Loop Quantum Gravity and Quantum Information

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- Lec. 1 LQG, ref: Ashtekar-Bianchi, "A short review..." Rep. Prog. Phys 2021 [2104.04394]
- $\boxed{\text{Lec. 2}}$ LQG & QI, ref: Bianchi-Livine, in "Handbook of QG" Springer 2023 [2302.05922]

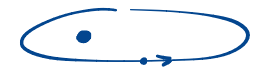
Cracow School of Theoretical Physics

Zakopane

June 17, 2024

QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Semiclassical regime of LQG

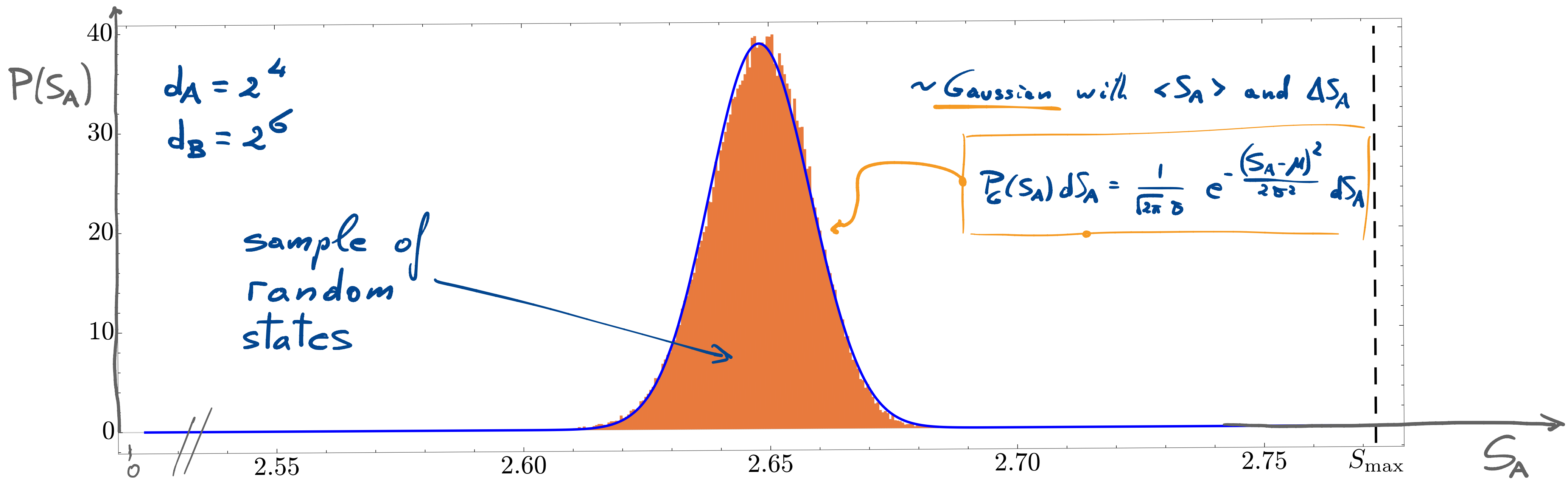
- Ground state ? c.f. Hydrogen atom: $|1s\rangle$ vs. 
- Semiclassical states with fixed $\langle\psi|\hat{O}|\psi\rangle$, $\langle\psi|\hat{O}_1\hat{O}_2|\psi\rangle$, ...

Quantum information — new tools:

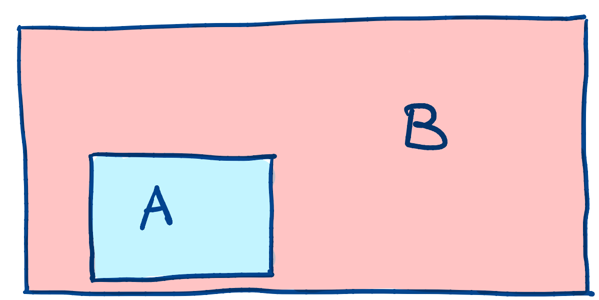
- semiclassical corner of the Hilbert space
- entanglement bounds on correlations
- subsystems from subalgebras
- causal origin of correlations

1 Random states and typical entanglement entropy

* Given a random pure state and a subsystem A, what is the probability of finding entanglement entropy S_A ?



Setup [Page PRL'1993]



- Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, $\dim = d_A \times d_B$
- Random pure state $|\psi\rangle = U\,|\psi_0\rangle$

  uniform distribution: random unitary (Haar) ; reference state
- Entanglement entropy $S_A(|\psi\rangle) = -\mathrm{Tr}_A(\rho_A \log \rho_A)$, $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$

☐ Exact formulas for the average and the variance of $P(S_A)$

☒ Average Entropy [Page, PRL'93] $1 < d_A \leq d_B$

$$\boxed{\langle S_A \rangle = \Psi(d_A d_B + 1) - \Psi(d_B + 1) - \frac{d_A - 1}{2 d_B}}$$

Gamma funct. $\Gamma(x)$

diGamma funct. $\Psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$

- Asymptotics: $d_B \gg d_A$ $\quad \langle S_A \rangle \sim \log d_A$ $\quad$ average entropy ≈ max entropy

