CREATION OF HAWKING QUANTA FAR AWAY FROM A BLACK HOLE

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Outline of the talk

1) Review of the Hawking's calculation

2) Modification of the Hawking's argument

3) Thermodynamic interpretation and a simple model of backreaction

Vaidya Metric

A simple model of the process of black hole formation:

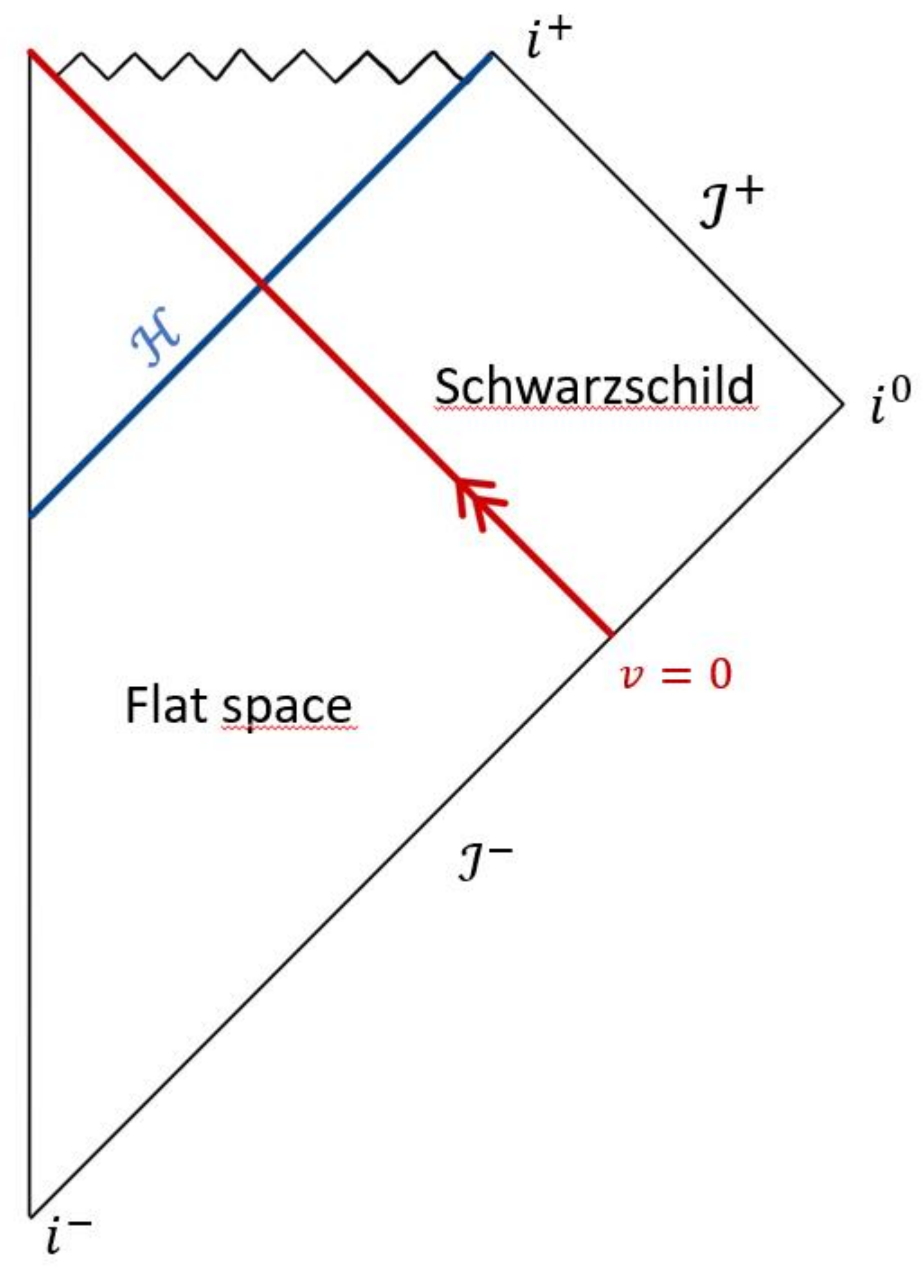
- The far past is an empty flat space

- At advanced time $v = 0$ a null shockwave with a total energy M is sent in. This infinitely thin collapsing shell of matter eventually forms a Schwarzschild black hole of mass M.

- Corresponding metric:

$$g = -\left(1 - \frac{2M}{r}\theta(v)\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

where $\theta(v)$ is the Heaviside step function.



Scalar perturbations in the Vaidya spacetime

- Take real, massless scalar field Φ. Equations of motion: $\nabla_\mu \nabla^\mu \Phi = 0$
- Mode decomposition:

$$\Phi(x) = \sum_{l,m} \int_0^\infty \mathrm{d}\omega \, \left(A_{\omega lm} p_{\omega lm}(x) + \text{ h. c.}\right)$$
$$= \sum_{l,m} \int_0^\infty \mathrm{d}\omega \, \left(B_{\omega lm} h_{\omega lm}(x) + \text{h. c.}\right) + \begin{pmatrix}\text{part supported} \\ \text{in the BH interior}\end{pmatrix}$$

