CDT from the IR to the UV

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4d (lattice) QG

Main goal (at least in 80ties) for QG

- Obtain the background geometry $\langle g_{\mu\nu} \rangle$ we observe
- Study the fluctuations around the background geometry
- The possibility to relate to other QG approaches like FRG

What 4d CDT offers:

- A non-perturbative QFT definition of QG*
- A background independent formulation
- An emergent background geometry $\langle g_{\mu\nu} \rangle$
- The possibility to study the quantum fluctuations around this emergent background geometry.

*QG = Quantum Gravity or QG = Quantum Geometry

Problems to confront for a (lattice) theory of QG

(1) How to define the quantum theory

(2) How to face the non-renormalizability of quantum gravity

(3) Provide evidence of a continuum limit (where the continuum field theory has the desired properties)

(1) How to define the quantum theory

The classical action in 3+1 dimensions:

$$S[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g(x)} \Big(R(x) - 2\Lambda \Big)$$

One may define the quantum theory via the path integral

$$Z[G,\Lambda] = \int \mathcal{D}[g] \; e^{iS[g]/\hbar}$$

But what precisely is meant by $\int \mathcal{D}[g]$?

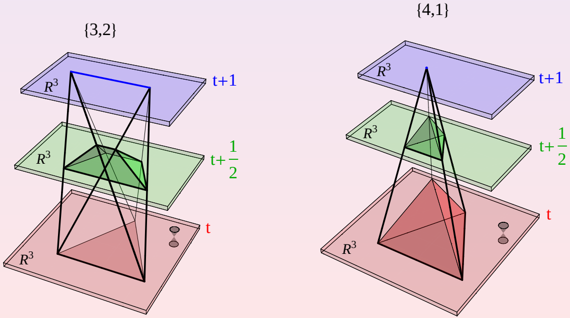
Classical GR refers to a manifold and a metric $g_{\mu\nu}$ with Lorentzian signature. Consider instead metrics with Euclidean signature. Then “classical” GR refers to smooth Riemannian manifolds.

The path integral in QM integrates not only over classical smooth paths, but over continuous paths. Should we integrate over continuous geometries and should we include an integration over different manifolds (e.g. manifolds with different topologies)?

CDT takes a minimalistic view on these problems: one fixes the manifold. Here I will discuss only the situation where $\mathcal{M} = R \times S^3$. We use a subset of continuous geometries, in fact a subset of so-called piecewise linear geometries, that can be constructed by gluing together identical building blocks (4-simplices). They are hopefully dense in the set of continuous geometries.

Inspired by canonical quantization we assume a Lorentzian signature, a hyperbolic spacetime and a time foliation. We triangulate it using the building blocks and use the Regge action for piecewise linear manifolds in the path integral.

The somewhat remarkable aspect is that one can actually rotate each such Lorentzian geometry to an Euclidean geometry, where each building block is then a 4-simplex where all links have the same length a. This length then plays the role of a UV cut-off, precisely as in ordinary lattice field theories.



Another remarkable feature is that the Regge action becomes very simple when using identical building blocks. For a given 4d triangulation T, denote by $N_4(T)$, $N_2(T)$ and $N_0(T)$ the number of 4-simplices, 2-simplices (triangles) and 0-simplices (vertices). Then

$$S[T] = -k_2 N_2(T) + k_4 N_4(T) = -k_0 N_0(T) + \tilde{k}_4 N_4(T)$$

where

$$k_2 = c_1 \frac{a^2}{G}, \quad k_4 = c_2 \frac{a^2}{G} + c_3 \frac{a^4 \Lambda}{G}.$$

$$k_0 = c_1' \frac{a^2}{G}, \quad \tilde{k}_4 = c_2' \frac{a^2}{G} + c_3' \frac{a^4 \Lambda}{G}.$$

$$\begin{aligned} S[T] &= -k_0 N_0(T) + k_{32} N_{32}(T) + k_{41} N_{41}(T) \\ &= -(k_0 + 6\Delta) N_0(T) + k_4 N_4(T) + \Delta N_{41}(T) \end{aligned}$$

$$Z_L(G,\Lambda) = \int \mathcal{D}[g_L]\, e^{iS_L[g_L]} \to Z^L_{CDT}(k_2,k_4) \to Z^E_{CDT}(k_2,k_4)$$

$$Z^E_{CDT}(k_2,k_4) = \sum_T \frac{1}{C_T}\, e^{-S[T]} = \sum_{N_4,N_2} e^{k_2 N_2 - k_4 N_4} \mathcal{N}(N_2,N_4),$$

$$\mathcal{N}(N_2,N_4) = \sum_{T(N_2,N_4)} \frac{1}{C_T}$$

The partition function for QG is the generating function for the number of abstract triangulations with N_4 4–simplices and N_2 2–simplices. QG is pure combinatoric!

