Introduction to asymptotically safe gravity

64th Kraków school of Theoretical Physics, Zakopane, July 2024

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Lecture 1: Introduction to asymptotically safe gravity

- **Problem of perturbative quantum gravity**
- **Asymptotically safe quantum gravity**
  - **Main idea**
  - **Tools/Techniques**
  - **Evidence**

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Lecture 1: Introduction to asymptotically safe gravity

- Problem of perturbative quantum gravity
- Asymptotically safe quantum gravity
  - Main idea
  - Tools/Techniques
  - Evidence

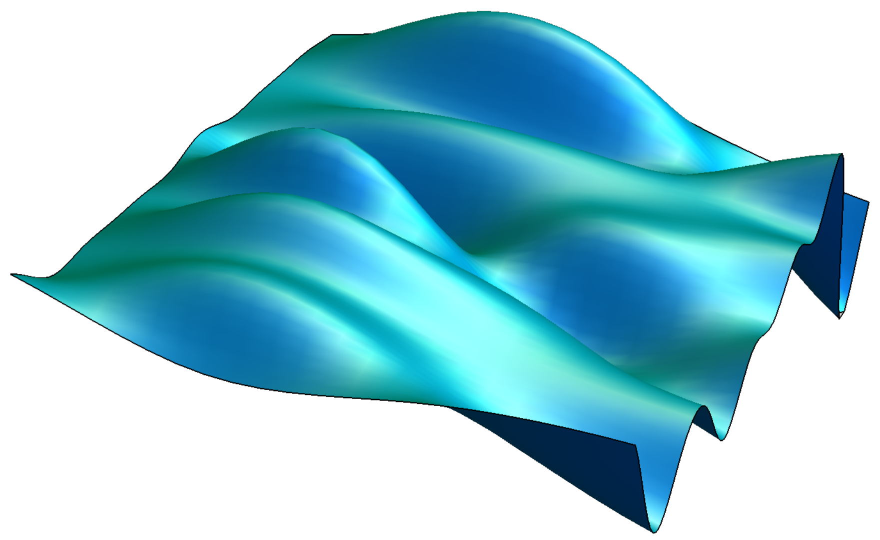
Lecture 2: Probing (asymptotically safe) gravity at all scales

- Asymptotically safe gravity and matter:
  - Effect of matter on gravity
  - Effect of gravity on Standard Model matter
  - Asymptotic safety in the dark sector

Quantum gravity

Classical gravity:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \quad \rightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \quad \rightarrow \quad g_{\mu\nu}$$



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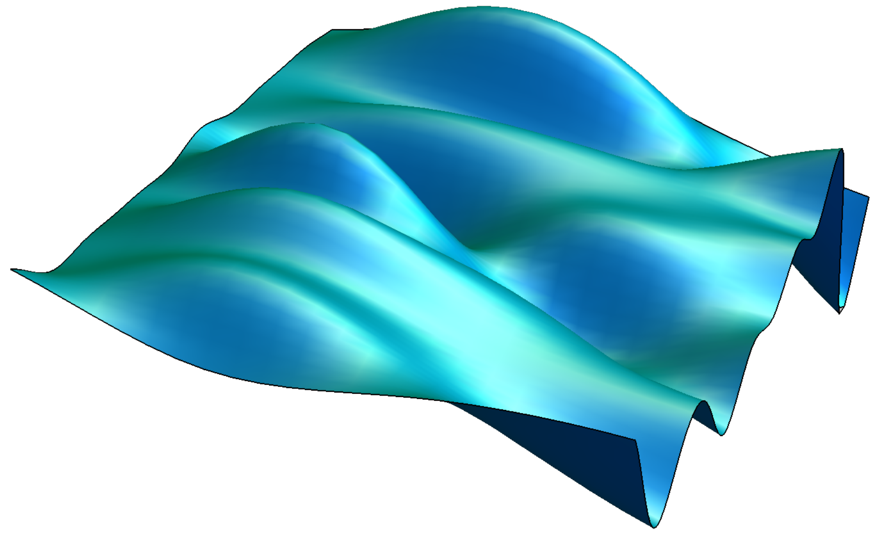

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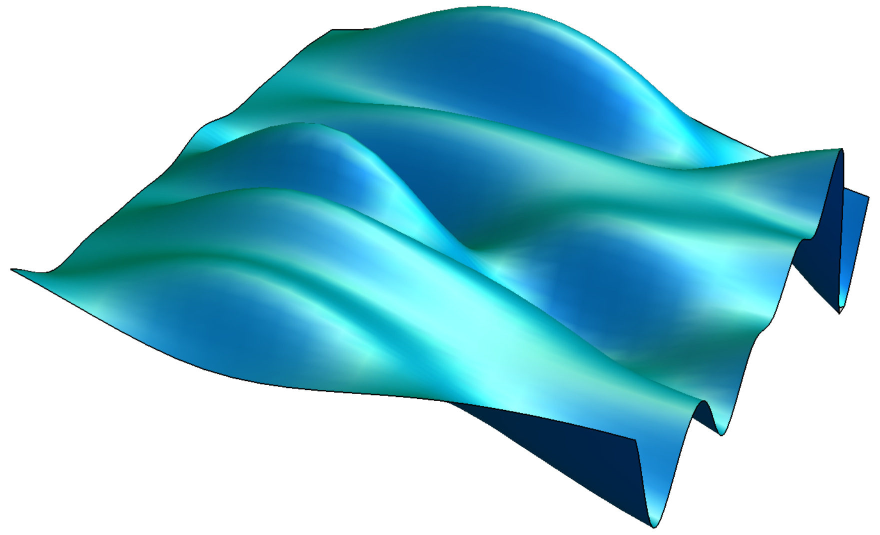

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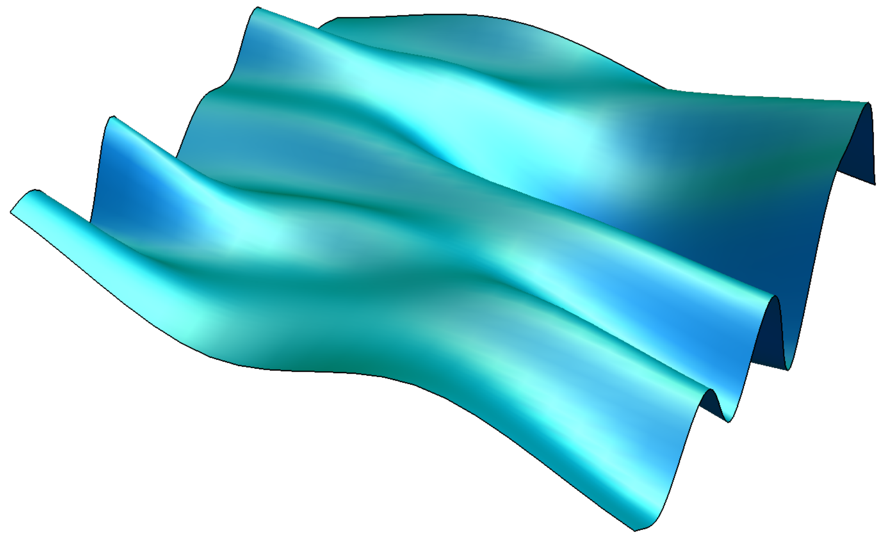

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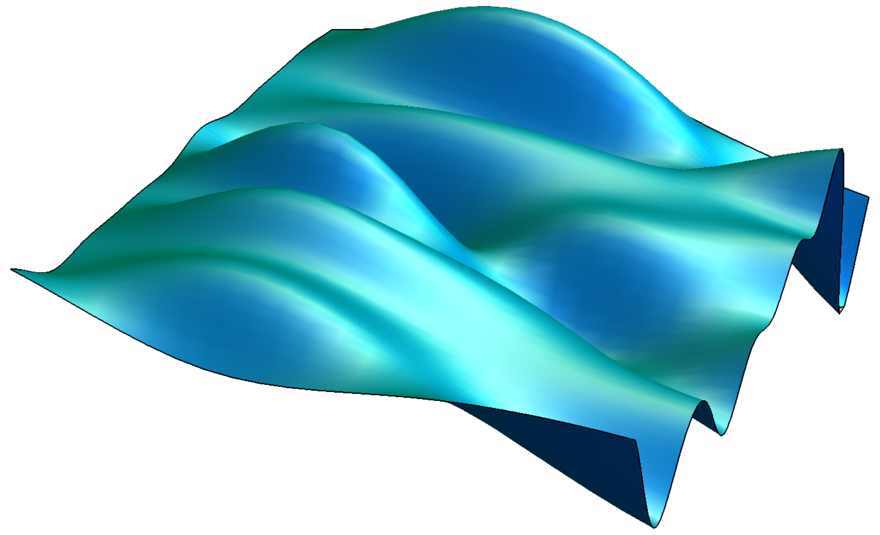

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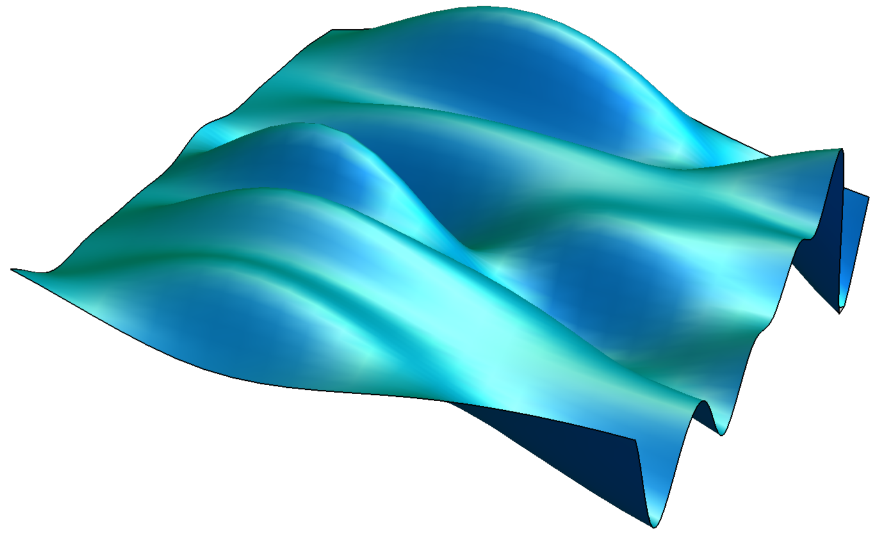

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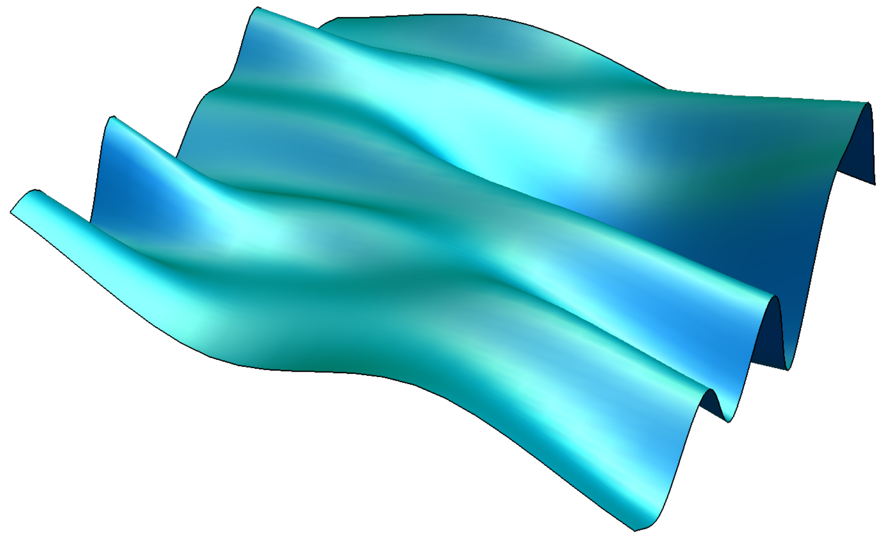

$\cdots$

$$\rightarrow \quad \langle g_{\mu\nu}\rangle$$

What is the effect of quantum fluctuations? New interactions!

The problem of perturbative quantum gravity

$$e^{i\Gamma[\langle g_{\mu\nu}]\rangle} = \int \mathscr{D} g_{\mu\nu}\, e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

with

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R$$

1-loop divergences

$$\sim \sqrt{-g},\ \sim \sqrt{-g}R,\ \sim \sqrt{-g}R^2,\ \sim \sqrt{-g}R_{\mu\nu}R^{\mu\nu}$$

2-loop divergences

$$\ldots \sim \sqrt{-g}R_{\mu\nu\kappa\lambda}R^{\kappa\lambda}{}_{\rho\sigma}R^{\rho\sigma\mu\nu}, \ldots$$

superficial degree of divergence:

$$D = 2L + 2$$

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perturbative non-renormalizability

⇒ loss of predictivity

2-loop divergences

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effective field theory: assume naturalness (all couplings $\frac{c_i}{M_{\mathrm{Pl}}^{d_i}}$ with $c_i \sim \mathcal{O}(1)$

⇒ higher-order interactions subleading for processes at energies $E \ll M_{\mathrm{Pl}}$

Restoring predictivity by demanding more symmetry

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Imposing a symmetry on an action relates couplings and/or sets them to zero

Example:

real scalar field theory without internal symmetry: allowed interactions $\lambda_i \phi^i, \ldots$

real scalar field theory with internal $\mathbb{Z}_2$ symmetry ($\phi \to -\phi$): allowed interactions $\lambda_{2i} \phi^{2i}, \ldots$

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Gravitational example: supergravity

Theory	Counterterm	Loop Order	divergence
$D=4,\ Q=32,\ \mathcal{N}=8$	$\mathcal{D}^8 R^4$	7	unknown
$D=4,\ Q=16,\ \mathcal{N}=4$	R^4	3	no
$D=4,\ Q=20,\ \mathcal{N}=5$	$\mathcal{D}^2 R^4$	4	no
$D=24/5,\ Q=32$	$\mathcal{D}^8 R^4$	5	yes
$D=5,\ Q=16$	R^4	2	no

from Z. Bern, J. J. Carrasco, M. Chiodaroli, H. Johansson and R. Roiban, ‘23

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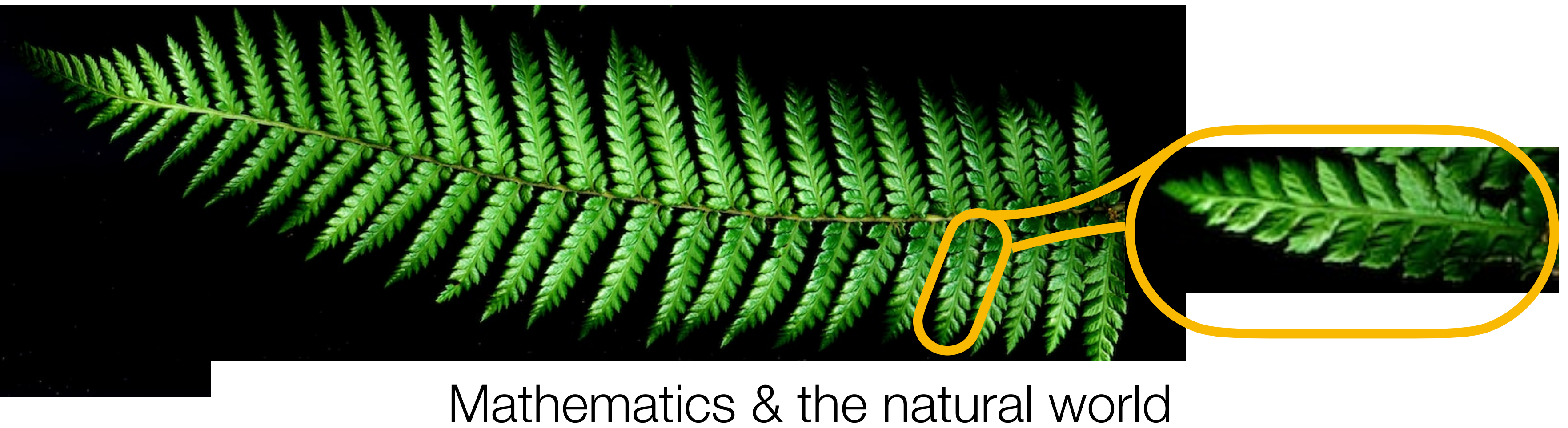
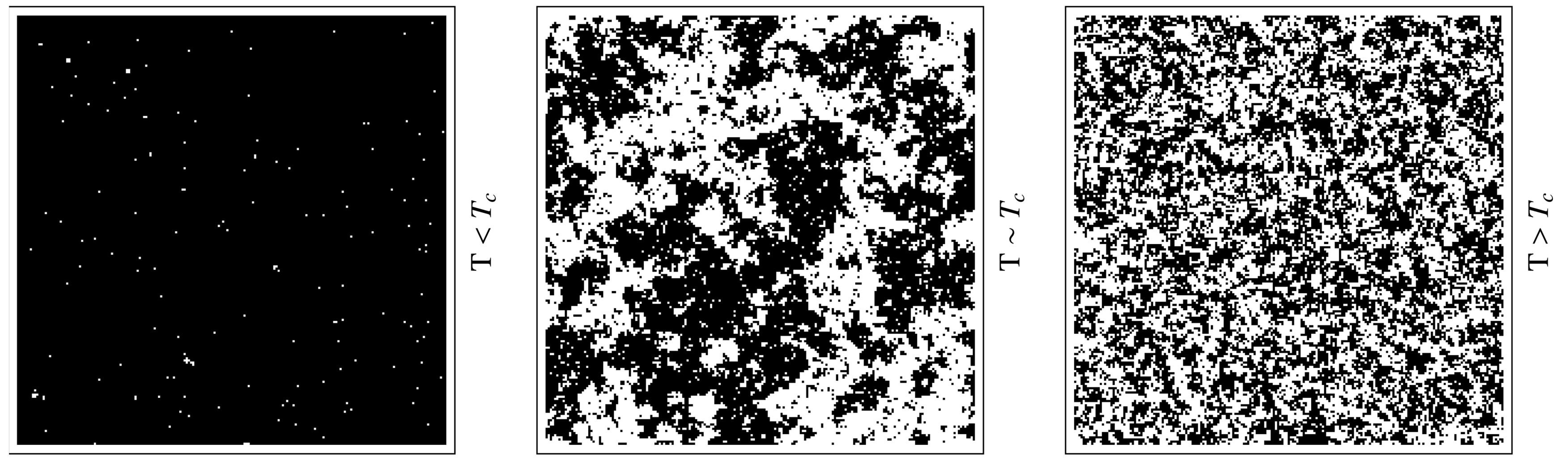
Usually: symmetry imposed at the classical level; checking for anomalies when quantizing
→ what about symmetries that *emerge* at the quantum level?

What symmetry? Scale symmetry!

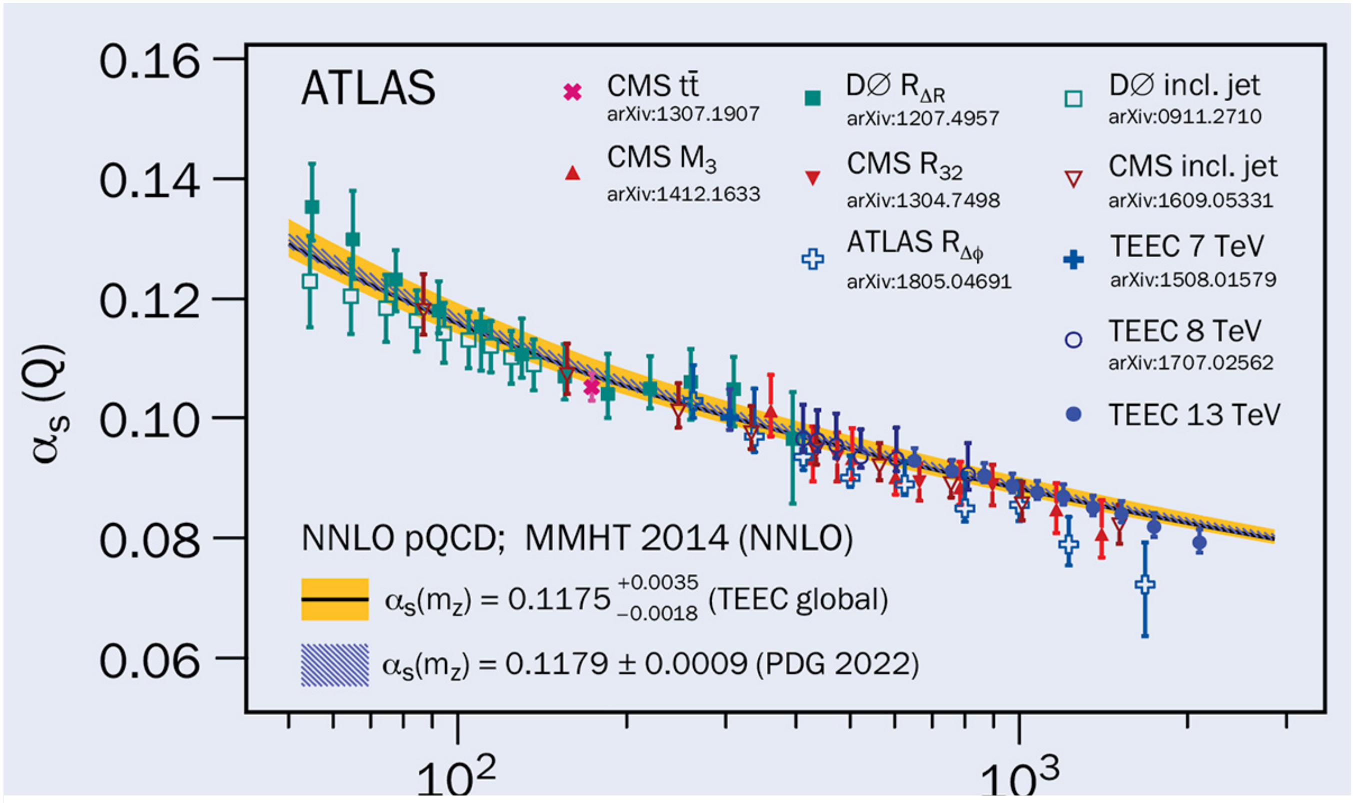
Hypothesis: The quantum structure of spacetime is described by an asymptotically safe quantum field theory of the metric - gravity exhibits quantum scale symmetry [Weinberg ’74]



