


# Gravitational waves from neutron stars: detection prospects and inferences for two distinct types of remnants

Lectures 1 & 2 - Binary neutron star mergers

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- Motivation
- Gravitational wave astronomy (LVK detections)
- BNS signals: GW170817 & GW190425
- Modelling of BNS waveforms
- Detection prospects and inferences:
  - Inspiral
  - Early postmerger
  - Late postmerger
  
- Few remarks on topics untouched

# Motivation

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Fields impacted from the study of BNS mergers:

- **Gravitational wave physics:** among strongest source.
- **Neutron star properties:** laboratory to study high-density matter; key to decipher NS properties (e.g. radius and EoS).
- **GRB physics:** central engine of short gamma-ray bursts. BNS merger GRB association.
- **Nucleosynthesis of heavy elements:** kilonova emission due to the radioactive decay of by-products of the r-processed matter from the material ejected in the merger.
- **Cosmology:** measure of Hubble parameter with GW information (BNS as standard sirens).

Most of the above firmly supported by the one single multi-messenger observation of **GW170817**.

# Motivation

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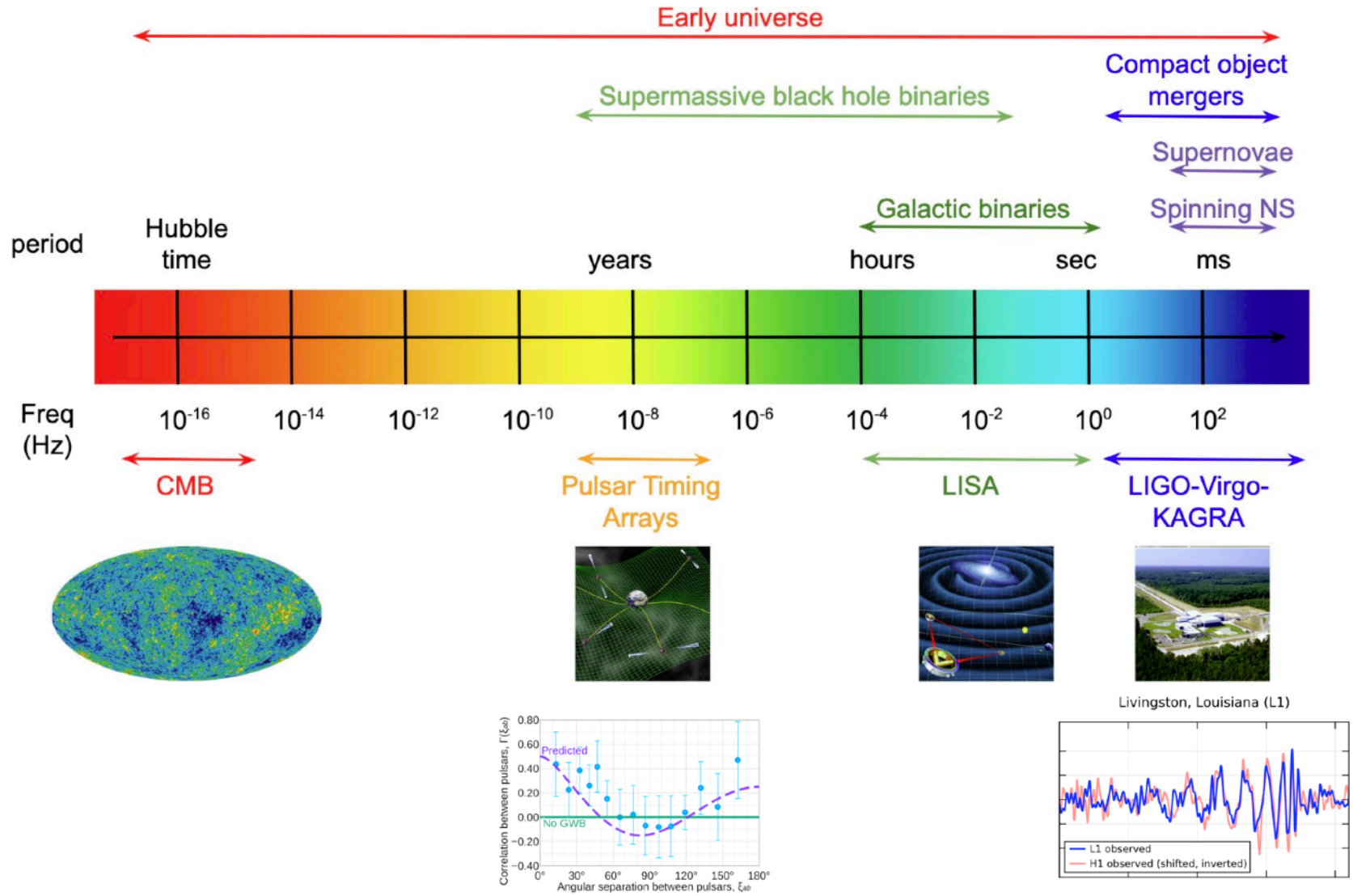
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# Gravitational-wave spectrum

Adapted from: Romano, J.D., Cornish, N.J.  
 Living Rev Relativ 20, 2 (2017).  
<https://doi.org/10.1007/s41114-017-0004-1>

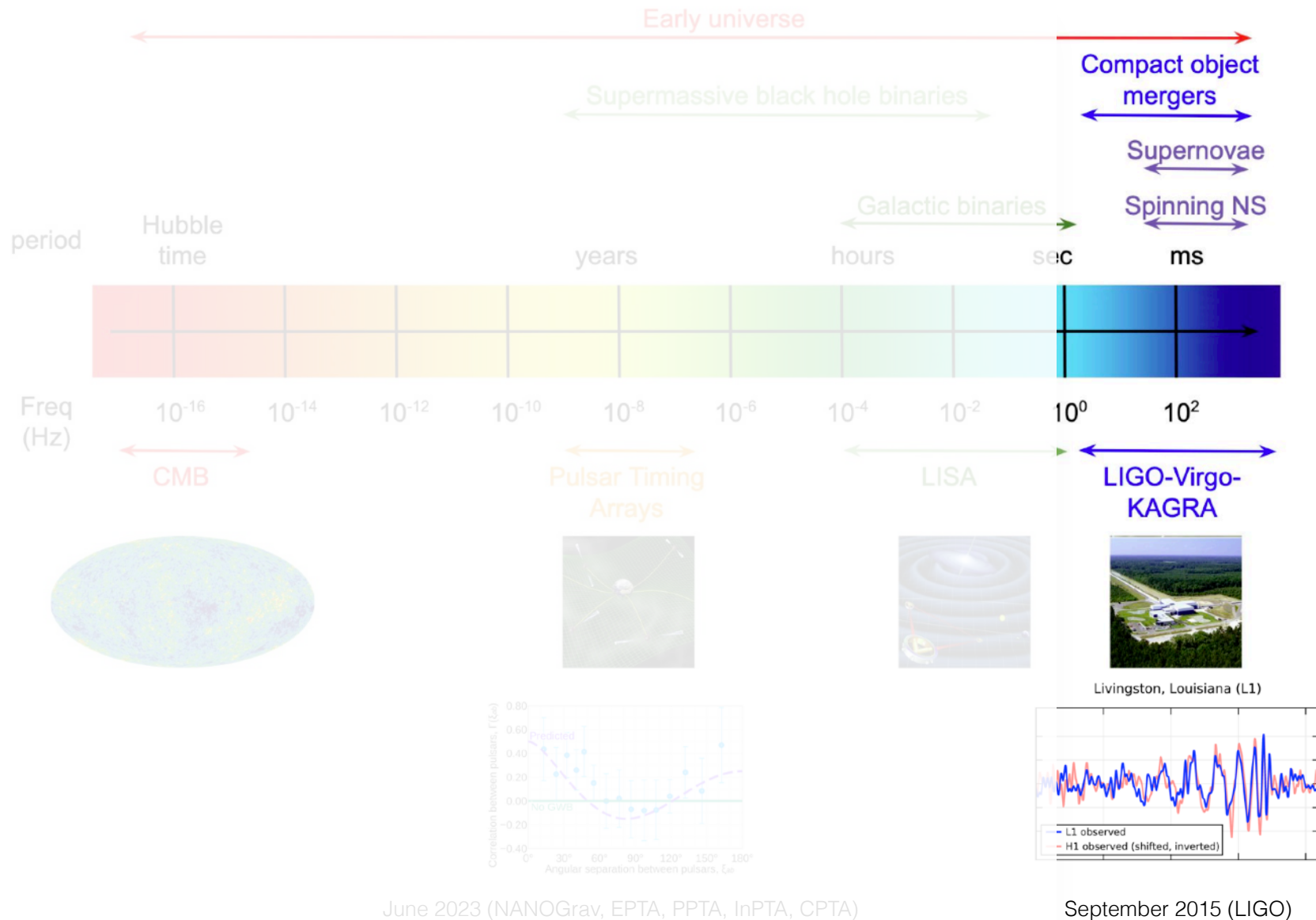


June 2023 (NANOGrav, EPTA, PPTA, InPTA, CPTA)

September 2015 (LIGO)

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# Gravitational waves: present-day detectability region

Prime astrophysical sources of GW: compact objects (neutron stars and black holes) with matter at relativistic speeds.

$$\mathcal{L} \sim \varepsilon^2 \frac{c^5}{G} \left( \frac{R}{R_S} \right)^{-2} \left( \frac{v}{c} \right)^6$$

$$\mathcal{L} \sim 10^{59} \text{ erg s}^{-1} \sim 10^{25} \mathcal{L}_{\odot} \sim 10^{51} \text{ W}$$

Gravitational radiation

$$f \leq \frac{c^3}{4\pi GM} \sim 10^4 \left( \frac{M}{M_{\odot}} \right)^{-1} \text{ Hz}$$

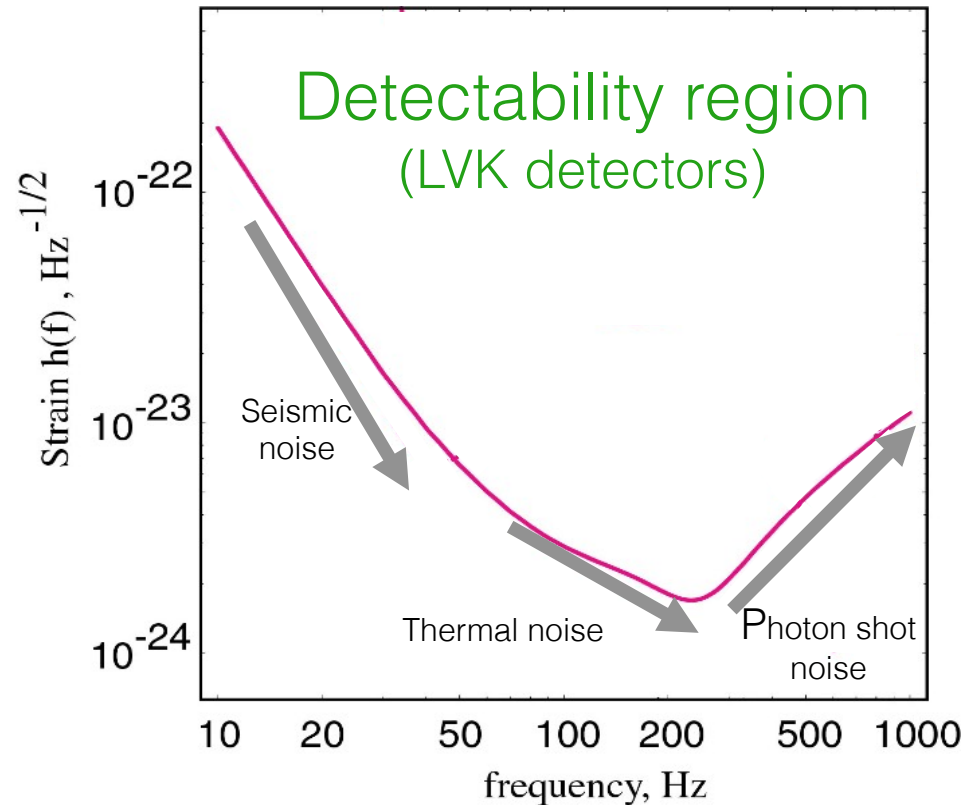
within the audible range

$$[20 \text{ Hz}, 20 \text{ kHz}]$$

Electromagnetic radiation

$$f \geq 10^7 \text{ Hz}$$

$$\text{visible light: } f \sim 10^{11} \text{ Hz}$$



# Orbital evolution B1913+16

Orbital evolution of the Hulse-Taylor binary pulsar agrees with that of a compact binary system that emits gravitational radiation according to GR.

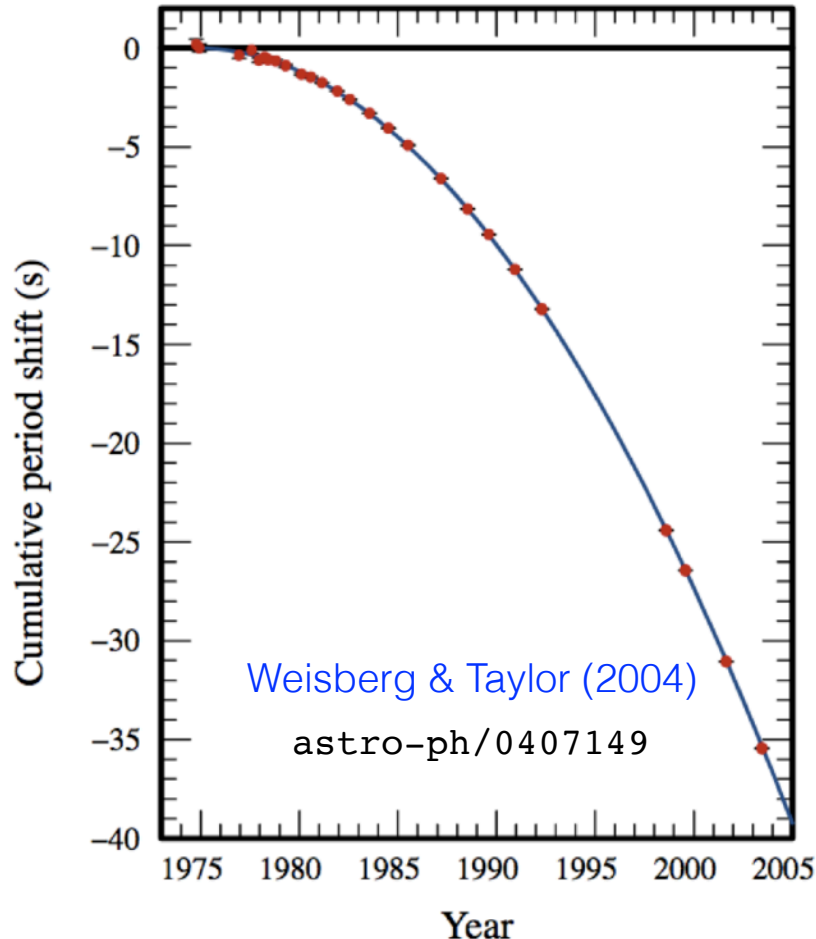


Hulse

Taylor



(1993)



$$t_{\text{coal}} = \frac{5}{256} \frac{c^5}{G^3} \frac{(l_0^{\text{in}})^4}{\mu M^2}$$
$$\approx 1542 \times 10^6 \text{ years}$$

See also [Burgay et al \(2003\)](#) for results on PSR J0737-3039



# Galactic compact BNS systems observed

Examples of Galactic BNS that will merge within a Hubble time (13.7 Gyr)

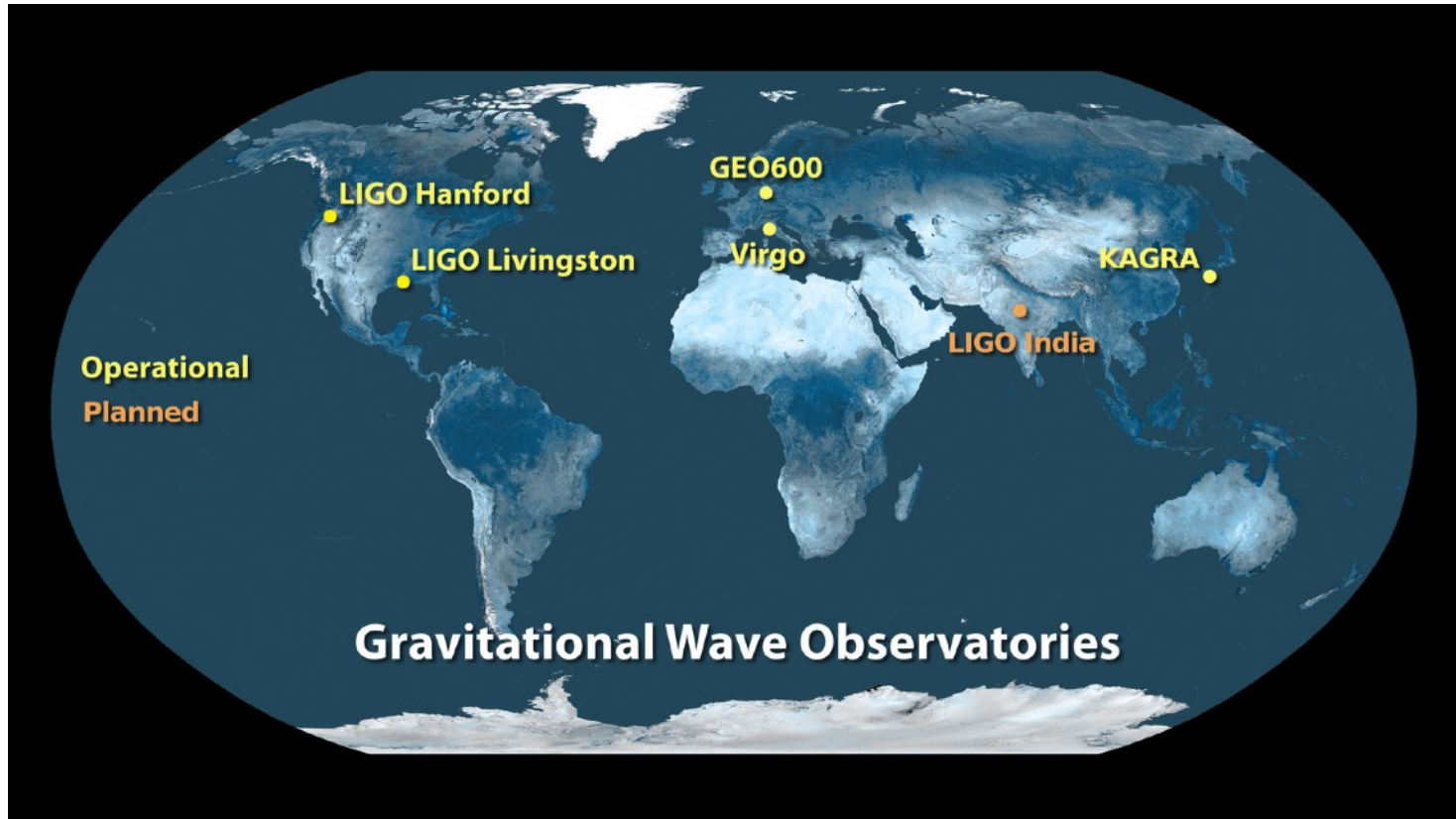
Lorimer (2008)

System	P(day)	e	$M_1(M_{\text{sun}})$	$M_2(M_{\text{sun}})$	$M(M_{\text{sun}})$	$\tau_{\text{GW}}(10^8 \text{yr})$
B1913+16	0,323	0,617	1.39	1.44	2.83	2.45
B1534+12	0,421	0,274	1.33	1.35	2.69	22.5
B2127+11C	0,335	0,681	1.35	1.36	2.71	2.2
J0737-3039	0,102	0,088	1.35	1.24	2.58	0.85
J1756-2251	0,320	0,180	1.31	1.26	2.58	1.69
J1906-0746	0,166	0,085	1.25	1.37	2.62	3.0

$$\tau_{\text{GW}} = \frac{5}{64} \frac{a^4}{\mu M^2} = 2.2 \times 10^8 q^{-1} (1+q)^{-1} \left( \frac{a}{R_{\odot}} \right)^4 \left( \frac{M_1}{1.4 M_{\odot}} \right)^{-3} \text{yr}$$

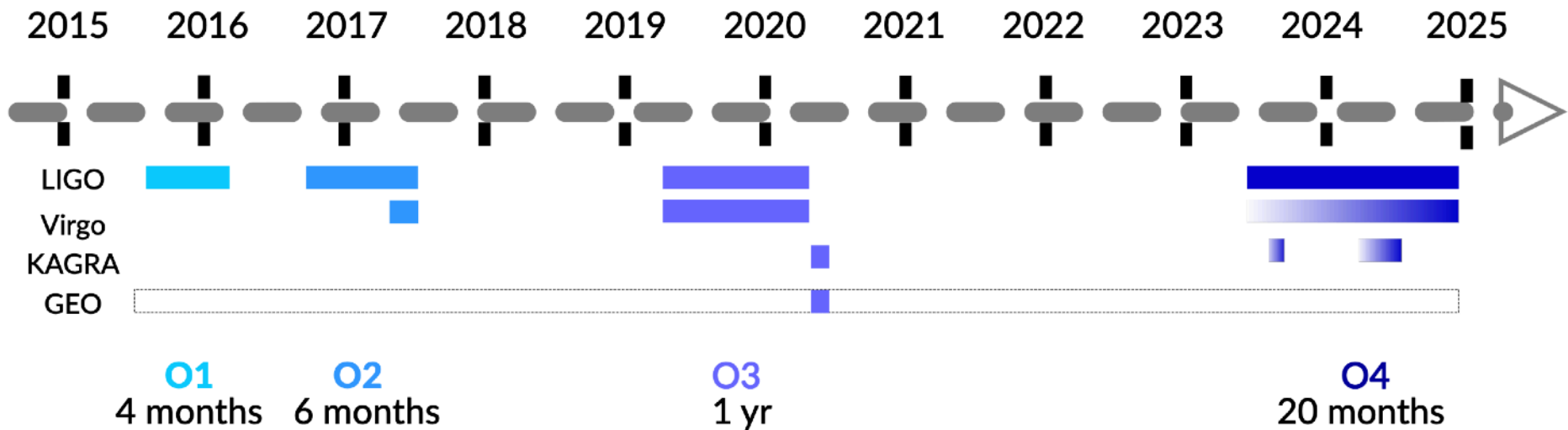
[according to lowest-order dissipative contribution from GR (2.5PN level); both NSs point masses.]

# A global GW detector network



- km-scale interferometers
- sensitive to GWs between a few Hz to a few kHz
- simultaneous detection increases detection confidence
- improved sky localisation and polarization

# Observing timeline



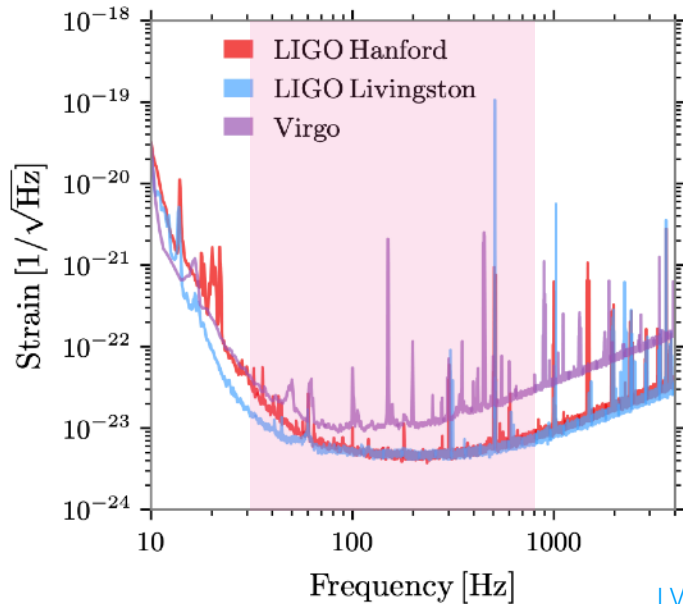
More details at: <https://observing.docs.ligo.org/plan/>

All data is **public**: Gravitational Wave Open Science Center ([www.gw-openscience.org](http://www.gw-openscience.org))

# Detectors' sensitivities and BNS range during O3

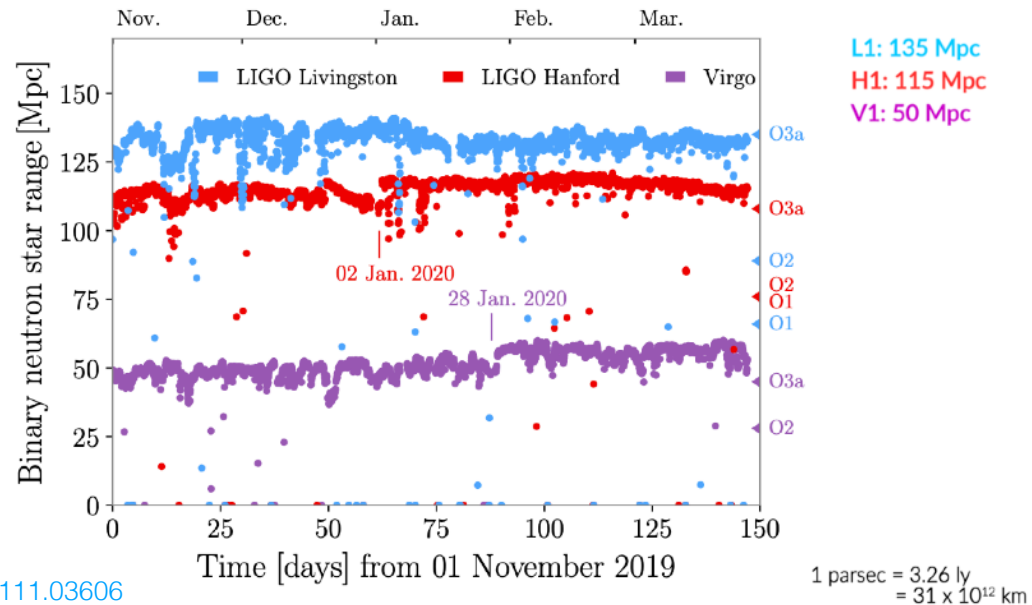


Sensitivities during O3  
(2019-2020)

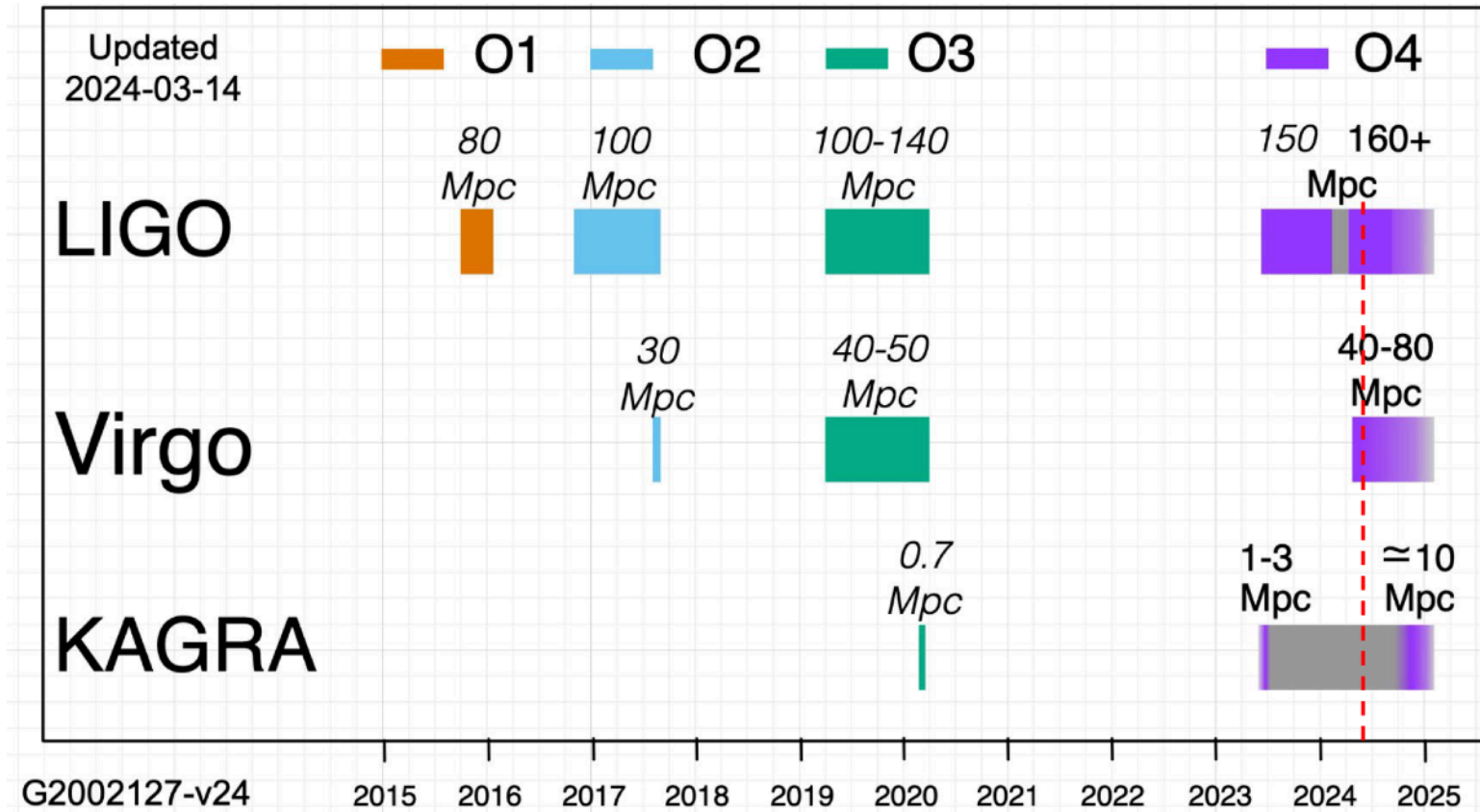


LVK arXiv:2111.03606

Distance range to BNS mergers  
(averaged over sky position and inclination)

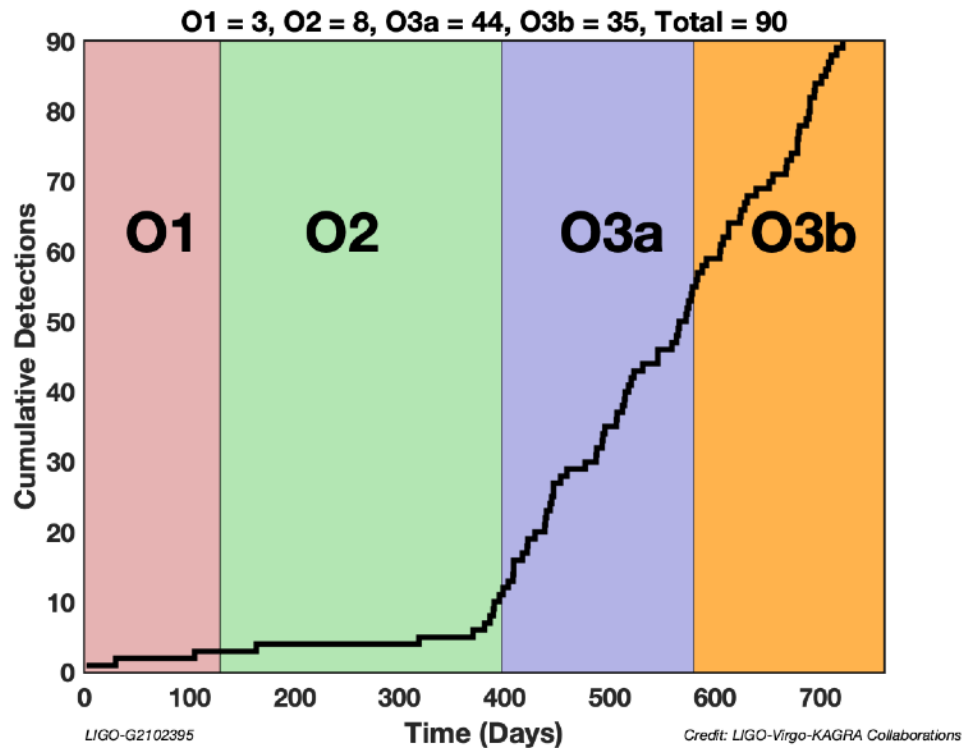


# Observing timeline



More details at: <https://observing.docs.ligo.org/plan/>

# GW observations



GWTC-1:

11 GW events from O1 & O2

Including GW150914 & GW170817

GWTC-2 & GWTC-2.1:

