# Lessons of the EFT Treatment of Quantum General Relativity

**John F. Donoghue**

1) EFT and General Relativity

2) Quantization and renormalization of GR

3) A couple of quantum gravity calculations

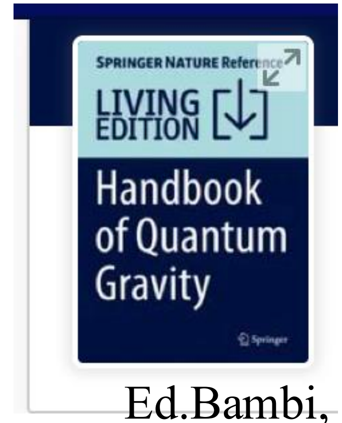
4) Seven Lessons of the gravitational EFT

5) Limits/limitations

Amherst Center for Fundamental Interactions
*Physics at the interface: Energy, Intensity, and Cosmic frontiers*
University of Massachusetts Amherst

Zakopane
6/21/2024

# Varieties of EFTs in Gravity




Ed.Bambi, Modesto, Shapiro

### Quantum General Relativity and Effective Field Theory

John F. Donoghue (Massachusetts U., Amherst) (Nov 17, 2022)

### Effective Field Theory for Large-Scale Structure

Mikhail M. Ivanov (Princeton, Inst. Advanced Study) (Dec 16, 2022)

### EFT for de Sitter Space

Daniel Green (UC, San Diego) (Oct 11, 2022)

### Effective Field Theory for Compact Binary Dynamics

Walter D. Goldberger (Yale U.) (Dec 13, 2022)

### Effective Field Theory and Applications: Weak Field Observables from Scattering Amplitudes in Quantum Field Theory

N. Emil J. Bjerrum-Bohr (Bohr Inst.), Ludovic Planté (Bohr Inst.), Pierre Vanhove (IPhT, Saclay) (Dec 17, 2022)

### Gravity, Horizons and Open EFTs

C.P. Burgess (McMaster U. and Perimeter Inst. Theor. Phys. and CERN and Dublin Inst.), Greg Kaplanek (Imperial Coll., London) (Dec 18, 2022)

### Soft-Collinear Gravity and Soft Theorems

Martin Beneke (Munich, Tech. U.), Patrick Hager (Munich, Tech. U.), Robert Szafron (Brookhaven) (

# General Relativitistic EFT

**Normal QFT with attention paid to the energy scales involved**

**Low energy symmetry and fields**

- general covariance and the metric as the active

**Path Integral with limits**

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[ i \int d^4x \sqrt{-g} \left( -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots \text{SM} \ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right) \right]$$

- note**: metric must be part of the PI**

# Recall: EFT techniques

**Quantum methods sample all energies**

- including where the EFT is incorrect

**But wrong part is local => like parameters in Lagrangian**

- calculations must respect symmetries (~ dim. reg.)
- match or measure renormalized parameters

**Nonlocal effects are reliable**

- only from low energy D.O.F. and interactions
- long distance propagation

**In calculations near Minkowski**:

- nonanalytic only from nonlocal

$$(q^2)^n \to \Box^n \delta(x)$$

$$\log(-q^2) \to L(x-y) = \langle x | \log \Box | y \rangle$$

# Quantizing general relativity

Feynman quantized gravity in the 1960's

Quanta = gravitons

Rules for Feynman diagrams given

<u>Subtle features</u>:

metric has 4x4 components – only 2 are physical DOF!

-need to remove effects of unphysical ones

Gauge invariance (general coordinate invariance)

- calculations done in some gauge

-need to maintain symmetry

In the end, the techniques used are very similar to other gauge theories

QUANTUM THEORY OF GRAVITATION*

By R. P. Feynman

(Received July 3, 1963)

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; for example, as far as quantum effects

## Feynman's tree theorem:

This made me investigate the entire subject in great detail to find out what the trouble is. I discovered in the process two things. First, I discovered a number of theorems, which as far as I know are new, which relate closed loop diagrams and diagrams without closed loop diagrams (I shall call the latter diagrams "trees"). The unitarity relation which I have just been describing, is one connection between a closed loop diagram and a tree; but I found a whole lot of other ones, and this gives me more tests on my machinery. So let me just tell

