Gravitational Waves and Stellar-Mass Black-Hole Mergers

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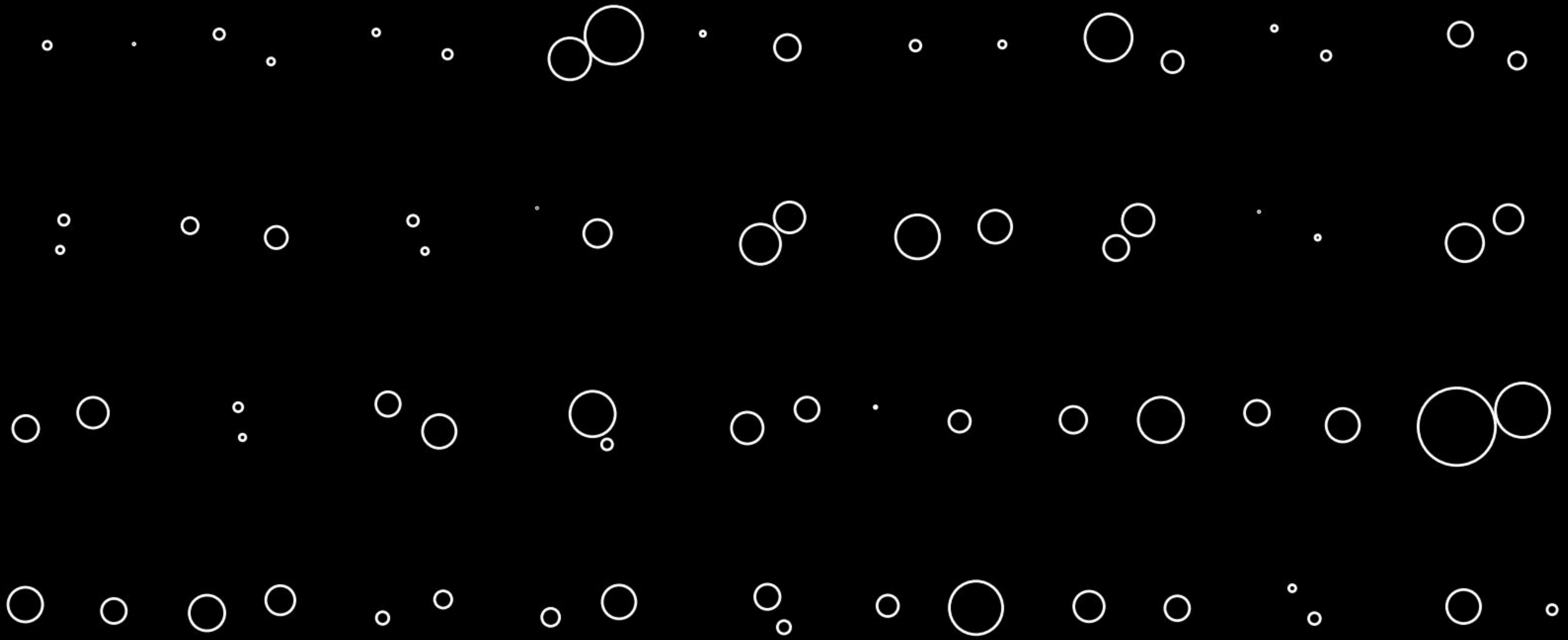
Northwestern
CIERA
CENTER FOR INTERDISCIPLINARY EXPLORATION AND RESEARCH IN ASTROPHYSICS

@actualdrdoctor

LIGO VIRGO KAGRA



Gain a working knowledge of gravitational wave detections, sources, and implications*

* Primarily relating to stellar-mass compact objects

Overview

01.
Theory of Gravitational waves

02.
Detecting Gravitational Waves

03.
Characterizing GW Sources

04.
Catalog of GW Sources

05.
Astrophysical Black-Hole Mergers

06.
Population Inference

A gentle introduction to GWs and binary sources

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta},$$

Small perturbations *h* to background metric

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}^{\alpha\beta} = -16\pi T^{\alpha\beta}.$$

Wave equation w/ strain tensor *T* (Lorentz gauge)

$$\bar{h}_{\alpha\beta} = A\mathbf{e}_{\alpha\beta} \exp(ik_\gamma x^\gamma),$$

Wave solutions in vacuum

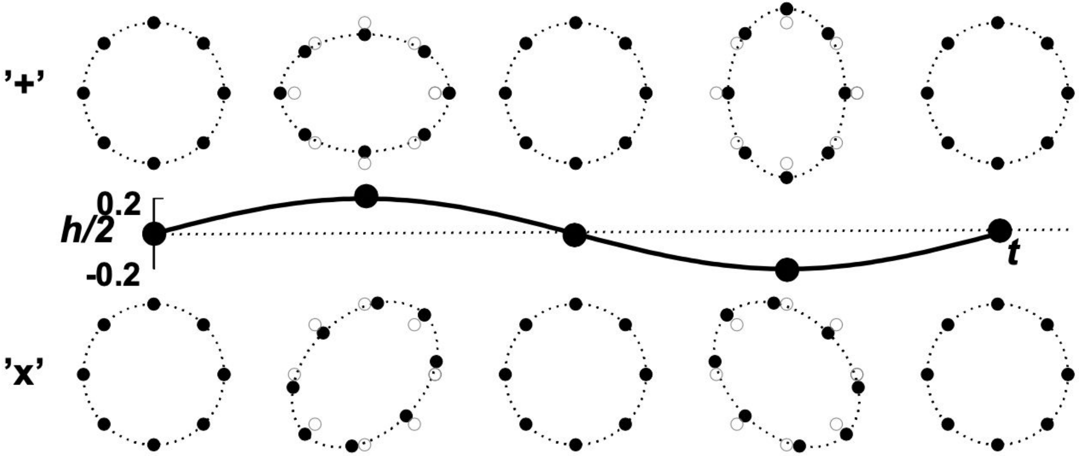
Schutz and Ricci arxiv:1005.4735

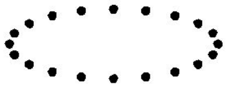
Transverse-Traceless Gauge

$$\mathbf{e}^{ij}k_j = 0$$ transverse

$$\mathbf{e}^i{}_i = 0$$ traceless

Two independent polarizations,
aka “plus” (+) and “cross” (x)





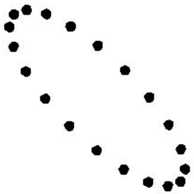


Image from https://arxiv.org/pdf/1005.4735

Quadrupole formula

$$\bar{h}^{\mathrm{TT}ij} = \frac{2}{r}\ddot{M}^{\mathrm{TT}ij}$$

At leading order, strain is sourced by time-varying quadrupole moment *M*

$$L_{gw}^{mass} = \frac{1}{5}\left\langle \dddot{\tilde{M}}^{jk}\dddot{\tilde{M}}_{jk}\right\rangle$$

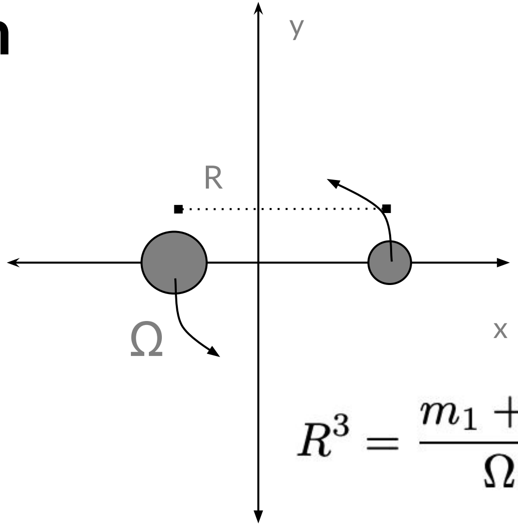
GW luminosity from mass quadrupole

GWs from a quasi-Circular Binary System

$$M_{xx} = \frac{1}{2}\mu R^2 \cos(2\Omega t)$$

$$M^{\mathrm{TT}xx} = M^{xx}/2.$$

Components of the quadrupole moment



$$R^3 = \frac{m_1 + m_2}{\Omega^2}$$

$$\bar{h}^{\mathrm{TT}xx} = -2^{1/3}\frac{\mathcal{M}^{5/3}\Omega_{gw}^{2/3}}{r}\cos\left[\Omega_{gw}(t-r)\right],$$

$$L_{gw} = \frac{4}{5\cdot 2^{1/3}}\left(\mathcal{M}\Omega_{gw}\right)^{\frac{10}{3}},$$

Solve for strain and luminosity

$$\mathcal{M} := \mu^{3/5}(m_1+m_2)^{2/5}$$

“Chirp mass”

Energy Balance of GW-Emitting Binary

GW luminosity = orbital energy loss rate

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left(\left(\frac{5}{96}\right)^3 \pi^{-8} f_{GW}^{-11} \dot{f}_{GW}^{3}\right)^{1/5}$$

Straightforward relation between observables and binary parameters!!!!!!!!

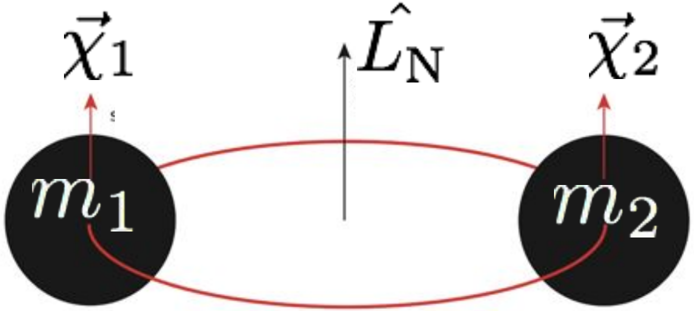
Chirping binaries are the ultimate astrophysical source

Unimpeded by foregrounds

No calibration to other astro sources

Straightforward astrophysical inference

Effect of spins (lowest order)

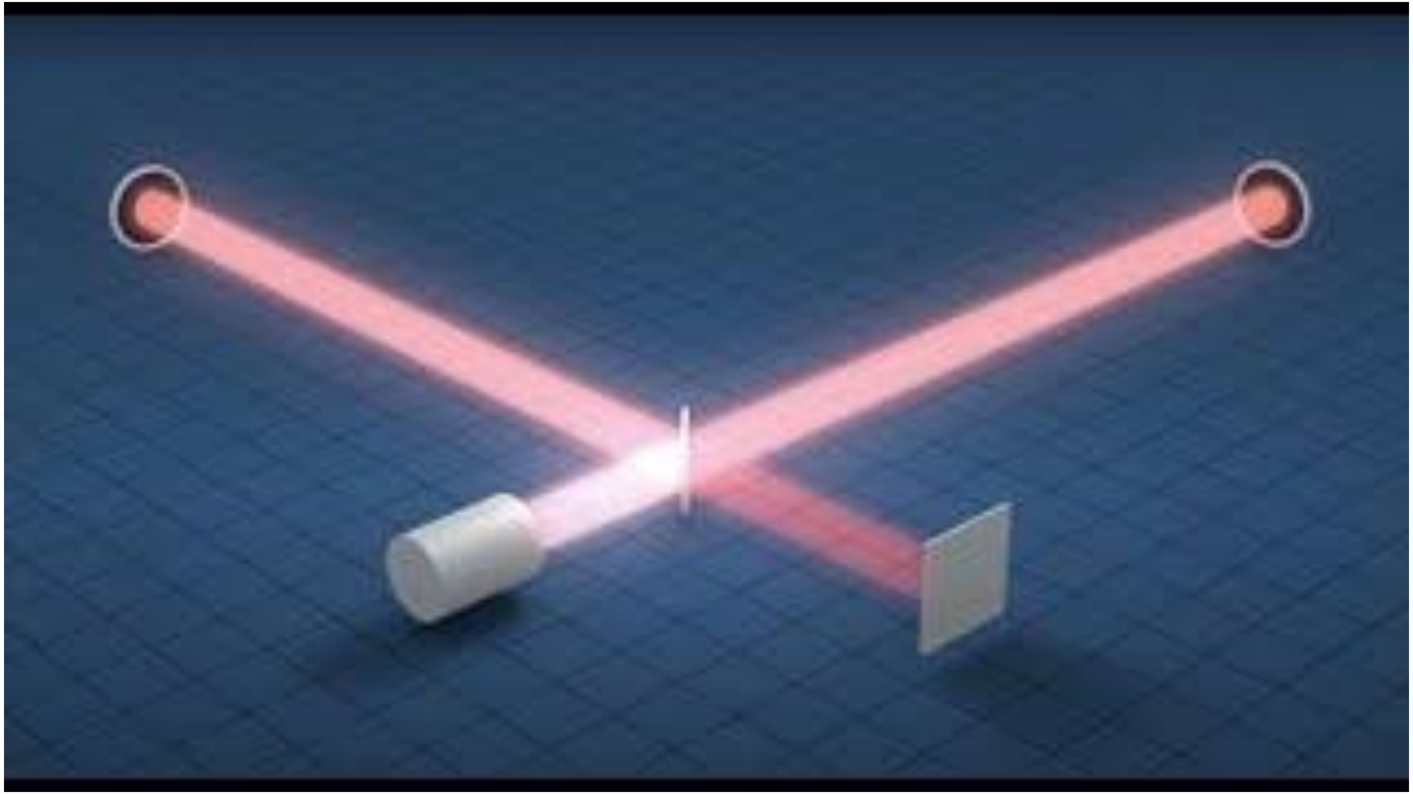


$$\chi_{\rm eff} = \frac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{L}_{\rm N}}{M}$$

Effective Inspiral Spin Parameter

How can we detect these GWs?

Interferometry

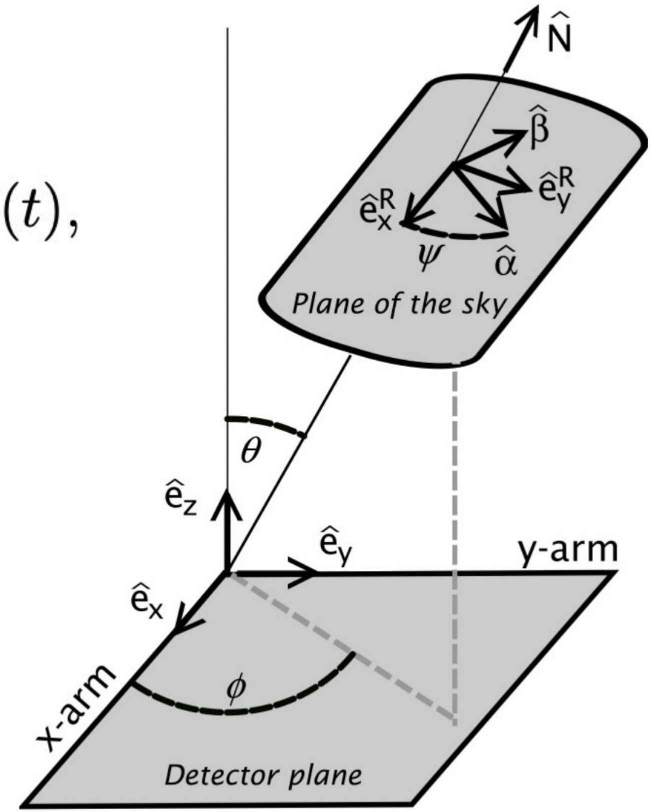


Credit: T. Pyle

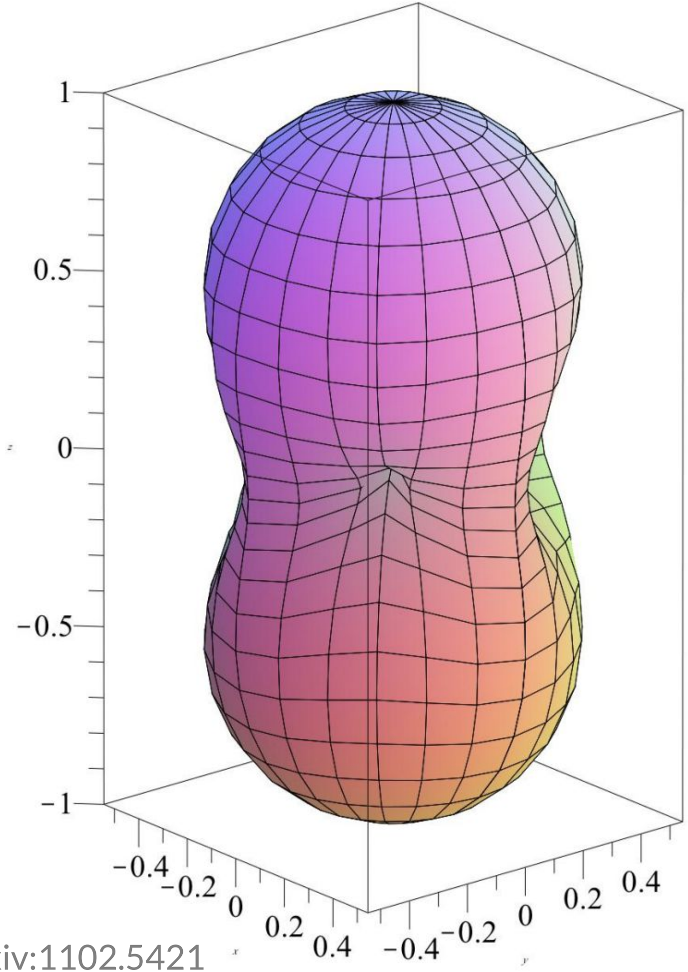
Interferometer response

$$\frac{\delta L(t)}{L} = F_+(\theta,\, \phi,\, \psi) h_+(t) + F_\times(\theta,\, \phi,\, \psi) h_\times(t),$$

$$F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi \cos 2\psi - \cos\theta \sin 2\phi \sin 2\psi,$$
$$F_\times = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi \sin 2\psi + \cos\theta \sin 2\phi \cos 2\psi.$$



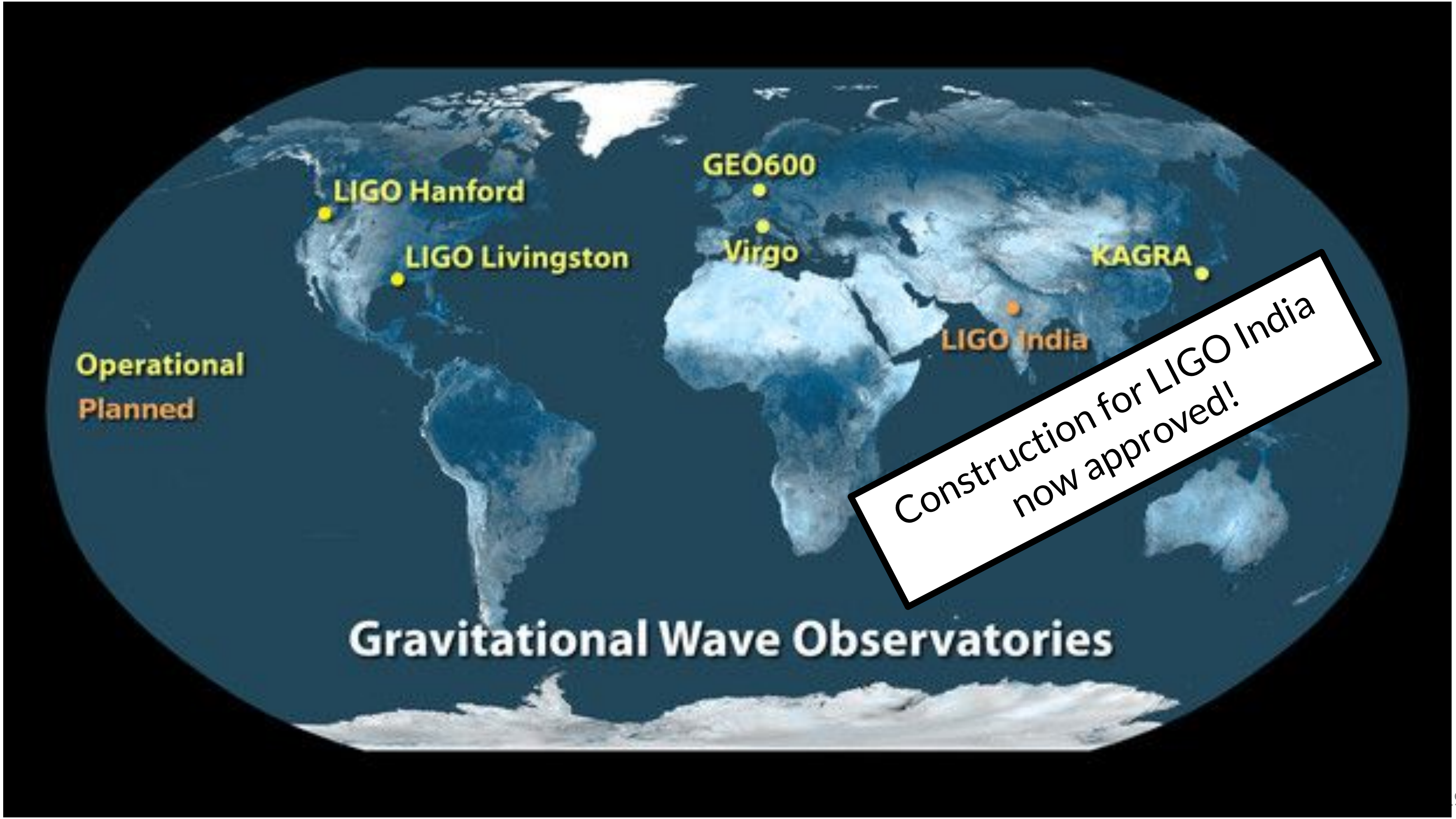
Credit: arxiv:1102.5421



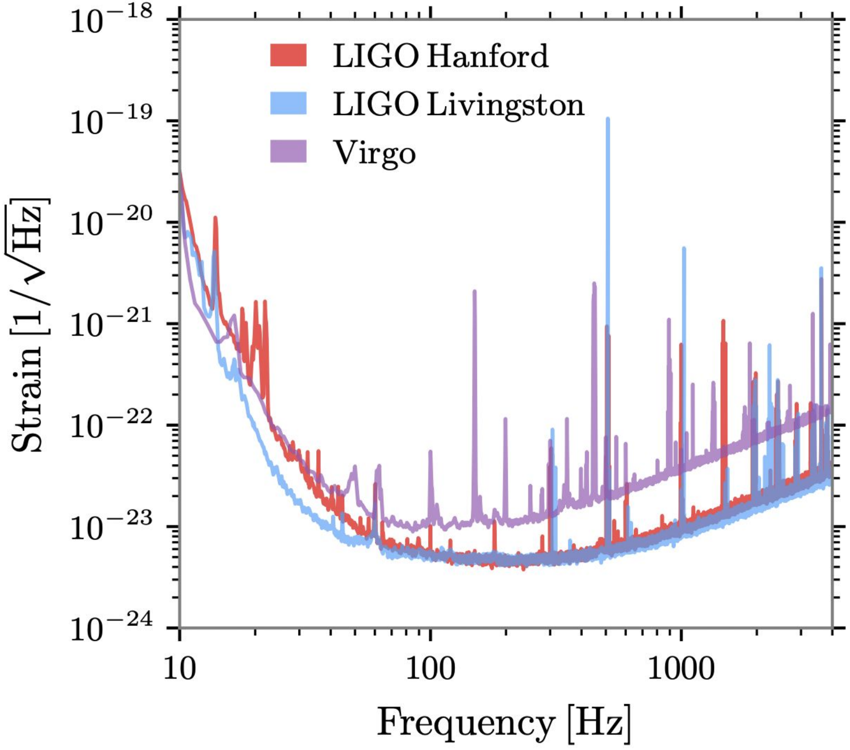
Amplitude response of interferometer
(averaged over polarizations)

Credit: arxiv:1102.5421


KAGRA


GEO600
LIGO Hanford
LIGO Livingston
Virgo
KAGRA
LIGO India
Operational
Planned
Gravitational Wave Observatories
Construction for LIGO India now approved!

