Estimating the Hubble constant from the mock GW data of Einstein Telescope

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Einstein Telescope

★ ET is a proposed Gen III gravitational wave detector

★ Tenfold better sensitivity than present Gen II detectors

★ GW bandwidth: 1 Hz – 10 kHz (LIGO: 10 – 1000 Hz)

★ Located underground at a depth of 100–300 m to reduce noise

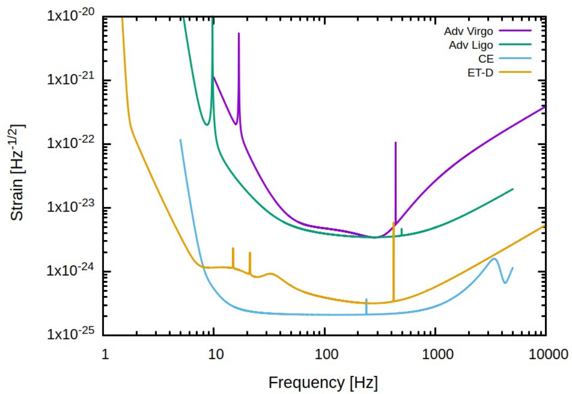


Figure 1: Amplitude spectral density of ET

- ★ Currently accepted ET design is called ET-D.
- ★ Equilateral triangle configuration with arm-length 10 km
- ★ 2-band xylophone design with 6 interferometers
- ★ Low Frequency (1 – 40 Hz); High Frequency (40 Hz – 10 kHz)
- ★ Sensitive to GW from all directions without any blind spot
- ★ Can generate null streams useful to eliminate glitches

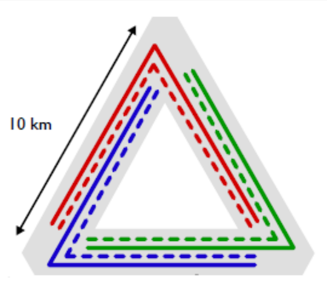


Figure 2: ET-D design

- Detect BH-BH mergers upto $z \sim 20$ @ 10^5–10^6 events/year
- Detect NS-NS mergers upto $z \sim 3$ @ 10^4–10^5 events/year
- H_0 measurement to 1% uncertainty in 1 year of ET (You et al. 2021)
- Two candidate locations: *Island of Sardinia, Italy* OR *Meuse–Rhine Euroregion (near Belgium, Germany, Netherlands)*

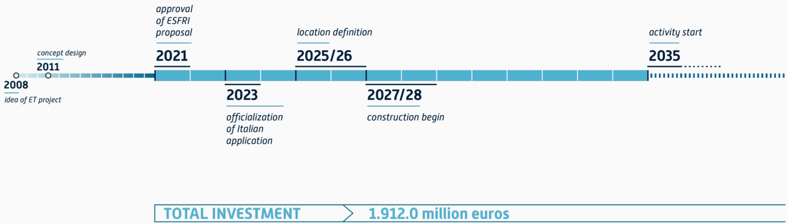


Figure 3: Timeline of ET
Image source: https://www.einstein-telescope.it/en/einstein-telescope-en/

Gravitational Wave Astronomy

- The strain (amplitude), h, in the interferometer arm of length, L, of a GW detector is given by

$$h(t) = \Delta L/L = \text{constant} \times \frac{\mathcal{M}^{5/3}}{d_L} f^{2/3}\, \Theta \cos \Phi$$

where $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$ is called the chirp mass of a binary.

- Due to the cosmological redshift of the incoming GW frequency, what we measure is the redshifted chirp mass, $\mathcal{M}_z = (1+z)\mathcal{M}$.
- Since a measured chirp mass can correspond to many chirp mass and redshift values, we get **mass-redshift degeneracy**.
- In the absence of an EM counterpart of the GW event, one can lift the degeneracy with the population method.

