gravitational field is treated quantum-mechanically, it induces fluctuations – noise – in the lengths of the arms of gravitational wave detectors. The characteristics of the noise depend on the quantum state of the gravitational field, and can be calculated exactly in several interesting cases. For coherent states the noise is very small, but it can be greatly enhanced in thermal and (especially) squeezed states. Detection of this fundamental noise would constitute direct evidence for the quantization of gravity and the existence of gravitons.

This essay was awarded First Prize in the 2020 Essay Competition of the Gravity Research Foundation.

[1]maulik.parikh@asu.edu
[2]frank.wilczek@asu.edu
[3]george.zahariade@asu.edu

"Quantum Mechanics of Gravitational Waves," PRL, arXiv:2010.08205

"Signatures of the Quantization of Gravity at Gravitational Wave Detectors," PRD, arXiv:2010.08208

Frank Wilczek

George Zahariade

Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - M\int d\lambda\sqrt{-g_{\mu\nu}\frac{dX^\mu}{d\lambda}\frac{dX^\nu}{d\lambda}} - m_0\int d\tau\sqrt{-g_{\mu\nu}\frac{dY^\mu}{d\tau}\frac{dY^\nu}{d\tau}}$$

Einstein-Hilbert action + action for two free-falling particles

Use Fermi normal coordinates, putting mass M on classical trajectory

$$X^\mu = (t, \vec{0})$$

Let the other particle be at

$$Y^\mu = (t, \vec{\xi})$$

Action

Next, insert metric in Fermi normal coordinates into particle action:

$$g_{00}(t,\xi) = -1 - R_{i0j0}(t,0)\xi^i\xi^j + O(\xi^3)$$
$$g_{0i}(t,\xi) = -\frac{2}{3}R_{0jik}(t,0)\xi^j\xi^k + O(\xi^3)$$
$$g_{ij}(t,\xi) = \delta_{ij} - \frac{1}{3}R_{ikjl}(t,0)\xi^k\xi^l + O(\xi^3)\ .$$

Inserting this into the particle action gives

$$-m_0\int d\tau\sqrt{-g_{\mu\nu}\frac{dY^\mu}{d\tau}\frac{dY^\nu}{d\tau}} \approx -m_0\int dt\left(\frac{1}{2}R_{i0j0}(t,0)\xi^i\xi^j - \frac{1}{2}\delta_{ij}\dot{\xi}^i\dot{\xi}^j\right)$$

Action

Expanding action to lowest order in metric perturbation, we have:

$$S = -\frac{1}{64\pi G}\int d^4x\ \partial_\mu h_{ij}\partial^\mu h^{ij} + \int dt \frac{1}{2} m_0 \left(\delta_{ij}\dot{\xi}^i\dot{\xi}^j - \dot{h}_{ij}\dot{\xi}^i\xi^j\right)$$

Write metric perturbation in Fourier modes:

$$h_{ij}(t,\vec{x}) = \frac{1}{\sqrt{\hbar G}}\sum_{\vec{k},s} q_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}}\epsilon^s_{ij}(\vec{k})$$

Then

$$S = \int dt \frac{1}{2} m_0 \dot{\xi}^2 + \int dt \sum_{\vec{k},s}\frac{1}{2} m\left(\dot{q}^2_{\vec{k},s} - \omega^2_{\vec{k}} q^2_{\vec{k}}\right) - g\int dt \sum_{\vec{k},s}\dot{q}_{\vec{k},s}\epsilon^s_{ij}(\vec{k})\dot{\xi}^i\xi^j$$

Geodesic Deviation in the Presence of a Graviton Mode

free particle

$$S_\omega = \int dt \left(\frac{1}{2} m (\dot{q}^2 - \omega^2 q^2) + \frac{1}{2} m_0 \dot{\xi}^2 - g \dot{q} \dot{\xi} \xi \right)$$

simple harmonic oscillator

cubic interaction term

where $g \equiv \dfrac{m_0}{2\sqrt{\hbar G}}$

Quantization Strategy

We will treat **both** the deviation/second particle/mirror and gravity **quantum mechanically.**

We will then **integrate out gravity,** giving the effective dynamics of the geodesic deviation in the presence of quantized gravity.