(2) Facing the non-renormalizability of QG

It is known how to make 4d QG renormalizable: add an R^2 term to the action (Stelle, 1977). It makes the theory asymptotically free. Problem with unitarity.

Another route is via the asymptotic safety scenario (ASS) (Weinberg 1979), implemented via the functional renormalization group (FRG). Here one investigates if there exists a non-perturbative UV fixed point in a Wilsonian formulated theory of QG. So far FRG results have provided support for this idea.

4d CDT has a reflection positive transfer matrix. Such lattice theories result in unitary theories if a continuum limit exists. Thus unitarity is probably not an issue in CDT.

Lattice field theories are well suited to investigate fixed points and the corresponding continuum limits. Thus 4d CDT seems ideally suited to study ASS.

Recall standard lattice field theory (LFT)

(1) Asymptotic free theories (Gaussian fixed points)

Prime example: YM theories in 4d. One (bare) coupling constant g_0. From perturbation theory we know that the fixed point is UV (the β-function is negative).

$$\beta(g_0) = -a\frac{dg_0}{da} = -\beta_1 g_0^3 - \cdots , \qquad a(g_0) = \frac{1}{\Lambda_{YM}} e^{-1/2\beta_1 g_0^2},$$

For a physical mass m_{ph} (from stringtension , glueball mass...)

$$m_0(g_0) = m_{ph}\, a(g_0) = \frac{m_{ph}}{\Lambda_{YM}} e^{-1/2\beta_1 g_0^2}$$

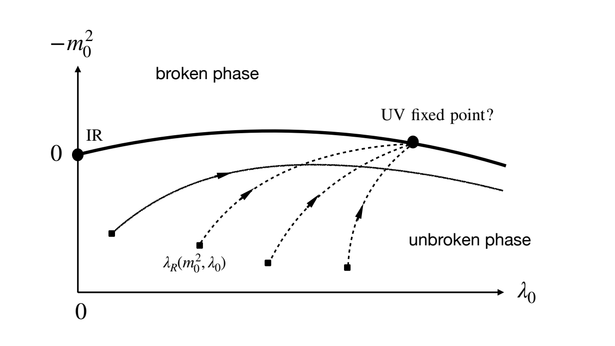
We can measure $m_0(g_0)$ on the lattice and thus reconstruct the β-function, even if we could not calculate it perturbatively.

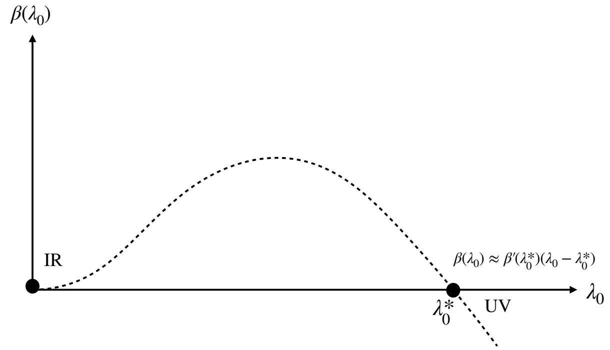
(2) Non-Gaussian UV fixed points

ϕ^4 theory in 4d. Two dimensionless coupling constants m_0^2, λ_0

$$S = \sum_n \Big(\sum_{\mu=1}^{4} (\phi(n+\mu) - \phi(n))^2 + m_0^2 \phi^2(n) + \lambda_0 \phi^4(n) \Big)$$

$$\lambda_R(m_0^2, \lambda_0) \propto \Gamma_0^{(4)}(p_i = 0; m_0^2, \lambda_0)$$



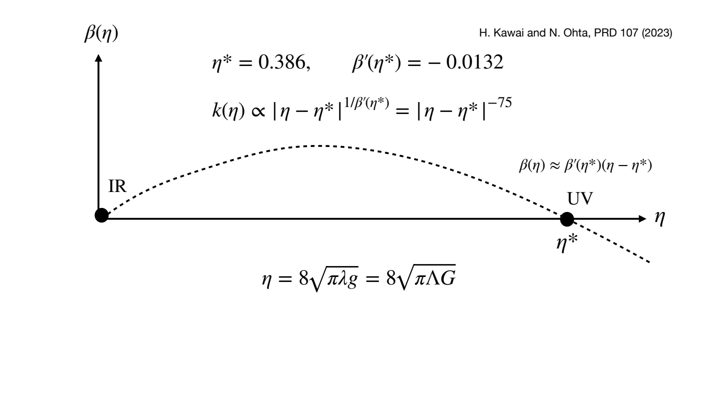


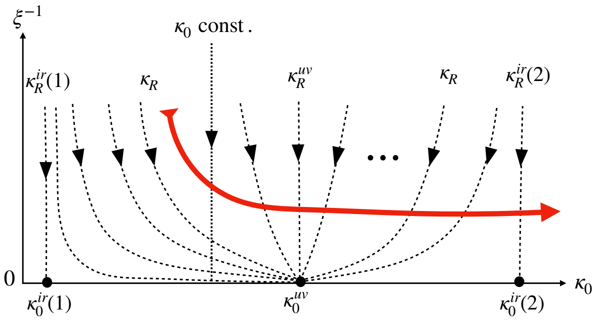
If there had been a UV fixed point:

$$-a\frac{d\lambda_0}{da} = \beta(\lambda_0) \approx \beta'(\lambda_0^*)(\lambda_0 - \lambda_0^*), \qquad a(\lambda_0) \propto |\lambda_0 - \lambda_0^*|^{-1/\beta'(\lambda_0^*)}$$