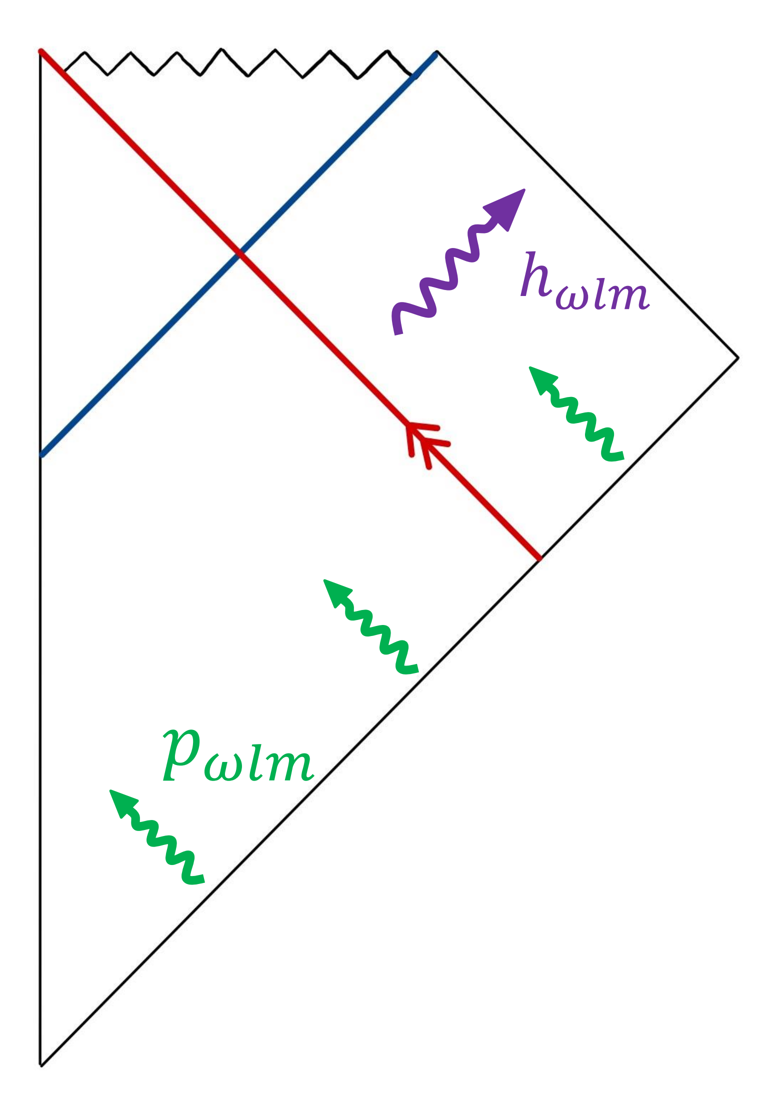
- Positive-frequency modes on $\mathcal{J}^-$:

$$p_{\omega lm}(x) \xrightarrow{x\to\mathcal{J}^-} \frac{1}{\sqrt{4\pi\omega}} \frac{e^{-i\omega(t+r)}}{r} Y_{lm}(\theta,\varphi)$$

- Positive-frequency modes on $\mathcal{J}^+$:

$$h_{\omega lm}(x) \xrightarrow{x\to\mathcal{J}^+} \frac{1}{\sqrt{4\pi\omega}} \frac{e^{-i\omega(t-r)}}{r} Y_{lm}(\theta,\varphi)$$

- Upon quantization, $A_{\omega lm}, B_{\omega lm}$ are annihilation operators, which allow us to formulate a definition of ”particles”.



Review of the Hawking's argument

- $|\text{state of the system}\rangle = |0\rangle$, s.t. $A_{\omega lm}|0\rangle = 0 \ \ \forall\, \omega, l, m.$

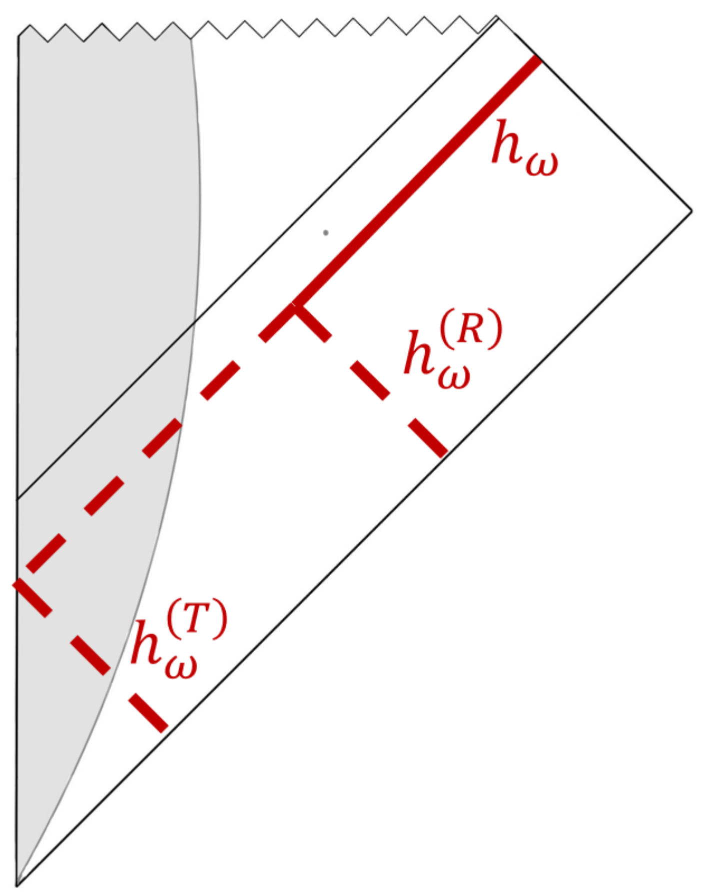
- Want to find find the Bogoliubov transformation between positive-frequency modes on $\mathcal{J}^-$ and $\mathcal{J}^+$:

$$h_\omega = \int_0^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'} p_{\omega'} + \beta_{\omega\omega'} p^*_{\omega'}\right)$$