Detectors are noisy!



The amplitude spectral density of noise in detectors

Sources of noise include:

- Shot noise
- Thermal noise
- Ground motion
- Newtonian noise

LVK Phys. Rev. X; 13(4):041039

Statistical distribution of detector noise

$$p(\{n(t_i)\}_i) = \frac{1}{\sqrt{2\pi|C|}} \exp\left\{-\frac{1}{2} n^\top C^{-1} n\right\}$$

Assume Gaussian noise*

$$p(\tilde{n}(f)) \sim \frac{1}{\int 2\pi S_n(f) df} e^{-\frac{1}{2}\langle n|n\rangle}$$

Covariance is diagonal (PSD S_n) in frequency domain

$$\langle a|b\rangle = 2\int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f) + \tilde{a}^*(f)\tilde{b}(f)}{S_n(f)} df$$

"Noise-weighted inner product"

* noise is not truly Gaussian current detectors

Matched Filtering

$$\hat{d}(t) = \int_{-\infty}^{\infty} d(t')K(t-t')dt'$$

Filter the data with a template *K* that maximizes the signal-to-noise ratio

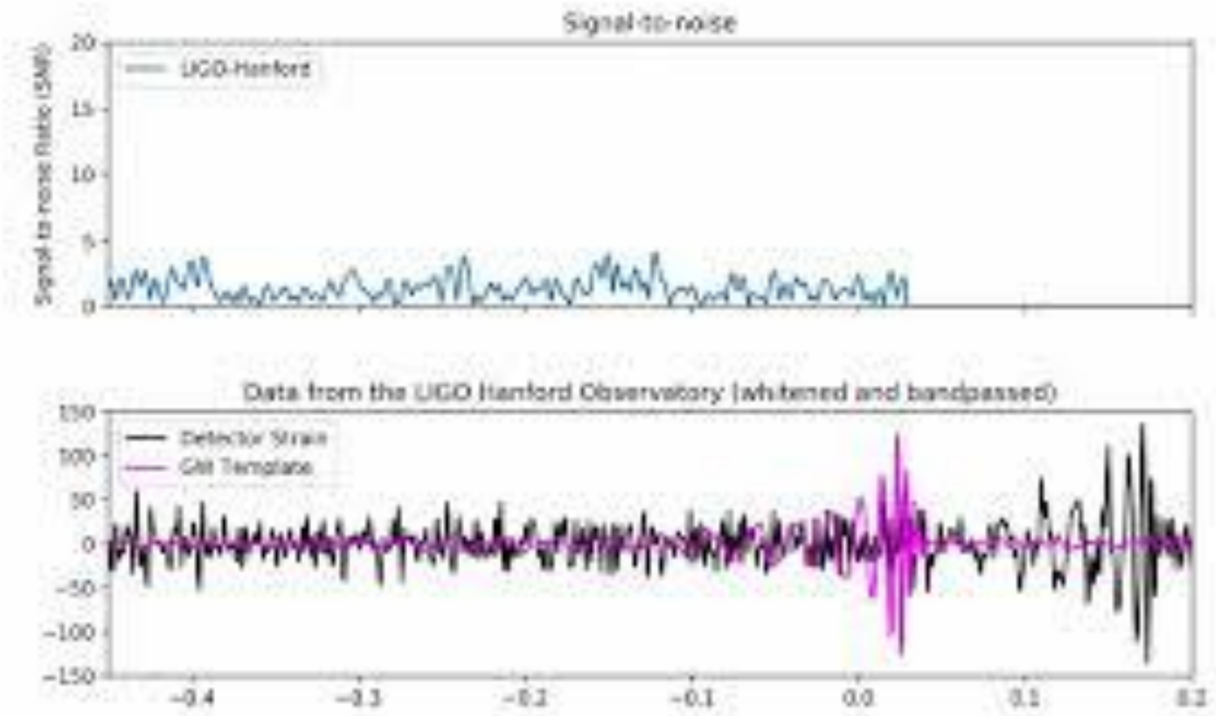
$$\tilde{K}(f) = c\frac{\tilde{h}(f)}{S_n(f)}$$

Optimal filter is the signal weighted by the PSD

$$\rho^2(t) = \frac{\langle d|h\rangle^2}{\langle h|h\rangle}$$

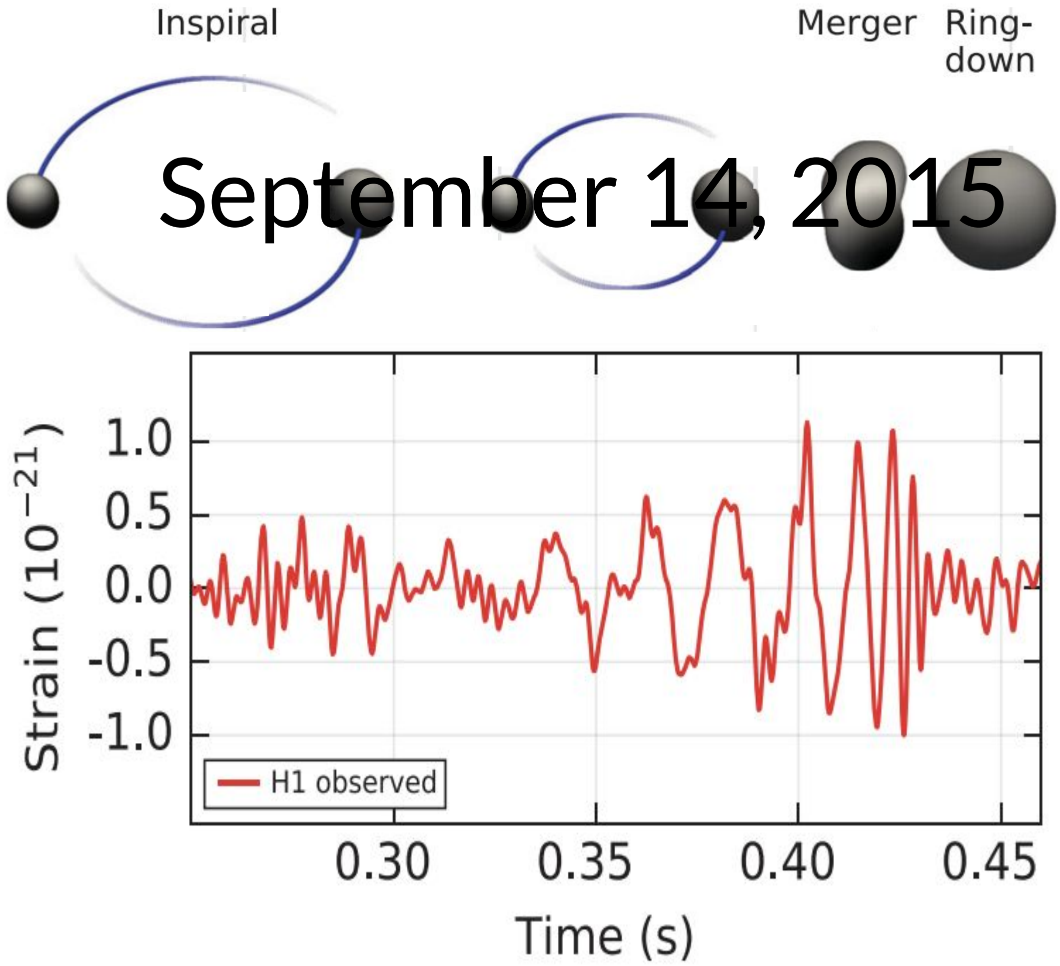
"Matched-filter signal-to-noise ratio"

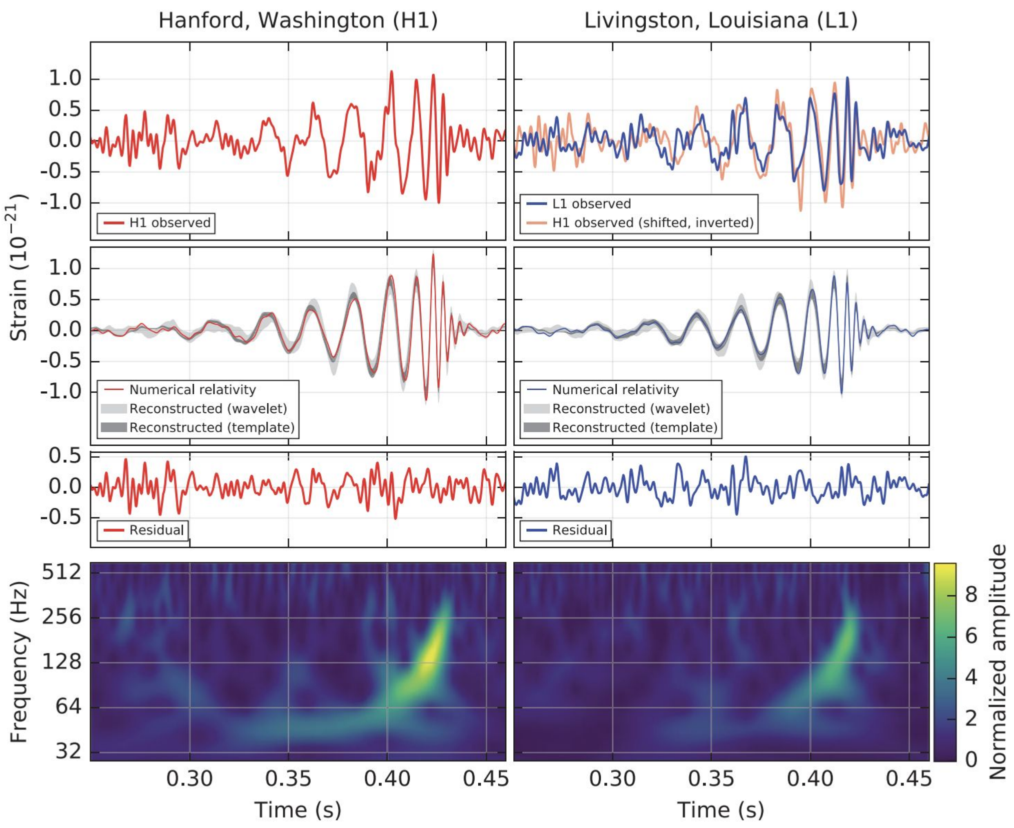
Matched Filtering in Action



Credit: Alex Nitz

September 14, 2015





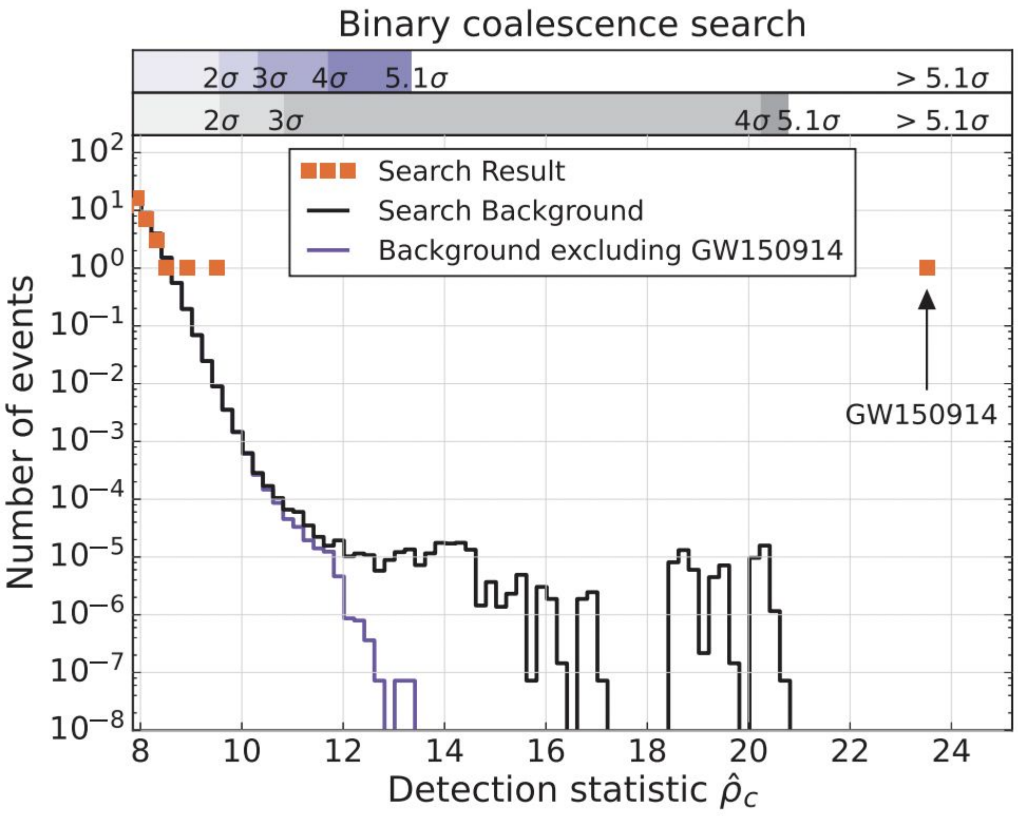
The first gravitational-wave detection:

GW150914

LIGO Scientific and Virgo Collaborations (2016), Phys. Rev. Lett. 116, 061102

Noise can produce high SNR…

…but loud GW will produce even higher SNR!



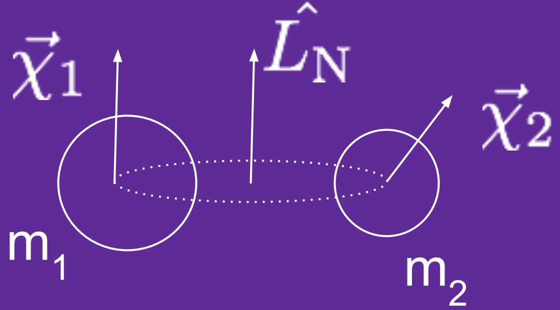
LIGO Scientific and Virgo Collaborations (2016), Phys. Rev. Lett. 116, 061102

- ~ 30 $M_\odot$ BHs 🏋️
- ~ 0.5 Gpc away (z~0.1) 📏

How do we measure source properties, including uncertainties?

Bayesian Estimation of Source Parameters

$$p(\vec{\vartheta}|\vec{d}) \propto p(\vec{d}|\vec{\vartheta})\pi(\vec{\vartheta})$$



Source Parameters ϑ: Masses (m_1, m_2) and 3-D dimensionless spin vectors (χ_1,χ_2) of the two coalescing objects, luminosity distance, sky position,...

Data *d*: Strain in all operating detectors

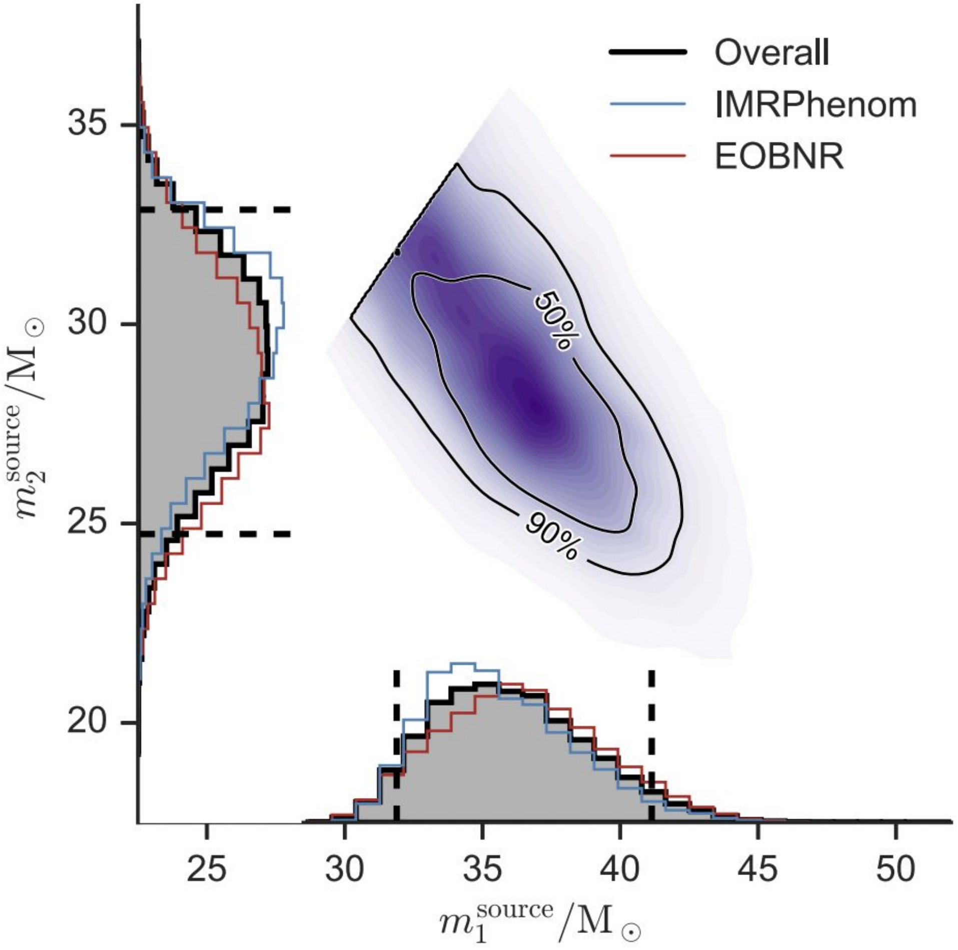
Likelihood: Gaussian in residuals between strain data and model

Priors: Up to analyst, but we will revisit…

Parameter Estimation in Practice

- GW source posterior is 15+ dimensions!
- Draw samples from the posterior distribution
- Use samples to perform monte carlo integrals over the posterior

$$\int f(\theta)p(\theta|d)d\theta \approx \langle f(\theta)\rangle_{\mathrm{samples}}$$

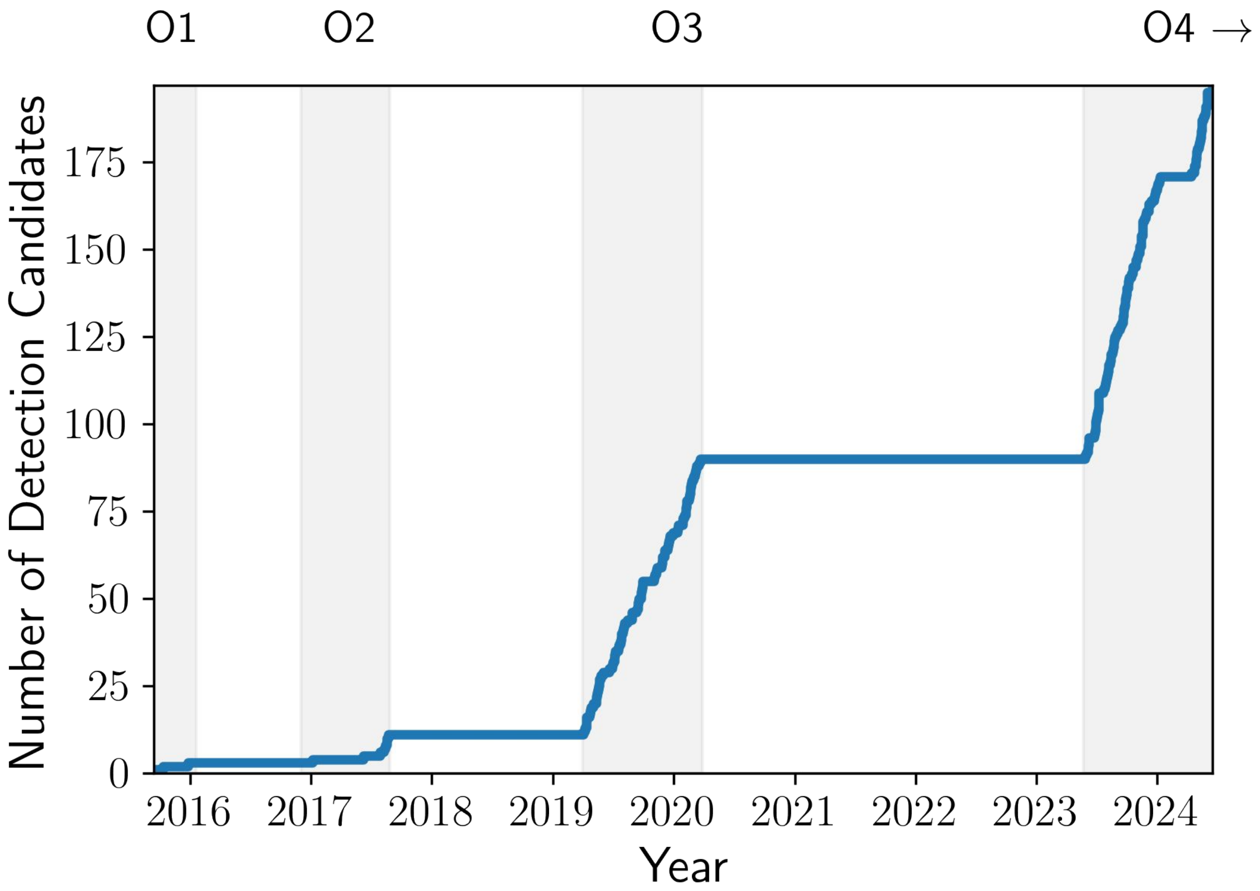


Use “**approximants**” to GR to quickly evaluate h(t) for any source parameters (m_1, m_2, …)

