EFT, GR & QG

John Donoghue
Zakopane 2024

Game plan:

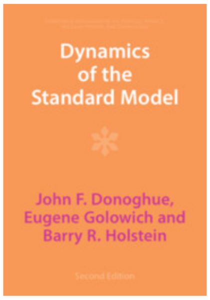
1) Effective Field Theory — pedagogic
   - beyond effective Lagrangians
   - EFT as a dynamical QFT

2) General Relativity as an EFT — rigorous
   - GR as a QFT – calculations
   - 7 lessons of EFT
   - limitations & unknowns

3) Quadratic Gravity — speculative
   - renormalizable QFT for gravity
   - emergent causality

Some resources:

1) "Dynamics of the Standard Model" is now free - download it



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2nd edition

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2) EPFL lectures

[Submitted on 1 Feb 2017]

EPFL Lectures on General Relativity as a Quantum Field Theory

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These notes are an introduction to General Relativity as a Quantum Effective Field Theory, following the material given in a short course on the subject at EPFL. The intent is to develop General Relativity starting from a quantum field theoretic viewpoint, and to introduce some of the techniques needed to understand the subject.

Comments: 70 pages

3) ISQG lectures

- 6 lectures on perturbative quantum gravity
+ others

Basics of Quantum Gravity

May 16, 2023 to November 16, 2023
Europe/Zurich timezone

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4) Web page with more resources

General Relativity as a Quantum Field Theory

MINI-COURSE FOR ISQG OVERVIEW TRISEP 2023 EPFL LECTURES ON GR AS A QFT NORDIC WINTER SCHOOL 2019 A COURSE EFFECTIVE FIELD THEORY LITERATURE

Collected references and lectures

Today: Effective Field Theory

I will assume that everyone knows "EFT 1.0"
- one can add "effective Lagrangian" to represent heavy physics
- local, nonrenormalizable

ex. $\mathcal{H}_W = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu$

<u>Message here: EFT is real QFT</u>

- loops
- renormalizing the nonrenormalizable
- nature of quantum prediction

<u>Subsections:</u>

1) Linear/nonlinear sigma model
   - explicit construction of EFT
   - calculations
   - why results are reliable

2) Background field method (briefly)
   - if time

Linear Sigma Model: $(\sigma, \phi_1, \phi_2, \phi_3)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\phi}\cdot\partial^\mu \vec{\phi} - \frac{\mu^2}{2}(\sigma^2+\vec{\phi}^2) + \frac{\lambda}{4}(\sigma^2+\vec{\phi}^2)^2$$

Symmetry $\Sigma \equiv \sigma + i\,\vec{\tau}\cdot\vec{\phi}$ $\qquad \sigma^2+\vec{\phi}^2 = \frac{1}{2}\mathrm{Tr}(\Sigma\Sigma^\dagger)$

Pauli

$$\Sigma \to \underline{V_L}^\dagger\, \Sigma\, \underline{V_R}$$

SSB $\quad \langle\sigma\rangle = v = \sqrt{\mu^2/\lambda}$

A) $\sigma \equiv v + \tilde{\sigma}$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu\tilde{\sigma}\,\partial^\mu\tilde{\sigma} - m_\sigma^2\tilde{\sigma}^2\right] + \frac{1}{2}\partial_\mu\vec{\phi}\,\partial^\mu\vec{\phi}$$
$$- \lambda v\tilde{\sigma}(\tilde{\sigma}^2+\vec{\phi}^2) - \frac{\lambda}{4}(\tilde{\sigma}^2+\vec{\phi}^2)^2$$

B) $\Sigma = \sigma + i\vec{\tau}\cdot\vec{\phi} = (v+s)\,e^{i\vec{\tau}\cdot\vec{\pi}/v} = (v+s)\,U$

$\tilde{\sigma} = s + \cdots \qquad \phi = \vec{\pi} + \cdots$

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu s\,\partial^\mu s - m_\sigma^2 s^2\right] + \frac{1}{4}(v+s)^2\,\underline{\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)}$$
$$- \lambda v s^3 - \frac{\lambda}{4}s^4$$

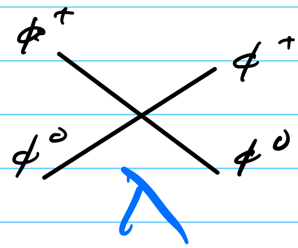
renorm.!