## FDFP Ghosts in YM and GR:

Incidentally I investigated further and discovered another very interesting point. There is another theory, more well-known to meson physicists, called the Yang-Mills theory, and I take the one with zero mass; it is a special theory that has never been investigated in great detail. It is very analogous to gravitation; instead of the coordinate transformation group being the source of everything, it's the isotopic spin rotation group that's the source of everything. It is a non-linear theory, that's like the gravitation theory, and so forth. At

Feynman uses tree theorem in reverse:

Well, what then, now you have the difficulty; how do you cure it? Well I tried the following idea: I assumed the tree theorem to be true, and used it in reverse. If every closed ring diagram can be expressed as trees, and if trees produce no trouble and can be computed

Introduces ghosts:

when I made it gauge invariant. But then secondly, you must subtract from the answer, the result that you get by imagining that in the ring which involves only a graviton going around, instead you calculate with a different particle going around, an artificial, dopey particle is coupled to it. It's a vector particle, artificially coupled to the external field, so designed as to correct the error in this one. The forms are evidently invariant,

DeWitt later works out the formal details:

DeWitt: Because of the interest of the tricky extra particle that you mentioned at the end, and its possible connection, perhaps, with some work of Dr Białynicki-Birula, have you got far enough on that so that you could repeat it with just a little more detail? The structure of it and what sort of an equation it satisfies, and what is its propagator? These are technical points, but they have an interest.

Feynman: Give me ten minutes. And let me show how the analysis of these tree diagrams, loop diagrams and all this other stuff is done mathematical way. Now I will show you that I too can write equations that nobody can understand. Before I do that I should

# Quantization

**Quantizing gravity**:

-Covariant quantization      Feynman deWitt

-gauge fixing

-ghosts fields

-Background field method     't Hooft Veltman

-retains symmetries of GR

-path integral

**Background field:**

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\ g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu_\lambda h^{\lambda\nu} + \ldots \end{aligned}$$

Expand around this background:

$$S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[ \frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]$$

$$\begin{aligned} \mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[ \bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\ \mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\ &\quad + \bar{R} \left( \frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left( 2h^\lambda_\mu h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu} \end{aligned}$$

Linear term vanishes by Einstein Eq.

$$\bar{R}^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$$

**Gauge fixing**:

-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}$$

$$h \equiv h^{\lambda}_{\lambda}$$

**Ghost fields**:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}}\eta^{*\mu}\left\{\eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu}\eta^{\nu}\right\}$$

vector fields
anticommuting,
in loops only

# <u>Feynman rules:</u>

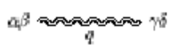
### A.1 Scalar propagator

The massive scalar propagator is:

$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

### A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:



$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]$$

### A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:

$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2}\left[p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left((p\cdot p') - m^2\right)\right]$$

### A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:

$$= \tau_2^{\eta\lambda\rho\sigma}(p, p', m)$$

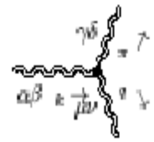
$$\begin{aligned}\tau_2^{\eta\lambda\rho\sigma}(p,p') = i\kappa^2\Bigg[&\left\{I^{\eta\lambda\alpha\delta}I^{\rho\sigma\beta}{}_{\delta} - \frac{1}{4}\left\{\eta^{\eta\lambda}I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma}I^{\eta\lambda\alpha\beta}\right\}\right\}(p_\alpha p'_\beta + p'_\alpha p_\beta)\\ &-\frac{1}{2}\left\{I^{\eta\lambda\rho\sigma} - \frac{1}{2}\eta^{\eta\lambda}\eta^{\rho\sigma}\right\}\left[(p\cdot p') - m^2\right]\Bigg]\end{aligned} \tag{61}$$