Cosmology

- The current standard model of the universe is called the Λ-Cold Dark Matter (ΛCDM) model.
- It is characterized by the parameters: H_0, Ω_m, Ω_Λ, Ω_rad, Ω_k, w
- Hubble constant, H_0, quantifies the expansion rate of the universe; lies around 70 km/s/Mpc.
- By construction, $\Omega_\mathrm{m} + \Omega_\mathrm{rad} + \Omega_k + \Omega_\Lambda = 1$
- In the minimal six-parameter model: $\Omega_\mathrm{rad} \sim 0$, $\Omega_k = 0$ (flat), $w = -1$ so that $\Omega_\Lambda + \Omega_\mathrm{m} = 1$

Luminosity distance, $d_L = \dfrac{c}{H_0}(1+z)\displaystyle\int_0^z \frac{dz'}{\sqrt{\Omega_\mathrm{m}(1+z')^3 + (1-\Omega_\mathrm{m})}}$

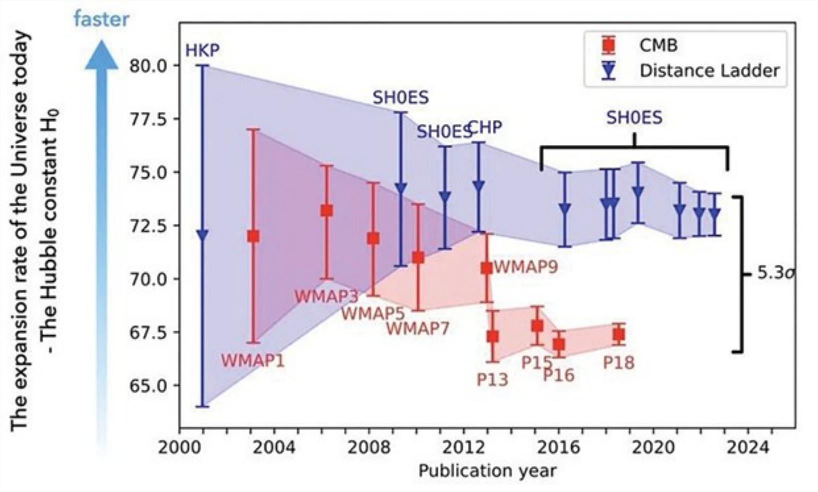


Figure 4: Plot showing conflicting H_0 measurements from two datasets (Image credit: William D'Arcy Kenworthy, Stockholm University)

- $\triangle$ Measurements from Cepheids and Type Ia Supernovae (late universe) $\rightarrow$ **73.0 ± 1.0 km/s/Mpc**
  (SH0ES Collaboration; Riess et al. 2022)
- $\bigtriangledown$ Measurements from Cosmic Microwave Background (CMB) (early universe) $\rightarrow$ **67.4 ± 0.5 km/s/Mpc**
  (Planck Collaboration; Aghanim et al. 2020)
- $\Diamond$ The divergence is found to be of $\sim 5\sigma$ significance.
- $\square$ Measurements from Tip of the Red Giant Branch (TRGB) (late universe) $\rightarrow$ **69.8 ± 2.2 km/s/Mpc** (Freedman 2021)
- $\bigcirc$ GW standard siren measurements can solve the discrepancy.

Method

- ☆ We generate a NS-NS binary merger population using the binary evolution code *StarTrack* (Belczynski et al. 2008, 2020).
- ☆ These NS binaries are analyzed using ET's design sensitivity.
- ☆ The ones which exceed the detection threshold (SNR_eff $>$ 8 and at least one SNR_i $>$ 3 for $i \in [1, 2, 3]$) are identified as events.
- ☆ Using a cosmological model, the luminosity distance for each detected event is measured from the observable quantities.

Given data: $P(\mathcal{M})$, $P(\mathcal{M}_z)$, $P(d_L)$, $\Omega_\mathrm{m} = 0.3$, $\Omega_\Lambda = 1 - \Omega_\mathrm{m} = 0.7$, $w = -1$

$$\because \mathcal{M}_z = \mathcal{M}(1+z) \implies z = \frac{\mathcal{M}_z}{\mathcal{M}} - 1$$

Probability distribution of z:

$$P(z) = \int d\mathcal{M}_z \, P(\mathcal{M}_z) \int d\mathcal{M} \, P(\mathcal{M}) \, \delta\left(z - \left(\frac{\mathcal{M}_z}{\mathcal{M}} - 1\right)\right)$$

$$\because H_0 \equiv H_0(z, d_L) = \frac{c}{d_L}(1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_\mathrm{m}(1+z')^3 + (1-\Omega_\mathrm{m})}}$$

Probability distribution of H_0:

$$P(H_0) = \int dz \, P(z) \int dd_L \, P(d_L) \, \delta\left(H_0 - H_0(z, d_L)\right)$$

We find the cumulative probability from the probability density as:

$$CP\,(H_0) = \int_0^{H_0} P(H_0')\, dH_0'$$

- This function ranges from 0 to 1 and equals 0.5 at the median of the probability distribution.
- For 90% confidence interval, we get the lower and upper bounds when $CP = 0.05$ and $CP = 0.95$ respectively.
- Next, we calculate the function, $CP \times (1 - CP)$, which ranges from 0 to 0.25 and peaks at the median of the probability distribution.
- We repeat the above steps for every event. Finally, we compute the product, $\prod CP \times (1 - CP)$, and scale it so that the peak is 0.25.