Quantum Mechanics

Suppose the gravitational field is initially in state $|\Psi\rangle$

We don't know what the final state of the field is.

Formally, we calculate the transition probability of the particle, or second mirror, to go from state A to state B in time T in the presence of a gravitational field that is initially in state $|\Psi\rangle$

$$P_\Psi(A \to B) = \sum_{|f\rangle} |\langle f, B|\hat{U}(T)|\Psi, A\rangle|^2$$

The relatively simple form of the action allows the calculation to be performed **exactly**

Transition Probability

wave-function dependent term

noise

auto-correlation function

$$P_\Psi = \mathcal{I}_{A,B} \int \tilde{\mathcal{D}}\xi \tilde{\mathcal{D}}\xi' \mathcal{D}\mathcal{N}_\Psi \exp\left[-\frac{1}{2}\int_0^T \int_0^T dt\, dt'\, A_\Psi^{-1}(t-t')\mathcal{N}_\Psi(t)\mathcal{N}_\Psi(t')\right] \times$$
$$\exp\left[\frac{i}{\hbar}\int_0^T dt \left\{\frac{1}{2}m_0\left(\dot{\xi}^2 - \dot{\xi}'^2\right) + \frac{1}{4}m_0\left(h(t) + \mathcal{N}_\Psi(t)\right)\left(X(t) - X'(t)\right)\right\}\right.$$
$$\left. - \frac{i m_0^2 G}{8\hbar}\int_0^T dt\, \left(X(t) - X'(t)\right)\left(\dot{X}(t) + \dot{X}'(t)\right)\right]$$

fluctuation term

classical wave

dissipation term

where $X(t) = \frac{d^2}{dt^2}(\xi^2)$

"Signatures of the Quantization of Gravity at Gravitational Wave Detectors," arXiv:2010.08208

Langevin Equation

Taking the classical limit for the geodesic deviation, we find

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{h} + \ddot{N}_{\Psi} - \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2\right)\xi$$