- Expectation value of the particle number on $\mathcal{J}^+$: $\langle N^+_\omega \rangle = \langle 0 | B^\dagger_\omega B_\omega | 0 \rangle = \int_0^\infty \mathrm{d}\omega' |\beta_{\omega\omega'}|^2$.

- Focus on wavepackets localized near the horizon. By a general ray-tracing argument one can show that the Bogoliubov coefficients satisfy $|\alpha_{\omega\omega'}| = e^{4\pi M\omega}|\beta_{\omega\omega'}|$. Then completeness relation $\sum_{\omega'}(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = 1$ implies:

$$\langle N^+_\omega \rangle = \sum_{\omega'} |\beta_{\omega\omega'}|^2 = \frac{1}{e^{2\pi\omega/\kappa} - 1}$$



Hawking quanta far away from the horizon

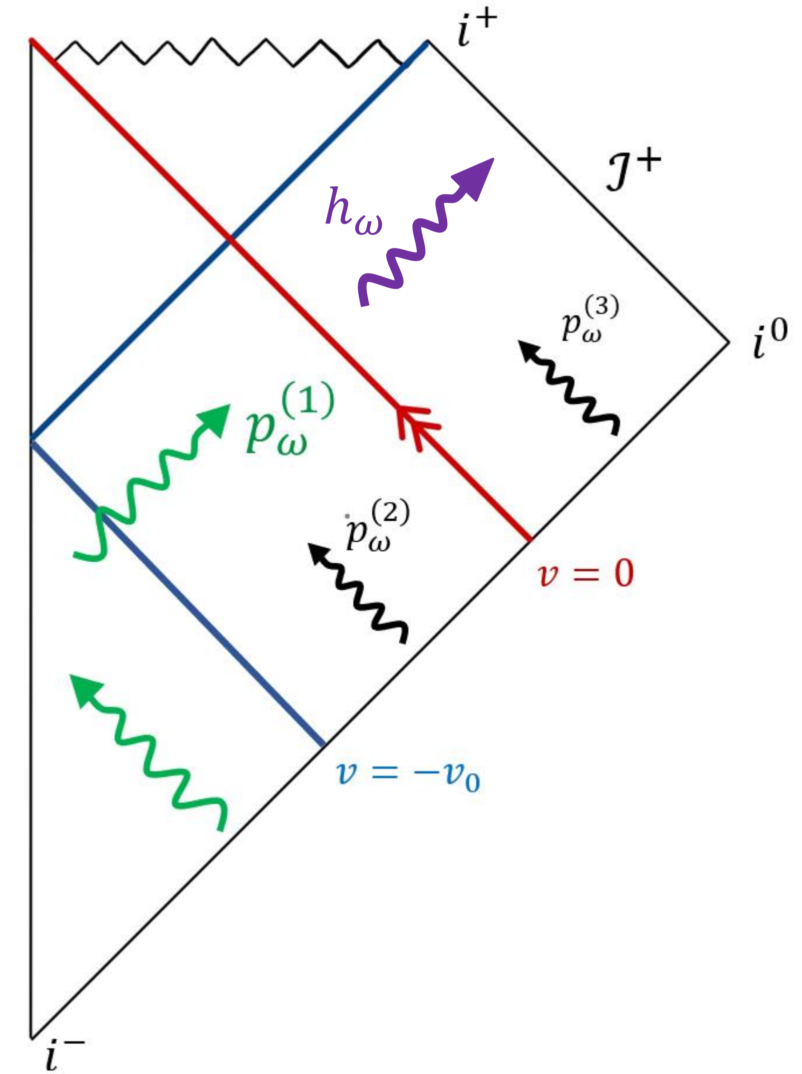
- For high frequencies $\omega \gg M^{-1}$ (WKB approx.) we can solve the Klein-Gordon equation without the near-horizon limit.

- For Vaidya spacetime we can infer the relations between $h_{\omega lm}$ and $p_{\omega lm}$ from a continuity condition across the shockwave $\{v = 0\}$.

$$h_\omega \Big|_{v=0} = \int_0^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'}\, p_{\omega'} + \beta_{\omega\omega'}\, p^*_{\omega'}\right)\Big|_{v=0}$$

$$= \int_\delta^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'}\, p^{(1)}_{\omega'} + \beta_{\omega\omega'}\, p^{(1)*}_{\omega'}\right)\Big|_{v=0} + \begin{pmatrix}\text{zero frequency} \\ \text{part}\end{pmatrix},$$

where $p_\omega = p^{(1)}_\omega + p^{(2)}_\omega + p^{(3)}_\omega$, $p^{(1)}_\omega = p_\omega \cdot \theta(v_0 - v)$ and $\delta \to 0$.