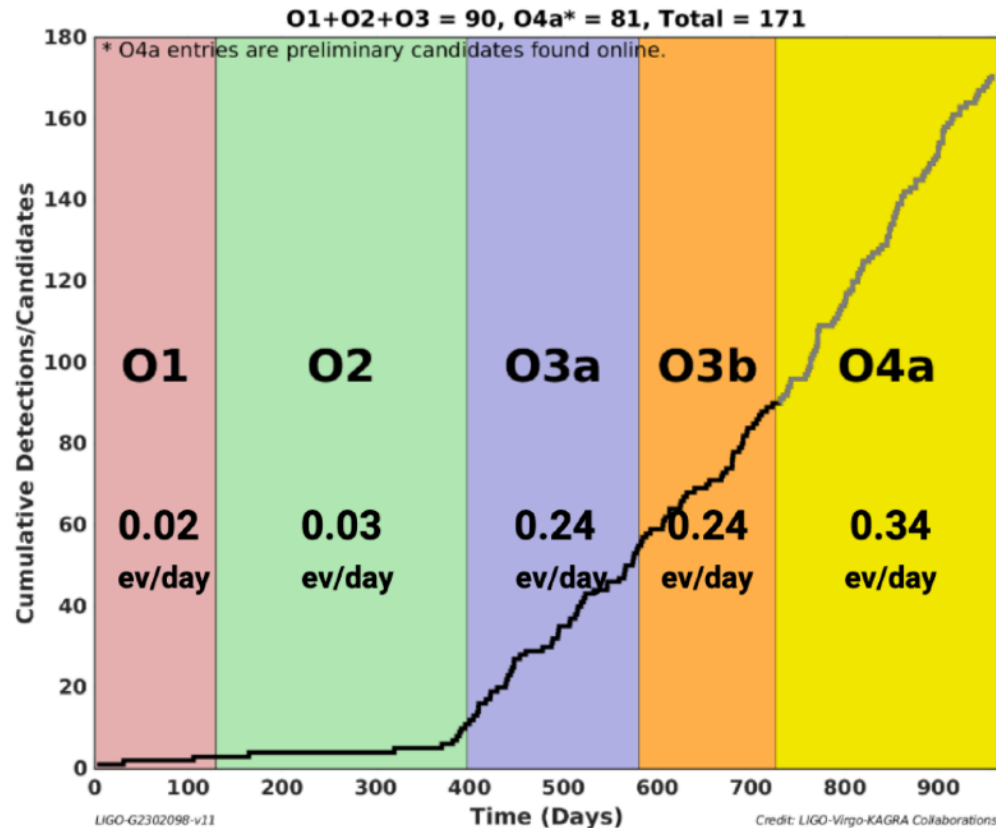
44 new GW events (O3a)

GWTC-3:

35 new GW events (O3b)

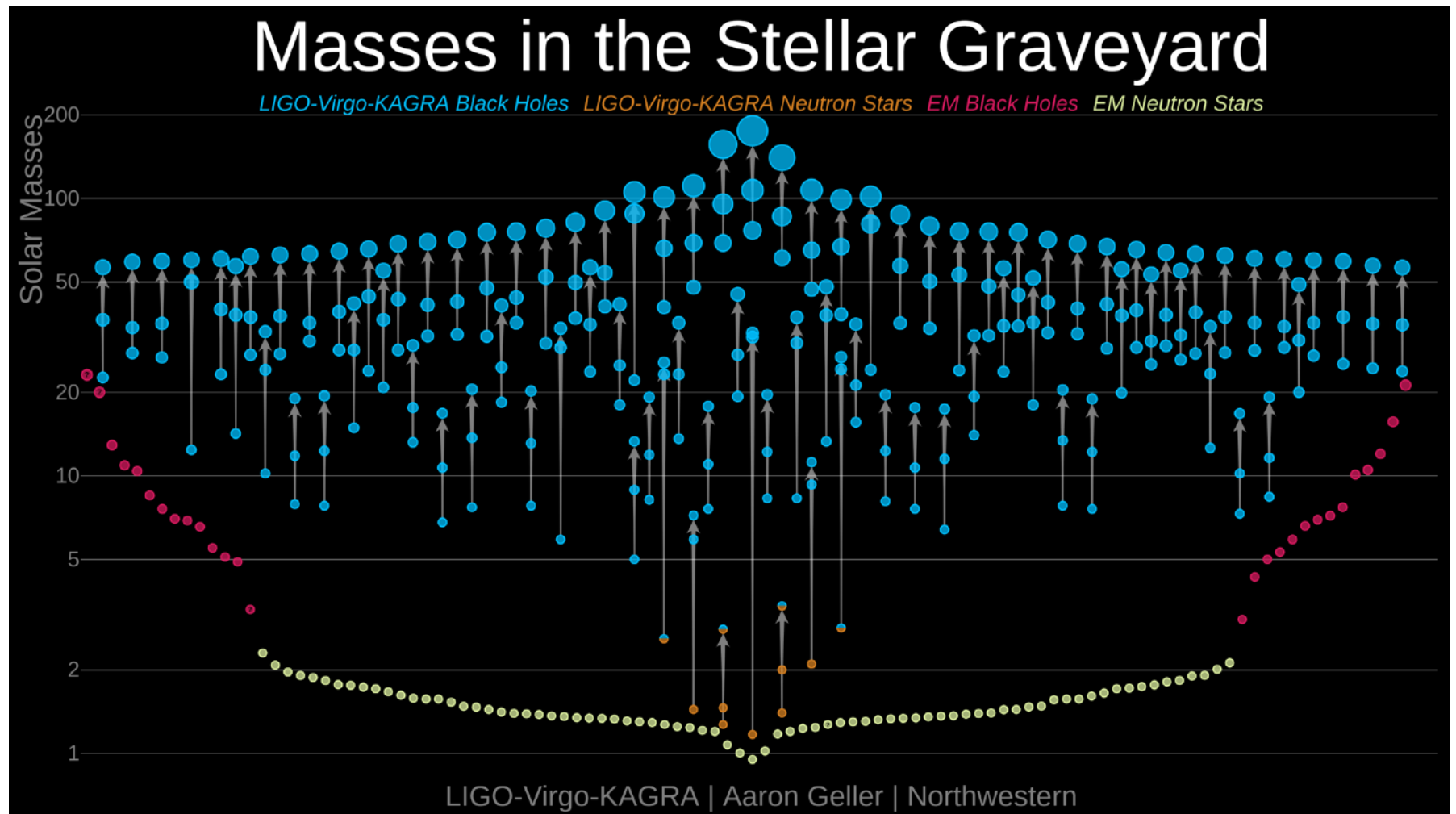
O3 detection rate  $\sim$  1 event every 5 days

# GW observations



O4a significant detection candidates: 81 (92 total, 11 retracted)

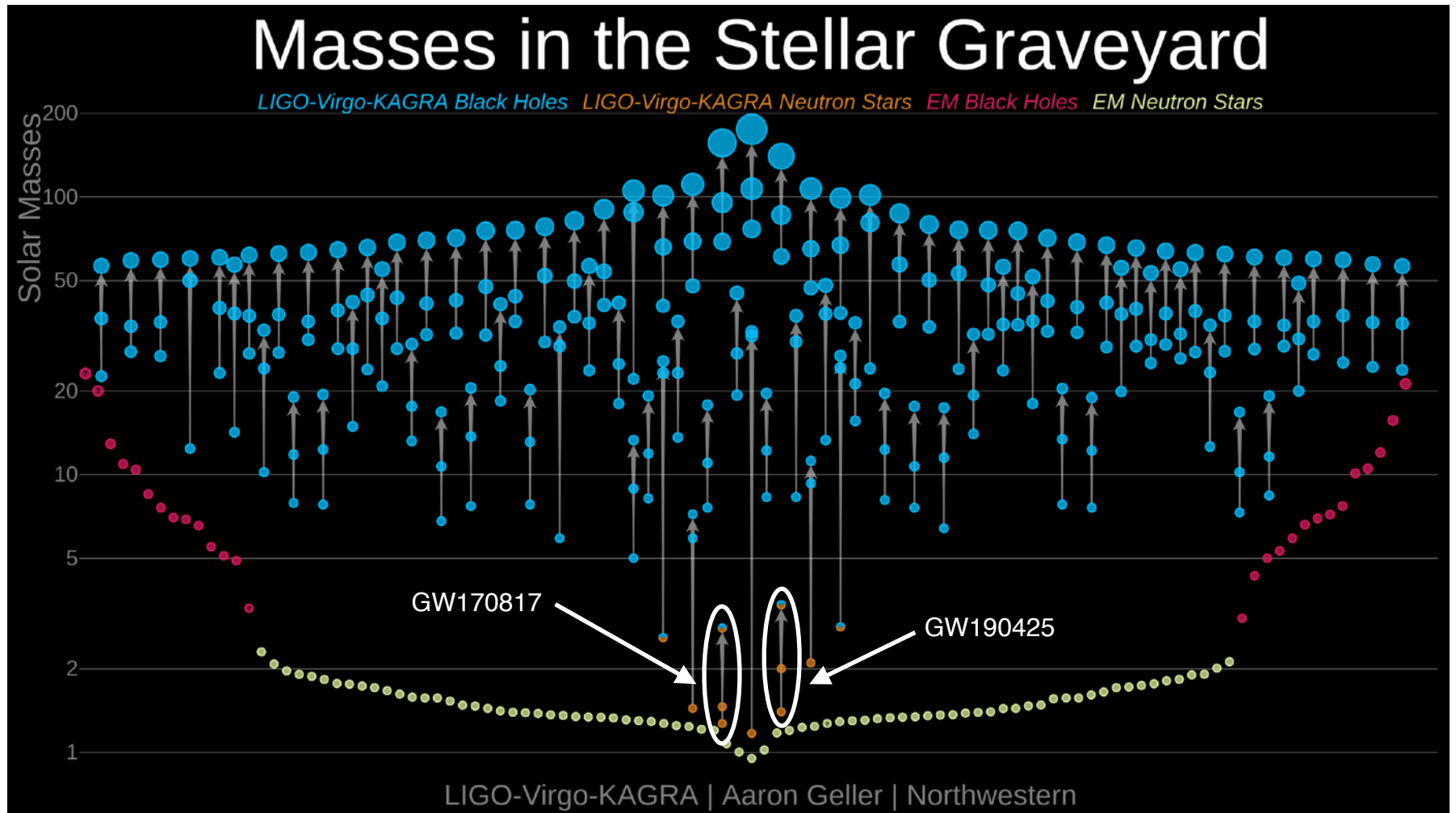
[O4a entries are preliminary candidates found online]



All events through end of O3 with  $p_{\text{astro}} > 0.5$



# GW observations (O1-O3)

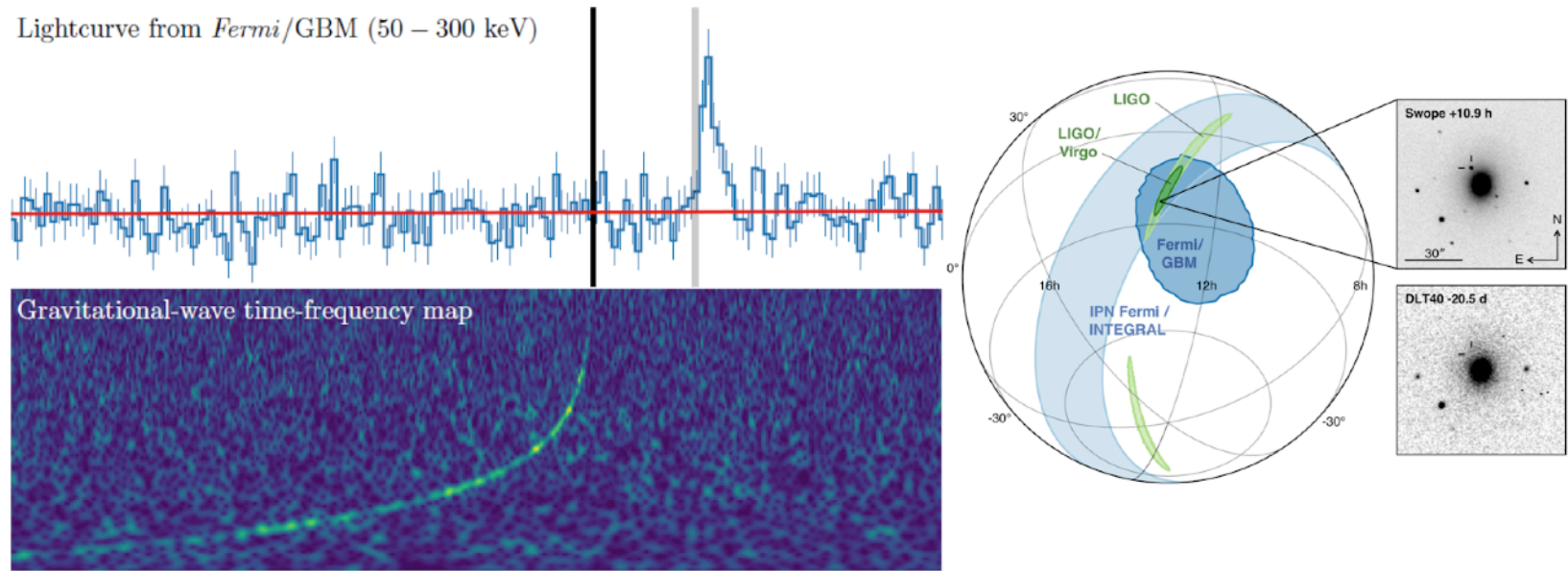


All events through end of O3 with  $p_{\text{astro}} > 0.5$

# LVK BNS mergers - GW170817

GW170817 & GRB170817A & AT2017gfo

Dozens of EM follow-up observations: **multi-messenger astronomy**



Large impact in astrophysics, cosmology, and nuclear physics:

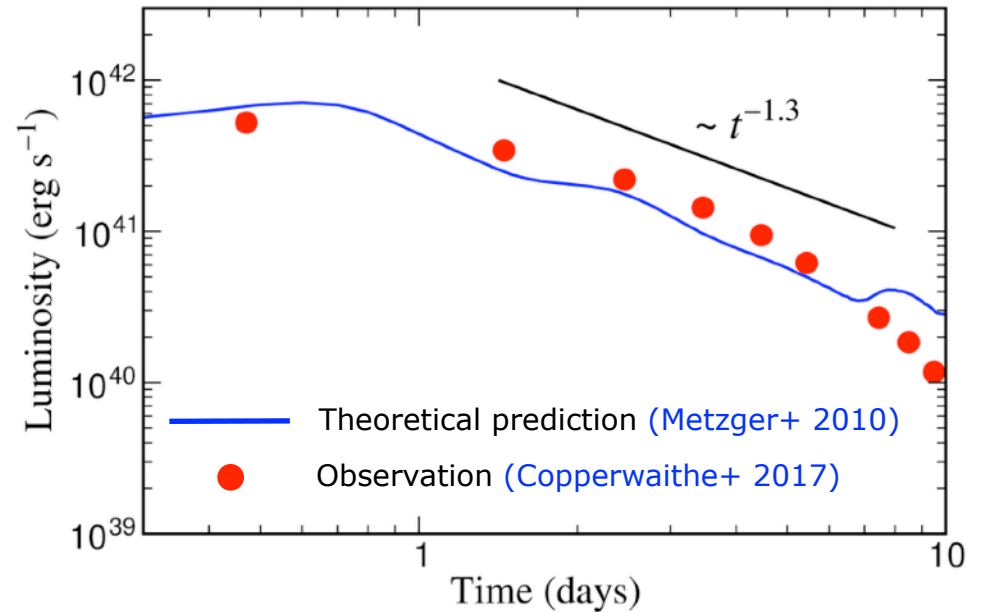
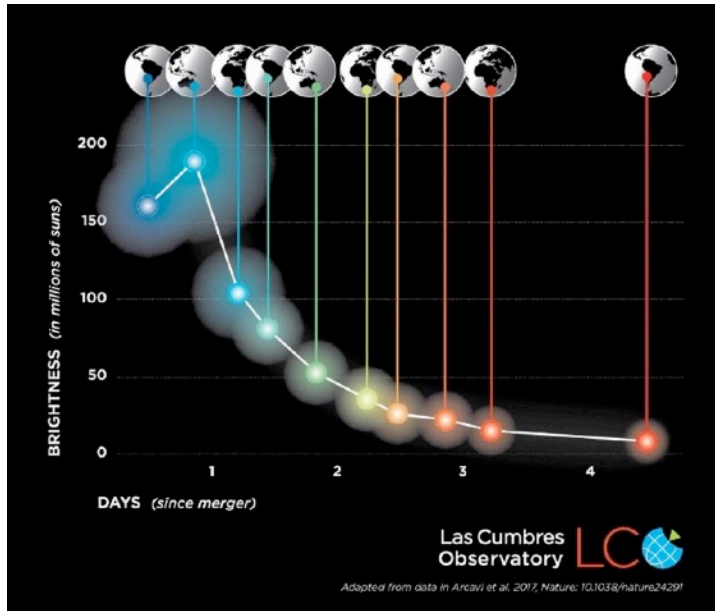
- BNS/sGRB association
- Support for the kilonova model, heavy element nucleosynthesis
- Measurement of  $H_0$  (BNS as standard siren)
- Constraints on EOS of high-density matter (tidal deformability)



# Zoom on the kilonova in NGC 4993



# Does the kilonova model work for GW170817?



Temporal evolution of the kilonova determined by radioactive decay of nuclei. Two components:

Blue: dominated by light elements ( $Z < 50$ )

Red: presence of de lanthanides ( $Z = 57-71$ ) and/or actinides ( $Z = 89-103$ )

GW170817  
0.06 solar masses ejected



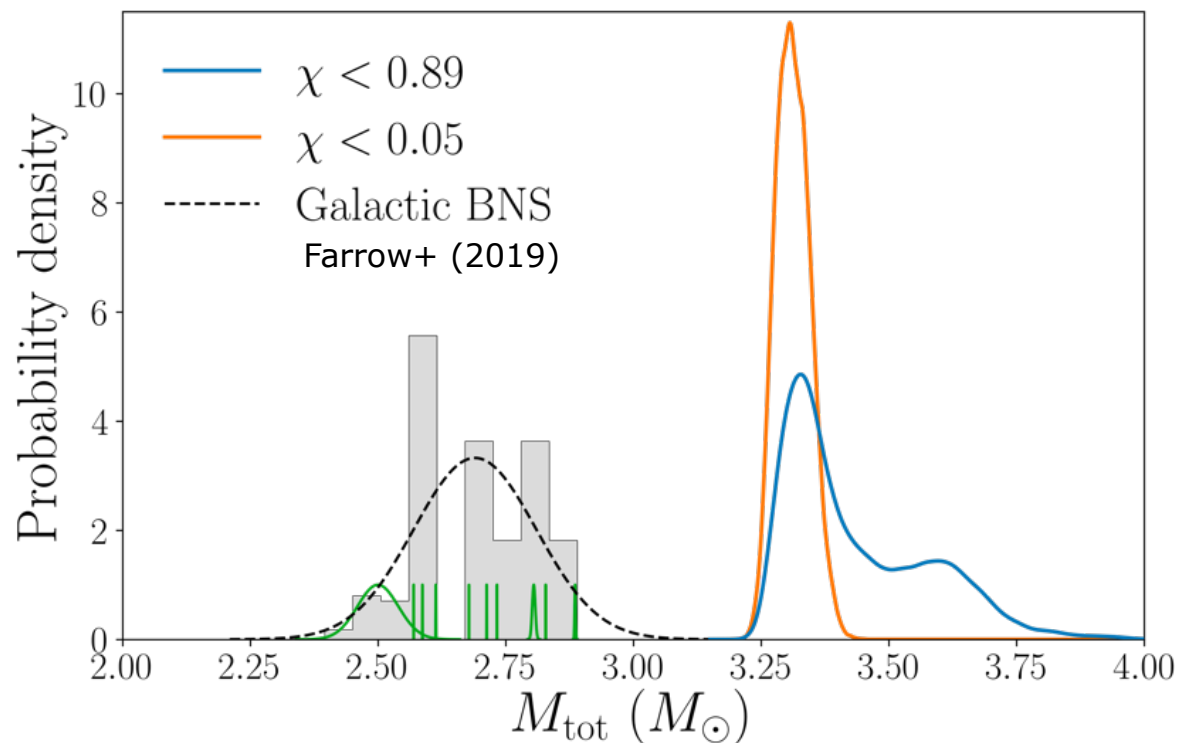
10x Earth mass of gold  
50x Earth mass of platinum  
5x Earth mass of uranium

# LVK BNS mergers - GW190425

GW190425: Observation of a Compact Binary Coalescence with Total Mass  $\sim 3.4 M_{\odot}$   
(LVC, ApJL, 892:L3, 2020)

Most likely 2<sup>nd</sup> BNS merger after GW170817 (BBH or NSBH cannot be ruled out)

2 interferometer detection: L1 + Virgo (poor sky localisation; no EM counterpart)

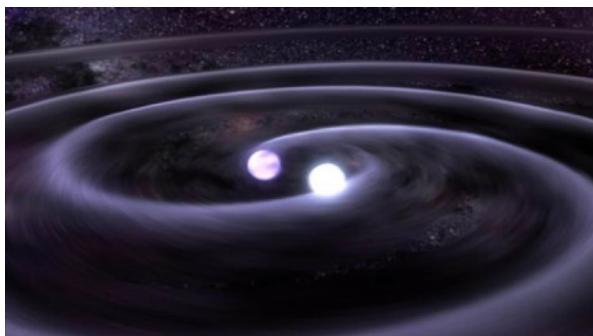


Total mass larger than any known system so far. A new population?

# LIGO-Virgo-KAGRA physics program

## Transient GW signals

Compact Binary Coalescences (CBC)  
modelled

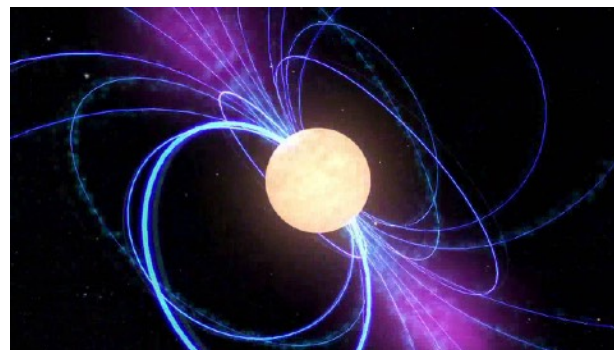


Bursts (e.g. supernovae) unmodelled

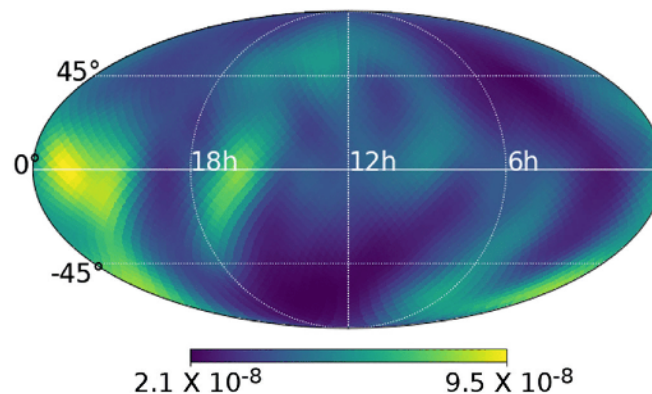


## Long(er) duration GW signals

Continuous waves  
(e.g. rotating neutron stars)



Stochastic GW background

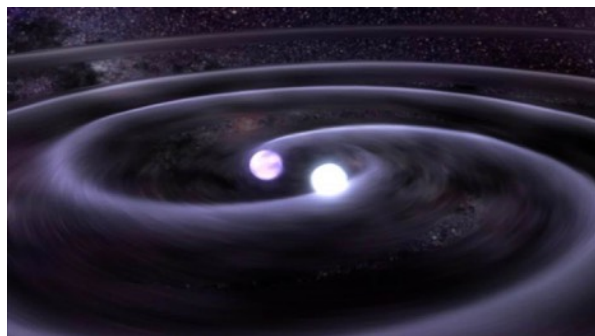


# LIGO-Virgo-KAGRA physics program

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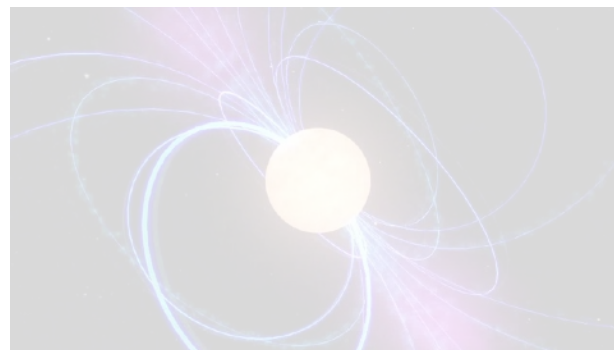


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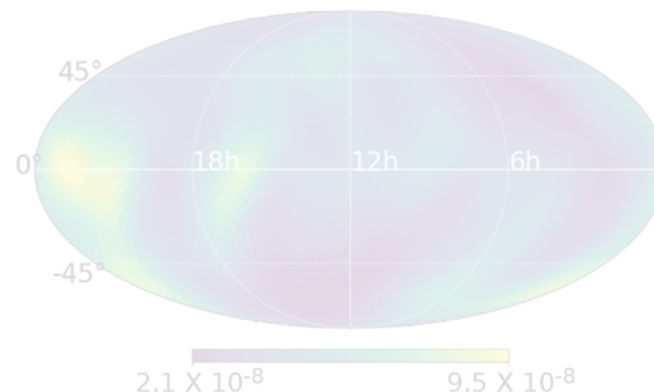


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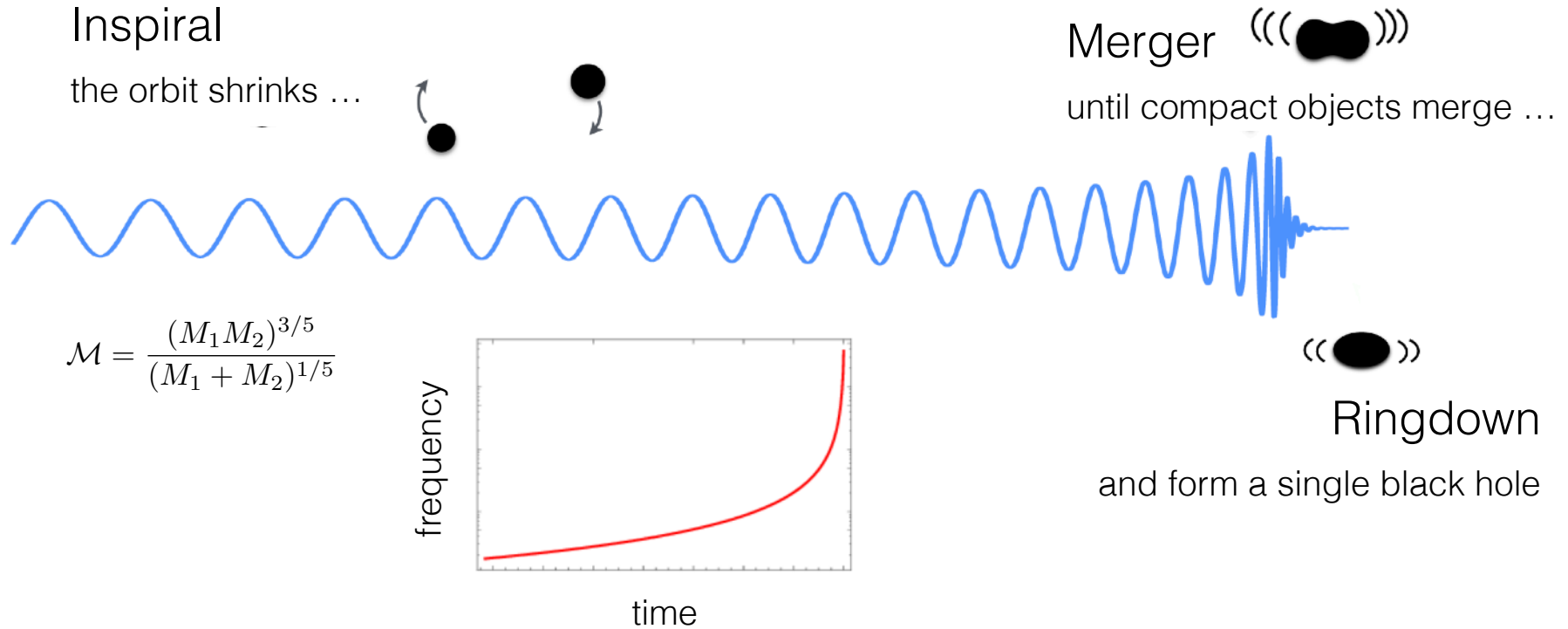
Stochastic GW background



# GW observations

All events detected so far are consistent with **compact binary coalescences**.

Signal “chirps” in the sensitivity band of the detector.

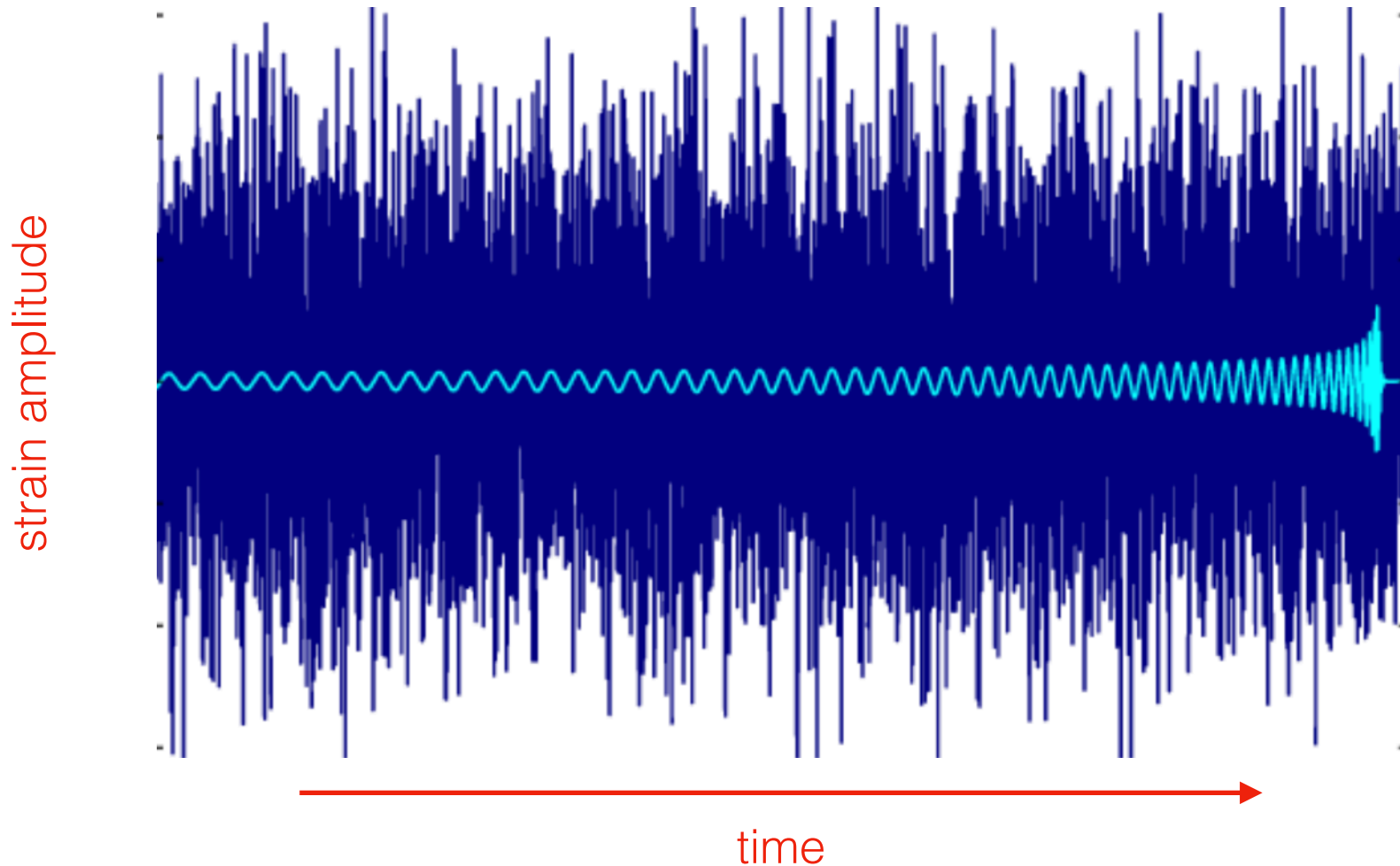


Until merger, B1913+16 will be emitting a distinctive “chirp” gravitational wave signal, a universal waveform for all coalescing, quasi-circular binary systems.



# CBC sources - modelled sources

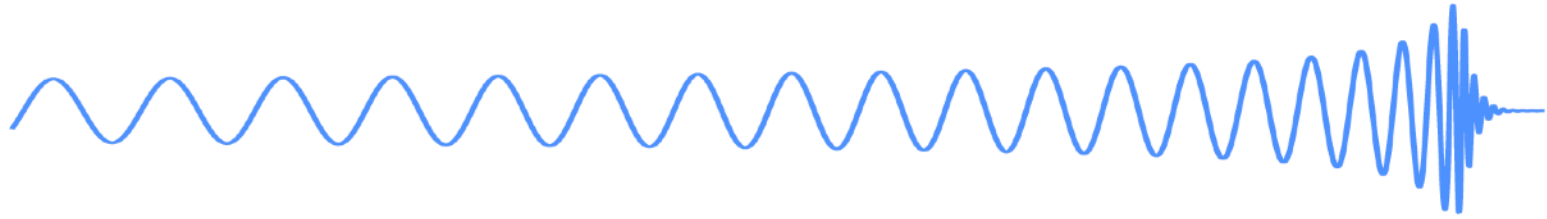
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GW signal buried into noise. To dig the signal out, searches are based on **matched-filtering** using template banks from GR.