classical gravitational wave

quantum noise

radiation reaction

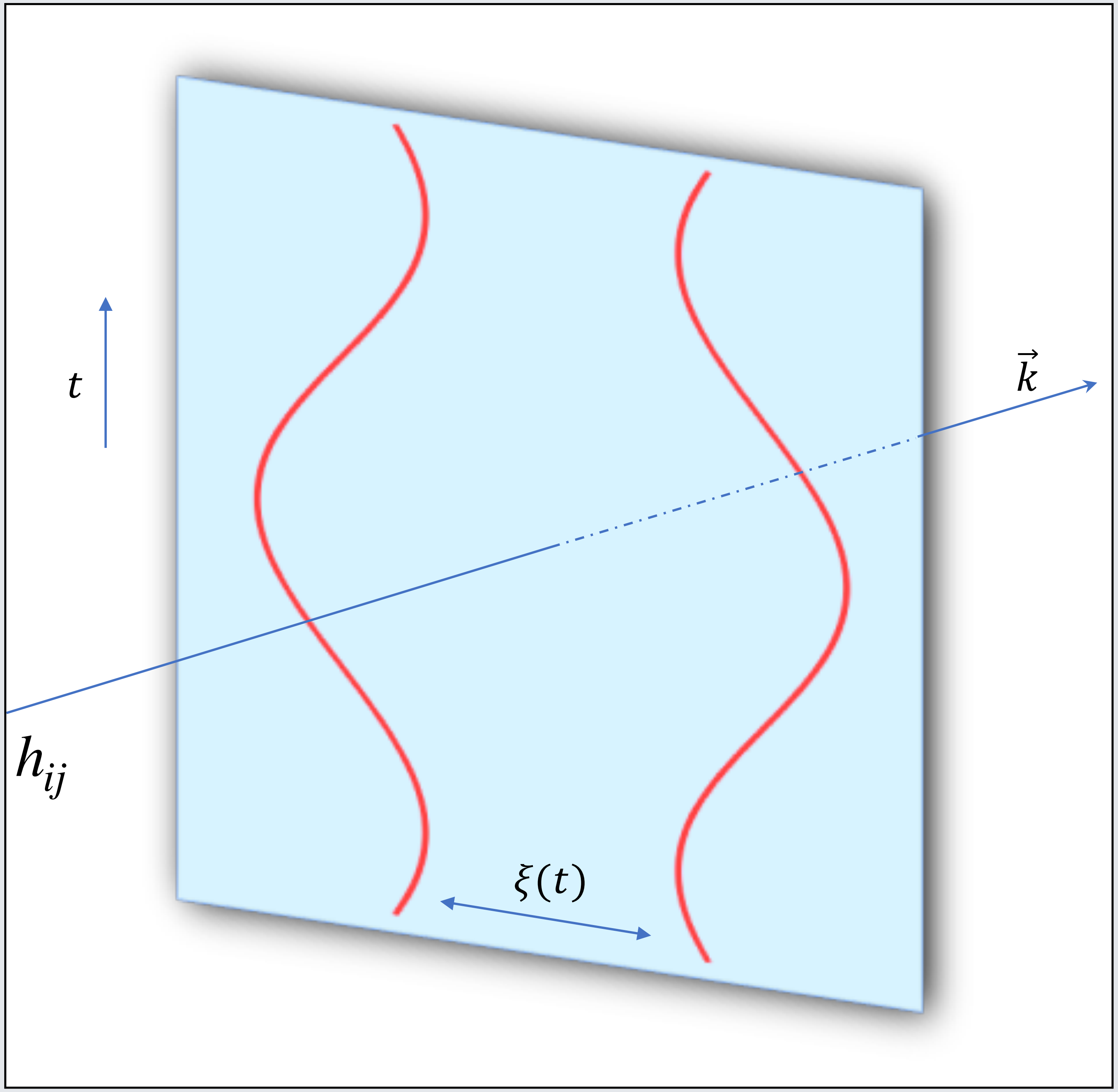
This is the geodesic deviation equation in **quantum** spacetime

Compare classical geodesic deviation: $\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$

