# From Skyrmions to holographic exotics

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# Dedicated to Michał Praszałowicz, on the occasion of His 70th birthday

# Prologue

- 1974 - Close encounter with Michał
- 1977-1983 - Period of unlimited appreciation of pQCD
- 1983-1985 - Lost illusions, farewell to pQCD
- [MAN, M Praszałowicz, W Słomiński; *A practical guide to the next-to-leading order of the perturbation expansion in QCD*, Annals of Physics 166, 433, 1986]
- 1983 - Revelation: M. Peskin brings preprints by E. Witten on Skyrme model to Zakopane School

# Skyrmion revisited

- Skyrme idea (1961): **Baryon as topological soliton** (isospin-space hedgehog) of the mesonic (pionic) fields

$$L = L_\sigma + L_{Skyrme}$$

- Witten, Adkins, Nappi (1982-83):
  - Link to large $N$
  - Anomalous term

$$L = L_\sigma + L_{Skyrme} + L_{WZW}$$

  - Quantization of collective modes from symmetries (moduli space) leads to

$$H = M_0 + \frac{\vec{I}^2}{2\Omega_{sol}}$$

where $\vec{I}(= \vec{J})$ are isospin (angular momentum) respectively.

(Technically, equivalent to algebraic hydrogen atom quantization done W. Pauli in 1921 (!))

# SU(3) extension

[PO Mazur, MAN, M Praszałowicz, *SU (3) extension of the Skyrme model*, Physics Letters B 147, 137, 1984]

- Quantization of the symmetric top with some local symmetry included
- States superselected by **triality zero** condition for representations of SU(3) (octet, decuplet)
- Challenges: third triality zero representation is antidecuplet: Michał notes that leads to a **light pentaquark**. Consequences for elusive $\Theta^+$.
- Challenges: Explicit chiral symmetry breaking does not work.

# SU(3) extension other way

- Already at the level of strange quarks symmetry breaking is so bad, so collective method fails.
- Proper way is to use Born-Oppenheimer approximation - **fast vibration of the kaon** in the $SU(2)$ solitonic background. Then **slow rotation** of the bound state happens in the presence of non-Abelian Berry phase originating from kaons.

$$H = M_0 + \text{binding} + \frac{(\vec{J} - (1 - c_k)tr(K\vec{I}K^\dagger))^2}{2\Omega_{sol}}$$

([isospin-spin transmutation in Callan-Klebanov 1985])

- Bound kaon behaves as s-quark, modulo baryon number

# HL Baryons

- For $c(b)$ baryons, similar picture, but with **two** Berry phases from $D$ and $D^*$, so one gets

$$H_1 = \frac{[(\vec{J} - \vec{S}_H) - (1 - c_D)tr(D\vec{I}D^\dagger) - (1 - c_{D^*})tr(D^*\vec{I}D^{*\dagger})]^2}{2\Omega_{sol}}$$

- In the infinitely heavy mass Berry phases *exactly* **cancel**
  Then

$$H_1 = \frac{(\vec{J} - \vec{S}_H)^2}{2\Omega_{sol}} = \frac{\vec{I}^2}{2\Omega_{sol}}$$

  (Realization of **Isgur-Wise symmetry** at the baryonic level)
- Also, soliton can capture more than one meson (double heavy baryons, exotica).

# Heavy-Light Effective Action Revisited

Combining chiral symmetry with heavy-spin symmetry leads to novel feature [MAN, Rho, Zahed (1992), Bardeen-Hill (1993)]
Both symmetries enforce the **presence of opposite parity** $(0^+, 1^+)$ multiplet $G = \frac{1+\not{v}}{2}(\tilde{D} + \gamma^\mu \gamma_5 \tilde{D}^*_\mu)$
in addition to standard $(0^-, 1^-)$ one $H = \frac{1+\not{v}}{2}(\gamma_5 D + \gamma^\mu D^*_\mu)$

- Consequence of chiral symmetry $[\not{v}, \gamma_5]_+ = 0$
- Doublers communicate only through axial current
- Physical split in axial couplings and masses, $m_G - m_H \sim O(\Sigma_l) \sim 350 MeV$
- Chiral doublers do not double the number of states in quark model, but *reorganize* them in different way.

# Several possibilities

(i) soliton captures $H$ meson (heavy baryon)
(ii) soliton captures $G$ meson (doubler of heavy baryon)
(iii) soliton captures $\bar{H}$ (heavy pentaquark)
(iv) soliton captures $\bar{G}$(doubler of heavy pentaquark)
(v) soliton captures more mesons.....