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

### A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]



$$= \tau_{3\,\alpha\beta\gamma\delta}^{\mu\nu}(k, q)$$

where

$$\begin{aligned}\tau_{3\,\alpha\beta\gamma\delta}^{\mu\nu}(k,q) = -\frac{i\kappa}{2}\times\Bigg(&\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right]\\ &+2q_\lambda q_\sigma\left[I_{\alpha\beta}{}^{\sigma\lambda}I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda}I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta}{}^{\nu\lambda}\right]\\ &+\left[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\lambda}\right)\right.\\ &\left.-q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\nu}\right) - \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\sigma\lambda}\right)\right]\\ &+\left[2q_\lambda\left(I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\nu}(k-q)^\mu + I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\mu}(k-q)^\nu - I_{\gamma\delta}{}^{\lambda\sigma}I_{\alpha\beta\sigma}{}^{\nu}k^\mu - I_{\gamma\delta}{}^{\lambda\sigma}\right.\right.\\ &\left.+q^2\left(I_{\alpha\beta\sigma}{}^{\mu}I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma}I_{\gamma\delta\sigma}{}^{\mu}\right) + \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}{}^{\lambda\rho}I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho}I_{\alpha\beta\rho}{}^{\sigma}\right)\right]\\ &+\Bigg\{(k^2 + (k-q)^2)\left[I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta\sigma}{}^{\nu} + I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\right]\\ &-\left(I_{\gamma\delta}{}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}{}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right)\Bigg\}\Bigg)\end{aligned} \tag{62}$$

# Summary – quantization

Quantization is no different from any other theory!

$$Z^{core} = \int [d\phi d\psi dA dg]_{\mathrm{Limits}} \exp\left[i \int d^4x \sqrt{-g}\left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots \mathrm{SM} \ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right)\right]$$

For more details:

[Submitted on 1 Feb 2017]

**EPFL Lectures on General Relativity as a Quantum Field Theory**

John F. Donoghue, Mikhail M. Ivanov, Andrey Shkerin

These notes are an introduction to General Relativity as a Quantum Effective Field Theory, following the material given in a short course on the subject at EPFL. The intent is to develop General Relativity starting from a quantum field theoretic viewpoint, and to introduce some of the techniques needed to understand the subject.

# Renormalization

**One loop calculation**: 't Hooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

dim. reg. preserves symmetry

**Renormalize** parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu} = 0$

**Note**: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

# What are the quantum predictions?

**Not the divergences**

- they come from the Planck scale
- unreliable part of theory

**Not the parameters**

- local terms in L
- we would have to measure them

**Low energy propagation**

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

# Example 1: Corrections to the gravitational potential

Scattering potential

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Full result is the full scattering amplitude
NR Potential is a useful way of illustrating result

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

# What to expect:

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2\right]$$

Non-analytic

Relation to position space:

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

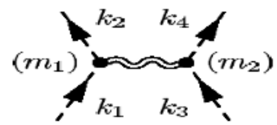
Classical quantum

# Result:

:

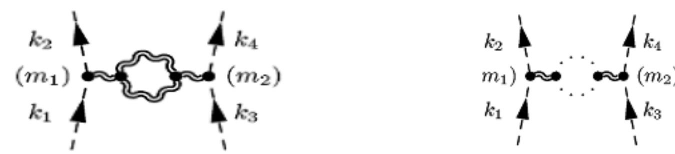
$$\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}$$

Lowest order:

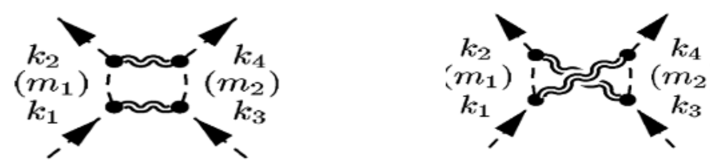



Vertex corrections:

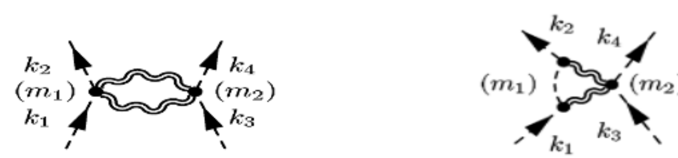

Vacuum polarization:
(Duff 1974)




Box and crossed box




Others:



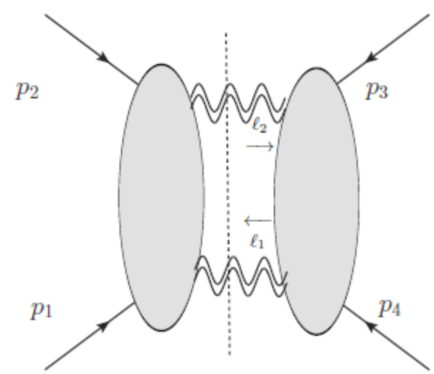
## On-shell techniques and loops from unitarity

