Loop Quantum Gravity and Quantum Information

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- Lec. 1 LQG, ref: Ashtekar-Bianchi, "A short review..." Rep. Prog. Phys 2021 [2104.04394]
- Lec. 2 LQG & QI, ref: Bianchi-Livine, in "Handbook of QG" Springer 2023 [2302.05922]

slides at: bit.ly/Zakopane-LQG

Cracow School of Theoretical Physics

Zakopane

June 16, 2024

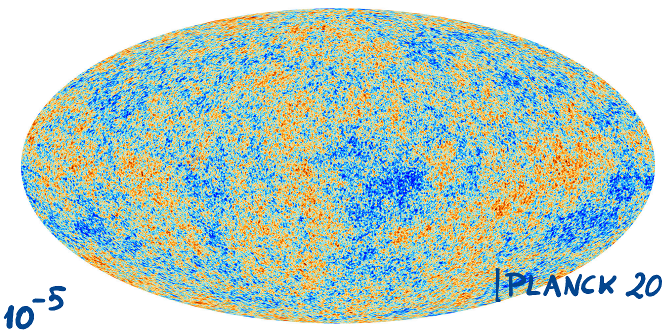
THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

1 Area is more fundamental than length

- Dimensional analysis $\boxed{\dfrac{G\hbar}{c^3}} \sim 10^{-70}\ m^2$ $c=1$, $k_B=1$ but keep $G, \hbar$

- Curvature perturbations and CMB

$$P_s(k) \sim \underline{\underline{G\hbar}}\ \frac{H_*^2}{\epsilon_*} \sim 10^{-10}$$



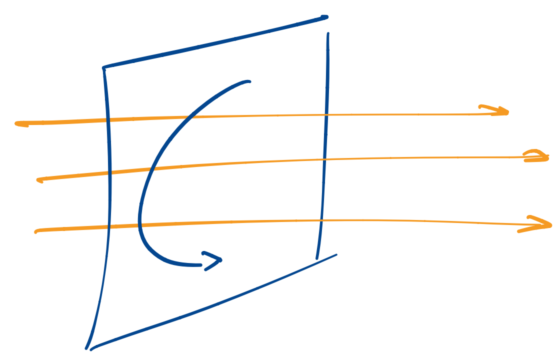
[PLANCK 2018]

$$\frac{\delta T}{T} \sim \sqrt{P_s(k)} \sim 10^{-5}$$

- Black hole entropy $S_{BH}(M,J) = \dfrac{\mathrm{Area}(M,J)}{4G\hbar}$

- Area spectrum and gravitational flux in LQG

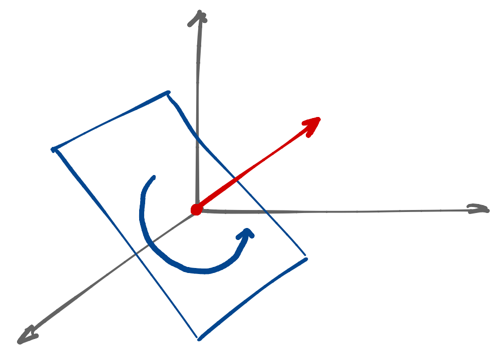
$$\boxed{A_j = 8\pi\, \underline{\underline{G\hbar}}\, \gamma \sqrt{j(j+1)}}$$



□ Angular momentum and quantum geometry

- Angular Momentum $\vec{L} = \vec{q} \times \vec{p}$

  $\longrightarrow$ identifies a plane in $\mathbb{R}^3$



- Phase Space : $\{L^i, L^j\} = \varepsilon^{ij}{}_k L^k$

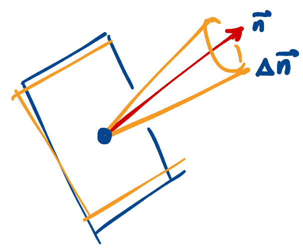
- Quantization $\rightsquigarrow$ Quantum Plane