Hawking quanta far away from the horizon

- We obtain:

$$\beta_{\omega\omega'} = \frac{1}{\pi}\left(\frac{\omega'}{\omega}\right)^{1/2} \int_{2M}^{\infty} \mathrm{d}r \, \left(\frac{r}{2M} - 1\right)^{4iM\omega} e^{2i(\omega+\omega')r}$$

and $\alpha_{\omega\omega'} = \beta_{\omega,-\omega'}$. The relation $|\alpha_{\omega\omega'}| = \exp\left(\frac{\pi\omega}{\kappa}\right)|\beta_{\omega\omega'}|$ is not satisfied!

- Expectation value of the number operator is logarithmically divergent at UV, so we need to introduce a UV-cutoff $\Lambda \gg \omega$. Then:

$$\langle N_\omega^+ \rangle = \int_0^\Lambda \mathrm{d}\omega' \, |\beta_{\omega\omega'}|^2 = \frac{2M}{\pi} \frac{1}{e^{\beta_H \omega} - 1} [\log(\Lambda/\omega) + \mathcal{O}(\Lambda^0)],$$

where $\beta_H = 8\pi M$ is the inverse Hawking temperature.

- We have a non-thermal dependence $\propto \log(\Lambda/\omega)$.

Kerr black hole radiation

One can do similar calculations for Kerr-Vaidya black hole:

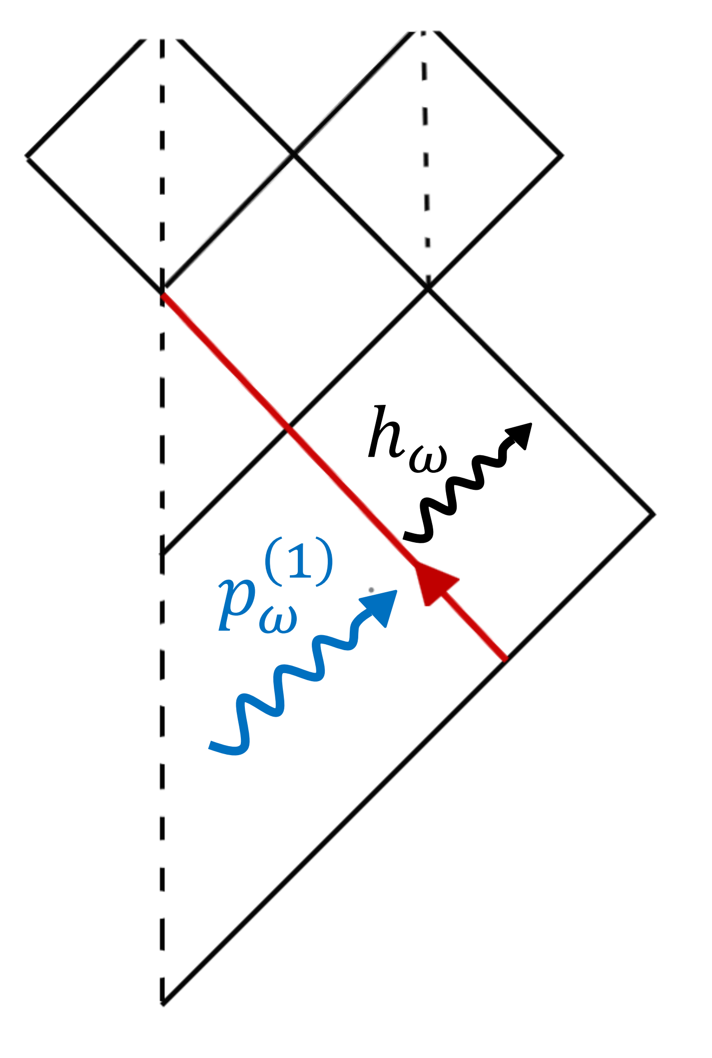
$$\beta_{\omega\omega'} = \frac{1}{2\pi}\left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}} \int_{r_+}^{\infty} \mathrm{d}r \left(1 + \sqrt{\frac{r^2 - K_{\omega lm}/\omega^2}{r^2 + a^2}}\right) \left(\frac{r - r_+}{r - r_-}\right)^{-\frac{im\Omega_+}{\kappa_+}} e^{im \arctan\left(\frac{r}{a}\right)} \exp\left[i\omega'\left(r + \int_{r_+}^{r} \mathrm{d}r' \sqrt{\frac{r'^2 - K_{\omega lm}/\omega^2}{r'^2 + a^2}}\right)\right] \times$$
$$\times \exp\left[i\omega'\left(r + \frac{1}{2\kappa_+}\log\left(\frac{r}{r_+} - 1\right) + \frac{1}{2\kappa_-}\log\left(\frac{r}{r_-} - 1\right) + \int_{r_+}^{r} \mathrm{d}r' \frac{\sqrt{r'^4 + a^2 r'(r' + 2M) - \Delta(r') K_{\omega lm}/\omega^2}}{(r' - r_+)(r' - r_-)}\right)\right],$$

Divergent part:

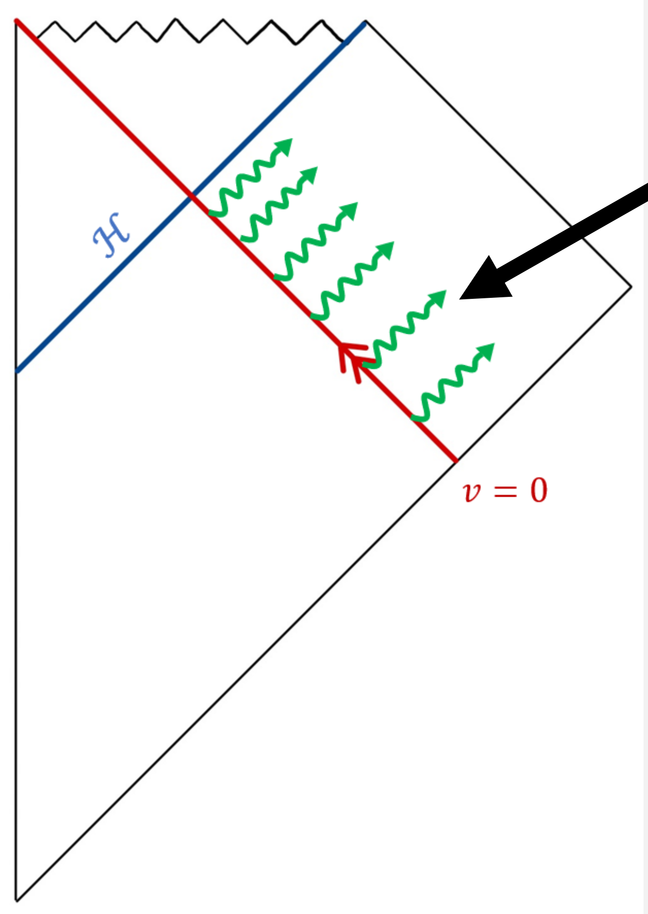
$$\beta_{\omega\omega'} \sim \left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}} \left(2ir_+(\omega + \omega')\right)^{1 + \frac{i\omega}{\kappa_+} - \frac{im\Omega_+}{2\kappa_+}} \Gamma\left(1 + \frac{i\omega}{\kappa_+} - \frac{im\Omega_+}{2\kappa_+}\right)$$

$$\langle N_\omega^+ \rangle \sim \log\left(\frac{\Lambda}{\omega}\right) \left[\exp\left(\frac{2\pi}{\kappa_+}\left(\omega - \frac{m\Omega_+}{2}\right)\right) - 1\right]^{-1}$$

Contribution from the angular momentum does not agree with the standard results!?



Thermodynamic interpretation



Think of h_ω as sum of N modes localized in position space - N boxes with photon gas. Assume that the box at position r_i has temperature

$$T_i = \frac{\hbar}{2\pi}\frac{M}{r^2}\left(1 - \frac{2M}{r}\right)^{1/2} = \frac{\hbar\alpha_i}{2\pi},$$

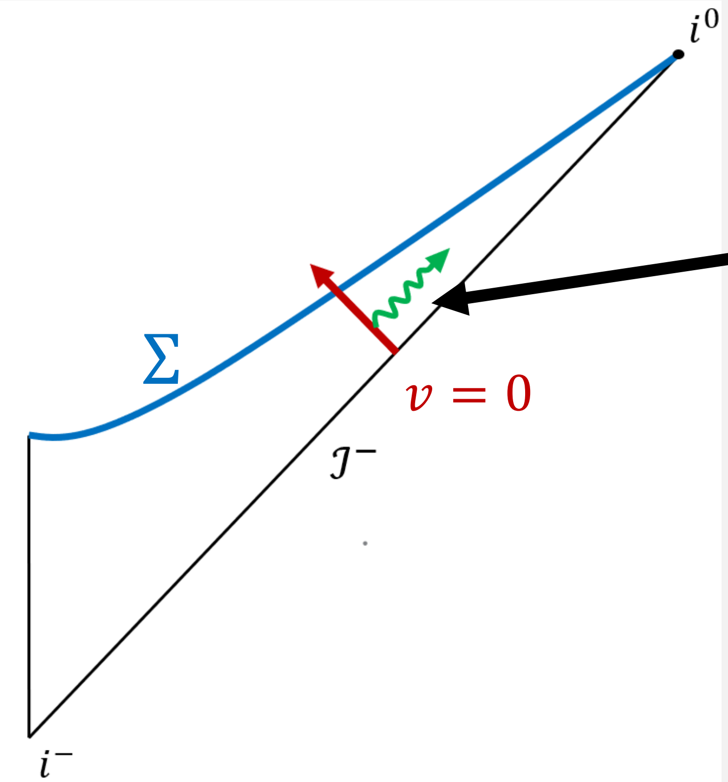
Take density of states: $\delta\rho_i(\omega)\mathrm{d}\omega = C \cdot \delta r \cdot \mathrm{d}\omega.$

Free energy:

$$F = \sum_i F_i = -\frac{\hbar C}{2\pi}\int_0^\infty \mathrm{d}\omega \log\left(\frac{\Lambda}{\omega}\right) \log\left(1 - e^{-\beta_H \hbar\omega}\right)$$

- Relation between position space and momentym space UV cutoffs: $b = 2M\left(\frac{\omega}{\Lambda}\right)^2 + \mathcal{O}(\Lambda^{-3})$
- Entropy $S = \sum_i S_i \propto \log(2M/b)$ resembles the formula for entanglement entropy of a 2d CFT

Simple model of backreaction – is formation of a non-extremal black hole possible?