- Short, unhappy life of $\tilde{\Theta}_c(3099)$
- [MAN, M. Praszałowicz, M. Sadzikowski, J. Wasiluk; *Chiral doublers of heavy-light baryons*, Phys. Rev. D70, 031502, 2004]
  *... In particular, we interpret the state recently reported by the H1 experiment at HERA as a chiral partner* $\tilde{\Theta}_c(3099)$ *of yet undiscovered ground state pentaquark* $\Theta_c(2700)$.

# Desperately seeking exotics ...

# Experimental revolution: Post-Babar-ian era

Abundance of exotic heavy-light particles

- $\bar{\mathbf{c}}\mathbf{du}\bar{\mathbf{s}}$ :$X(2866)$, $X_1(2904)$
- $\mathbf{c}\bar{\mathbf{c}}\mathbf{q}\bar{\mathbf{q}}$ : $\chi_{c1}(3872)$
- $\mathbf{c}\bar{\mathbf{c}}\mathbf{u}\bar{\mathbf{d}}$: $Z_c(3900)$, $Z_c(4020)$, $Z_c(4050)$, $X(4100)$, $Z_c(3985)$, $Z_c(4430)$, $R_{c0}(4240)$
- $\mathbf{c}\bar{\mathbf{c}}\mathbf{u}\bar{\mathbf{s}}$: $Z_{cs}(3985)$, $Z_{cs}(4000)$, $Z_{cs}(4220)$
- $\mathbf{b}\bar{\mathbf{b}}\mathbf{u}\bar{\mathbf{d}}$: $Z_b(10610)$, $Z_b(10650)$
- $\bar{\mathbf{c}}\mathbf{c}\bar{\mathbf{c}}\mathbf{c}$: $X(6900)$
- $\mathbf{cc}\bar{\mathbf{u}}\bar{\mathbf{d}}$: $T_{cc}^{+}(3875)$, also $T_{cc}^{0}$, $T_{cc}^{++}$ (preliminary)
- Pentaquarks $\mathbf{c}\bar{\mathbf{c}}\mathbf{uud}$: $P_c((4380),(4450)$
  $\rightarrow [(4440),(4457)],(4312)]$, $P_c(4337)$ $(3\sigma)$ significance
- $\mathbf{c}\bar{\mathbf{c}}\mathbf{uds}$: $P_{cs}(4459)$
- possibilities of further **heavy-light "chemistry": many more expected (?)**

Input from lattice

# Theoretical revolution: Gravity/Gauge duality (holography, AdS/CFT)

- In the 70' QCD became **fundamental theory of quarks and gluons**, strings (flux tubes) appear as **effective**, e.g. Lund model
- Maldacena pointed that gauge theory is 4 dim is equivalent to string theory in higher dimensions: **Towards two fundamental theories of strong interactions (?!)**
- Various versions - conformal window, lower-dimensions (solid state physics)
- Witten (1998) applied duality to QCD: pure YM in $3+1$ at large N and $\lambda = g_{YM}^2 N$
  Surprising similarity to spectrum of glueballs at large N lattice

# Sakai-Sugimoto breakthrough 2005

- Adding $N_f$ massless fermions - geometric SB$\chi$S
- Low energy limit $S = S_{YM} + S_{CS}$ where $S_{YM} \sim \int d^4xdz \mathrm{Tr} \left( \frac{1}{2} k(z)^{-1/3} F^2_{\mu\nu} + k(z) F^2_{\mu z} \right)$ where $k(z) = 1 + z^2$
- Mode expansion: $A_\mu(x^\mu, z) = \sum_n B^{(n)}_\mu(x_\mu)\Psi_n(z)$, $A_5(x^\mu, z) = \sum_n \phi^{(n)}(x^\mu)\Phi_n(z)$
- Keeping only $\phi^{(0)}$ yields $L = L_\sigma + L_{Skyrme} + L_{WZW}$. Adding $B^{(1)}_\mu \sim \rho$ and $B^{(2)}_\mu \sim a_1$ yield hidden gauge model
- **Successful phenomenology with very few parameters**