Mathematics & the natural world



Phase transitions



Particle physics

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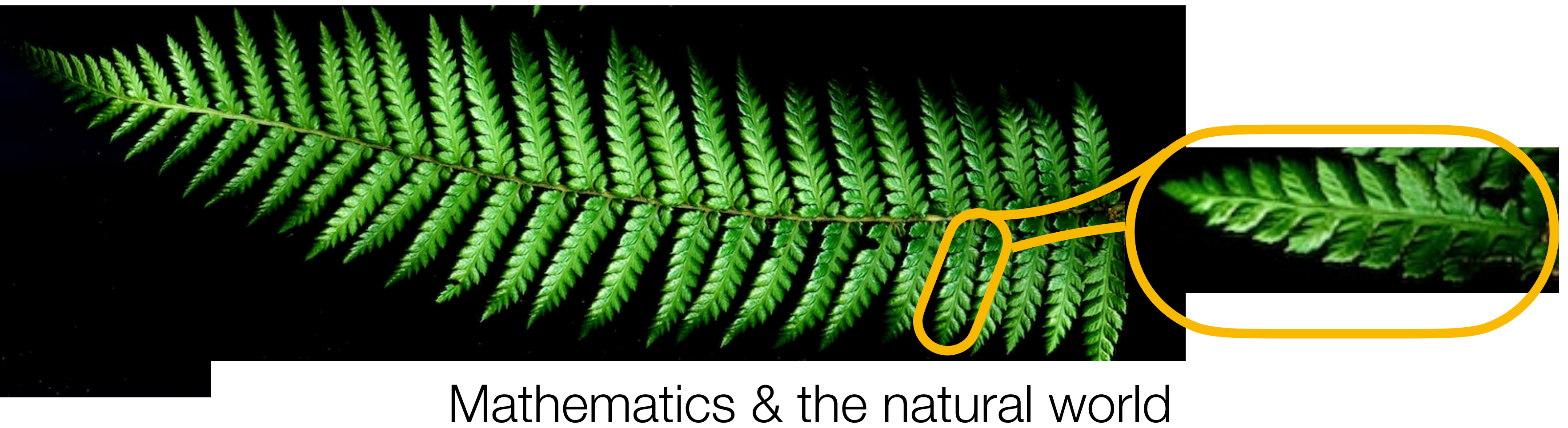

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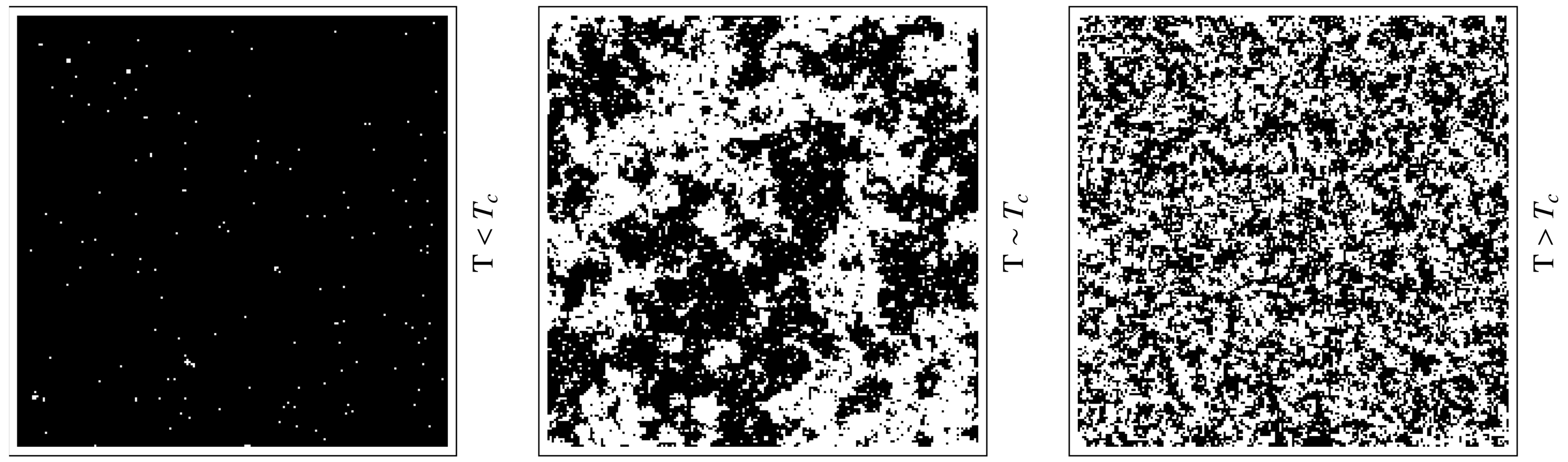

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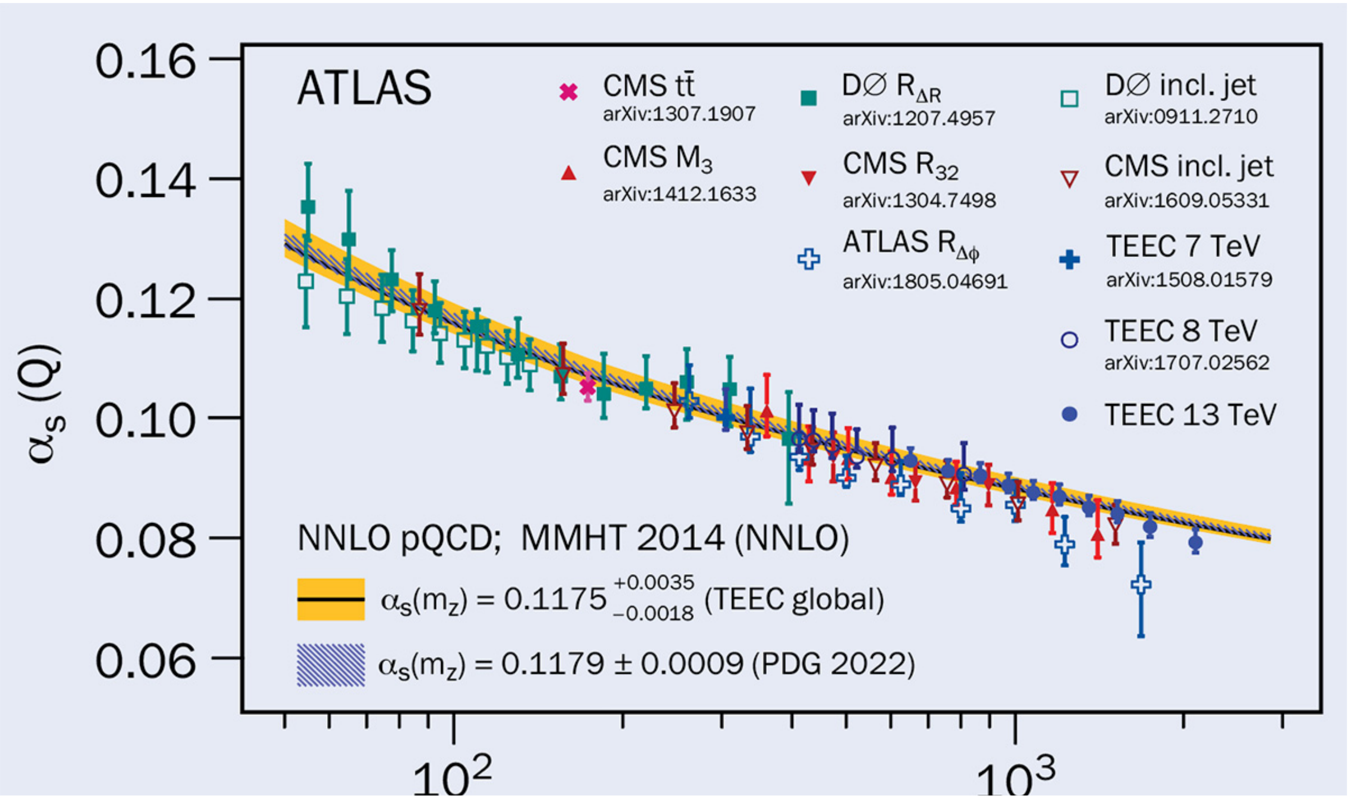
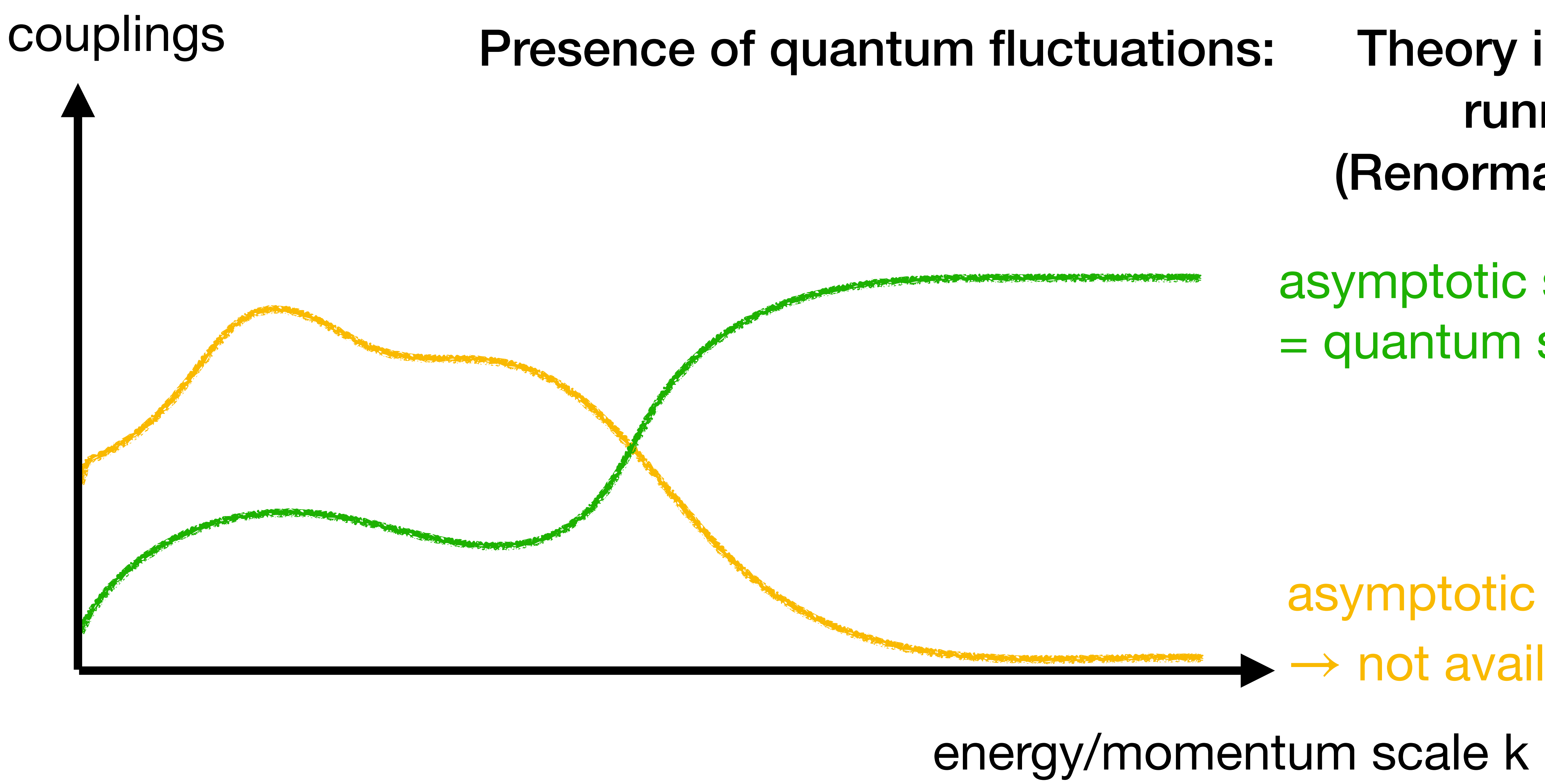


Particle physics

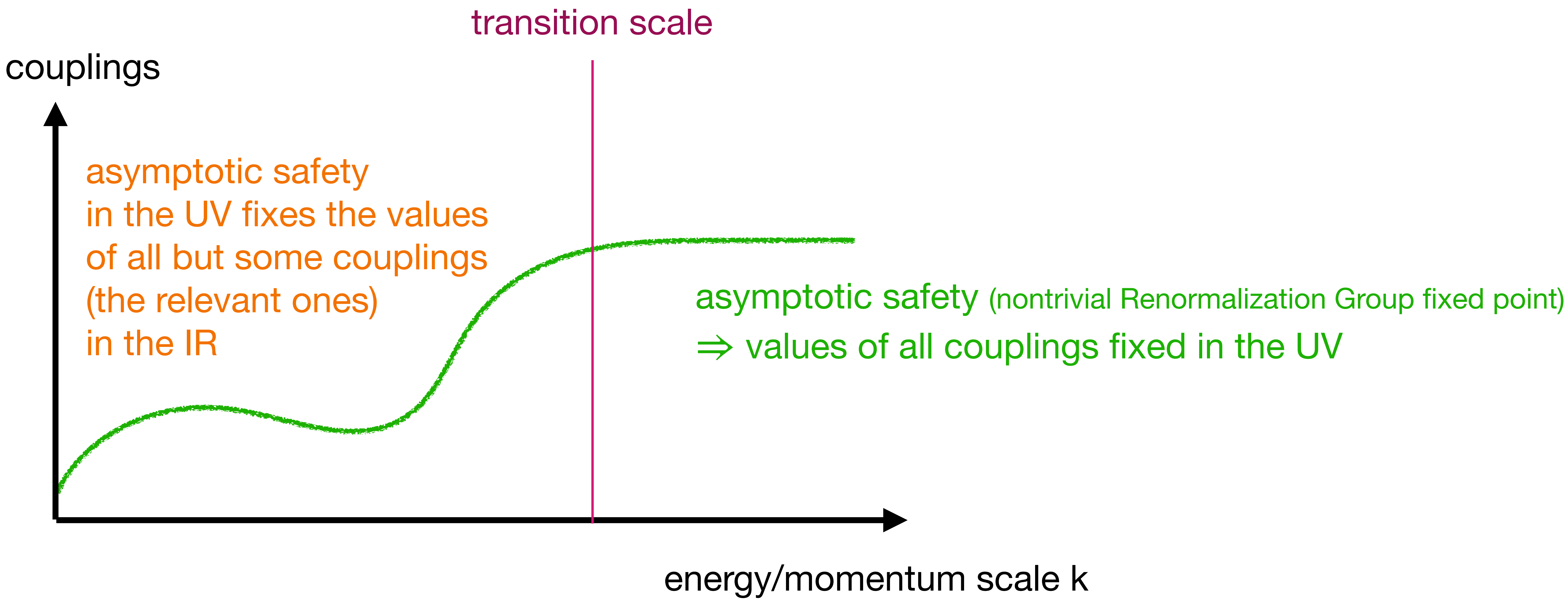
Presence of quantum fluctuations: **Theory is scale dependent; running couplings (Renormalization Group flow)**



asymptotic safety (nontrivial Renormalization Group fixed point)
= quantum scale symmetry

asymptotic freedom (trivial Renormalization Group fixed point)
→ not available for Einstein gravity

Predictivity in asymptotic safety



Predictivity at all scales: UV

Asymptotic safety (quantum scale symmetry): $\beta_{g_i} = k\partial_k\, g_i(k) = 0 \;\forall i$

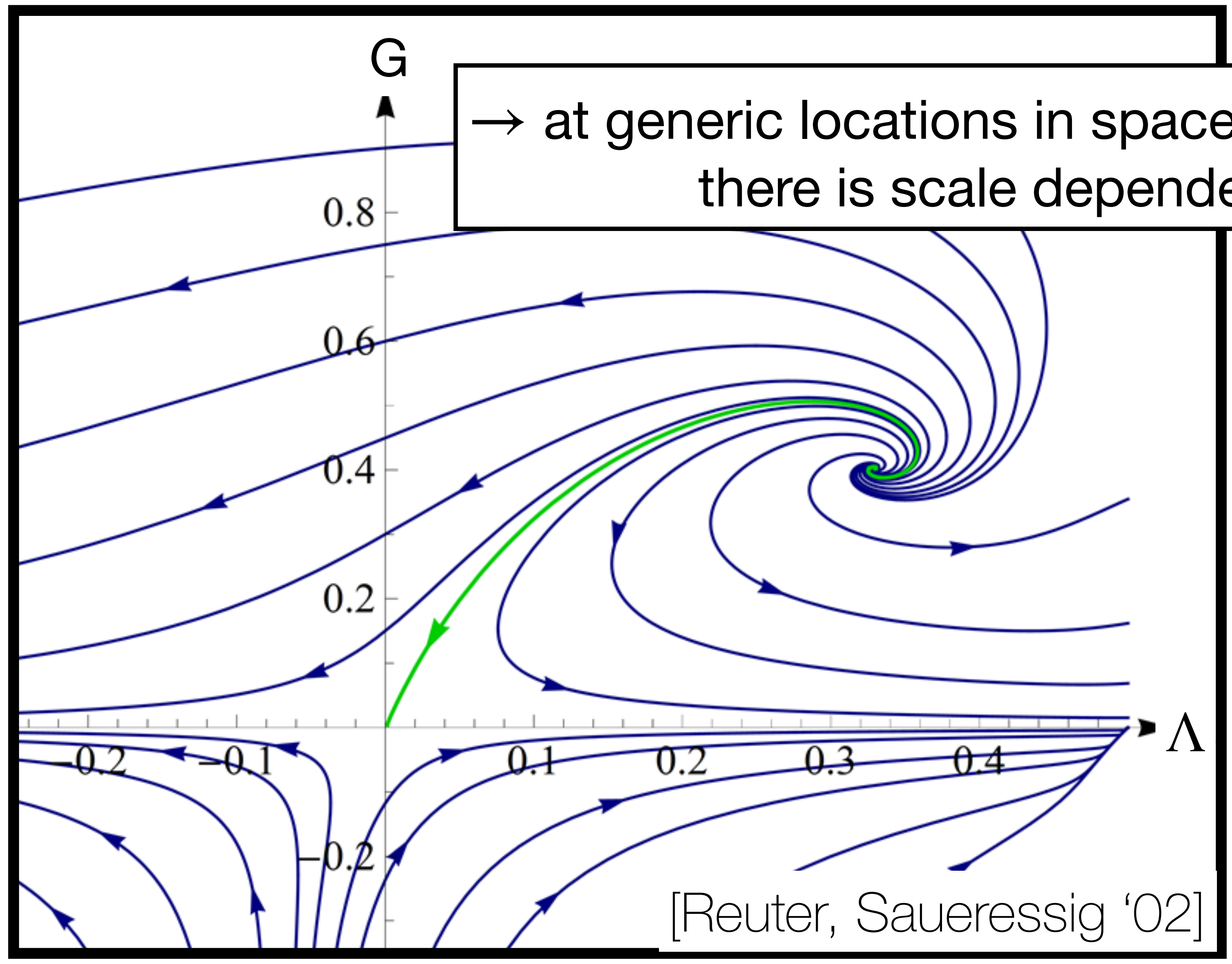
→ system of coupled algebraic equations

→ generically, besides $g_i = 0 \,\forall i$ (free fixed point, which is guaranteed to exist), at best a finite number of real zeros

Example: $\Gamma_k = \int d^4x \sqrt{g} \sum_{n=0}^{8} g_n R^n$

n	$\tilde{\Lambda}_*$	$\tilde{G}_*$	$\tilde{\Lambda}_*\tilde{G}_*$	$10^3\times$								
				$\tilde{g}_{0*}$	$\tilde{g}_{1*}$	$\tilde{g}_{2*}$	$\tilde{g}_{3*}$	$\tilde{g}_{4*}$	$\tilde{g}_{5*}$	$\tilde{g}_{6*}$	$\tilde{g}_{7*}$	$\tilde{g}_{8*}$
1	0.1297	0.9878	0.1282	5.226	-20.140							
2	0.1294	1.5633	0.2022	3.292	-12.726	1.514						
3	0.1323	1.0152	0.1343	5.184	-19.596	0.702	-9.682					
4	0.1229	0.9664	0.1188	5.059	-20.585	0.270	-10.967	-8.646				
5	0.1235	0.9686	0.1196	5.071	-20.538	0.269	-9.687	-8.034	-3.349			
6	0.1216	0.9583	0.1166	5.051	-20.760	0.141	-10.198	-9.567	-3.590	2.460		
7	0.1202	0.9488	0.1141	5.042	-20.969	-0.034	-9.784	-10.521	-6.048	3.421	5.905	
8	0.1221	0.9589	0.1171	5.066	-20.748	0.088	-8.581	-8.926	-6.808	1.165	6.196	4.695

[Codello, Percacci, Rahmede '08]



→ at generic locations in space of couplings, there is scale dependence

[Reuter, Saueressig '02]