# CBC waveform modelling

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**Huge** amounts of template waveform banks required for both **detection** and **parameter estimation**. Banks **incomplete** in some regions of the parameter space (large mass ratio, precession, non QC orbits, ...).

Numerical relativity is our best tool to model CBC waveforms. **But:** incomplete physics, insufficient resolution, memory and computer power limitations. Very expensive.

Synergy between numerical relativity and analytical relativity fundamental.

**Machine Learning** can help complete template banks, aid searches and PE.

Aspects to improve:

- increase mass ratio and harmonic content
- decrease WF systematics
- non-QC binaries (eccentricity, dynamical captures, hyperbolic encounters)
- WF models for exotic compact objects?
- WF models beyond GR?

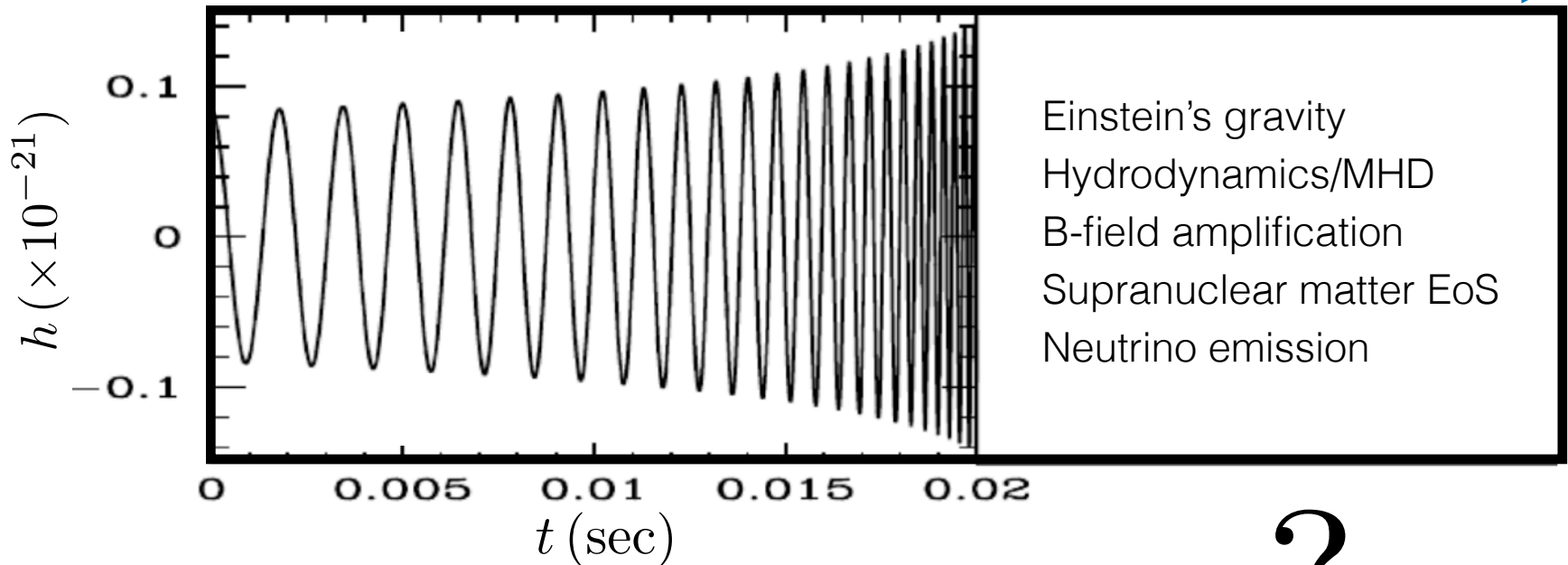
# BNS mergers

Before merger ( $20 \text{ Hz} < \nu < \sim \text{kHz}$ )

After merger ( $\nu > \sim \text{kHz}$ )

post-Newtonian / EOB

Numerical Relativity



PN waveform approximants currently include **high-order tidal corrections, dynamical tidal effects, and spin-tidal couplings**.

**Present-day tidal waveform models** for BNS mergers are mostly based on the EOB approach.

?

Post-merger waveform is the current frontier

# NR waveform approximants - A challenging numerical problem

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## Mathematically & Physically:

- **strong gravitational fields**
- matter motion with **relativistic speeds**
- **shock waves**
- **strong magnetic fields**
- **Nuclear physics:** EoS, radiation transport, nuclear reaction networks

## Numerically:

- intrinsic **multidimensional** character
- inherent issues in Einstein's gravity: **coordinate degrees of freedom** and **curvature singularities** (BH formation)
- **spatial scales:** near zone vs wave zone

## Computationally:

- Time-dependent problem: Major resources needed in terms of memory and CPU. Large parameter space. **Supercomputers.**

Despite difficulties, major progress achieved during ~ **last 25 years** in NR simulations of BNS mergers.

# Numerical Relativity: Our basic theoretical model

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• **Einstein's field equations**  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = 8\pi T_{\mu\nu}$

Formulations: BSSN, Z4, GHF (hyperbolicity).

Methods: high-order finite differencing, (pseudo-)spectral methods.

• **Hydrodynamics equations**  $\nabla_{\mu} T^{\mu\nu} = 0 \quad \nabla_{\mu}(\rho u^{\mu}) = 0$

Flux-Conservative hyperbolic formulations (“Valencia”).

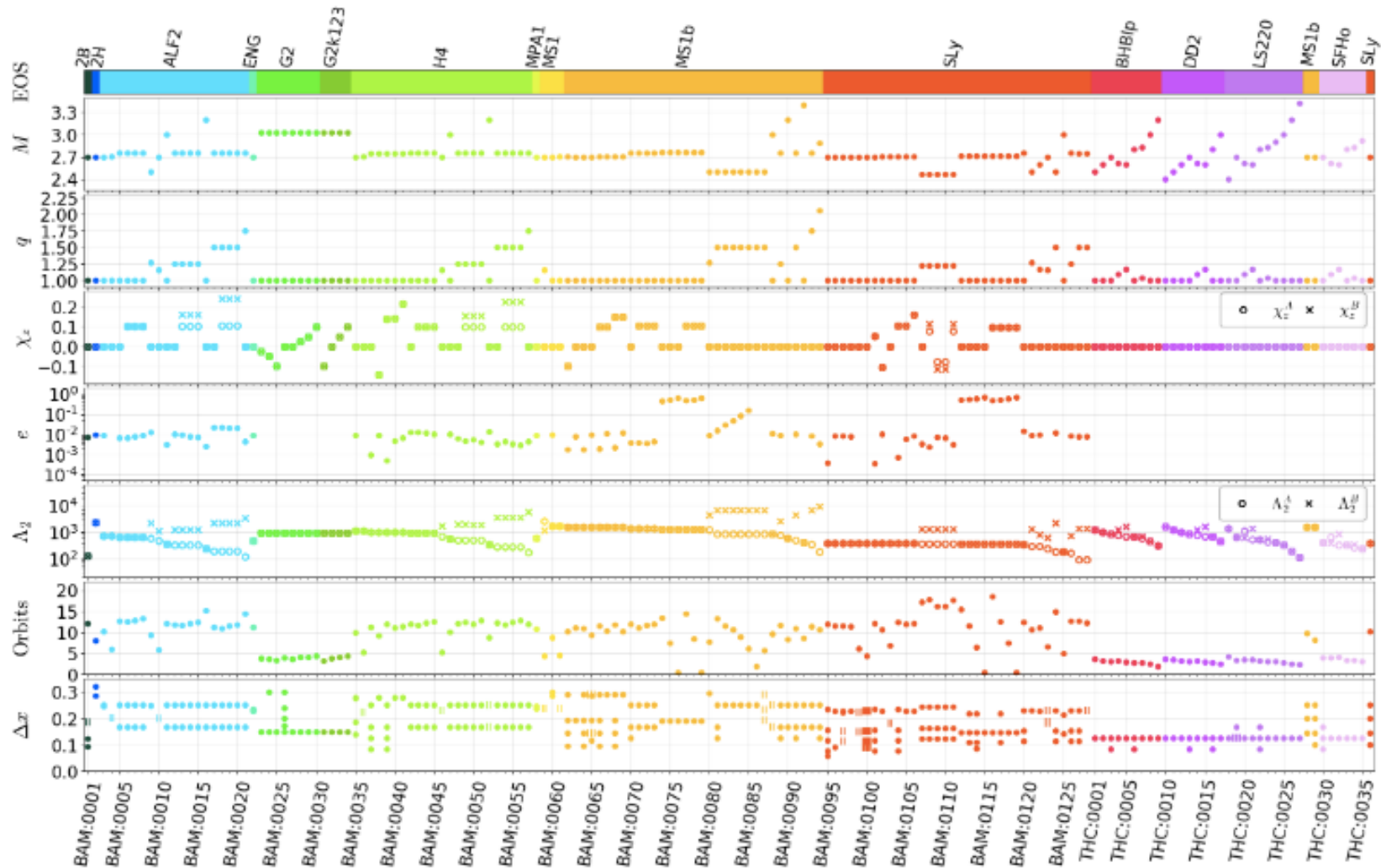
Methods: high-order shock-capturing finite volume.

## Current frontier:

- initial data (eccentric orbits, spins, precession)
- microphysics for tabulated thermal EOS
- magnetic fields (KHI, MRI, amplification, jet formation)
- dissipative (non ideal) fluids
- neutrino radiation transport (full Boltzmann, M1)
- nucleosynthesis (nuclear reaction network)

**Most simulations:** hybrid (piece polytropic + ideal gas) EoS; few include tabulated thermal EoS, neutrino effects, MHD (B-field amplification), and viscosity.

Example: Computational Relativity (CoRe) collaboration's public database of GWs from BNS mergers. ([www.computational-relativity.org](http://www.computational-relativity.org))

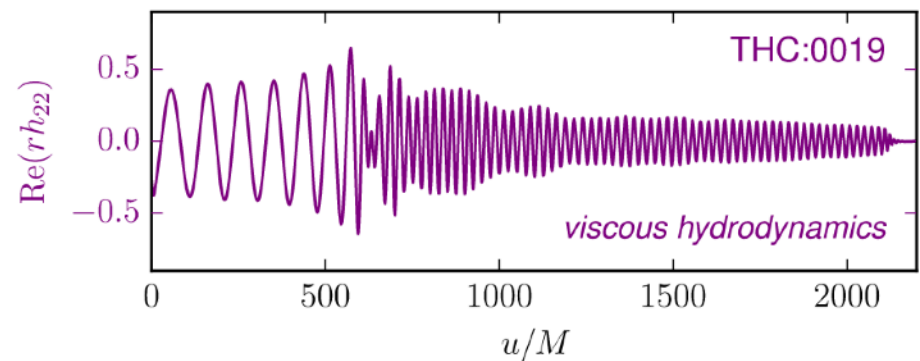
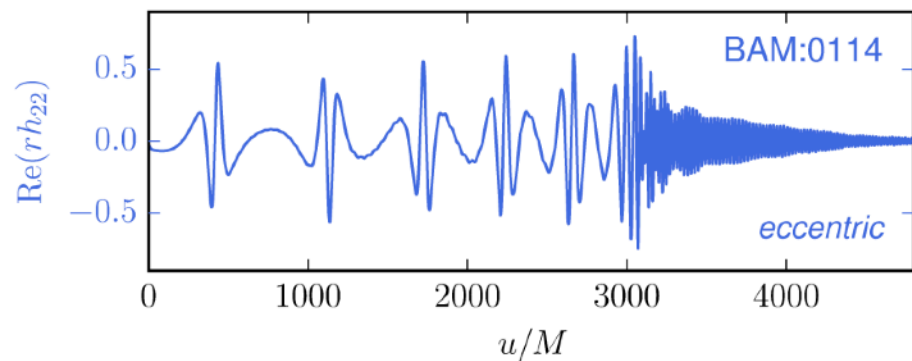
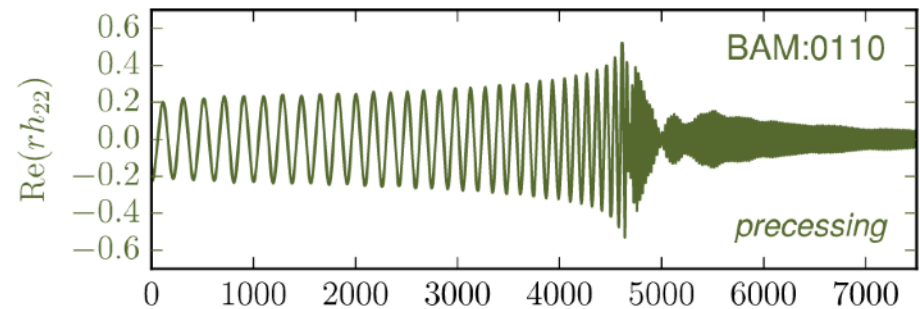
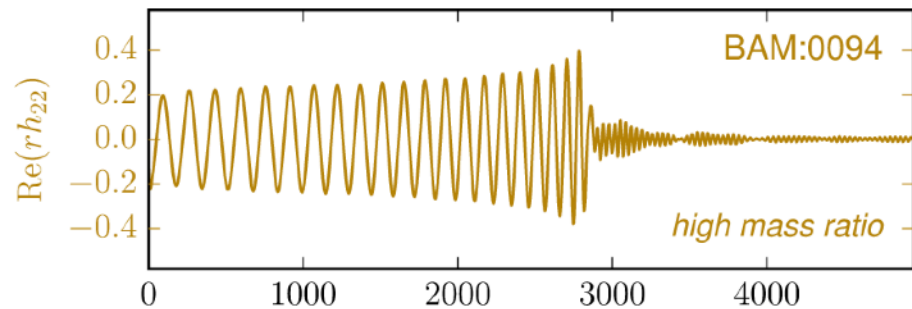
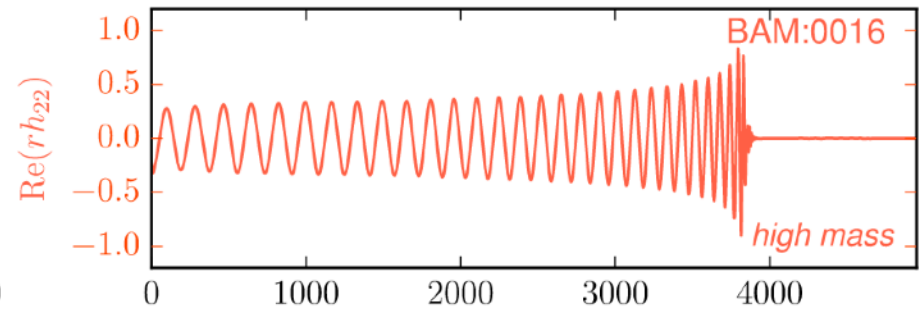
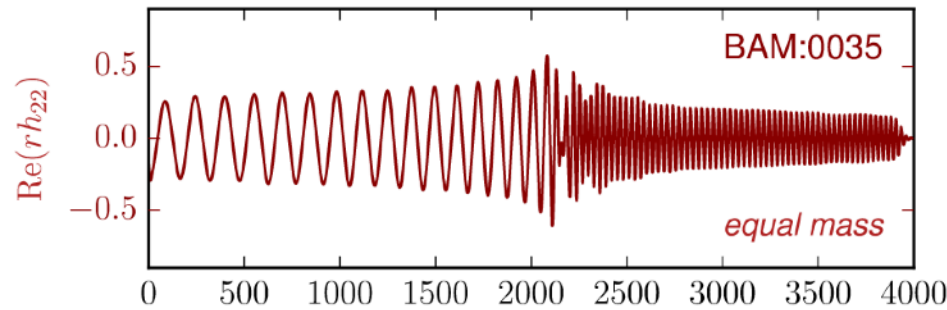


**367 waveforms, 150 million CPU-hours on supercomputers**

NR waveforms fundamental to validate LIGO-Virgo PE pipelines

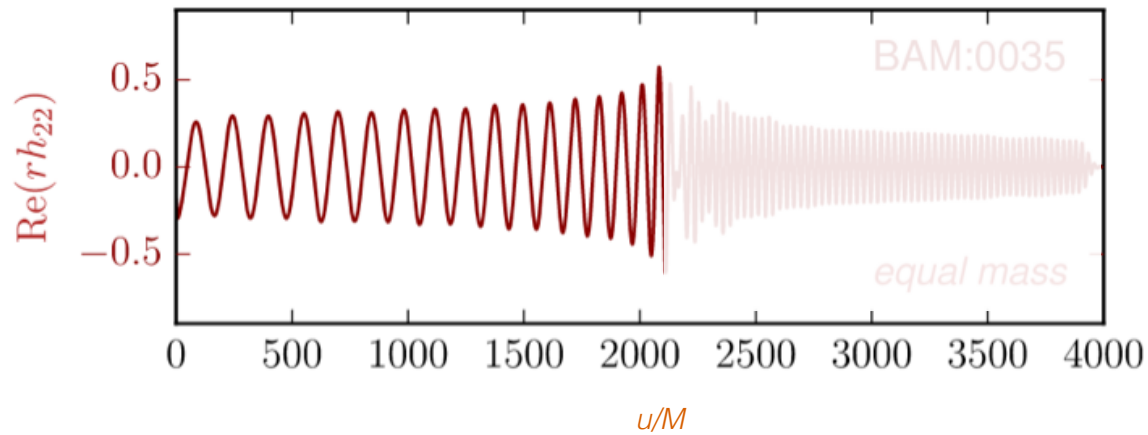
Dietrich+ 2018  
1806.01625

# CoRe BNS database ([www.computational-relativity.org](http://www.computational-relativity.org))



Dietrich+ 2018  
1806.01625

2nd release of the database (590 waveforms)  
González+ 2023  
2210.16366



(Late) inspiral



# Tidal deformability and measurability of NS properties

NS in binary systems produce mutual tidal stresses that deform the metric around the star in an EoS-dependent manner, through the **tidal deformability parameter**.

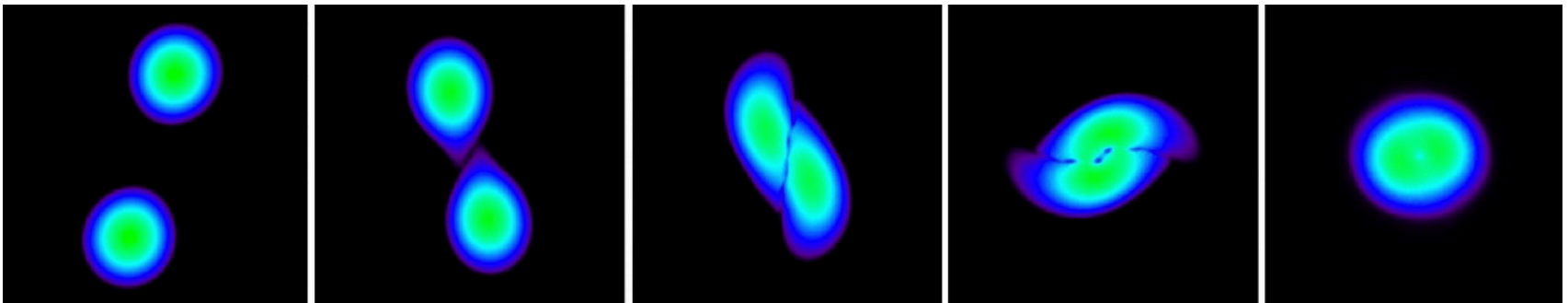
$$\Lambda = \frac{2}{3} k_2 \left( \frac{R}{M} \right)^5$$

$k_2$  quadrupolar Love number

Both  $R$  and  $k_2$  are fixed for a given stellar mass  $M$  by the EoS ( $k_2 \approx 0.05\text{--}0.15$  for realistic neutron stars;  $k_2=0$  for BHs).

The tidal deformability parameter describes the degree to which a local metric suffers quadrupole deformations when in the tidal field of a companion.

Large tidal effects on BNS mergers observed in numerical simulations of late inspiral.



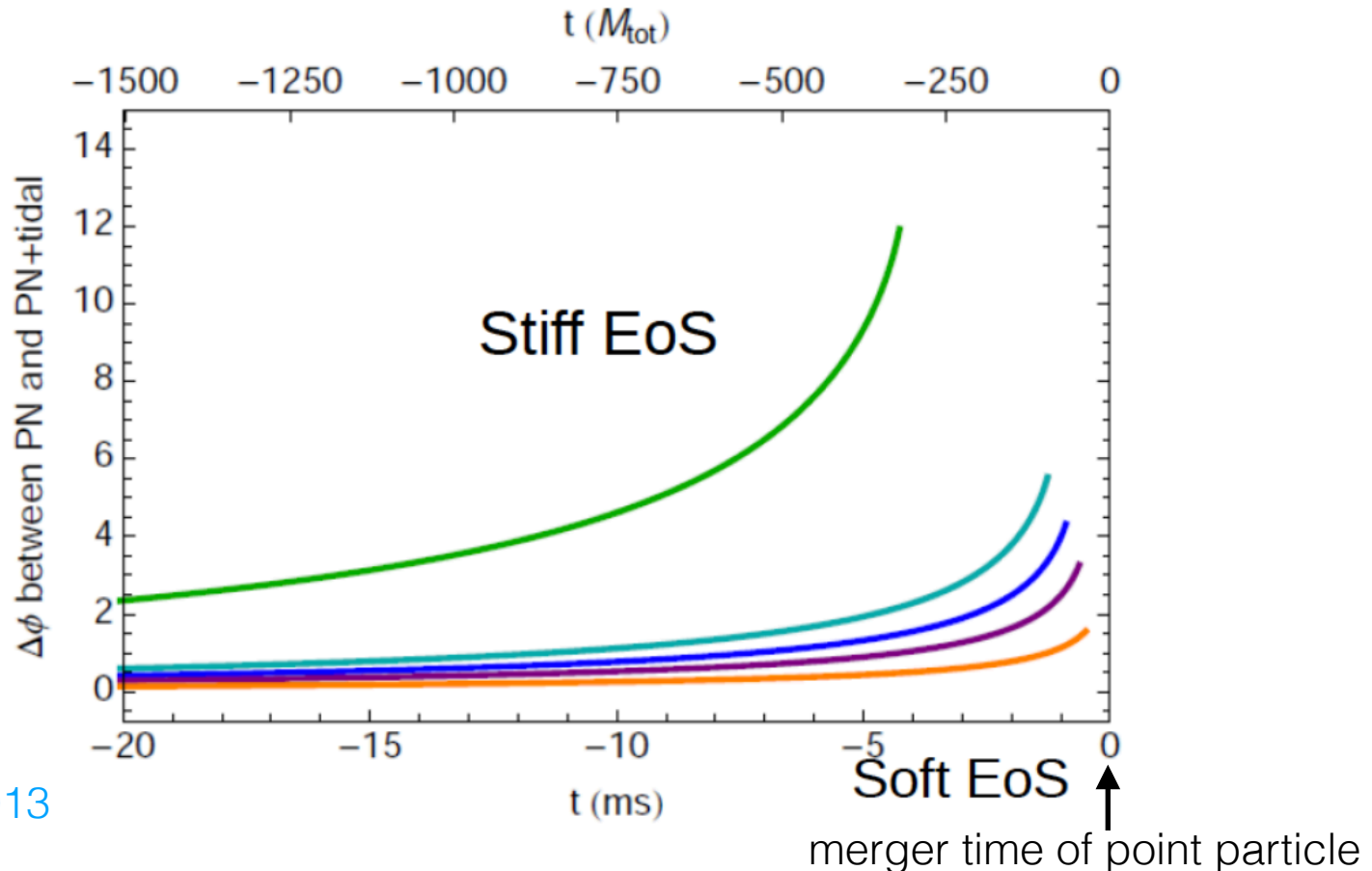
(cf. Bauswein)

It determines the  $(\ell, m) = (2, 0)$  departure of the asymptotic metric from spherical symmetry and the departure of **waveform phase evolution** from point-particle form.

# Tidal deformability and measurability of NS properties

**Orbital phase evolution** affected by tidal deformability, as GW frequency increases. Effect becomes significant above  $f_{\text{GW}} \sim 600$  Hz (only last orbits before merger), potentially observable.

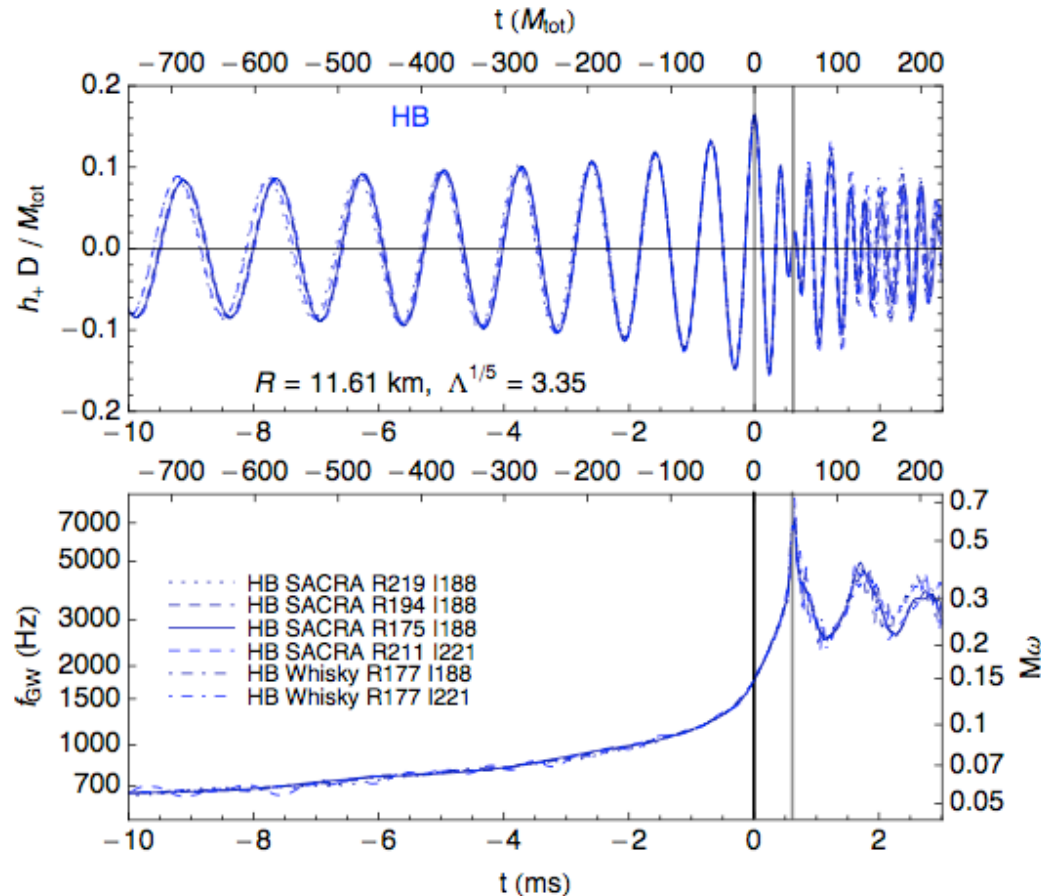
Inspiral **accelerated** compared to point-particle inspiral for larger  $\Lambda$ .



Read+ 2013

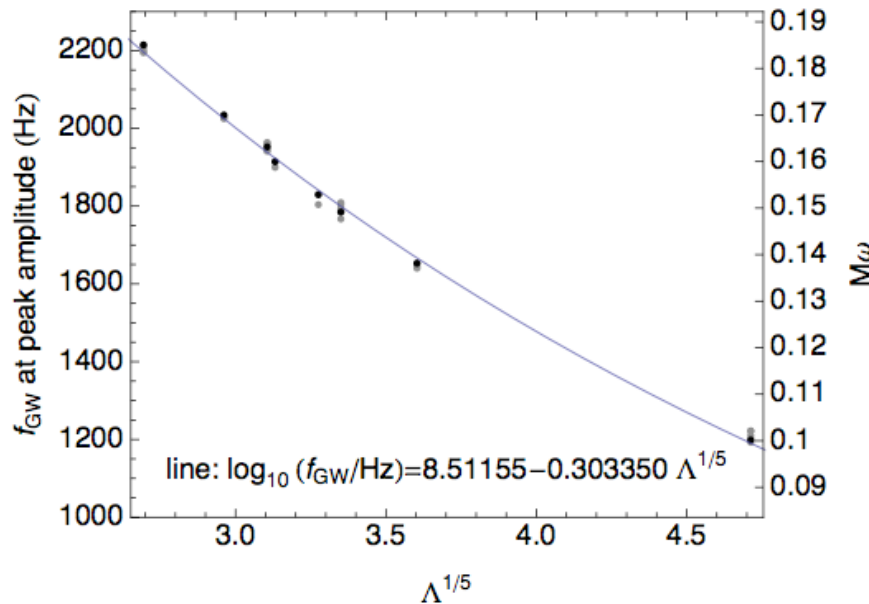
# Tidal deformability and measurability of NS properties

[Read+ \(2013\)](#): systematic investigation of inspiral using extended set of EOS and multiple-code effort (SACRA & Whisky) to generate NR waveforms. Goal: improve data-analysis estimates of the measurability of matter effects in BNSs.



# Tidal deformability and measurability of NS properties

$\Lambda$  effectively characterizes properties of the star at the peak of the GW amplitude at the end of the inspiral phase.



Quasi-universal relation found between peak GW frequency at merger and tidal deformability.

Tidal deformability and radii scale tightly but not perfectly.

Quasi-universality implies that once  $f_{\text{GW}}$  at peak amplitude is measured, so is the tidal deformability, hence  $I$ ,  $M/R$ . **Procedure:** (1)  $f_{\text{GW}}$  and component masses measured from inspiral GW signal. (2) quasi-universal relation yields values of component Lambdas; (3) analytic relation between Lambda and  $R$  yields individual radii.

With a single source at  $\sim 100$  Mpc, the NS radius or  $\Lambda$  could be constrained to about 10%.

# Constraining the NS radius & EoS with GW170817

Properties of GW source inferred by matching the data with predicted waveforms.

Frequency domain post-Newtonian waveform model.

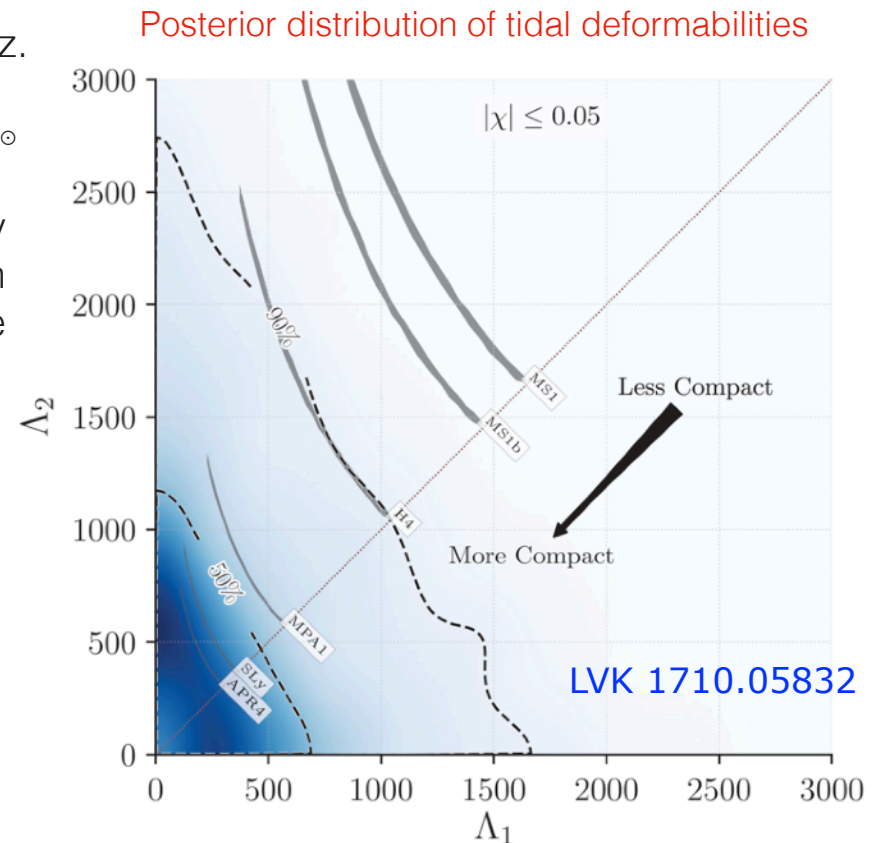
Bayesian analysis in the frequency range 30–2048 Hz.