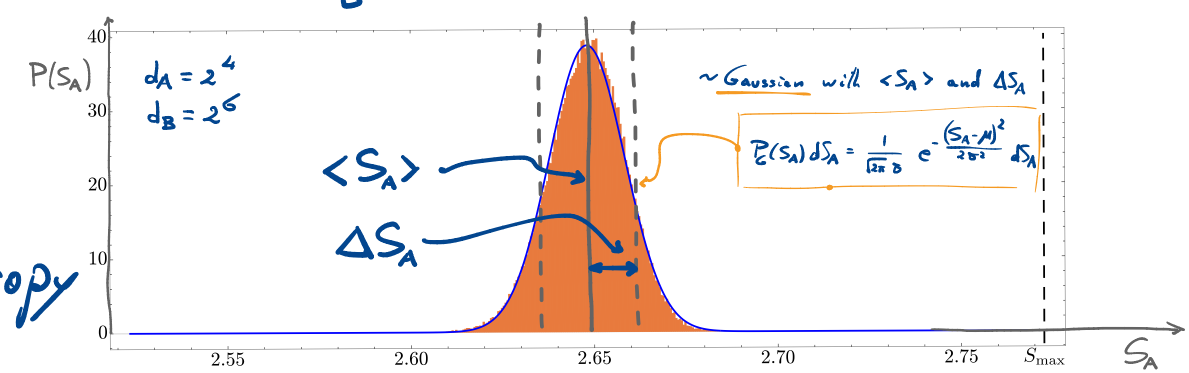
☒ Variance [Bianchi-Donà, PRD'19]

$$(\Delta S_A)^2 = \langle S_A^2 \rangle - \langle S_A \rangle^2$$

$$\boxed{(\Delta S_A)^2 = \frac{d_A + d_B}{d_A d_B + 1} \Psi'(d_B + 1) - \Psi'(d_A d_B + 1) - \frac{(d_A - 1)(d_A + 2 d_B - 1)}{4 d_B^2 (d_A d_B + 1)}}$$

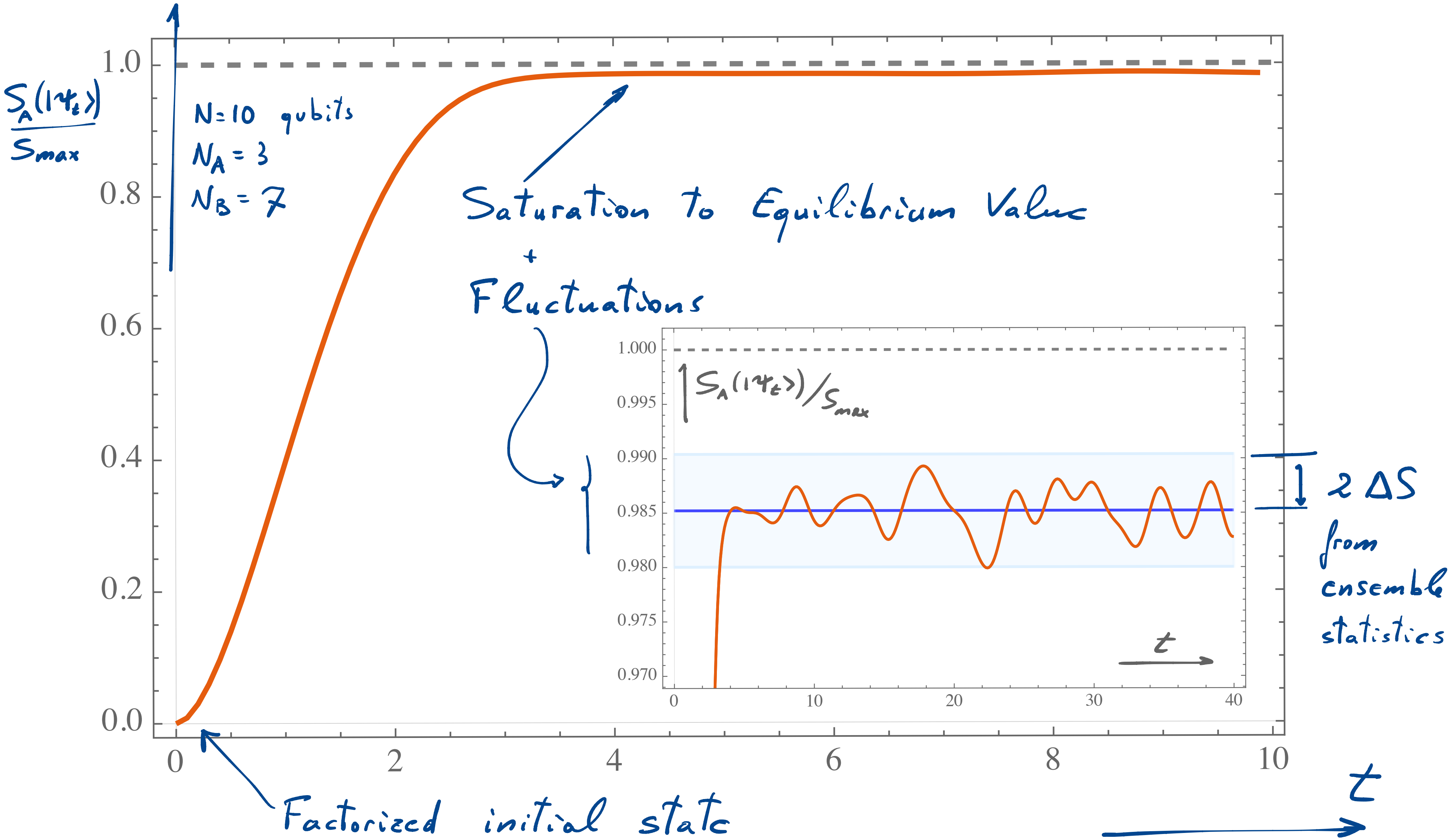
- Asymptotics: $d_B \gg d_A$ $\quad (\Delta S_A)^2 \sim \frac{1}{2 d_B^2}$ $\quad$ narrow distribution