Results

We simulated 50000 NS-NS mergers of which 5940 were marked as detected. Measurables for those events constitute the mock data.

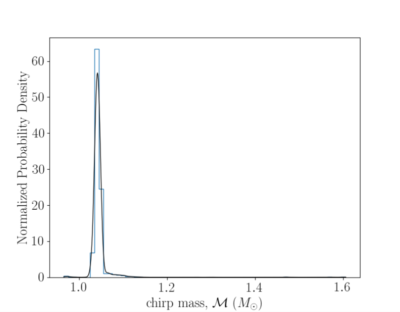


Figure 5: $P(\mathcal{M})$ used in the study based on the binary evolution model M30.B. Here, $\mathcal{M}_{\text{min}} = 0.96\ M_\odot$ and $\mathcal{M}_{\text{max}} = 1.60\ M_\odot$. The distribution is represented by the blue histogram with binsize $0.01\ M_\odot$.

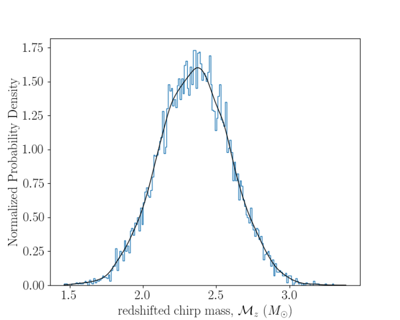


Figure 6: $P(\mathcal{M}_z)$ for one of the events. The distribution is represented by the blue histogram with binsize $0.01\ M_\odot$. This $P(\mathcal{M}_z)$ is to be used as input together with $P(\mathcal{M})$ to determine $P(z)$ for this specific event.

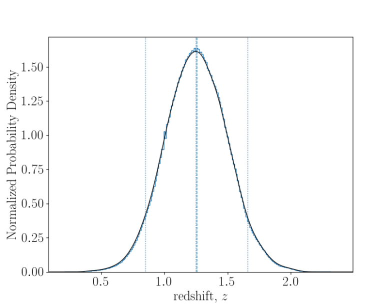


Figure 7: $P(z)$ for the same event. Middle dotted line shows the median of the distribution. The left and right dotted lines show the 90% confidence interval. Dashed line shows the injected value. The distribution is shown by the blue histogram with binsize 0.01.

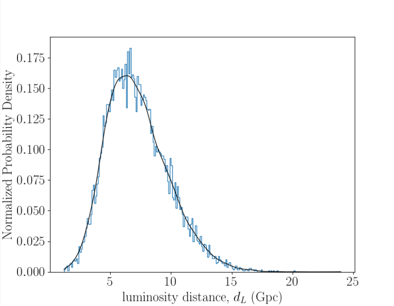


Figure 8: $P(d_L)$ for the same event. The distribution is shown by the blue histogram with binsize 0.1 Gpc. This $P(d_L)$ is to be used as input together with $P(z)$ to determine $P(H_0)$ for this specific event.

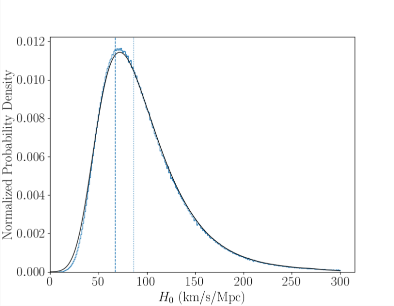


Figure 9: $P(H_0)$ for the same event. Dotted line shows the median of the distribution. Dashed line shows the injected value. The distribution is represented by the blue histogram with binsize 1 km/s/Mpc. The smooth curve denotes the distribution with binsize 0.1 km/s/Mpc.

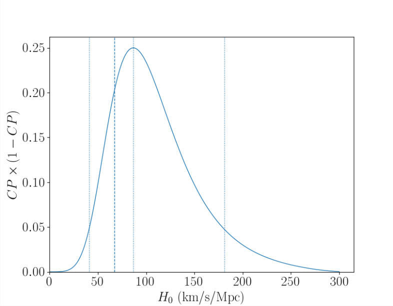


Figure 10: $CP \times (1 - CP)$ for H_0 (normalized) with stepsize 0.1 km/s/Mpc. Middle dotted line shows the median of the distribution. Left and right dotted lines mark the 90% confidence interval. Dashed line shows the injected value of 67.3 km/s/Mpc.