Because of the noise term, the new equation is a **stochastic** equation

Classical Geodesic Separation by Gravitational Waves

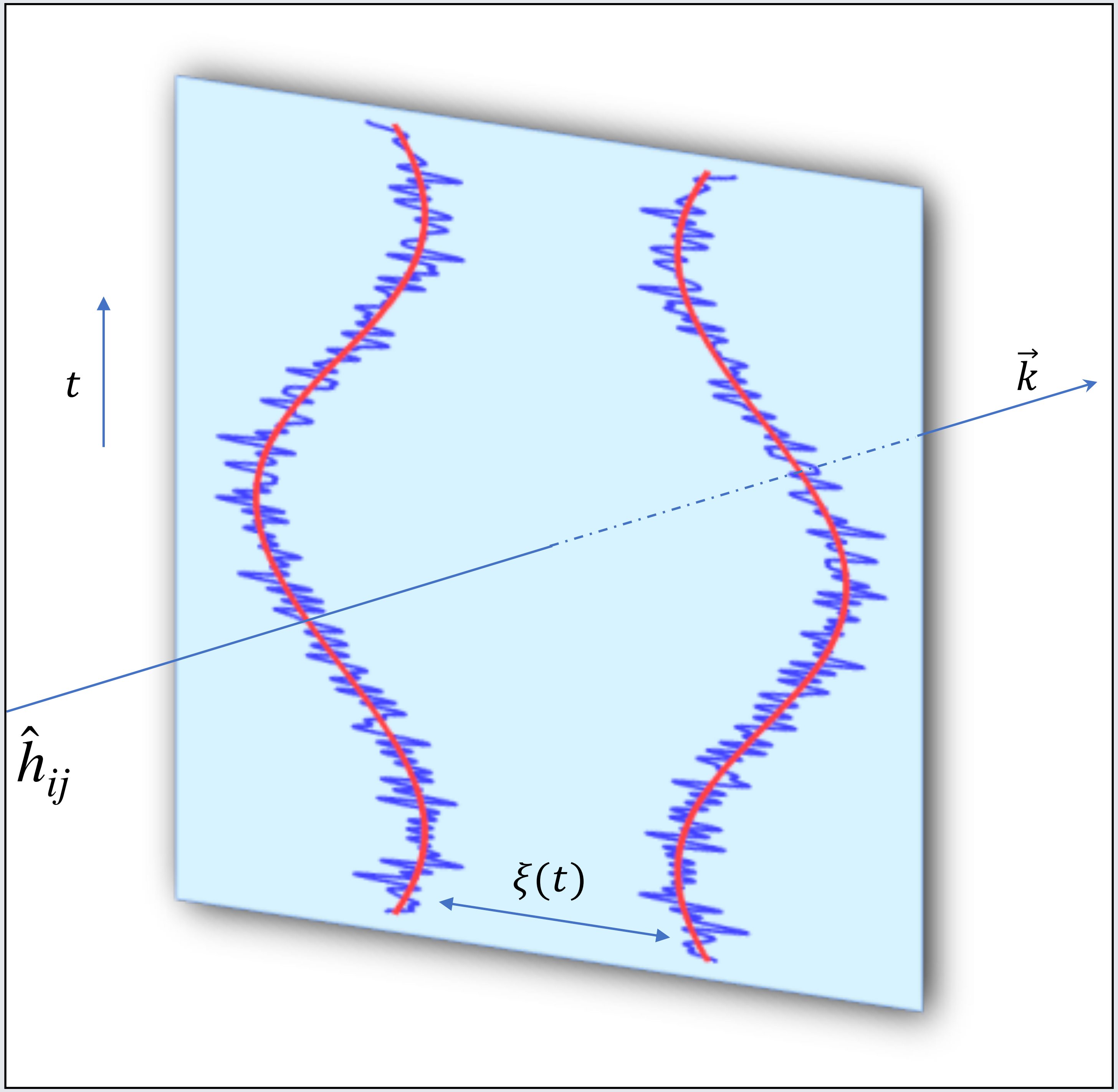
$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$



classical gravitational wave

The Noise of Gravitons

$$\ddot{\xi} \approx \frac{1}{2}\left(\ddot{h} + \ddot{N}_{\Psi}\right)\xi$$



quantized gravitational wave

Main Message

The signal of **quantum gravity** is in the **noise.**

Is The Noise Detectable?

For the noise to be detectable:

1. Its **spectrum** should be distinguishable from other sources of noise

2. Its **amplitude** should not be too small

Noise Spectrum

The auto-correlation function and noise power spectrum are **exactly calculable** for many classes of quantum states
(the vacuum, thermal states, coherent states, squeezed states…)

$$A_{\text{vac}}(t,t') \approx \frac{1}{(t-t')^2}$$

$$S_{\text{vac}}(\omega) = \frac{16G\hbar}{15\pi^2}\omega$$

$$A_{\text{squeezed}}(t,t') \approx \cosh(r)\frac{1}{(t-t')^2}$$

$$S_{\text{squeezed}}(\omega) \approx e^r\,\frac{16G\hbar}{15\pi^2}\omega$$

$$A_{\text{thermal}}(t,t') \approx \frac{1}{(t-t')^2} - \frac{\pi^2 k_B^2 T^2}{\hbar^2 \sinh^2(\frac{\pi k_B T (t-t')}{\hbar})}$$