$\Rightarrow$ Typical value of the entanglement entropy



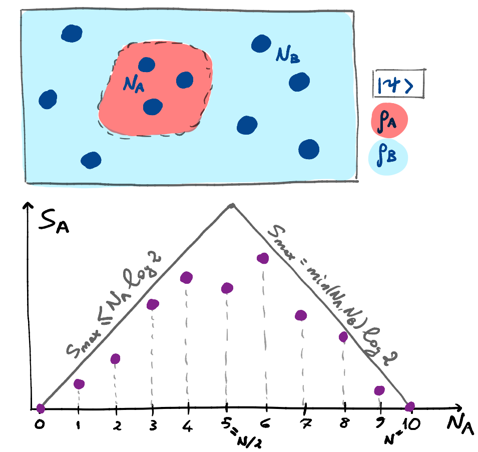
▨ Entanglement Entropy and Thermalization

- Random factorized initial state $|\psi_0\rangle = |\phi_A\rangle|\chi_B\rangle$ (quantum quench)
- Evolution with chaotic Hamiltonian, $|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$



Typical entanglement with constraits

- Page '93: qubit model of unitary BH evaporation
  * note: no Hamiltonian



* Assumptions:

- Finite dimension $\quad \dim \mathcal{H}$ $\quad \left\{ \begin{array}{ll} \text{qubits} & \checkmark \\ \text{QFT} & \times \\ \text{QG} & ? \end{array} \right.$

- Factorization $\quad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ $\quad \left\{ \begin{array}{lll} \text{qubits} & \checkmark & \\ \text{QFT} & \times & \text{(Reeh-Schlieder Thm)} \\ \text{QG} & \times & \end{array} \right.$

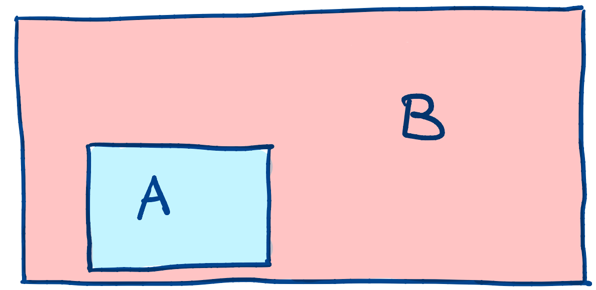
Constraints: $G^i |\psi\rangle = 0$

$$\mathcal{H}^{(j)} = \bigoplus_{\ell} \left(\mathcal{H}_A^{(\ell)} \otimes \mathcal{H}_{\bar{A}}^{(j,\ell)} \right)$$

finite dim, non-factorized

e.g.: qubits → fixed N excitations
cold atoms in optical lattice → N atoms
QFT → fixed energy
QG → Diff & H constraints
BH mass → fixed energy

☐ Typical entanglement with constraits : abelian example



- Hilbert Space $\quad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

- Hamiltonian $\quad H = H_A + H_B \quad$ with $\quad [H_A, H_B] = 0$

- Basis $\quad |\varepsilon_A, \alpha\rangle \, |\varepsilon_B, \beta\rangle$, energy eigenspace $\; E = \varepsilon_A + \varepsilon_B$

- Direct sum decomposition and energy conservation

$$\mathcal{H} = \bigoplus_E \mathcal{H}^{(E)} \quad , \qquad \boxed{\mathcal{H}^{(E)} = \bigoplus_{\varepsilon_A} \left(\mathcal{H}_A^{(\varepsilon_A)} \otimes \mathcal{H}_B^{(E-\varepsilon_A)} \right)}$$

- Energy eigenstates and block-diagonal density matrix

$$H\,|\psi_E\rangle = E\,|\psi_E\rangle \quad \Rightarrow \quad [\rho_A, H_A] = 0 \quad \Rightarrow \quad \rho_A = \begin{pmatrix} \blacksquare & & & \\ & \blacksquare & & \\ & & \blacksquare & \\ & & & \ddots \end{pmatrix} = \bigoplus_{\varepsilon_A} p(\varepsilon_A)\, \rho(\varepsilon_A)$$

with $\quad \rho_A = \mathrm{Tr}_B |\psi_E\rangle\langle\psi_E|$

- Entanglement entropy

$$S_A(|\psi_E\rangle) \equiv -\mathrm{Tr}\left(\rho_A \log \rho_A\right) = \sum_{\varepsilon_A} p(\varepsilon_A) \left(-\mathrm{Tr}\, \rho(\varepsilon_A) \log \rho(\varepsilon_A)\right) - \sum_{\varepsilon_A} p(\varepsilon_A) \log p(\varepsilon_A)$$