Source-frame chirp mass constrained to be  $\sim 1.186M_{\odot}$

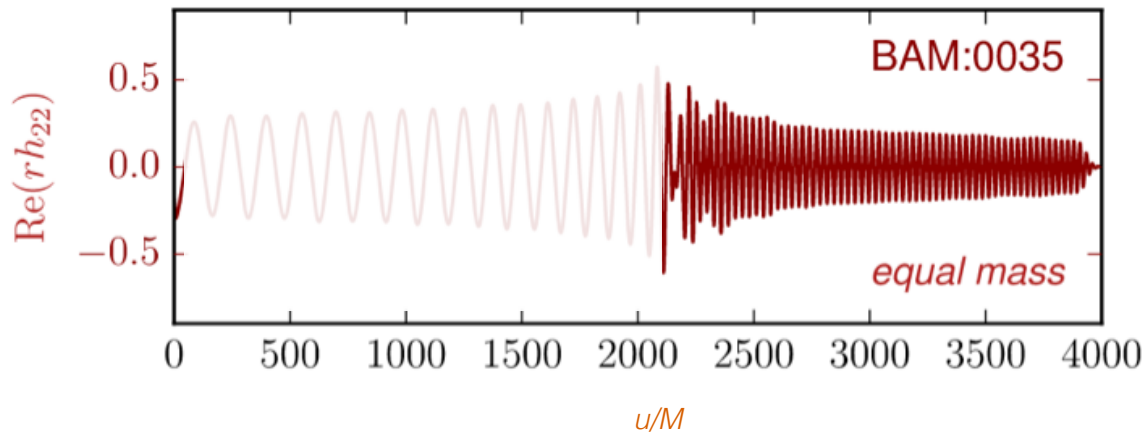
Estimates of component masses affected by degeneracy between mass ratio and aligned spin components. Assumptions made on admissible values of spins.

Low-spin prior (consistent with observed population) yield component masses  $M_1 \in (1.36, 1.60)M_{\odot}$  and  $M_2 \in (1.17, 1.36)M_{\odot}$ .

A single function  $\Lambda(m)$  computed from the static  $l=2$  perturbation of a TOV solution.



LVK constraints on  $\Lambda_1$  and  $\Lambda_2$  **disfavor (stiff) EoS** that predict less compact stars, such as MS1 and MS1b, since mass range recovered generates  $\Lambda$  values outside the 90% probability region. Consistent with radius constraints from X-ray observations of neutron stars.

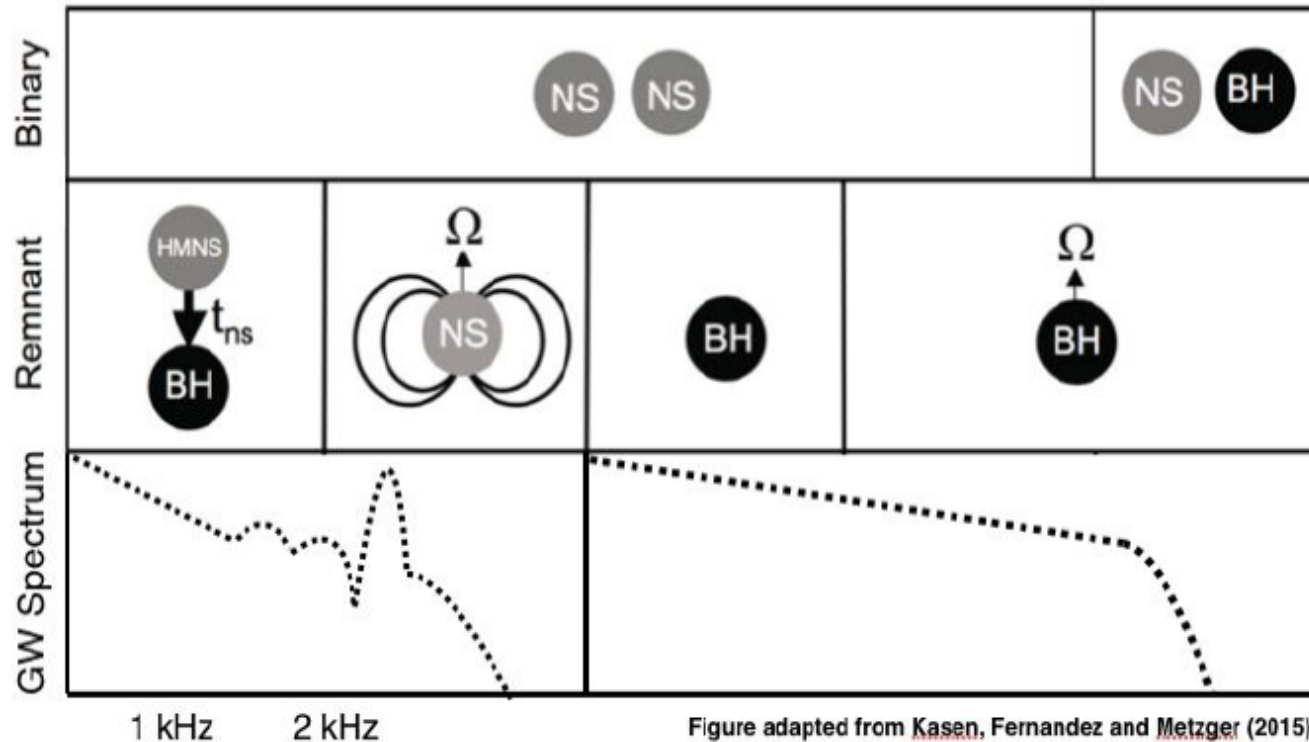


## Postmerger

Inspiral phase only probes cold EoS

Thermal effects not accessible

# Fate of postmerger remnant



## Possible outcome:

- Prompt collapse (BH ringing > 6 kHz)
- HMNS ( $t_{\text{GW}} \sim \text{few-10 ms}$ )
- SMNS ( $t_{\text{GW}} \sim 10\text{-}100 \text{ ms}$ )
- Stable remnant ( $t_{\text{GW}} \sim 100 \text{ ms, minutes, weeks+}$ )

**HMNS:** “Hypermassive” star. Remnant supported by differential rotation and thermal gradients.

**SMNS:** “Supramassive” star. Mass small enough to be supported by rigid rotation.

# Fate of postmerger remnant

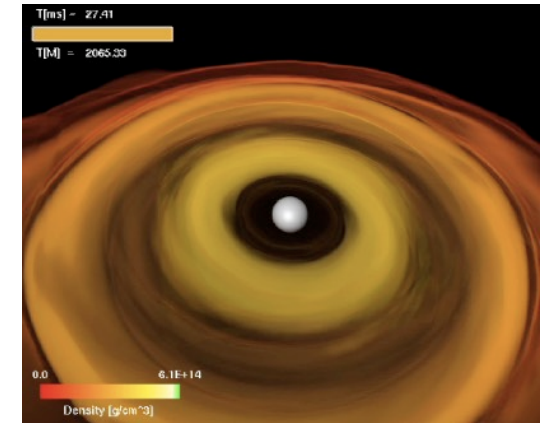
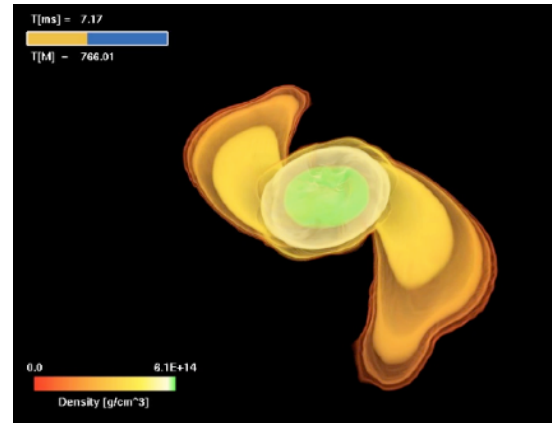
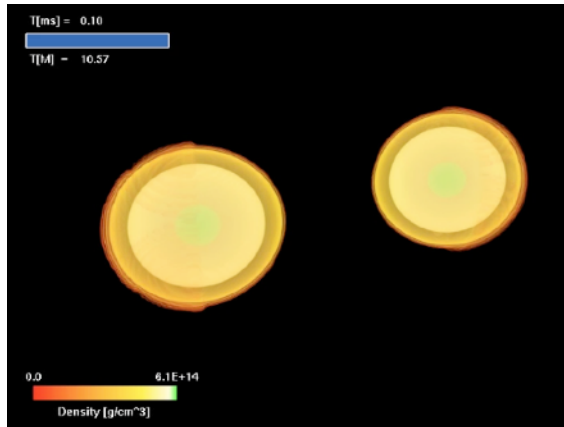
BNS merger



HMNS



BH+torus



Credit: AEI/Frankfurt

Variations on this **general trend** are produced by:

- differences in the **total mass** for the same EOS: a binary with smaller mass will produce a HMNS which is further away from the stability threshold and will collapse at a later time.
- differences in the **EOS** for the same mass: a binary with an EOS allowing for a larger thermal internal energy (ie hotter after merger) will have an increased pressure support and will collapse at a later time.



# Searches of postmerger signals

---

*Search for post-merger GW from the remnant of the binary neutron star merger GW170817* (LVK, Abbott+ 2017)

Search for signals of short duration ( $\lesssim 1$  s) (cWB; Klimenko+ 2016) and intermediate duration ( $\lesssim 500$ s) (STAMP Thrane+ 2011 y cWB) in the LIGO/Virgo data, including emission from HMNS or SMNS, respectively.

No GW signal from post-merger remnant found.

Upper limits (root-sum-square) of the amplitude, between 1-4 kHz, at a 50% detection efficiency

Short signal: 
$$h_{\text{rss}}^{50\%} = 2.1 \times 10^{-22} \text{ Hz}^{-1/2}$$

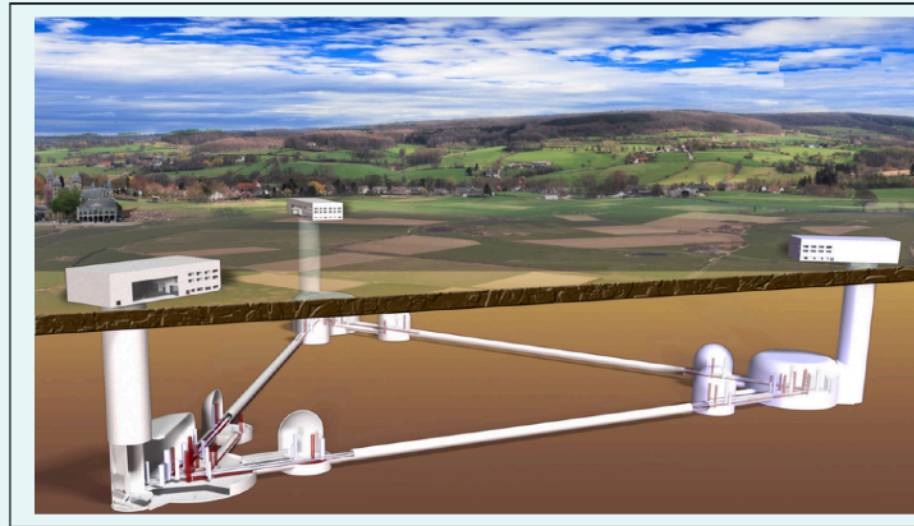
Intermediate signal: 
$$h_{\text{rss}}^{50\%} = 8.4 \times 10^{-22} \text{ Hz}^{-1/2} \quad (\text{ms magnetar model})$$

$$h_{\text{rss}}^{50\%} = 5.9 \times 10^{-22} \text{ Hz}^{-1/2} \quad (\text{bar-mode model})$$

Post-merger emission from a GW170817-like event may be detected when LIGO/Virgo reach design sensitivity or with 3G detectors (ET, CE).

# 3G Detectors

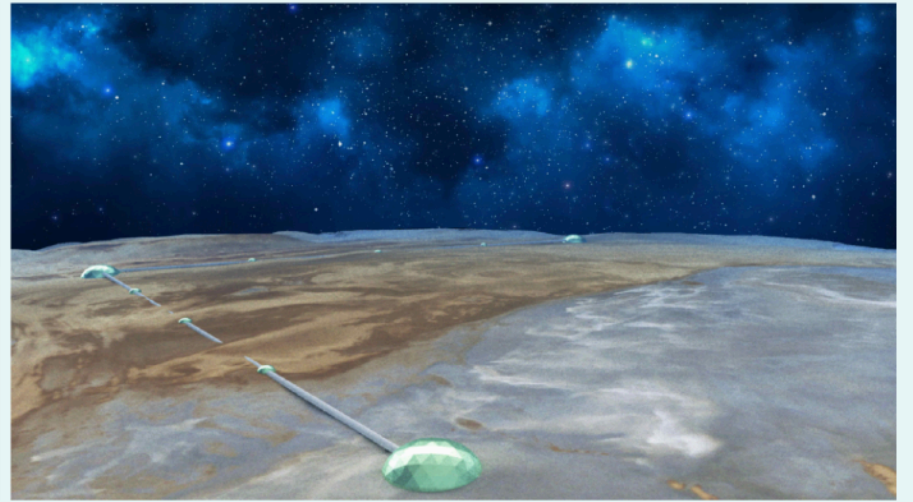
## Einstein Telescope



Einstein Telescope conceived to be six, V-shaped, underground interferometers, formed out of 10 km sides of an equilateral triangle

<http://www.et-gw.eu>

## Cosmic Explorer

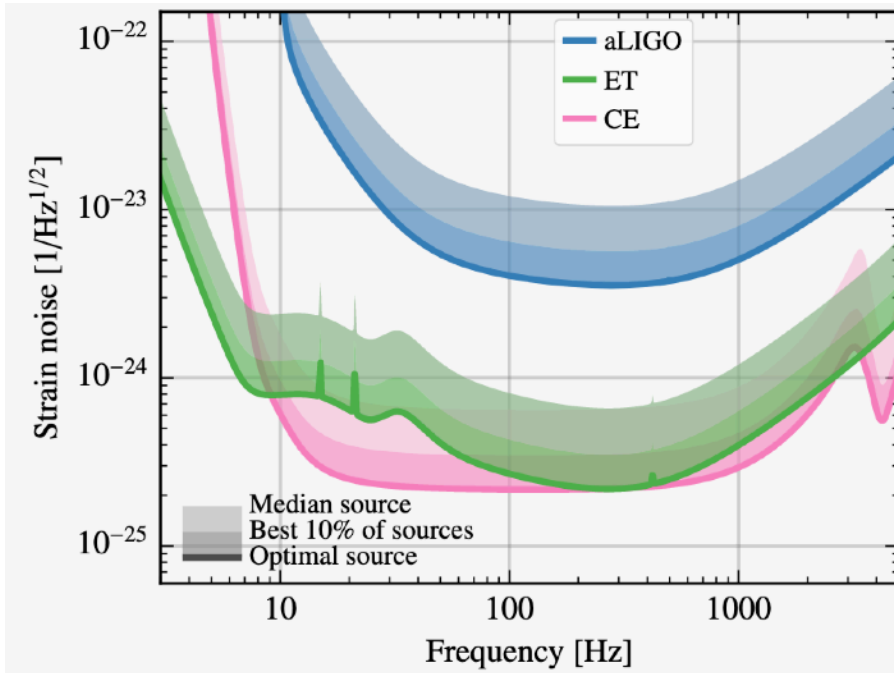


Cosmic Explorer conceived to be an L-shaped, overground interferometer, with 40 km arms

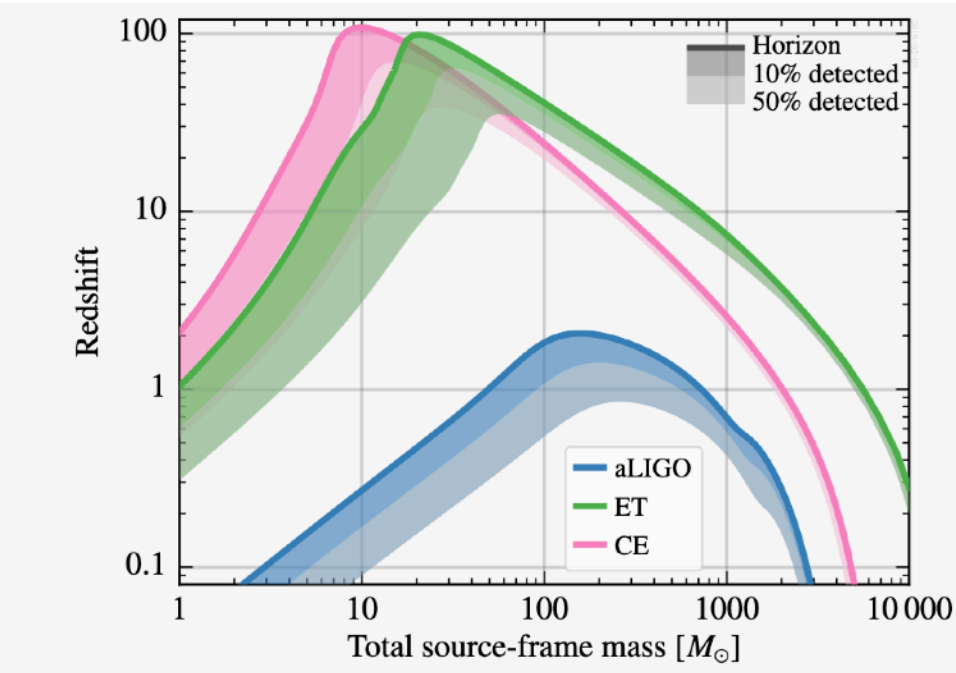
<https://dcc.cosmicexplorer.org/P2100003/public>

GWIC 3G reports <https://gwic.ligo.org/3Gsubcomm/>

# 3G Detectors



Sensitivity of ET and CE compared to Advanced LIGO



Reach for equal-mass non spinning binaries for 3G Observatoires

GWIC 3G reports <https://gwic.ligo.org/3Gsubcomm/>

# 3G Detectors

With ET and CE, GW observatories will leap from monitoring only the nearby Universe to surveying the entire Universe for BH mergers

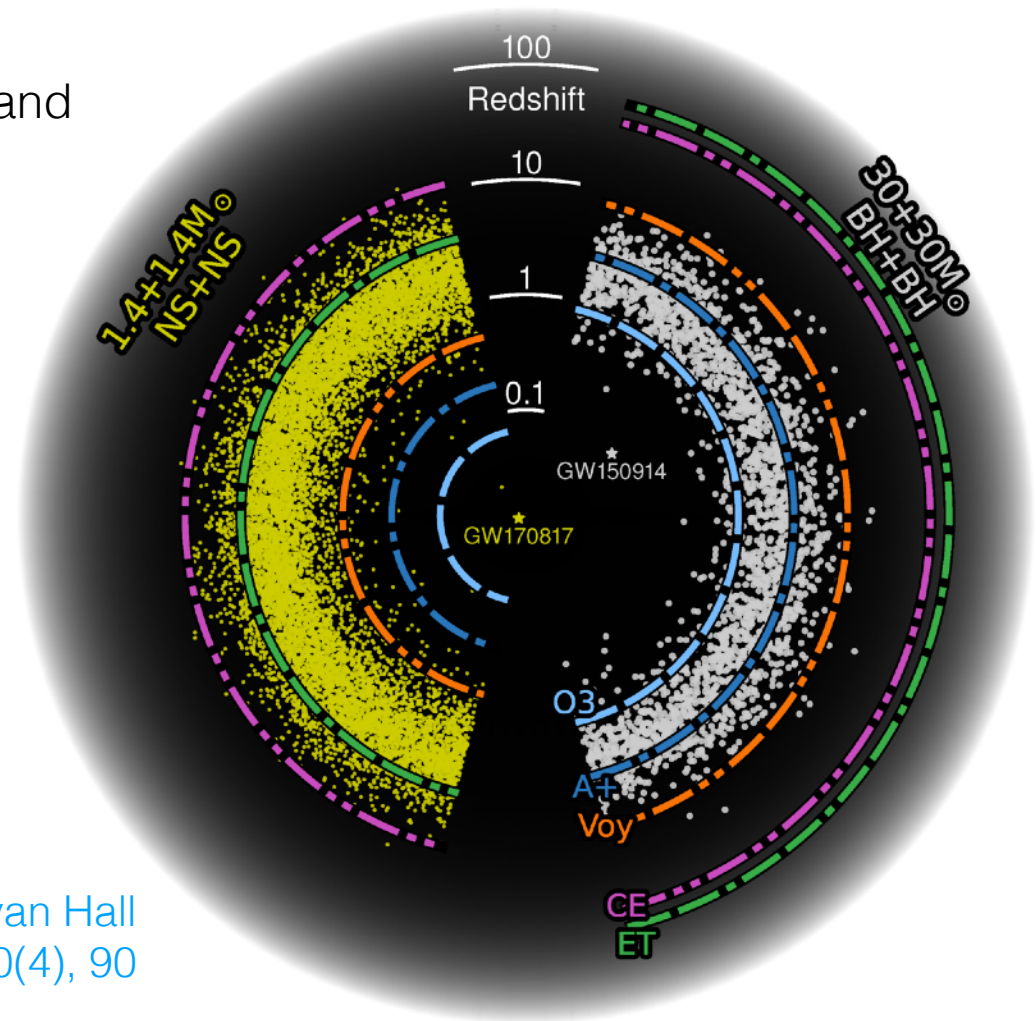
Redshift reach of LIGO Voyager, ET, and CE. Shown are the redshifts for:

- BNS mergers
- BBH mergers

Assumptions:

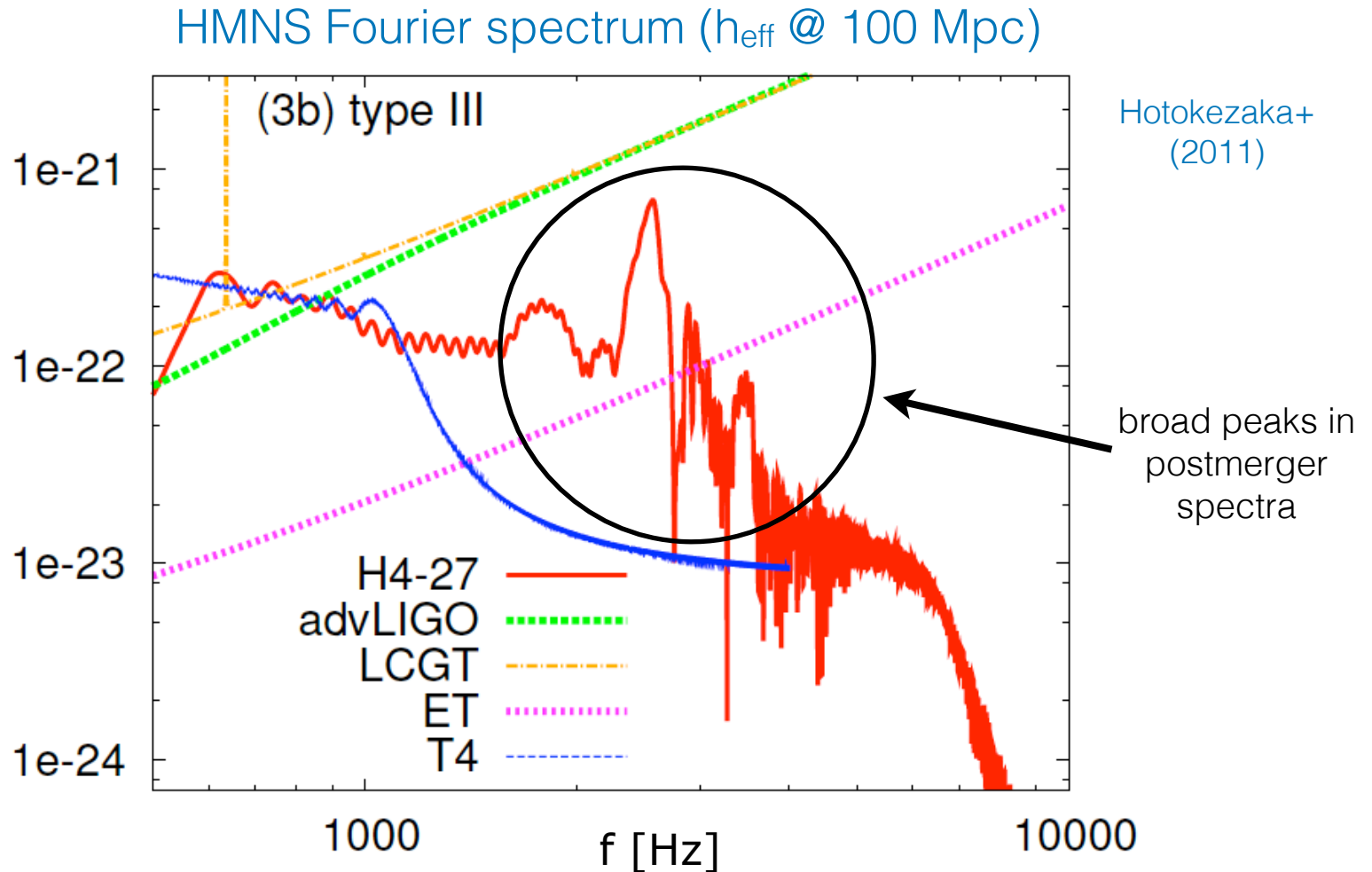
- Madau-Dickinson SFR
- Time from binary formation to merger is 100 Myr

Most binaries merge at  $z \sim 2$



Evan Hall  
Galaxies **2022**, 10(4), 90

# Characterisation of the properties of postmerger signal



2-4 kHz peaks detectable by 3G detectors or 2G detectors if source close enough (20 Mpc).

# Characterisation of the properties of postmerger signal

---

Postmerger phase has rich GW phenomenology; good prospects for constraining EoS.

**HMNS asteroseismology:** interpretation of the characteristics of oscillation modes of the postmerger remnant in terms of the physical properties of the stellar interior.

Goal: use GW spectra to find **quasi-universal relations** between oscillation frequencies (spectra peaks) and NS properties (mass, radius, tidal deformability)

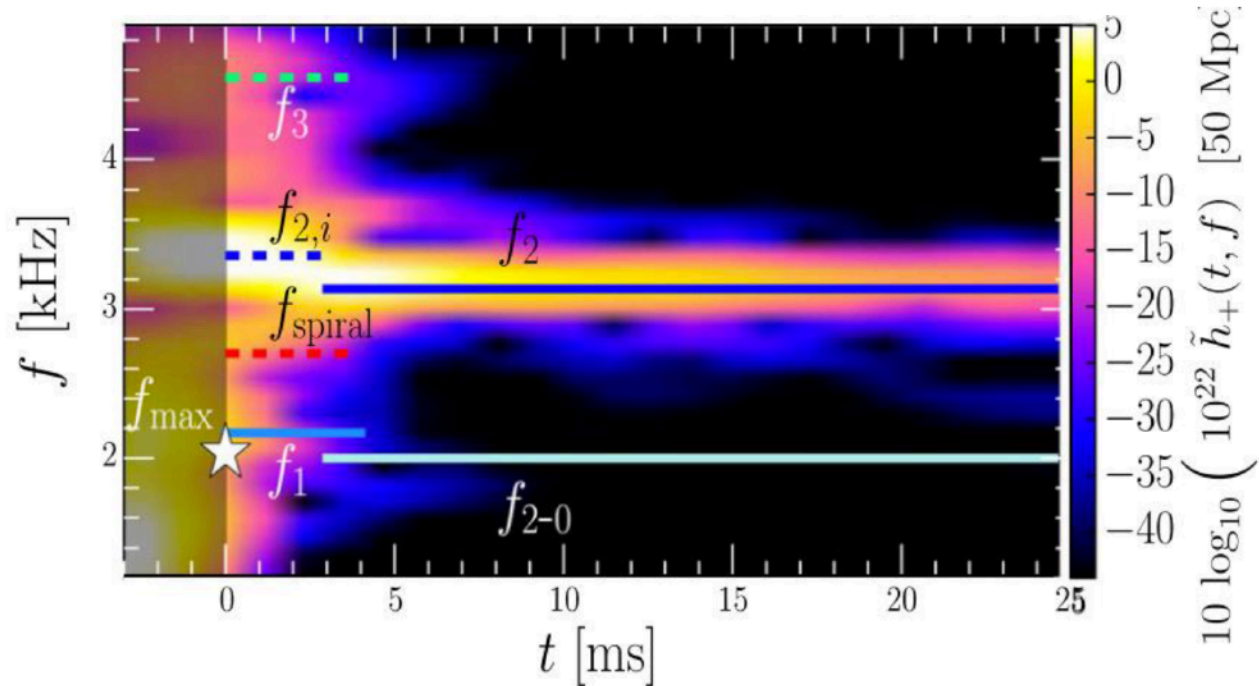
NR simulations have revealed that a number of peculiar frequencies are related with the properties of the binary through quasi-universal relations.

Relations suggested for:

1. **the GW frequency at merger** (e.g. [Read+ 2013](#); [Bernuzzi+ 2014](#); [Takami+ 2015](#); [Rezzolla & Takami 2016](#); [Most+ 2019](#); [Bauswein+ 2019](#); [Weih+ 2020](#); [Gonzalez+ 2022](#); [Topolski+ 2024](#); [Guerra+ 2024](#))
2. **the dominant peak frequency in the postmerger spectrum** (e.g. [Oechslin & Janka 2007](#); [Bauswein & Janka 2012](#); [Read+ 2013](#); [Rezzolla & Takami 2016](#); [Gonzalez+ 2022](#); [Topolski+ 2024](#), [Guerra+ 2024](#))
3. **other frequencies** identifiable in the transient period right after the merger ([Bauswein & Stergioulas 2015](#); [Takami+ 2015](#); [Rezzolla & Takami 2016](#))

# HMNS asteroseismology

Many attempts to empirically fit HMNS mode frequencies vs Love number: [Bauswein & Stergioulas \(2013\)](#), [Read+ \(2013\)](#), [Takami+ \(2015\)](#), [Rezzolla & Takami \(2016\)](#), [Vretinaris+ \(2019\)](#), [Lioutas+ \(2021\)](#), ...



- $$\log_{10} \left( \frac{f_{\max}}{\text{kHz}} \right) \approx a_0 + a_1 (\kappa_2^T)^{1/5} - \log_{10} \left( \frac{2\bar{M}}{M_\odot} \right)$$

$$a_0 = 4.186, \quad a_1 = -0.195.$$

- $$f_{2,i} \approx 6.401 - 1.299 (\kappa_2^T)^{1/5} \text{ kHz},$$

- $$f_2 \approx 5.832 - 1.118 (\kappa_2^T)^{1/5} \text{ kHz}.$$

**Polar fluid modes** most important for GW emission:

- **Fundamental mode (f-mode)**: exists only for non-radial oscillations and describes surface waves. Due to the interface between the star and its surroundings. Eigenfunction has no nodes inside the star.

$$f \propto \sqrt{\bar{\rho}} \propto \sqrt{M/R^3} \quad f \sim 2 \text{ kHz}, \tau < 1 \text{ s}$$

- **Pressure modes (p-modes)**: exist for both radial and non-radial oscillations. Infinitely many of them. Pressure is the restoring force. Oscillations nearly radial. Frequencies depend on the travel time of acoustic waves across the star (standing sound waves).

$$f > 4 \text{ kHz}, \tau > 1 \text{ s}$$

- **Gravity modes (g-modes)**: only exist for finite temperature stars. Infinitely many of them. Buoyancy is the restoring force. Oscillations nearly tangential.

$$f < 500 \text{ Hz}, \tau > 5 \text{ s}$$

**Axial fluid modes:**

- Rotation modes (r-modes).  
Subclass of inertial modes.

**Polar and axial spacetime modes:**

- Wave or curvature modes (w-modes).

$$f > 6 \text{ kHz}, \tau \sim 0.1 \text{ ms}$$



# GW asteroseismology of pulsating NS

Idea already put forward in seminal work by [Andersson & Kokkotas \(1998\)](#).

Computed linear eigenfrequencies of the oscillation modes of NS most important for GW astronomy, the **f-mode**, the first **pressure p-mode** and the first **GW w-mode**.

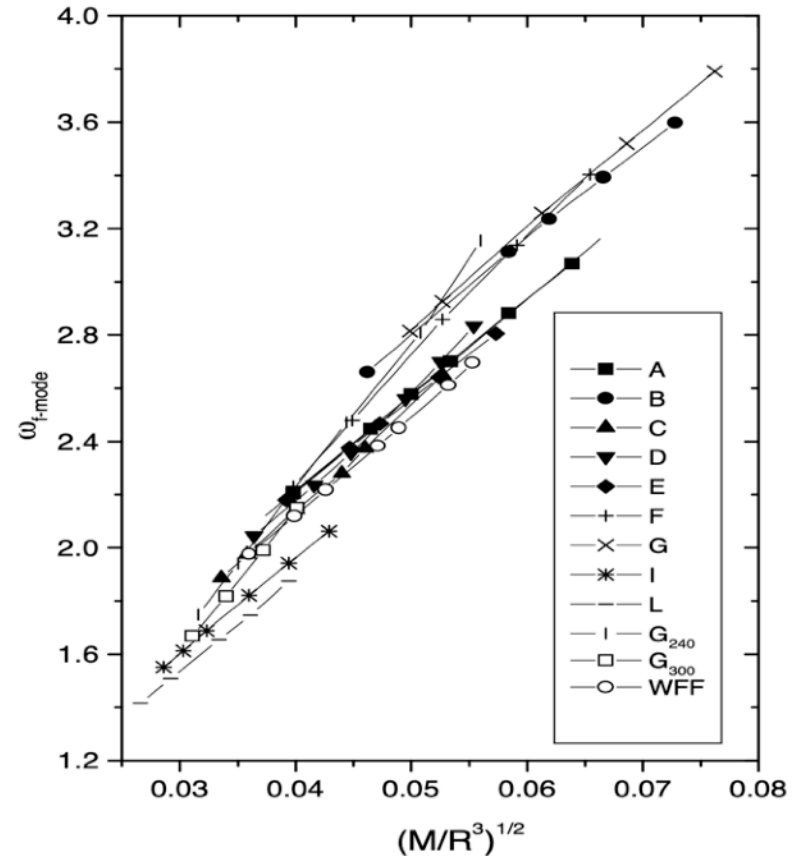
Used NS models with **twelve realistic EoS**.