- On-shell amplitudes only
- No ghosts needed – axial gauge
- Exhibits "double copy" relations
- Both unitarity cuts and dispersion relation methods

$$iM^{1-\text{loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\sum_{\lambda_1,\lambda_2} M^{\text{tree}}_{\lambda_1\lambda_2}(p_1,p_2,-\ell_2^{\lambda_2},\ell_1^{\lambda_1})(M^{\text{tree}}_{\lambda_1\lambda_2}(p_3,p_4,\ell_2^{\lambda_2},-\ell_1^{\lambda_1}))^*}{\ell_1^2\ell_2^2}\Big|_{cut},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

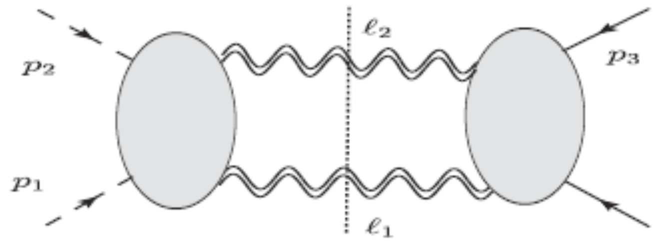



**Confirm results for gravitational potential**

- gauge invariance check

# Example 2: Light bending at one loop

Again using unitarity methods




$$
\begin{aligned}
i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2\Bigg[&\frac{\kappa^2}{t} + \kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}} + \hbar\kappa^4\frac{15}{512\pi^2} \\
&\times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right) \\
&+ \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) \\
&+ \kappa^4\frac{M\omega\, i}{8\pi\ t}\log\left(\frac{-t}{M^2}\right)\Bigg], \qquad (11)
\end{aligned}
$$

Can convert amplitude to bending angle using eikonal method

Result different for scalars, photons and gravitons

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$ for (scalar photons gravitons)

# Hawking Radiation

Appears to be a property of the low energy theory

1) Hambli-Burgess: Pauli Villars regulators

-flux from local limit of Green's function (Haag Fredenhagen)

$$\mathcal{F} \equiv -\langle T_t{}^r \rangle = -\langle T_{tr^\star} \rangle$$
$$= -\frac{1}{2} \lim_{x' \to x} \left( \frac{\partial}{\partial t'} \frac{\partial}{\partial r^\star} + \frac{\partial}{\partial r^{\star\prime}} \frac{\partial}{\partial t} \right) G(x, x').$$

-dependence on regulator vanishes exponentially

2) Agullo, Navarro-Salas, Olmo, Parker

- Also test cutoff sensitivity – find robustness in prediction

$$\langle \text{in} | N^{\text{out}}_{i_1 i_2} | \text{in} \rangle = - \frac{t_{l_1}(w_1) t^*_{l_2}(w_2) \delta(w_1 - w_2)}{2\pi \sqrt{w_1 w_2}} \int_{-\infty}^{+\infty} dz$$
$$\times e^{-i[(w_1+w_2)/2]z} \left[ \frac{(\frac{\kappa}{2})^2}{(\sinh \frac{\kappa}{2} z)^2} - \frac{1}{z^2} \right]$$

3) Also Jacobson and Unruh with sonic analogues.

# Seven Lessons of the EFT

1) Universality of the NR gravitational interaction

2) Classical physics from loops

3) No "test particle" limit for quantum effects

4) "Quantum corrected metric" is not a valid quantum concept

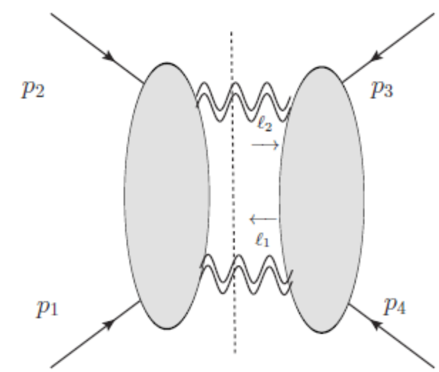
5) Trajectories of massless particles are not universal

6) CC and G are not running parameters

7) Lightcones/ Penrose diagrams etc likely uncontrolled approximations

# 1) Universality of the NR Gravitational Interaction

**Soft theorems extend to some loop effects**




Recall on-shell unitarity method

On-shell amplitudes satisfy soft theorems
- Low, Weinberg and Gross-Jackiw

The relevant cuts are exactly these universal pieces

Then the leading loop results are also universal
- first found painfully by Holstein and Ross
- then true for particles, molecules, the Moon etc.