Bianchi-Donà [1904.08370] PRD 2019

Bianchi-Hackl-Kieburg-Rigol-Vidmar [2112.06959] PRX 2022

□ Exact formulas for:

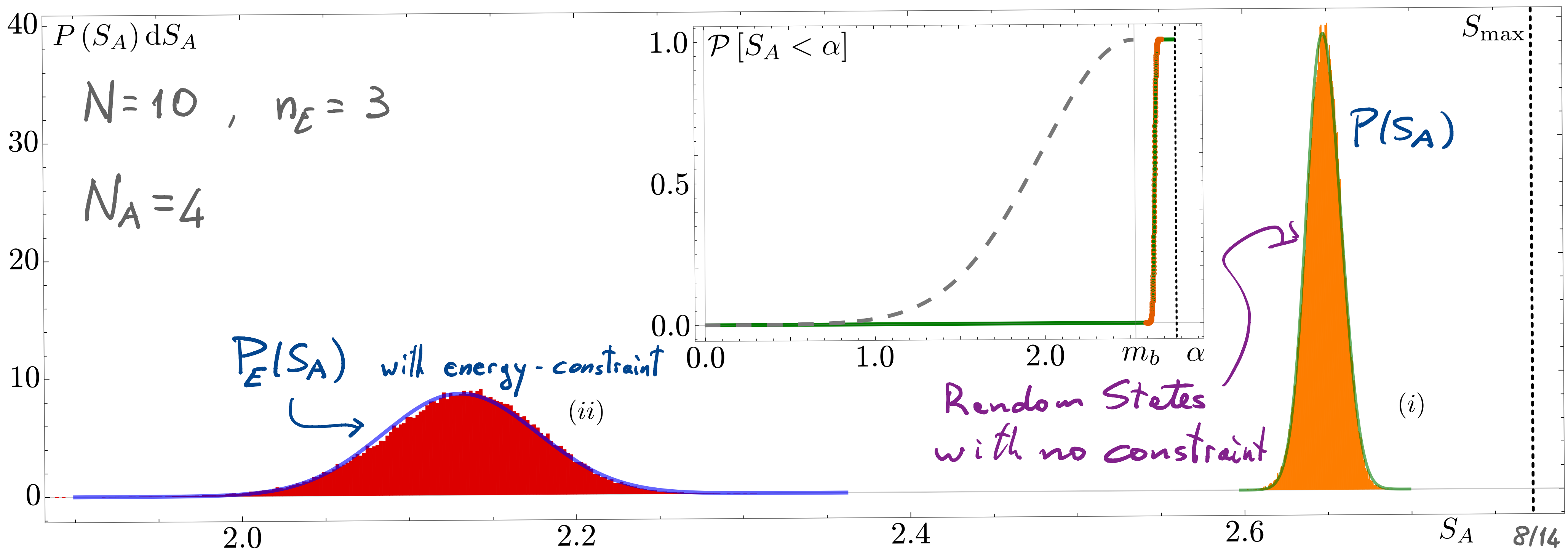
$$\boxed{\mathcal{H}^{(E)} = \bigoplus_r \left(\mathcal{H}_A^{(r)} \otimes \mathcal{H}_{\bar{A}}^{(r)}\right)} \qquad d_r = \dim \mathcal{H}_A^{(r)}\,, \quad b_r = \dim \mathcal{H}_{\bar{A}}^{(r)}\,, \quad D = \sum_r d_r b_r$$

- Average entanglement entropy

$$\boxed{\langle S_A \rangle_E = \sum_r \frac{d_r b_r}{D}\left(\Psi(D+1) - \Psi(\max(d_r, b_r)+1) - \min\left(\frac{d_r-1}{2b_r}, \frac{b_r-1}{2d_r}\right)\right)}$$