- Effective one-body (EOB) family: include strong-field effects in test particle limit + calibration to NR
- IMRPhenom family: stitch PN and EOB results to NR
- Surrogate: interpolate NR simulations

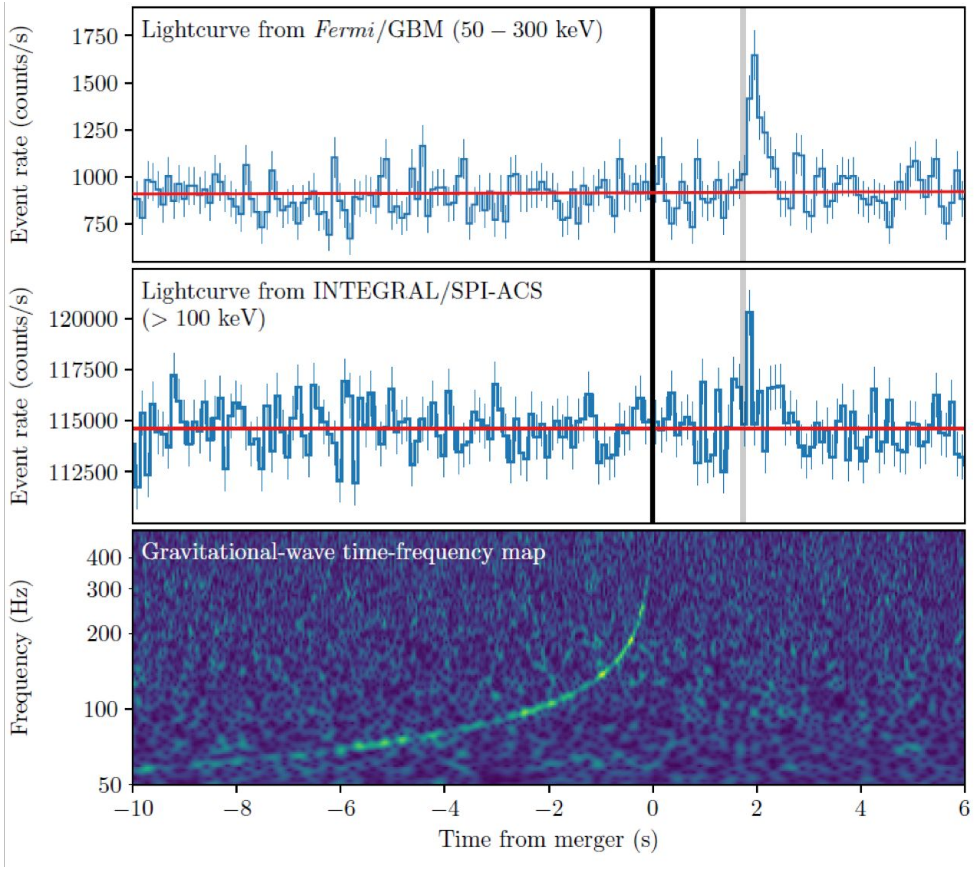
LIGO Scientific and Virgo Collaborations 10.1103/PhysRevLett.116.241102

Catalog of Gravitational Wave Detections

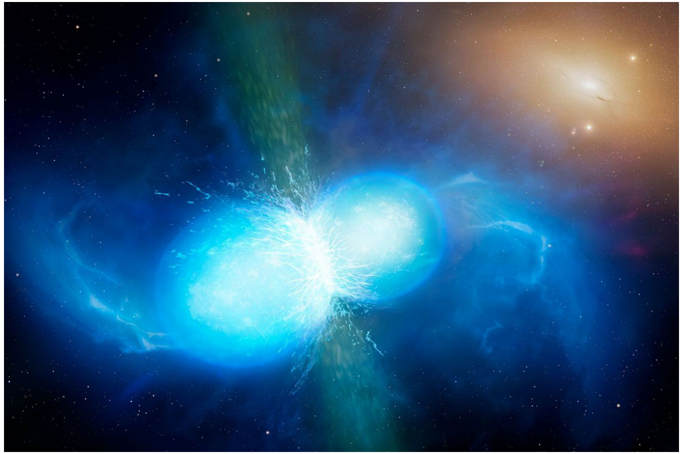


https://git.ligo.org/zoheyr-doctor/plot-gracedb-events

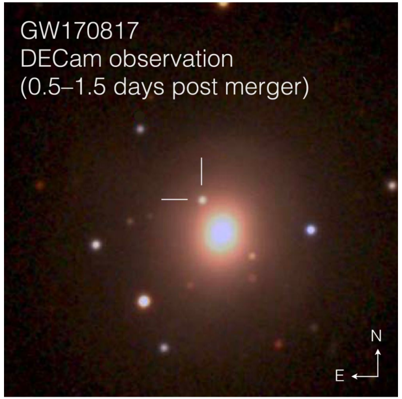
GW170817: Neutron-Star Merger w/ EM counterpart!



GW170817 Multi-messenger ApJL 848 L12 (2017)

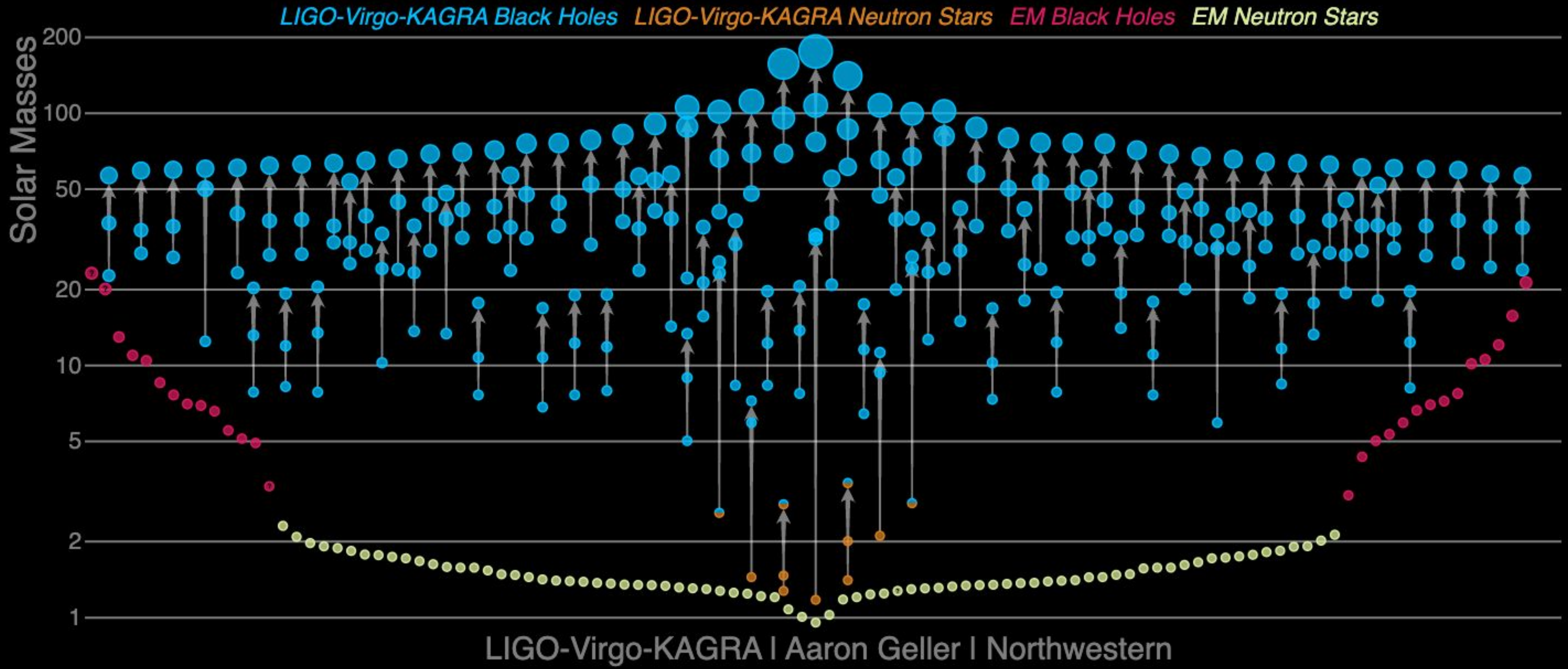


Credit: University of Warwick/Mark Garlick

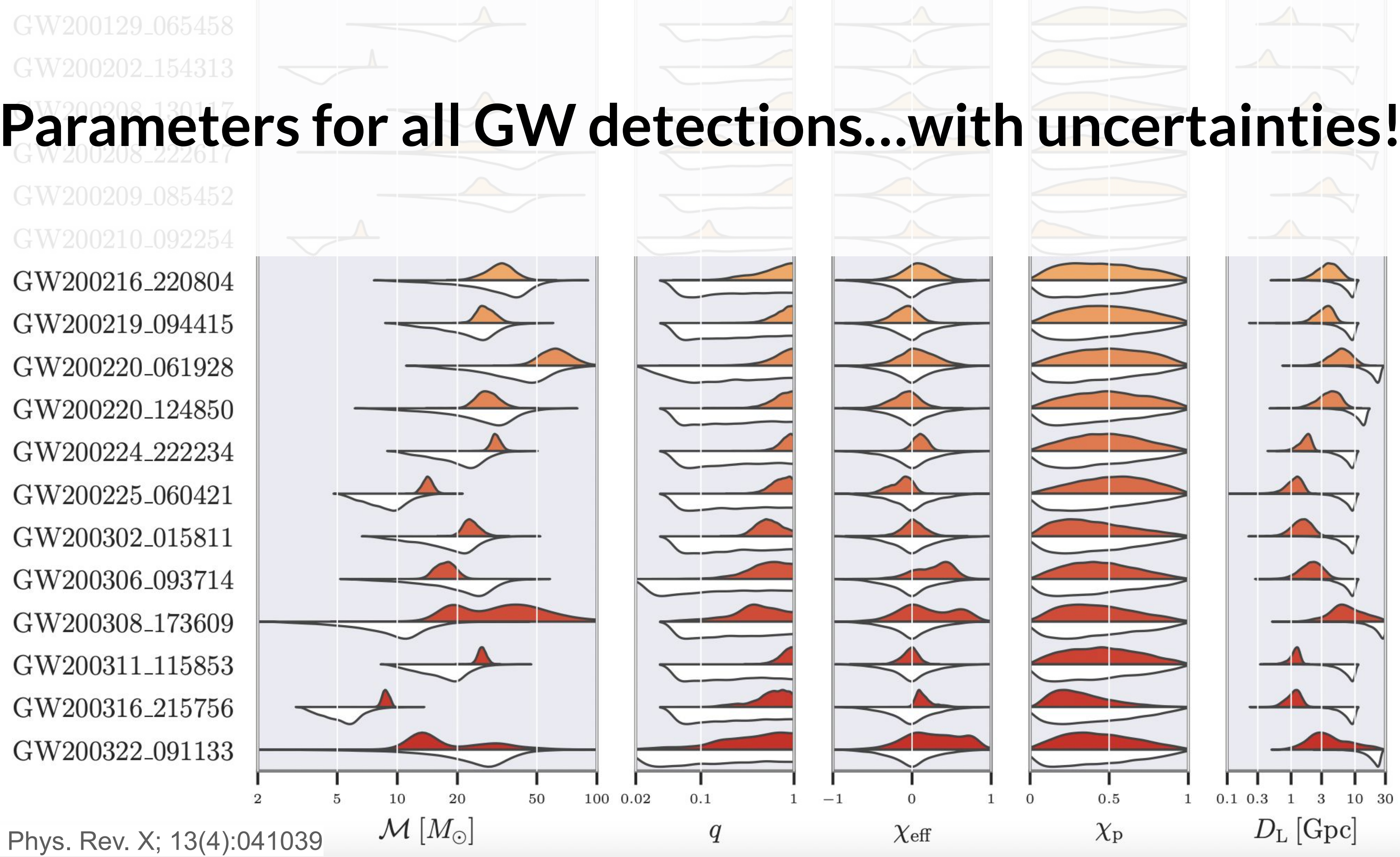


Soares-Santos,..., **ZD**,... ApJL 848 L16 (2017)

Masses in the Stellar Graveyard

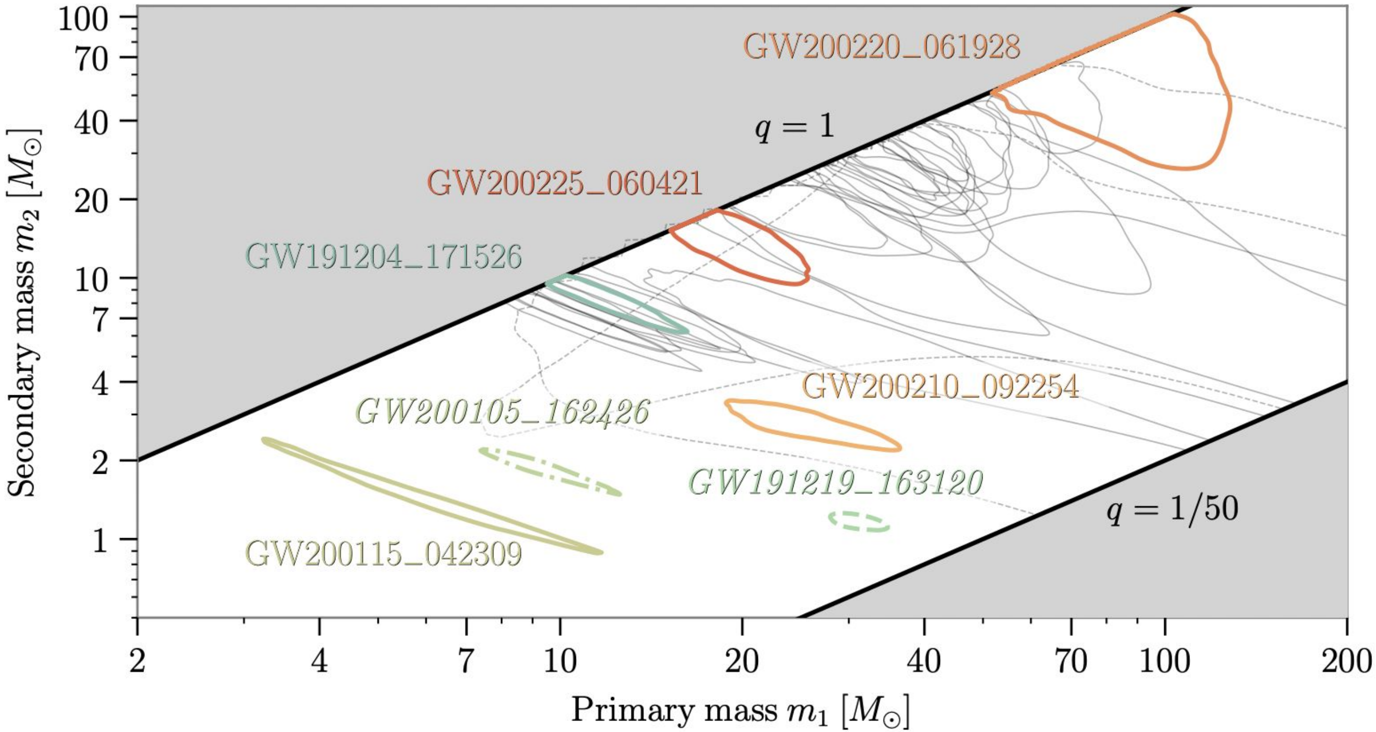


Parameters for all GW detections…with uncertainties!



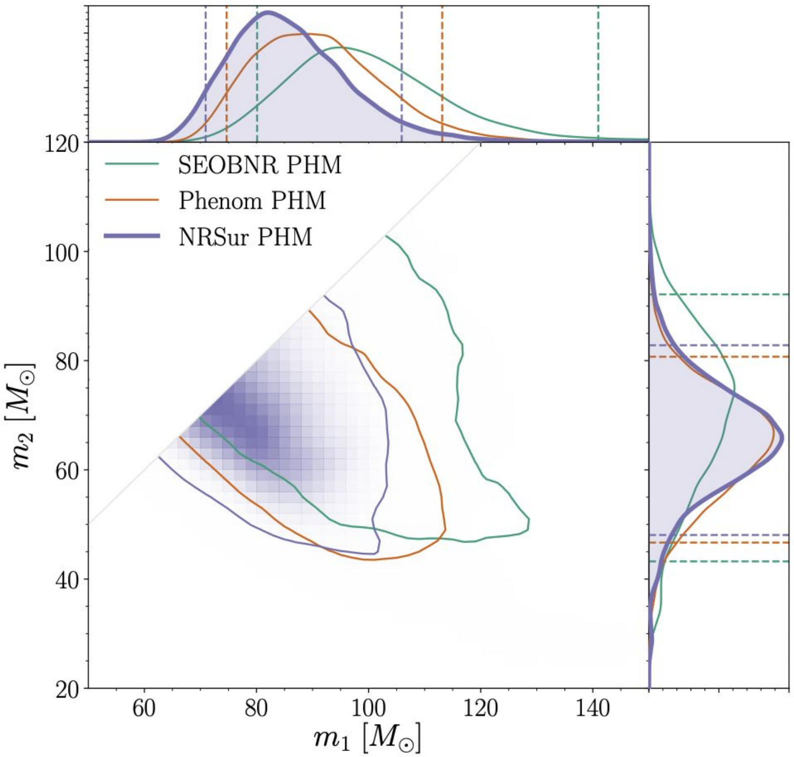
LVK, Phys. Rev. X; 13(4):041039

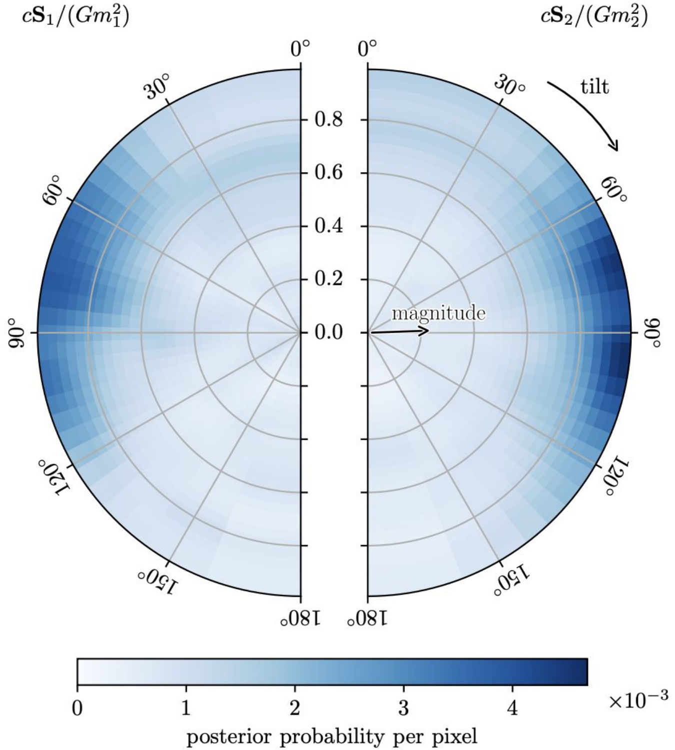
Joint posterior samples for all events and parameters



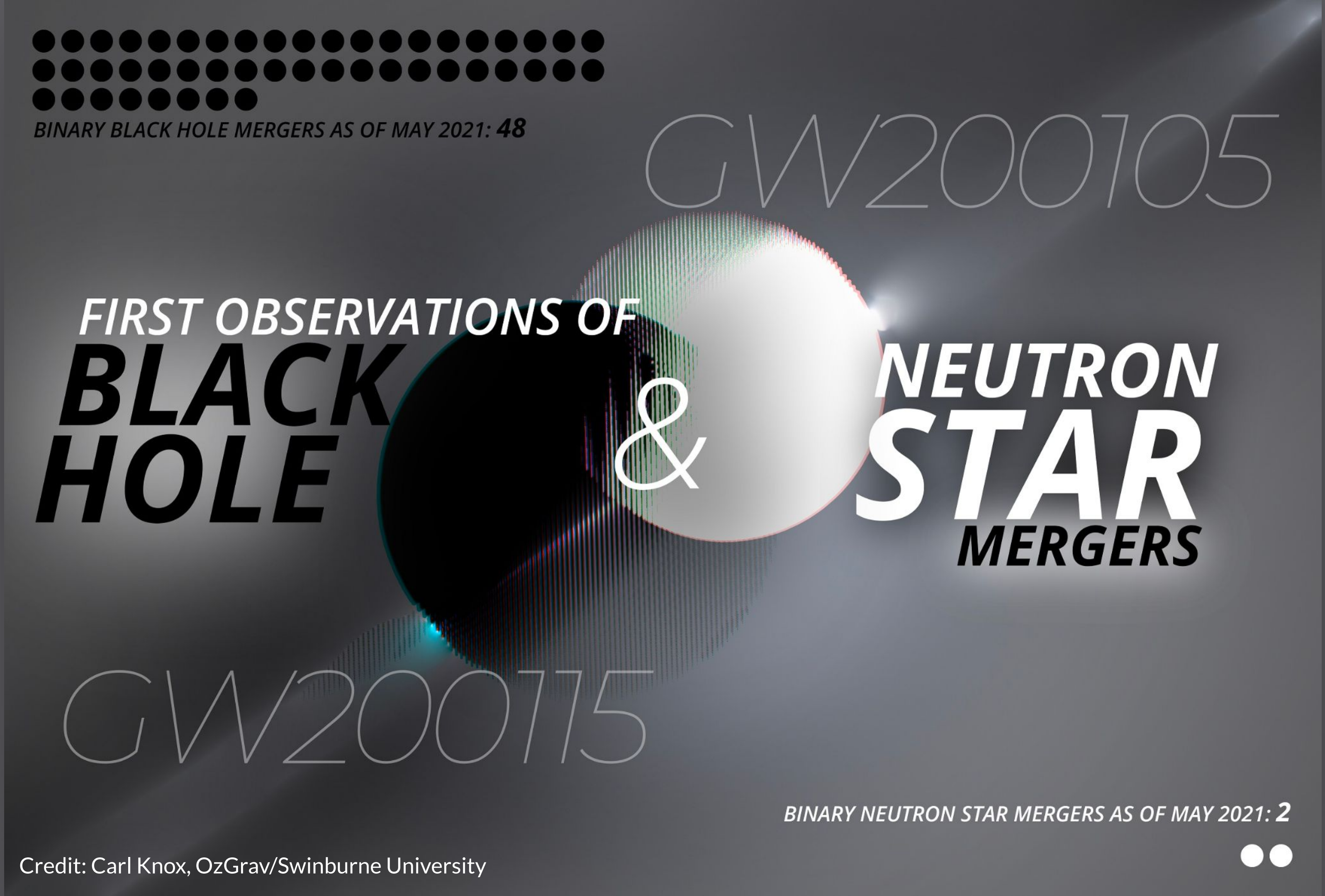
LVK, Phys. Rev. X; 13(4):041039

GW190521 - Heavy BH Merger!