## 2) Classical physics from loops

**Folk theorem** – the loop expansion is the $\hbar$ expansion

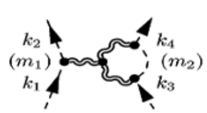
- not true
- classical physics also present in loop expansion
- hidden factors of hbar

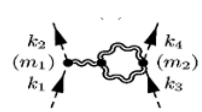
$$\mathcal{L} = \hbar\bar{\psi}\left(i\partial\!\!\!/ - \frac{m}{\hbar}\right)\psi$$

- at one loop, present in $\sqrt{q^2}$ non-analyticity

$$\sqrt{\frac{m^2}{-q^2}} \to \hbar\sqrt{\frac{m^2}{-\hbar^2 q^2}}$$

- both classical and quantum present in some diagrams

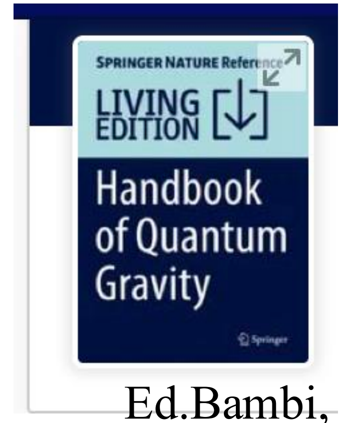



$$\begin{aligned} M_{5(a)+5(b)}(\vec{q}) &= 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2\right) \\ M_{5(c)+5(d)}(\vec{q}) &= -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2 \end{aligned}$$

This has become a vibrant subfield

# Varieties of EFTs in Gravity




Ed.Bambi, Modesto, Shapiro

### Quantum General Relativity and Effective Field Theory

John F. Donoghue (Massachusetts U., Amherst) (Nov 17, 2022)

### Effective Field Theory for Large-Scale Structure

Mikhail M. Ivanov (Princeton, Inst. Advanced Study) (Dec 16, 2022)

### EFT for de Sitter Space

Daniel Green (UC, San Diego) (Oct 11, 2022)

### Effective Field Theory for Compact Binary Dynamics

Walter D. Goldberger (Yale U.) (Dec 13, 2022)

### Effective Field Theory and Applications: Weak Field Observables from Scattering Amplitudes in Quantum Field Theory

#5

N. Emil J. Bjerrum-Bohr (Bohr Inst.), Ludovic Planté (Bohr Inst.), Pierre Vanhove (IPhT, Saclay) (Dec 17, 2022)

### Gravity, Horizons and Open EFTs

C.P. Burgess (McMaster U. and Perimeter Inst. Theor. Phys. and CERN and Dublin Inst.), Greg Kaplanek (Imperial Coll., London) (Dec 18, 2022)

### Soft-Collinear Gravity and Soft Theorems

Martin Beneke (Munich, Tech. U.), Patrick Hager (Munich, Tech. U.), Robert Szafron (Brookhaven) (

## 3) There is no "test particle" limit for quantum effects

All quantum corrections are of the same form


(a) (b)

| | | | |
|---|---|---|---|
| Classical | $\frac{GM}{r}$ | $\frac{Gm}{r}$ | - test particle |
| Quantum | $\frac{G\hbar}{r^2}$ | $\frac{G\hbar}{r^2}$ | - no test particle possible |

Also visible in the results for massless particle scattering

# 4) “Quantum corrected metric” is not a valid quantum item

Tempting to ask for eg. “quantum corrections to Schwarzschild”

- mea culpa

But not a well-defined quantum question

**Specific objection– not field redefinition independent** (Kirilin)

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad \rightarrow \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + a h_{\mu\lambda} h^{\lambda}_{\nu}$$

- explicit calculation to demonstrate this

Haag’s theorem only guarantees field redefinition independence for **on-shell** matrix elements

Metric is only part of a full quantum calculation

# 5) Trajectories of massless particles are not universal

Recall:

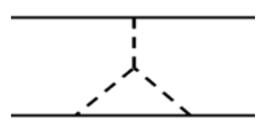
$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$$ for scalars, photons, gravitons