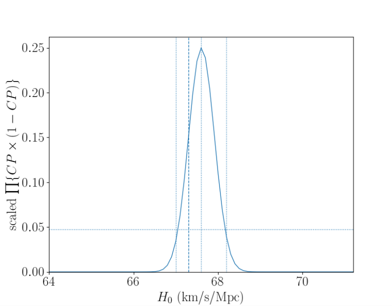


Figure 11: Combined $CP \times (1 - CP)$ for H_0 with stepsize 0.1 km/s/Mpc. Middle dotted line shows the median of the H_0 distribution. Left and right dotted lines mark the 90% confidence interval. Dashed line shows the injected value of 67.3 km/s/Mpc. The estimate obtained is $H_0 = 67.6 \pm 0.6$ km/s/Mpc.

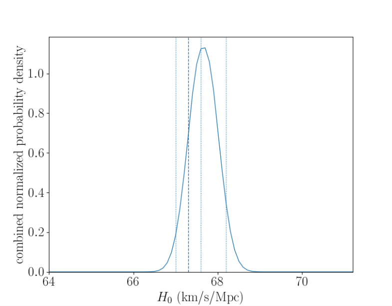
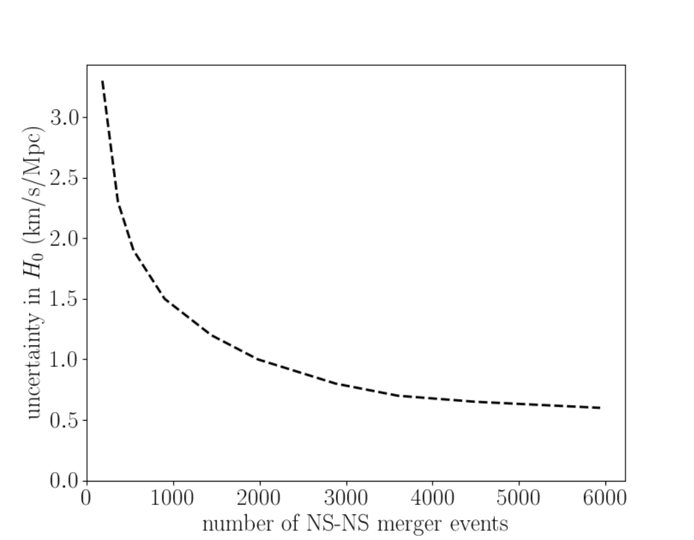


Figure 12: Combined $P(H_0)$ (normalized) with stepsize 0.1 km/s/Mpc. Middle dotted line shows the median of the distribution. Left and right dotted lines mark the 90% confidence interval. Dashed line shows the injected value of 67.3 km/s/Mpc. The estimate obtained is $H_0 = 67.6 \pm 0.6$ km/s/Mpc.

The uncertainty in H_0 drops inversely as $\sim 1/\sqrt{N}$ with number of events, and becomes less than 1% for more than ~ 5000 events.



- If the true chirp mass lies at the extremes of the intrinsic chirp mass distribution, the mass-redshift degeneracy is not lifted.
- In that case, as $P(\mathcal{M}) \approx 0$ so that $P(z) \approx 0$ at the actual z, and we get an entirely wrong redshift, and thus, a bad H_0 estimate.
- We encountered $\sim$15 such instances out of the total 5940 events. But we did not eliminate any. Negligible effect on final result.
- NS-NS events are very few and restricted to low redshifts. Hence, evolution of H_0 with z is hard to ascertain through them.