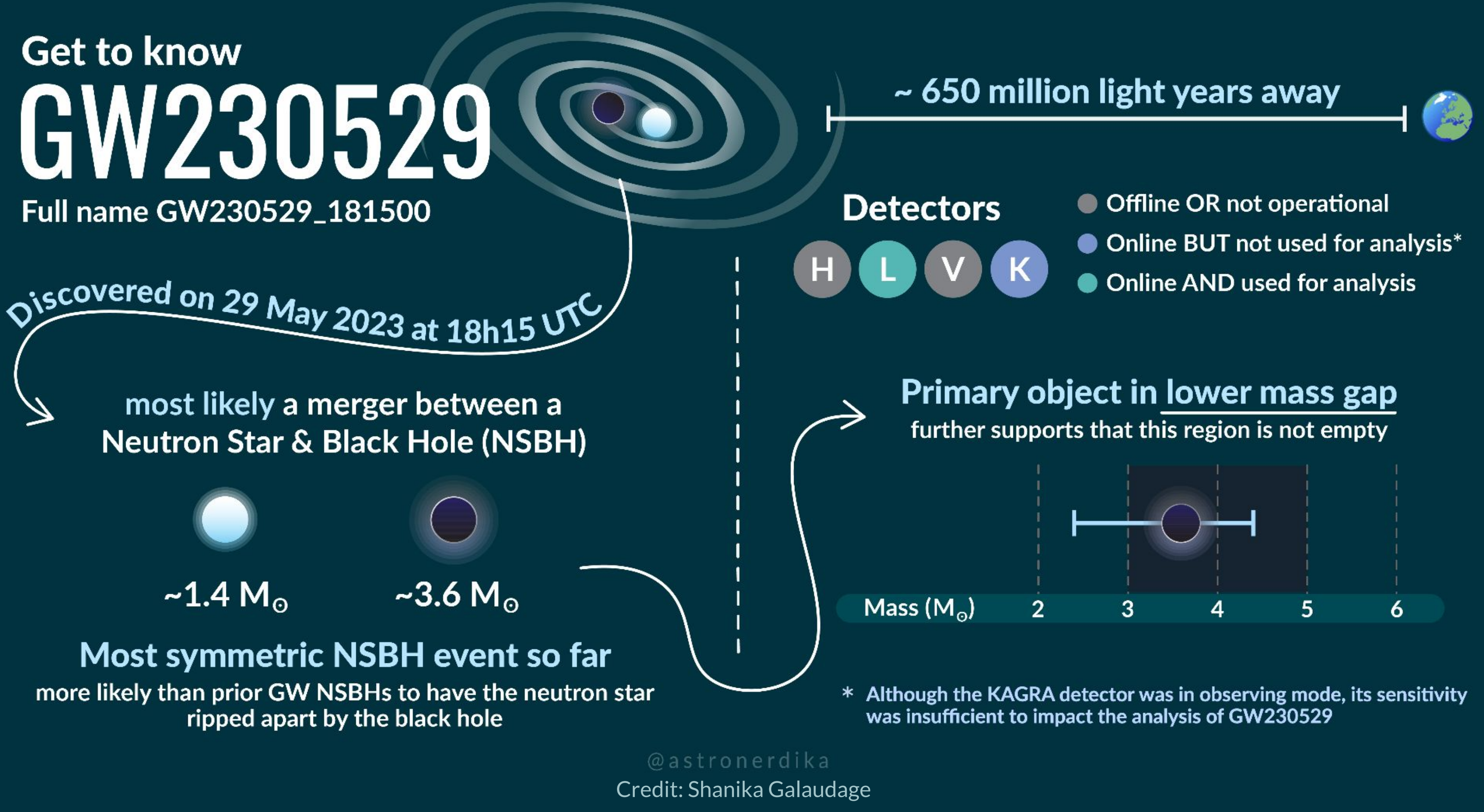


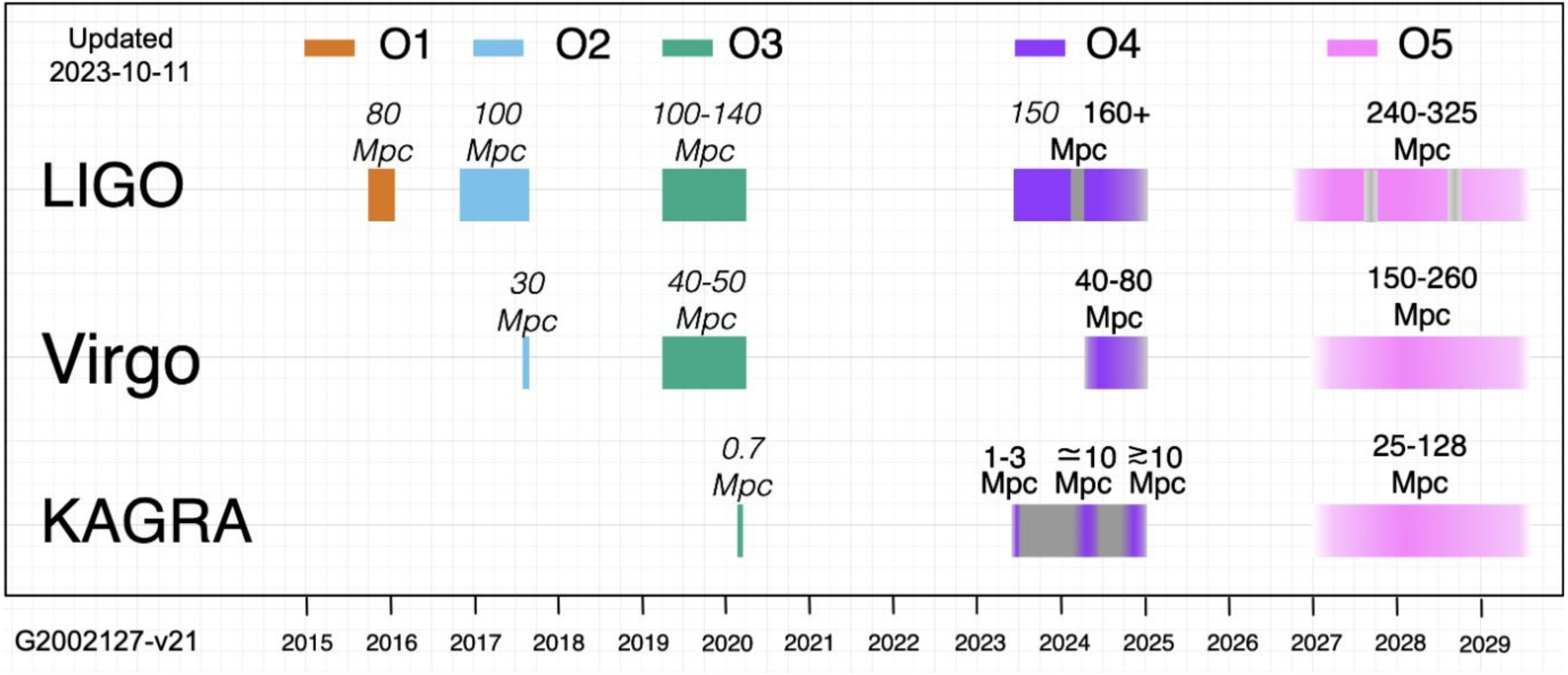


LVK, ApJL, 900, 1, id.L13, 27 pp.



Credit: Carl Knox, OzGrav/Swinburne University


Get to know
GW230529
Full name GW230529_181500
Discovered on 29 May 2023 at 18h15 UTC
most likely a merger between a
Neutron Star & Black Hole (NSBH)
~1.4 M☉
~3.6 M☉
Most symmetric NSBH event so far
more likely than prior GW NSBHs to have the neutron star ripped apart by the black hole
~ 650 million light years away
Detectors
H
L
V
K
Offline OR not operational
Online BUT not used for analysis*
Online AND used for analysis
Primary object in lower mass gap
further supports that this region is not empty
Mass (M☉)
2
3
4
5
6
* Although the KAGRA detector was in observing mode, its sensitivity was insufficient to impact the analysis of GW230529
@astronerdika
Credit: Shanika Galaudage


Updated
2023-10-11
O1
O2
O3
O4
O5
LIGO
80 Mpc
100 Mpc
100-140 Mpc
150 160+ Mpc
240-325 Mpc
Virgo
30 Mpc
40-50 Mpc
40-80 Mpc
150-260 Mpc
KAGRA
0.7 Mpc
1-3 Mpc
≃10 Mpc
≳10 Mpc
25-128 Mpc
G2002127-v21
2015
2016
2017
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2029

October 12, 2020

AARON JONES

Post a Comment

January 19, 2021

FRANCISCO LLAMAS

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September 28, 2020

DRIPTA BHATTACHARJEE

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October 30, 2019

WEYL E. COYOTE

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October 08, 2019

JOCELYN READ

Post a Comment

October 22, 2019

DEBNANDINI MUKHERJEE

Post a Comment

August 12, 2019

MAYA FISHBACH

Post a Comment

What have we learned from individual events?

Strain data

Source parameters

$$p(\vec{\vartheta}|\vec{d}) \propto p(\vec{d}|\vec{\vartheta})\pi(\vec{\vartheta})$$

posterior likelihood prior

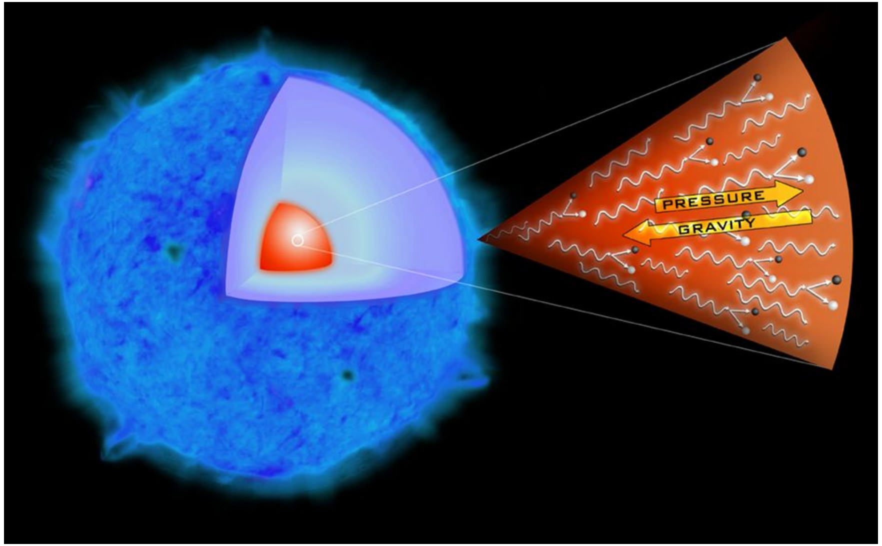
- BBH, BNS, NSBH can merge in a Hubble time!
- Some merging black holes spin
- The merging objects can have unequal masses
- BH and NS from GWs have different properties than those observed through EM

How are compact-object mergers produced?

First we need to form compact objects…

Stellar Progenitors of Compact Objects

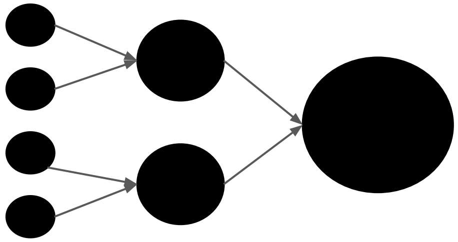
- Feature at transition from neutron stars to black holes?
- Feature at pair-instability supernova mass?
- Dips/peaks from non-linear mass - compactness relation of progenitor stars?



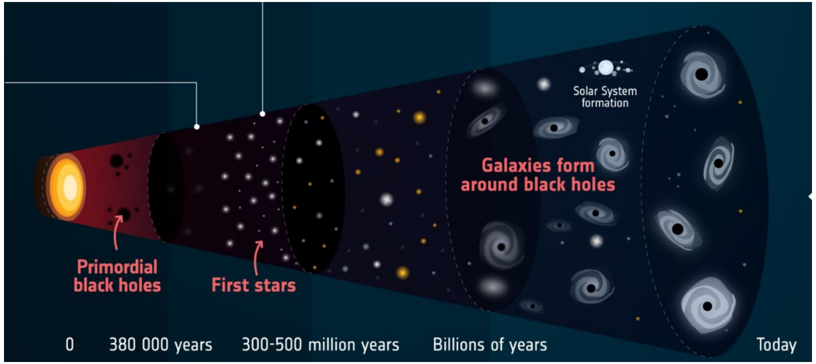
Pair instability supernova (credit NASA)

More Exotic Compact-Object Formation Scenarios

Hierarchical formation



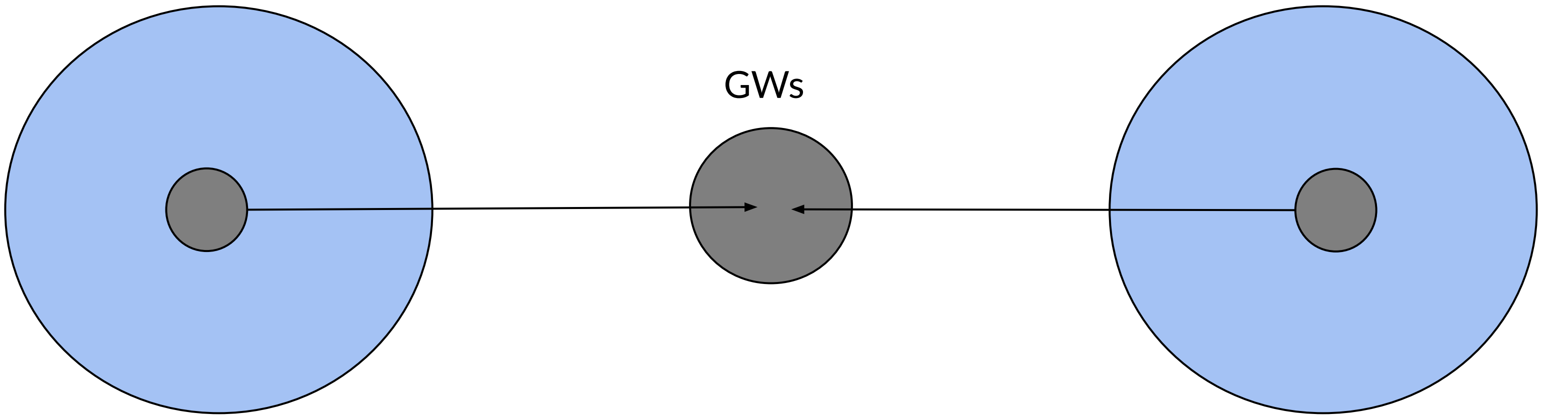
Primordial Formation



Credit: ESA

…then we need compact objects to merge in a Hubble time.

What DOESN'T Work…



The Big Mystery…

Peters (1964):

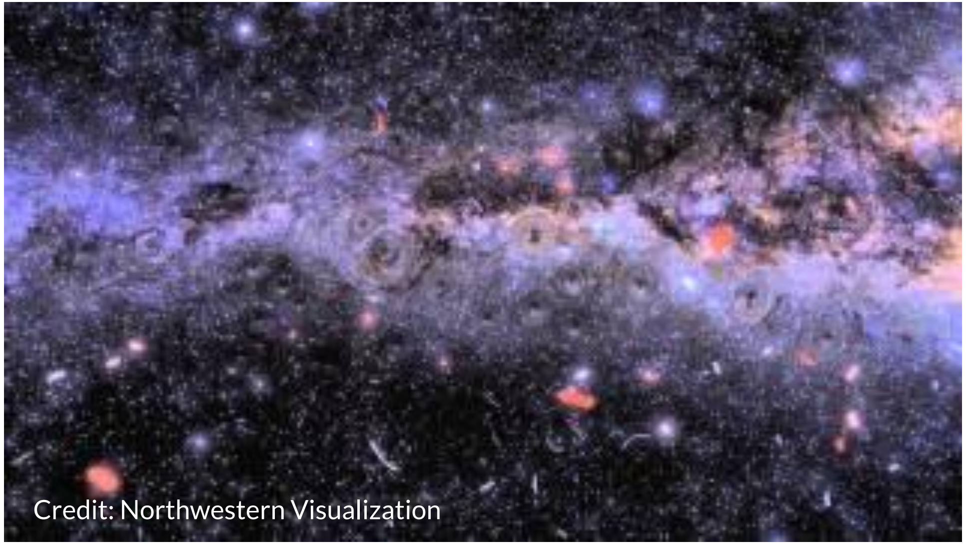
$$a = \left(\frac{64G^3}{5c^5} M^3 t_{\rm merge} \right)^{\frac{1}{4}} \sim 50\,\mathrm{R}_\odot \left(\frac{M}{60\,\mathrm{M}_\odot} \right)^{\frac{3}{4}} \left(\frac{t_{\rm merge}}{14\,\mathrm{Gyr}} \right)^{\frac{1}{4}}$$

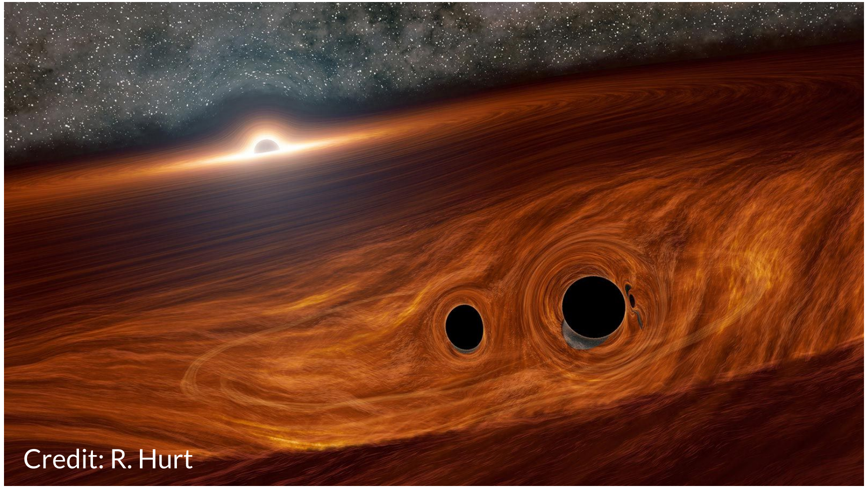
To merge stellar-mass COs in a Hubble time, they must be closer than the radii of their progenitor stars!

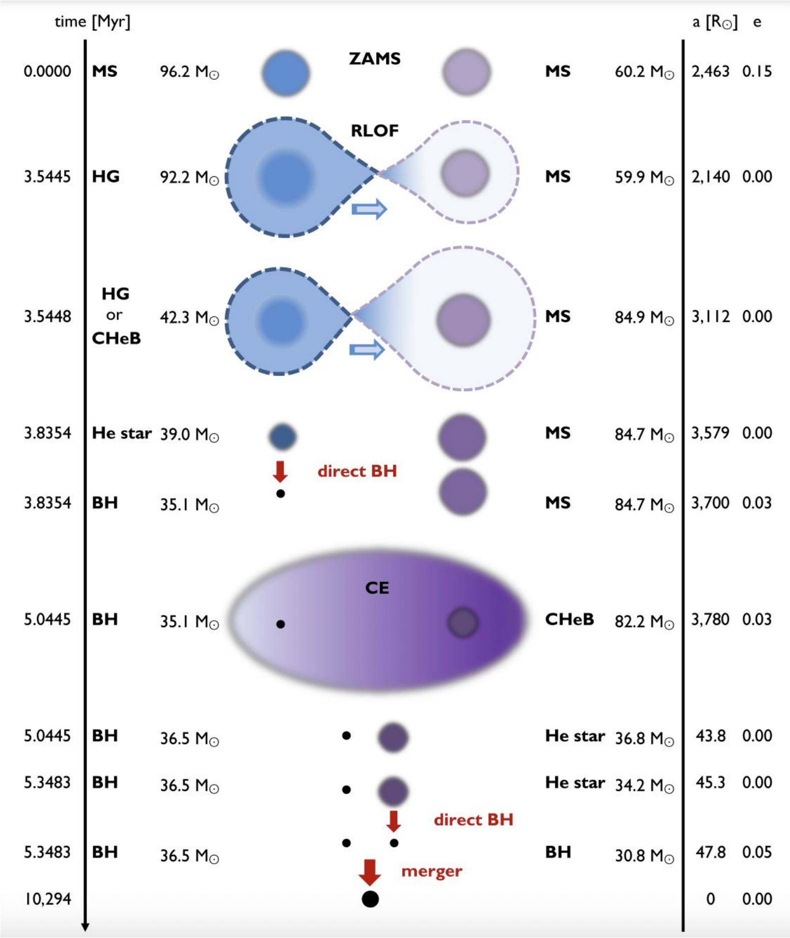
Two Families of Compact-Object Merger Channels

Dynamical Channel

Isolated Binary Channel


Credit: Northwestern Visualization


Credit: R. Hurt



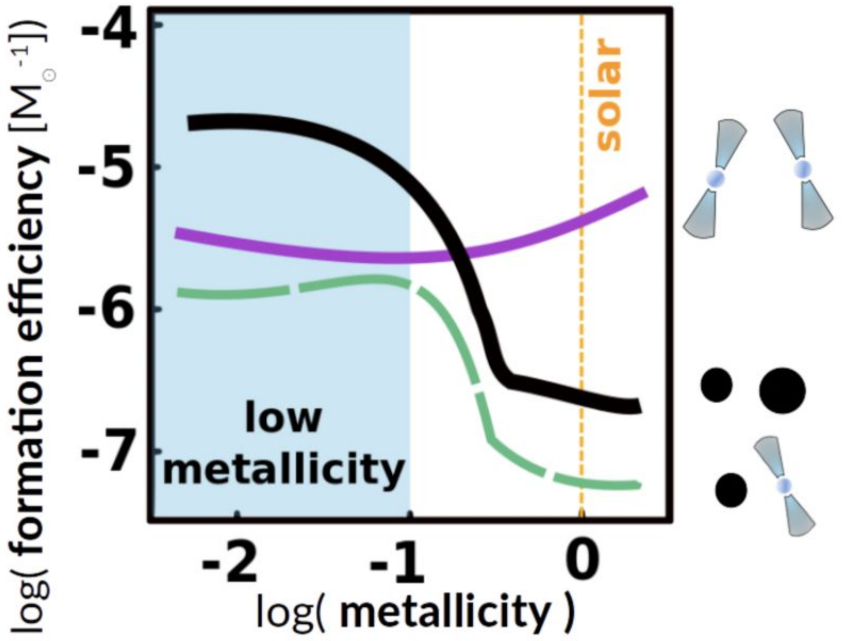
Belczynski, Holz, Bulik & O'Shaughnessy. Nature (2016)