  - observables $\hat{L}^i$ with $[\hat{L}^i, \hat{L}^j] = i\hbar\, \varepsilon^{ij}{}_k \hat{L}^k$

  - states $|\psi\rangle \in \mathcal{H} = \bigoplus_j \mathcal{H}^{(j)}$, o.n. basis $|j, m\rangle$

- Coherent States (peaked on direction $\vec{n}$ at $j \gg 1$)

  $|j, \vec{n}\rangle$

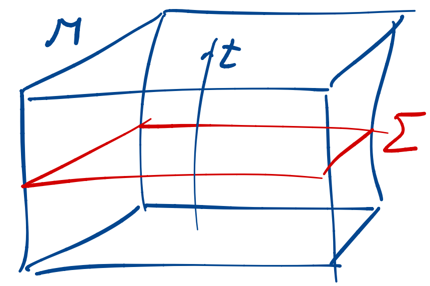


- Superposition of planes $|\psi\rangle = \alpha\, |j, \vec{n}\rangle + \beta\, |j, \vec{n}'\rangle$

$\boxed{2}$ Action, fluxes and loops

- Action
$$S_{GR}[e^I, \underbrace{\omega^{IJ}}_{SO(1,3)}] = \frac{1}{8\pi G}\int_M \underbrace{\frac{1}{2} e^I \wedge e^J}_{\text{Area 2-form}} \wedge \left(\frac{1}{2}\epsilon_{IJKL}\, F^{KL}(\omega) - \frac{1}{\gamma} F_{IJ}(\omega)\right)$$

- 3+1 foliation



$$S_{GR} = \frac{1}{8\pi G\gamma}\int dt \int_\Sigma \underbrace{\frac{1}{2} e^i \wedge e^j}_{p} \wedge \underbrace{\frac{d}{dt}(\omega_{ij} + \gamma K_{ij})}_{\dot{q}} - \dots$$

$$= \frac{1}{8\pi G\gamma}\int dt \int_\Sigma \underbrace{E_i} \wedge \frac{d}{dt} \underline{\underline{A^i}} - \dots$$

Ashtekar connection
$A = \frac{\sigma_i}{2} A^i_a \, dx^a$

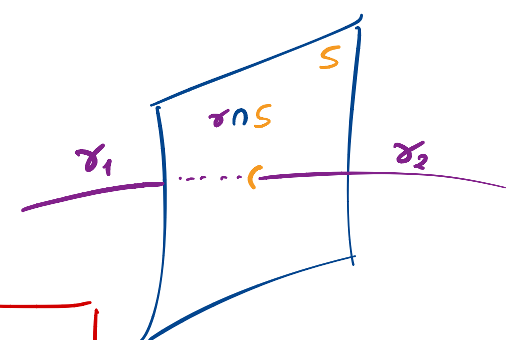
$SL(2,\mathbb{C}) \simeq SO(1,3)$
$\downarrow$
$SU(2) \simeq SO(3)$

- Canonical conjugate variables

$$\{A^i_a(x), E^b_j(y)\} = 8\pi G\gamma\, \delta^i_j\, \delta^a_b\, \delta^{(3)}(x,y)$$