Predictivity at all scales: IR

Sources and sinks of the Renormalization Group flow

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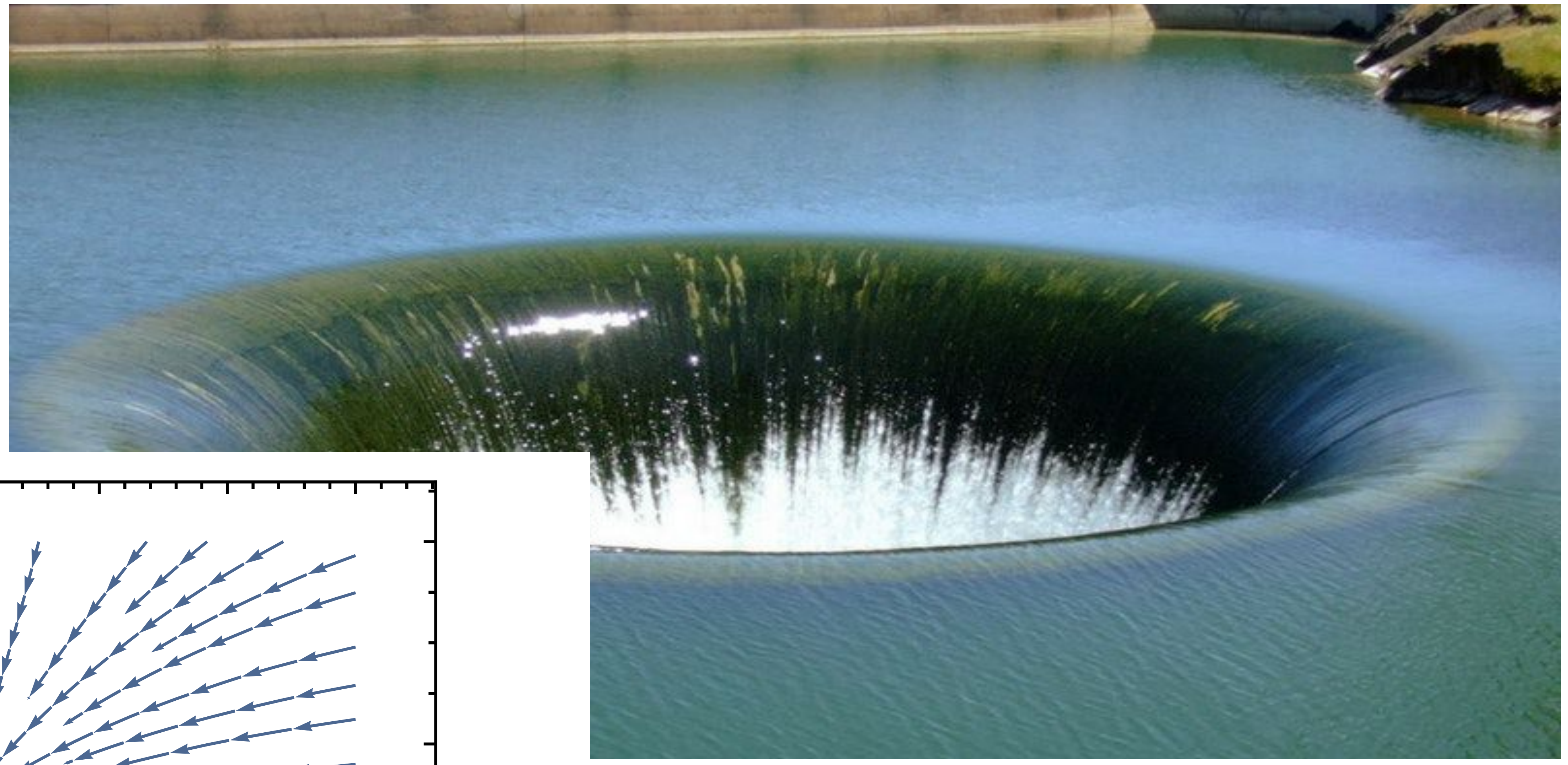
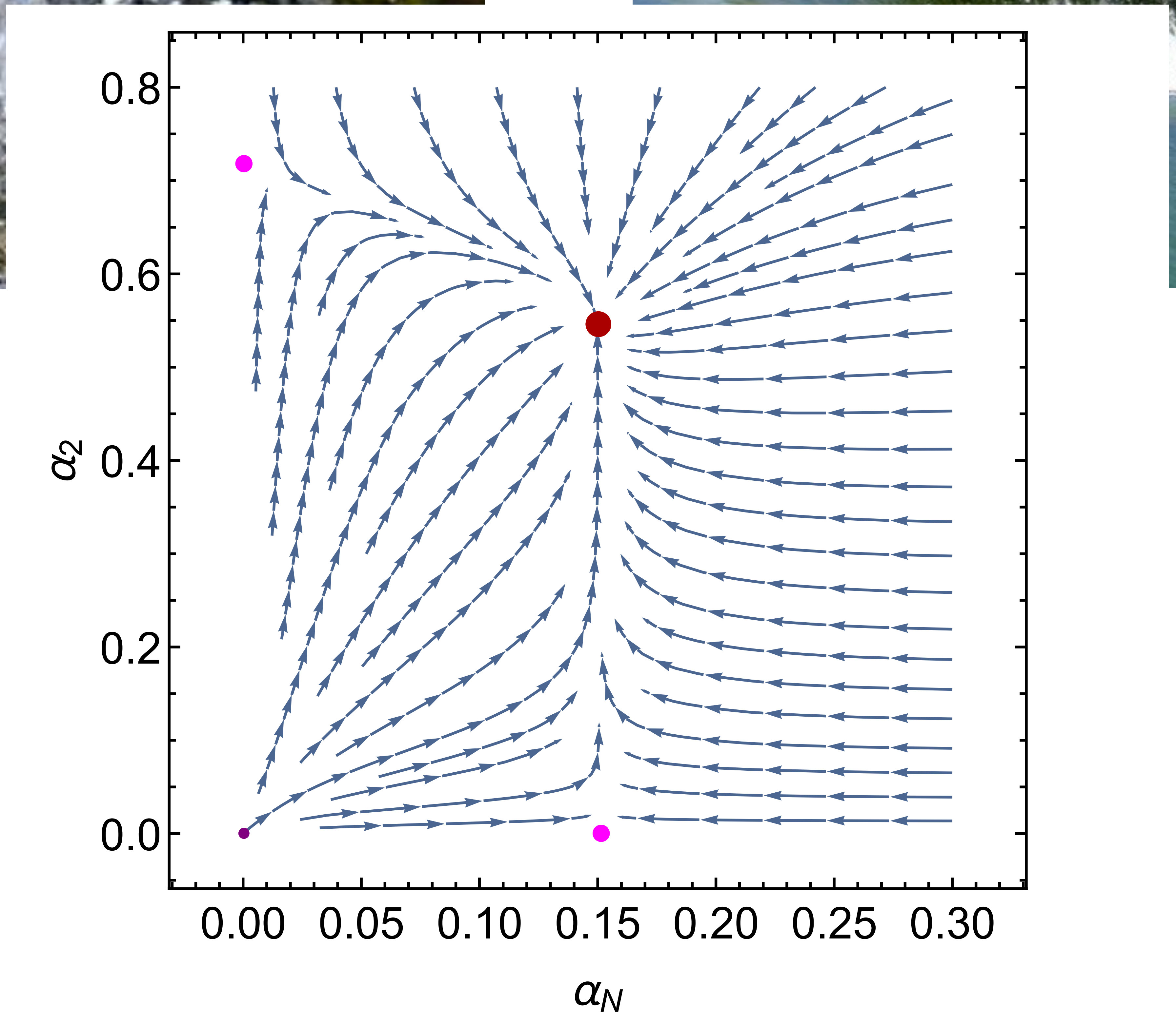




Predictivity at all scales: IR

Quantum fluctuations screen or antiscreen interactions

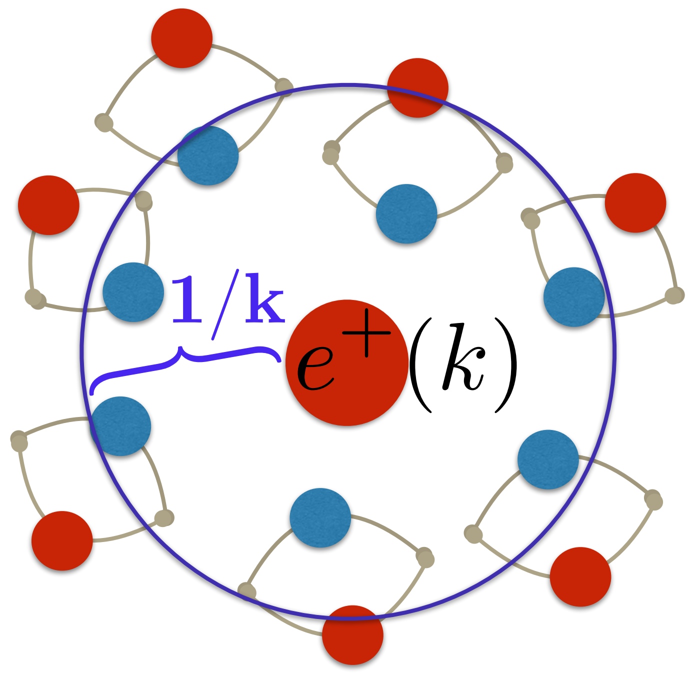
Standard Model examples:

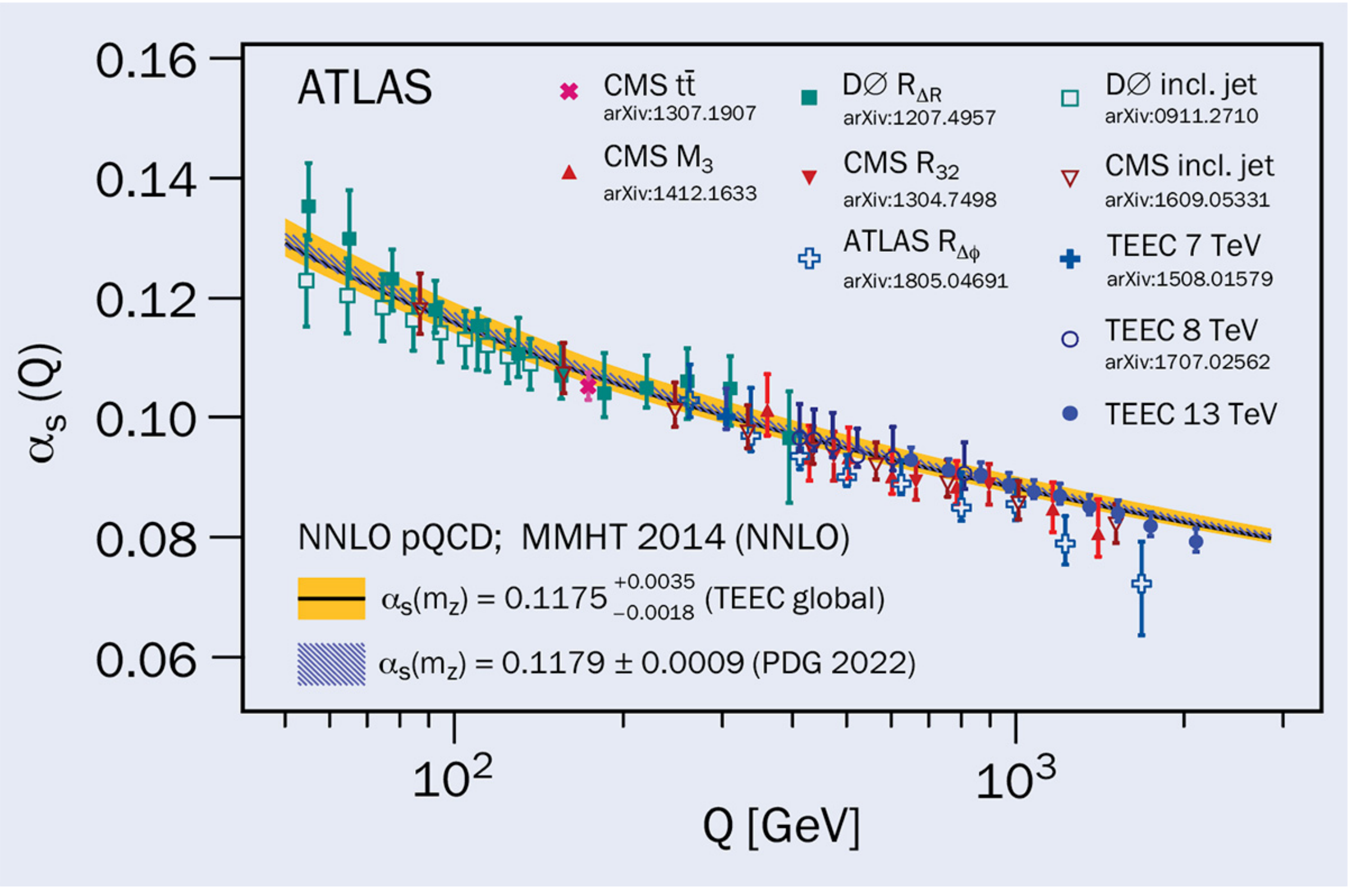
QED: $\beta_e = k\,\partial_k\, e(k) = \frac{1}{12\pi^2}e^3 + \dots$

→ $e(k)$ decreases as k is lowered

QCD: $\beta_g = k\,\partial_k\, g(k) = -\frac{7}{16\pi^2}g^3 + \dots$

→ $g(k)$ increases as k is lowered





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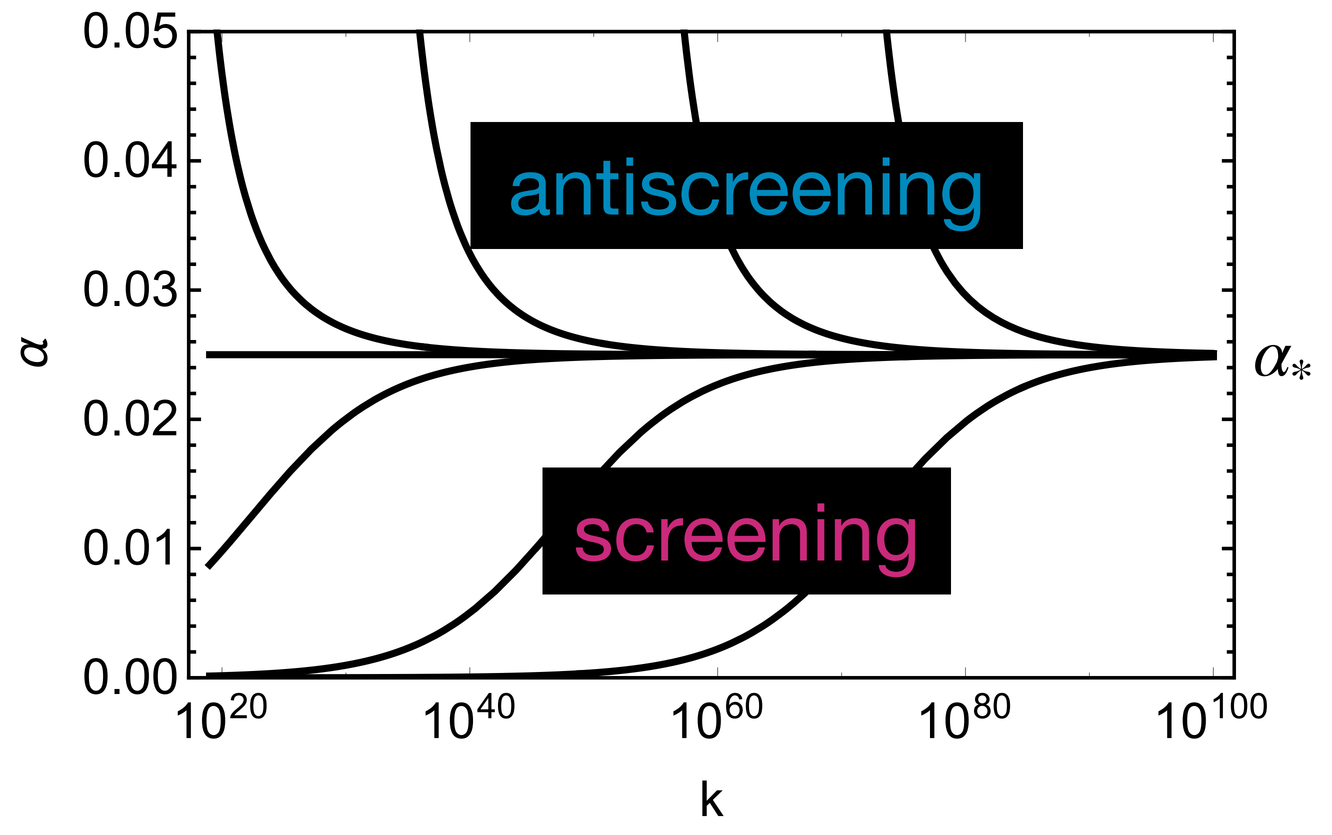
relevant couplings



$\beta_\alpha = \alpha\,(\alpha_* - \alpha)$

quantum fluctuations drive coupling **away from** scale symmetry

→ a range of coupling values achievable at the Planck scale

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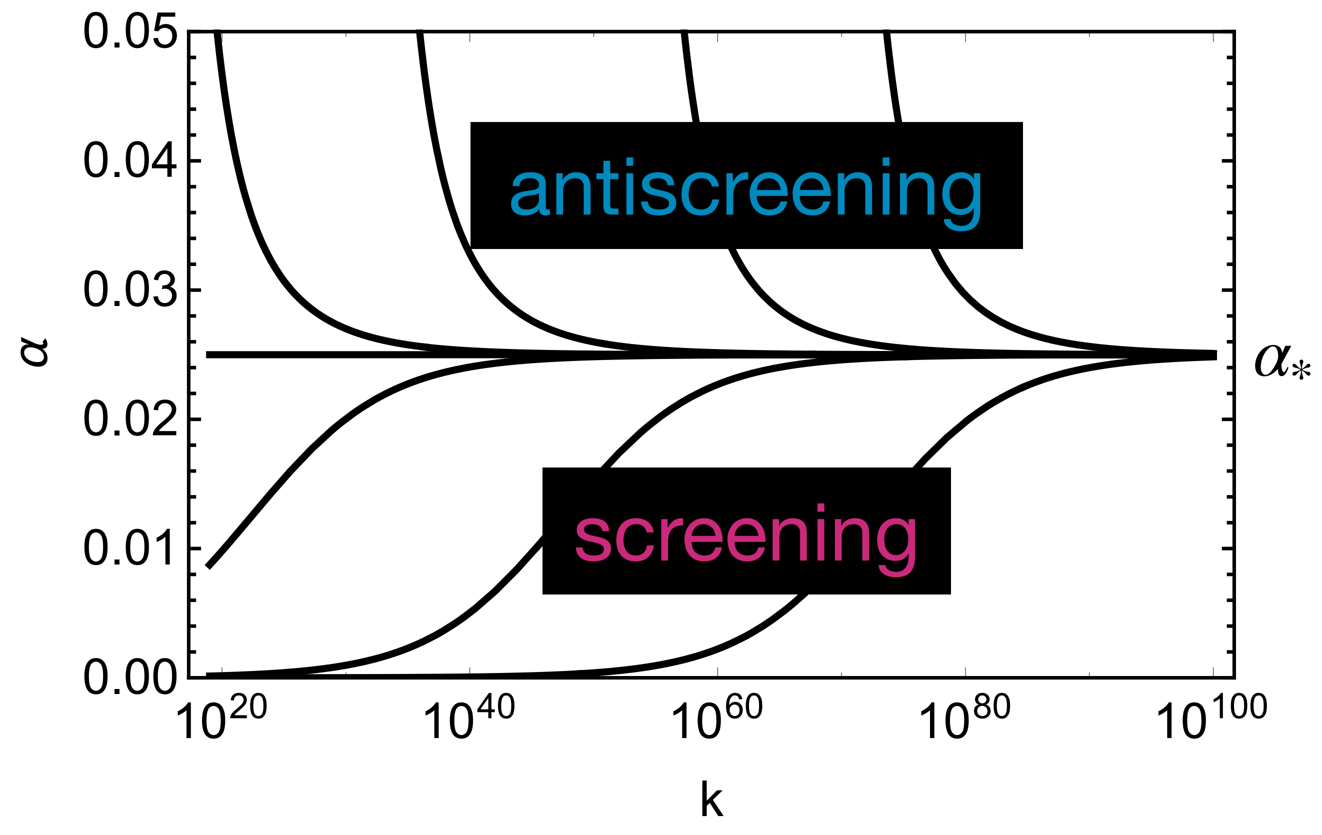

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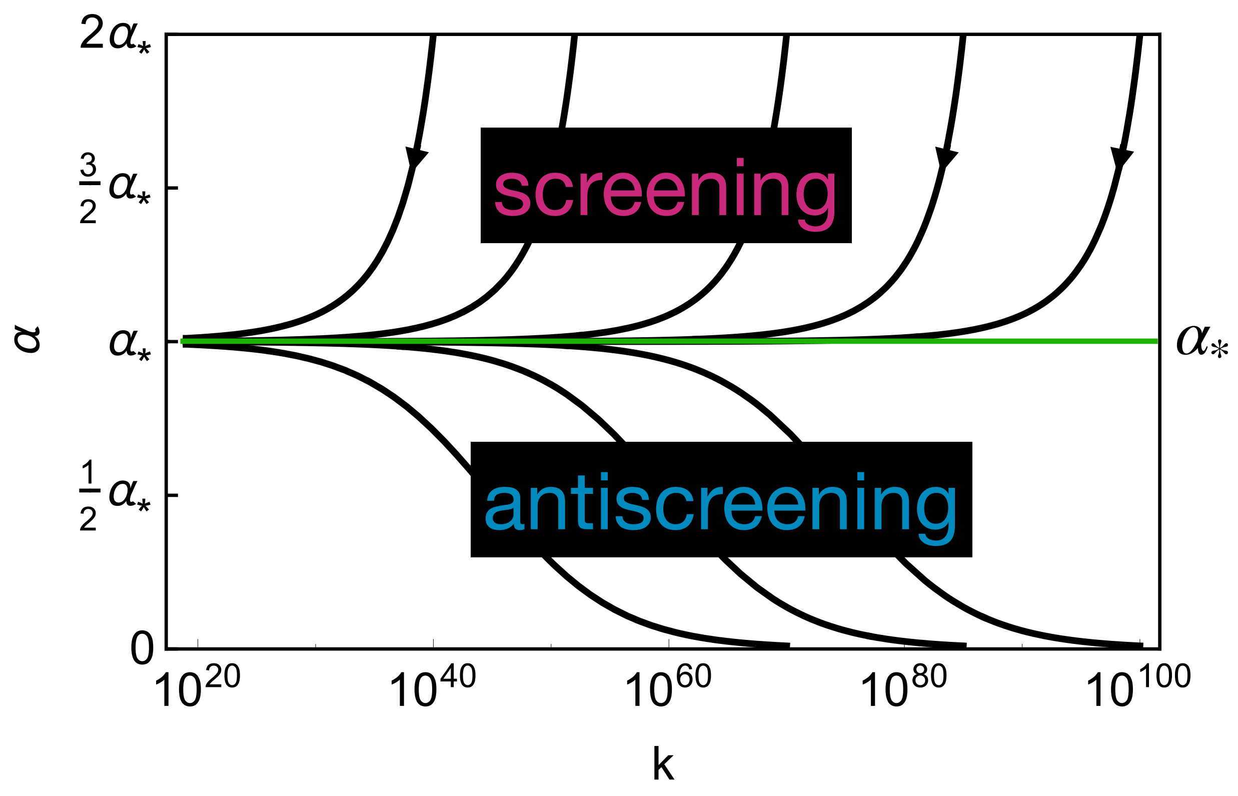
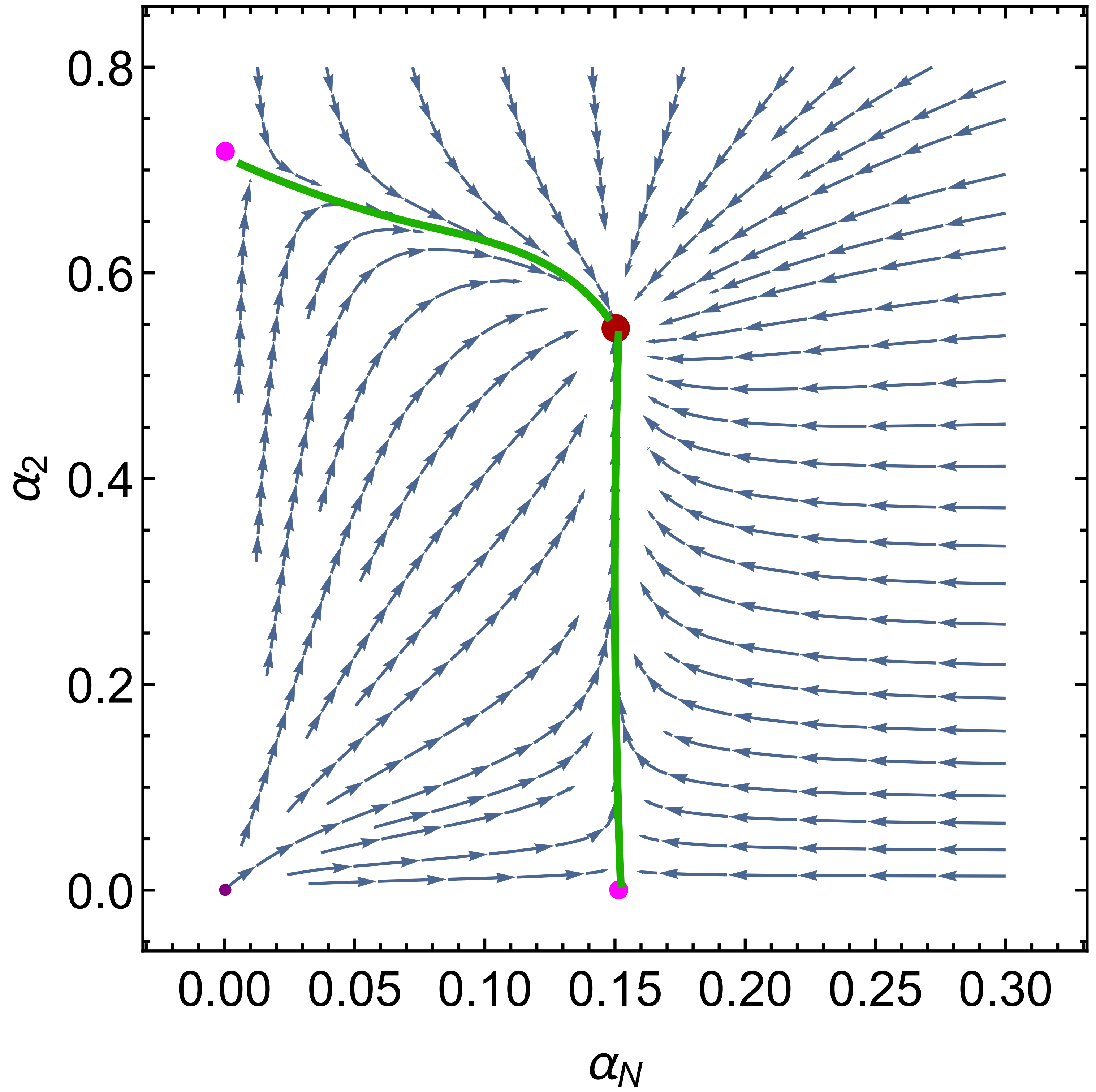