Noise Correlations

We can consider **two** separated detector arms

Since our noise originates directly from the source, both arms will detect the **same** noise

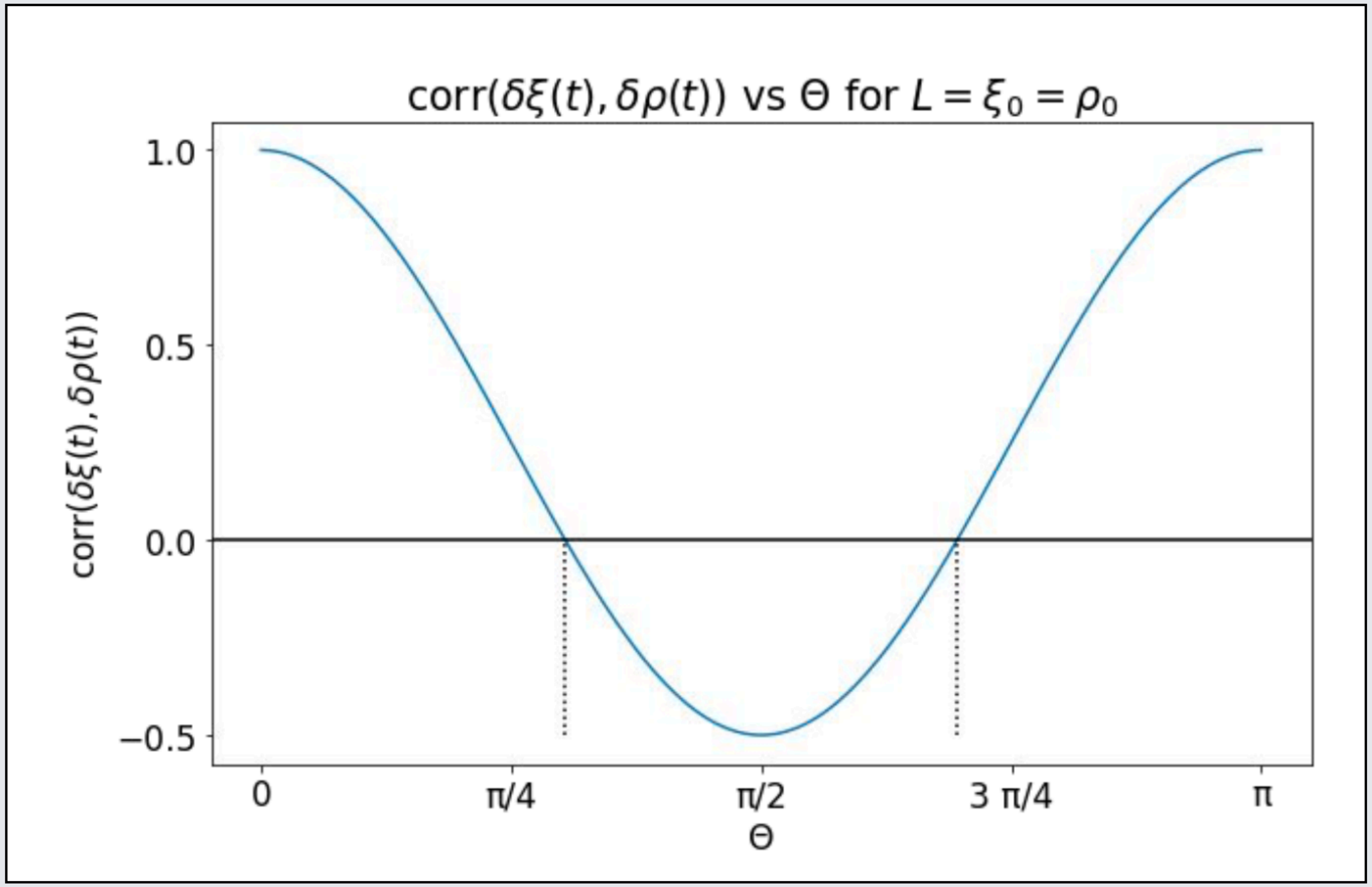
Quantum-gravitational noise is **correlated** across detectors