Symmetry visible $\quad U \to V_L^\dagger\, U\, V_R$

Haag's theorem ~ names don't matter – on shell

Check – same scattering amplitudes

A)



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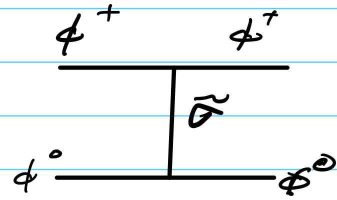
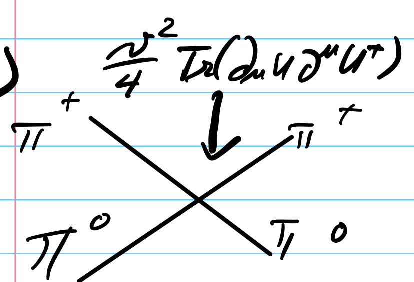


$$= -2i\lambda\left[1 + \frac{2\lambda v^2}{q^2 - m_\sigma^2}\right]$$

$m_\sigma^2 = 2\lambda v^2$

$$= i\frac{q^2}{v^2}\left[1 + \frac{q^2}{m_\sigma^2} + \cdots\right]$$

B)



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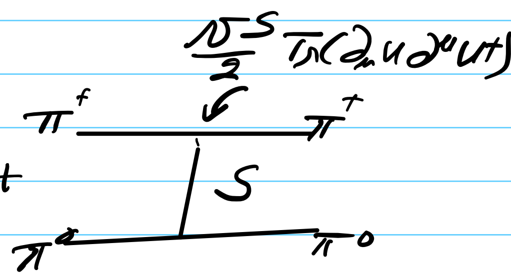
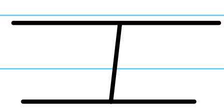


$$= i\frac{q^2}{v^2} \quad + \quad i\frac{q^4}{v^2 m_\sigma^2} \quad + \cdots$$

Effective Lagrangians   S is heavy

Lowest order $\mathcal{L}_2 = \frac{v^2}{4}Tr(\partial_\mu U \partial^\mu U^+)$

Next order   Propagator —— $= -1/m_\sigma^2$



$$\mathcal{L}_4 = \frac{v^2}{8m_\sigma^2}\left[Tr(\partial_\mu U \partial^\mu U^+)\right]^2$$

$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4$ energy expansion (tree level matching)

This is EFT 1.0
– holds for all matrix elements

But now Loops

Problems:

- Equiv. only at low E
- loops sample all E $\Longrightarrow$ wrong
- $\mathcal{L}_{eff}$ is non-renormalizeable

Solution: Uncertainty Principle

- wrong parts of loops look local
  $\Longrightarrow$ in local $\mathcal{L}_{eff}$
- correct parts of loops from low E
  - distinct from local $\mathcal{L}_{eff}$

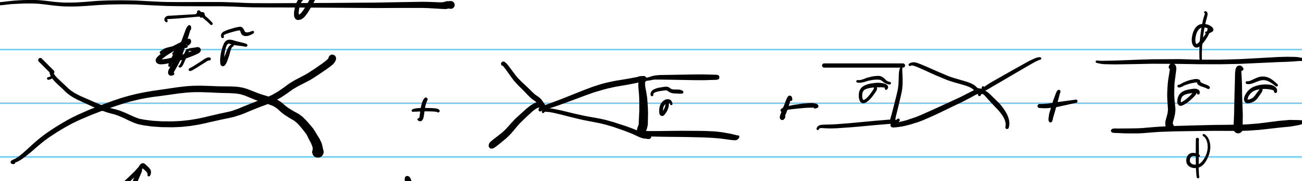
Explicit Example:

General $\mathcal{L}_{eff}$

$$\mathcal{L}_{eff} = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \ell_1 \left[\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)\right]^2$$

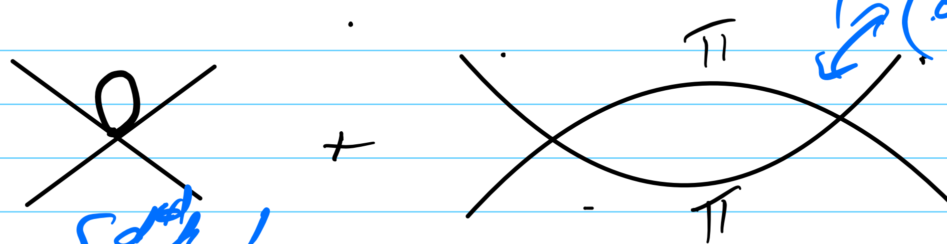
$$+ \ell_2 \mathrm{Tr}\left(\partial_\mu U \partial_\nu U^\dagger\right) \mathrm{Tr}\left(\partial^\mu U \partial^\nu U^\dagger\right)$$