A set of “**empirical relations**” between the mode-frequencies and the parameters of the star (the radius  $R$  and the mass  $M$ ) was inferred.

For example, for the f-mode:

$$\omega_f \sim 0.78 + 1.635 \left( \frac{M}{1.4M_\odot} \right)^{1/2} \left( \frac{10 \text{ km}}{R} \right)^{3/2}$$

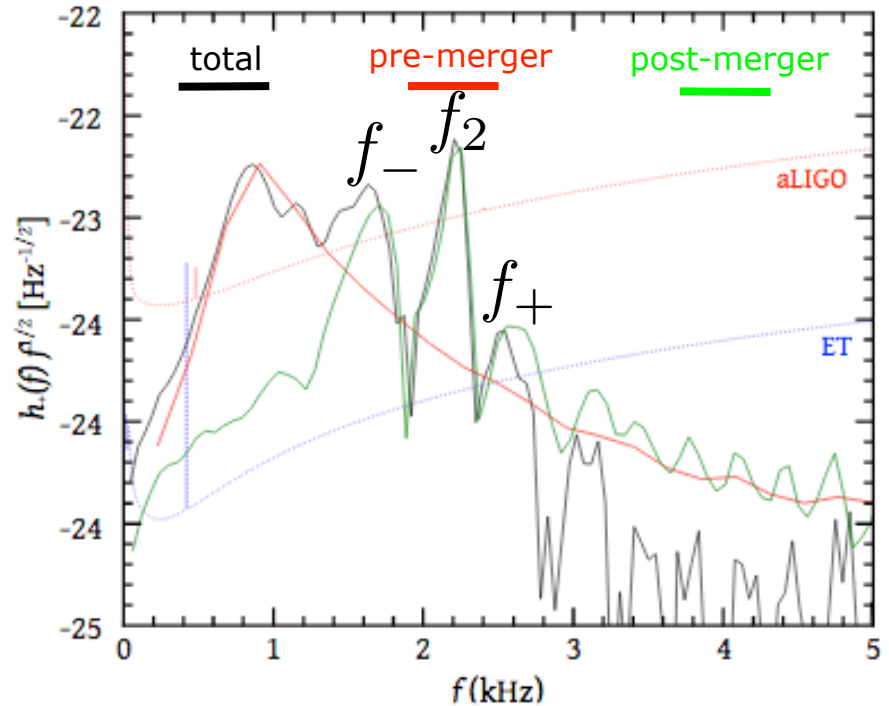
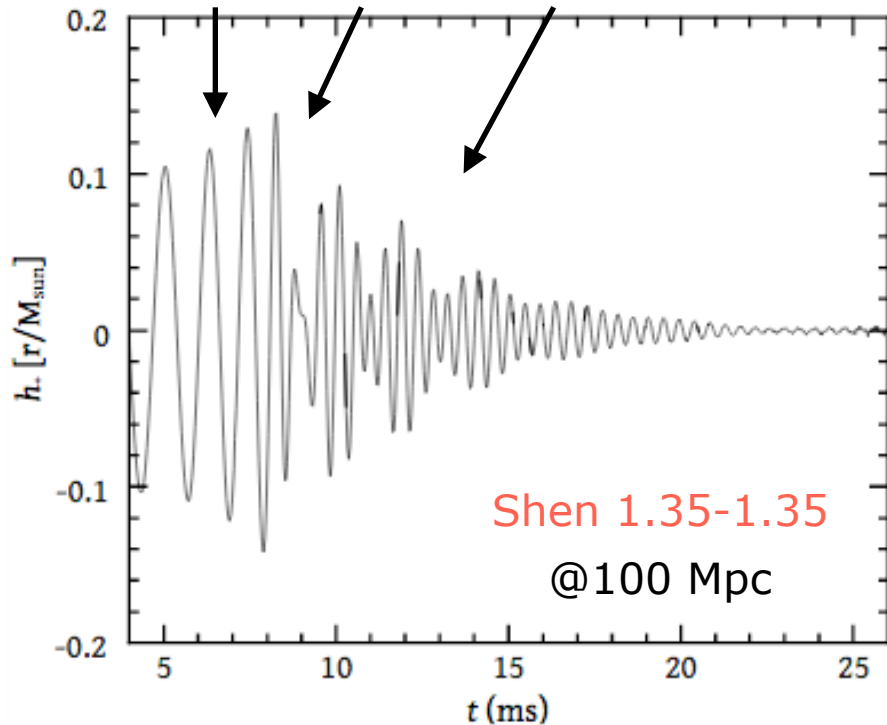
Proof-of-principle that those empirical relations can be used to extract the details of the star from observed oscillation modes.



# Oscillation modes in HMNS (early postmerger)

GW signal divided in three distinct phases:  
 inspiral, merger, postmerger oscillations

Stergioulas+ (2011)



Triplet of frequencies identified in GW spectrum.

Future GW observations may extract  $f_2$  &  $f_0$

GW **asteroseismology** with post-merger remnant to **constrain the EOS**.

$f_{\text{peak}} = f_2$       fundamental  $l=m=2$  f-mode oscillation

$f_- = f_2 - f_0$       quasilinear combination tones

$f_+ = f_2 + f_0$       quasilinear combination tones

# Detectability postmerger signal - **Bayeswave**

---

**Bayeswave** (Cornish & Littenberg 2015) provides a robust method to characterize the GW emission from the remnant of a BNS merger.

Bayesian GW data analysis pipeline developed within the LVK Collaboration.

`docs.ligo.org/lscsoft/bayeswave`

**Bayeswave** employs a morphology-independent approach to **reconstruct** the postmerger GW signal through a sum of appropriate basis functions (sine-Gaussians wavelets) and provide the full **posterior probability distribution** of the underlying waveform.

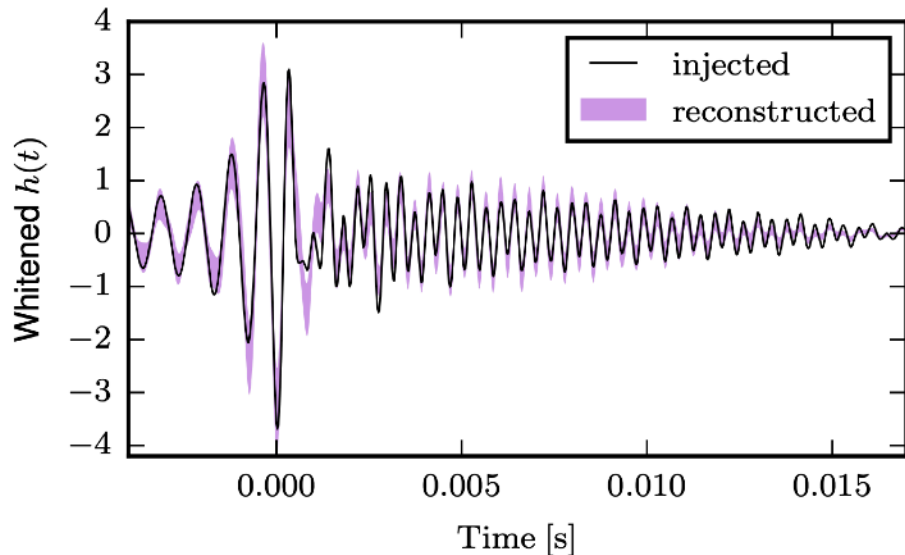
**Bayeswave** has been applied on **simulated data** from NR simulations of BNS mergers injected into a network of advanced ground-based detectors

Clark+ 2014, 2016; Chatziioannou+ 2017; Bose+ 2018; Miravet-Tenés+ 2023, 2024

Results demonstrate the GW signal reconstruction capabilities of the pipeline. Posterior distributions for the **GW peak frequency** and for the **NS radius**.

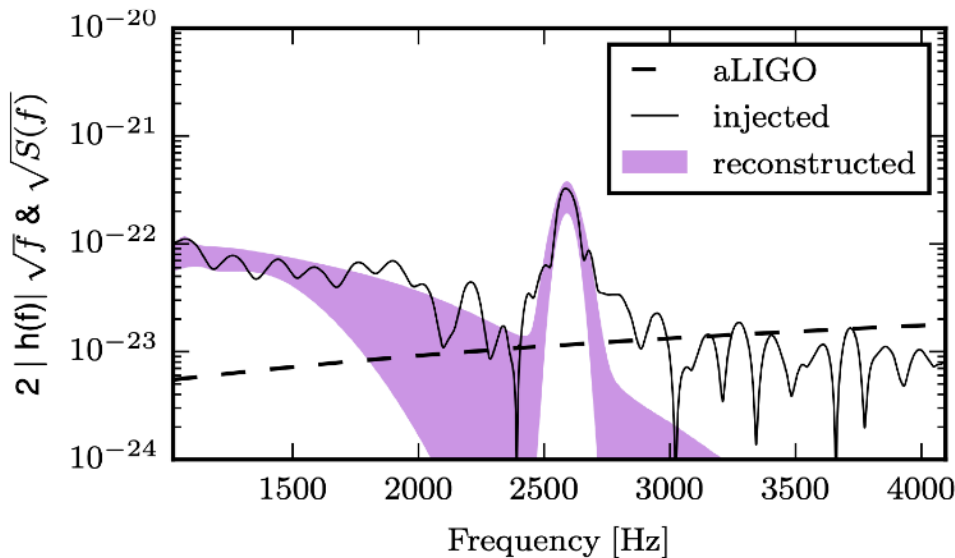
# Detectability postmerger signal - Bayeswave

Reconstruction of waveform and spectrum



Waveform from a NR simulation of two nonspinning,  $(1.35, 1.35)M_{\odot}$  NS with DD2 EoS.

Injected at a post-merger SNR of 5 (roughly corresponds to a distance of 20Mpc).



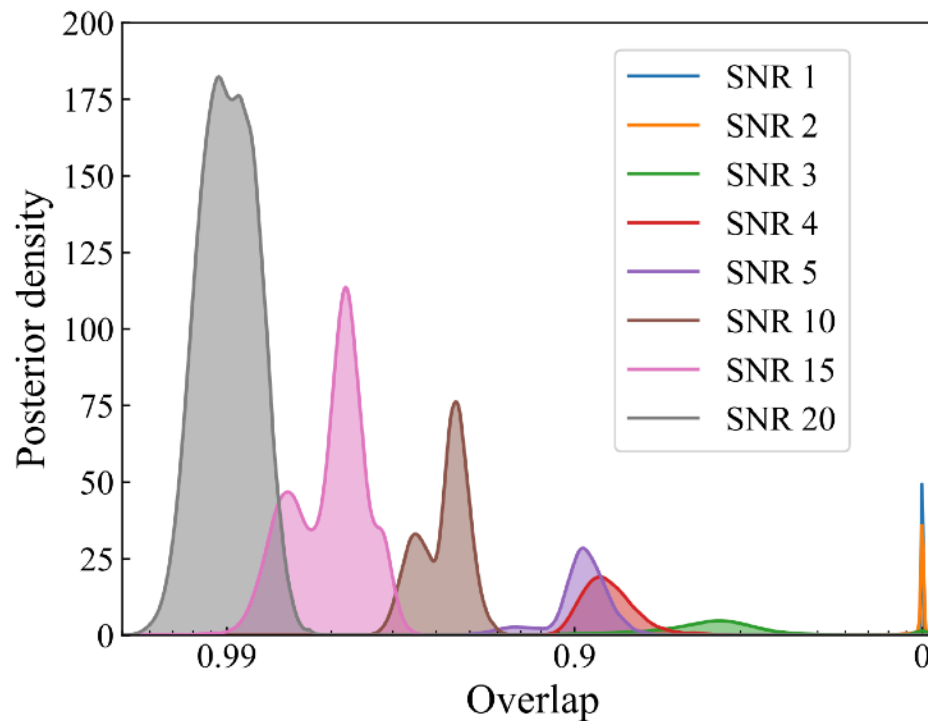
Shaded region denote 90% CI of the reconstruction.

[Chatziioannou+ 2017](#)

# Detectability postmerger signal - Bayeswave

Quality of reconstruction measured by the **overlap** between signal  $s$  and model  $h$

$$\mathcal{O} \equiv \frac{\langle s, h \rangle}{\sqrt{\langle s, s \rangle} \sqrt{\langle h, h \rangle}} \quad \langle a, b \rangle \equiv 2 \int_0^\infty \frac{a(f)b^*(f) + a^*(f)b(f)}{S_h(f)} df$$



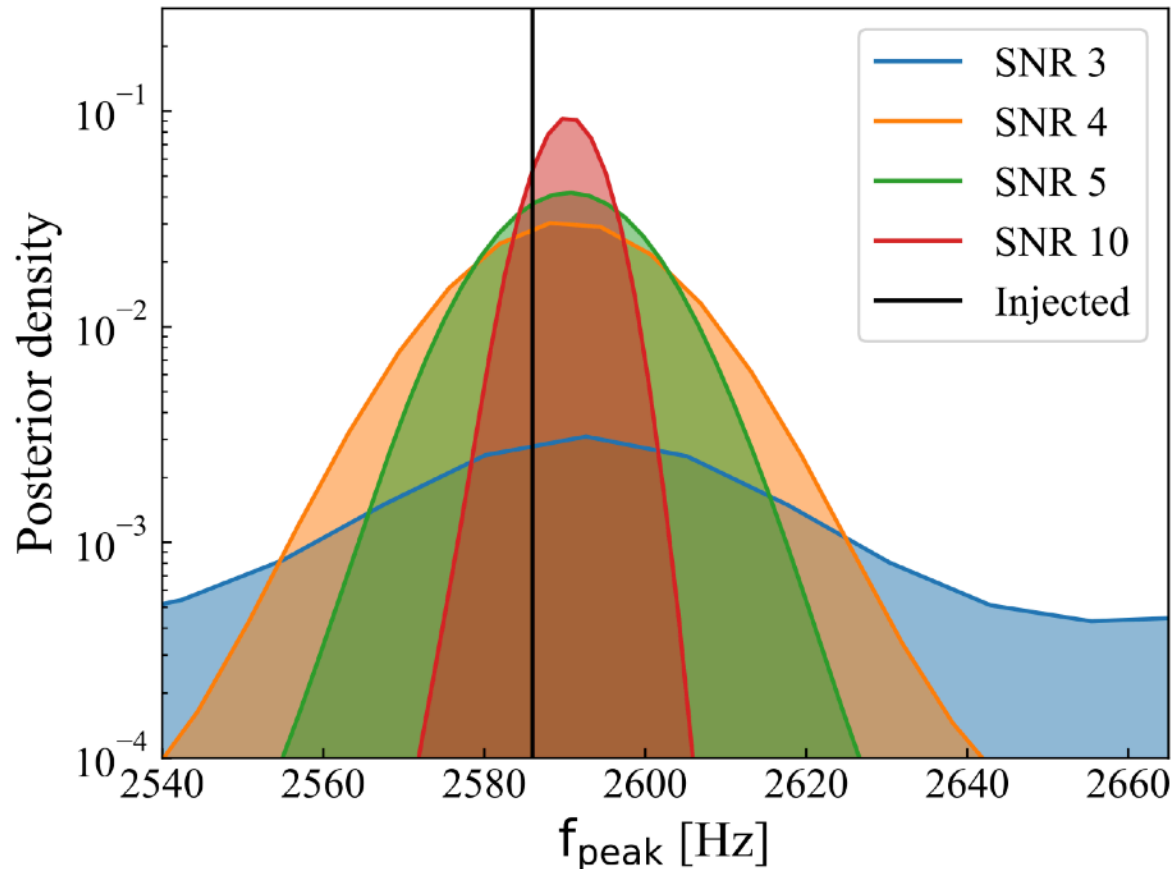
Chatziioannou+ 2017

$\mathcal{O} = 1$  perfect match between injected and reconstructed signals

$\mathcal{O} = 0$  no match

# Detectability postmerger signal - **Bayeswave**

Posterior density function of the GW peak frequency

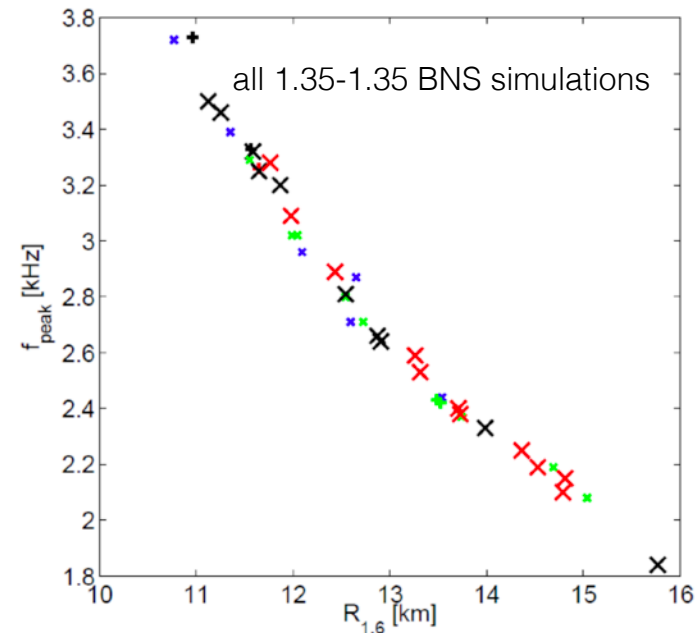
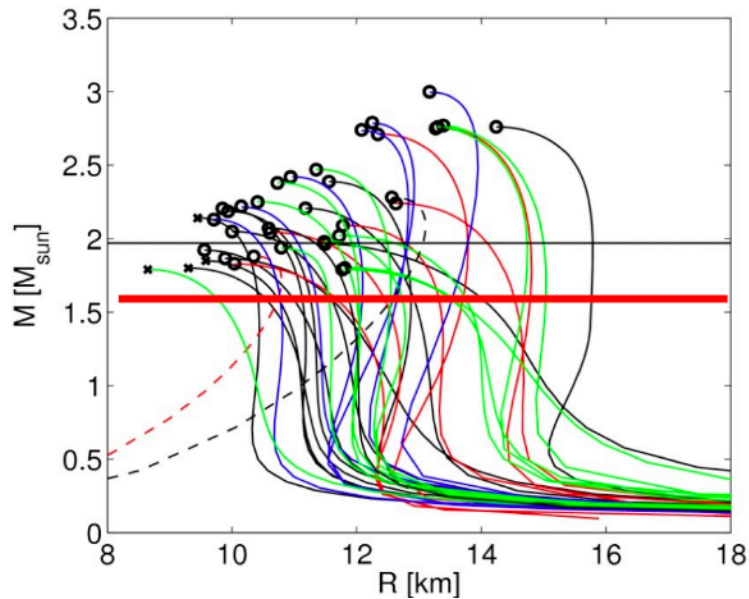


Chatziioannou+ 2017

As SNR increases, **Bayeswave** achieves a more accurate reconstruction of the signal and the posterior peaks at the correct value for  $f_{\text{peak}}$ .

# Detectability postmerger signal - Bayeswave

Bauswein+ (2012): peak frequency of  $(1.35-1.35)M_{\odot}$  BNS merger correlates with the radius of a  $1.6M_{\odot}$  nonrotating NS in an EoS-independent manner (density regimes are comparable). Similar relations found for other binary masses and other radii ( $R_{1.35}$  or  $R_{1.8}$ ) (Bauswein+ 2012, 2016)



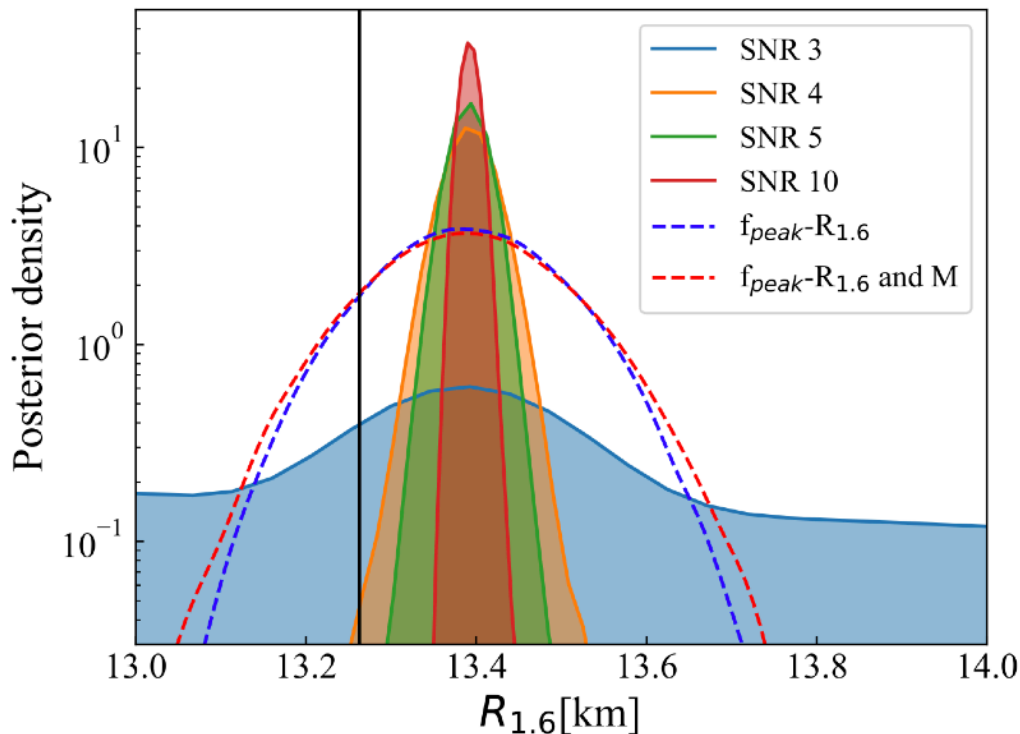
Therefore, a potential measurement of  $f_{\text{peak}}$  from postmerger signal can be used to obtain an estimate on  $R_{1.6}$ , a quantity that can be used to directly constrain the EoS.

The smaller the scatter ( $<200$  m) the smaller the error in radius measurement.

# Detectability postmerger signal - Bayeswave

Inference on the radius: posteriors for the  $f_{\text{peak}}$  converted to posteriors for  $R_{1.6}$ .

For different binary total mass  $M$ , the deviation from perfect universality in the  $f_{\text{peak}}/M - R_{1.6}$  relation is taken into account. Systematic uncertainties dealt with through marginalisation.



**Shaded regions:** posteriors for best-fit model (systematic uncertainties ignored)

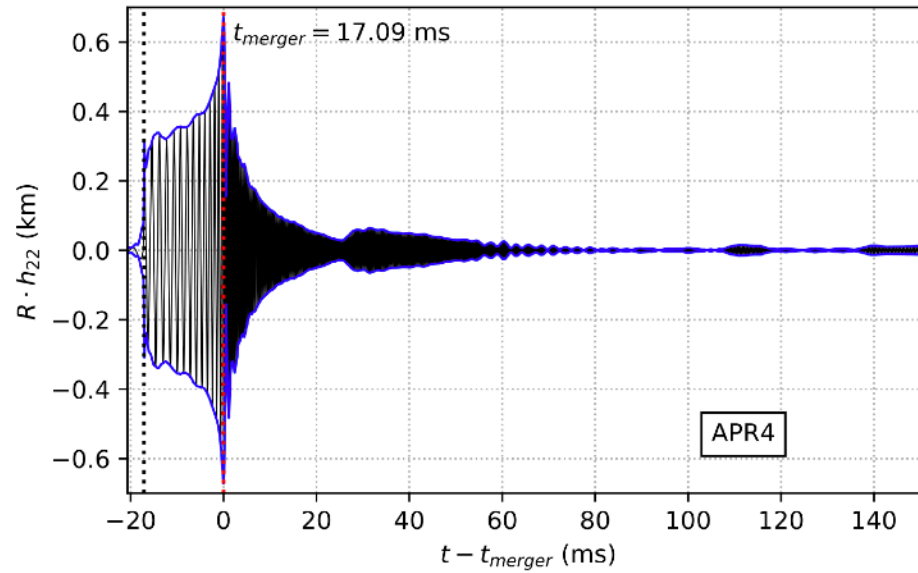
**Dashed lines:** marginalised posteriors including systematic uncertainties in measurements in  $f_{\text{peak}}/M - R_{1.6}$  and total mass  $M$ .

Chatziioannou+ 2017

Measurement of  $R_{1.6}$  to within (300–700) m at the 90% CL.

The statistical error in the NS radius measurement from the postmerger signal comparable to the corresponding error from the inspiral signal.



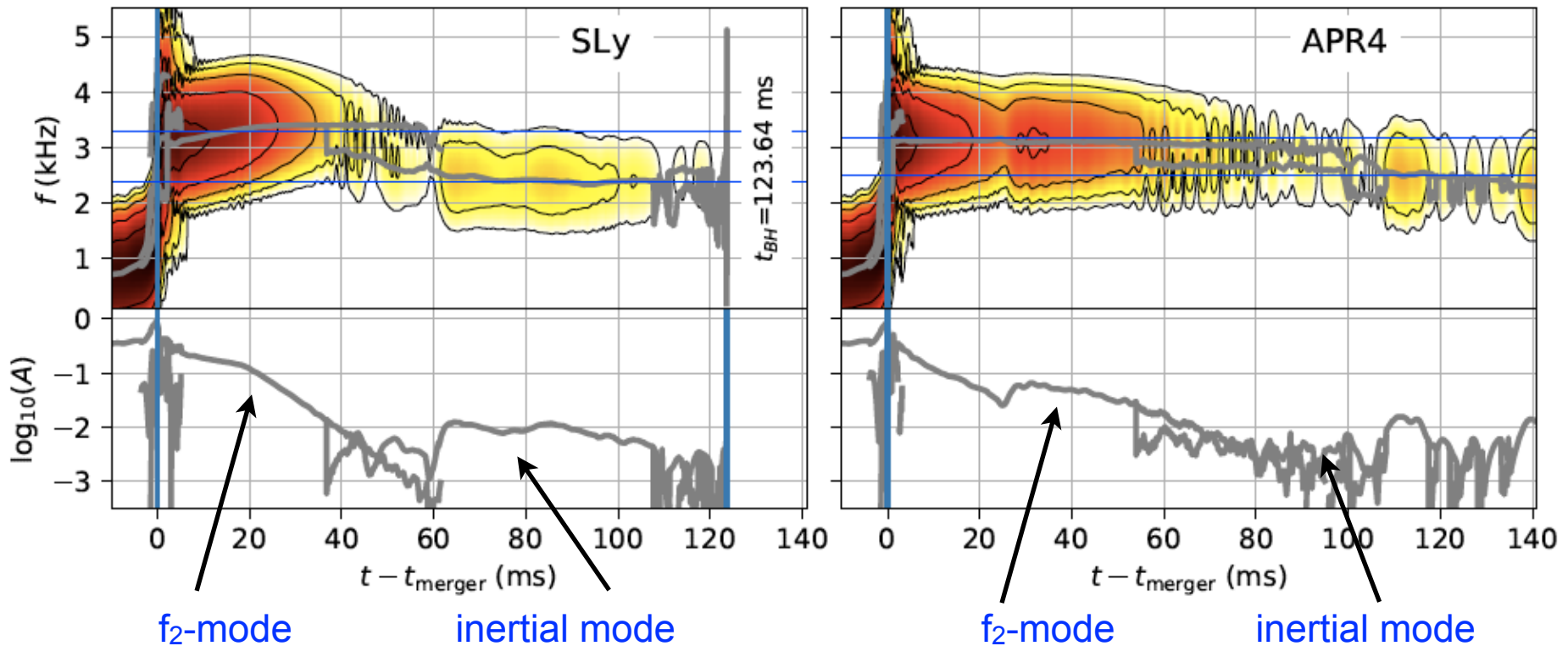


Late time postmerger

# Late postmerger: convective excitation of inertial modes

De Pietri+ (2018, 2022) reported results on the **longest term simulations** of BNS mergers thus far performed (up to 140 ms post-merger).

De Pietri+ (2018)

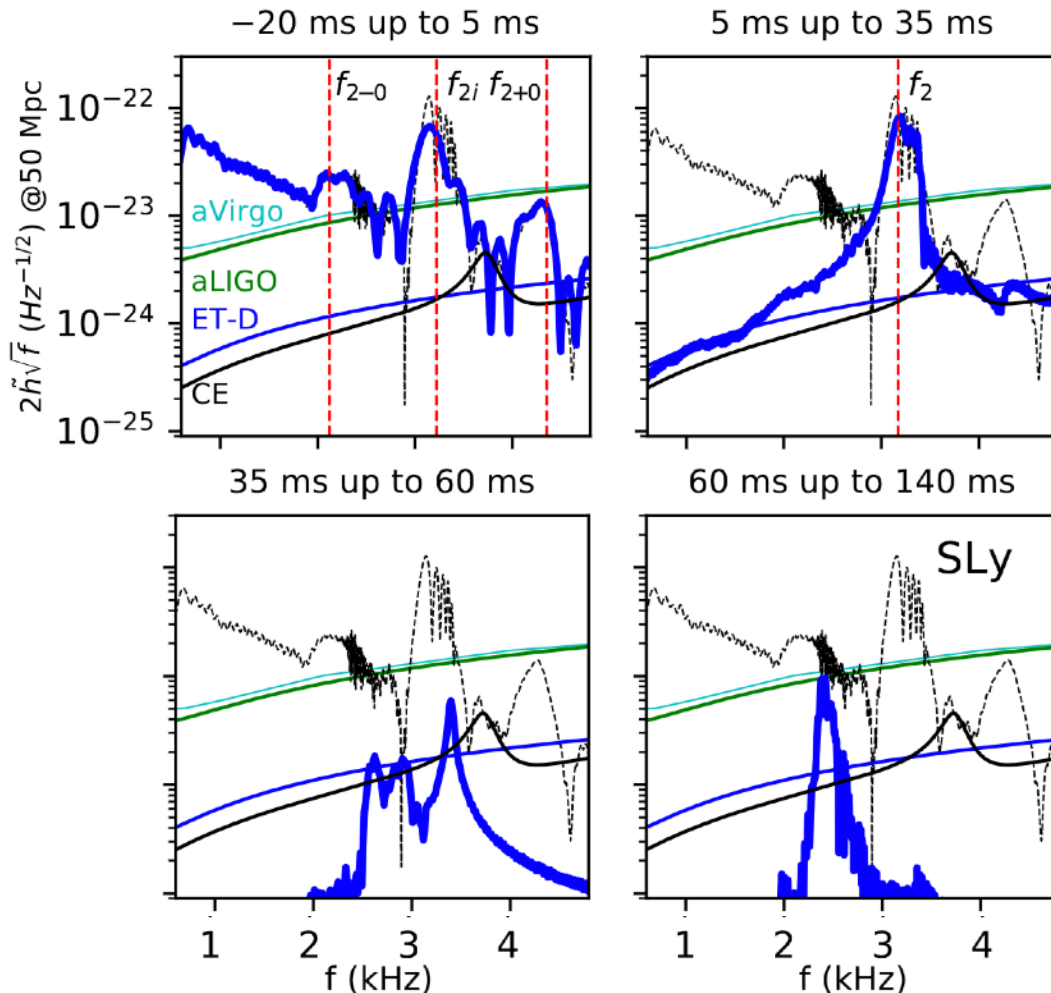


Early post-merger GW emission dominated by the f-mode.

Late post-merger GW emission dominated by a low-frequency (gravito-)inertial mode (due to convection).