"Graviton Noise Correlation in Nearby Detectors," MP, F. Setti, arXiv:2312.17335

Angular Dependence of Noise Correlations

$$\mathrm{corr}(\delta\xi, \delta\rho) = \frac{\mathrm{cov}(\delta\xi, \delta\rho)}{\sqrt{\mathrm{Var}(\delta\xi)}\sqrt{\mathrm{Var}(\delta\rho)}} \approx \frac{\xi_0^2\rho_0^2}{L^4}\left[\frac{1}{2}(3\cos^2\theta - 1)\right]$$

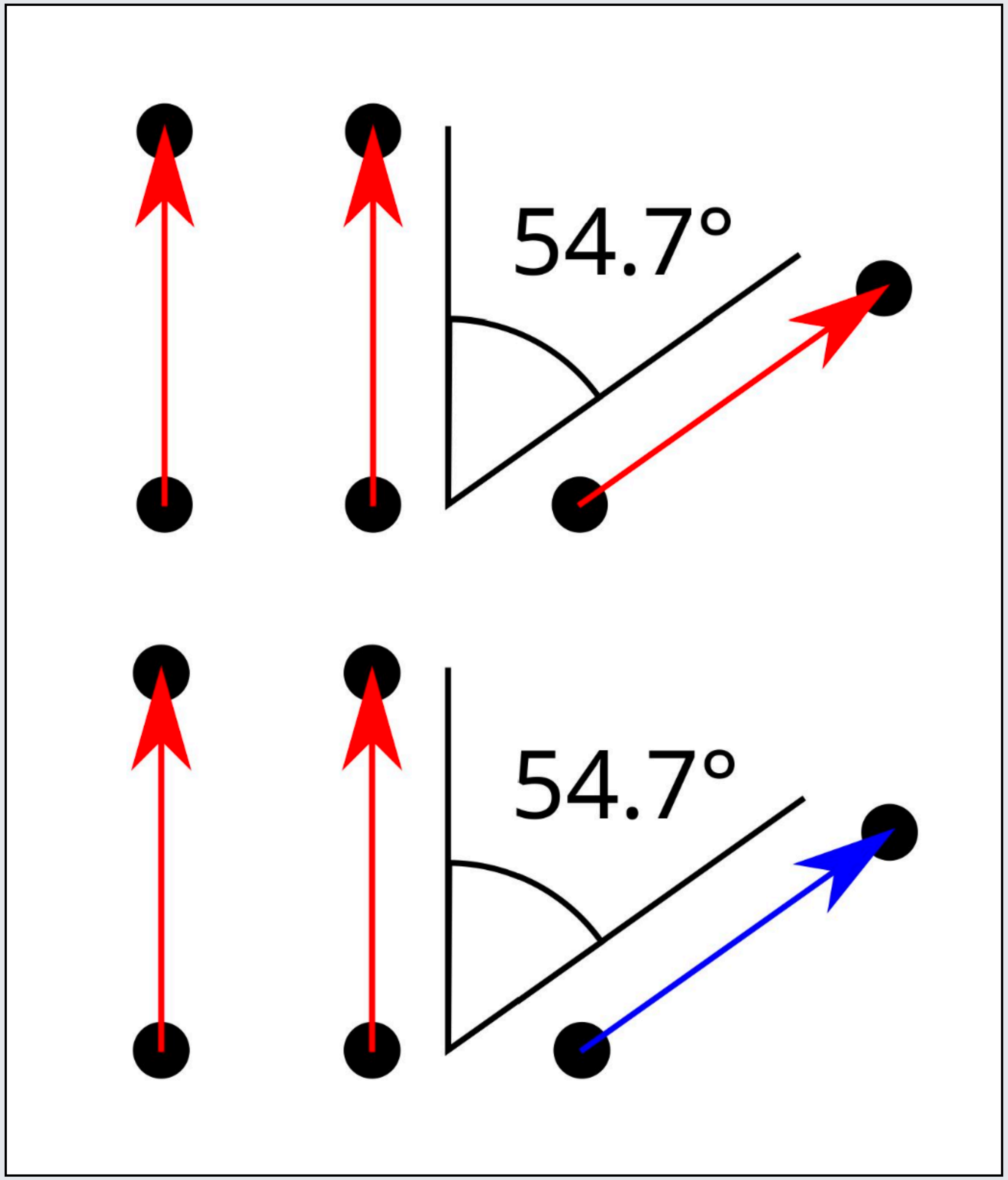
The correlation for quantum-gravitational noise is maximal when the detectors are aligned and zero when they are at an angle of 54.7 degrees