Dynamical part of the spacetime. A little bit of Hawking radiation is created here, at early times

Energy of the shockwave is decreased:

$$M \to M - \delta U$$

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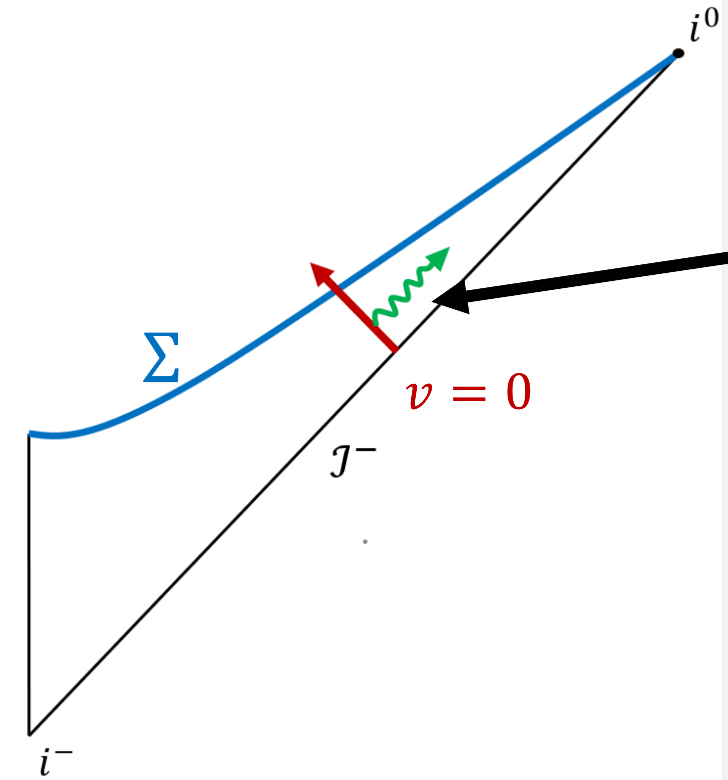

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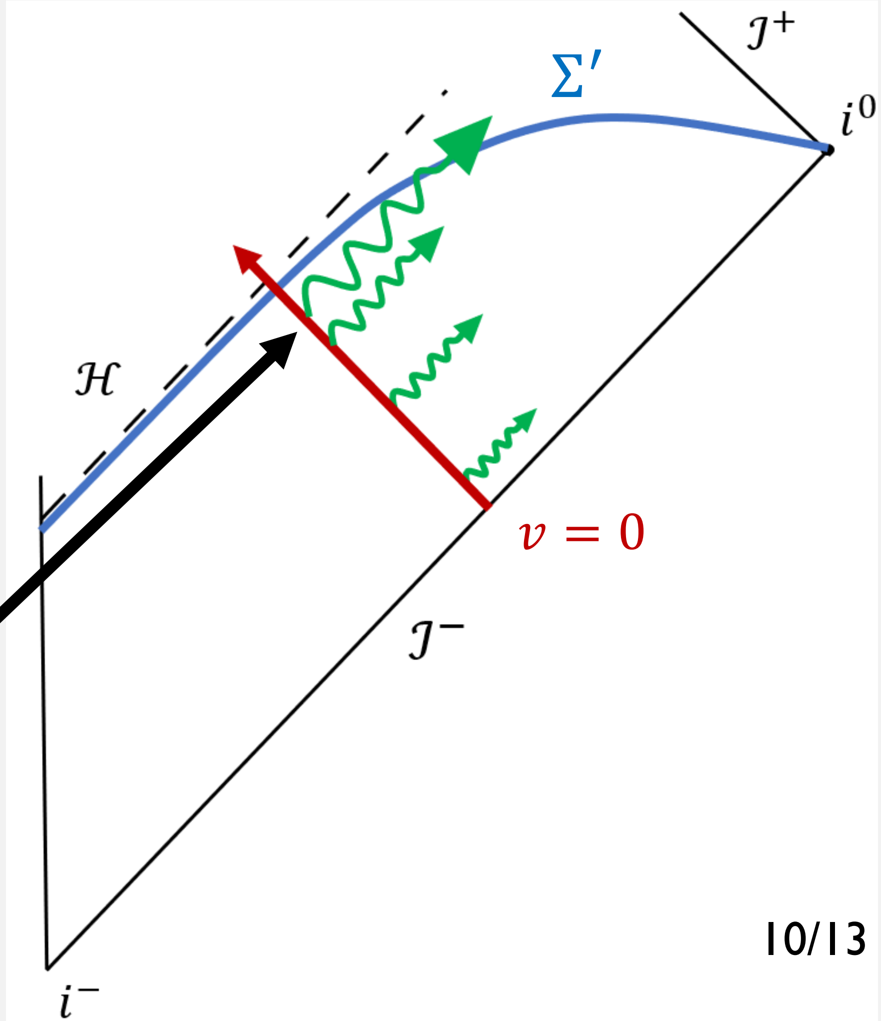