The quantum corrections amount to tidal forces

-long range propagation



- sample gravitational fields at more than one position

**Not geodesic motion**

# 6) Cosmological constant and G are not running parameters

-at least in EFT region

Most obviously – **no power-law running in physical processes**
- i.e. $\Lambda_{cc} \sim (\Lambda_{\text{cutoff}})^4 \qquad G \sim (\Lambda_{\text{cutoff}})^2$
- physical running with kinematic quantities $\sim q^2, R$
- energy expansion of Lagrangian
- no universal repackaging as running parameters

But also **not log running** with energy scale
- kinematic logs not related to renormalization of CC or R

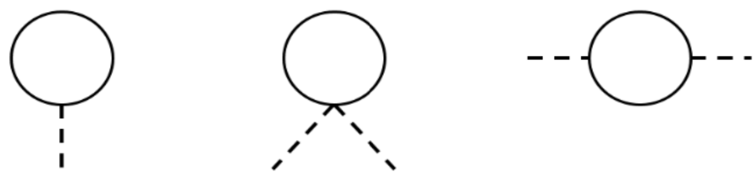
**Some points**:

a) Renormalizaton of CC and (non) running
b) Non-local effective actions
c) Non-local partners

## a) Example: Renormalization of CC from massive particle

Can be probed by individual metric couplings

$$\sqrt{-g}\Lambda = \sqrt{-\bar{g}}\Lambda(1 + \frac{1}{2}h^{\sigma}_{\sigma} + \frac{1}{8}(h^{\sigma}_{\sigma})^2 - \frac{1}{4}h_{\sigma\lambda}h^{\sigma\lambda} + ...)$$



(a) (b) (c)

$$-i\mathcal{M} = i\frac{m^4}{32\pi^2}\left(\frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{\mu\nu}h_{\mu\nu}\right)\left[\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m^2} + \frac{3}{2}\right]$$

Tadpole diagram can have no momentum flow through it

But also $\mu\frac{\partial\mathcal{M}}{\partial\mu} \neq 0$ does not imply physical running

No kinematic variable involved
Logarithm disappear when renormalized

## b) Nonlocal effective actions and running

### Example QED

$$S = \int d^4x \; -\frac{1}{4} F_{\rho\sigma} \left[ \frac{1}{e^2(\mu)} + b_i \ln\left(\Box/\mu^2\right) \right] F^{\rho\sigma}$$

With

$$\langle x | \ln\left(\frac{\Box}{\mu^2}\right) | y \rangle \equiv L(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \ln\left(\frac{-q^2}{\mu^2}\right) \quad .$$

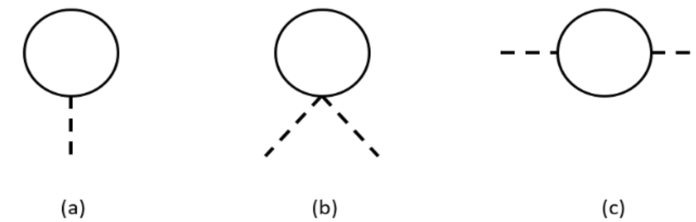
**There is true running in gravity at order $R^2$** (Barvinsky Vilkovisky)

$$S \sim \int d^4x \sqrt{-g} \left[ ... + c_1(\mu_R) R^2 + b_1 R \log(\Box/\mu_R^2) R + ... \right]$$

But these constructions do not work with CC and R

i.e. $\frac{2}{\kappa^2} R + b \Gamma_\mu \log(\Box/\mu_R^2) \Gamma^\mu$ is not covariant

## c) Non-local "partners"




There are residual energy scale dependences
- starting at order $h^2$

$$\mathcal{M}_{\mu\nu\alpha\beta} = \frac{1}{160\pi^2 q^4}\left(Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\alpha}\right)\left[m^4 J(q^2) + \frac{1}{6}m^2 q^2 - 3m^2 q^2 J(q^2)\right]$$

with $\quad Q_{\mu\nu} = q_\mu q_\nu - \eta_{\mu\nu} q^2 \quad$ and $\quad J(q^2) = \int_0^1 dx\ \log\left[\frac{m^2 - x(1-x)q^2}{m^2}\right]$

This is zeroth order in the derivative expansion (like cc)
- but only active above the scale $m$