$\beta_\alpha = \alpha\,(-\alpha_* + \alpha)$

quantum fluctuations drive coupling **towards** scale symmetry

→ a unique coupling value achievable at the Planck scale

Predictivity at all scales: IR

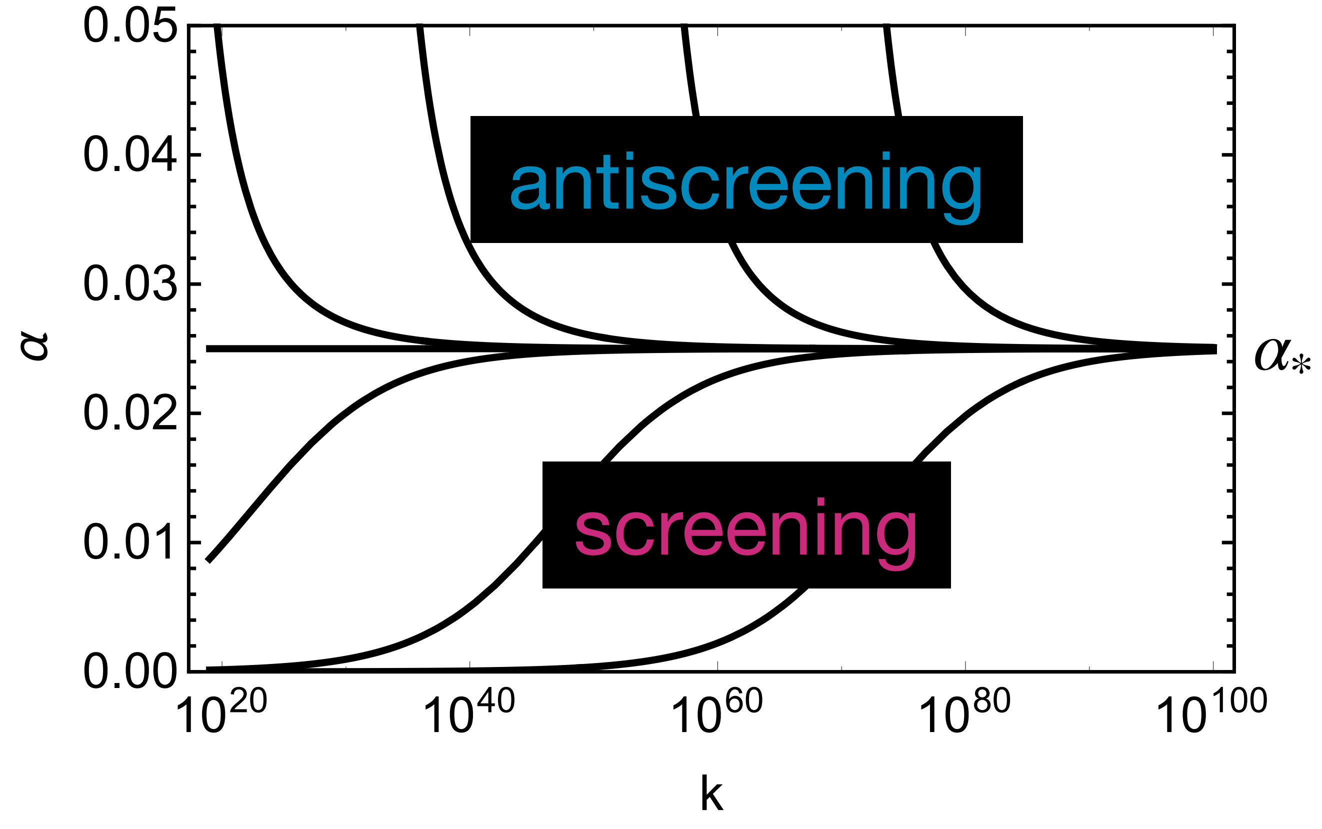


→ free fixed point: two relevant directions, α_2 and α_N are both free parameters

→ partially interacting fixed points: one relevant direction, α_2 and α_N become functions of one another

→ fully interacting fixed point: no relevant directions, no free parameters

relevant couplings



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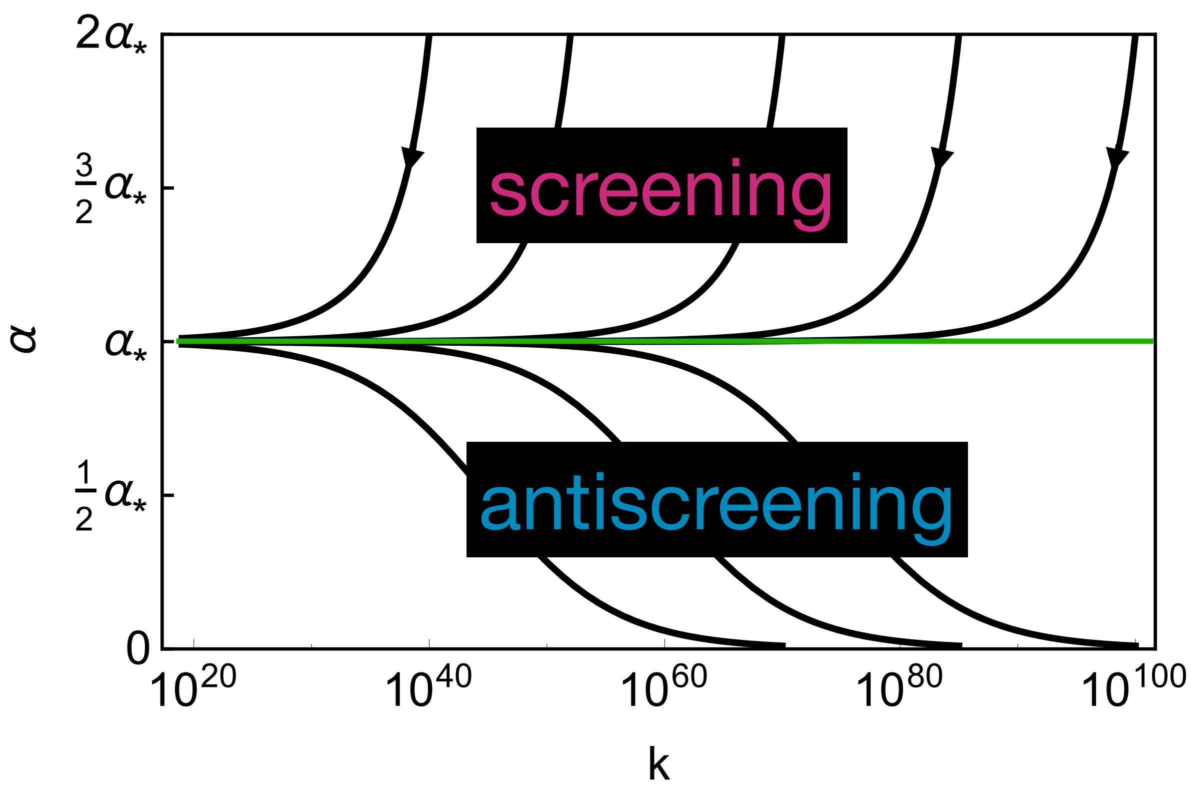

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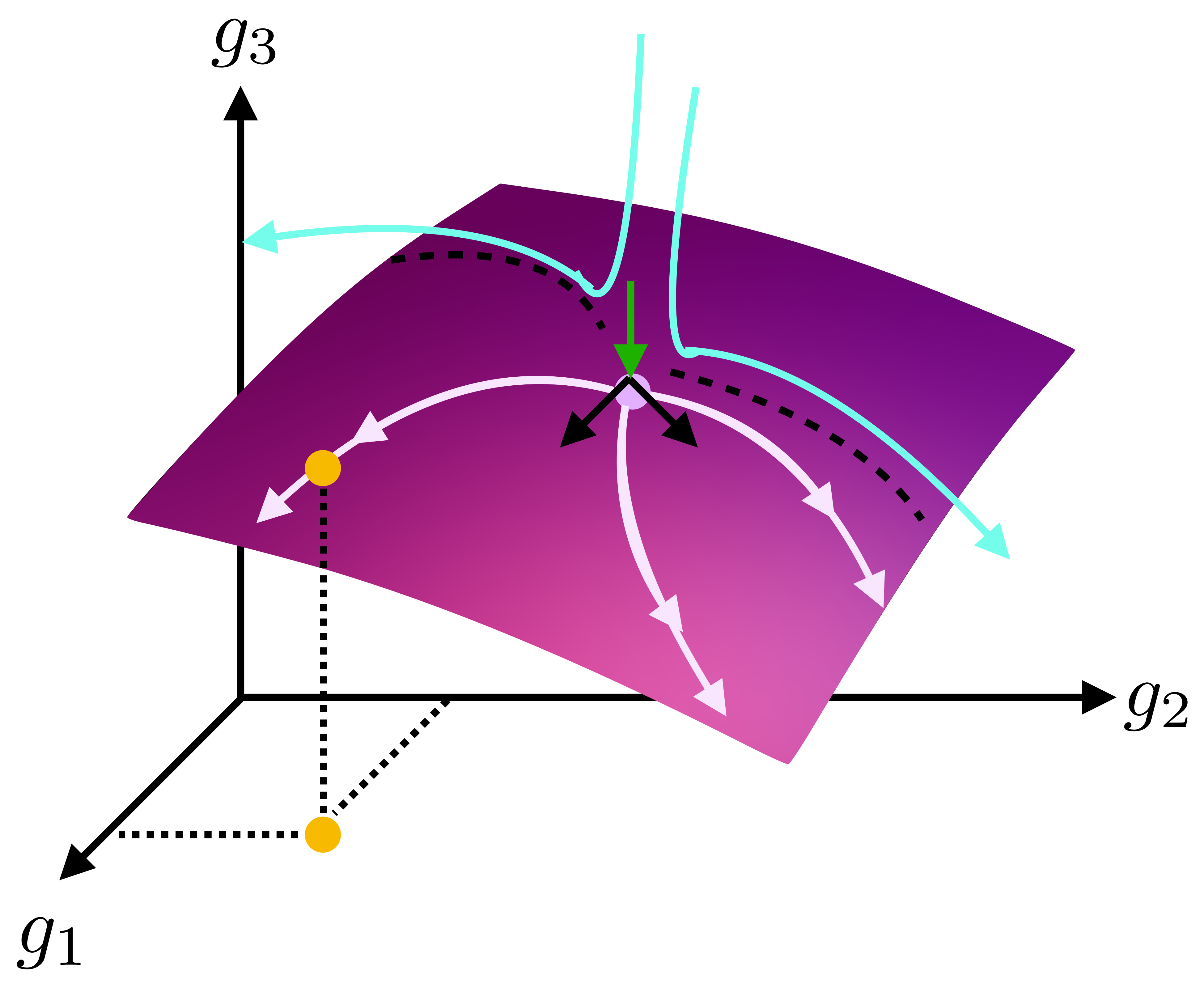
Counting free parameters: Critical exponents



Linearize beta functions

$$\beta_{g_i} = \beta_{g_i}(\vec{g} = \vec{g_*}) + \sum_j \frac{\partial \beta_{g_i}}{\partial g_j}\Big|_{\vec{g}=\vec{g_*}} \left(g_j - g_{j*}\right) + \mathcal{O}\left(g_j - g_{j*}\right)^2$$

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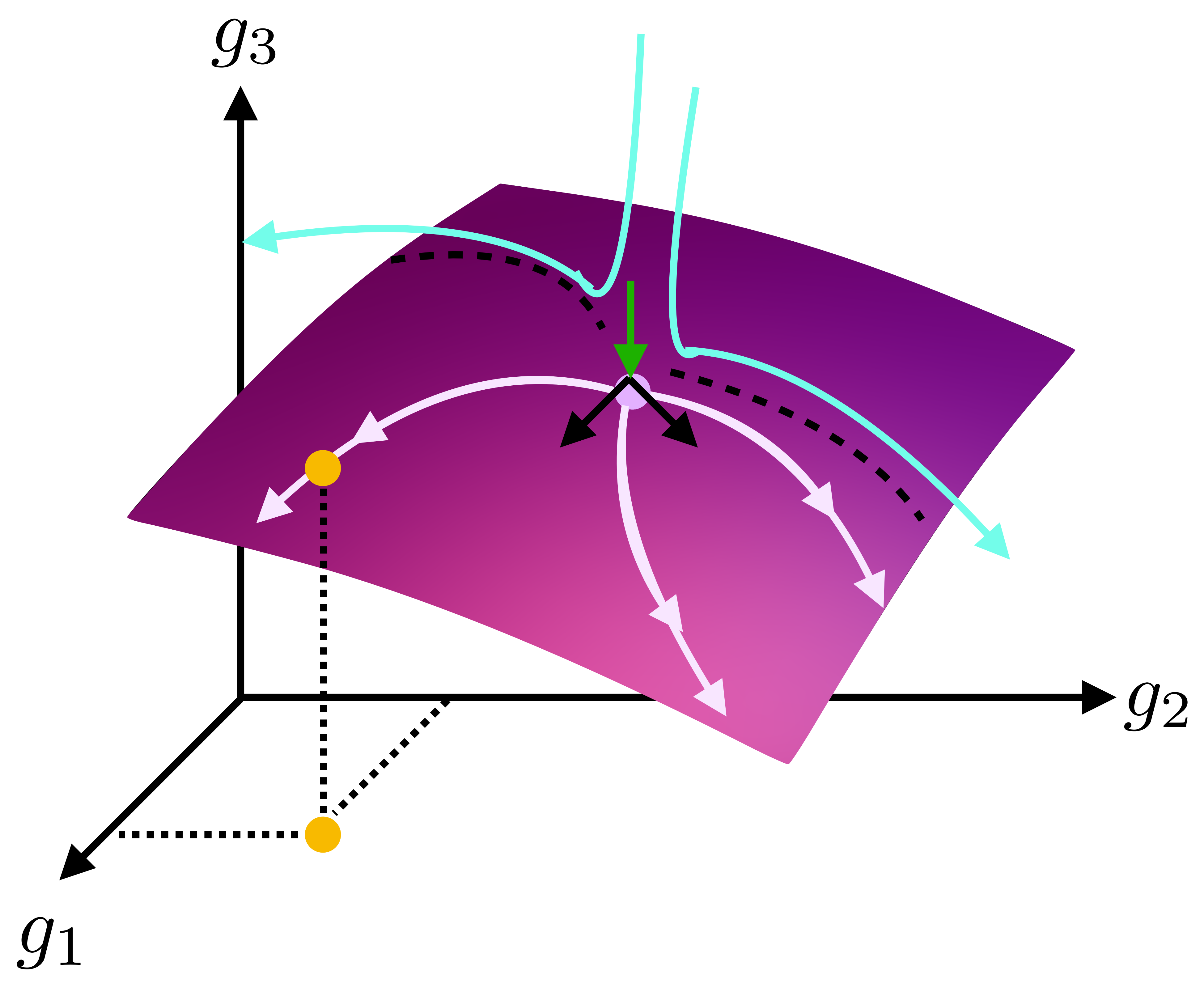

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solution: $g_i(k) = g_{i*} + \sum_I C_I V_i^I \left(\frac{k}{k_0}\right)^{-\theta_I}$ with $\theta_I = -\,\mathrm{eig}\left(\frac{\partial \beta_{g_i}}{\partial g_j}\right)$

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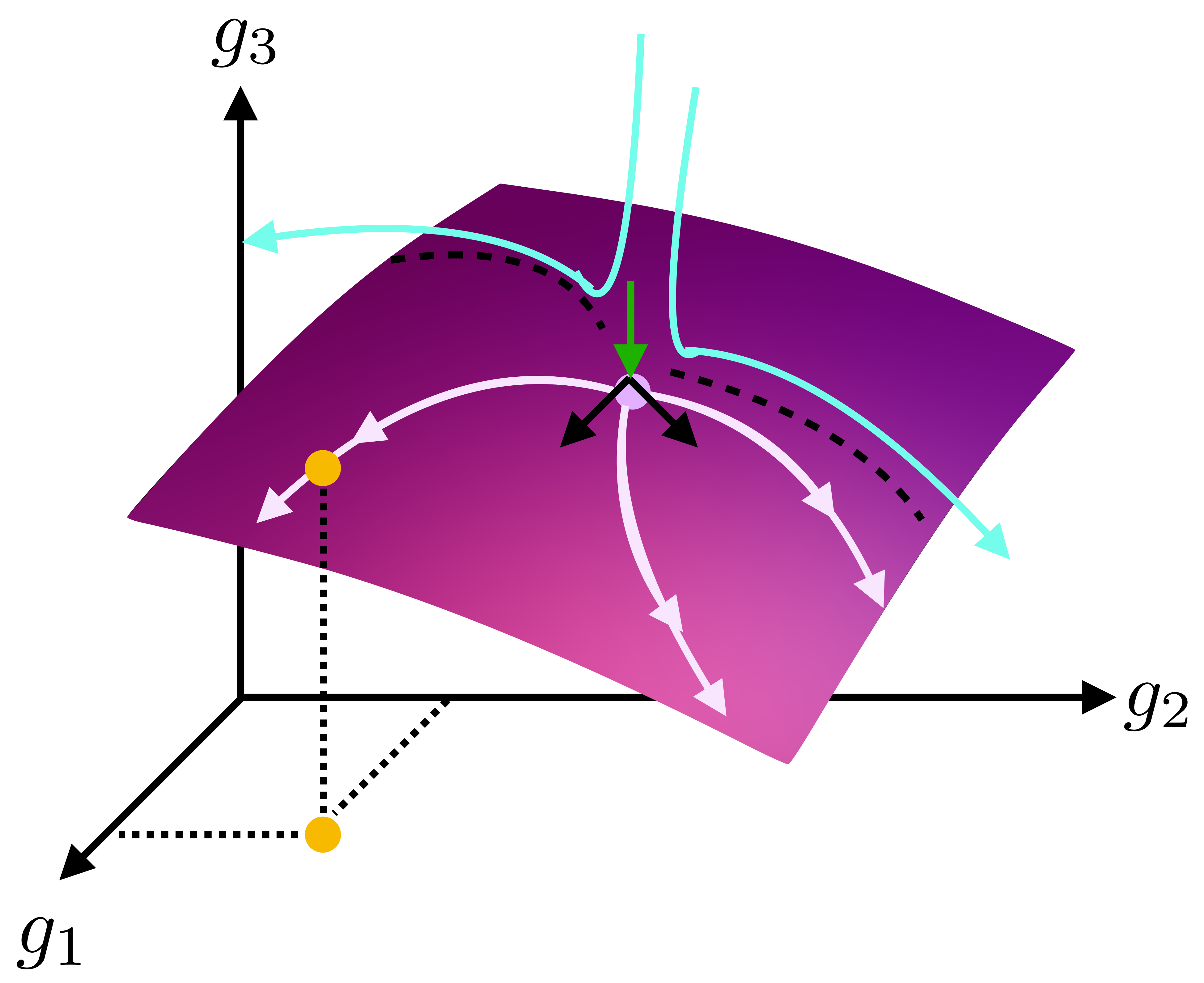