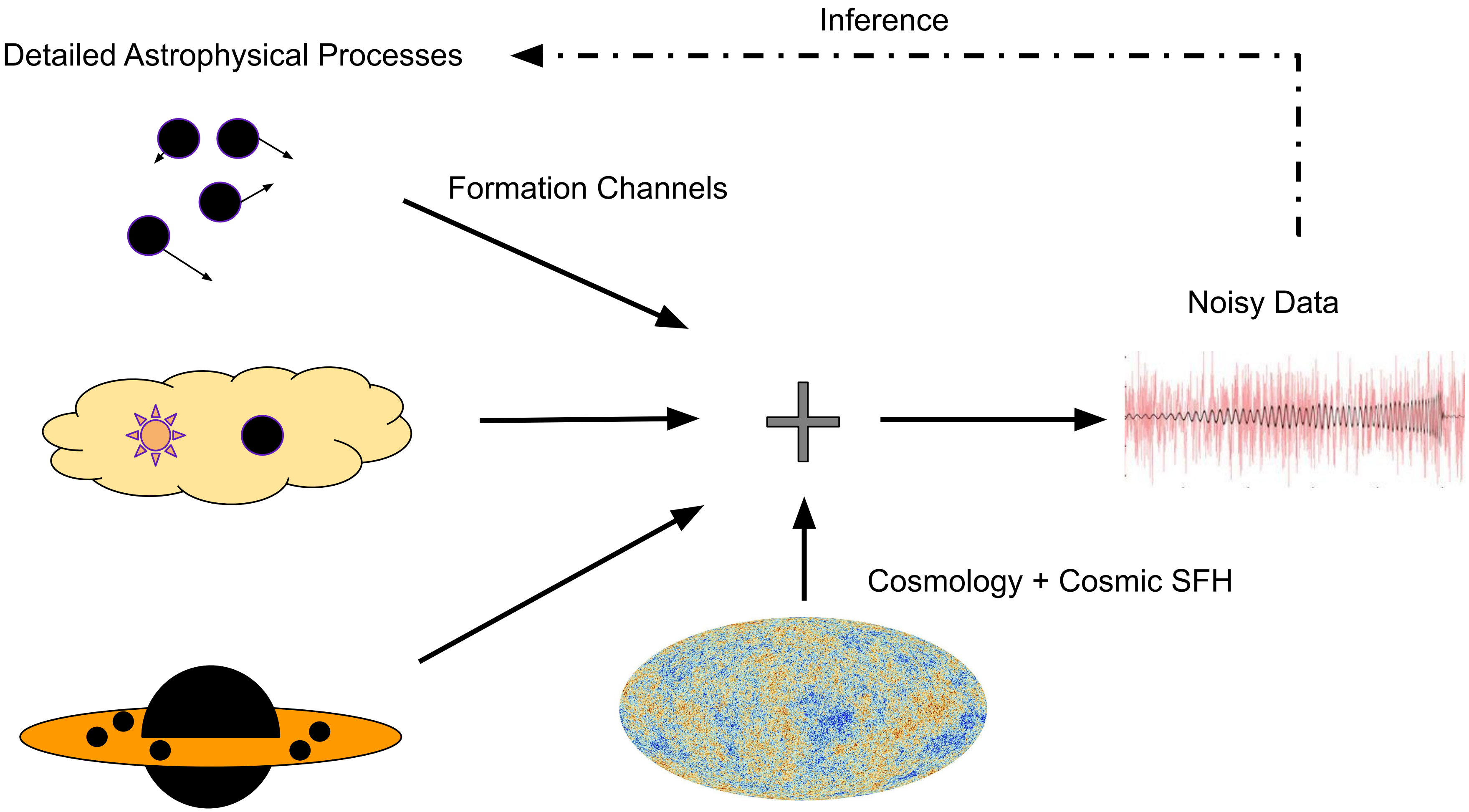
Many models, many knobs!

- Initial mass function of stars / COs
- Stability of mass transfer
- Cluster potential
- Metallicity evolution
- Accretion efficiency
- …

Double Compact Object Formation Depends Strongly on Metallicity

More metals…
→ more lines
→ more stellar winds
→ **smaller compact objects**



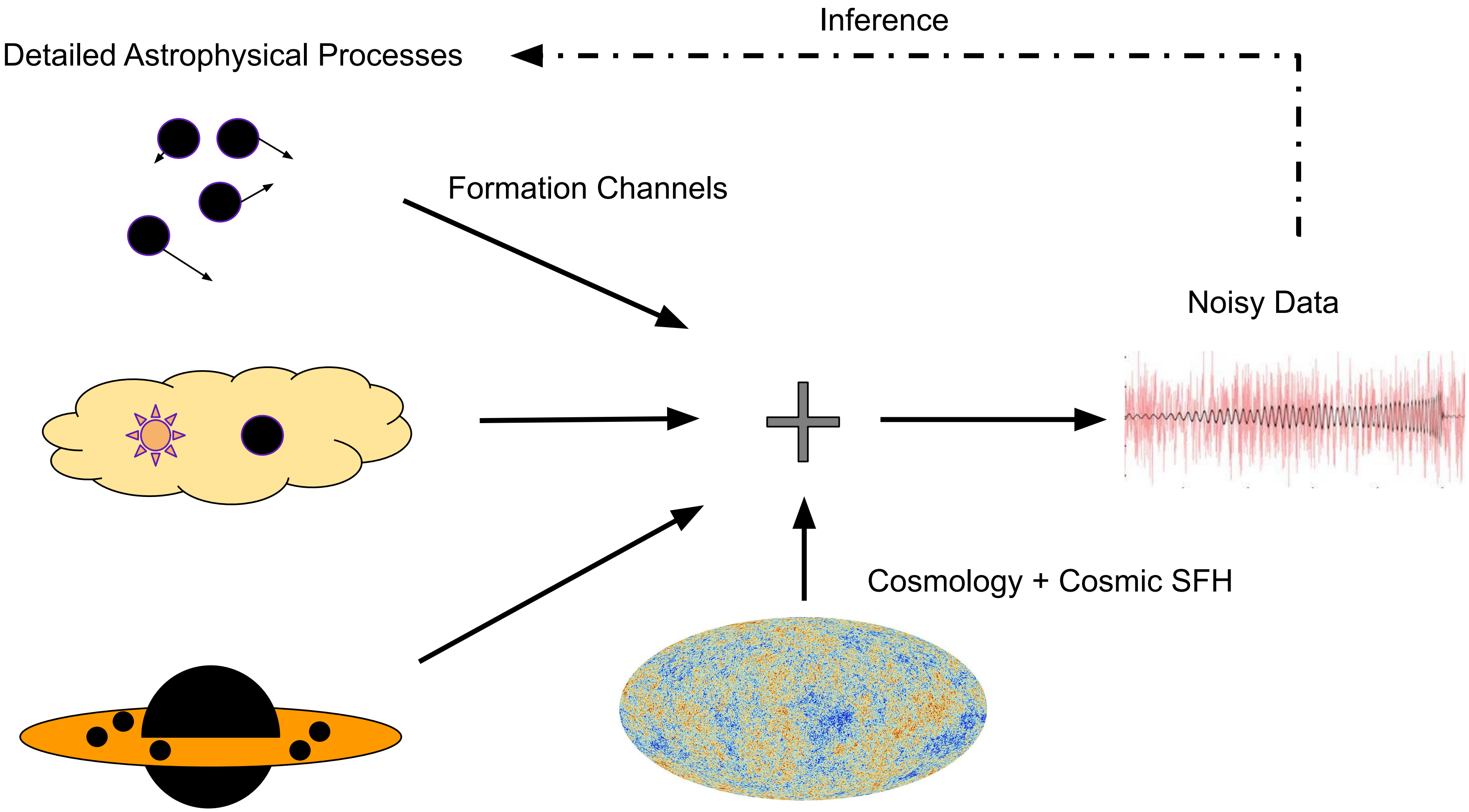

Inference
Detailed Astrophysical Processes
Formation Channels
Noisy Data
Cosmology + Cosmic SFH

High-Dimensional Source Parameter Space!

Noisy Data!

Systematic Uncertainties!

Should explain full catalog of GWs!


Inference
Detailed Astrophysical Processes
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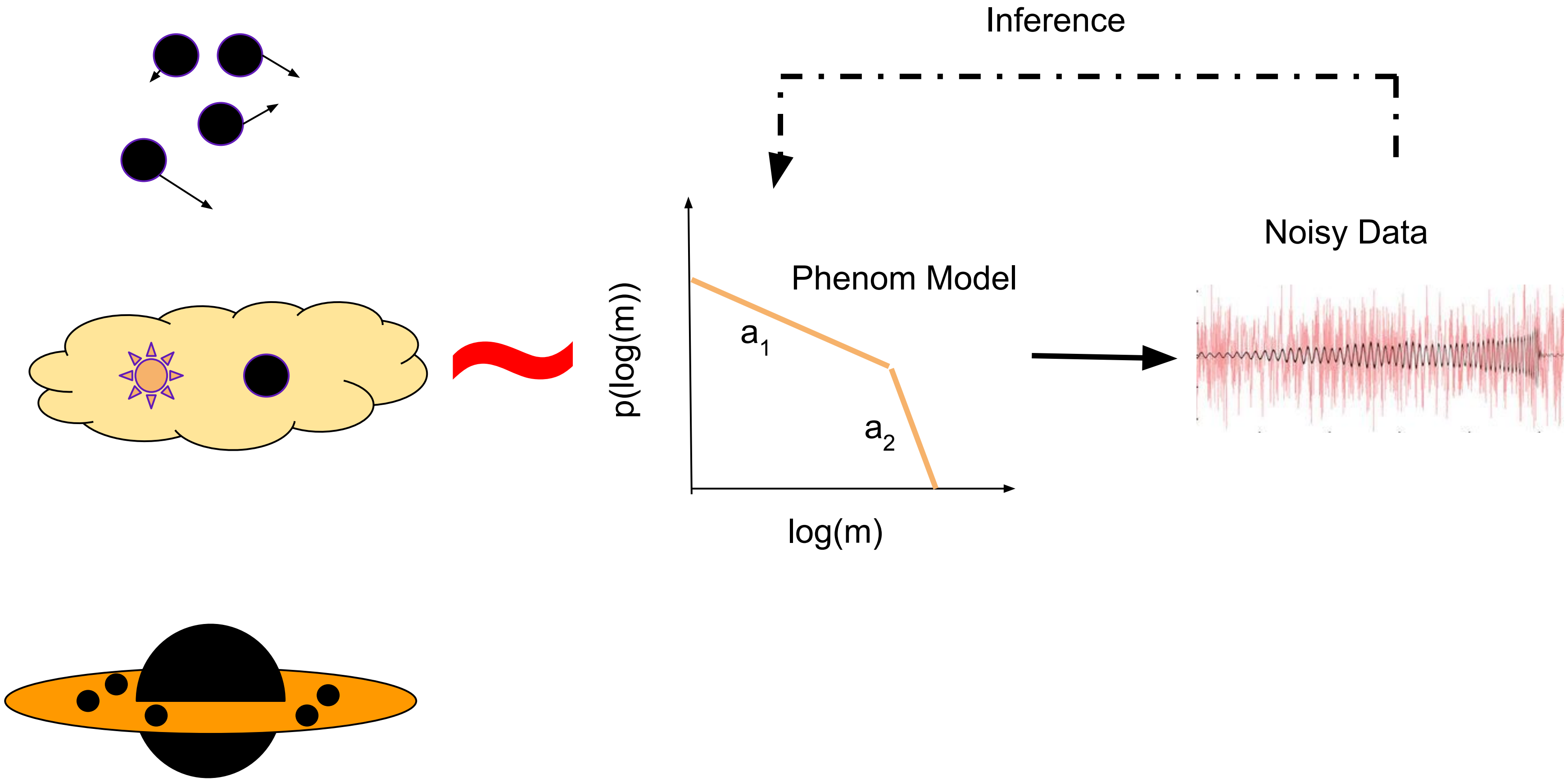
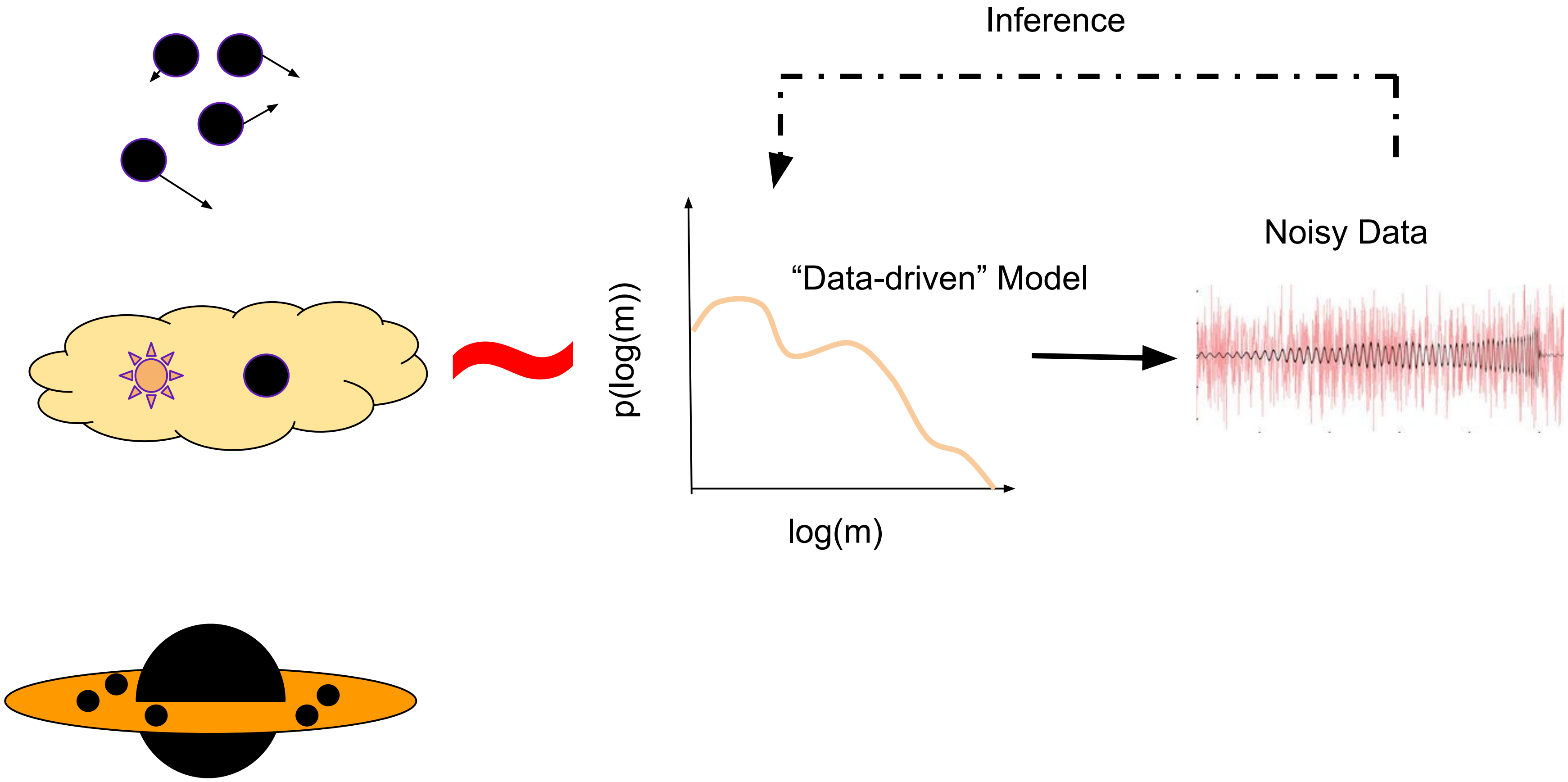
Detailed Astrophysical Processes



Detailed Astrophysical Processes



Simple Models

- Targeted questions (e.g. mass gaps)
- Easy to write down (e.g. power laws)
- Somewhat agnostic to astrophysical details
- Could miss important features

SYNERGY!

Detailed Astrophysical Models

- Can include our best understanding of BHs + interactions
- Can be tuned via other data sets
- Hard to write down
- Many parameters
- Possible systematic errors

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Data-Driven Models

- Find unexpected features in the population
- Corroborate results of simple models
- Compare with features in detailed models
- Black-box predictions

Simple Models

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- Could miss important features

Strain data segments that trigger GW search pipelines

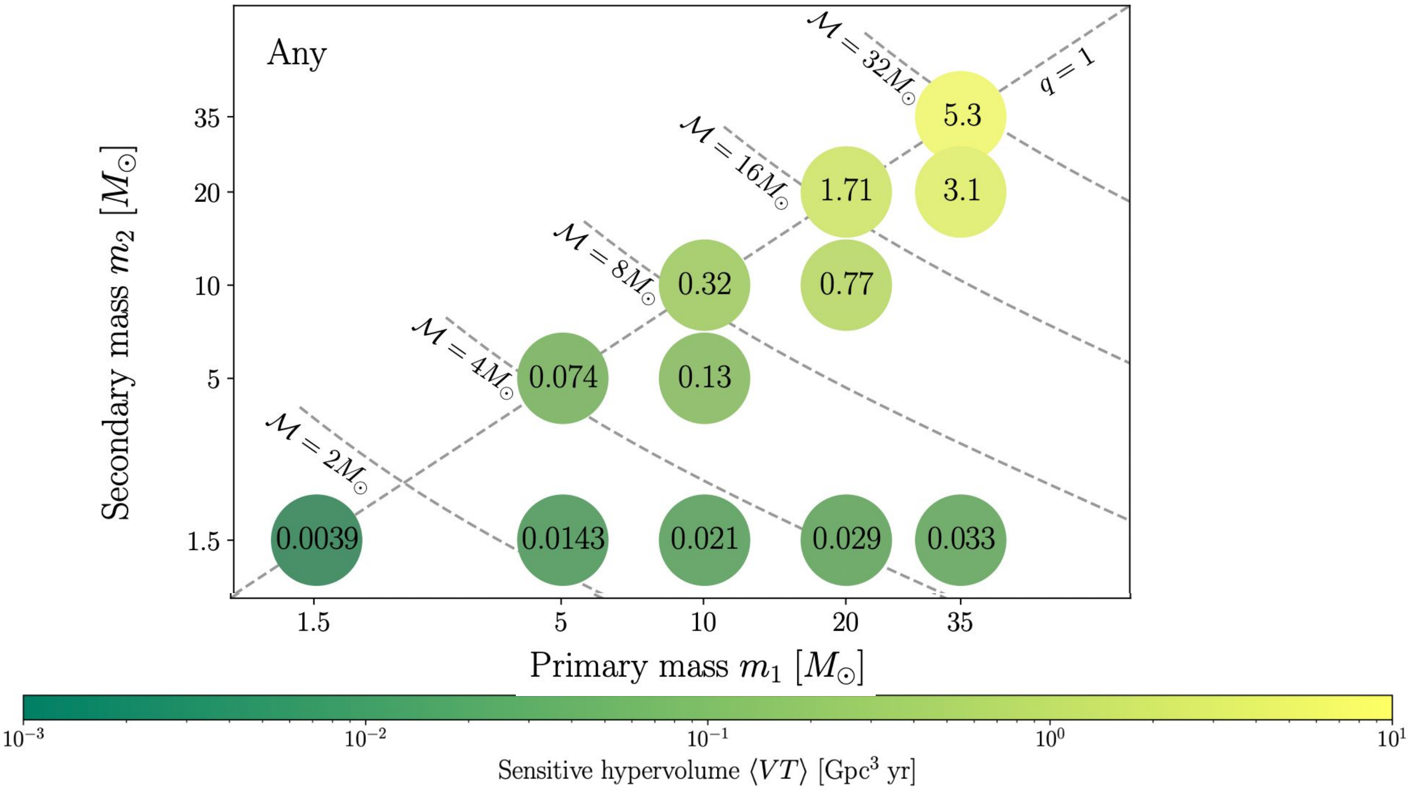
Astrophysical Models + Selection Effects

$$\mathcal{L}(\{d\}, N_{\mathrm{det}} | \Lambda, N_{\mathrm{exp}}) \propto N^{N_{\mathrm{det}}} e^{-N_{\mathrm{exp}}} \prod_{i=1}^{N_{\mathrm{det}}} \int \mathcal{L}(d_i|\theta)\pi(\theta|\Lambda) d\theta.$$