Large number of Hawking quanta is created at later times:

$$U_{Hawking} \sim M$$

Semi-classical horizon is shifted to $r_H' = 2(M - U_{Hawking})$

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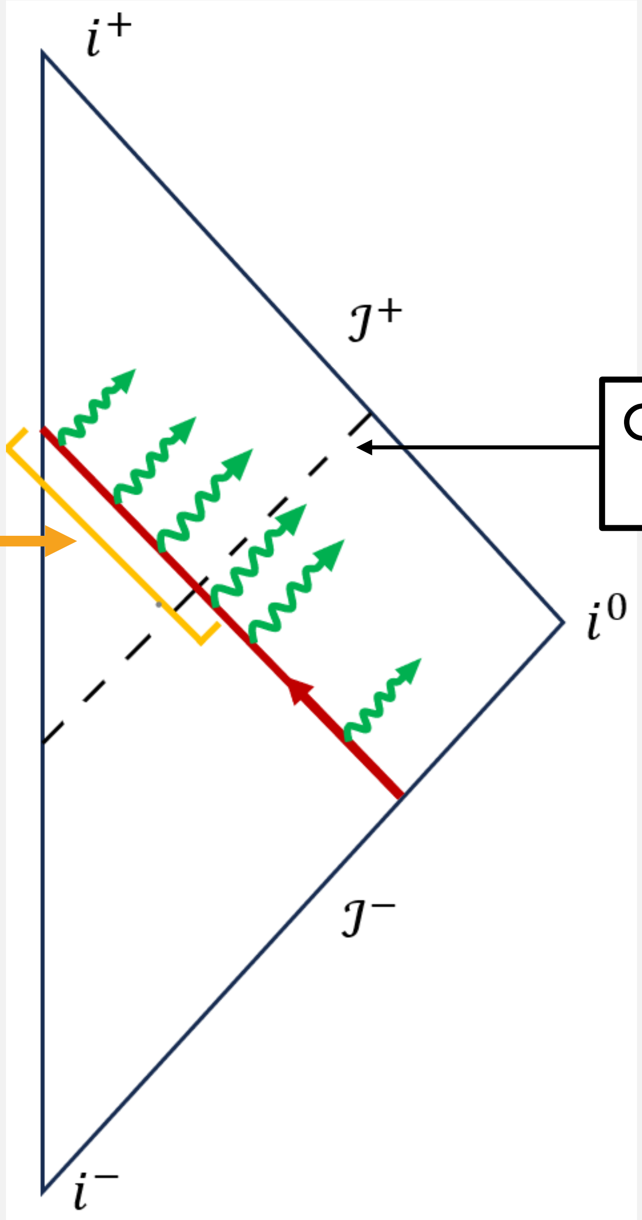
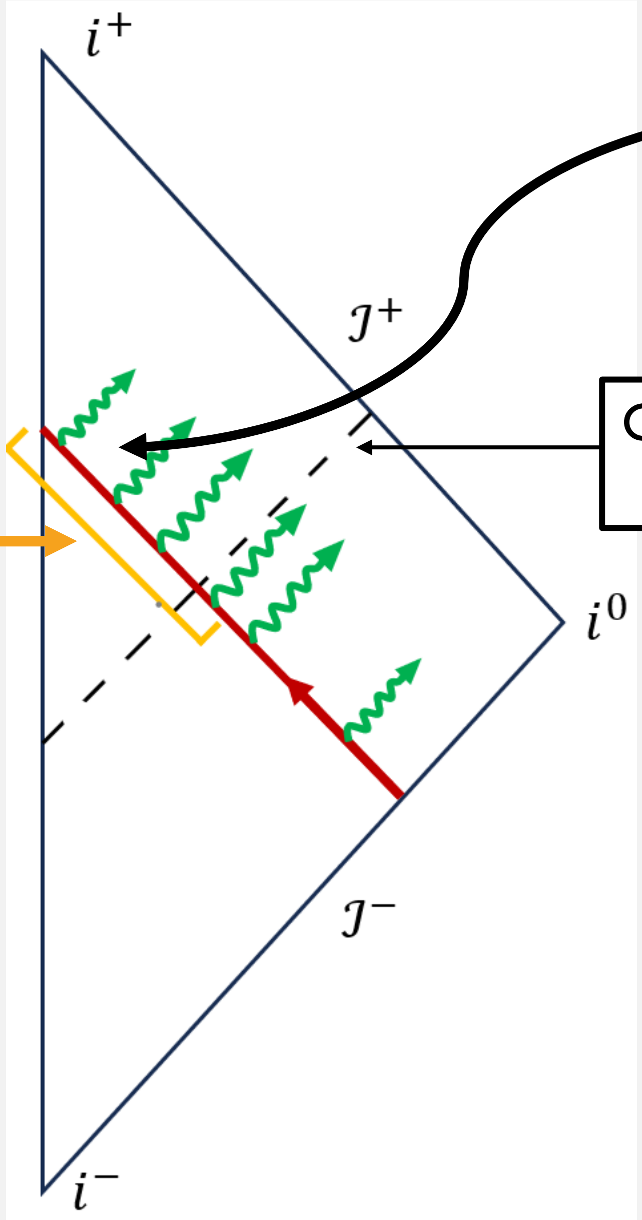


Region where large number of Hawking quanta is created.

Classical event horizon, $r = 2M$

Simple model of backreaction – is formation of a non-extremal black hole possible?