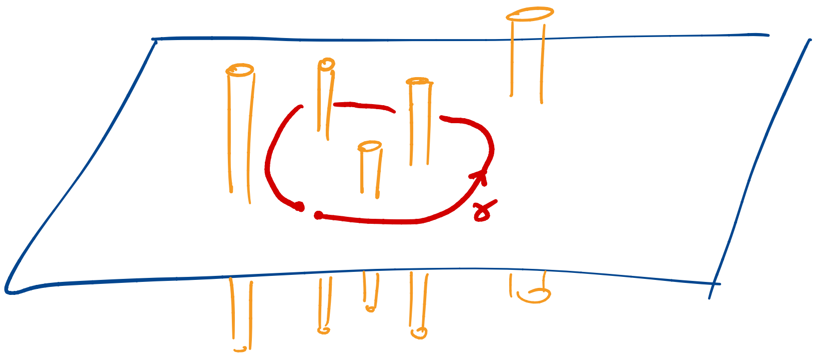
$\Downarrow$

- Holonomy-flux algebra

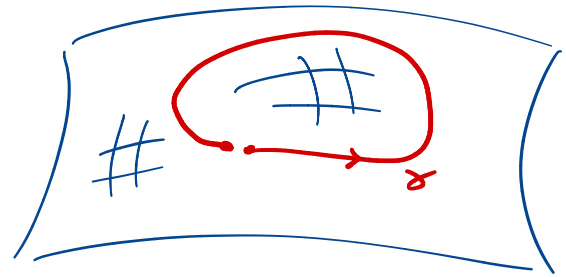


$$\boxed{\{\underline{h_\gamma[A]}, \underline{F^i_S[A,E]}\} = 8\pi G\gamma \; h_{\gamma_1}[A]\, \frac{\sigma^i}{2}\, h_{\gamma_2}[A]}$$

□ Loops are more fundamental than curvature



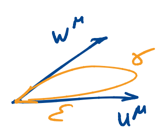
vs



$$h_\gamma{}^\mu{}_\nu = P\exp\left(\int_\gamma \Gamma^\mu_{\alpha\nu}\, dx^\alpha\right) = \delta^\mu{}_\nu + \int_S R^\mu{}_{\nu\alpha\beta}\, dx^\alpha \wedge dx^\beta + \dots$$

$$h_\gamma[A] = P\exp\left(i \int_\gamma A^i \frac{\sigma_i}{2}\right)$$

Proof:



$$= \delta^\mu{}_\nu + \int_\gamma \Gamma^\mu_{\alpha\nu}\, dx^\alpha + \iint \left(\Gamma^\mu_{\alpha\lambda}\Gamma^\lambda_{\beta\nu} - \Gamma^\mu_{\beta\lambda}\Gamma^\lambda_{\alpha\nu}\right) dx^\alpha \wedge dx^\beta + \dots$$

Stokes thm

$$= \delta^\mu{}_\nu + \int_S \left(\partial_\alpha \Gamma^\mu_{\beta\nu} - \partial_\beta \Gamma^\mu_{\alpha\nu} + \Gamma^\mu_{\alpha\lambda}\Gamma^\lambda_{\beta\nu} - \Gamma^\mu_{\beta\lambda}\Gamma^\lambda_{\alpha\nu}\right) dx^\alpha \wedge dx^\beta + \dots$$

s.t. $\partial S = \gamma$

▪ LQG: quantization of holonomies and fluxes

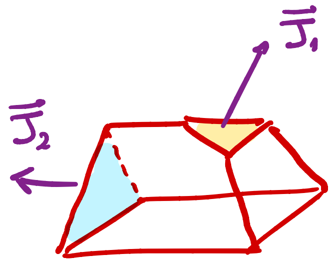
$$\boxed{\begin{aligned} [h_\gamma, F_S^i] &= 8\pi G\hbar\gamma \; h_{\gamma_1} \frac{i\sigma^i}{2} h_{\gamma_2} \\ [F_S^i, F_S^j] &= 8\pi G\hbar\gamma \; i\varepsilon^{ij}{}_k F_S^k \end{aligned}}$$