An Experiment to Isolate Quantum-Gravitational Noise

If the noise in all 3 detectors is correlated, it is **not** quantum-gravitational noise

If the noise in the parallel detectors is correlated, and the noise in the third detector is not correlated, it **is** likely quantum-gravitational noise



Noise for Quantum States of Gravitational Field

The magnitude of the noise depends on the quantum state as well as on the detector

$$\sigma^2 \sim \xi_0^2 \int_0^{\omega_{\max}} d\omega S(\omega)$$

variance of fluctuation

arm length

detector sensitivity

noise power spectrum

Note: the noise is not intrinsically small

Preliminary results indicate a **cumulative** effect: the fluctuations **grow with time** $\sigma \sim t^2$

Best Case

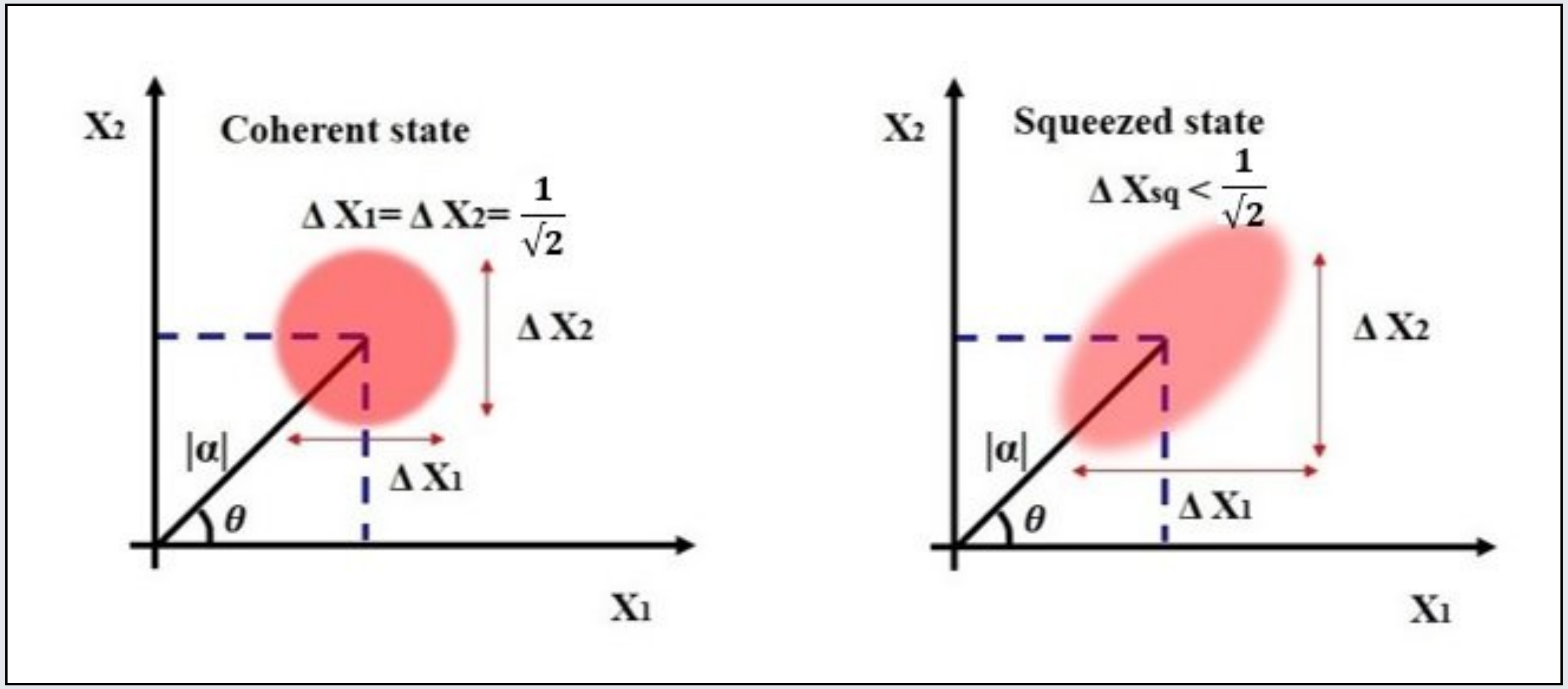
Quantum-gravitational noise is greatest when the state of the gravitational field is far from a coherent state, like a squeezed state