# Convective excitation of inertial modes

De Pietri+ (2018)

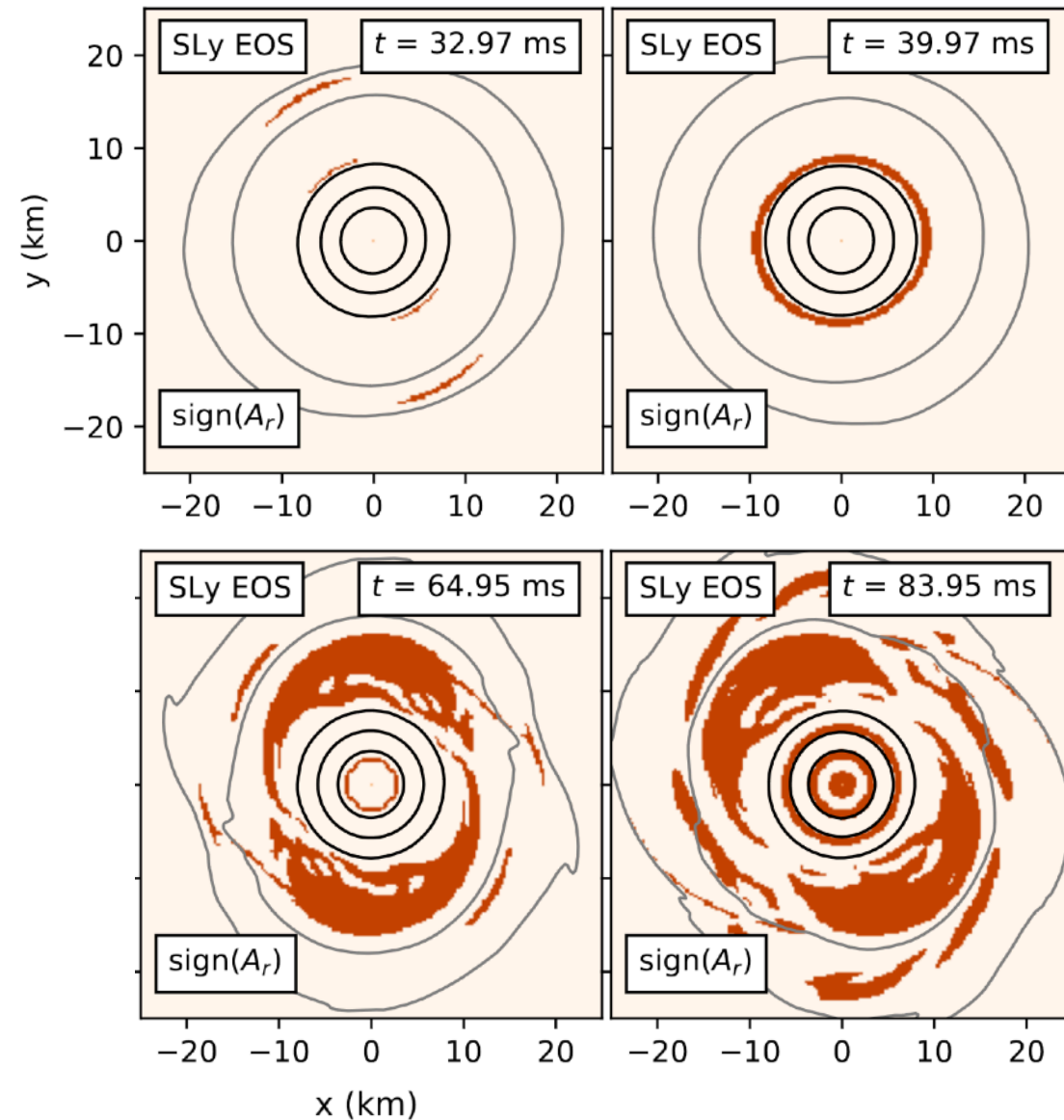


Early postmerger GW emission dominated by the  $f_2$ -mode.

Late postmerger emission dominated by a low-frequency (gravito-)inertial mode (due to convection).

Results shown for SLy EoS only. Holds for all EoS models.

# Sign of Schwarzschild discriminant (>0 unstable=dark)



$$A_\alpha = \frac{1}{\varepsilon + p} \nabla_\alpha \varepsilon - \frac{1}{\Gamma_1 p} \nabla_\alpha p$$

$$\Gamma_1 = \Gamma_{\text{th}} + (\Gamma_i - \Gamma_{\text{th}}) \frac{K_i \rho^{\Gamma_i}}{p}$$

About 30-50 ms after merger, parts of the HMNS become **convectively unstable**, exciting **inertial modes**.

Might be **potentially detected** by 3G detectors.

Opportunity to infer rotational and thermal properties of BNS remnants.

Treatment of thermal effects in postmerger

hybrid EoS vs tabulated EoS

# Thermal conditions of BNS mergers

---

**Late inspiral phase:** neutron stars are **cold**.

Neutron stars with age  $> 10^7$  years have undergone long-term cooling by neutrinos and photons.

$$T < 10^5 \text{ K} \sim 10 \text{ eV} \ll E_F \sim 100 \text{ MeV}$$

Hence, neutron stars can be modelled by cold EOS.

**Problem:** such EOS is still unknown; **need for a systematic survey.**

**Merger phase:** neutron stars are **hot**. Shock heating increases temperature to about

$$kT \sim 0.1 - 0.2 E_F \sim 10 \text{ MeV}$$

New effects likely to play important dynamical role: finite temperature effects, lepton fraction, neutrino thermal pressure, neutrino cooling.

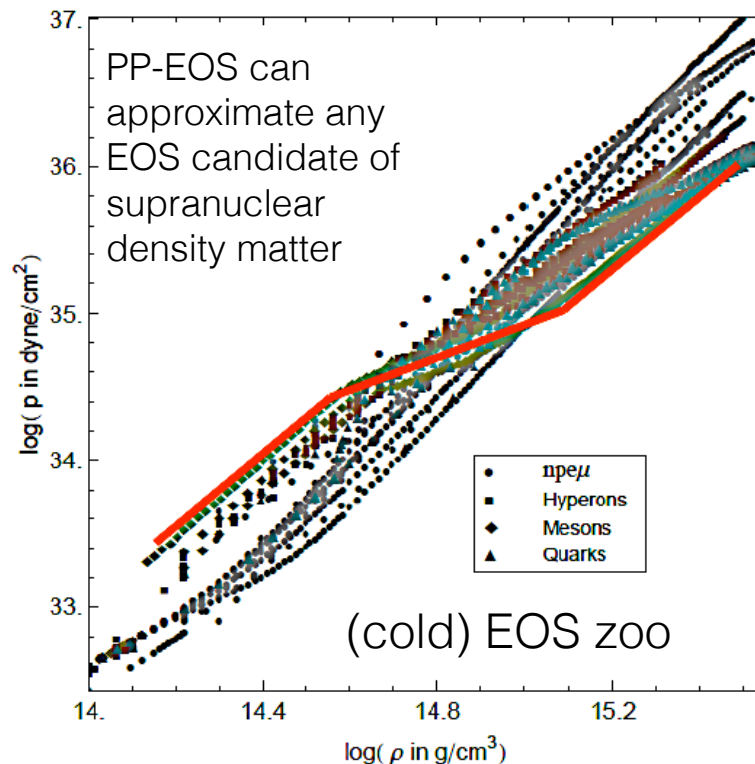
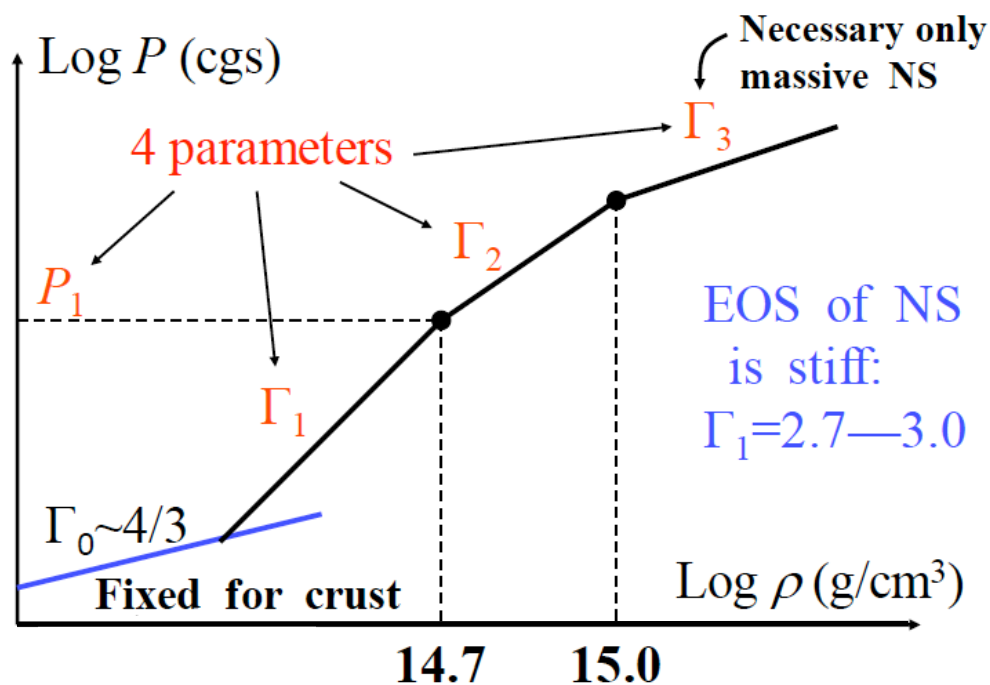
Neutron stars modeled by thermal EOS.

**Problem:** **systematic survey as well; few EOS available, more needed.**

# Dependence on the nuclear EOS

$$P(\rho, \varepsilon) = P_{\text{cold}}(\rho) + P_{\text{th}}(\rho, \varepsilon)$$

Piecewise-polytropic EOS for the cold part (Read et al 2009)



Thermal part of the pressure (shock heating) for hot, merged NS ( $T \sim 10\text{MeV}$ ) given by

$$P_{\text{th}} = (\Gamma_{\text{th}} - 1)(\varepsilon - \varepsilon_{\text{cold}})\rho, \quad \Gamma_{\text{th}} = 1.357 - 1.8$$

# Thermal conditions of BNS mergers

---

Finite temperature adds **further pressure support** that may change the internal structure of the postmerger remnant and its subsequent evolution.

[Lim & Holt \(2019\)](#) showed that above  $0.5n_0$  ( $n_0 \approx 0.16 \text{ fm}^{-3}$ ),  $\Gamma_{\text{th}}$  strongly depends on the nucleon effective mass.

Hence, hybrid approach may overestimate the thermal pressure by a few orders of magnitude ([Raithel+ 2021](#)) and may induce significant changes in the GW frequencies ([Bauswein+ 2010](#), [Figura+ 2021](#))

To overcome this limitation, some BNS simulations incorporate thermal effects through full finite-temperature EOS:

- CFC gravity: [Oechslin+ 2007](#), [Bauswein+ 2010](#)
- full GR: [Sekiguchi+ 2011](#), [Espino+ 2022](#), [Fields+ 2023](#), [Werneck+ 2023](#), [Guerra+ 2023](#)

**Main questions** we are interested in addressing:

**Q1:** is treatment of thermal effects imprinted in the GW spectra of the HMNS signal?

**Q2:** what are the differences in the spectra of finite-temperature models compared to piecewise-polytropic (hybrid) models?



# Comparison between hybrid and tabulated EoS treatment

- equal-mass BNSs in a quasiequilibrium circular orbit generated using **LORENE**
- binaries consist of two identical irrotational NSs modeled by
  1. **hybrid** EOS (**SLy**, **APR4**, **H4** & **MS1**): 7-piece PWP + thermal part  $\Gamma_{\text{th}} = 1.8$ .
  2. corresponding **tabulated** models. Tables by **Schneider–Roberts–Ott (2017)** freely available at [stellarcollapse.org](http://stellarcollapse.org)

Case		$T$	$M$	$C$	$\Lambda$	$M_{\text{ADM}}$	$J_{\text{ADM}}$	$\Omega$
SLy4 <sub>TAB</sub>	SLy4	0.01	1.28	0.13	304.75	2.54	6.63	1.77
DD2 <sub>TAB</sub>	DD2	0.01	1.29	0.11	700.24	2.56	6.73	1.77
HShen <sub>TAB</sub>	HShen	0.01	1.30	0.10	1170.90	2.58	6.82	1.78
LS220 <sub>TAB</sub>	Ls220	0.01	1.29	0.12	511.89	2.55	6.68	1.77
SLy4 <sub>PWP</sub>	SLy4	-	1.23	0.13	304.75	2.54	6.62	1.77
DD2 <sub>PWP</sub>	DD2	-	1.25	0.11	700.24	2.56	6.73	1.77
HShen <sub>PWP</sub>	HShen	-	1.26	0.10	1170.90	2.58	6.82	1.78
LS220 <sub>PWP</sub>	LS220	-	1.24	0.12	511.89	2.55	6.69	1.77

Simulations performed using the NR open-source **EinsteinToolkit** (Löffler+ 2012).

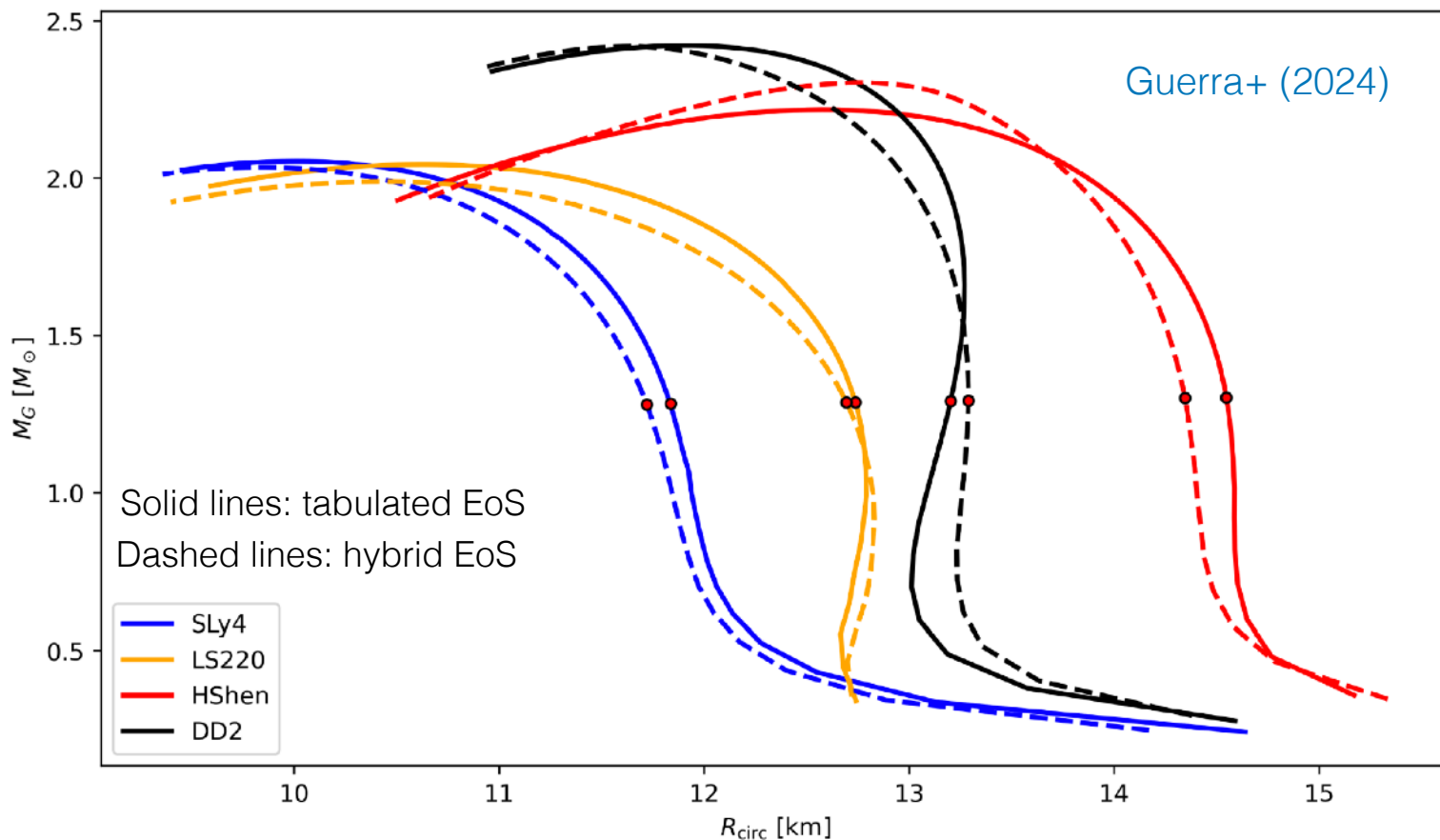


einstein  
toolkit

[einsteintoolkit.org](http://einsteintoolkit.org)

[illinoisgrmhd.net](http://illinoisgrmhd.net)

# Initial Data



[Soft EoS]

Sly4: Chabanat+ (1998)

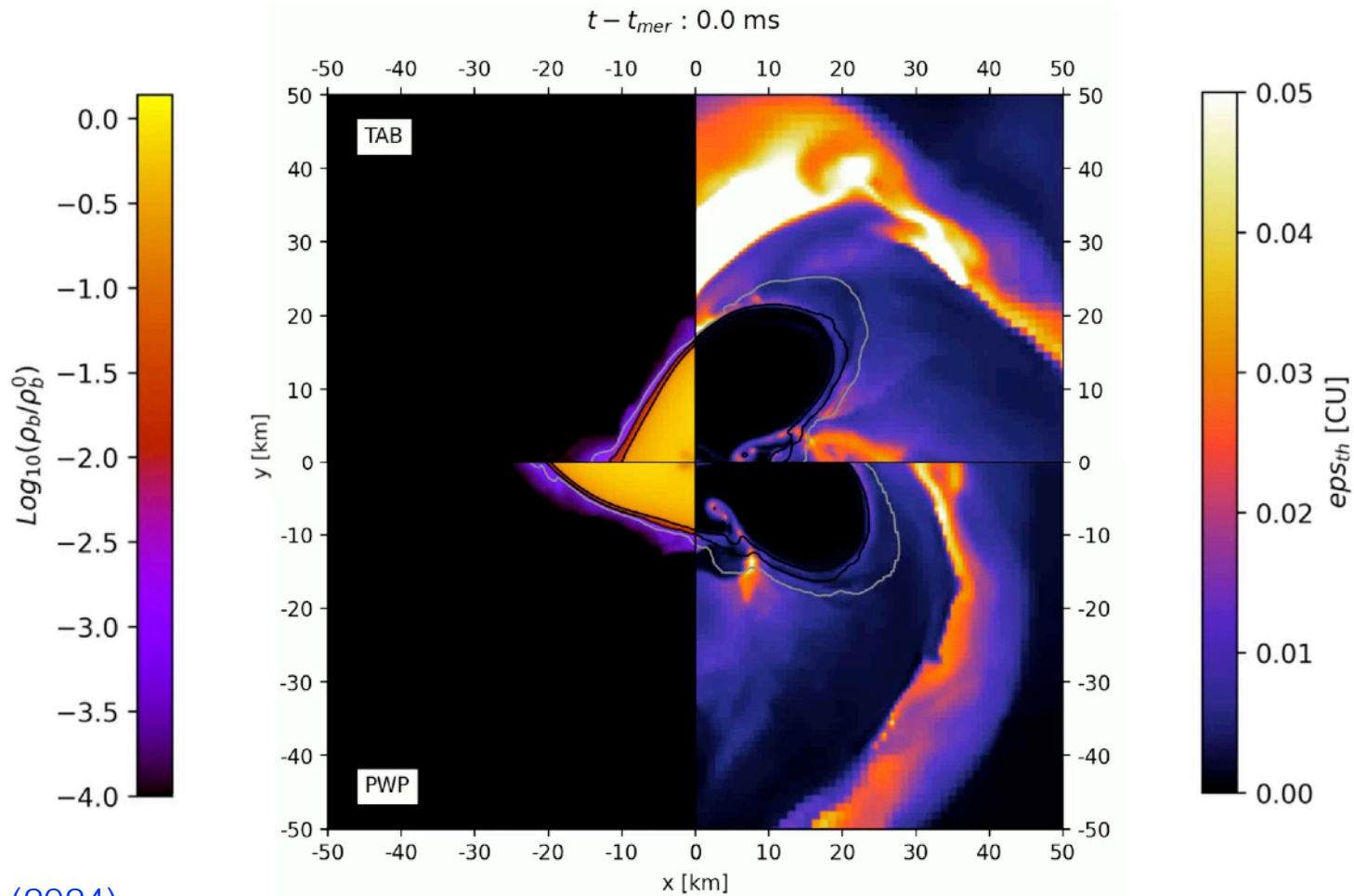
LS220: Lattimer & Swesty (1991)

[Stiff EoS]

DD2: Typel+ (2010)

HShen: Shen+ (2011)

Postmerger evolution of rest-mass density and internal energy for HShen EoS



Guerra+ (2024)

Top half: tabulated model  
Bottom half: hybrid model

Left half: rest-mass density  
Right half: internal energy

$t - t_{\text{mer}} = 5 \text{ ms}$

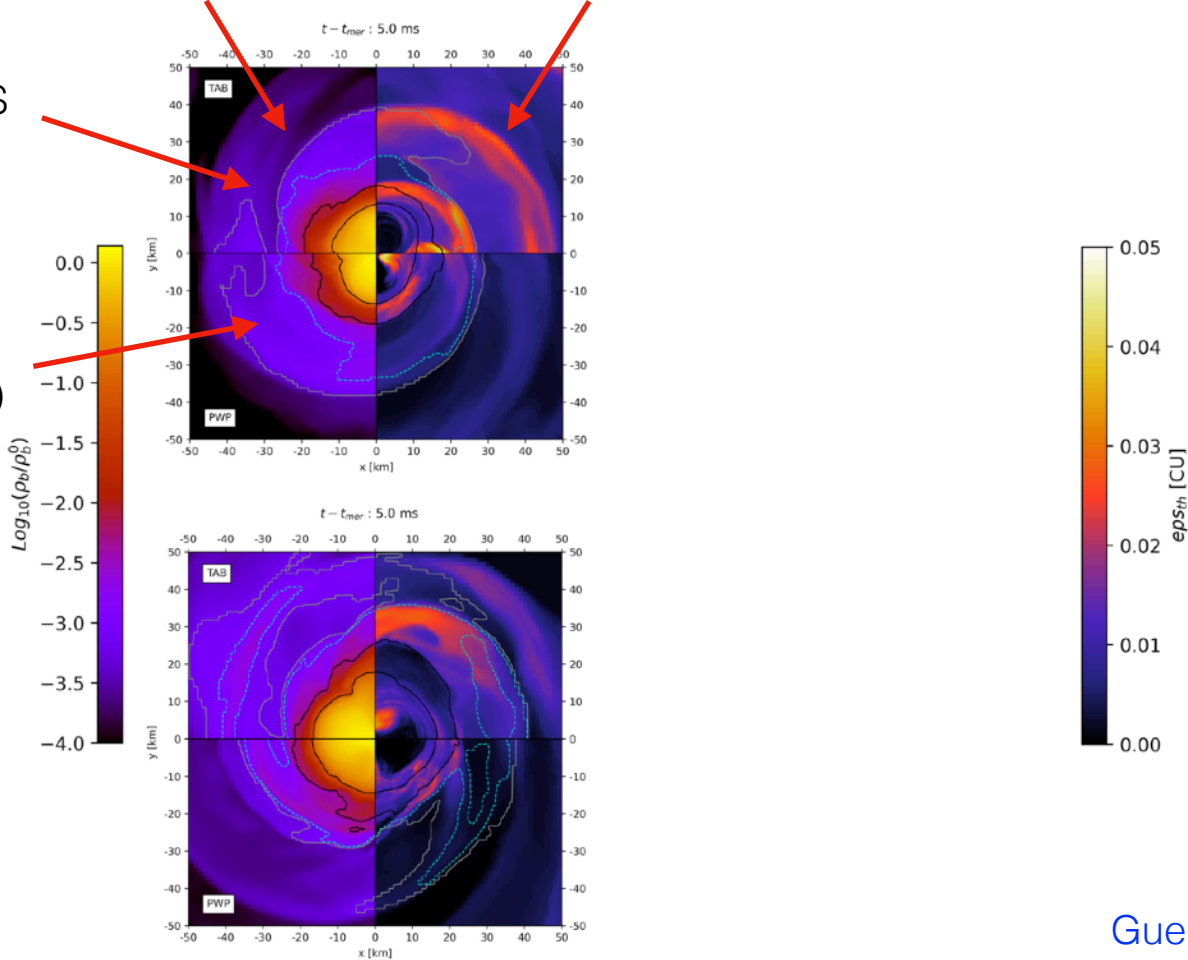
rest-mass density (left half)      internal energy (right half)

Tabulated EoS (top half)

DD2 EoS

Hybrid EoS (bottom half)

HShen EoS



Guerra+ (2024)

A few ms after merger, the production of heat (increase in T) much more significant in tabulated EoS models. Similar density distribution in both (hybrid and tabulated) models.

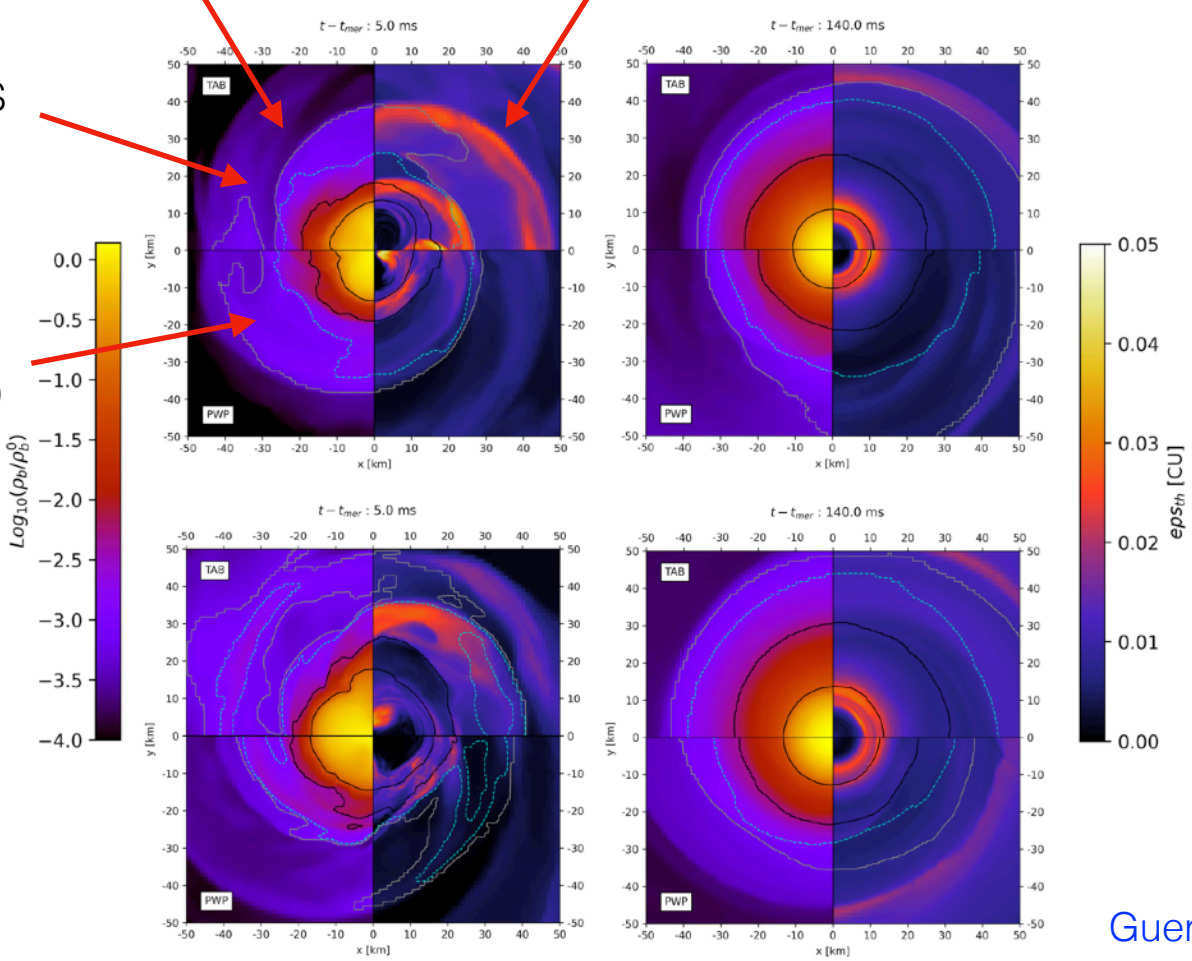
$t - t_{\text{mer}} = 140 \text{ ms}$

rest-mass density (left half)      internal energy (right half)

Tabulated EoS (top half)

DD2 EoS

Hybrid EoS (bottom half)

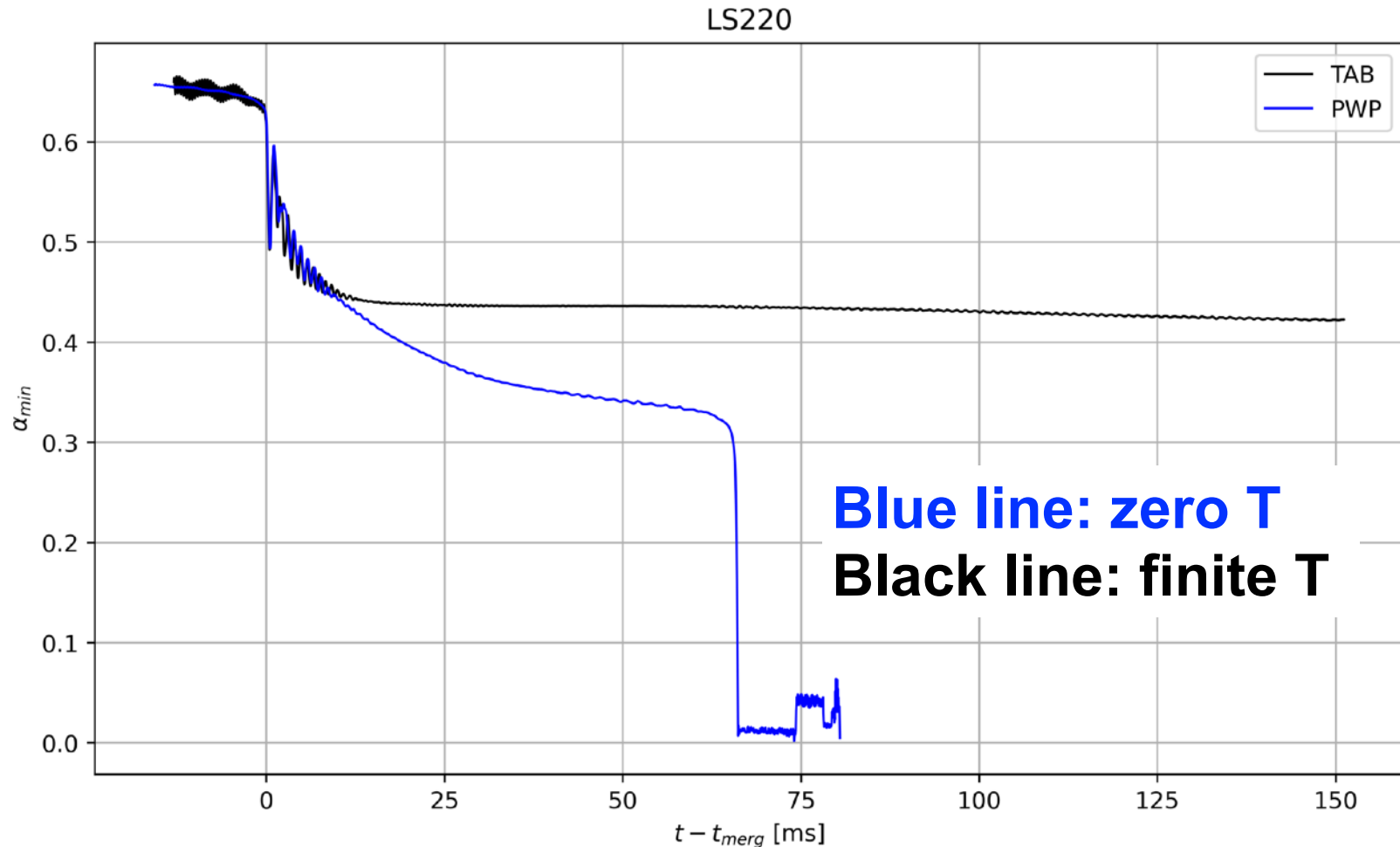


HShen EoS

Guerra+ (2024)

At late time, bulk radius of HMNS smaller for PWP models (higher compactness) and for both EoS (~10 km decrease for HShen). Ring of higher T surrounding HMNS for thermal models.