# Baryon in Sakai-Sugimoto scenario

- 4 dim pion $\rightarrow$ Skyrmion (static solution)
- 5 dim gauge field $\rightarrow$ **BPST instanton in** $x_1, x_2, x_3, z$ **in flavor**
- Topological number $\equiv$ baryon number
- Direct realization of 1989 Atiyah-Manton idea
  $U(\vec{x}) = P \exp(i \int dz A_z(\vec{x}, z))$
- 8 zero modes lead to moduli space quantization
  $M = M_0 + \left( \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N^2} + \sqrt{\frac{2}{3}}(n_\rho + n_Z + 1) \right) M_{KK}$
  where $l = 2I = 2J = 1, 3, 5\ldots$

# HL baryons in holographic scenario: Liu, Zahed, 2017

- CK-like scheme with heavy-spin symmetry
- $M = M_0 + (N_Q + N_{\bar{Q}})m_H + (\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N^2(1 - \frac{15(N_Q - N_{\bar{Q}})}{4N} + \frac{5(N_Q - N_{\bar{Q}})^2}{3N^2})^2})M_{KK} + \sqrt{\frac{2}{3}}(n_\rho + n_Z + 1))M_{KK}$
- Various combinations of q-numbers give all types of HL hadrons
  $N_Q = 1, N_{\bar{Q}} = 0$ yield **hll**
  $N_Q = 2, N_{\bar{Q}} = 0$ yield **hhl**
  $N_Q = N_{\bar{Q}} = 1$ yield pentaquarks **h$\bar{\mathbf{h}}$lll**
  $n_Z \neq 0$ yield excited (Roper-like), $n_\rho \neq 0$ yield odd parity
- **Three parameters**: $M_{KK}, M_0, m_H \sim M_D(M_B)$

# Adding spin effect (subleading in $m_H^{-1}$) Liu, MAN, Zahed, 2021

- 3 parameters, $M_0 \to m_N$, $M_{KK} \to m_{\Lambda_c}$, $m_H \sim M_D(M_B)$ for $c(b)$
- 3 pentaquarks $\frac{1}{2}, \frac{1}{2}^-(S=1)$, $\frac{1}{2}, \frac{1}{2}^-(S=0)$,$\frac{1}{2}, \frac{3}{2}^-(S=1)$, $IJ^\pi$ ($\frac{1}{2}, \frac{5}{2}^\pm$ ruled out), consistent with $P_c(4312, 4440, 4457)$ [LHCb]
- Recently reported $P_c(4337)$ at $3\sigma$ significance is not supported
- Open and hidden decay widths (Liu, MAN, Zahed, 2021) e.g. $P_c \to \Lambda_c + \bar{D}$, $\Gamma(S=0, J=\frac{1}{2}) : \Gamma(S=1, J=\frac{1}{2}) : \Gamma(S=1, J=\frac{3}{2}) = \frac{1}{2} : \frac{5}{6} : \frac{1}{3}$
- Formfactors (Liu, Mamo, MAN, Zahed; 2021), consistent with recent GLUEX results on $\gamma p \to (P_c^+) \to J/\psi p$

# Tetraquark puzzle

- **$hh\bar{l}\bar{l}$** - several predictions (positive/negative $\pm 200$ $MeV$)
- Measurement of $\Xi^{++}_{cc}(3621)$ fixed the normalization for $bb\bar{u}\bar{d}$ tetraquark
(Karliner, Rosner (2017); Eichten, Quigg (2017)), bound up to $200MeV$ (!)
- $T^{+}_{cc}$ $(01^{+})$ (LHCb), narrow, bounded at $-360keV$, $\Gamma \sim 50keV$
- Holographic picture: instanton-antiinstanton "molecule" binds two mesons (Liu, MAN, Zahed (2019)), $b(c)$ tetraquarks bounded by 80(40) $MeV$
- Normalizing mass to $T^{+}_{cc}$, (Liu, MAN, Zahed (2022)) predict mass of $T_{bc}$ and $T_{bb}$, and estimate very narrow width.
- Stronger bindings in chiral quark soliton model [Michał Praszałowicz, 2023-2024]

# Summary

- **Strongly coupled QCD could be approached via duality from string theory in large $N$ and large $\lambda$ limit, including spectra of heavy-light hadrons**
- **Few parameters and very restrictive predictions, so models are confutable**
- Approach based on confinement, SB$\chi$S and heavy spin symmetry, in the limit of large $N$ and $\lambda$.
- "High brow" theory boils down to relatively simple QM in moduli space (top-down approach)
- **Astonishing and deep analogies to "old" physics, including joint work with Michał!**

**Happy Birthday, Michał !!!!!!**