Single event likelihood

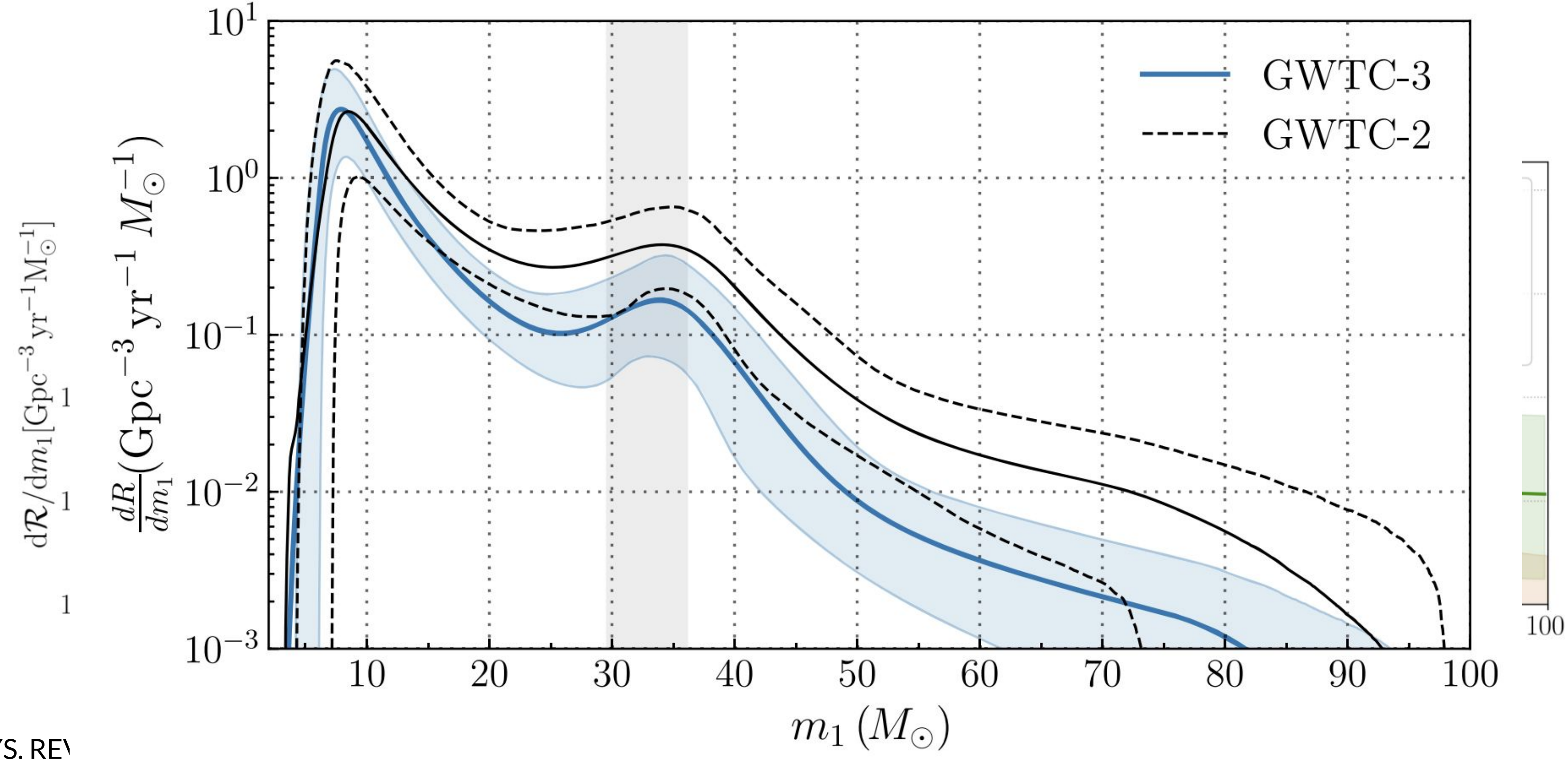
Single event parameter prior under Λ

Higher mass mergers are “louder” -> Selection effect



Mass Spectrum of Compact Object Mergers

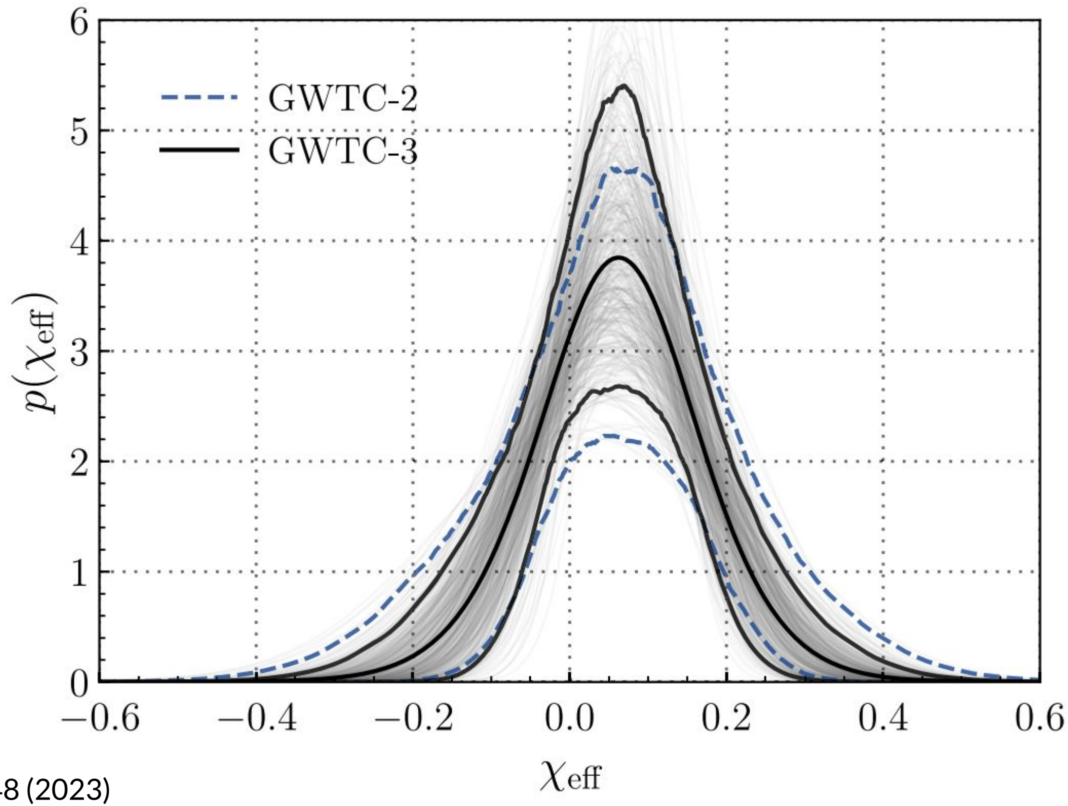
Structure in the binary black hole mass distribution



LVK PHYS. REV

Spin Spectrum of Compact Object Mergers

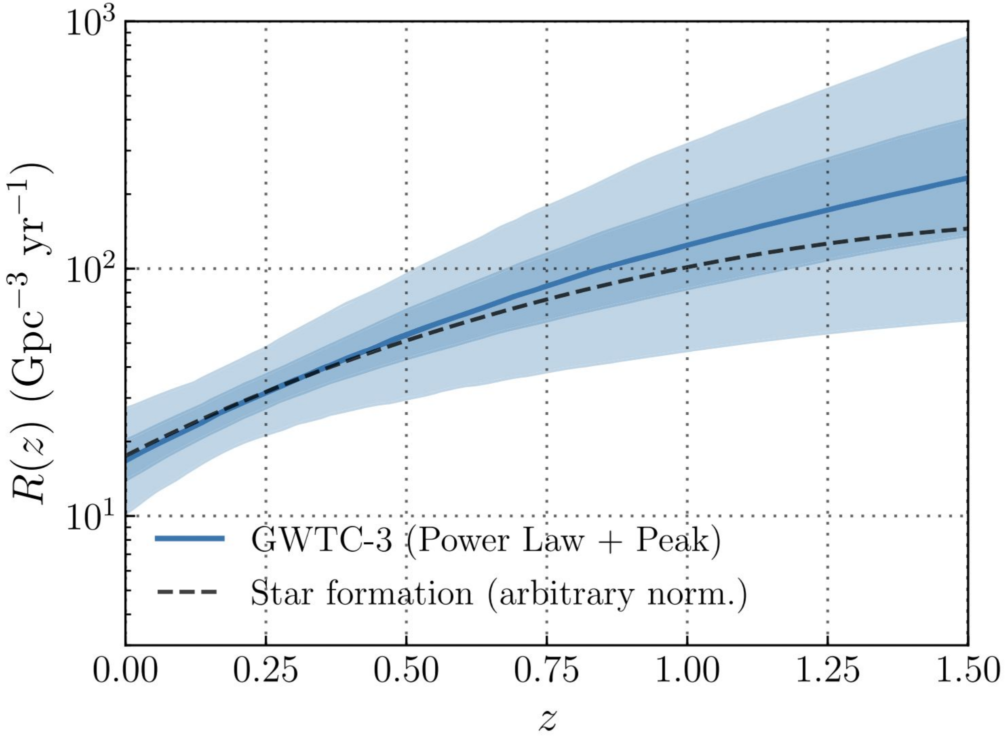
Binary black hole spin distribution



LVK PHYS. REV. X 13, 011048 (2023)

Merger rate with redshift

Merger rate is increasing with redshift



LVK PHYS. REV. X 13, 011048 (2023)

Population-level correlations

“Data-Driven” Models

Models with lots of flexibility but “agnostic” to the astrophysics enable…

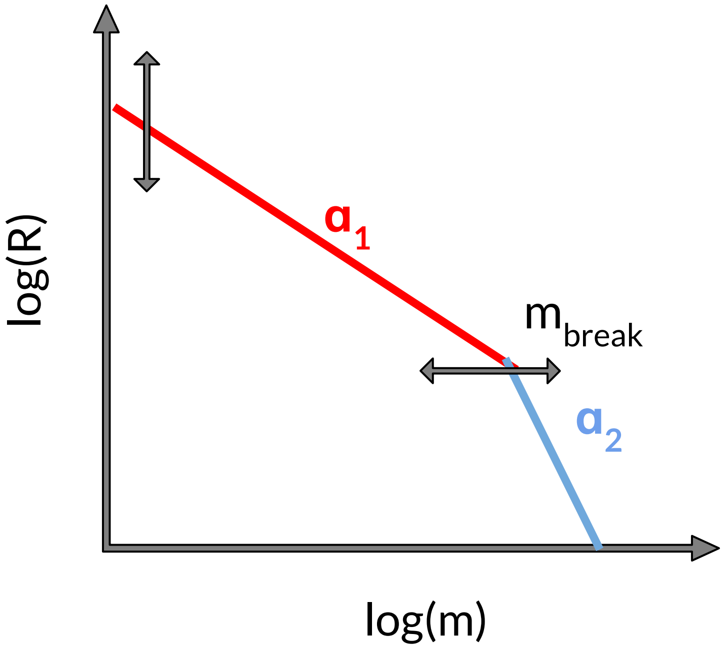
- Finding unexpected features in the population
- Corroborating results of simple models
- Comparison with features in detailed models
- Black-box predictions for other applications

Let’s use these different modeling approaches to study gravitational-wave populations!

Simple Models

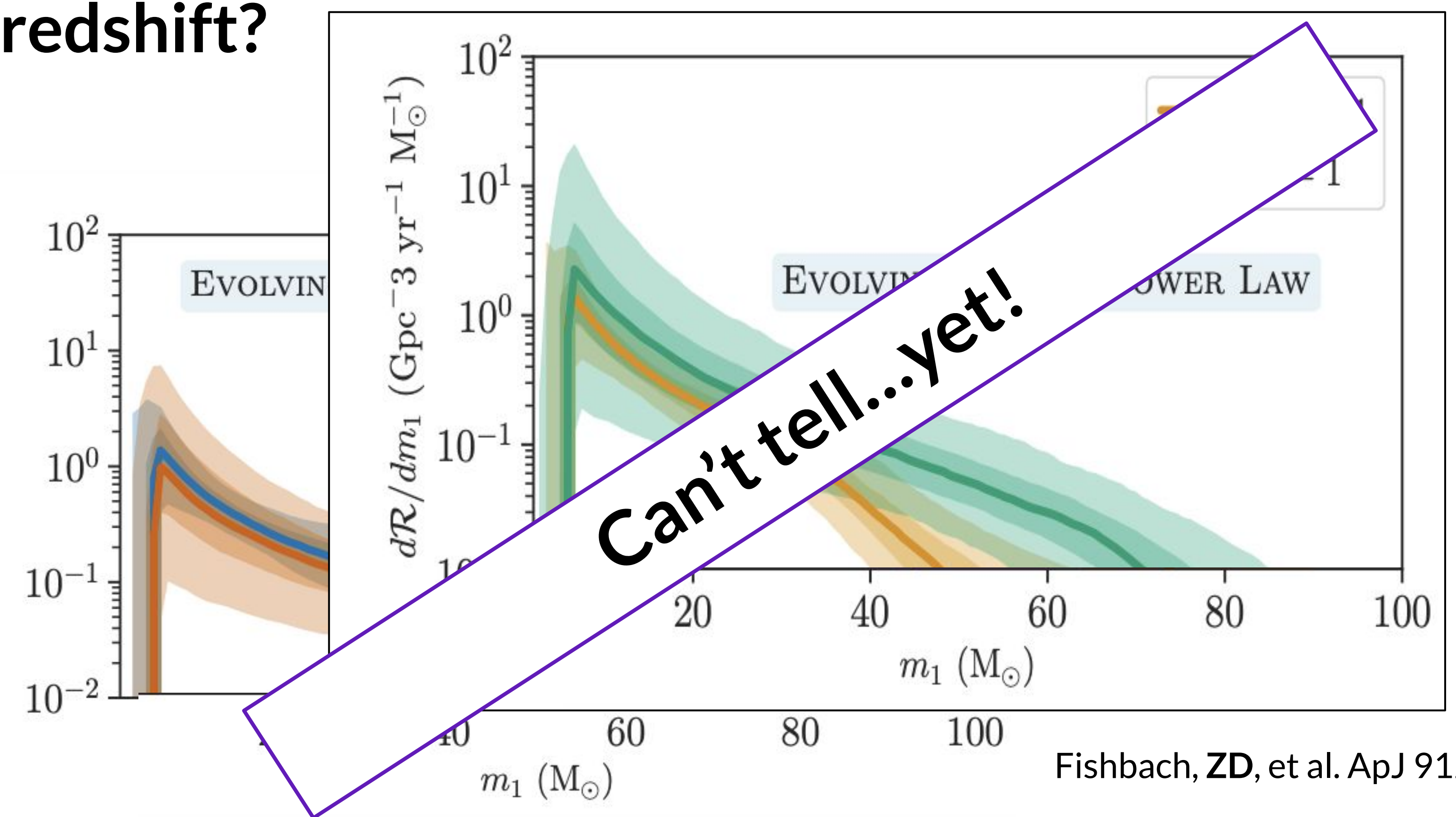
Fishbach & Holz (2017)
Talbot & Thrane (2018)
Wysocki, Lange, and O'Shaughnessy (2019)
Doctor et al (2020)
Kimball et al (2021)
Landry & Read (2021)
Farah, Fishbach, Essick et al (2021)
…

Does the mass distribution of black holes change with redshift?



Fishbach, **ZD**, et al. ApJ 912 98 (2021)

Does the mass distribution of black holes change with redshift?



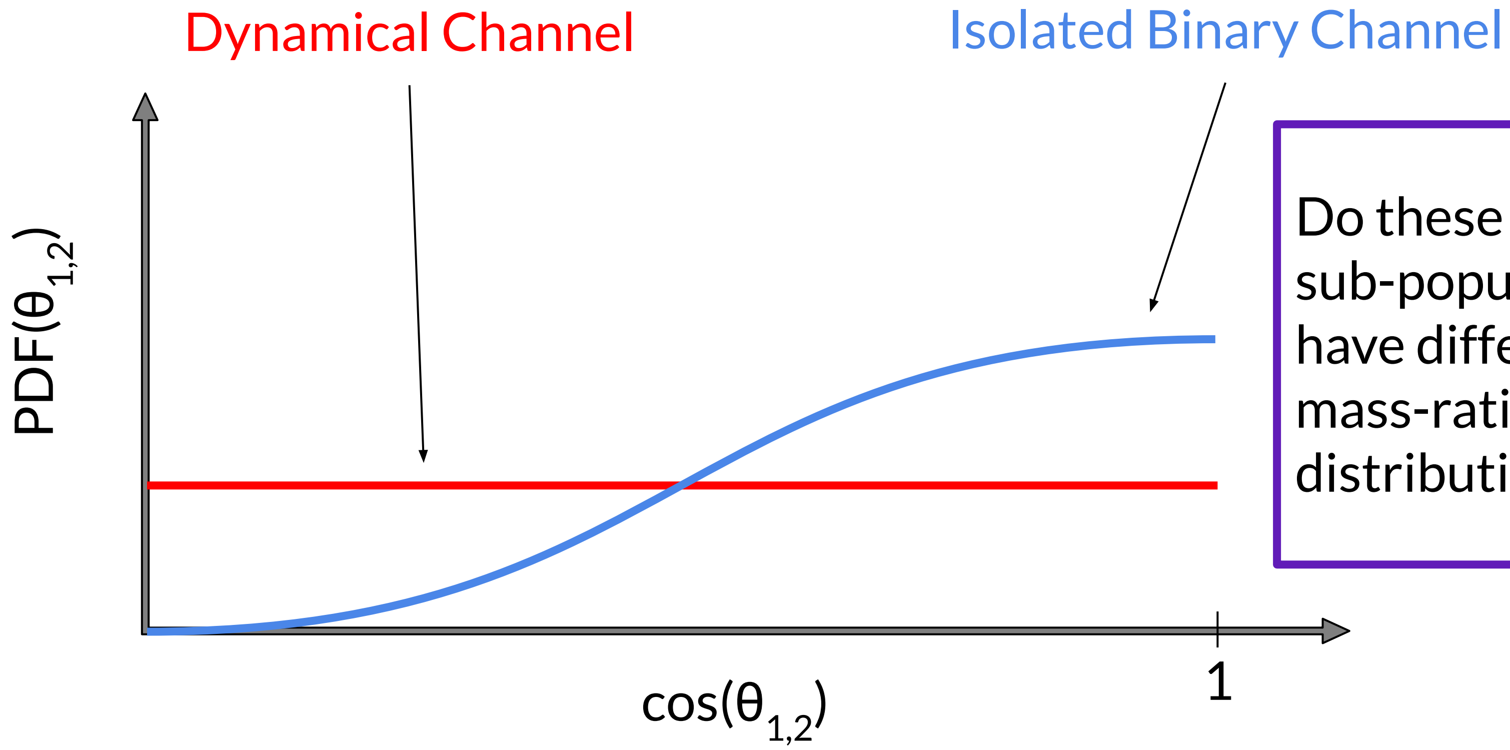
Fishbach, **ZD**, et al. ApJ 912 98 (2021)

Do BH sub-populations of spin have different masses?

Dynamical Channel

Isolated Binary Channel

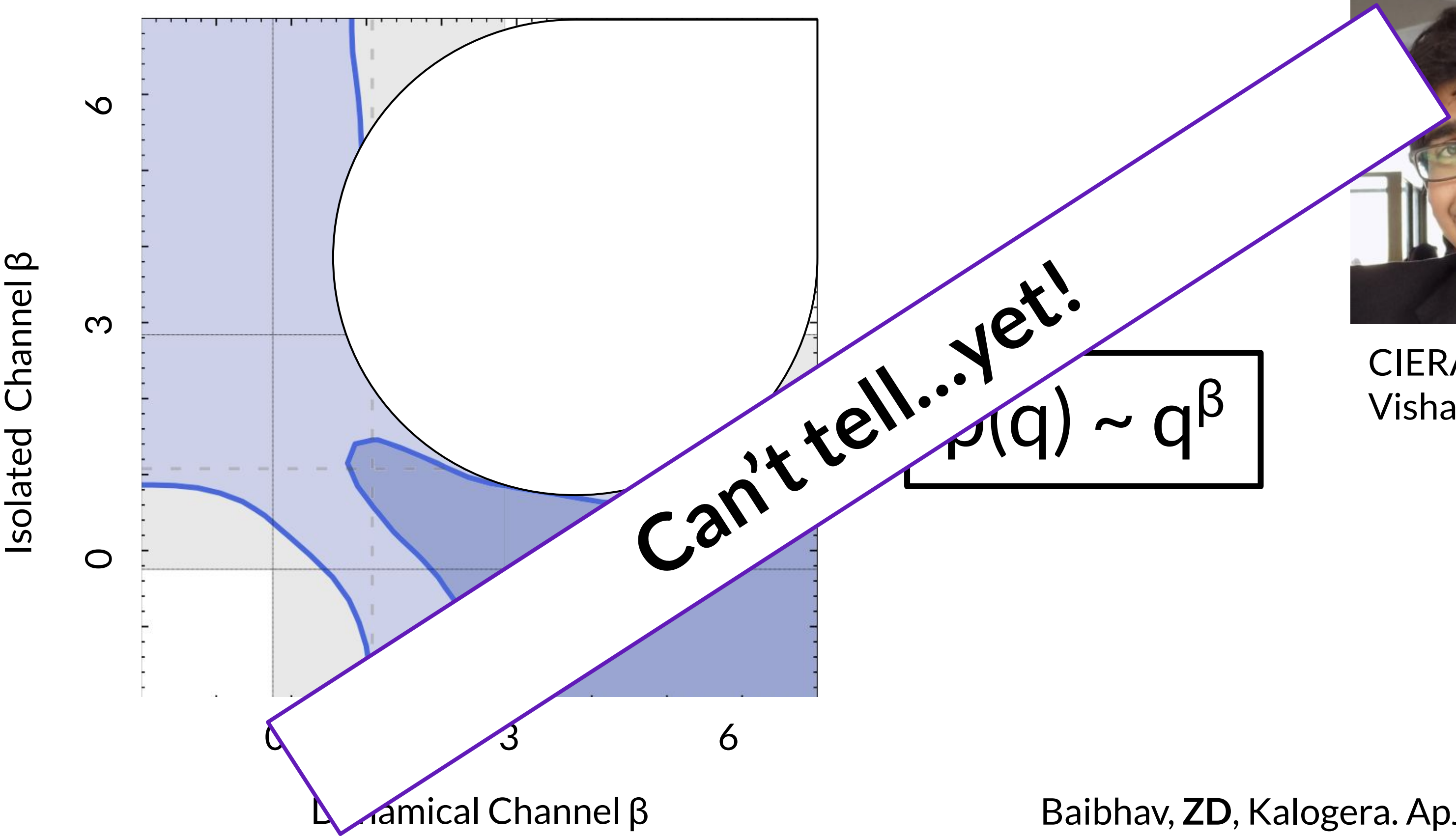
Do BH sub-populations of spin have different masses?

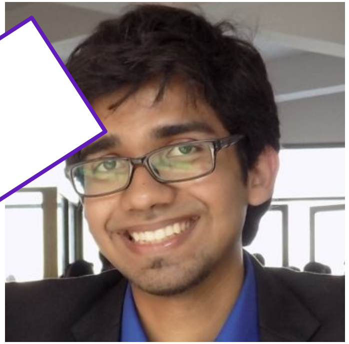


Do these putative sub-populations have different mass-ratio distributions?

Baibhav, ZD, Kalogera (2023)

Do BH sub-populations of spin have different masses?