# Evolution of the lapse function: **LS220**

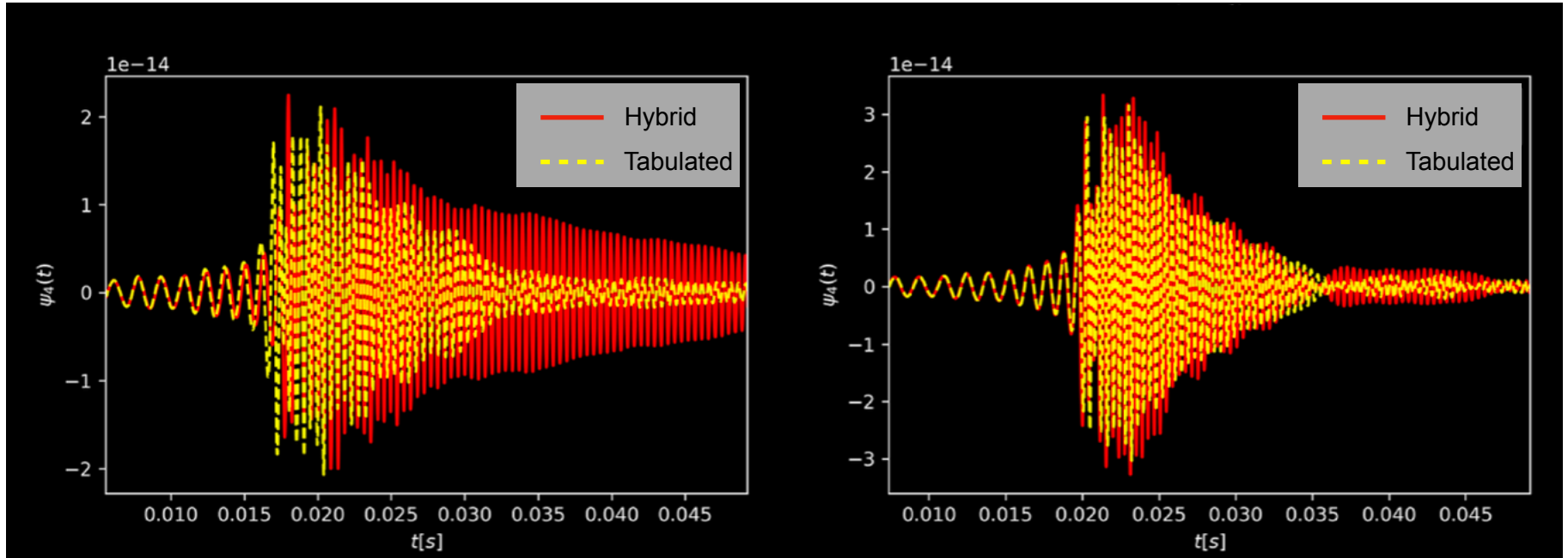


Most important dynamical difference found for **LS220** EoS:

- Finite T model produces a stable HMNS
- Zero T model collapses to a BH

# Differences on waveforms

Guerra+ (2024)

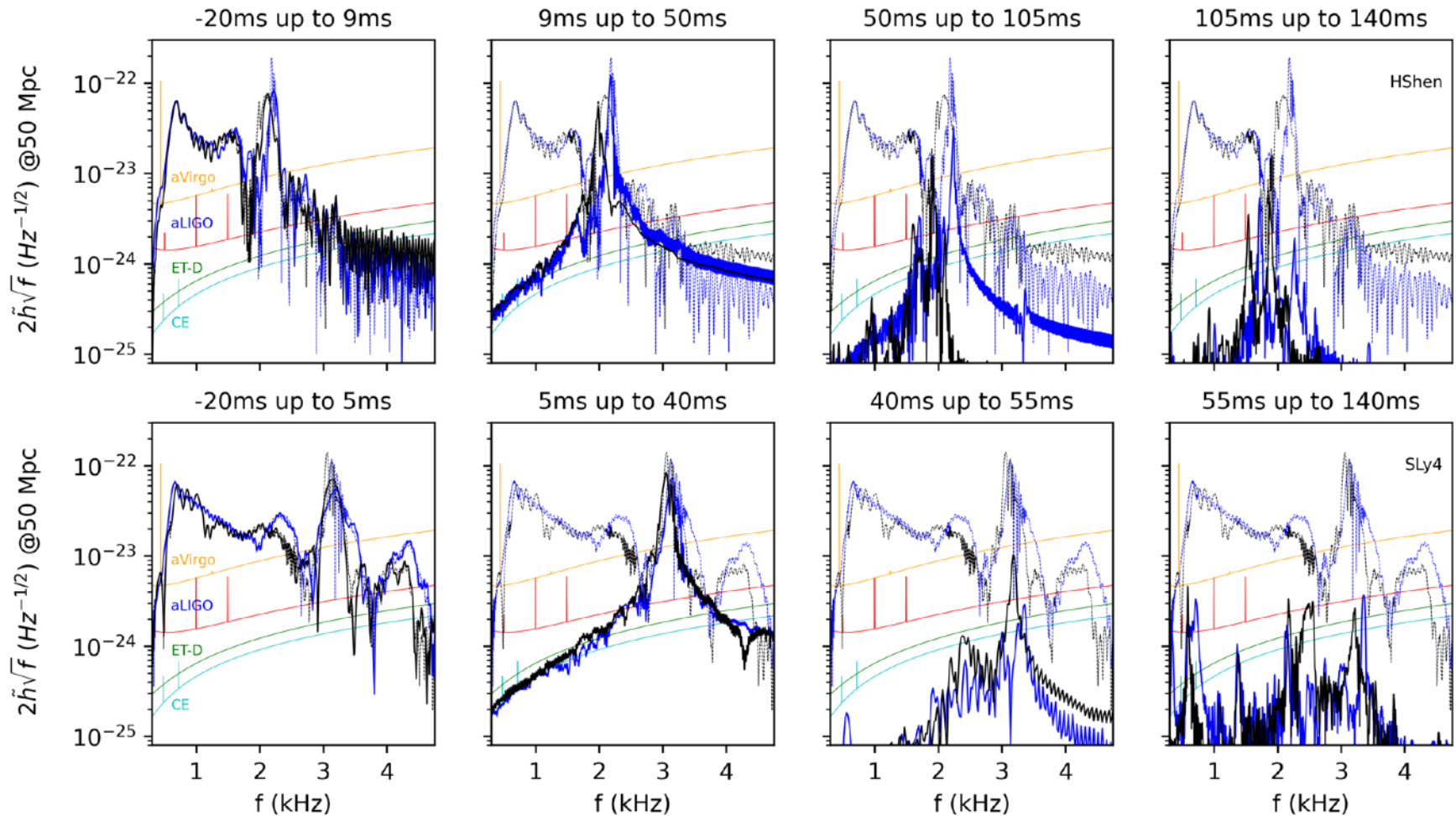


HShen

DD2

Most significant differences in postmerger waveform found for HShen EoS.  
HMNS oscillations damped more rapidly for tabulated EoS model.

# GW spectra: **HShen** (top) & **SLy4** (bottom)



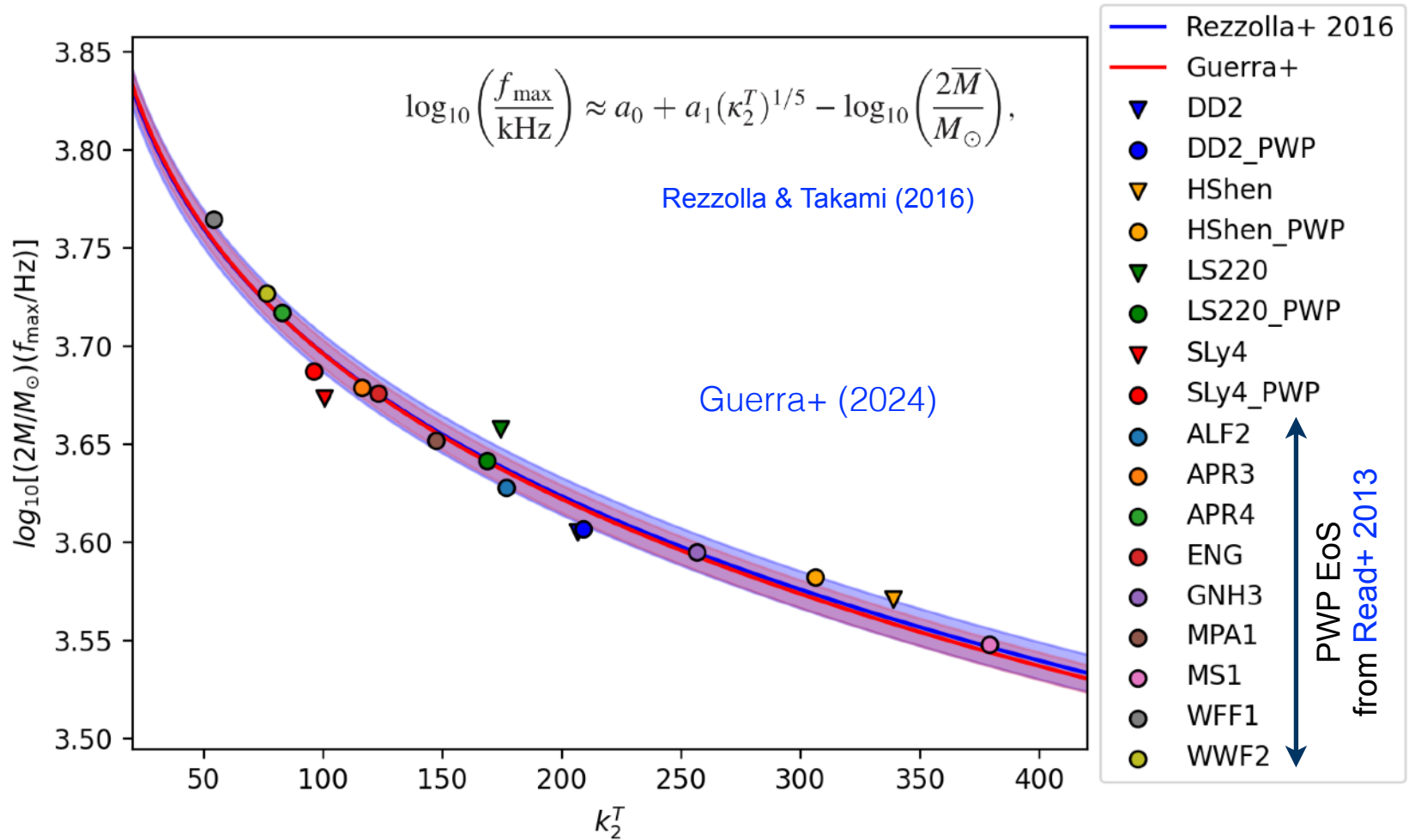
**Blue curves: PWP. Black curves: TAB**

Dashed lines: full PSD. Solid lines: windowed PSD

Peaks for  $f_2$ ,  $f_{2i}$ ,  $f_{inertial}$  shifted to lower frequencies for tabulated EoS models.



# HMNS asteroseismology - treatment of thermal effects



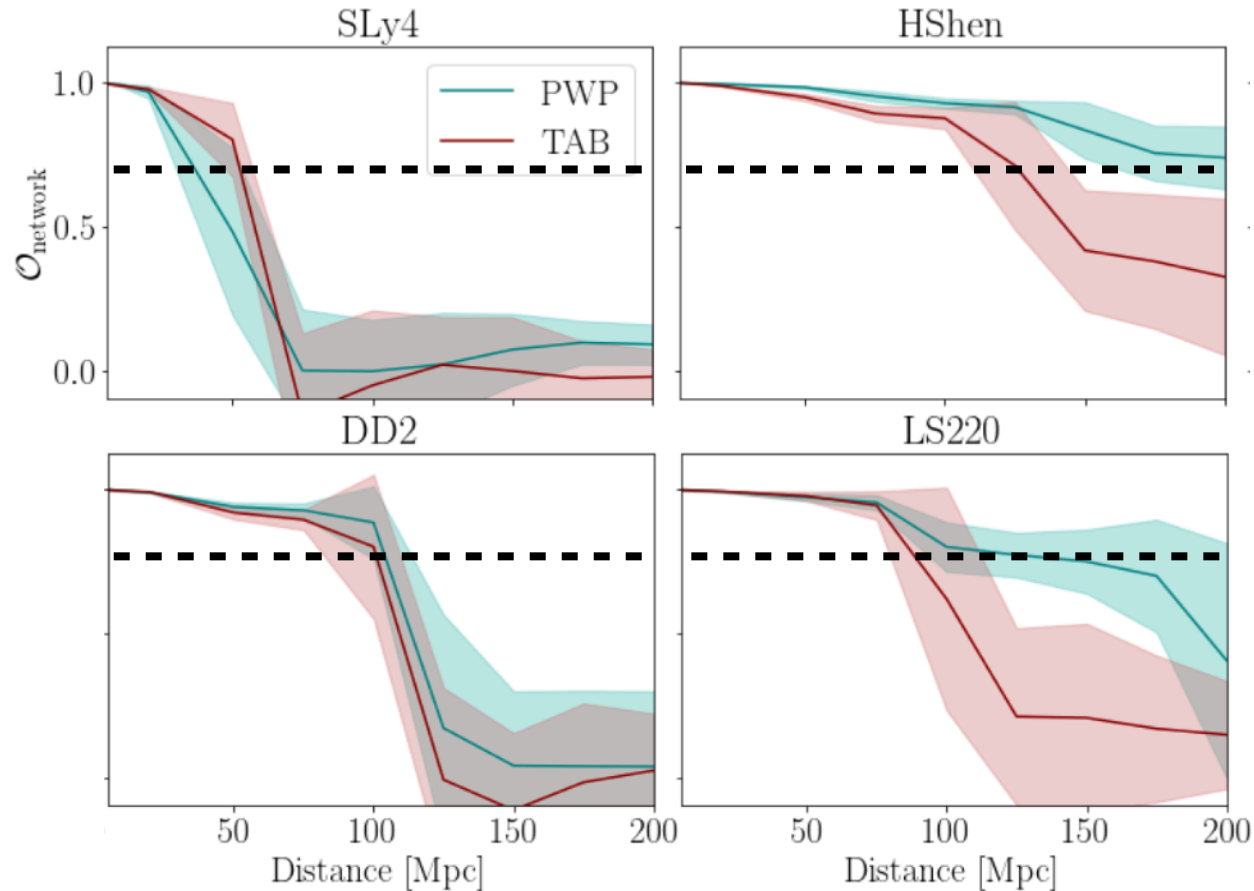
Hybrid models (circles):  
similar STD for both fits (0.0095, blue shaded region)

Tabulated models (triangles):  
Smaller STD (0.007; purple shaded region)

# Detectability of thermal EoS treatment - Bayeswave

Network overlap of early post-merger signal ( $f_2$ -mode)

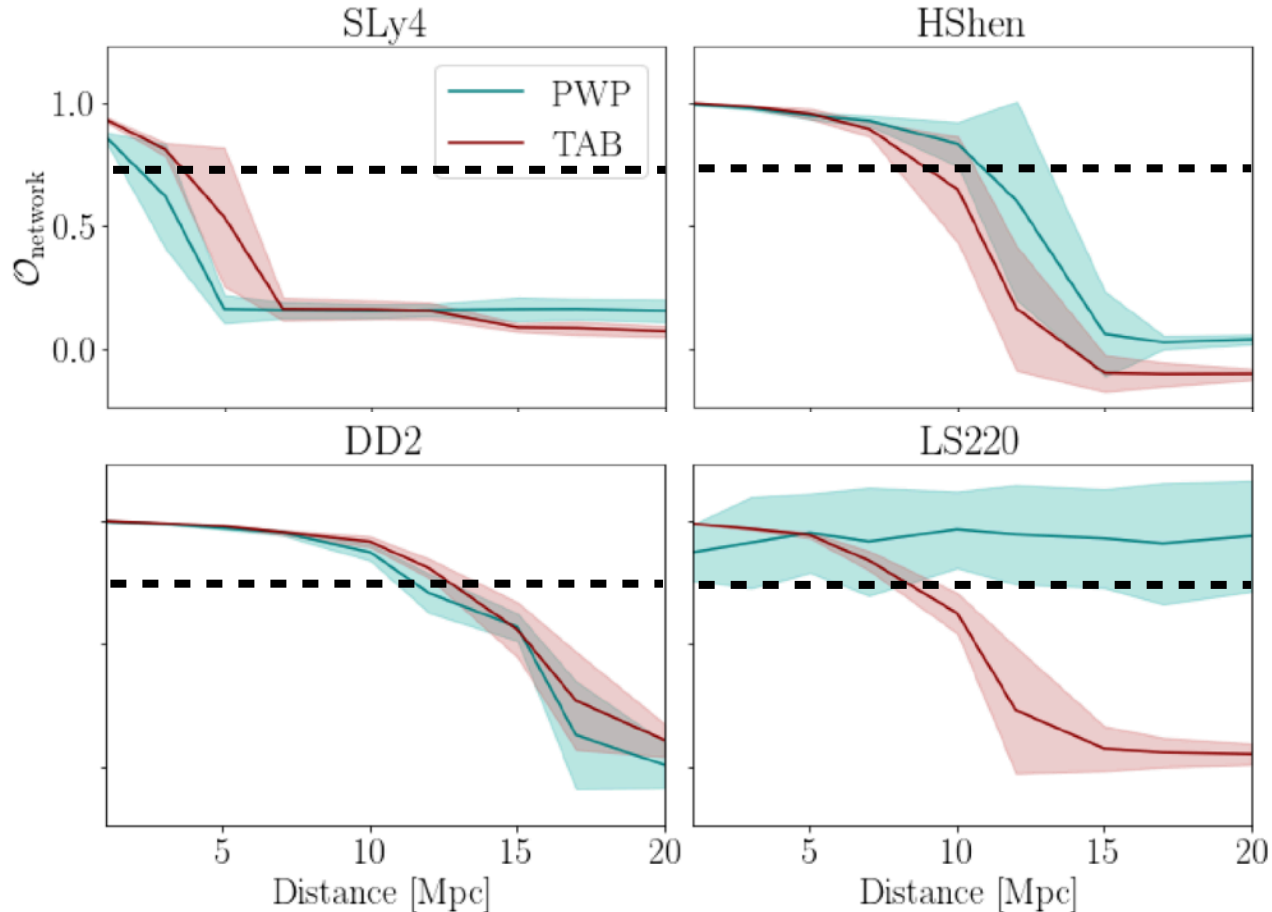
Miravet-Tenés+ (2024)  
2401.02493



No large differences between hybrid and tabulated models.  
ET overlap  $> 0.7$  for all EoS except **SLy4** even @100 Mpc.

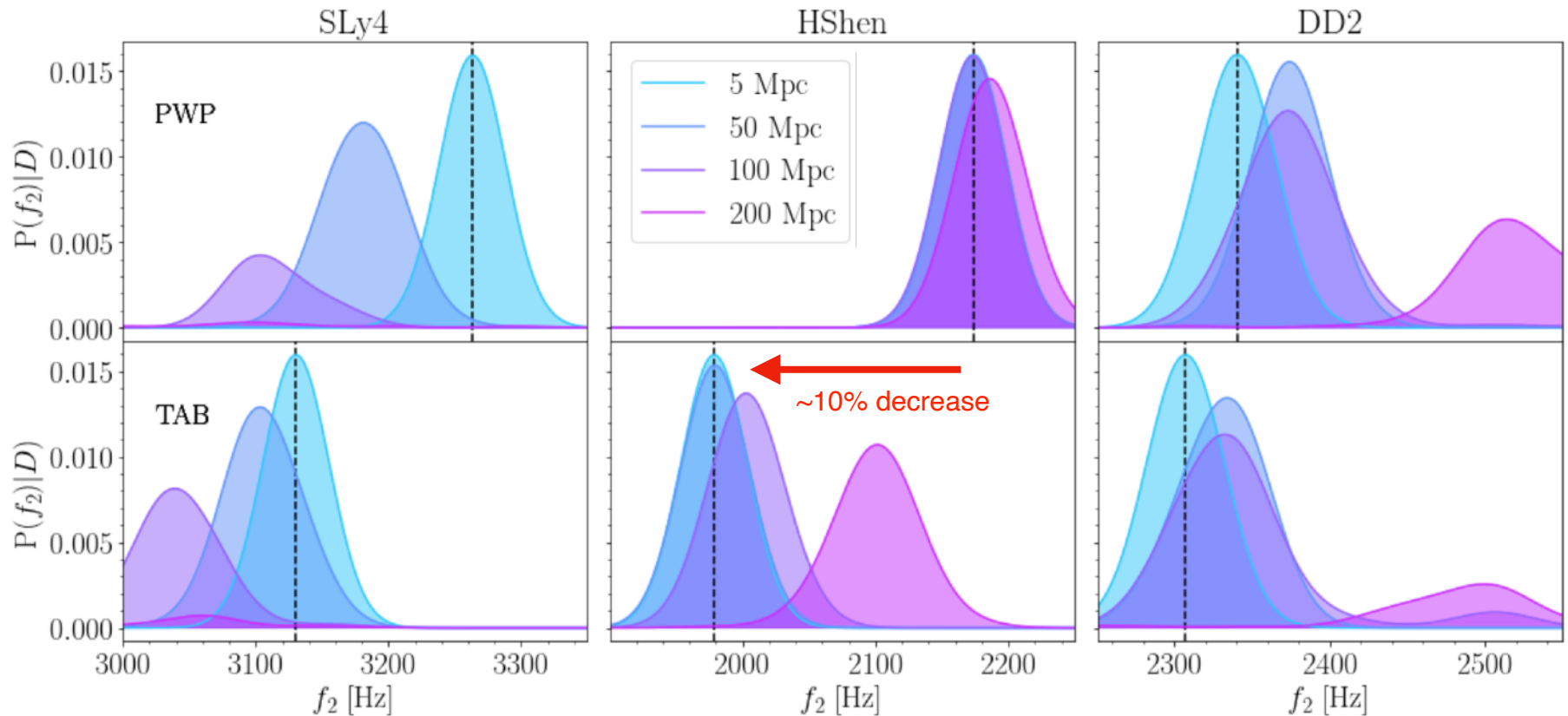
# Detectability of thermal EoS treatment - Bayeswave

Network overlap of late post-merger signal (inertial mode) [Miravet-Tenés+ \(2024\)](#)  
2401.02493



Inertial modes reconstructed properly up to **significantly closer distances** than the fundamental quadrupolar mode (10s of Mpc vs 100s of Mpc).

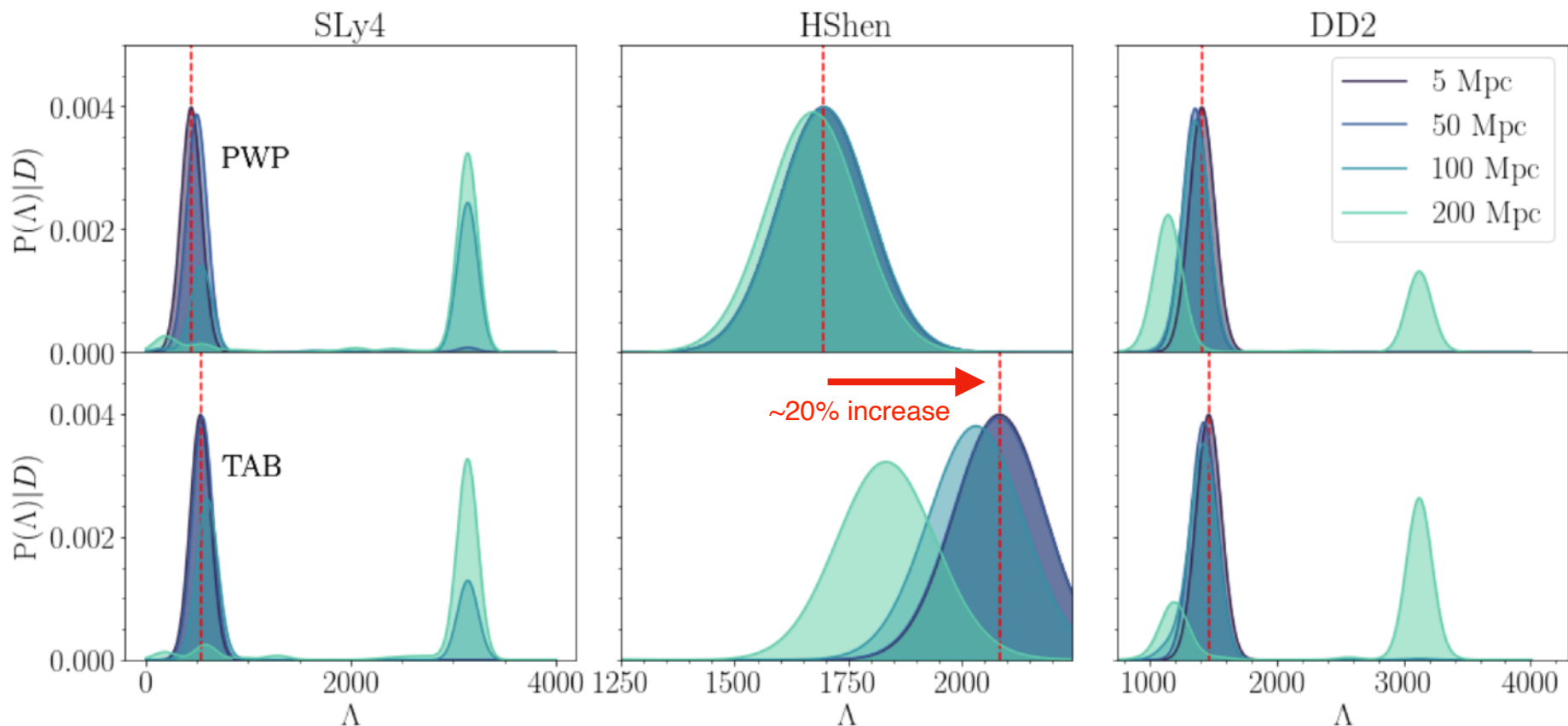
# Inference on the $f_2$ peak



Posterior probability of the peak frequency of the  $f_2$  mode, for different distances and PWP and TAB EoS. [Dashed vertical line: injected value]

**HShen** EoS shows the best results up to ~100-200 Mpc, for both types of models.

# Inference on the tidal deformability



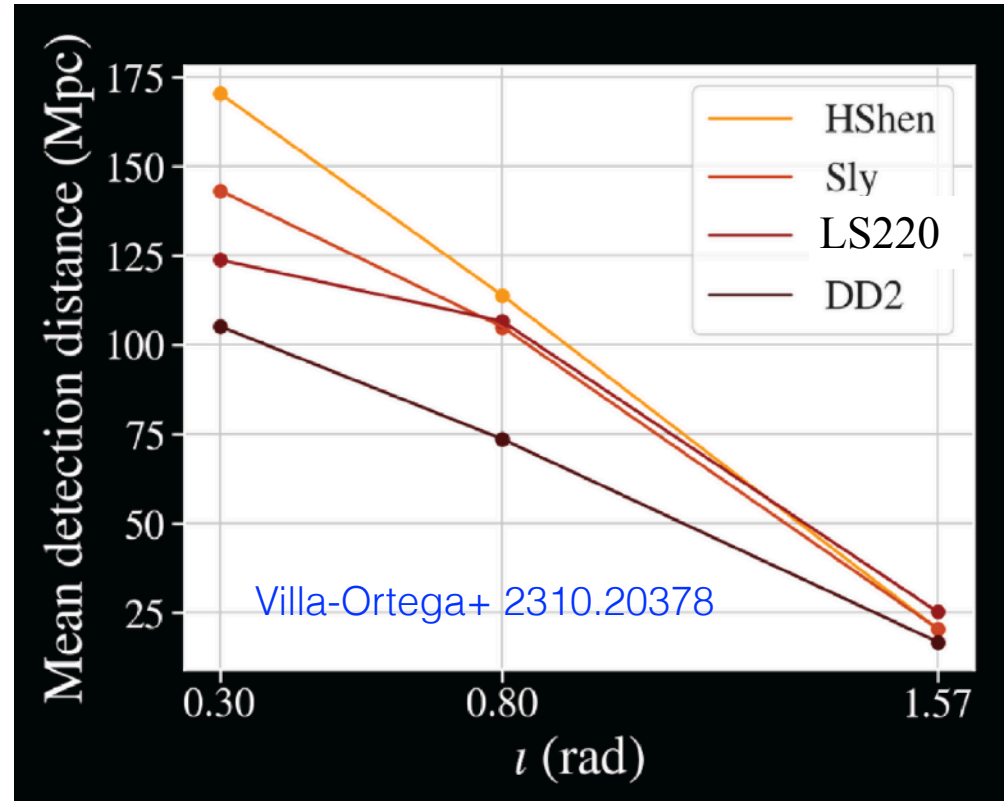
Posterior probability of tidal deformability obtained from the  $f_2$  mode, for different distances and PWP and TAB EoS. [Dashed red vertical line: injected value].

Largest **differences between hybrid and tabulated models** found for **HShen**.

# Detectability of thermal effects treatment

EOS	$\log \mathcal{B}_{\text{Cold}}^{\text{Th}}$
HShen	16.2
Sly	10.1
LS220	9.7
DD2	5.6

Log of relative Bayes Factors between tabulated and hybrid models (inclination 0.3, distance 100 Mpc)



Mean detection distances (averaged over sky angles) for a mean Bayes Factor  $\log \mathcal{B}_{\text{cold}}^{\text{th}} \sim 5$

Thermal EoS treatment detectable with  $\log \mathcal{B}_{\text{cold}}^{\text{th}} \geq 5$  at average distances of  $\sim 50$  Mpc for source inclinations  $\iota \leq 0.8$  regardless of the EoS.

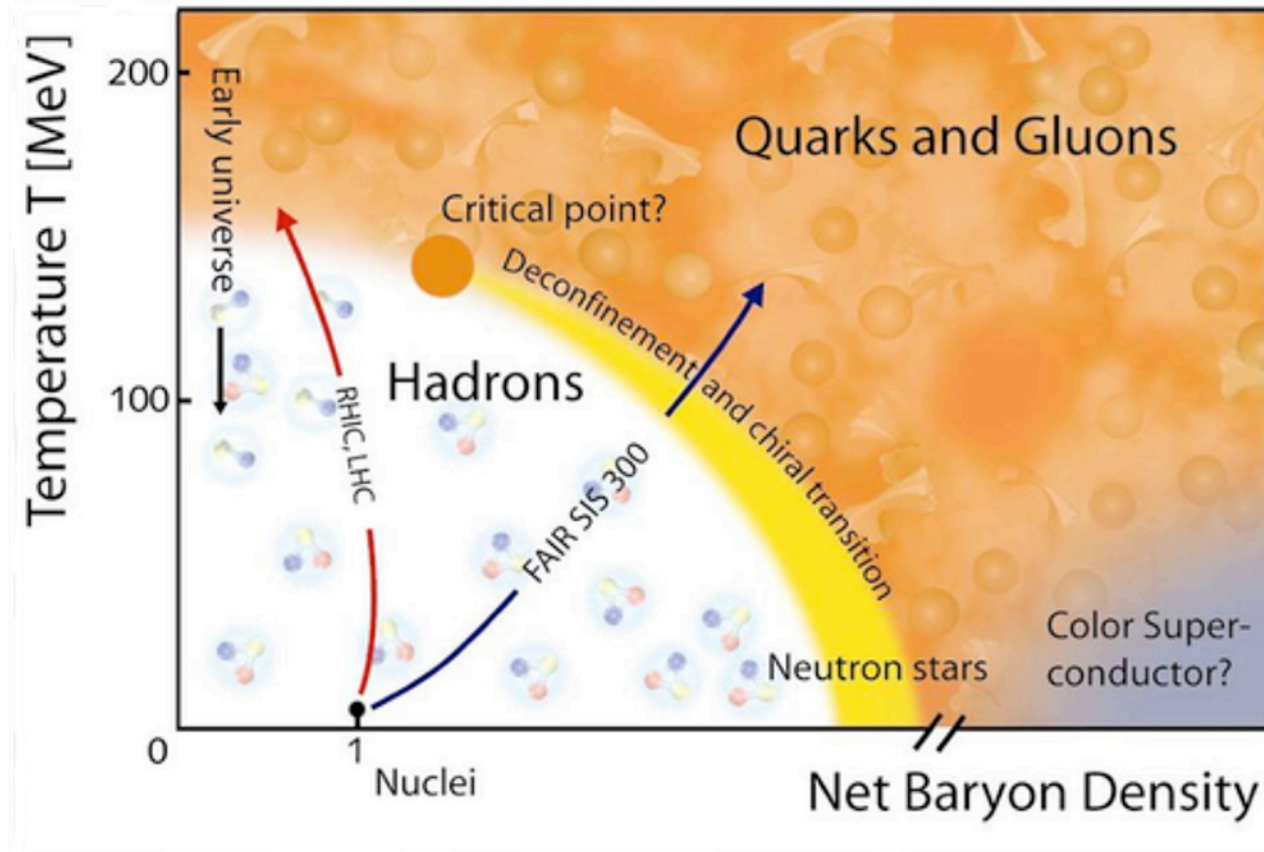
# Non-convex dynamics in BNS mergers

G. Riveccio, D. Guerra, M. Ruiz & JAF (2024)  
[arXiv:2401.06849](https://arxiv.org/abs/2401.06849)

# QCD phase diagram

QCD phase diagram determines the form of hadronic matter depending on temperature and matter density.

(Credit: GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt/Germany)



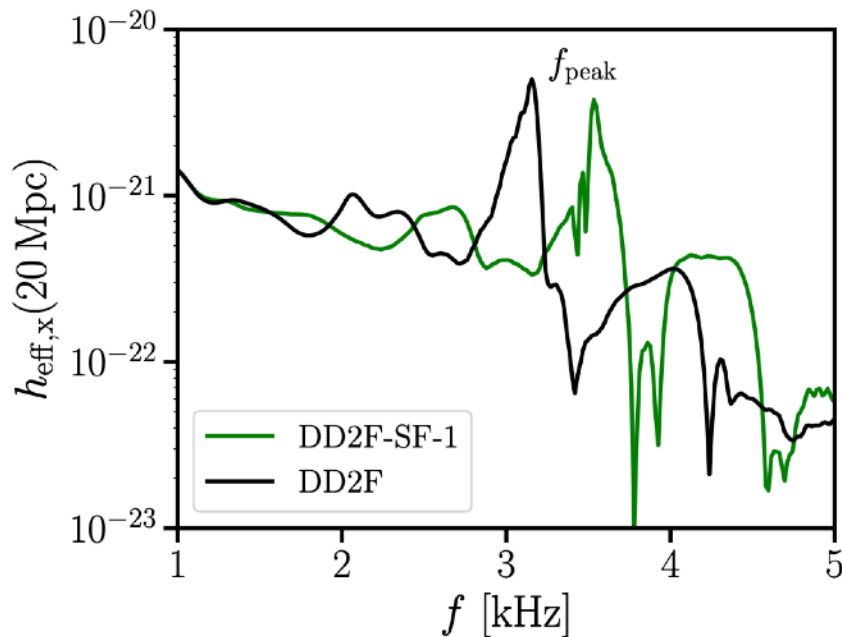
Is there a first order transition (from ordinary nuclear matter to a quark gluon plasma) and a critical point at finite density?