Some diagrams:

A) 

renorm of λ
+ finite

$$i\mathcal{M}_{\text{full}} = \int \frac{d^4k}{(2\pi)^4}\left[-2i\lambda + (-2i\lambda v)^2 \frac{i}{(k+p_+)^2 - m_\sigma^2}\right]\frac{i}{(k+p_+ + p_0)^2}\,\frac{i}{k^2} \times \left[-2i\lambda + (-2i\lambda v)^2 \frac{i}{(k+p'_+)^2 - m_\sigma^2}\right]. \tag{3.4}$$

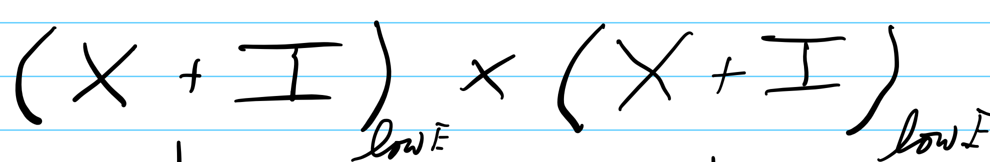
B) 

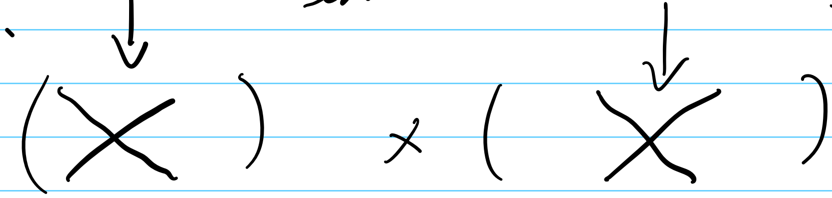
$T_\mu(\partial_\mu \alpha\, \partial^a U^*)$

$\int d^4k \, \frac{1}{k^2 + i\epsilon}$

$$i\mathcal{M}_{\text{eff}} = \int \frac{d^4k}{(2\pi)^4}\,\frac{i(k+p_+)^2}{v^2}\,\frac{i}{(k+p_+ + p_0)^2}\,\frac{i}{k^2}\,\frac{i(k+p'_+)^2}{v^2}.$$

Why can this work?

A) 

B) 

$\Longrightarrow$ Low E parts are correct

But 1) B is divergent
2) B is not the renorm of λ *

Results

$s = (p_1 + p_2)^2 = E_{cm}^2$
$t = q^2 = (p_1 - p_3)^2$
$u = (p_1 - p_4)$

A) The full theory at low energy

$$\mathcal{M}_{\text{full}} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2$$
$$- \frac{1}{144\pi^2 v^4}[s(s-u) + u(u-s)]$$
$$- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]$$

B) The EFT

$$\mathcal{M}_{\text{eff}} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4}$$
$$+ \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4$$
$$- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right]$$

if we identify

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$
$$\ell_2^r = \ell_2 + \frac{1}{192\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

Matching

– identical if

$$\ell_1^r = \frac{v^2}{8m_\sigma^2} + \frac{1}{192\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2} - \frac{35}{6}\right]$$
$$\ell_2^r = \frac{1}{384\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2} - \frac{11}{6}\right].$$

Various points:

renormalized a "nonrenormalizeable" theory

- locality

What are the EFT predictions?

- not the l_i
- **logs** – not local $\mathcal{L}_{eff}$

Limits of EFT

- at best $\frac{E^2 \log E^2}{96\pi^2 v^2} \sim 1$
- but could be lower

$$\frac{E^2}{m_\sigma^2} \sim 1$$

Same renorm works for all reactions

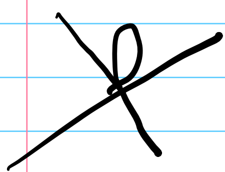
- not yet obvious
- symmetry
- Background Field Method

v^2 is not a running coupling

- $O(E^2)$ corrections not running
- power counting theorem
  - one loop ~ $O(E^4)$

Caution on cutoff renorm:

$$\mathcal{L}_{eff} = \frac{1}{\Lambda^2}\partial_\mu\pi\,\partial^\mu\pi\,\pi^2 + \frac{1}{\Lambda^4}\partial_\mu\pi\,\partial^\mu\pi\,\pi^4 + \cdots$$



$$\sim \int d^4k \, \frac{k^2}{k^2+i\varepsilon} + \int d^4k \, \frac{1}{k^2+i\varepsilon} E^2$$

$$\sim \underbrace{\Lambda^4 \times \text{const}}_{\text{violates symmetry}} + \underbrace{\Lambda^2 E^2}_{\text{renorm } \Lambda^2}$$

resolution: new Feynman rule $\sim \delta^4(0)$
— Gerstein, Jackiw, Lee, Weinberg PRD 3, 2486 (1971)