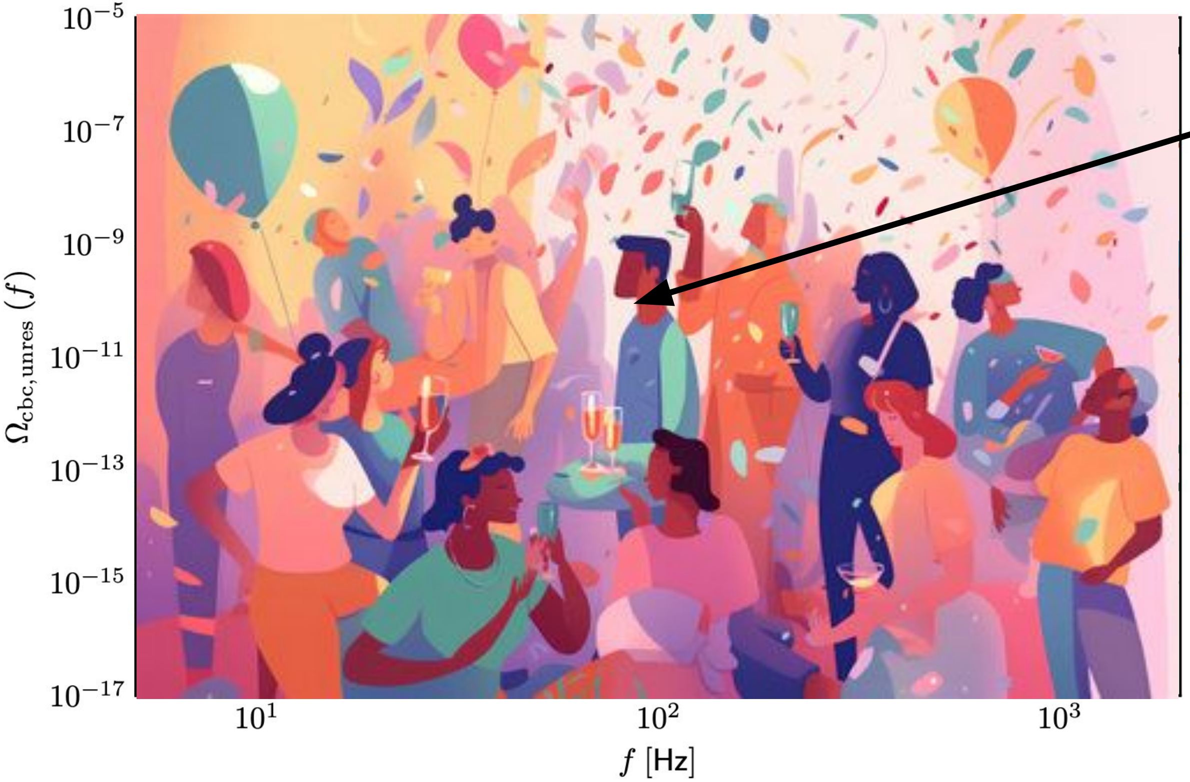


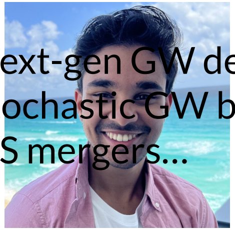


CIERA Fellow
Vishal Baibhav

Baibhav, **ZD**, Kalogera. ApJ 946 50 (2023)

The Stochastic GW Background from NS Mergers





Next-gen GW detectors will see stochastic GW background from NS mergers…

Darsan S. Bellie
NSF Graduate Fellow

…but the GW background may obscure other exotic GW signals

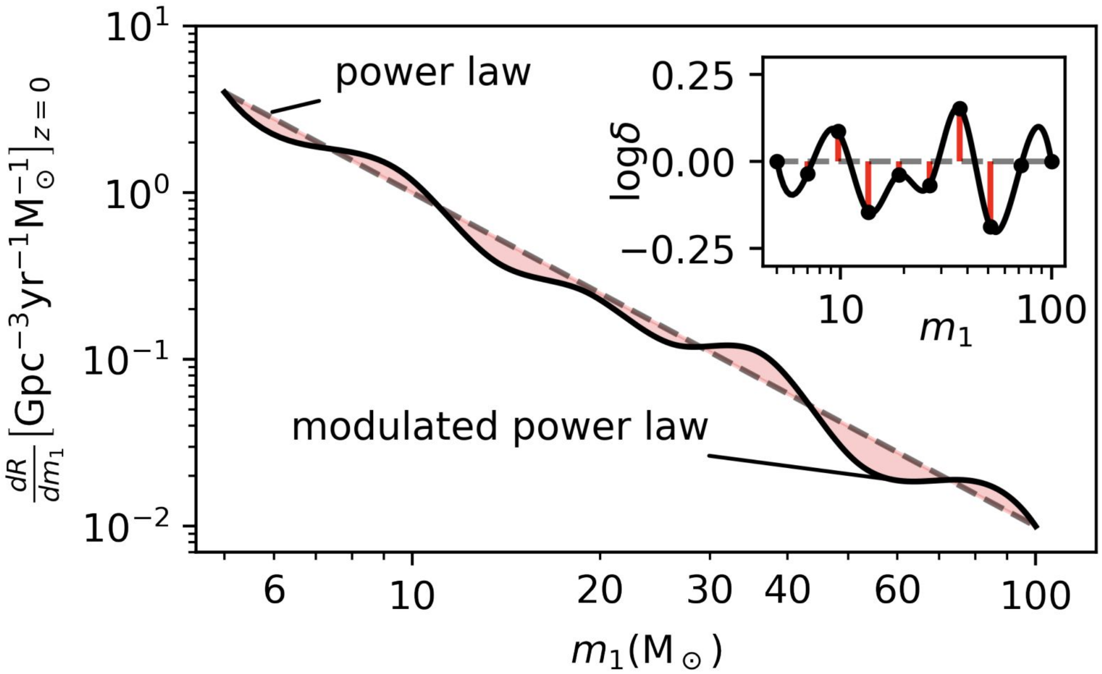
Dr. Sharan Banagiri
CIERA Fellow

Bellie, Banagiri, **ZD**, Kalogera. arXiv: 2310.02517 (2023)

Data-Driven Models

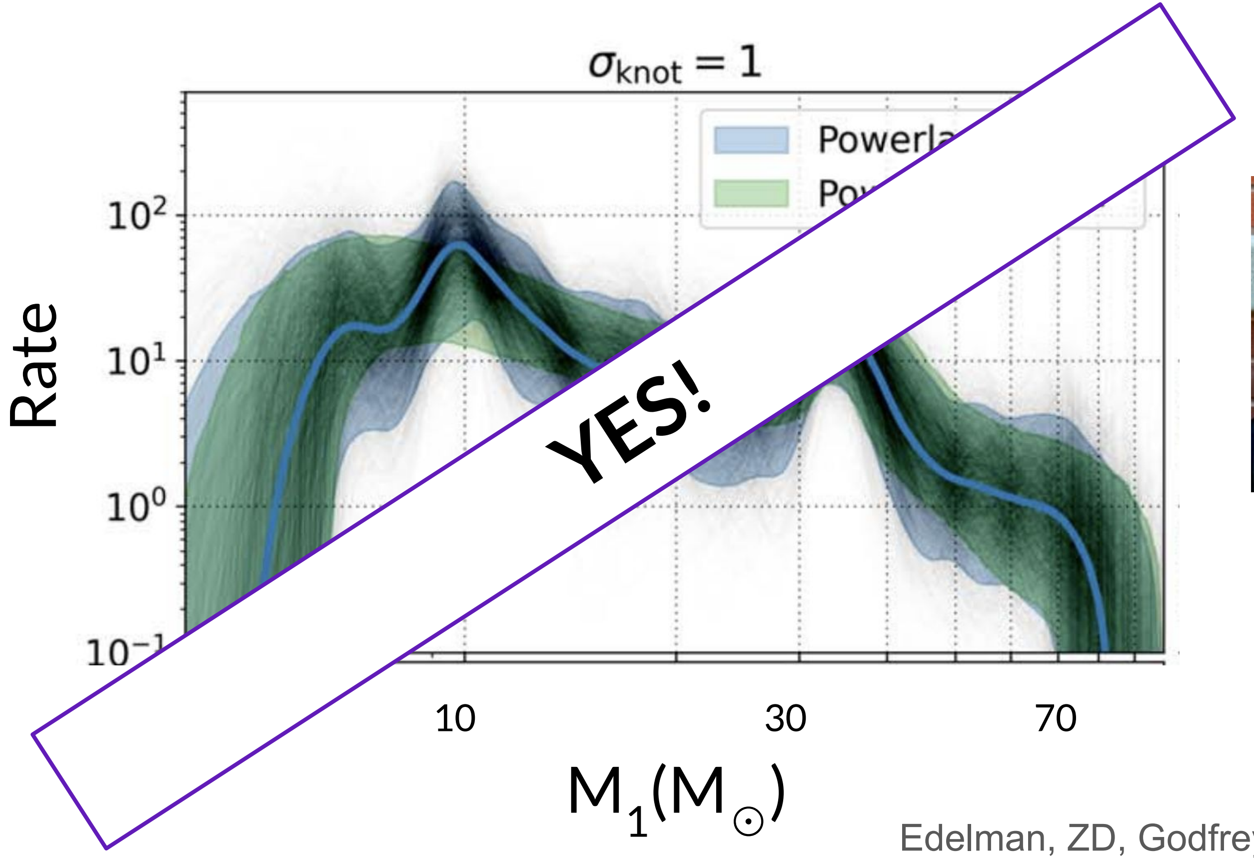
Mandel et al (2016)
Farr et al (2018)
Powell et al (2019)
Tiwari (2021)
Rinaldi et al (2021)
Sadiq et al (2021)
Godfrey et al (2023)
…

Is there structure in the BBH mass distribution?



Edelman, ZD, Godfrey, Farr (2022)

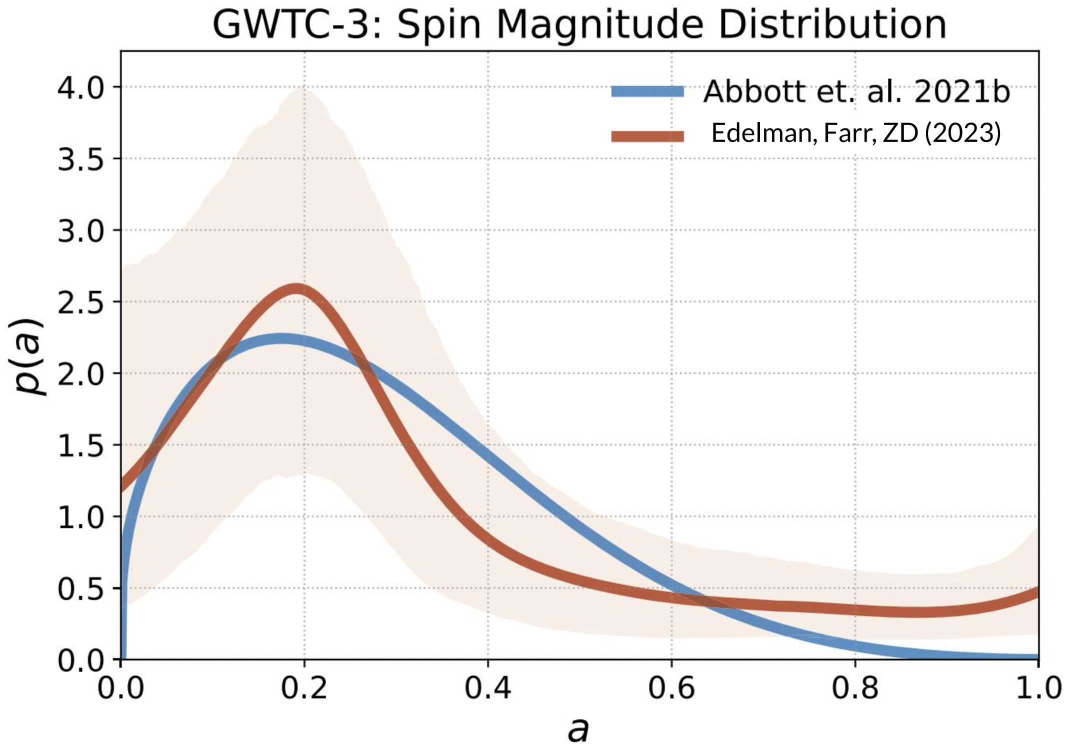
Is there structure in the BBH mass distribution?



Bruce Edelman
Research Software Engineer
(UOregon)

Edelman, ZD, Godfrey, Farr (2022)

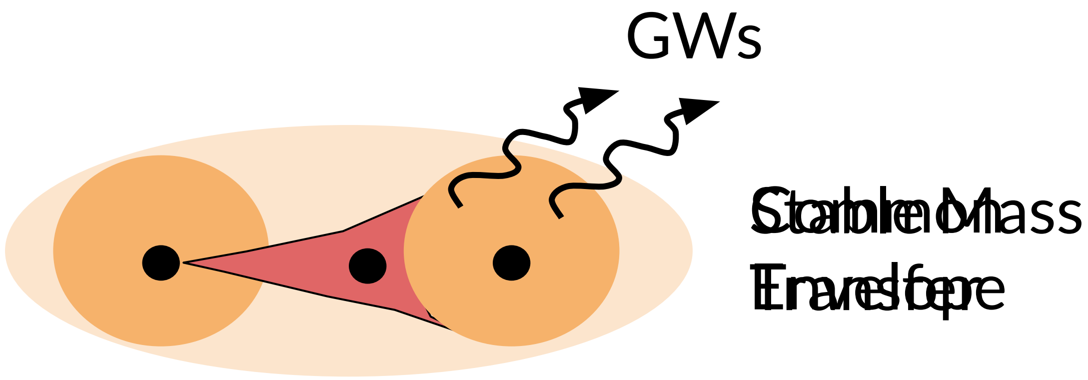
Is there structure in the BBH spin distribution?



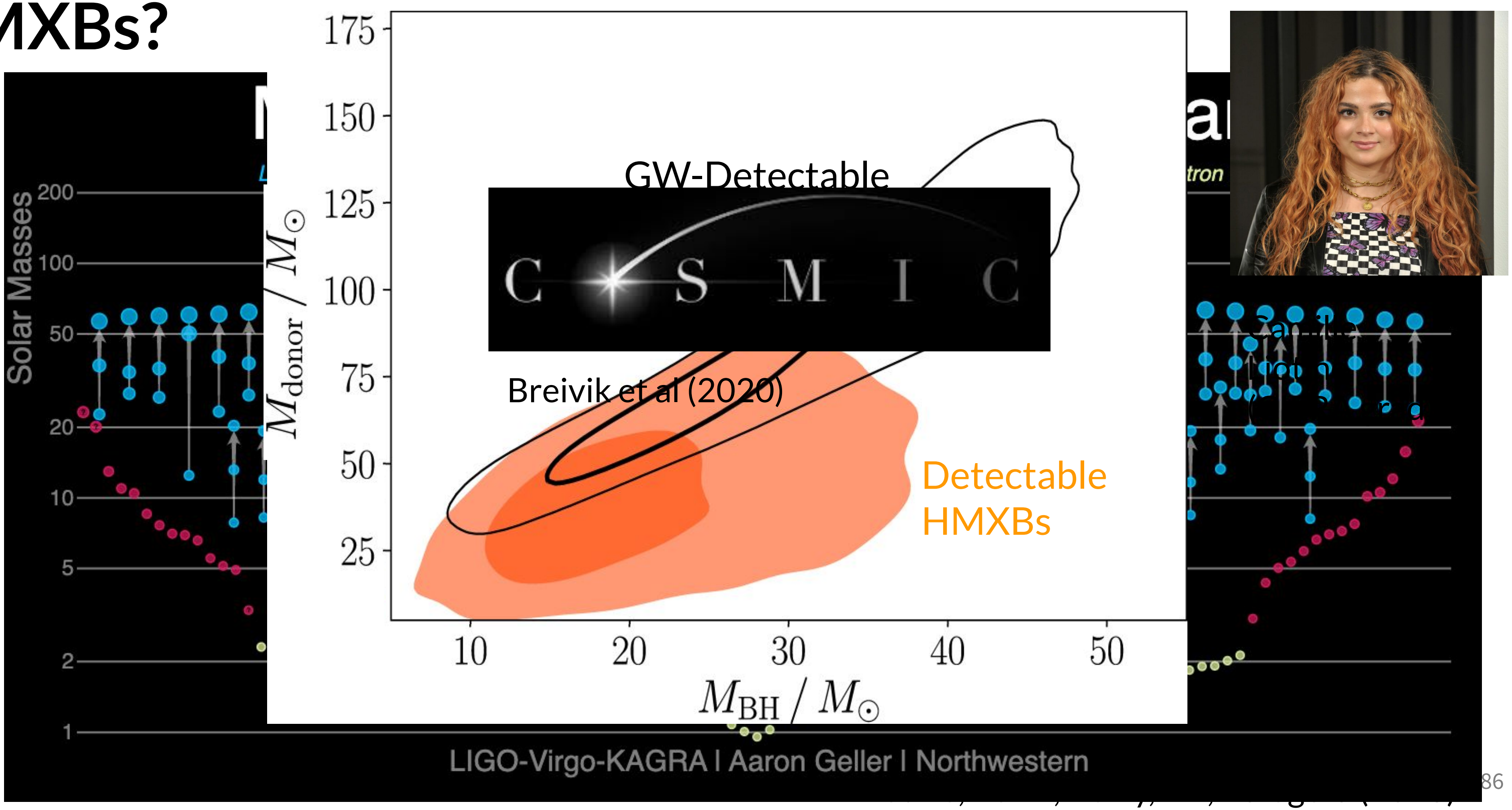
Detailed Models

BPASS
BSE
CMC
ComBinE
COMPAS
COSMIC
MOBSE
POSYDON
SEVN
StarTrack …

The Isolated Binary Channel



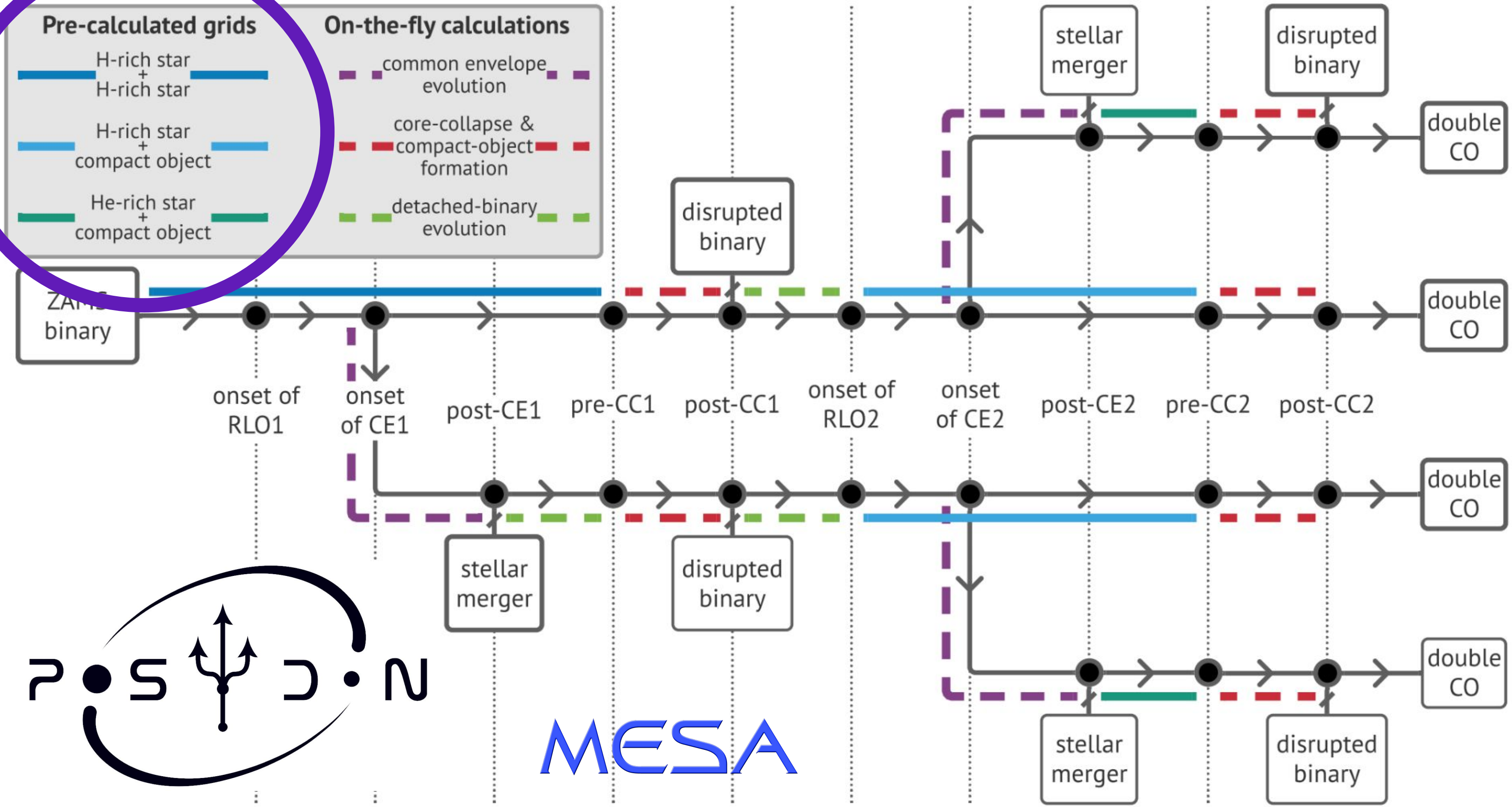
How do merging BH masses compare to those in HMXBs?



How do merging BH masses compare to those in HMXBs?

- $\lesssim$ 3% of detectable HMXBs have a BH with > 35 $M_\odot$
- Probability detected HMXB will merge as a BBH in a Hubble time is $\lesssim$ 1%
- Discrepant BH masses from GWs and HMXBs are expected!