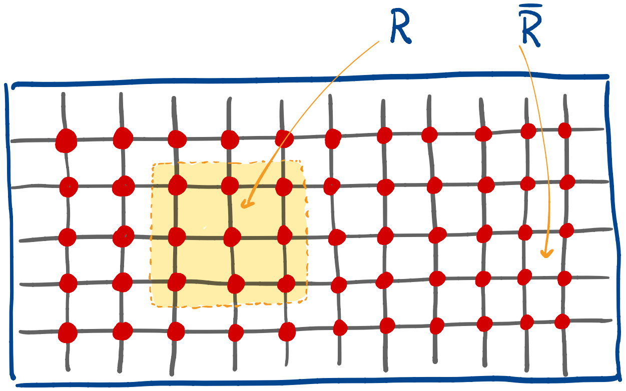
- Variance

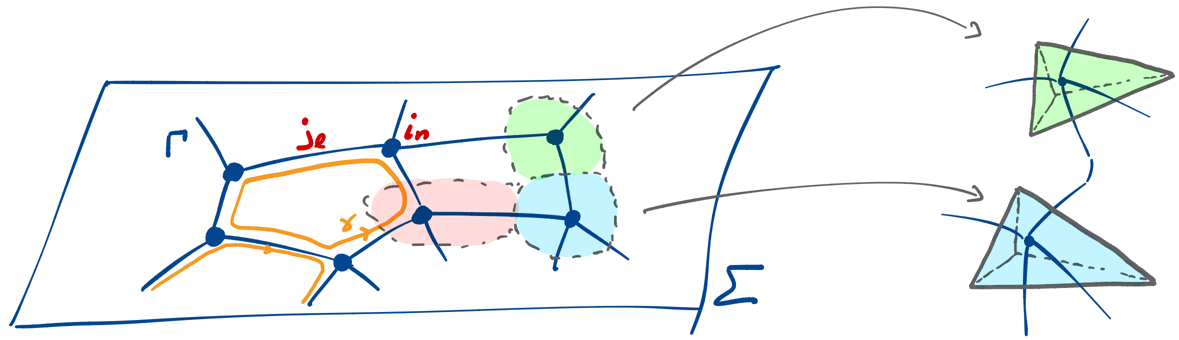
$$\boxed{(\Delta S)_E = \dots}$$



2 Hierarchy of states: Volume law, Area law, Zero law

- Many-body system $\mathcal{H} = \bigotimes_{n=1}^{N} \mathcal{H}_n = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$ vs LQG





a Volume-Law States $S_R(|\psi\rangle) = \gamma\, V_R + \dots$

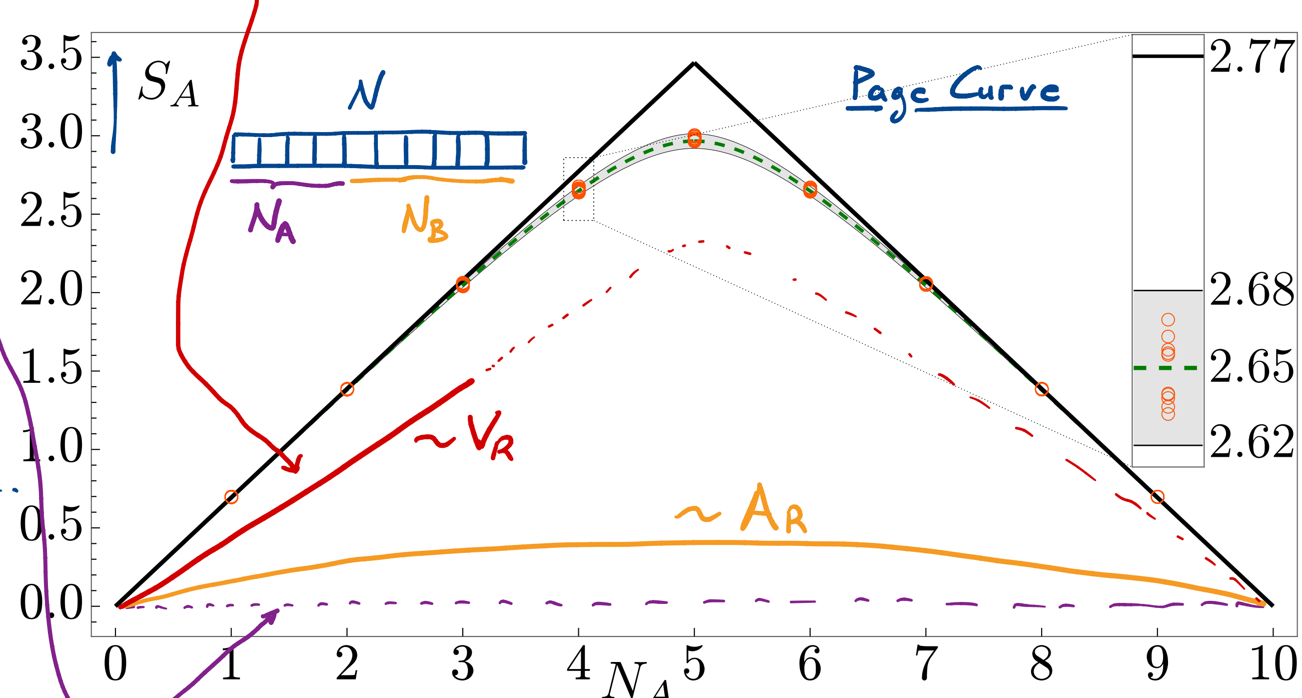
- Random States
- High Temperature

b Zero-Law States $S_R(|\psi\rangle) = c_0 + \dots$

- QG basis states
- High Energy (no T)

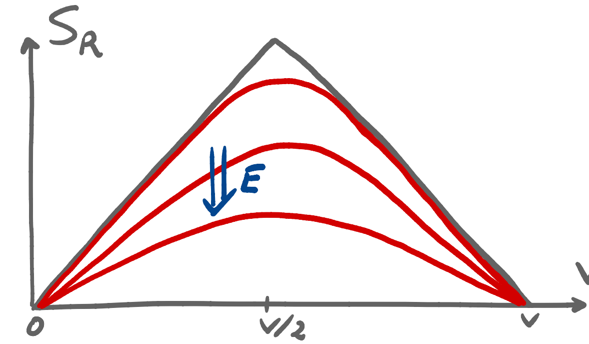
c Area-Law States $S_R(|\psi\rangle) = \alpha\, A_R + \dots$

- Ground State of Local H
- Long-Range Correlations



Area Law & Semiclassicality

- In CM & QFT, as we lower the energy, we transition from volume-law to area-law
- In CM, zero law states are high-energy (not Fock in QFT)