<u>also</u>: Λ^2 renorm of Λ^2 is not running of the physical amplitude

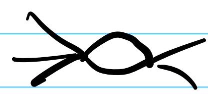
Background Field Method (briefly)

Utility

- same renorm. for all processes
- symmetry clear

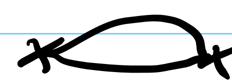
Basic idea:

Normally expand $\mathcal{L}_{int}$ first, then identify loop particle

$V(\phi) \rightarrow$ 

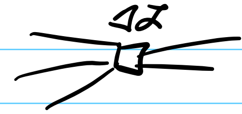
BFM: write $\phi = \bar{\phi} + \delta\phi \leftarrow$ loop particle

$$V \sim V''(\bar{\phi})\,(\delta\phi)^2$$

do loop first  $\Rightarrow \Delta\mathcal{L}$

then take matrix elements

$\Delta\mathcal{L}$



Example: NLSM

$$U = \bar{U}\, e^{i\vec{\tau}\cdot\vec{\Delta}/v}$$

$$\mathcal{L} = \mathcal{L}(\bar{U}) + \mathcal{L}_1(\bar{U},\Delta) + \mathcal{L}_2(\bar{U},\Delta)$$

$\mathcal{L}_1$: first order, vanish by EoM

$\mathcal{L}_2$: second order in Δ

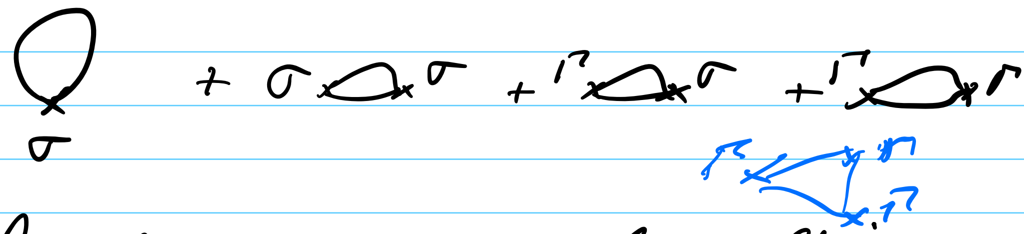
General form:

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu \Delta^i \partial^\mu \Delta^i + \Delta^i f_\mu^{ij}(\bar{u})\,\partial^\mu \Delta^j + \Delta^i g^{ij}(\bar{u})\,\Delta^j$$

$$= \frac{1}{2}\Delta^i \left[(\partial_\mu + \Gamma_\mu)(\partial^\mu + \Gamma^\mu) + \sigma\right]^{ij} \Delta^j$$

Now can do loops:

— one way is usual Feynman diagrams



In dim reg, general result is

$$\Delta \mathcal{L} = \frac{1}{16\pi^2 \varepsilon}\left[\frac{1}{12}\Gamma_{\mu\nu}\Gamma^{\mu\nu} + \frac{1}{2}\sigma^2\right]$$

$$\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu]$$

Recall $\Gamma_\mu = \Gamma_\mu(\bar{U})$, $\sigma = \sigma(\bar{U})$ in NLSM

After some algebra

$$\Delta \mathcal{L} = \frac{1}{384\pi^2}\frac{1}{\varepsilon}\left\{\left[\mathrm{Tr}(\partial_\mu \bar{U} \partial^\mu \bar{U}^\dagger)\right]^2 + 2\,\mathrm{Tr}(\partial_\mu \bar{U} \partial_\nu \bar{U}^\dagger) \times \mathrm{Tr}(\partial^\nu \bar{U} \partial^\mu \bar{U}^\dagger)\right\}$$

Symmetry ✓

Renorm all processes ✓

Notes:

— Universal method

— "Heat kernel" method easier

— Appendix B of DSM

— Also can find logs as nonlocal action

$$\int d^4x\, d^4y\; \Gamma_{\mu\nu}(x) \langle x | \ln \Box | y \rangle \Gamma^{\mu\nu}(y)$$

Barvinsky

Vilkovisky

$$\langle x | \ln \Box | y \rangle = \int \frac{d^4 q}{(2\pi)^4} \ln(-q^2)\, e^{i q\cdot(x-y)}$$