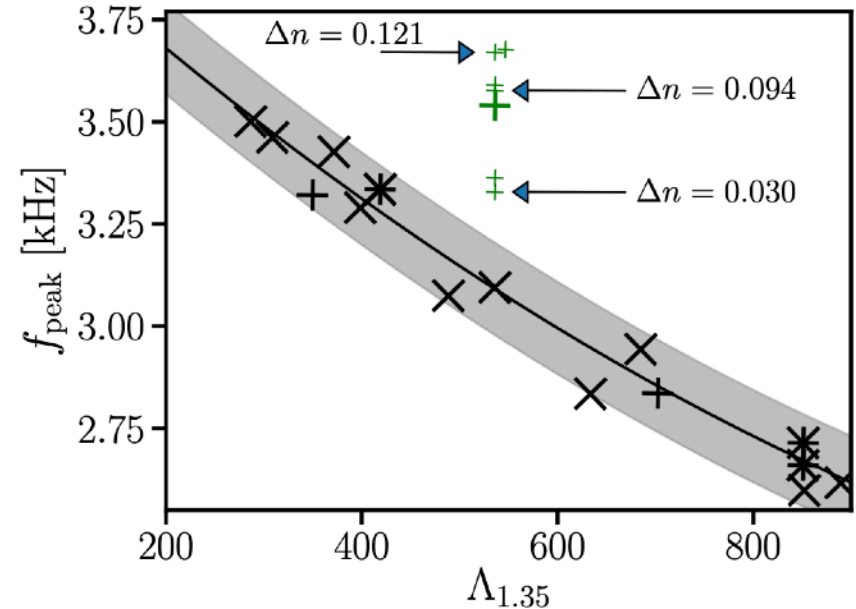
# BNS mergers with hadron-quark phase transitions

Bauswein+ (2019) identified an **observable imprint** of a first-order hadron-quark PT at supranuclear densities on the GW emission of BNS mergers.

Dominant postmerger GW frequency  $f_{\text{peak}}$  may exhibit a **significant deviation** from an empirical relation between  $f_{\text{peak}}$  and tidal deformability if a first-order PT leads to the formation of a stable extended quark matter core in the postmerger remnant.



(Bauswein+ 2019)



Green + : DD2F-SF EoS (PT to deconfined quarks)

Black markers: purely hadronic EoS

EoS from Wroclaw group (Fischer+ 2018, Bastian+ 2018)

Q1: Could this shift in frequency be explained by a different reason?

Q2: Could the anomalous dynamics be triggered by a **non-convex EoS**?

---

In classical fluid dynamics, the convexity of the system is determined by the EoS, (Menikoff & Plohr 1989; Godlewski & Raviart 1996), more specifically, by the so-called **fundamental derivative**:

$$\mathcal{G}_{(C)} \equiv -\frac{1}{2} V \frac{\frac{\partial^2 p}{\partial V^2}}{\frac{\partial p}{\partial V}} \quad \mathcal{G}_{(C)} = 1 + \frac{\partial \log c_s}{\partial \log \rho} = \frac{1}{2} \left( 1 + \Gamma_1 + \frac{\partial \log \Gamma_1}{\partial \log \rho} \right)$$

(all derivatives computed at constant entropy)

$\Gamma_1$  adiabatic index. Characterizes stiffness of EoS at a given density, showing a local maximum above nuclear matter density.

Pioneer work on EoS-driven, non-convex thermodynamics made by [Bethe \(1942\)](#), [Zel'dovich \(1946\)](#), and [Thompson \(1971\)](#). The latter introduced the concept of fundamental derivative. Fluids attaining negative values of the fundamental derivative are called Bethe-Zel'dovich-Thompson fluids, or **BZT fluids** (e.g. van der Waals EoS; see [Voss 2005](#)).

For relativistic fluid dynamics, [Ibáñez+ \(2013\)](#) found a quantity analogous to the classical fundamental derivative, the relativistic fundamental derivative.

$$\mathcal{G}_{(R)} = \mathcal{G}_{(C)} - \frac{3}{2} c_{s(R)}^2$$

# “Exotic” fluid dynamics

The fundamental derivative measures the **convexity of the isentropes** in the  $p - \rho$  plane.

- If  $\mathcal{G} > 0$ , isentropes in the  $p - \rho$  plane are convex, leading to expansive rarefaction waves and compressive shocks (Thompson 1971). This is the usual regime in which many astrophysical scenarios develop.
- However, some EoS may display regimes in which  $\mathcal{G} < 0$  and the EoS is non-convex. The non-convexity of isentropes in the  $p - \rho$  plane yield e.g. **compressive rarefaction waves and expansive shocks**.

These “exotic” phenomena have been observed experimentally

Cinnella & Congedo (2007)  
Cinnella+ (2011).

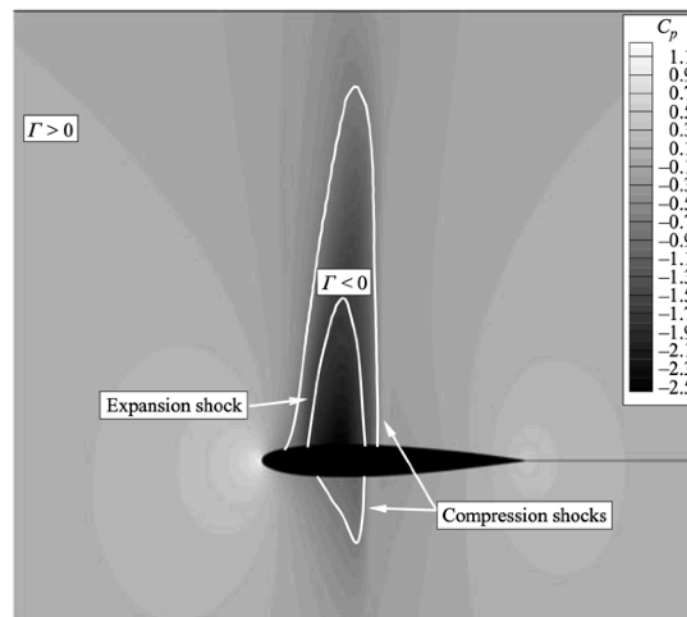
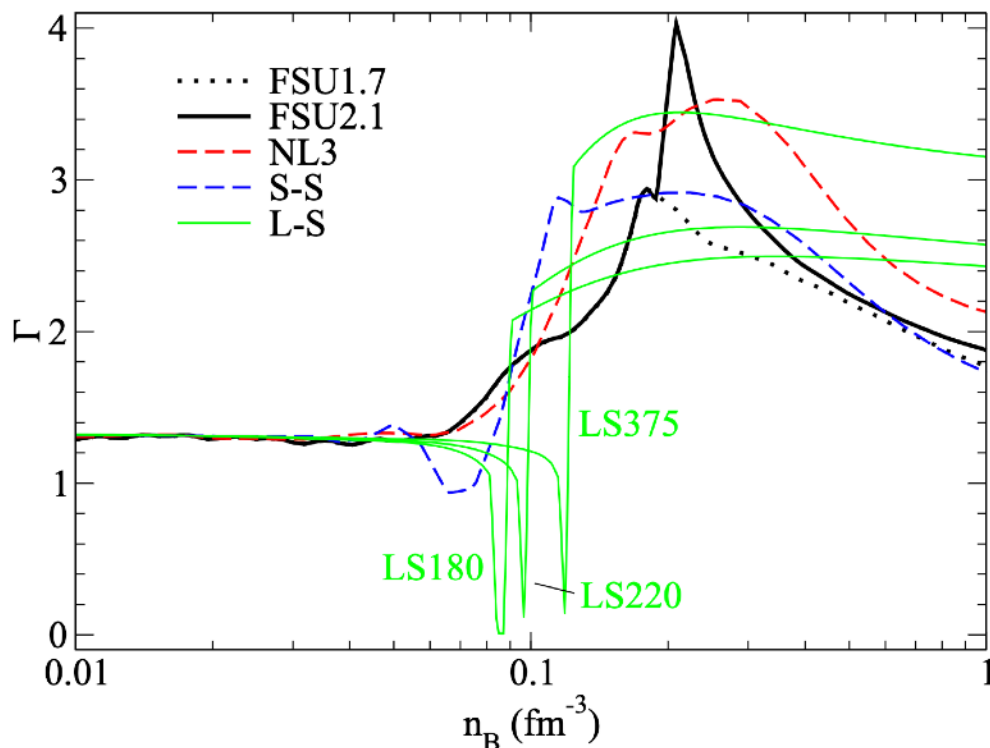


FIGURE 11. Pressure coefficient contours and  $\Gamma = 0$  contours for operating conditions  $p_\infty/p_c = 1.05$ ,  $\rho_\infty/\rho_c = 0.877$ ,  $\Gamma_\infty = 1.31$  (low-pressure transonic BZT regime).

At densities higher than nuclear saturation density  $n_0$  nuclear/hadronic matter undergoes a **phase transition** into a quark-gluon plasma.

The HotQCD Collaboration ([Bazavov+ 2014](#)) and the Wuppertal-Budapest Collaboration ([Borsányi+ 2014](#)) have shown that the EoS energy density in the crossover region is about  $1.2-3.1n_0$ . Within this temperature range  $[145 \leq T(\text{MeV}) \leq 163]$  the **sound speed is non-monotonic**.

Similar conclusions achieved by [Badaque & Steiner \(2015\)](#)



Non-monotonicity of sound speed can also result from the behaviour of the adiabatic index (see [Haensel & Potekhin 2004](#)).

[Shen+ \(2011\)](#)

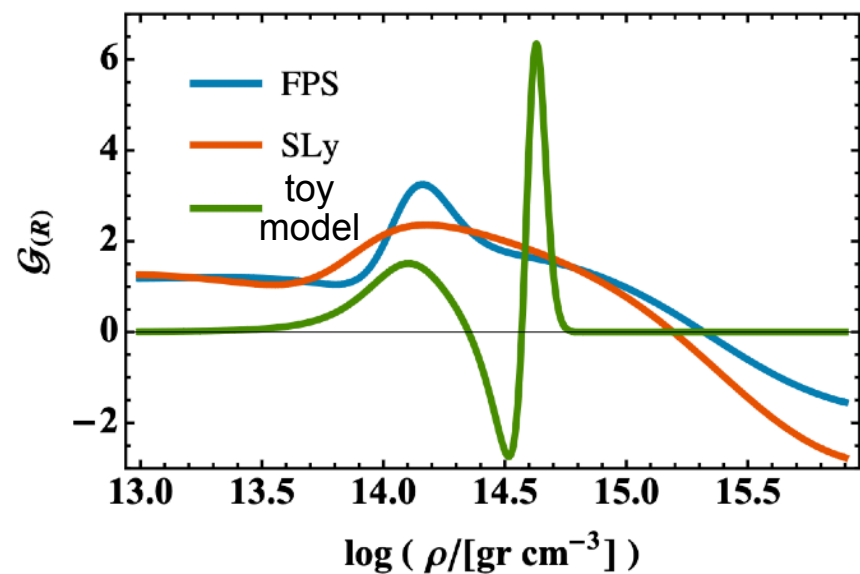
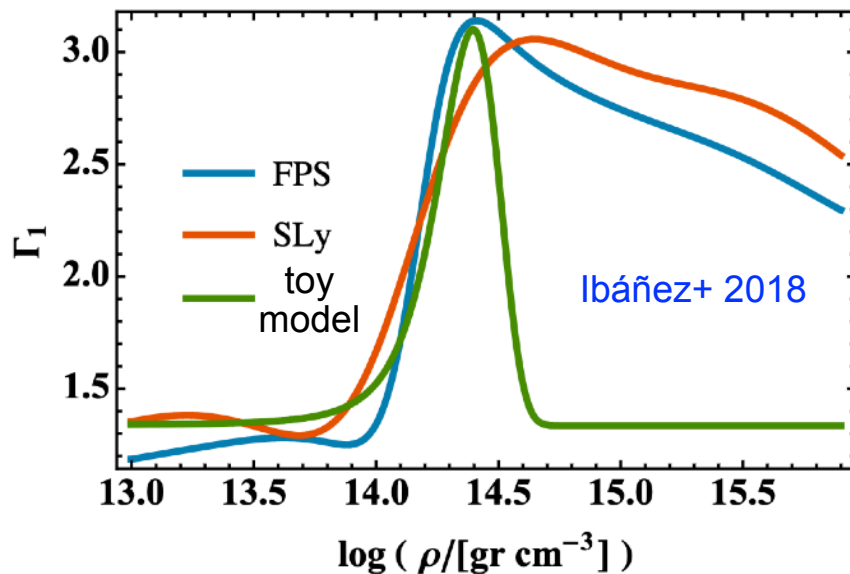
**non-monotonic behaviour of adiabatic index** for various EoS broadly used in numerical simulations of core collapse supernovae and BNS mergers.

The **non-monotonicity** of the adiabatic index (sound speed) should be considered as a genuine feature of matter at a few times nuclear saturation density.

Under such conditions, and particularly if there are phase transitions to exotic components, the fundamental derivative could be negative, implying that the EoS be non-convex in that regime. This would lead to **non-convex dynamics**.

Following [Ibáñez+ 2018](#) and [Aloy+ 2018](#) we illustrate the effects a non-convex EoS may produce on the dynamics of BNS mergers with a **phenomenological EoS**. This **toy-model** EoS mimics the loss of convexity resulting from a non-monotonic behavior of the adiabatic index with density.

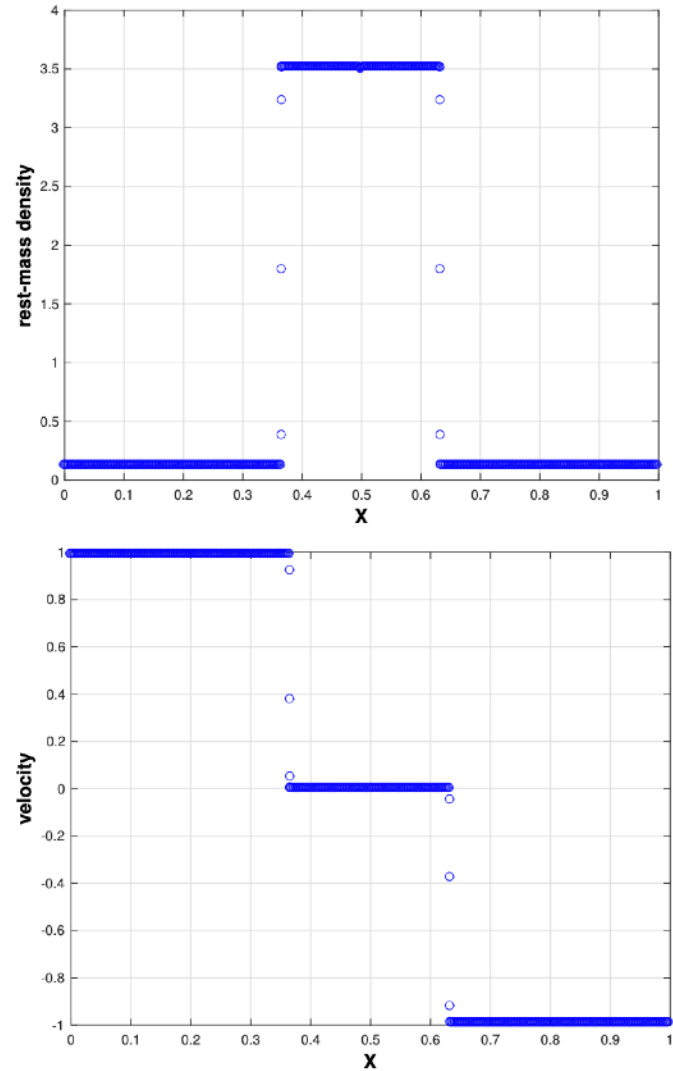
$$p = (\Gamma - 1)\rho\epsilon \quad \Gamma = \gamma_0 + (\gamma_1 - \gamma_0)e^{-\frac{(\rho - \rho_1)^2}{\sigma^2}} \quad \text{4 free parameters: } \gamma_0, \gamma_1, \sigma, \rho_1$$



# Example Riemann problem: relativistic blast wave collision

Ibáñez+ 2018

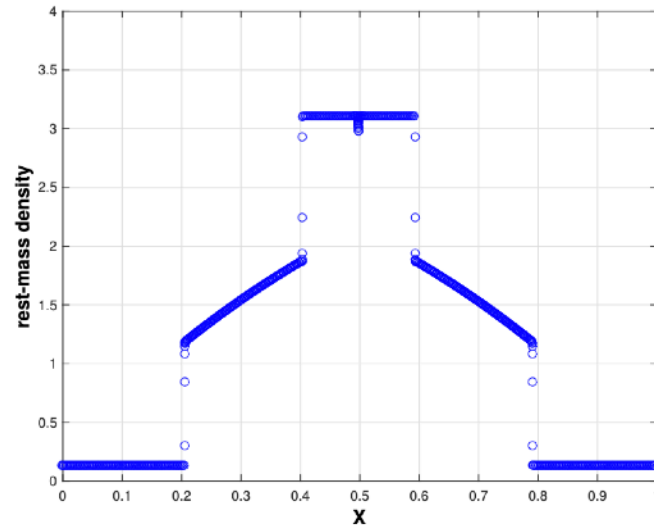
Convex dynamics  
(Ideal gas EoS)



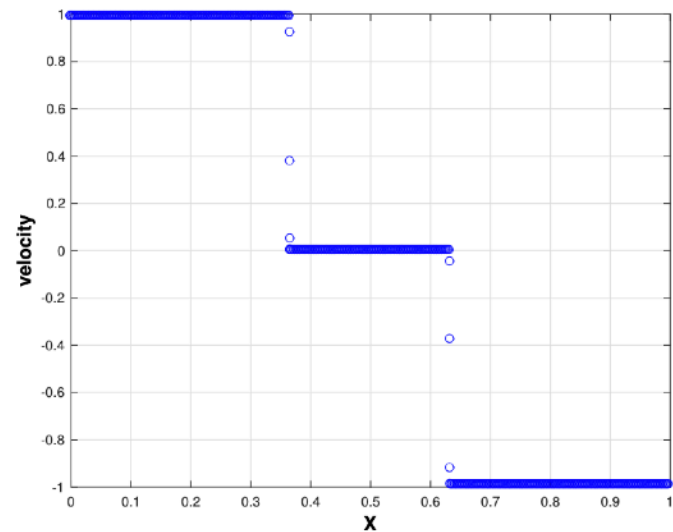
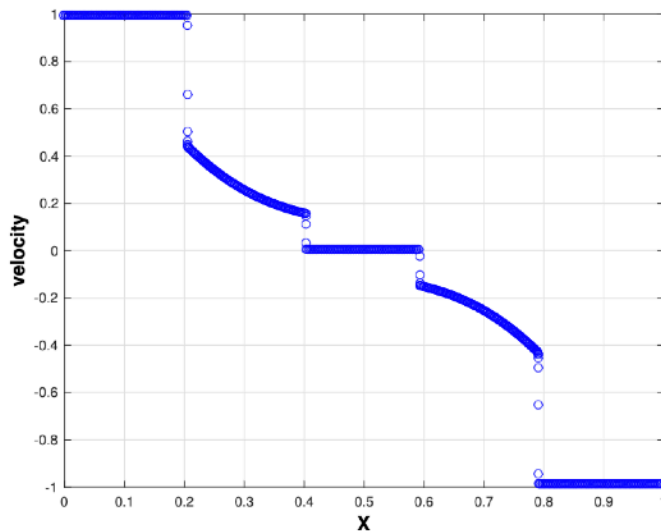
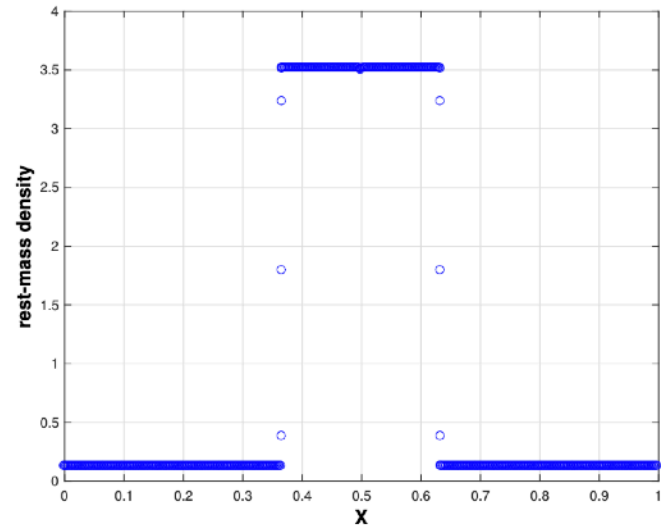
# Example Riemann problem: relativistic blast wave collision

Ibáñez+ 2018

Non-convex dynamics  
(toy model EoS)



Convex dynamics  
(Ideal gas EoS)



# The $p - \rho$ plane for a representative BNS simulation

$$p = (\Gamma - 1)\rho\epsilon$$

$$\Gamma = \gamma_0 + (\gamma_1 - \gamma_0)e^{-\frac{(\rho - \rho_1)^2}{\sigma^2}}$$

$$\begin{aligned} \gamma_0 &= 1.8 \\ \gamma_1 &= 3.0 \\ \rho_1 &= 0.91 \times 10^{15} \text{ (cgs)} \\ \sigma &= 0.35\rho_1 \end{aligned}$$

Blue: convex dynamics

$$(\mathcal{G}_C > 0, \mathcal{G}_R > 0)$$

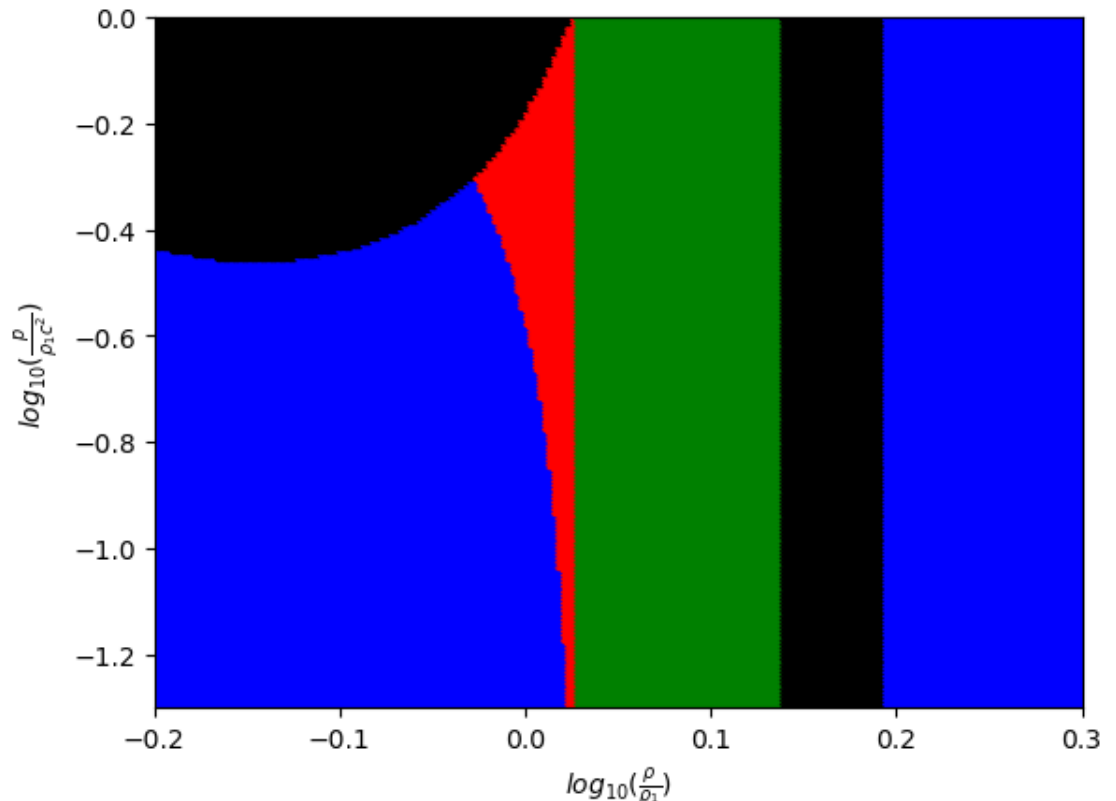
Red: convex (classical)  
non-convex (relativistic)

$$(\mathcal{G}_C > 0, \mathcal{G}_R < 0)$$

Green: non-convex dynamics

$$(\mathcal{G}_C < 0, \mathcal{G}_R < 0)$$

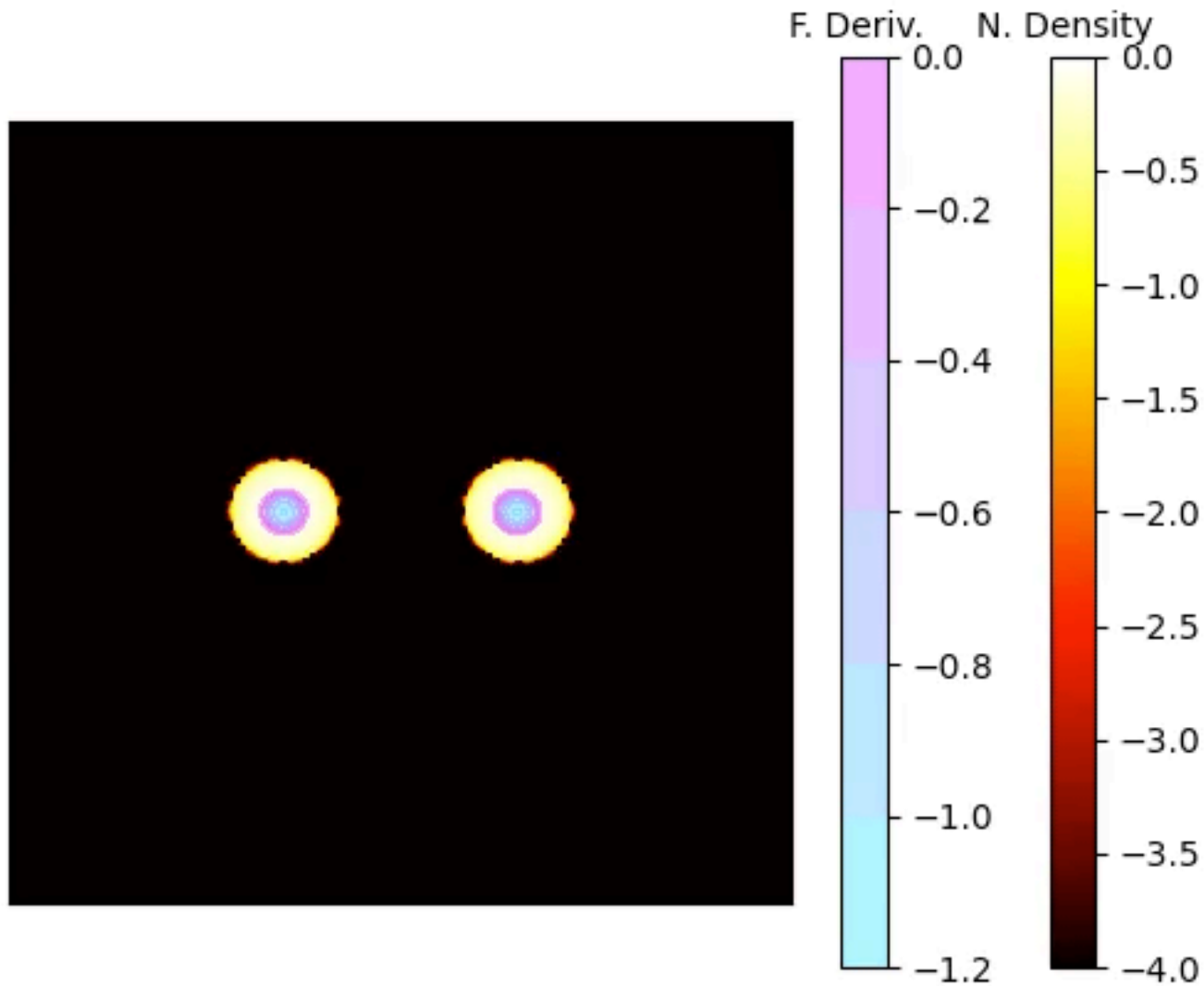
Black: sound speed out of bounds





# Non-convex BNS simulation

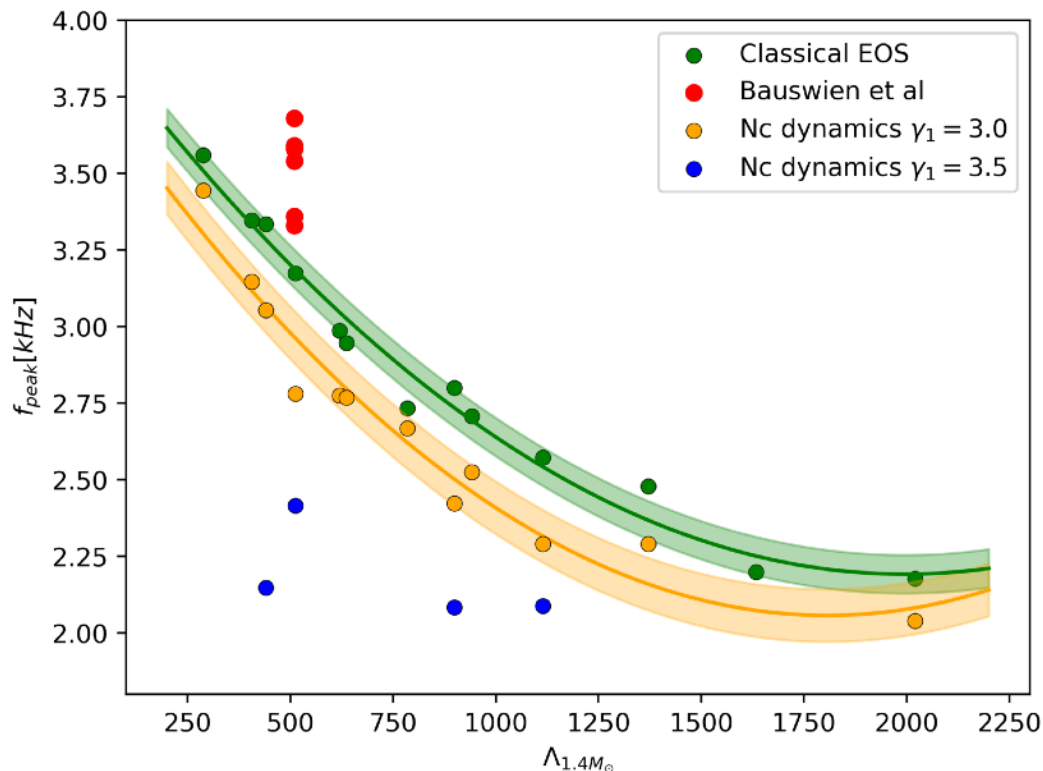
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# Empirical universal relations affected by non-convex dynamics

## Set of models considered

EOS	$M[M_{\odot}]$	$\mathcal{C}$	$\Lambda$	$M_{\text{ADM}}[M_{\odot}]$	$J_{\text{ADM}}[M_{\odot}^2]$	$\Omega$
WFF1	1.26	0.18	406.07	2.50	6.45	1.76
WFF2	1.27	0.17	1115.06	2.52	6.54	1.76
APR4	1.28	0.17	440.75	2.52	6.56	1.77
SLy4	1.28	0.16	512.57	2.54	6.62	1.77
APR3	1.28	0.16	620.00	2.53	6.61	1.77
ENG	1.28	0.16	636.35	2.53	6.60	1.77
MPA1	1.28	0.15	784.52	2.54	6.64	1.77
LS220	1.29	0.15	899.69	2.55	6.69	1.77
ALF2	1.29	0.15	941.42	2.54	6.66	1.77
DD2	1.29	0.13	1115.1	2.56	6.73	1.78
HShen	1.30	0.14	1633.39	2.58	6.82	1.78
GNH3	1.30	0.13	1371.15	2.58	6.81	1.78
MS1	1.30	0.13	2020.75	2.58	6.83	1.79



**Significant shifts in frequency observed** for  $f_2$  mode.

They can be as large as  $\sim 500$  Hz for some choice of the parameters of the toy model EoS.

Do these findings hold for realistic EoS?

# Going further

Postmerger dynamics and waveforms can also be affected by

- Effect of B-field amplification
- Effect of viscosity
- Neutrinos

Waveform generation (both inspiral and postmerger) and parameter estimation to be aided by **AI and Deep Learning**.

# Thank you for your attention!

## Main collaborators:

Davide Guerra (PhD student @ UV)  
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Giuseppe Riveccio (PhD student @ UV)  
Milton Ruiz (senior post-doc @ UV)  
Pablo Cerdá-Durán (faculty @ UV)

Roberto De Pietri (faculty @ Parma University)  
Nikolaos Stergioulas (faculty @ AUTH)

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