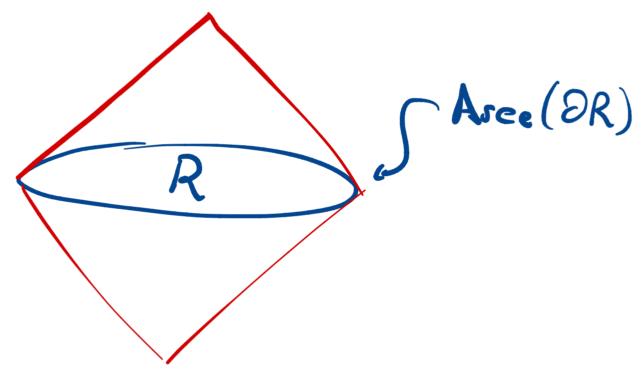


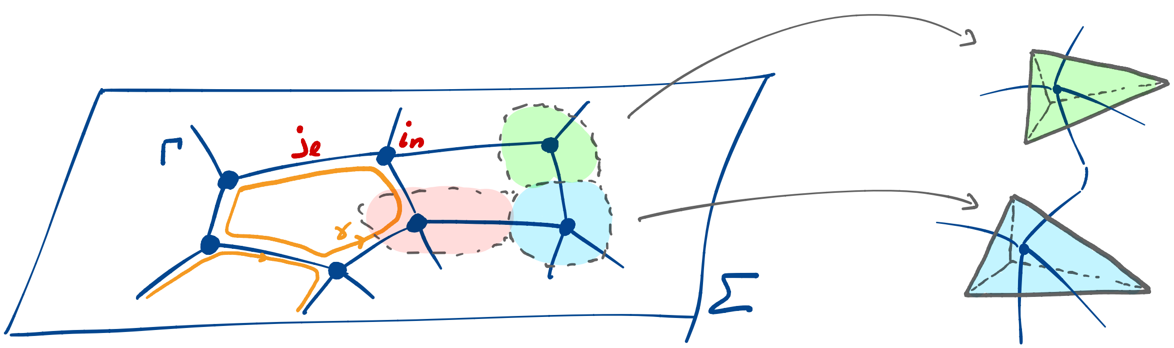
- In QG, we don't have an immediate notion of energy or energy-density

$\Longrightarrow$ Reverse Perspective: Entanglement as a Probe

Architecture Conjecture (Bianchi-Myers [1212.5183] CQG)

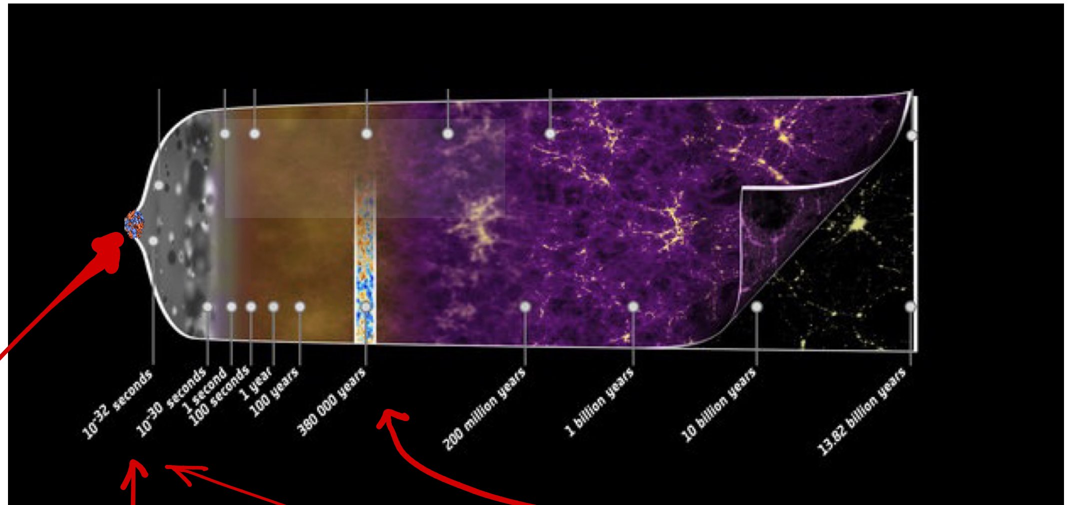
$$\boxed{\begin{array}{l}\text{Semiclassical } |\psi\rangle \text{ in QG} \\ \text{belong to the area-law corner of } \mathcal{H}_{phys} \\ \\ S_R(|\psi\rangle) = 2\pi \dfrac{\langle \mathrm{Area}(\partial R)\rangle}{\ell_p^2} + \dots\end{array}}$$