Detailed Models



POSYDON

Fragos et al (2022)

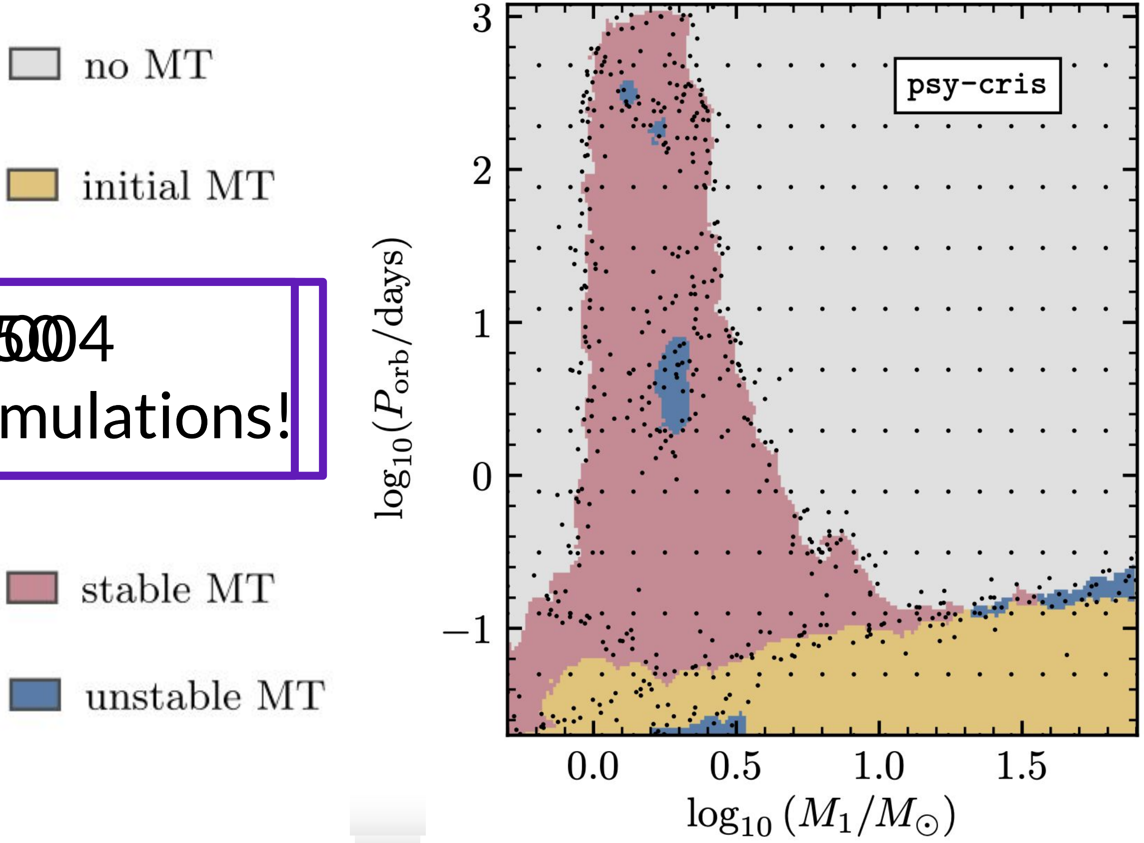
MESA

Paxton et al (2011)

Detailed Models

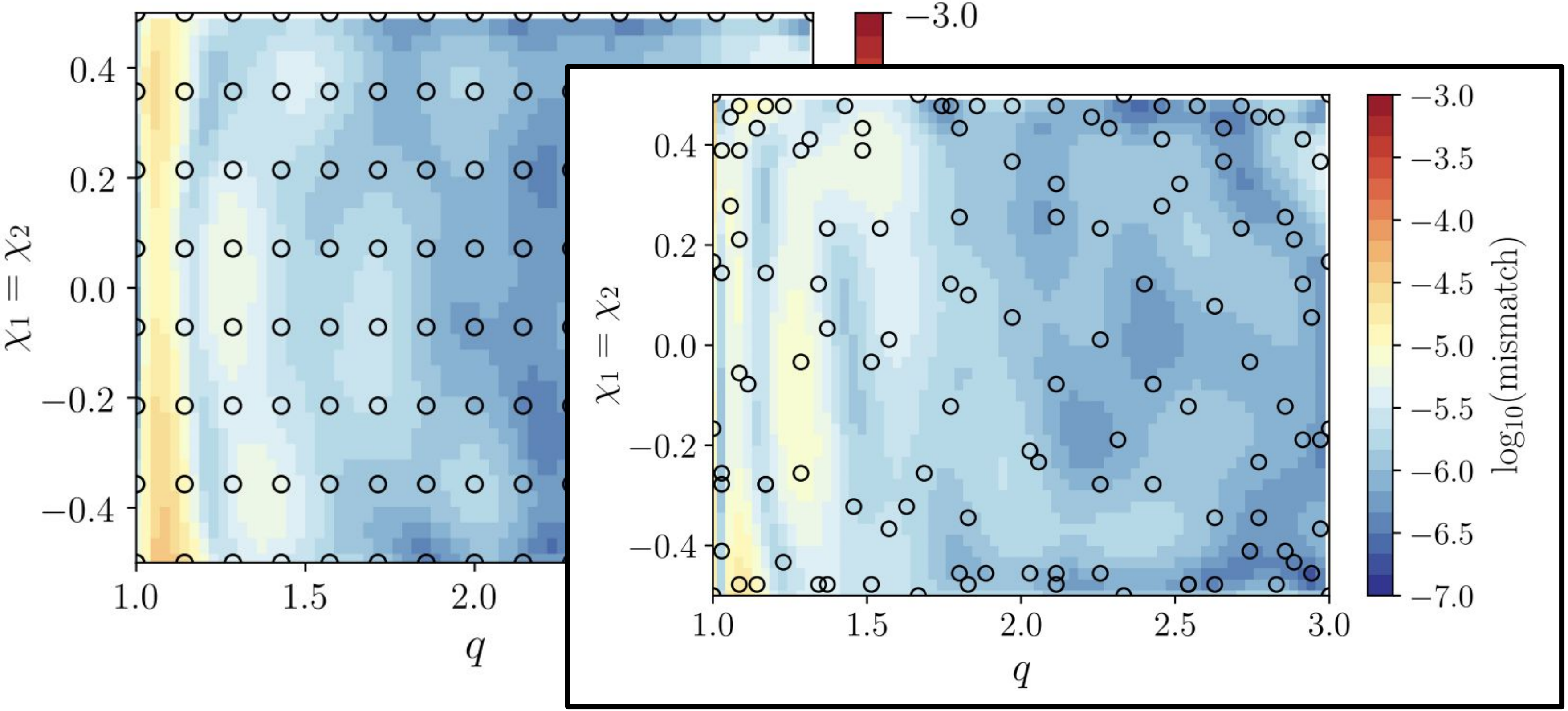
Minimize how many simulations are needed

2,500
7,004
simulations!



Kyle Rocha
CIERA grad

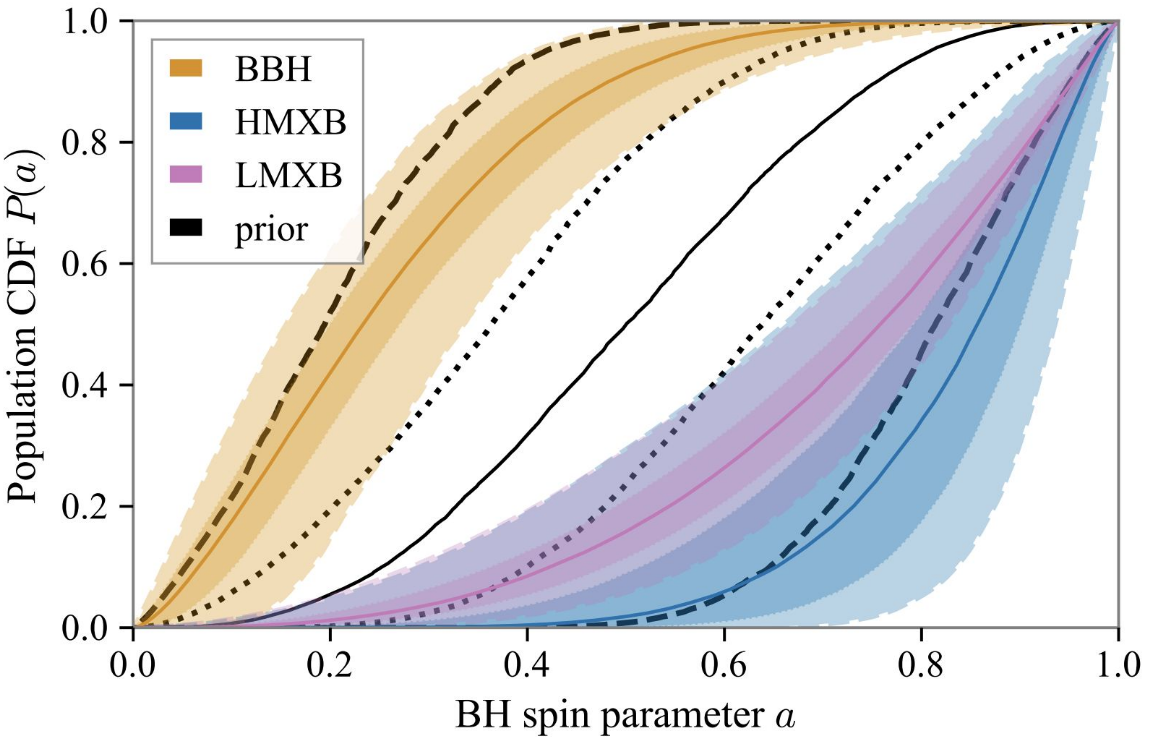
Rocha, …, ZD, … (2022)



A different problem with a similar solution

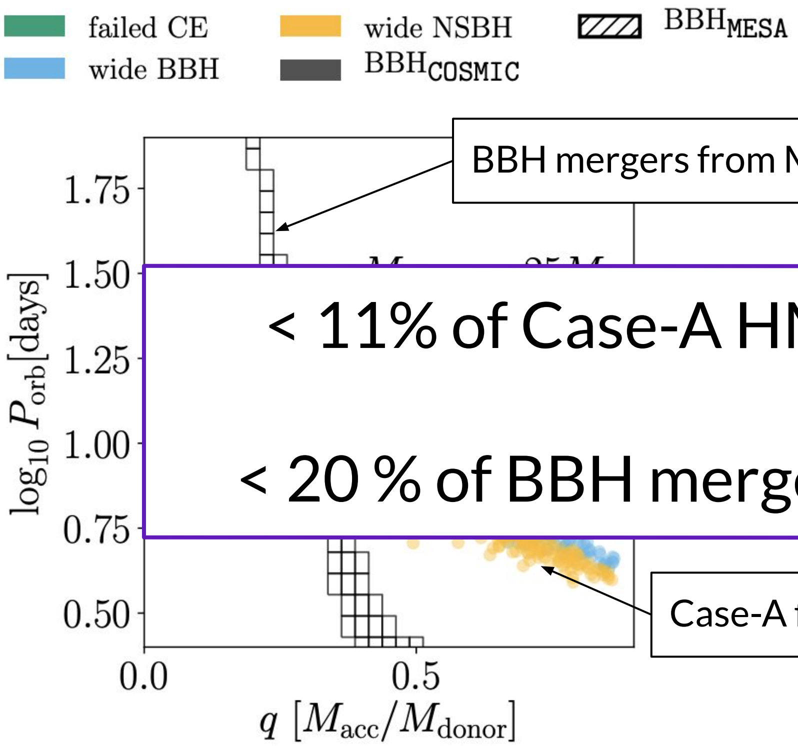
ZD, Farr, Holz, Puerrer (2017)

Do High-Spin HMXBs become High-Spin Merging BBHs?



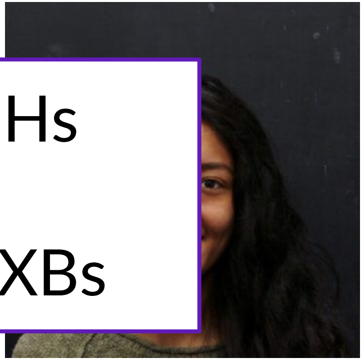
Fishbach & Kalogera (2022)

Do High-Spin HMXBs become High-Spin Merging BBHs?



< 11% of Case-A HMXBs -> Merging BBHs

< 20 % of BBH mergers from Case-A HMXBs

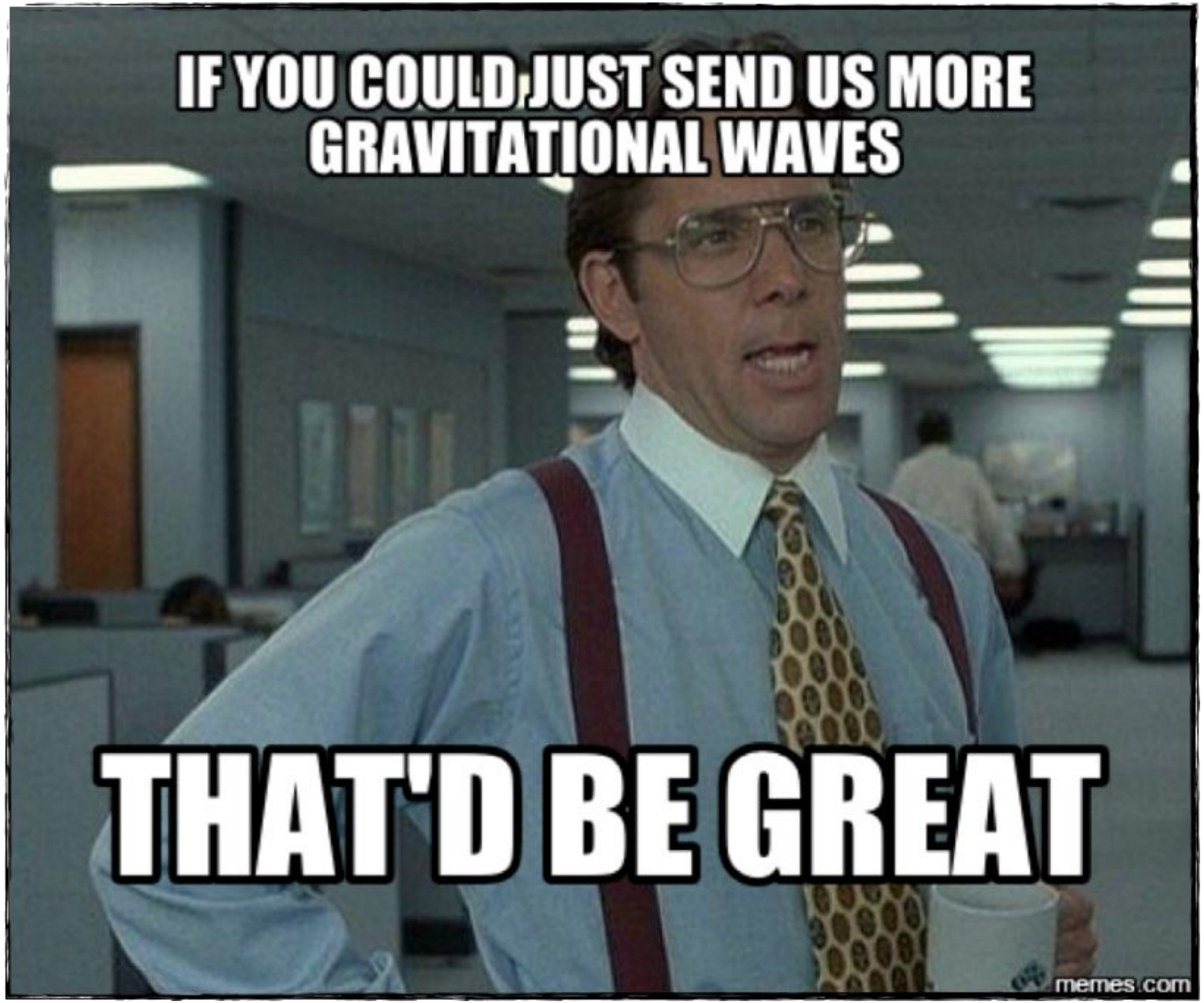


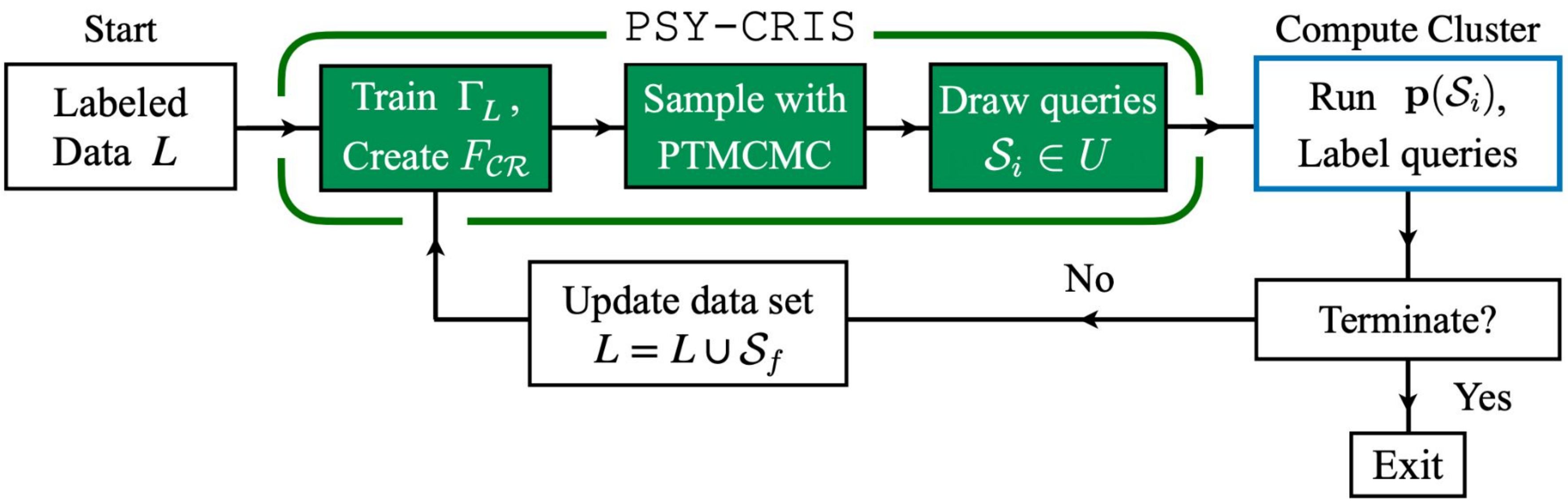
Monica Gallegos-Garcia
CIERA Grad

Gallegos-Garcia, Fishbach, Kalogera, Berry, ZD (2022)

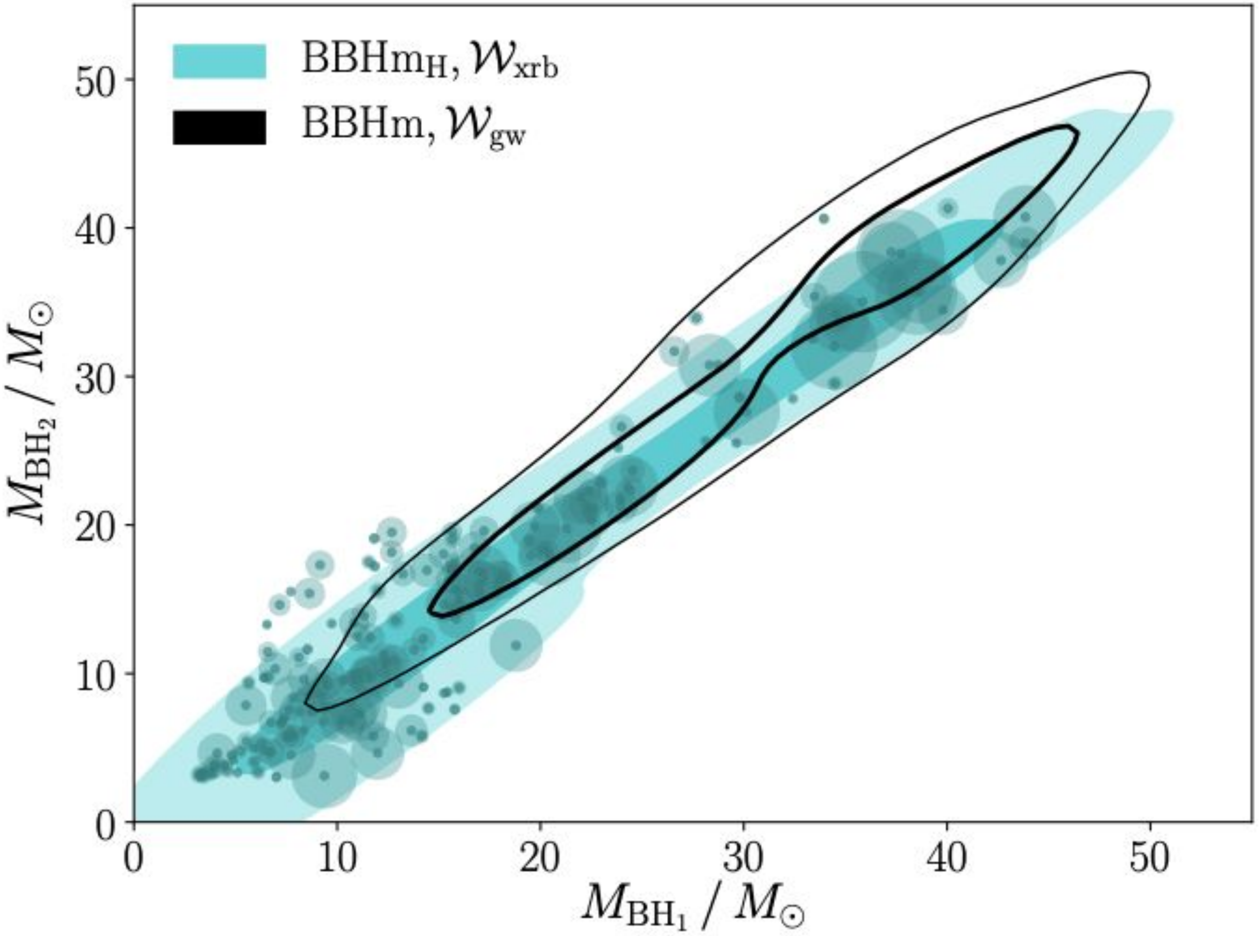
Remarks

- Multiple population modeling approaches enable us to understand CO mergers from different angles
- Detailed models: Incorporate our full astrophysical picture, but expensive and many systematics
- Simple parametric models: Empirically test specific questions
- Data-driven models: Look for the unexpected
- Team effort! A rich set of problems for everyone to get involved in
    - Stars, dynamics, statistics, machine learning, detectors and instrumentation…


IF YOU COULD JUST SEND US MORE GRAVITATIONAL WAVES
THAT'D BE GREAT
memes.com


Start
Labeled Data L
PSY-CRIS
Train Γ_L, Create $F_{\mathcal{CR}}$
Sample with PTMCMC
Draw queries $\mathcal{S}_i \in U$
Compute Cluster
Run $\mathbf{p}(\mathcal{S}_i)$, Label queries
Terminate?
No
Update data set $L = L \cup \mathcal{S}_f$
Yes
Exit

to confusion, we use *merging BBHs* for clarity. To identify high-spin HMXBs in simulations, we assume the spin of the first-born BH is imparted by the scenario of Case-A mass transfer (MT) while both stars are on the main sequence (MS; Valsecchi et al. 2010; Qin et al. 2019). In this scenario, the donor star, which is also the progenitor of the first-born BH, could form a high-spin BH following a combination of (i) MT that prevents significant radial expansion; (ii) strong tidal synchronization at low orbital periods, and (iii) inefficient AM transport within the massive star post MS. We do not



BBHm$_H$, $\mathcal{W}_{xrb}$
BBHm, $\mathcal{W}_{gw}$
50
40
30
20
10
0
0
10
20
30
40
50
M_{BH_2} / $M_\odot$
M_{BH_1} / $M_\odot$