Conclusion

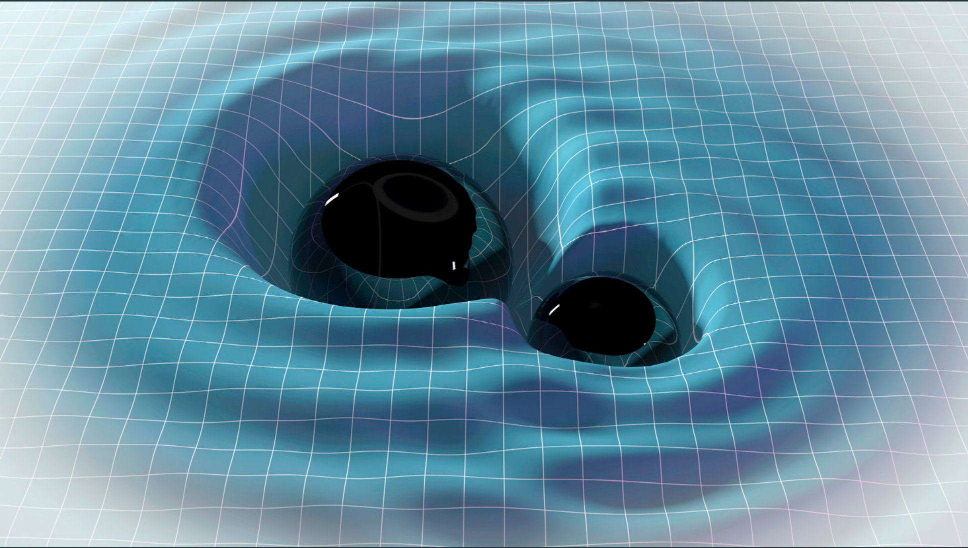
- We demonstrated a method of determining H_0 using only GW data from Einstein Telescope.
- We found that uncertainty will fall below 1% if more than 5000 NS-NS events are detected with ET (i.e. 1 month of observation).
- We will analyze ET mock data generated for various other binary evolution models.
- We will do similar analyses for BH-BH events and compare the result with that obtained from NS-NS events.
- We will estimate other cosmological parameters and quantify the accuracy that can be achieved.

Belczynski et al.
Evolutionary roads leading to low effective spins, high black hole masses, and O1/O2 rates for LIGO/Virgo binary black holes
Astronomy & Astrophysics, 2020

Singh & Bulik
Constraining parameters of coalescing stellar mass binary black hole systems with the Einstein Telescope alone
Physical Review D, 2021

Singh & Bulik
Constraining parameters of low mass merging compact binary systems with Einstein Telescope alone
Physical Review D, 2022

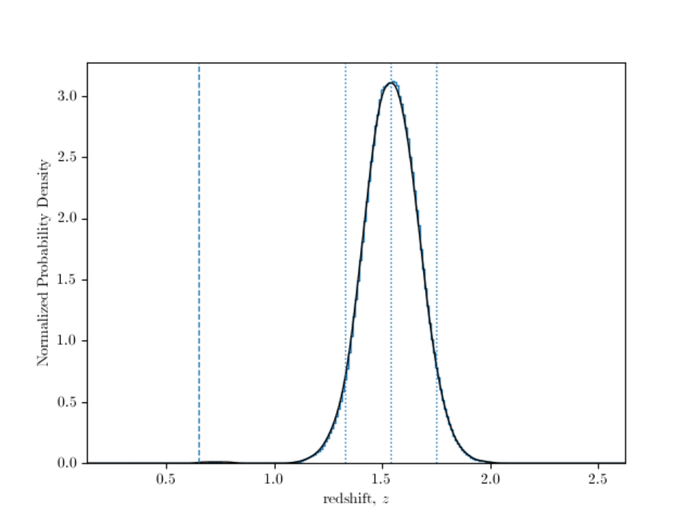
That’s all !



QUESTIONS ?

Results - exception

The injected redshift value is 0.65 and recovered value is 1.55 because of an extreme chirp mass value of 1.6 $M_\odot$.



- GW are spacetime deformations that move at the speed of light.
- Astrophysical sources: compact binaries, rotating neutron stars, short duration bursts and stochastic GW background.
- GW distort the plane transverse to the propagation axis in two ways: h_+ and $h_\times$ polarizations.
- This property is used to detect them using km-scale Michelson interferometers and high-power lasers.

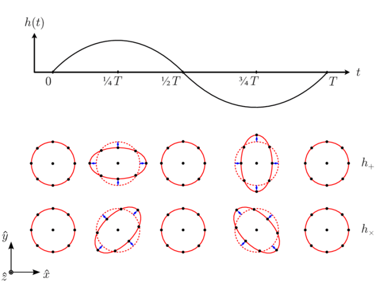


Figure 13: GW polarizations (Le Tiec & Novak 2016)

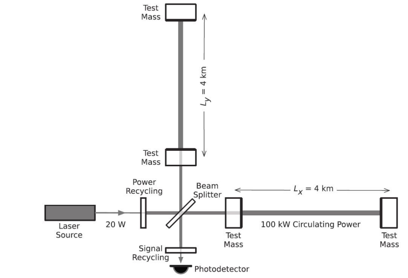


Figure 14: Laser Interferometer (Abott et al. 2016)