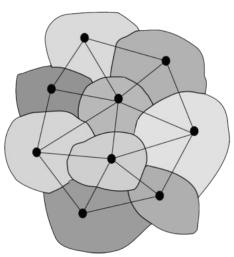
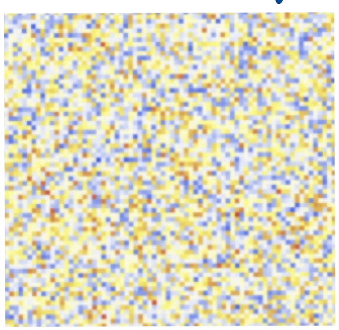
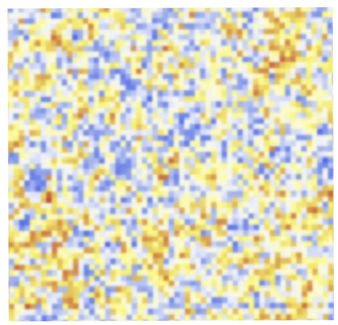
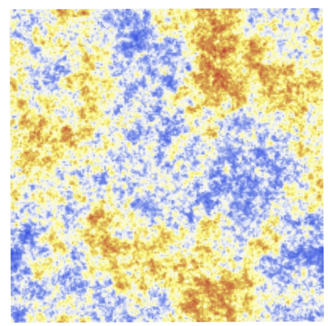
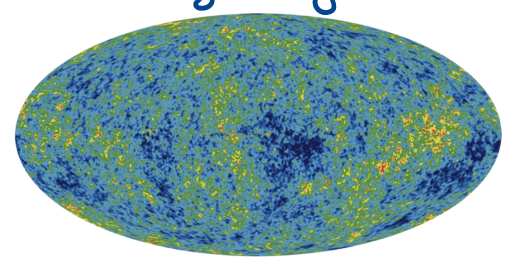


Volume Law and Zero Law States are genuine quantum geometries far from classical spacetime + quantum perturbation

$\boxed{3}$ Zero law and causal origin of entanglement



Planck scale → Pre-Inflationary Phase → Inflation → Hot Big Bang & CMB

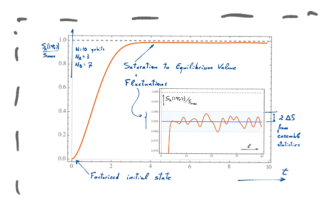






$|\psi_0\rangle$

Zero-law state, cosmological quench

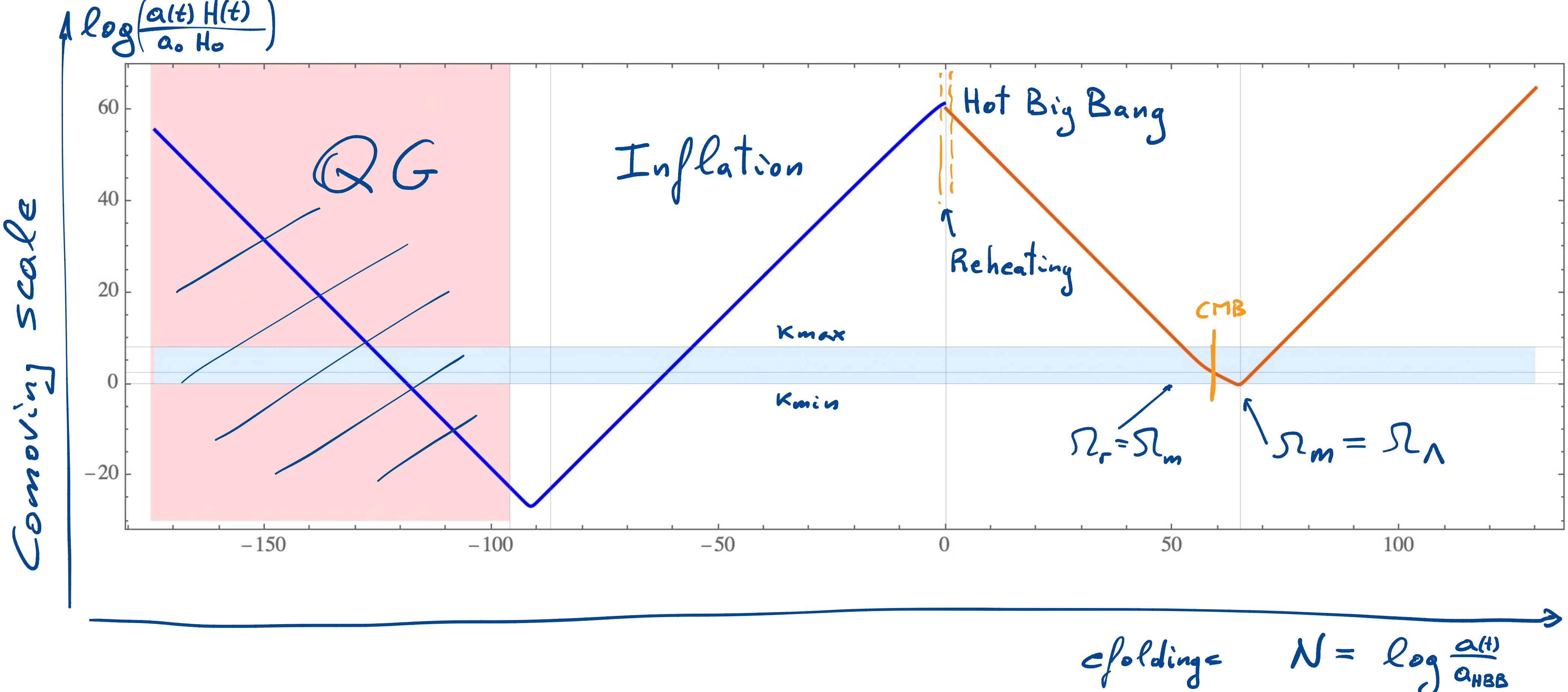
and quantum BKL conjecture

c.f.:



Bianchi-Hackl-Yokomizo [1512.08959] (IJMPD)

☐ Entanglement Dynamics in the Primordial Universe



- Inflation $\longrightarrow$ Gaussian Squeezing
- Before Inflation $\longrightarrow$ QG-to-QFT Transition

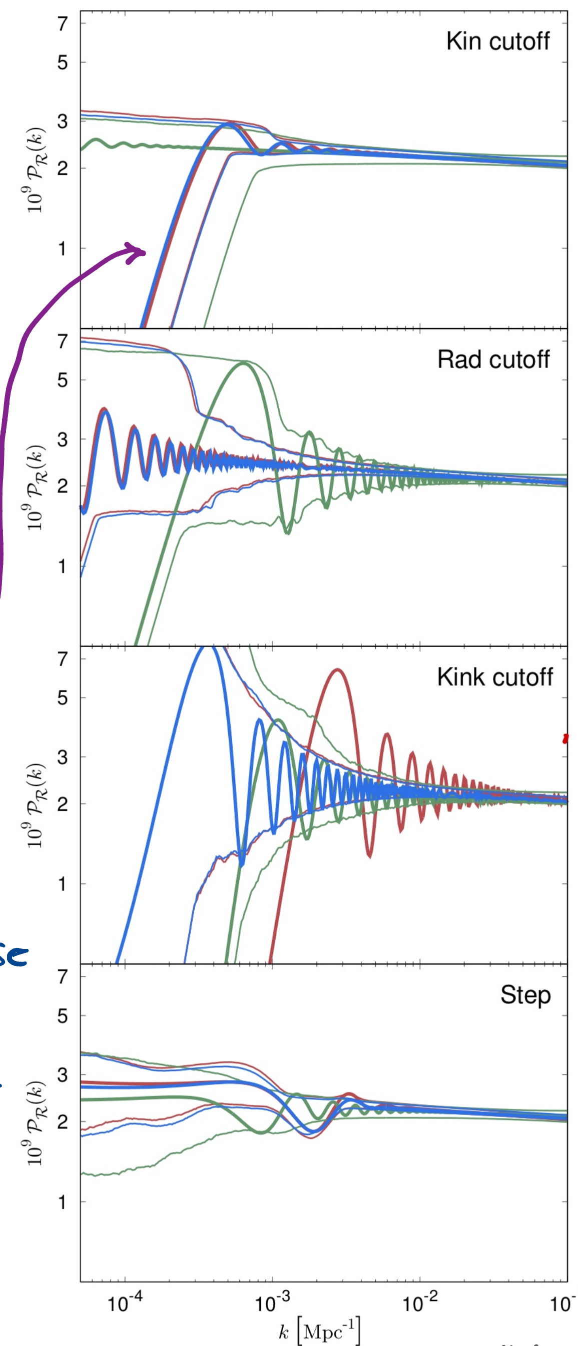
Power Spectrum & Initial State of Perturbations

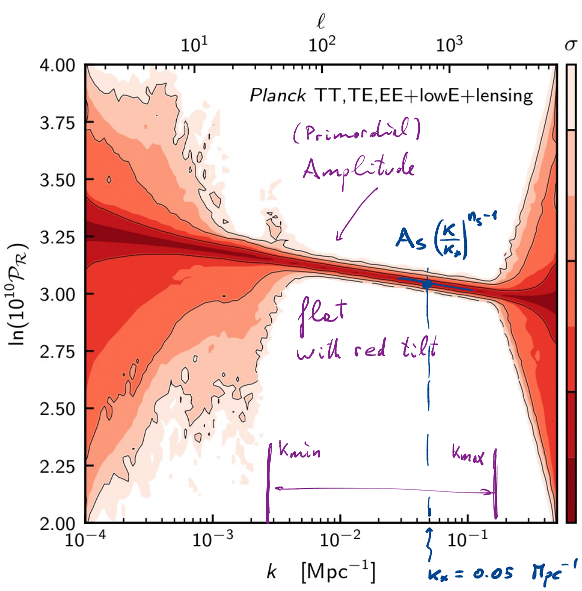
Planck 2018 results. X. Constraints on inflation [1807.06211]

- Initial state $|\psi_0\rangle$: Bunch-Davies vacuum vs excited
- Pre-inflationary phase can prepare excited state
- If inflation doesn't last too long ($N_{infl} \sim 60$)

  $\Rightarrow$ relic of the initial state at large scales (at low k)

- Zero-law state $\Rightarrow$ power suppression





* Note:

- degenerate with kinetic phase
- PLANCK 2018 low-ℓ anomaly
- Statistical significance?

Other observables?

□ Lec. 1 LQG, ref: Ashtekar-Bianchi, "A short review..." Rep. Prog. Phys 2021 [2104.04394]

- Area is more fundamental than length
- Loops are more fundamental than curvature
- QG as TQFT with 2d defects

□ Lec. 2 LQG & QI, ref: Bianchi-Livine, in "Handbook of QG" Springer 2023 [2302.05922]

- Hierarchy of entanglement in CMT and QFT
- Area law in LQG
- Primordial entanglement