Grav. flux = spin

$$\Longrightarrow \quad \boxed{F_S^i = 8\pi G\hbar\gamma\, J^i}$$

3 Quantum Polyhedron: Area element ⇝ spin $\vec{J}$

Polyhedron in $\mathbb{R}^3$ with N faces of fixed area

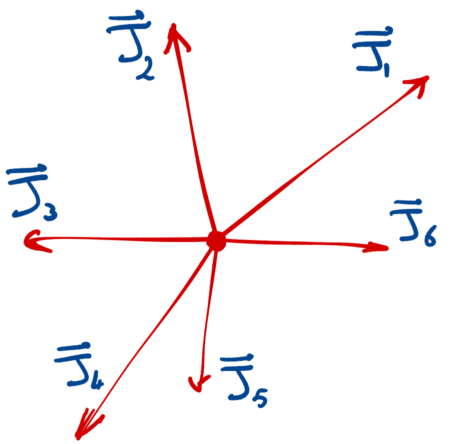


$\Longrightarrow$

$\Longleftarrow$

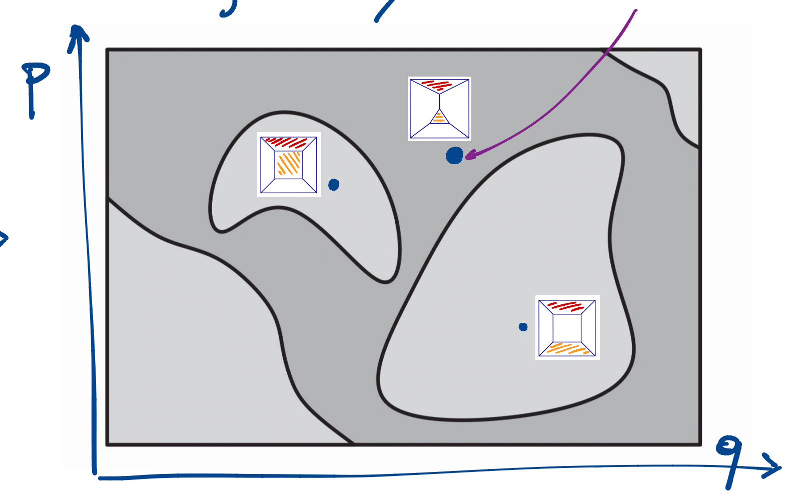
(Minkowski 1897)

N vectors with constraint $\sum_{n=1}^{N} \vec{J}_n = 0$



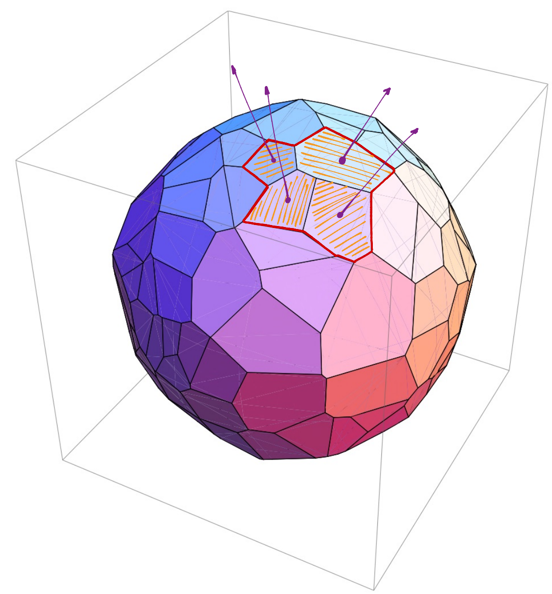
$\Longleftrightarrow$

Phase space (Kapovich-Millson) and adjacency basins

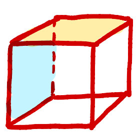
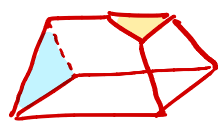


- **Quantization** → N spins with constraint $\boxed{\sum_{n=1}^{N} \vec{J}_n = 0}$