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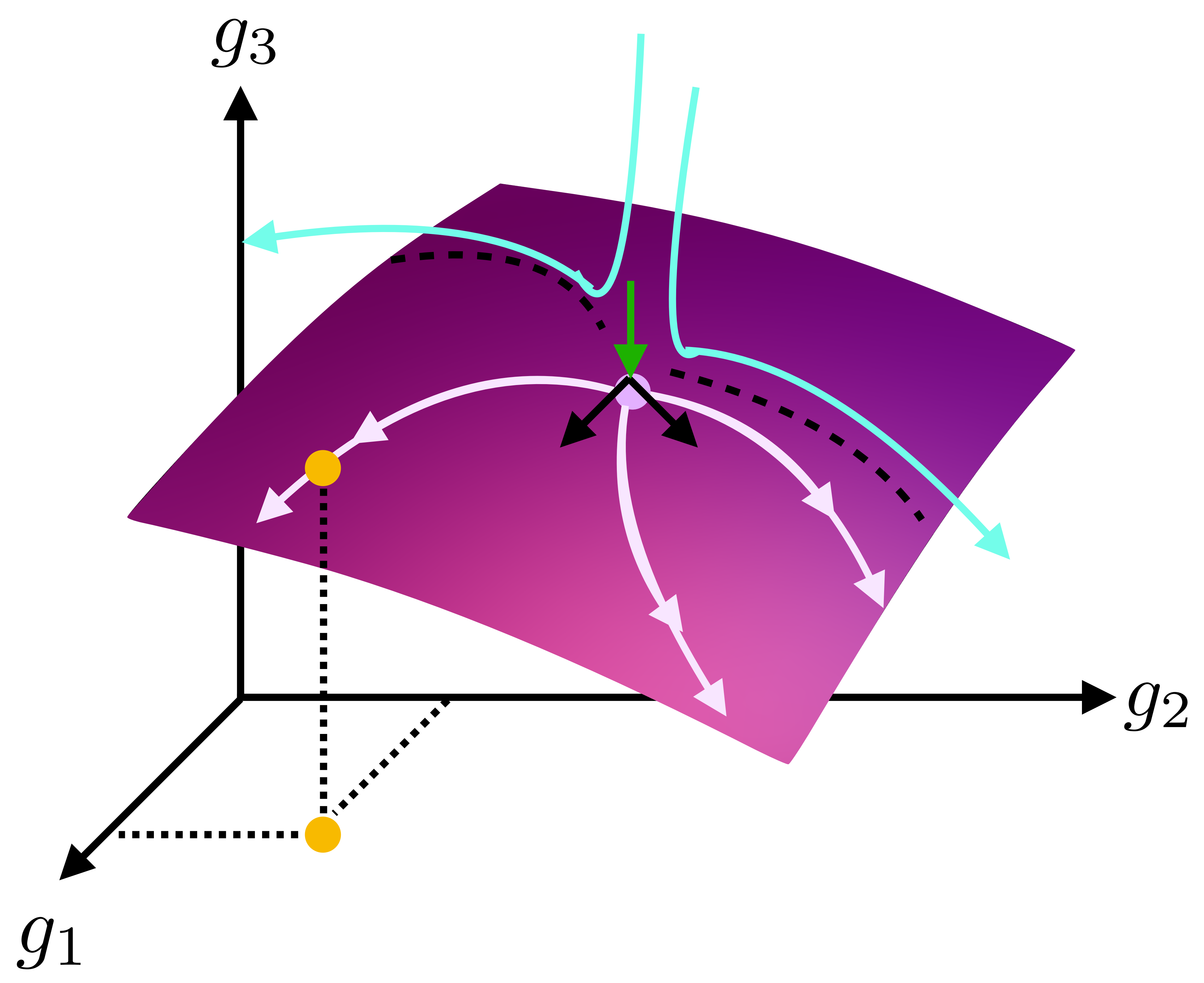

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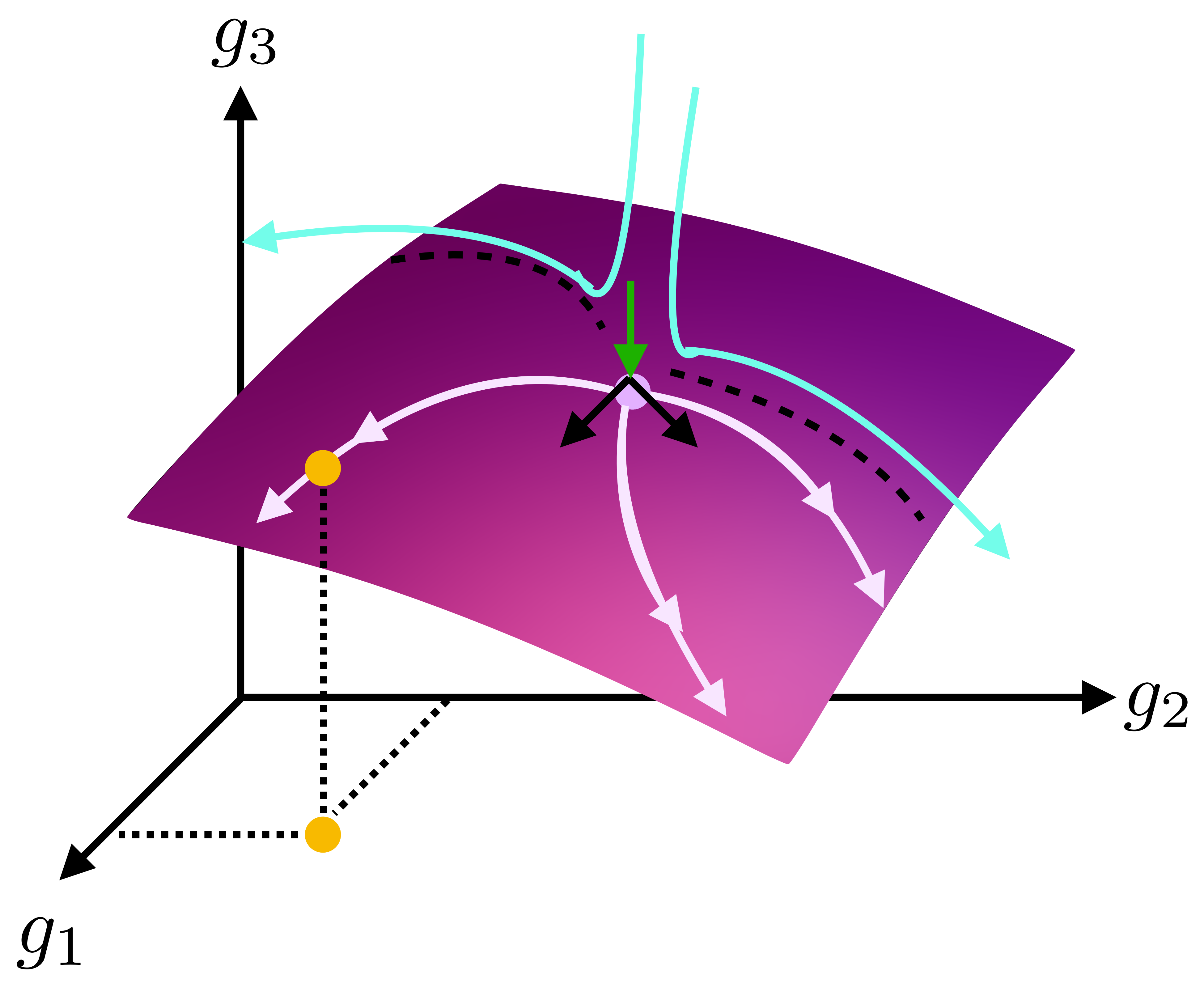

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(Note: beta functions not universal, but critical exponents are)

Counting free parameters at free and interacting fixed points

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Counting free parameters at free and interacting fixed points

Linearize beta functions

$$\beta_{g_i} = \beta_{g_i}(\overrightarrow{g} = \overrightarrow{g_*}) + \sum_j \frac{\partial \beta_{g_i}}{\partial g_j}\Big|_{\overrightarrow{g}=\overrightarrow{g_*}} \left(g_j - g_{j*}\right) + \mathcal{O}\left(g_j - g_{j*}\right)^2$$

solution: $g_i(k) = g_{i*} + \sum_I C_I V_i^I \left(\frac{k}{k_0}\right)^{-\theta_I}$ with $\theta_I = -\,\mathrm{eig}\left(\frac{\partial \beta_{g_i}}{\partial g_j}\right)$

Free fixed point:

$$\theta_i = -\frac{\partial \beta_i}{\partial g_i}\Big|_{g=0} = -\frac{\partial}{\partial g_i}\left(k\partial_k\left(\bar{g}_i\, k^{-d_{\bar{g}_i}}\right)\right)\Big|_{g=0} = d_{\bar{g}_i},$$

couplings with positive mass dimension are relevant

couplings with vanishing mass dimension are marginally (ir)relevant

couplings with negative mass dimension are irrelevant

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- can be more or less predictive than the free fixed point
- no way to know free parameters a priori

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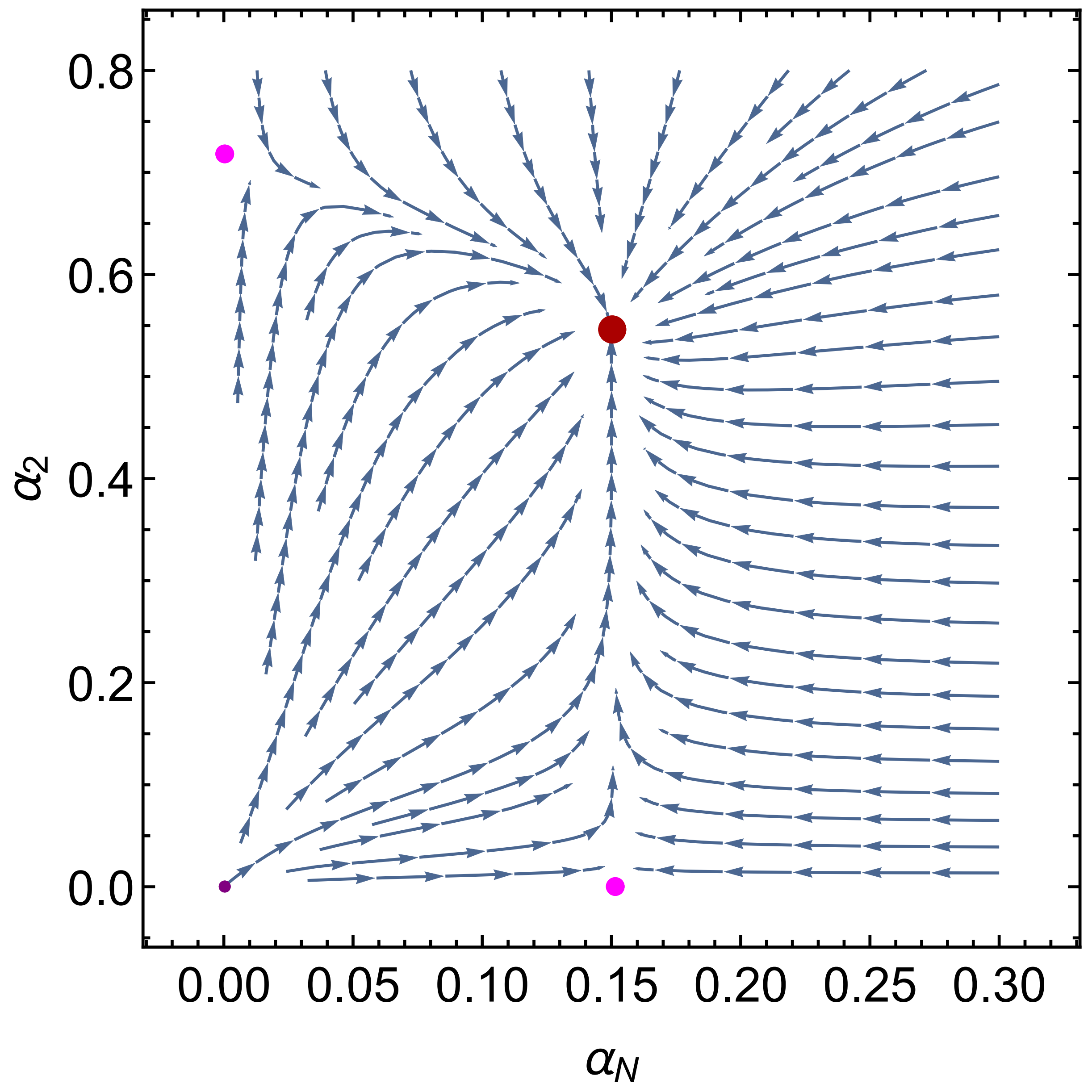

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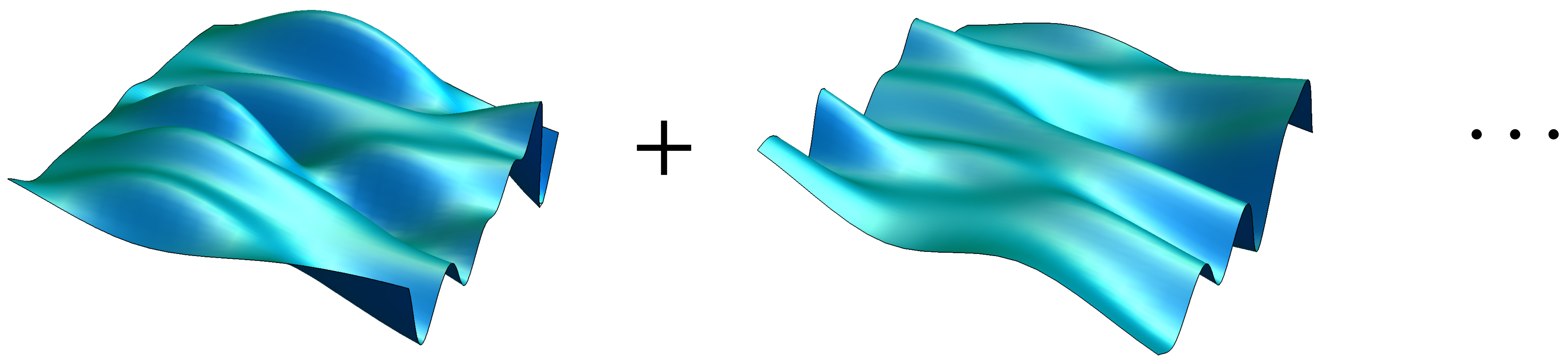
Asymptotic safety in gravity - key concepts

$$Z = \int \mathcal{D} g_{\mu\nu} e^{i\, S[g_{\mu\nu}]}$$



$$S[g_{\mu\nu}] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g}\, \left(R - 2\bar{\Lambda}\right) + \dots$$

G_N : interactions of gravity with matter
& nonlinear gravitational self-interactions

quantum effects: $G_N \to G_N(k)$,
similarly for higher-order interactions

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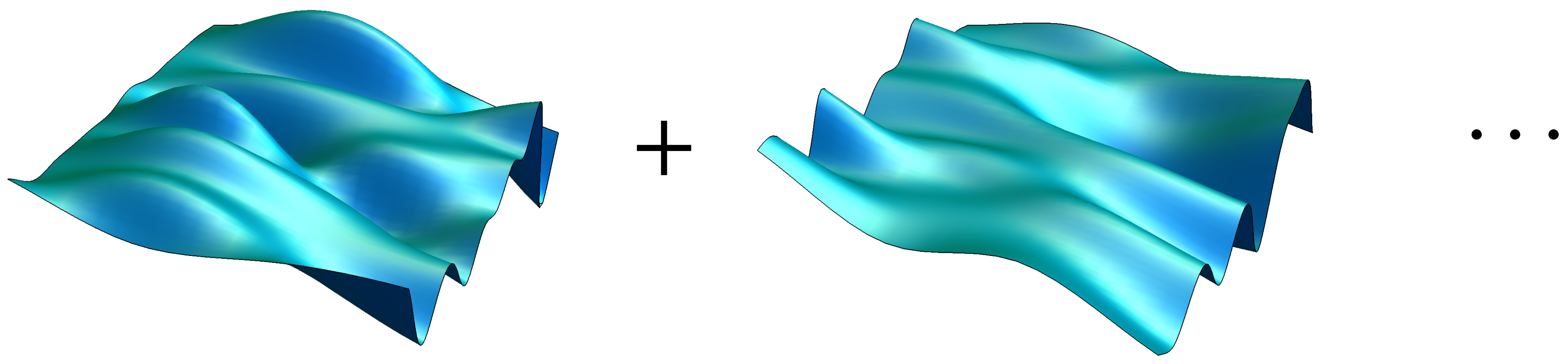

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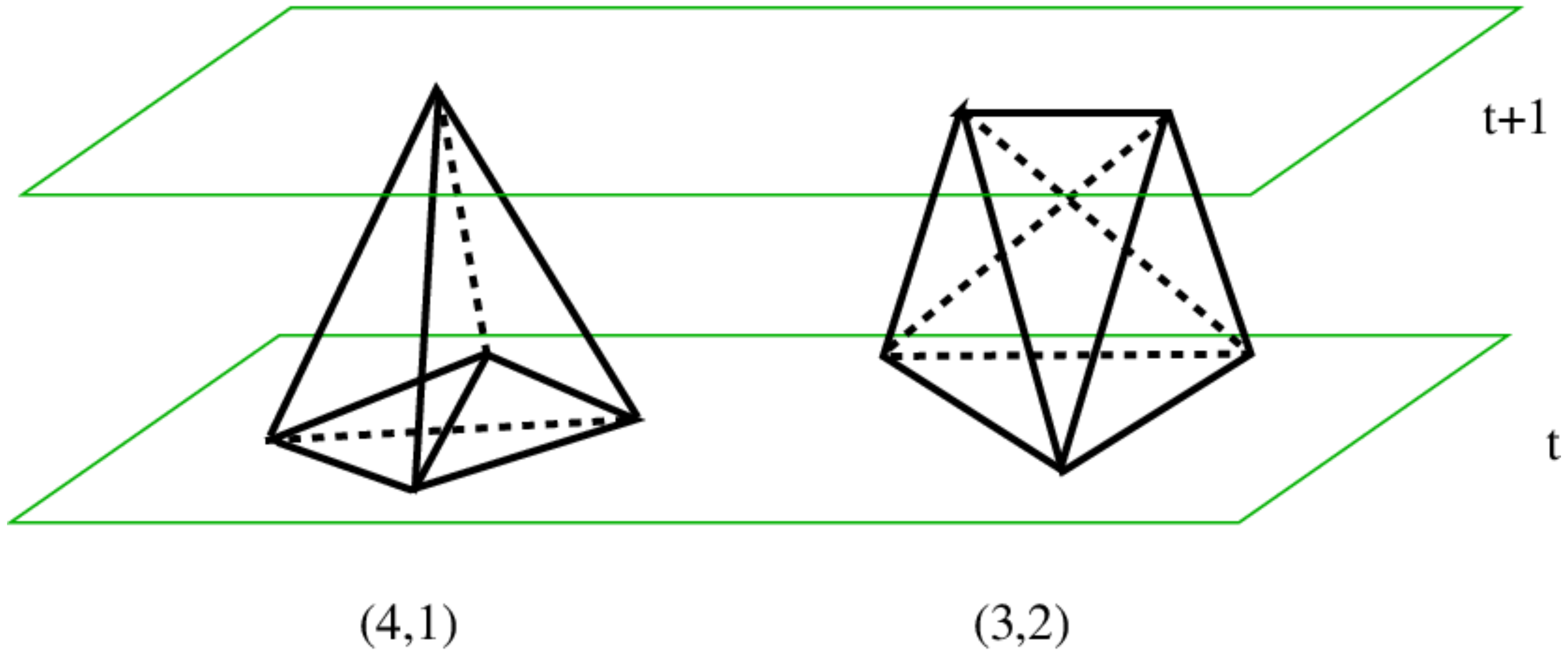
similarly for higher-order interactions

Most conservative approach to quantum gravity:

- metric carries gravitational degrees of freedom
  $\to$ works at low energies, so only give up if shown to fail
- Standard quantum field theory framework for quantization
  $\to$ works for the other fundamental forces, so only choose different framework for gravity if the standard framework fails

Tools to search for asymptotic safety: Lattice

Example: Causal Dynamical Triangulations



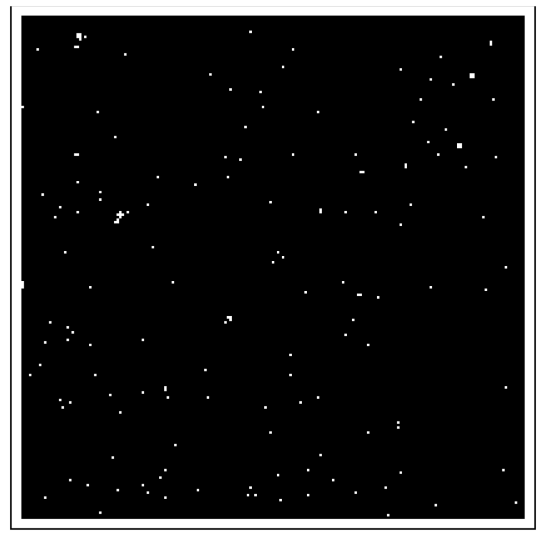
Discrete building blocks

Asymptotic safety: can take continuum limit, because at fixed-point values of the couplings, there is (quantum) scale symmetry

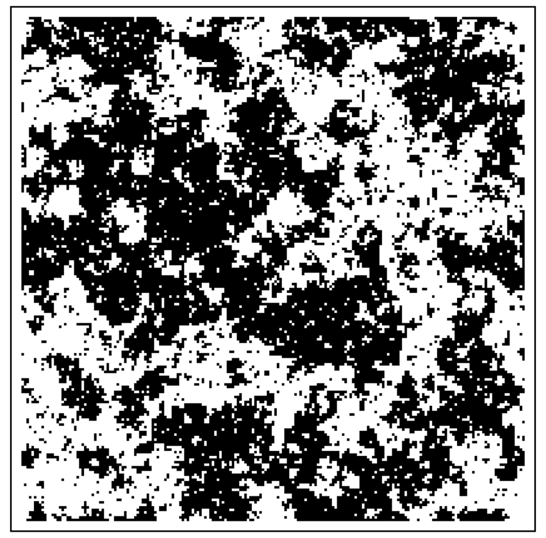
Scale symmetry in lattice theories: second order phase transition

→ interacting RG fixed points: ubiquitous in statistical physics

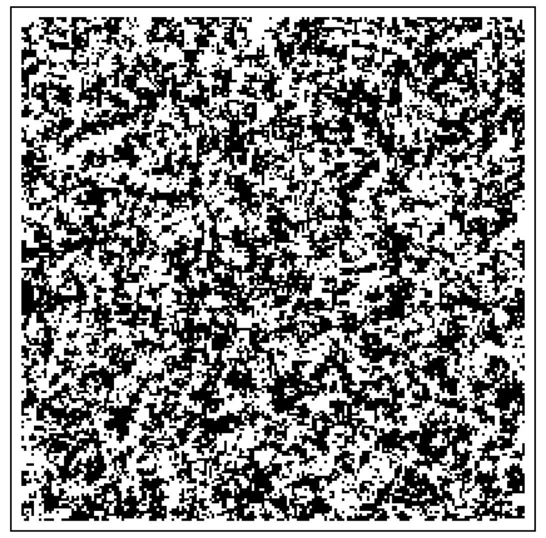
[Example: Ising model]