When completed ala Barvinsky Vilkovisky:

$$\begin{aligned}\mathcal{L} \ &= \ \frac{m^4}{40\pi^2}\left[\left(\frac{1}{\Box}R_{\lambda\sigma}\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R^{\lambda\sigma}\right) - \frac{1}{8}\left(\frac{1}{\Box}R\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R\right)\right] \\ &+ \ \frac{m^2}{240\pi^2}\left[R_{\lambda\sigma}\frac{1}{\Box}R^{\lambda\sigma} - \frac{1}{8}R\frac{1}{\Box}R\right]\end{aligned}$$

## 7) Light cones etc likely uncontrolled approximations

Evident from bending calculations above

Corrections are tiny at low energy

But eventually become of order unity as EFT fails

Classical concepts seem to fail
- lightcones
- geodesics
- Penrose diagrams
- manifold structure
- causality ?

"Gravity is geometry" is a classical notion
- perhaps not best for the quantum theory

# Limits of the EFT - High Energy

## Expect GREFT to fail below or around $M_P$

- becomes strongly coupled $\frac{q^2}{M_P^2}\log q^2$

**Example:** QCD and Chiral Perturbation Theory

$\Lambda_\chi \sim 0.6$ GeV , $4\pi F_\pi \sim 1.2$ GeV , quark, gluon DOF $\sim 2$ GeV

But, parametrically decoupled

## Full field theory encoded in coefficients

**Example**: ChPTh

$$L_\mu = U\partial_\mu U^\dagger$$

$$\mathcal{L} = \frac{F_\pi^2}{4} Tr(L_\mu L^\mu) + c_1 [Tr(L_\mu L^\mu)]^2 + c_2 Tr([L_\mu, L_\nu][L^\mu, L^\nu])$$

- linear sigma model $c_1 \sim F_\pi^2/m_\sigma^2 \quad c_2 \sim 0$
- QCD $c_1 \sim 0 \quad c_2 \sim F_\pi^2/m_\rho^2$

**For GREFT,**

Large $c_1, c_2$ implies lower energy breakdown

## Limitations and Technical Challenges

**But also low energy challenges**

- basically gravity effects build up
- local terms use curvature expansion
- metric as variable
- metric grows between regions of small curvature
- nonlocal terms sample metric at distant points
- issue even for classical gravity

Not completely unique to gravity

- Skyrmions in chiral theories

But crucial for possibility of large quantum effects

## Consider Reimann normal coordinates

Taylor expansion in a local neighborhood:

$$g_{\mu\nu}(y) = \eta_{\mu\nu} + \frac{1}{3}R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}(y_0)y^\alpha y^\beta y^\gamma$$
$$+ \left[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta}(y_0) + \frac{2}{45}R_{\alpha\mu\beta\lambda}(y_0)R^\lambda{}_{\gamma\nu\delta}(y_0)\right] y^\alpha y^\beta y^\gamma y^\delta + \mathcal{O}(\partial^5)$$

**Even for small curvature,**
**there is a limit to a perturbative treatment of long distance:**

$$R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta \ll 1$$

# Horizons are extreme example:

- locally safe – we could be passing a BH horizon right now
    - local neighborhood makes a fine EFT
    - can be small curvature

But quantum effects sample long distance

Recent work on classical BH and decoherence
- Danielson, Satishchandran, Wald
- issue for all quantum theories

**EFT has some difficulties at long distances**
- what is the parameter governing the problem?
- integrated curvature?

## Favorite Quotes:

"A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

Frank Wilczek
Physics Today
2002

**Another thoughtful quote:**

"I also question the assertion that we presently have no quantum field theory of gravitation. It is true that there is no *closed*, internally consistent theory of quantum gravity valid at all distance scales. But such theories are hard to come by, and in any case, are not very relevant in practice. But as an *open* theory, quantum gravity is arguably our *best* quantum field theory, not the worst. ….

*{Here he describes the effective field theory treatment}*

From this viewpoint, quantum gravity, when treated –as described above- as an effective field theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances."

J.D. Bjorken

# Summary:

Phrasing issue as "QM incompatible with GR" is misleading

GR is a very normal quantum EFT

There are lessons about quantum gravity here

But there are also limitations / technical challenges

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i \int d^4x \sqrt{-g}\left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots\text{SM}\ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right)\right]$$