Quantum Polyhedron = SU(2) Intertwiner Space



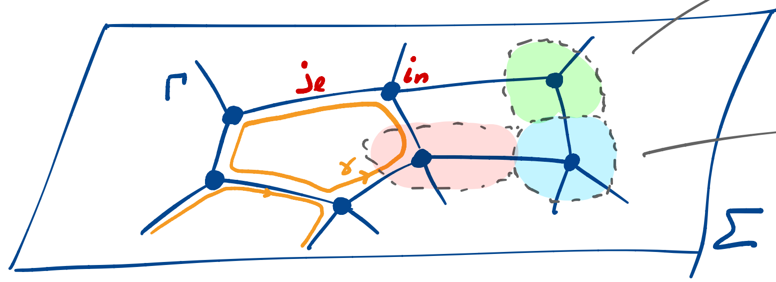
- States of the quantum geometry

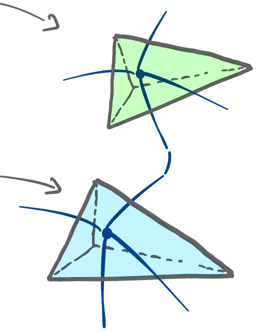
$|\psi\rangle = \alpha\ |$  $\rangle + \beta\ |$  $\rangle$

$\boxed{4}$ Quantum space and quantum spacetime

- Spin-network state = quantum space

$$|\Gamma, j_e, i_n\rangle =$$



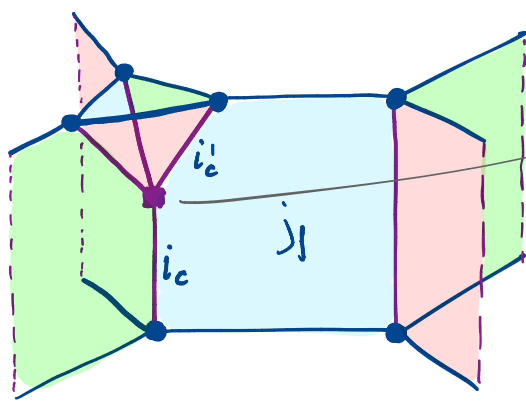


SU(2) irreps j

- Spin-foam transition amplitude = quantum spacetime

$$A_C(|\Gamma, j_e, i_n\rangle \to |\Gamma', j_e', i_n'\rangle) \;=\; \text{"}\int_{h_{ab}}^{h'_{ab}} \mathcal{D}g_{\mu\nu}\; e^{\frac{i}{\hbar} S_{GR}[g_{\mu\nu}]}\text{"}$$

$=$



SL(2, $\mathbb{C}$) irreps

$$\begin{cases} p = \gamma j \\ k = j \end{cases}$$

□ Gravity as a topological theory with constraints

	$S_{BF} = \int \underbrace{B_{IJ}} \wedge F^{IJ}(\omega)$	$S_{GR} = \int \underbrace{\frac{1}{16\pi G}\left(\frac{1}{2}\epsilon_{IJKL}\, e^K \wedge e^L - \frac{1}{\gamma}\, e_I \wedge e_J\right)} \wedge F^{IJ}(\omega)$
Fields	$B_{IJ}\,,\ \omega^{IJ}$	$e^I\,,\ \omega^{IJ}$
Gauge	Diff(M) $SO(1,3)$ $B^{IJ} \to B^{IJ} + \nabla\theta^{IJ}$ topological shift	Diff(M) $SO(1,3)$ ×
d.o.f.	zero locally only topological dof	2 per point $= 2 \times \infty$ unfreeze d.o.f. in a controlled way

- GR as topological theory with constrained $B^{IJ} =$ γ-simple 2-form
- LQG as 4d TQFT with γ-simple constraint on 2d defects

$B^{IJ} \rightsquigarrow J^{IJ}$ $SL(2,\mathbb{C})$ generator : γ-simple irreps $\vec{K}^2 - \vec{L}^2 = \left(\gamma - \frac{1}{\gamma}\right) \vec{K}\cdot\vec{L}$

$\Longrightarrow \boxed{p = \gamma K}$

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  - Area is more fundamental than length
  - Loops are more fundamental than curvature
  - QG as TQFT with 2d defects

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- Lec. 2 LQG & QI, ref: Bianchi-Livine, in "Handbook of QG" Springer 2023 [2302.05922]
  - Hierarchy of entanglement in CMT and QFT
  - Area law in LQG
  - Primordial entanglement