$T < T_c$



$T \sim T_c$



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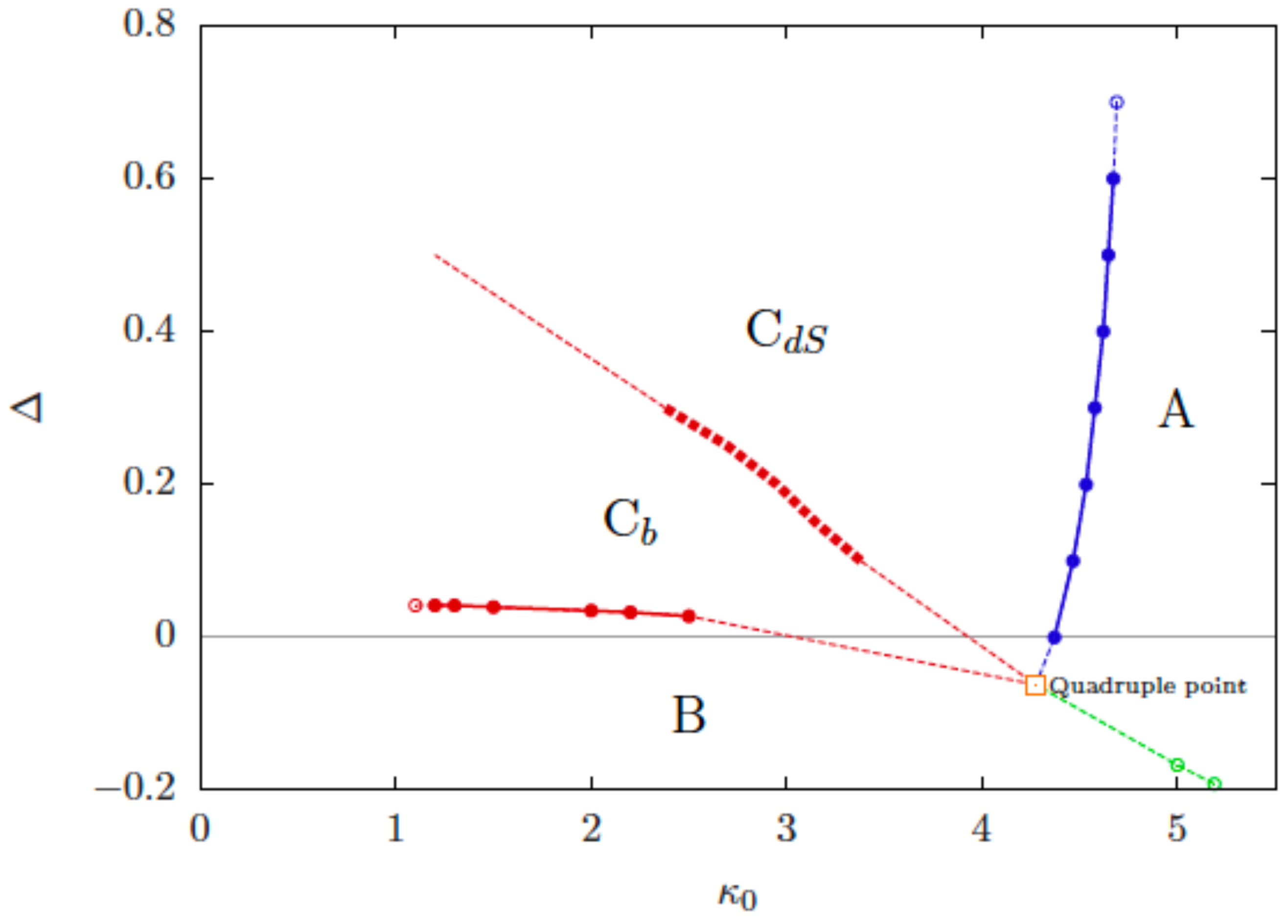


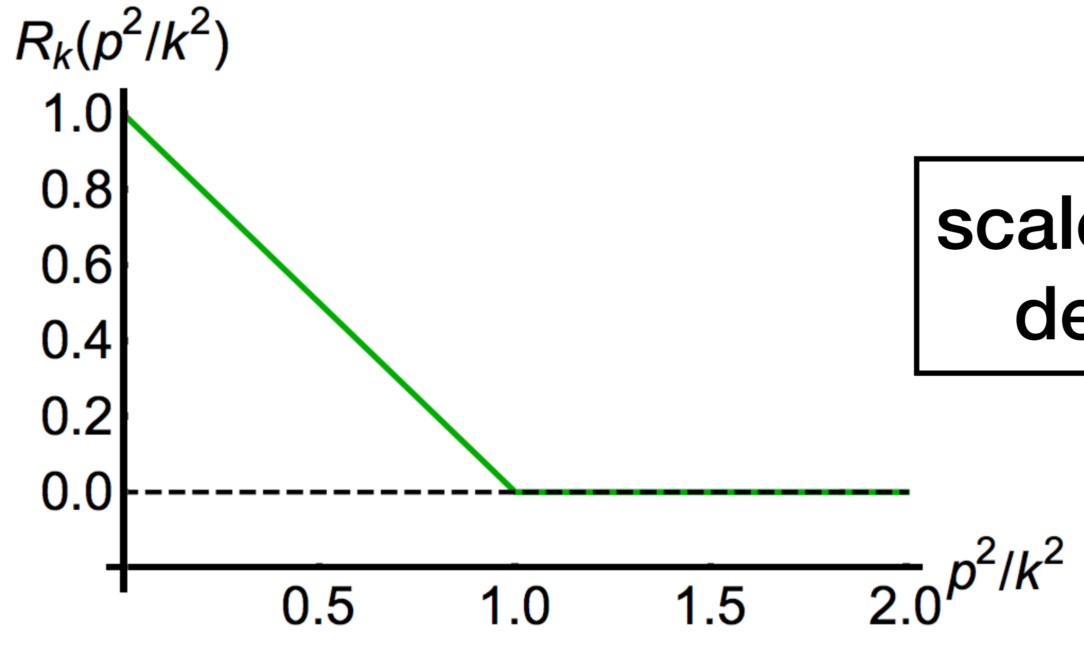
[Ambjørn, Coumbe, Gizbert-Studnicki, Görlich,Jurkiewicz ’17]

Tools to search for asymptotic safety: Functional Renormalization Group

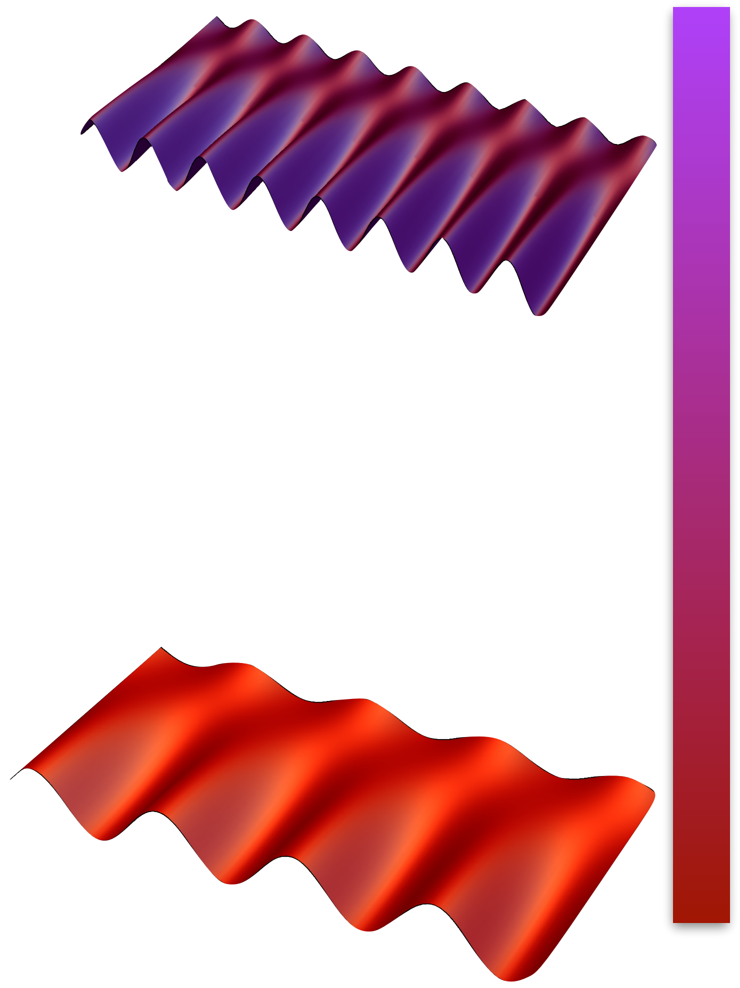
Tools to search for asymptotic safety: Functional Renormalization Group

probe scale dependence of QFT

$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \frac{1}{2}\int \varphi(-p) R_k(p) \varphi(p)}$$



scale- and momentum-dependent ``mass”



S

Γ_k contains effect of quantum fluctuations above k

$\Gamma_{k\to 0}$

$$\Gamma_k = \sum_i g_i(k) \int d^d x \, \mathcal{O}^i$$

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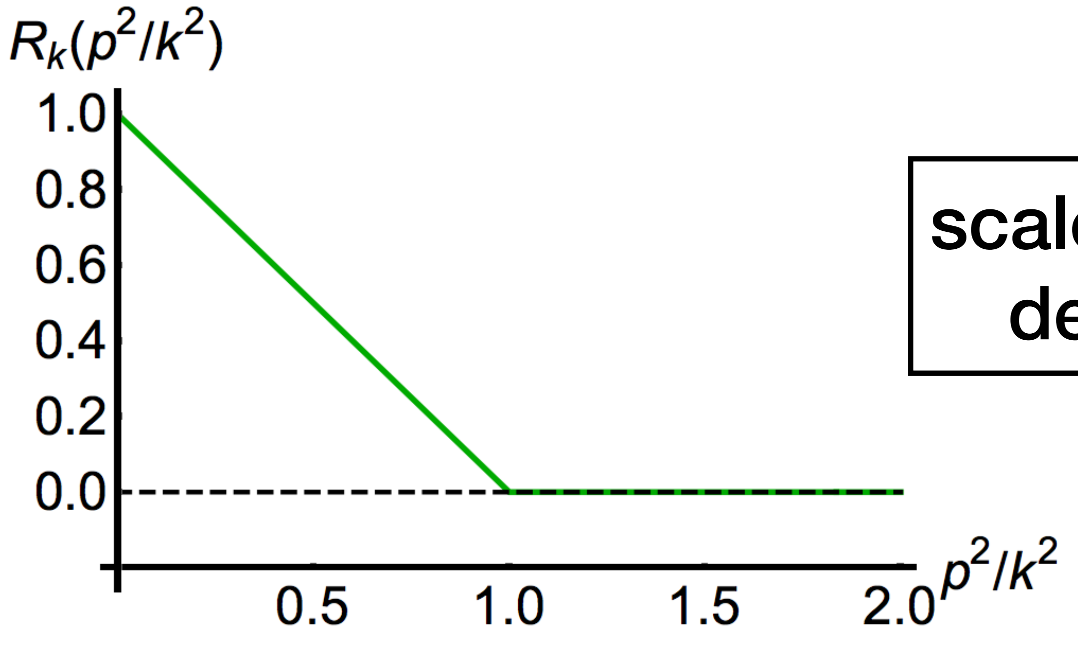

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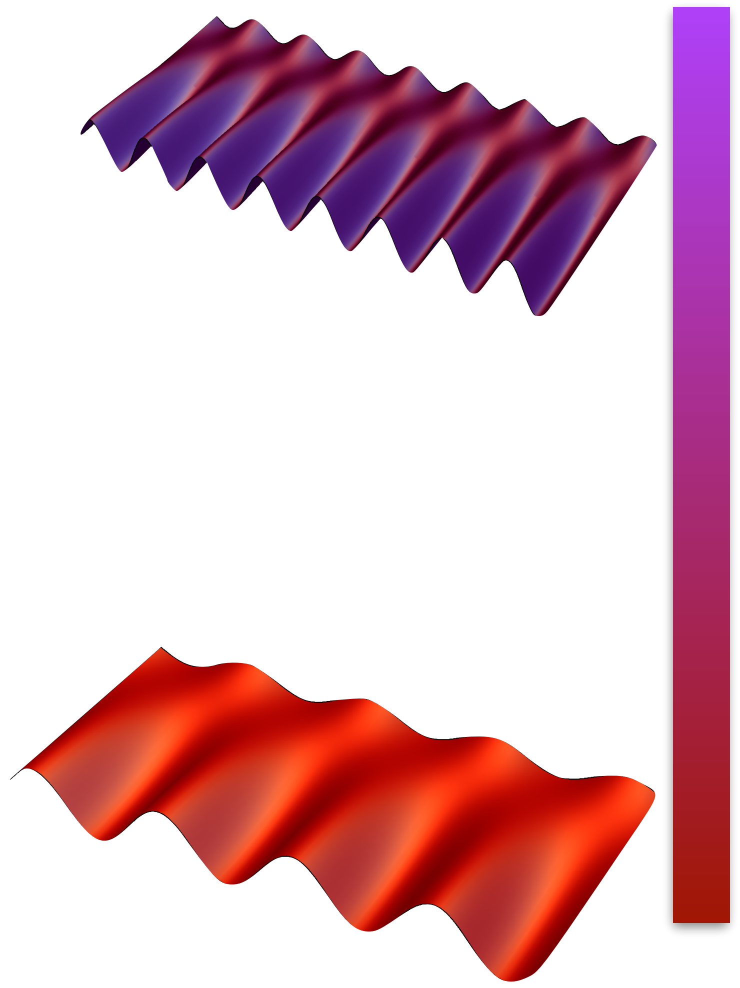

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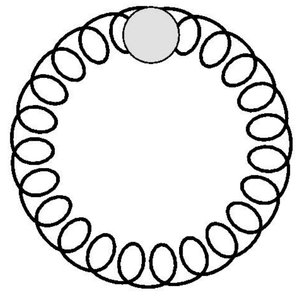
$$\Gamma_k = \sum_i g_i(k) \int d^dx\, \mathcal{O}^i \quad \to k\partial_k \Gamma_k = \sum_i \beta_{g_i} \int d^dx \mathcal{O}^i$$

Wetterich equation: $\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k =$ 

Wetterich '93, Reuter '96

Tools to search for asymptotic safety: Functional Renormalization Group

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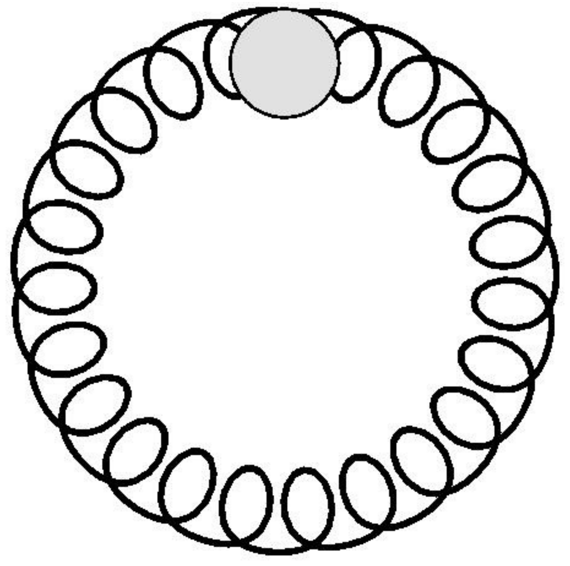

exact one-loop equation

$$\Gamma_k = \sum_i g_i(k) \int d^d x\, \mathcal{O}_i \longrightarrow \text{(infinite) tower of coupled equations } \beta_{g_i}$$

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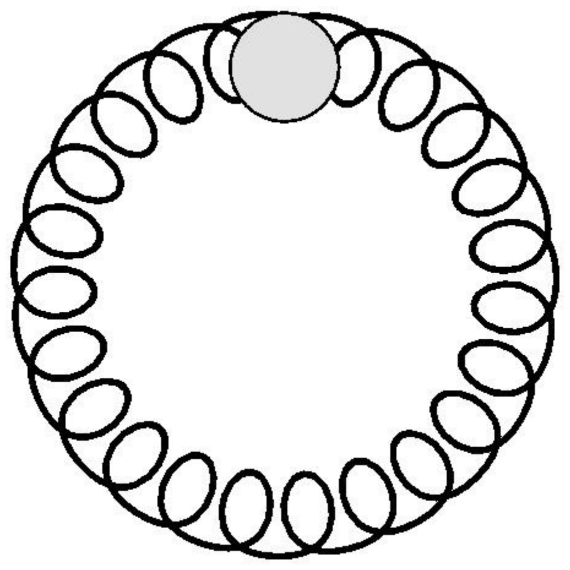

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strategy

- truncate to (finite) set of equations
- search for fixed point solutions
- enlarge truncation
- convergent results?

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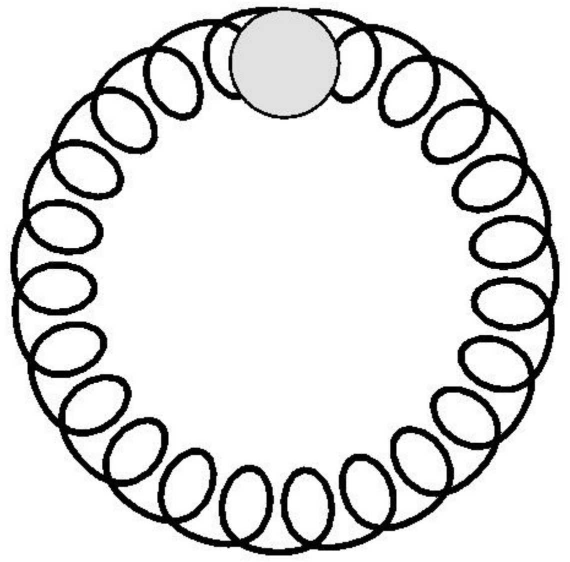

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successfully used in particle physics, statistical physics/condensed matter,
e.g., Quantum Chromodynamics, BEC-BCS-crossover,
strongly-correlated fermion systems,
Wilson-Fisher universality classes & beyond, ….. [see Dupuis, Canet, AE, et al. ‘20]

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example: Ising universality class

scaling exponent of the correlation length: $\xi \sim |T - T_c|^{-\nu}$

$\nu = 0.62999(5)$	conformal bootstrap	[Showk et al. ’14]
$\nu = 0.63002(10)$	Monte Carlo	[Hasenbusch ’10]
$\nu = 0.6304$	ε-expansion to seven loops	[Guida, Zinn-Justin ’98]
$\nu = 0.643$	FRG: LPA	[Berges, Tetradis, Wetterich ‘00]
$\nu = 0.6307$	FRG: $\mathcal{O}(\partial^2)$	[Canet, Delamotte, Mouhanna, Vidal ‘03]
$\nu = 0.63007(10)$	FRG: $\mathcal{O}(\partial^6)$	[Balog et al. ‘19]

Functional Renormalization Group in gravity

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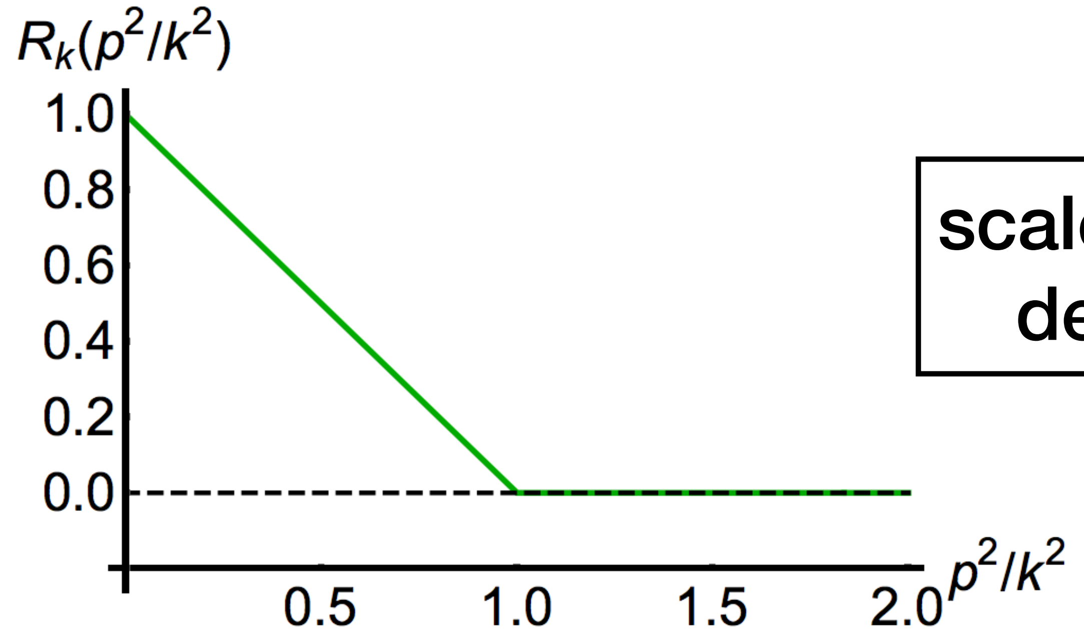

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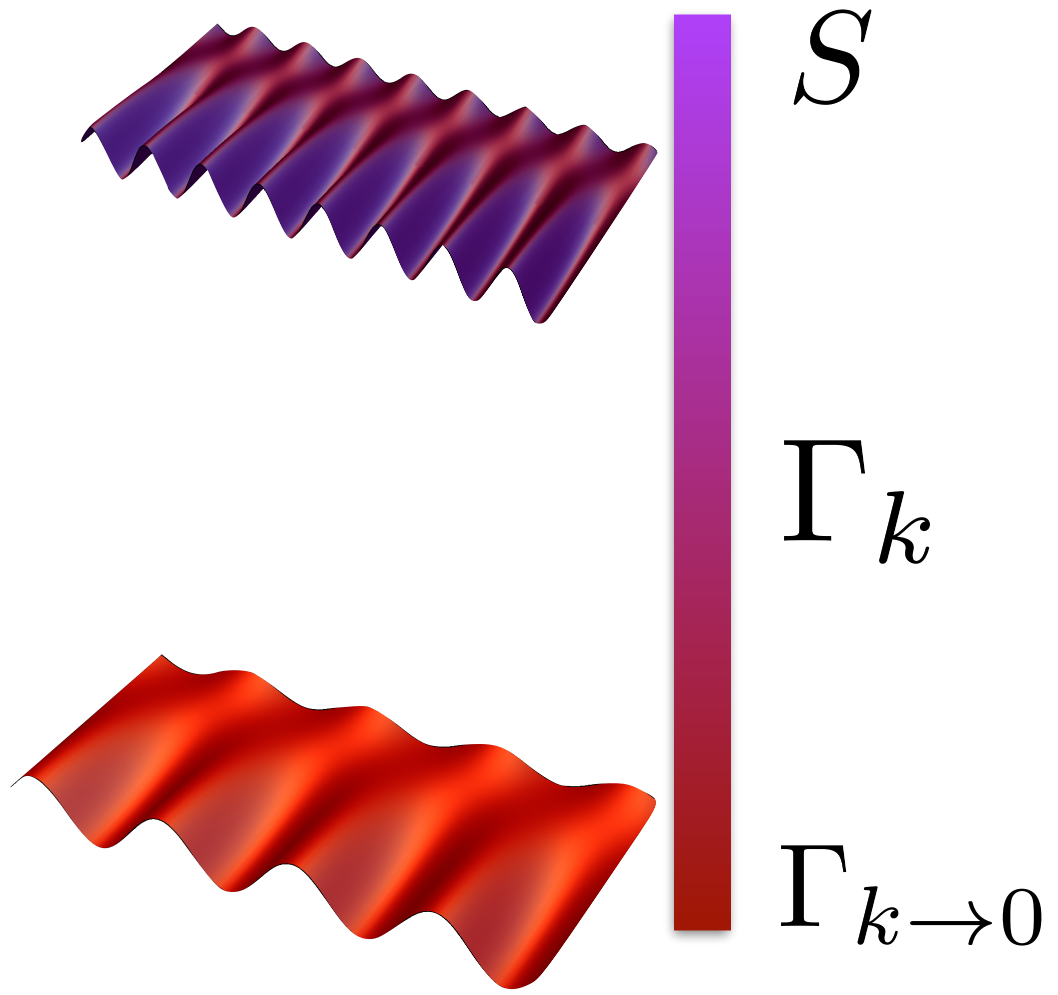


How to distinguish long and short ``wavelength”?