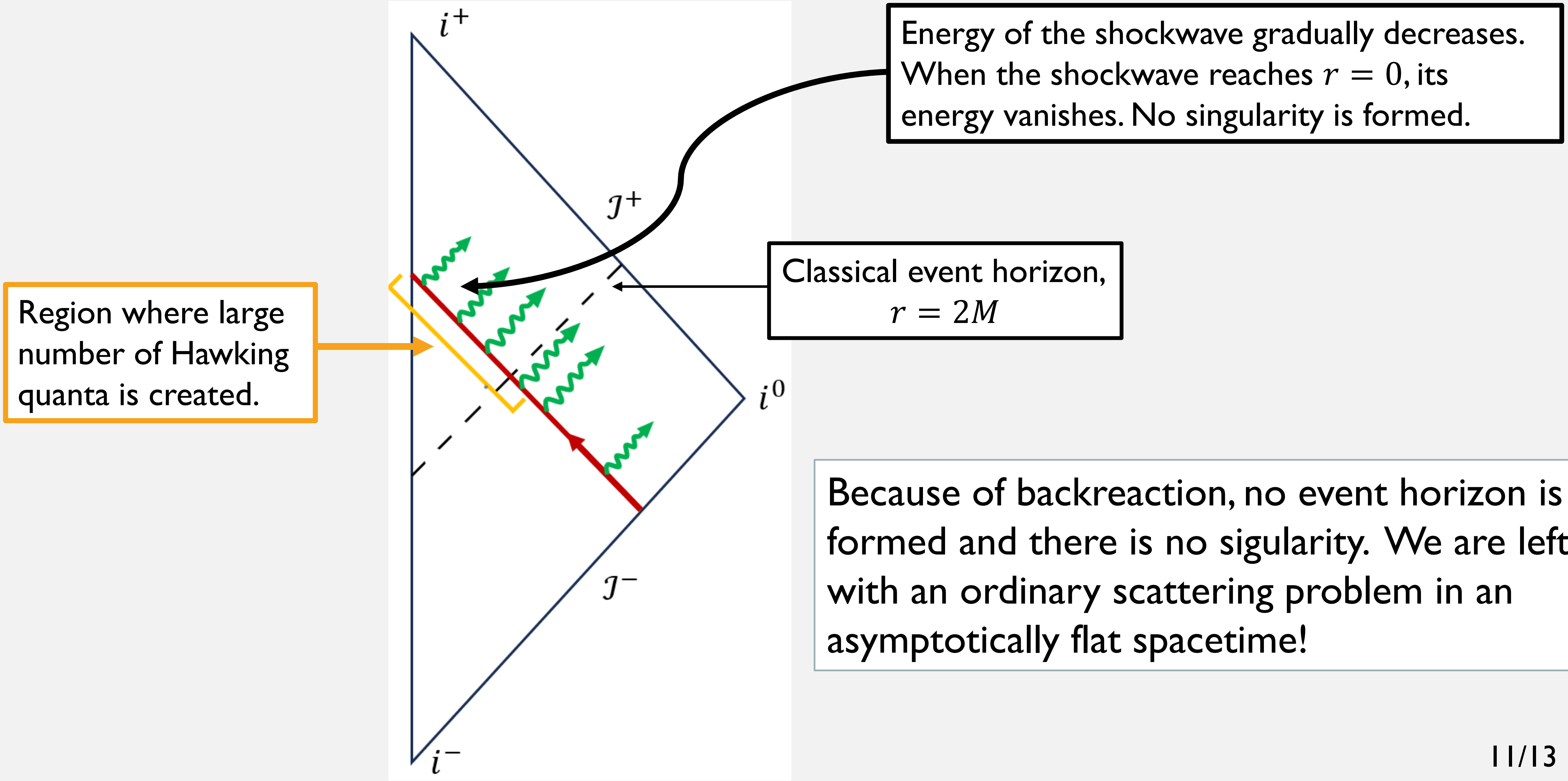


Energy of the shockwave gradually decreases. When the shockwave reaches $r = 0$, its energy vanishes. No singularity is formed.

Classical event horizon, $r = 2M$

Region where large number of Hawking quanta is created.

Simple model of backreaction – is formation of a non-extremal black hole possible?



Because of backreaction, no event horizon is formed and there is no sigularity. We are left with an ordinary scattering problem in an asymptotically flat spacetime!

Simple model of backreaction – is formation of a non-extremal black hole possible?

Divide a surface $v = \text{const.} > 0$ into small compartments of fixed affine length δr, and assume that the compartment at $r = r_i$ is filled with an ideal gas at Unruh temperature T_i.

Energy of the system at position r_i:

$$\delta U_i = \delta r_i \cdot \int_0^\infty \mathrm{d}\omega\, \rho(\omega)\, \frac{\omega}{\exp(\hbar\omega/T_i) - 1}$$

Primitive model of backreaction:

$$M(r) = M(\infty) + \int_\infty^r \mathrm{d}r' \int_0^\infty \mathrm{d}\omega\, \rho(\omega)\, \frac{\omega}{\exp(\hbar\omega/T_i) - 1}.$$

For $\rho(\omega) = c_0 \cdot \omega$, with suitable c_0 we can make the whole black hole evaporate

$$M(r = 0) = 0,$$

and recover the Bekenstein-Hawking formula for the black hole entropy from the standard thermodynamic formula:

$$S = -\frac{\partial F}{\partial T_H} = 4\pi M^2.$$

Thank you for your attention!