$$\left(\frac{\Delta L}{L}\right)_{\text{squeezed}} \sim e^{r/2} \left(\frac{\Delta L}{L}\right)_{\text{vacuum}}$$

exponential enhancement in squeezing parameter

Squeezed gravitational states could be produced by nonlinearities, inflation…

Coherent States and Squeezed States



Coherent state

Squeezed state

$$|\alpha\rangle \sim e^{\alpha a^\dagger - \alpha^* a}|0\rangle$$

$$|z\rangle \sim e^{\frac{1}{2}(z^* a^2 - z a^{\dagger 2})}|0\rangle$$

Squeezed States?

But why should the gravitational field be in a **squeezed state?**

Squeezed States from Nonlinear Couplings

If we have a field coupled to a classical source $H_{\rm int} = J\varphi$

where $\varphi \sim a + a^\dagger$

then turning on the coupling naturally produces a **coherent state**:

$$|0\rangle \to e^{-i\int H_{\rm int}dt}|0\rangle \sim e^{-i\int Ja^\dagger dt}|0\rangle$$

However if the coupling is **non-linear** $H_{\rm int} = J\varphi^2$

we produce roughly a **squeezed state**: $|0\rangle \to e^{-i\int Ja^\dagger a^\dagger dt}|0\rangle$

Gravity Naturally Produces Squeezed States

So we need nonlinear couplings to produce squeezed states $H_{\text{int}} = J\varphi^2$

But gravity naturally has such nonlinear couplings!

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S_{\text{E-H}} = \int d^4x\sqrt{-g}R = \int d^4x\sqrt{-\bar{g}}\left(\bar{R} + \frac{1}{2}(\bar{\nabla}h)^2 + \bar{R}_{abcd}h^{ac}h^{bd} + ...\right)$$

This should produce squeezed states during the **merger phase** of black hole collisions

A. Das, MP, F. Wilczek, R. Wutte, in progress.

Summary

Quantum fluctuations of spacetime produce **noise** in the deviation of particles

$$\ddot{\xi} \approx \frac{1}{2}\left(\ddot{h} + \ddot{N}_{\Psi}\right)\xi$$

The characteristics of the noise depend on the **quantum state** of gravity

For a **squeezed** state, the enhanced noise might even be **observable**

$$S_{\text{squeezed}}(\omega) \approx \cosh(2r) 4G\hbar\omega$$

Detectors with very high frequency sensitivity might be able to detect **enhanced correlated noise with a linear power spectrum** during the **merger phase** of black holes