→ metric

But metrics are summed over in the path integral?

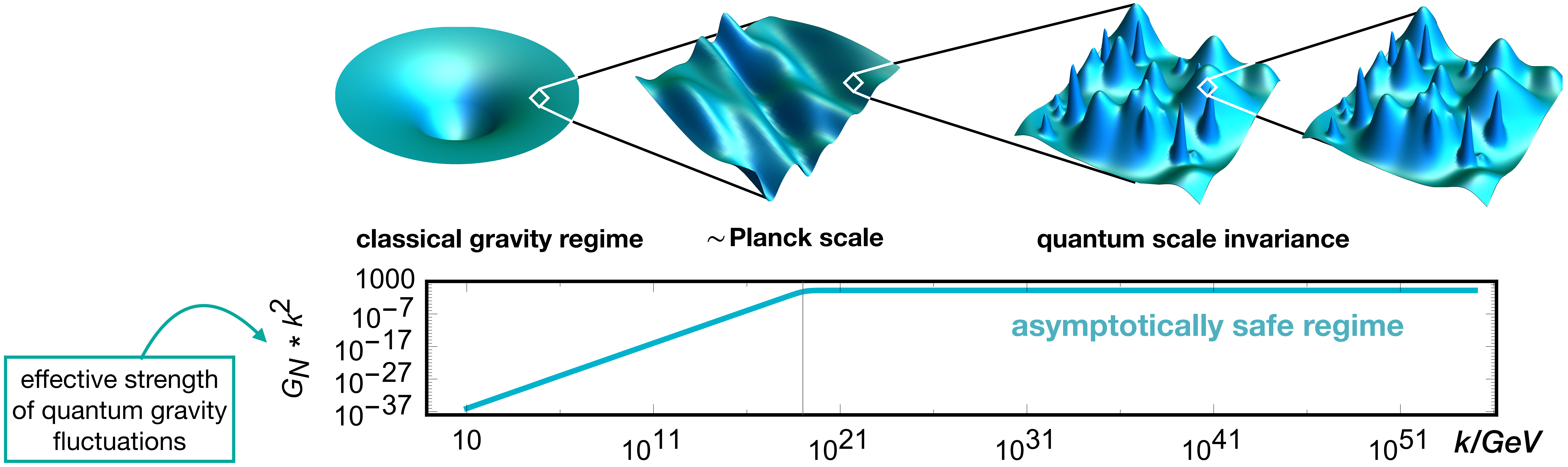
→ auxiliary background metric $\bar{g}_{\mu\nu}$

$$\rightarrow \quad e^{-\Gamma_k[\langle g_{\mu\nu}\rangle]} = \int \mathscr{D}h_{\mu\nu} e^{-S[g_{\mu\nu}]-\frac{1}{2}\int h_{\mu\nu} R_k(-\bar{D})^{\mu\nu\kappa\lambda} h_{\kappa\lambda}}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Asymptotic safety in gravity: Results

Asymptotic safety in quantum gravity: Newton coupling



$$\beta_G = 2G - \#\, G^2 + \ldots$$

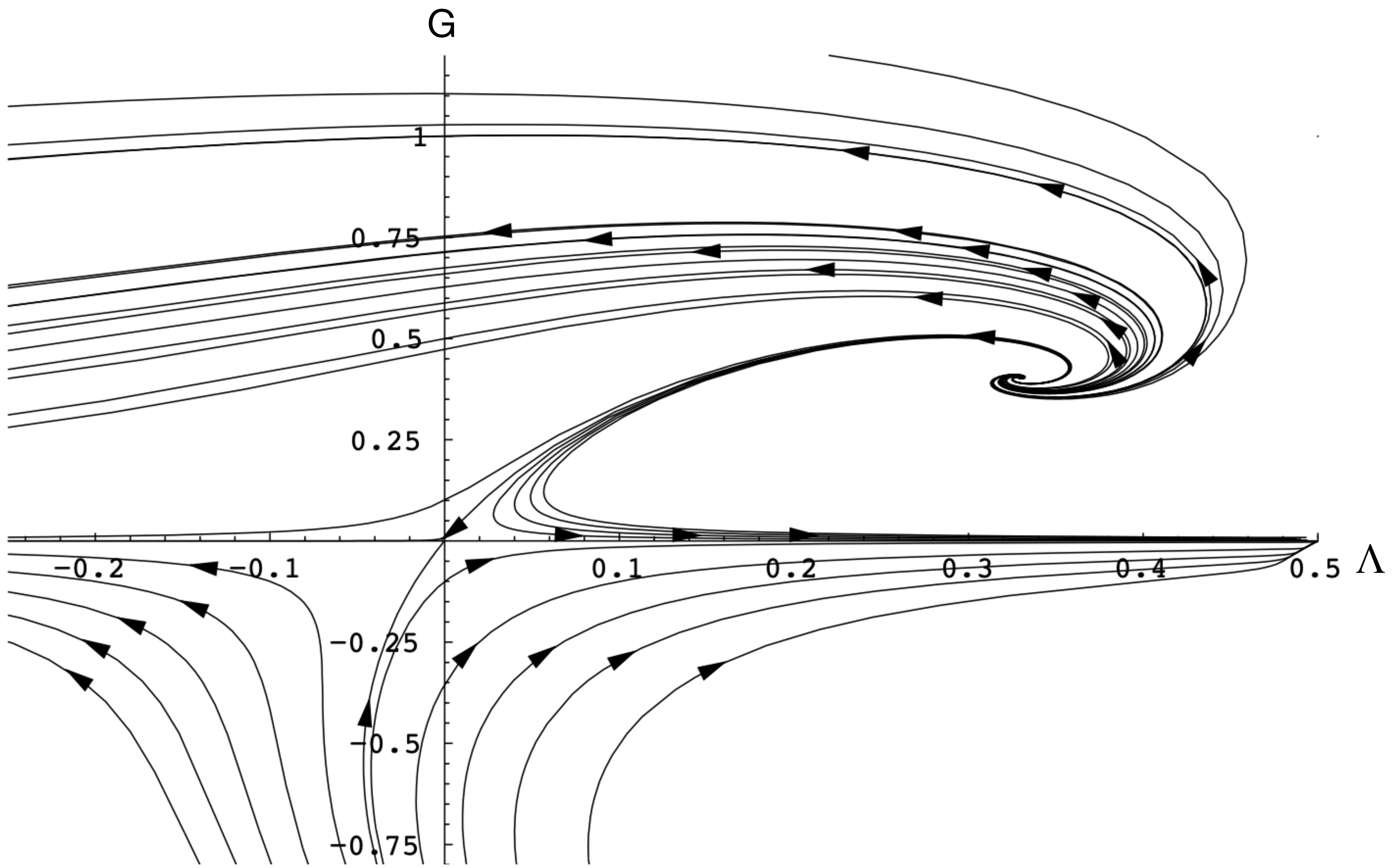
[Reuter '96; Souma '99]

$\# > 0$ depends on regulator function

$$G_* = \frac{2}{\#}$$ non-universal

$$\theta = -\left.\frac{\partial \beta_G}{\partial G}\right|_{G=G_*} = 2$$ universal

Asymptotic safety in quantum gravity: Newton coupling and cosmological constant



[Reuter, Saueressig ‘02]

asymptotically safe fixed point

two relevant directions

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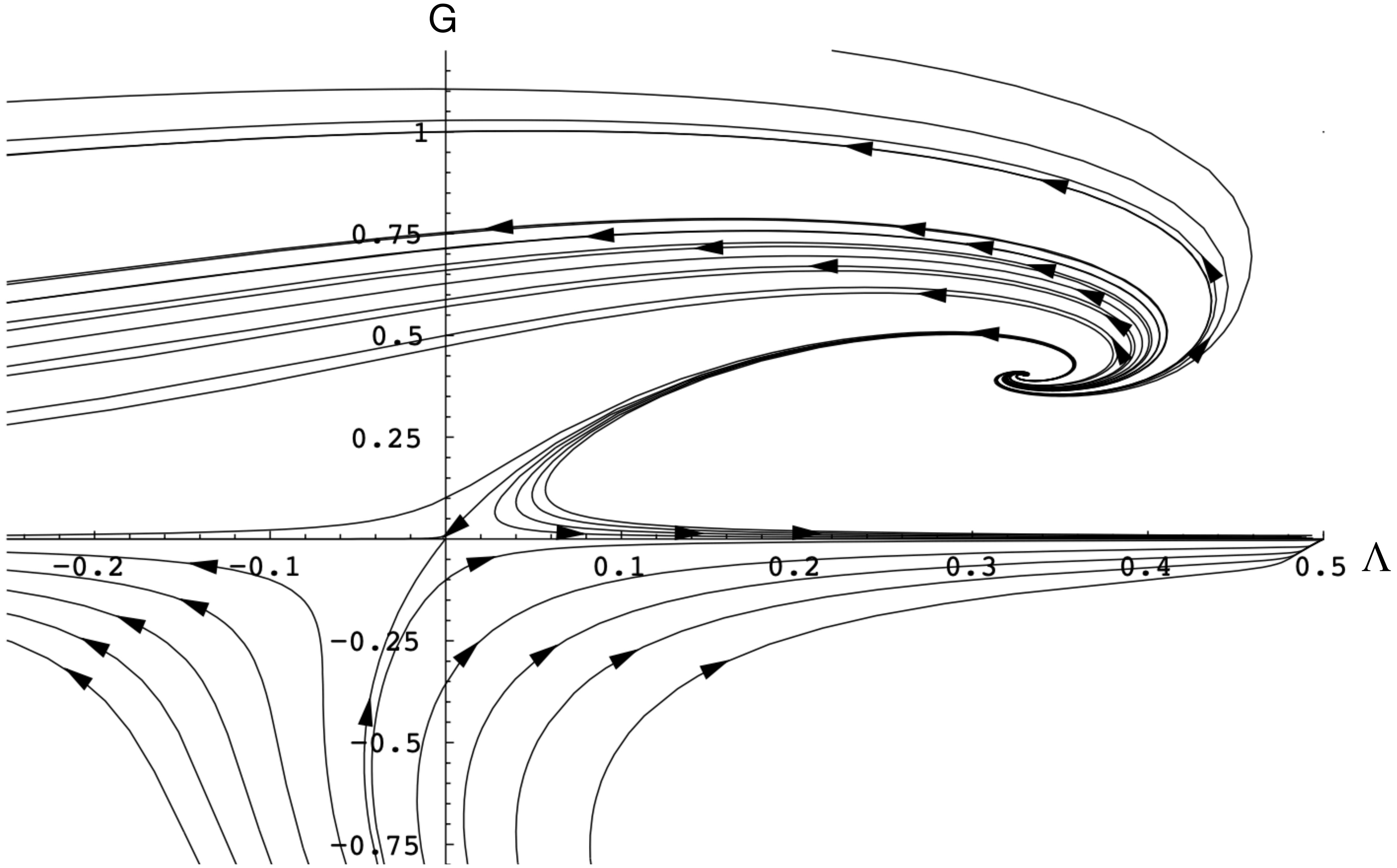

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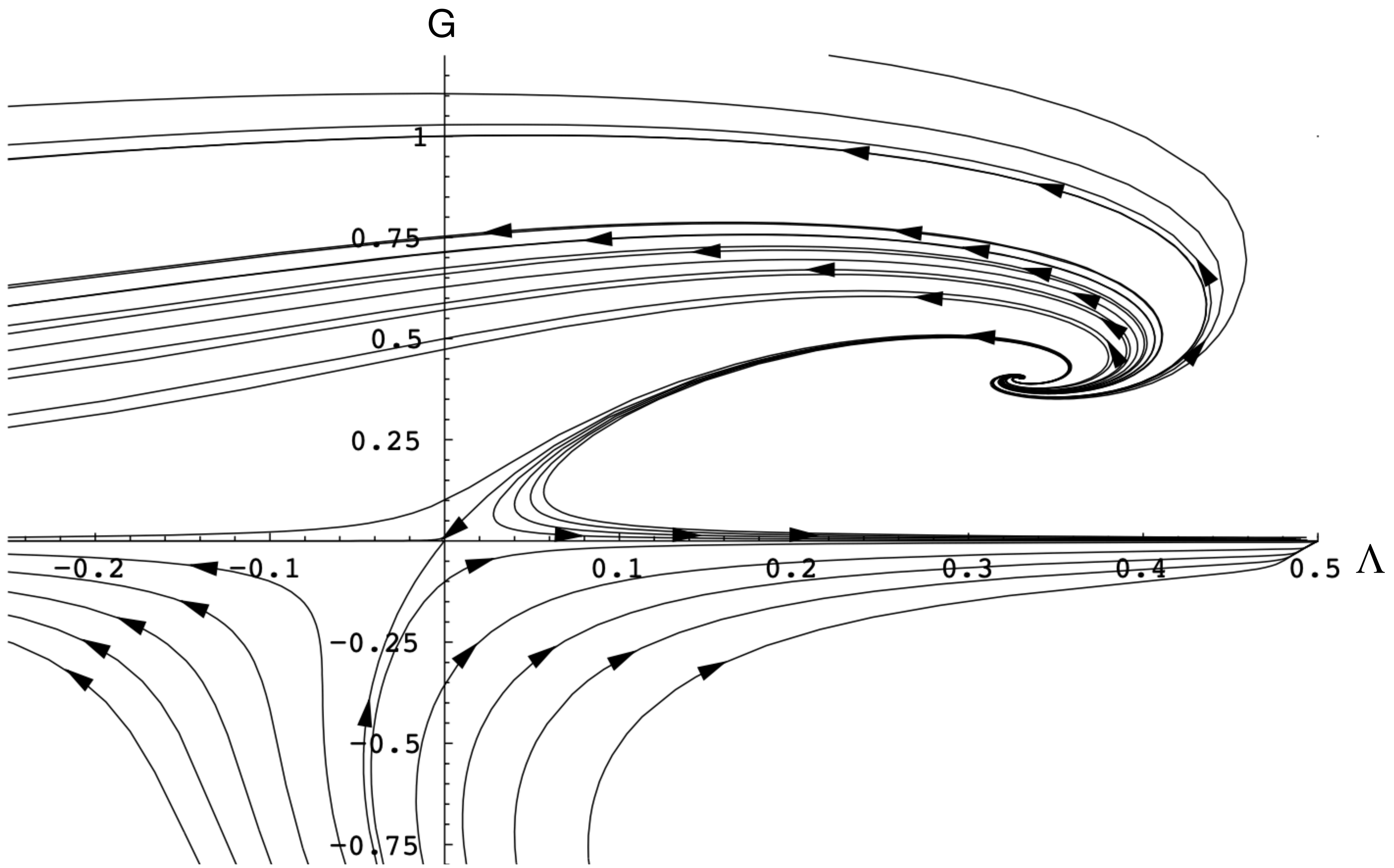

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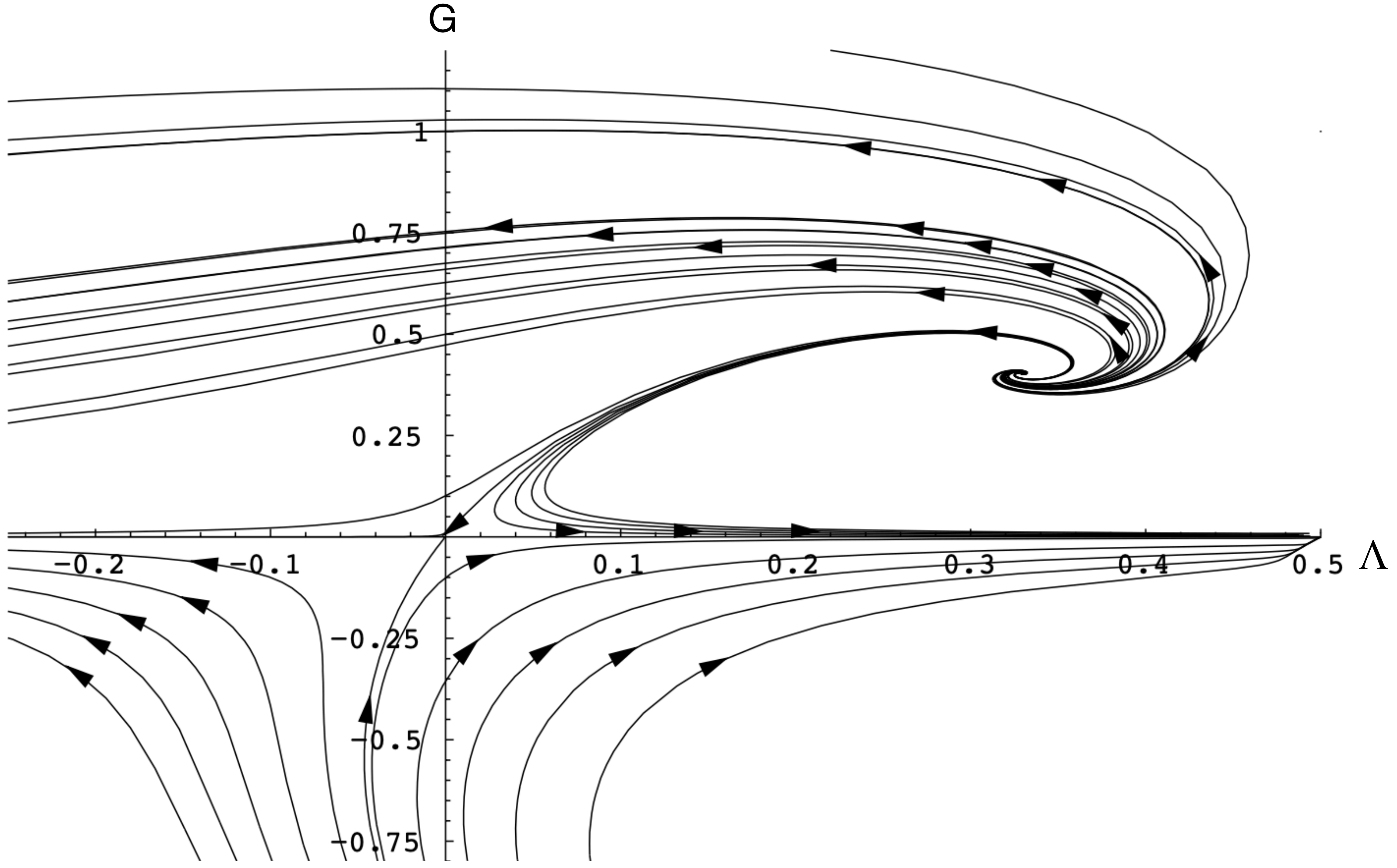

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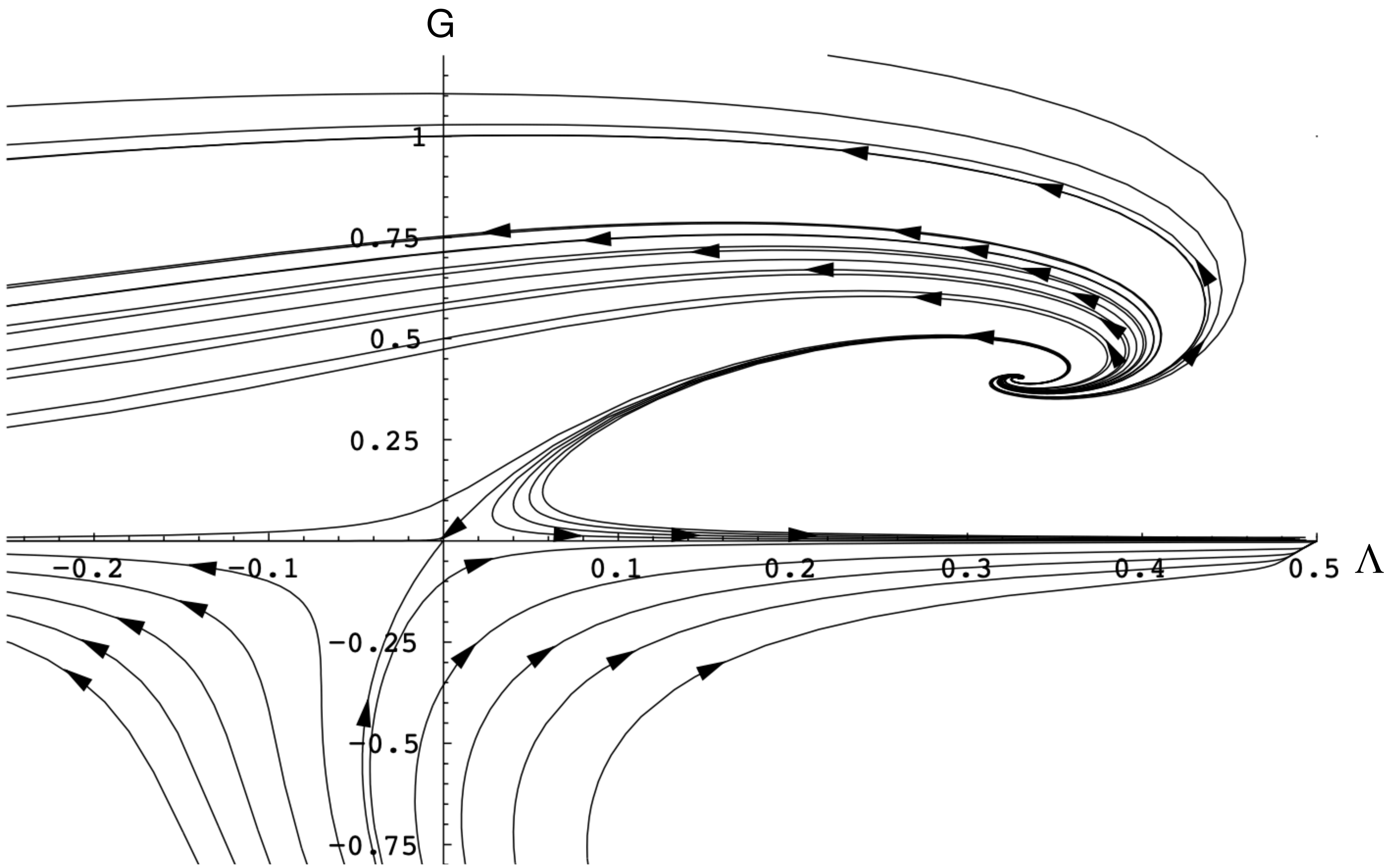

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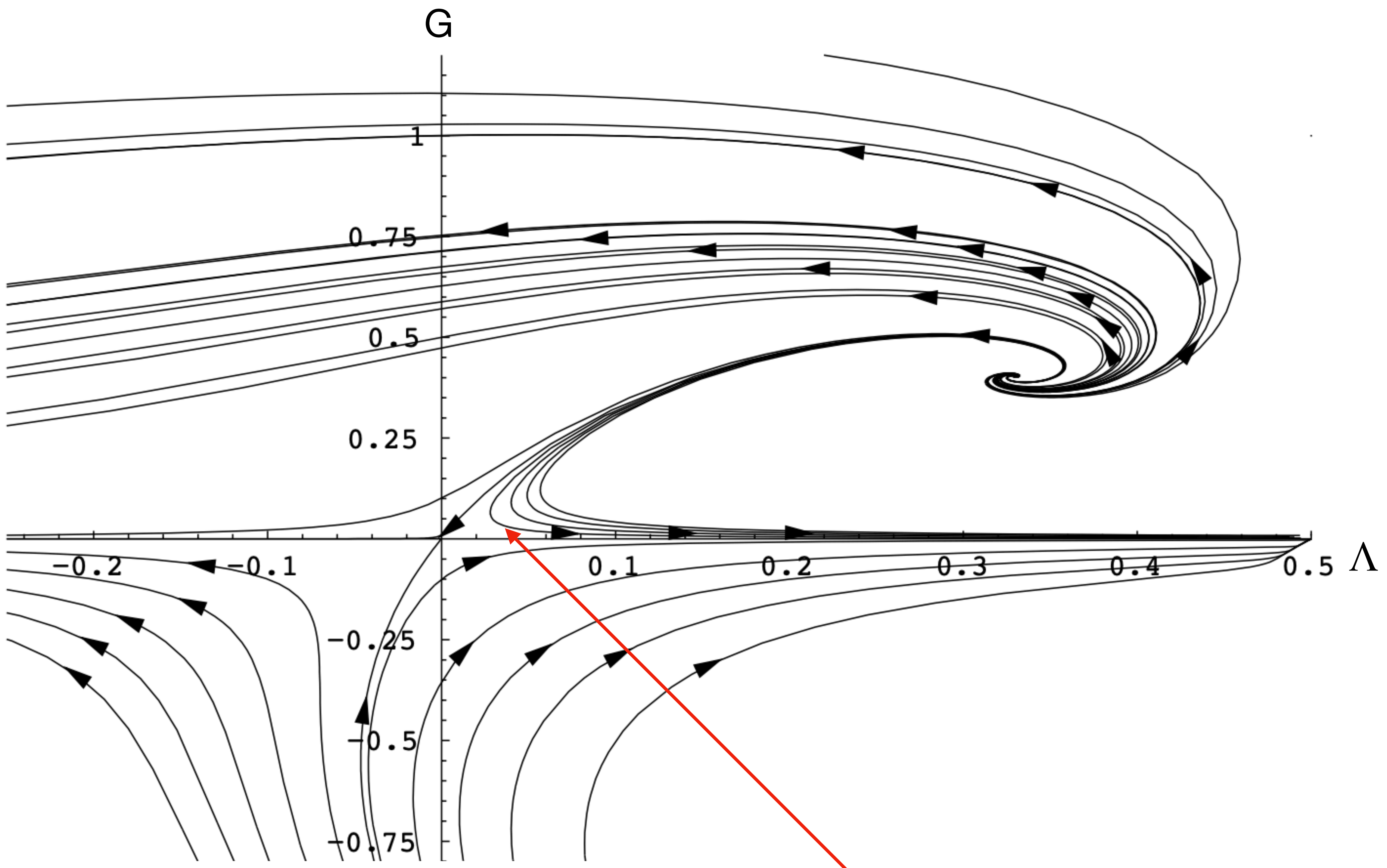

[Reuter, Saueressig '02]

RG trajectory ``of our universe"

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$\Rightarrow G \cdot \Lambda = \text{const}$

→ first phenomenological test: fixed point in the UV connected to classical gravity in IR

→ cosmological-constant ``problem": correct values of $G_N, \bar{\Lambda}$ realized on one particular trajectory

Asymptotic safety in quantum gravity: Curvature-squared results

curvature-squared terms:

$$bR^2, a\, R_{\mu\nu}R^{\mu\nu}, \frac{1}{\rho}E$$

Einstein-Hilbert + curvature squared:

4 couplings of local terms and one topological term

→ fixed point with three relevant directions, one irrelevant and one marginal

[Benedetti, Machado, Saueressig '09;
Falls, Ohta, Percacci '20]

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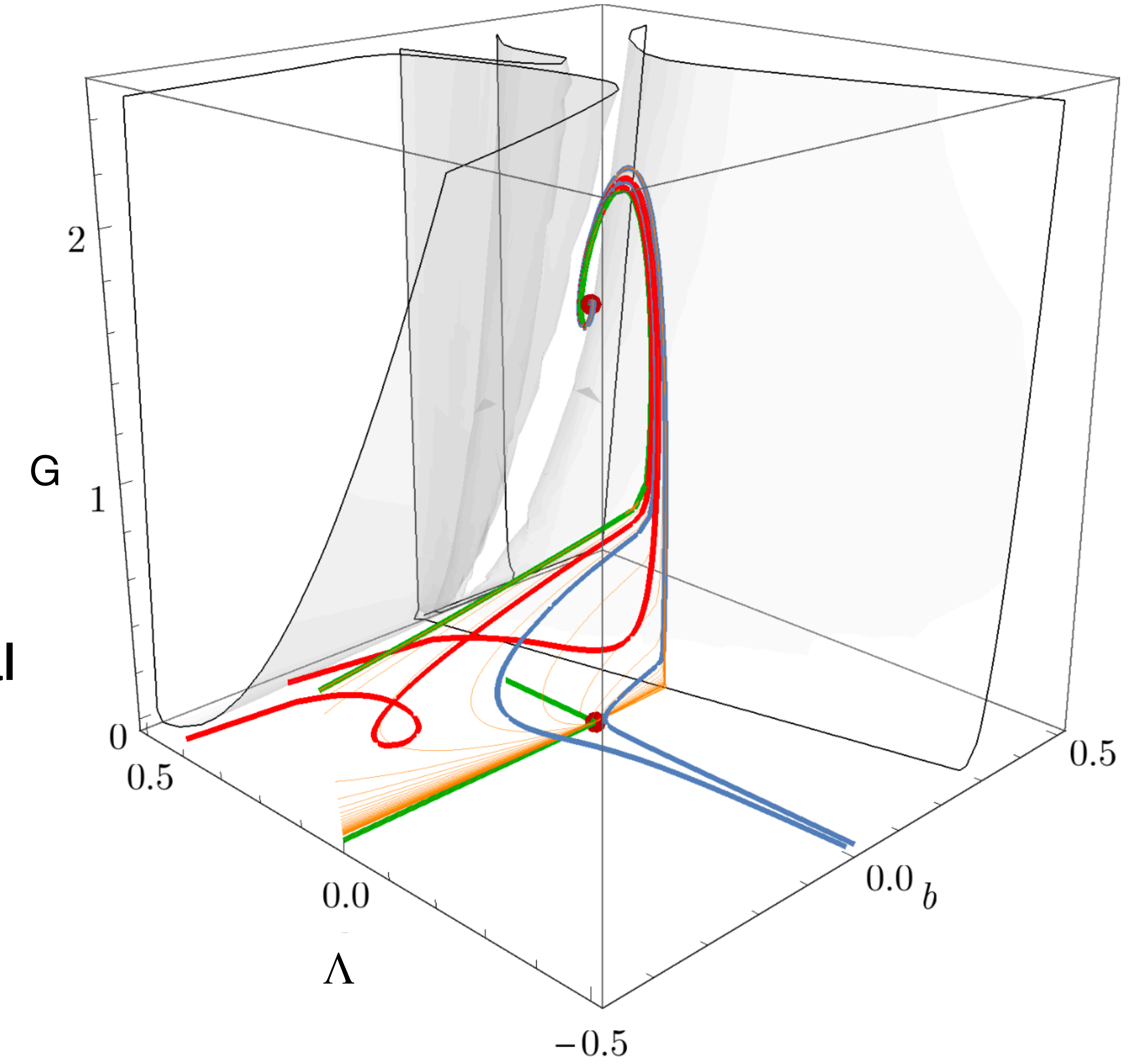


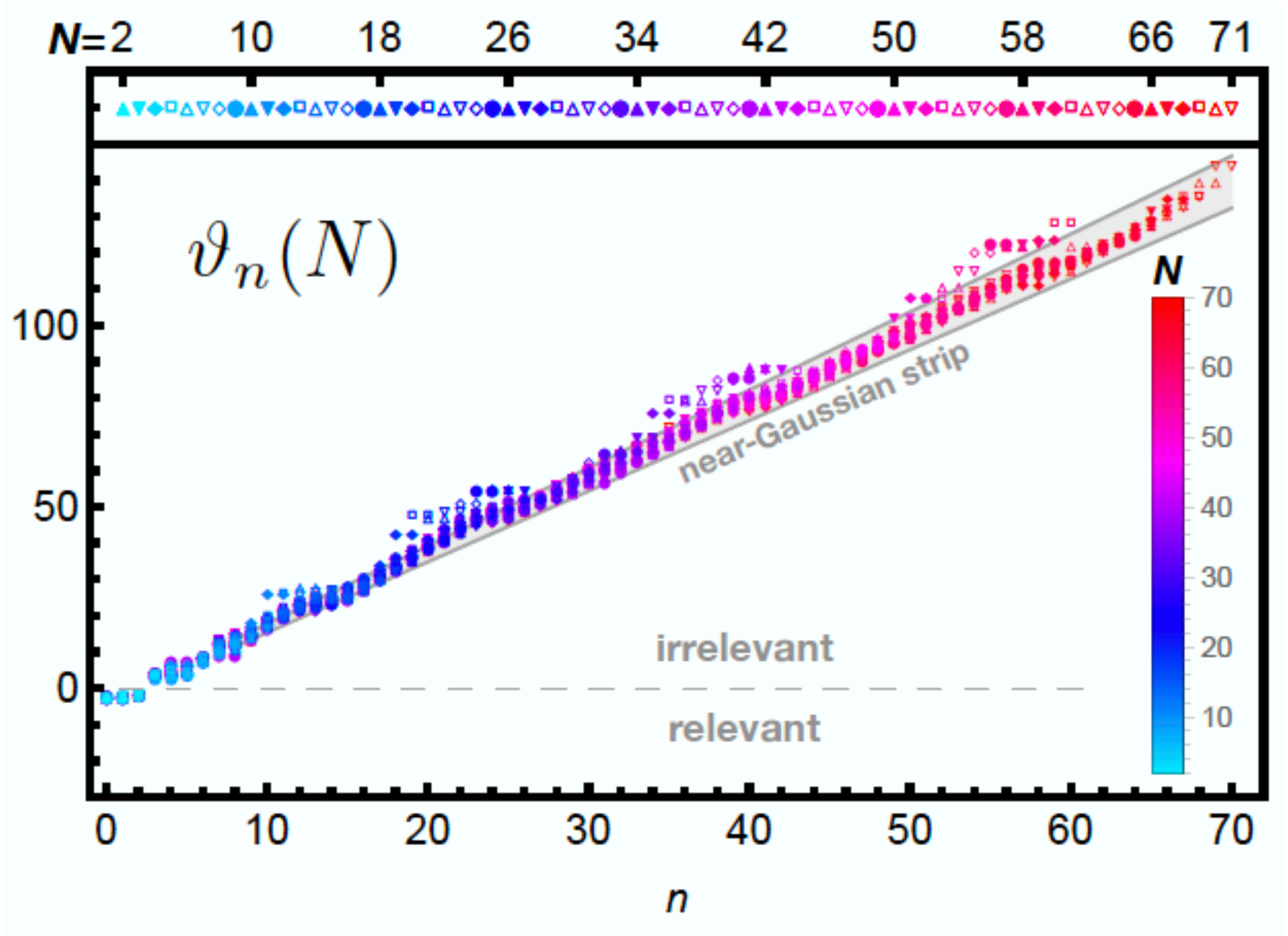
[Gubitosi, Oijer, Ripken, Saueressig '18]

Classical gravity + Starobinsky inflation, driven by bR^2 ?

→ fixed point is connected to a low-energy regime with classical gravity and Starobinsky inflation (but the latter is not a must)

Asymptotic safety in quantum gravity: Higher order in curvature

scaling exponents of $R^n, n = 0,\ldots,70$

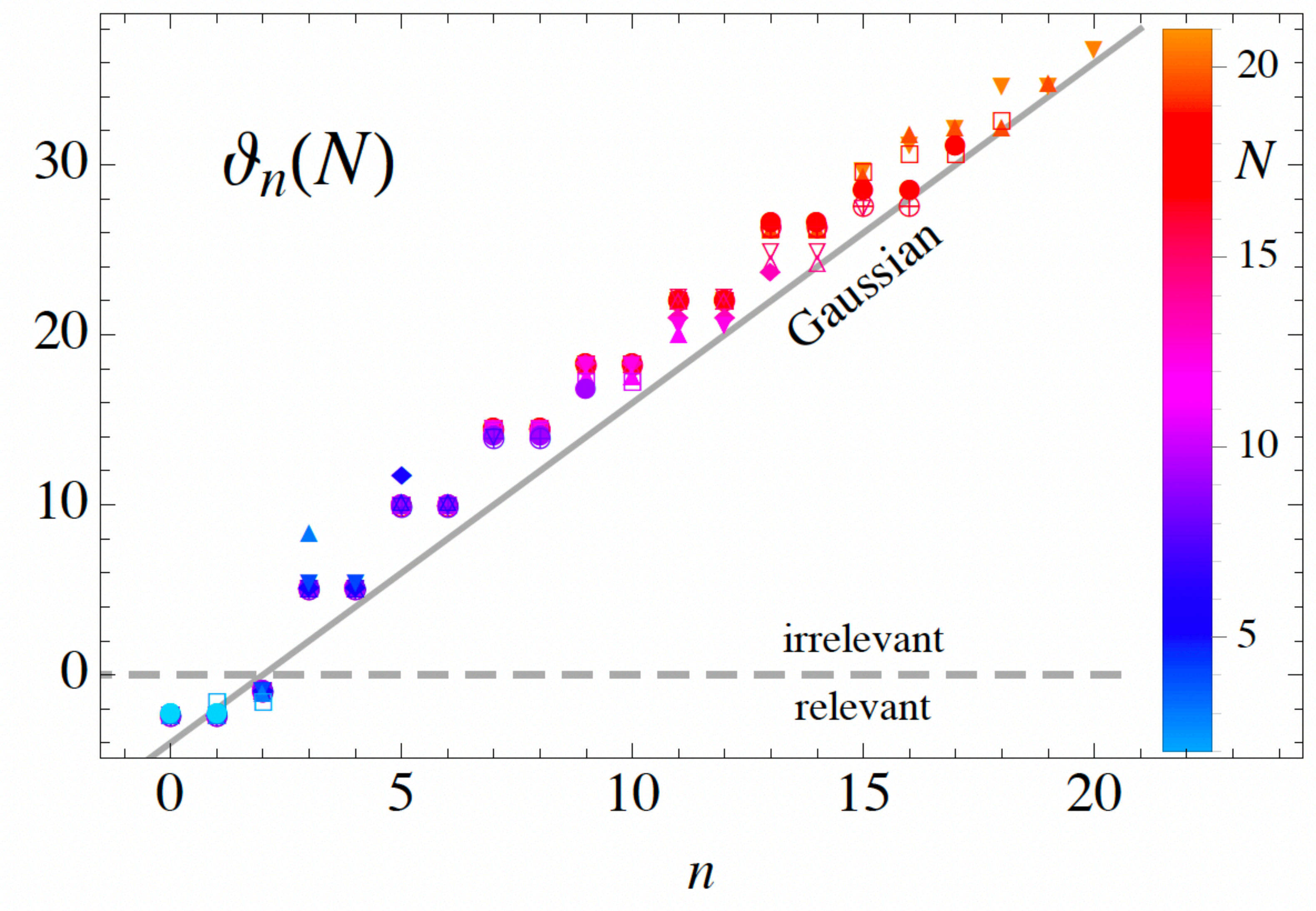


[Falls, Litim, Schröder '19], see also [Falls, Litim et al '13, '14…]

$$\vartheta_n = -\theta_n \approx -2n + 4$$

scaling exponents of

$F(R_{\mu\nu}R^{\mu\nu})^n + (R_{\mu\nu}R^{\mu\nu})^n, n = 0,\ldots,10$



[Falls, King, Litim, Rahmede '18]

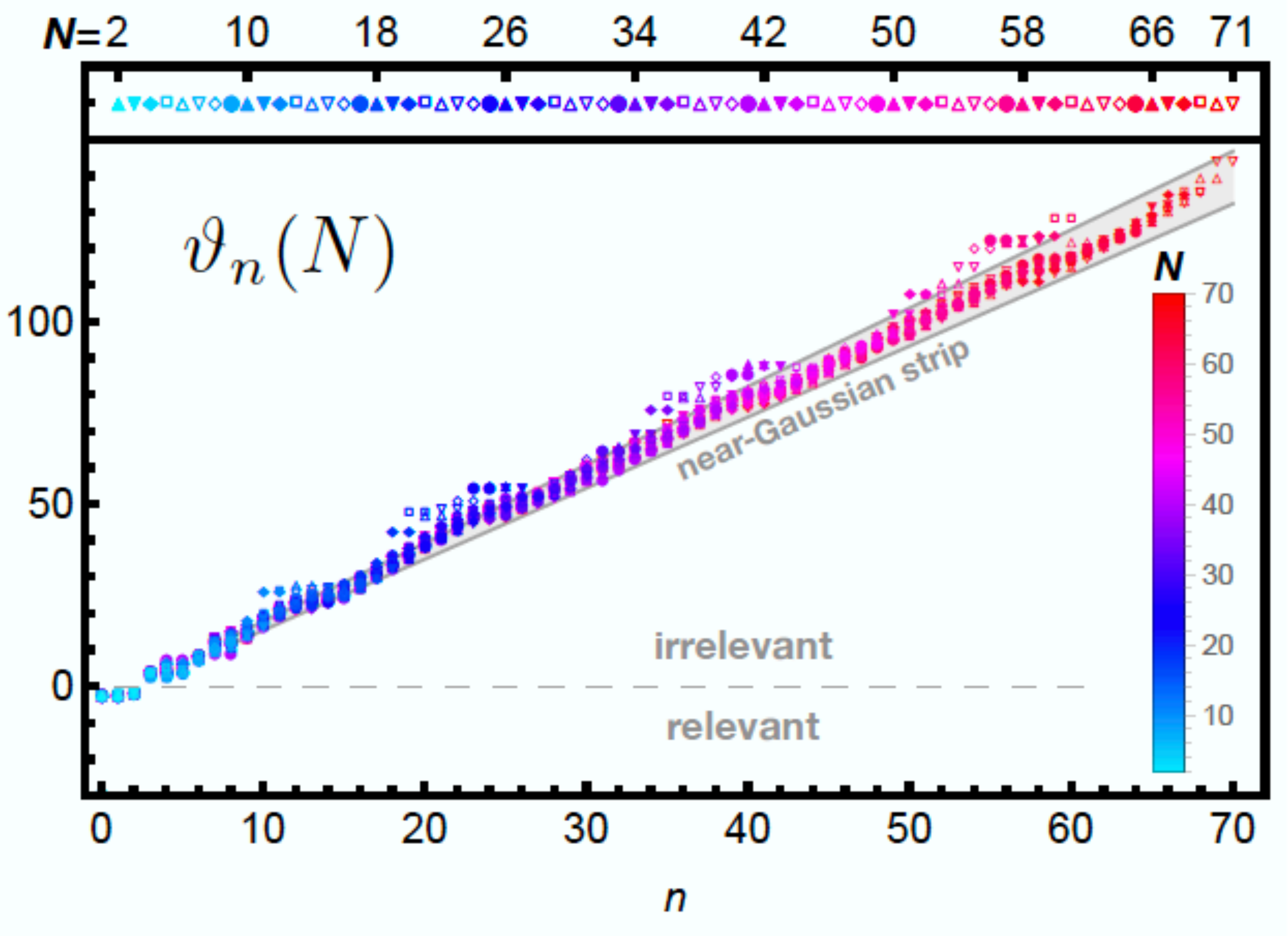
full $f(R)$?

[Benedetti, Caravelli '12; Dietz, Morris '12,
Demmel, Saueressig, Zanusso '14, '15;
Gonzalez-Martin, Morris, Slade '17;
Christiansen, Falls, Pawlowski, Reichert '17]

Three free parameters & near-perturbative nature

References	Gauge	Cutoff	Operators included beyond Einstein-Hilbert	# rel. dir.	# irrel. dir.	Reθ_1	Reθ_2	Reθ_3
Reuter and Saueressig, 2002	$\alpha=1,\beta=0$	exp.	-	2	-	1.94	1.94	-
Litim, 2004	$\alpha=0$	Litim (Litim, 2000, 2001)	-	2	-	1.67	1.67	-
Lauscher and Reuter, 2002	$\alpha=0,\beta=0$	exp.	$\sqrt{g}R^2$	3	0	28.8	2.15	2.15
Machado and Saueressig, 2008	$\beta=1,\alpha=0$	Litim	$\sqrt{g}R^2, \sqrt{g}R^3$	3	1	2.67	2.67	2.07
Codello et al., 2009	$\alpha=1,\beta=1$	Litim	$\sqrt{g}R^2, \sqrt{g}R^3$	3	1	2.71	2.71	2.07
Machado and Saueressig, 2008	$\beta=1,\alpha=0$	Litim	$\sqrt{g}R^2, \sqrt{g}R^6$	3	1	2.39	2.39	1.51
Codello et al., 2009	$\alpha=1,\beta=1$	Litim	$\sqrt{g}R^2, ..., \sqrt{g}R^8$	3	6	2.41	2.41	1.40
Falls et al., 2013, 2016	$\alpha=0,\beta=0$	Litim	$\sqrt{g}R^2, ..., \sqrt{g}R^{34}$	3	32	2.50	2.50	1.59
Benedetti et al., 2009	$\alpha=0$, h/o	Litim	$\sqrt{g}R^2, \sqrt{g}R_{\mu\nu}R^{\mu\nu}$	3	1	8.40	2.51	1.69
Gies et al., 2016	$\beta=\alpha=1$	Litim	$\sqrt{g}C^{\mu\nu\kappa\lambda}C_{\kappa\lambda\rho\sigma}C^{\rho\sigma}{}_{\mu\nu}$	2	1	1.48	1.48	-

[from AE '18]



[Falls, Litim, Schröder '19], see also [Falls, Litim et al '13, '14…]

- Choose truncations according to canonical power-counting
- Calculate critical exponents
- Check whether they are close to canonical power-counting