For $\beta'(\lambda_0^*) < 0$ we can define a continuum limit for $\lambda_0 \to \lambda_0^*$.

This situation is precisely what one would expect from the UV fixed point in QG. A suitable dimensionless coupling constant is ΛG and its β-function has been calculated in RFG. The only problem translating the scenario to a lattice theory is to the relation between the lattice cut off a and the FRG scale parameter k. We will return to this later.





$$0 = \xi \frac{d}{d\xi} \kappa_R(\kappa_0(\xi), \xi) = \xi \frac{\partial \kappa_R}{\partial \xi}\Big|_{\kappa_0} + \frac{\partial \kappa_R}{\partial \kappa_0}\Big|_{\xi} \, \xi \frac{d\kappa_0}{d\xi}\Big|_{\kappa_R}$$

$$\beta_0(\kappa_0) = \xi \frac{d\kappa_0}{d\xi}\Big|_{\kappa_R}, \quad \beta_R(\kappa_R) = -\xi \frac{\partial \kappa_R}{\partial \xi}\Big|_{\kappa_0}, \quad \beta_R(\kappa_R) = \frac{\partial \kappa_R}{\partial \kappa_0}\, \beta_0(\kappa_0)$$

The CDT lattice theory

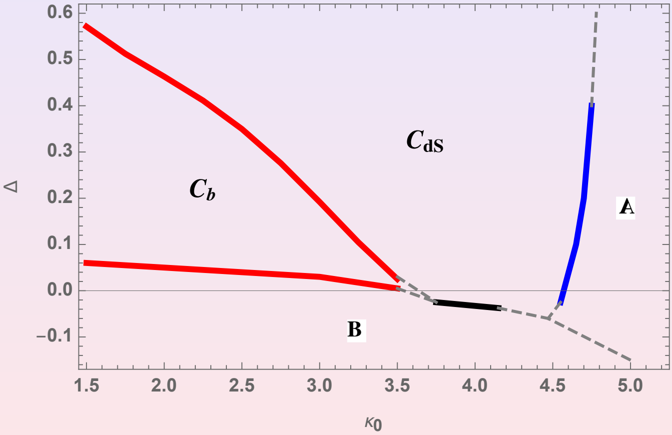
Three coupling constants, k_0, Δ, k_4

$$S[T] = -(k_0 + 6\Delta) N_0(T) + k_4 N_4(T) + \Delta N_{41}(T)$$

Not only is the connectivity of the lattice not fixed but also the spacetime volume N_4 is not fixed. However, for computer technical reasons we trade k_4 for a fixed N_4. We can recover the partition function as a function of k_4 by a (discrete) Laplace transformation

$$Z(k_4) = \sum_{N_4} \mathrm{e}^{-k_4 N_4} Z(N_4)$$

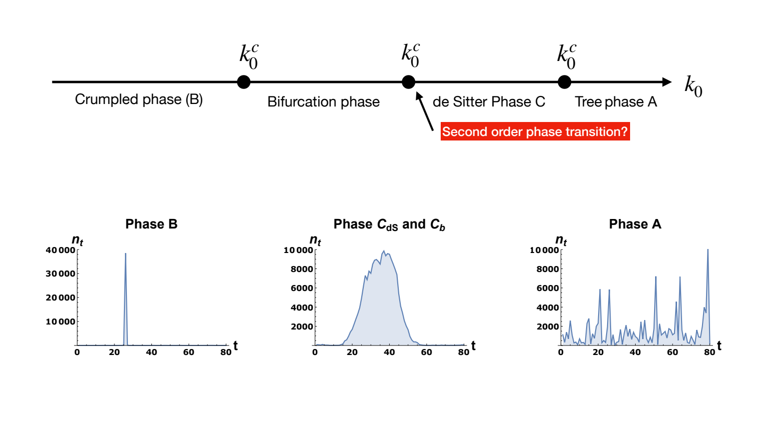
For a fixed N_4 we then have the following CDT phase diagram, where the precise phase transition lines are functions of N_4. Of course we only have "real" phase transitions for $N_4 = \infty$. For finite N_4 we have "pseudo-critical" phase transition lines.



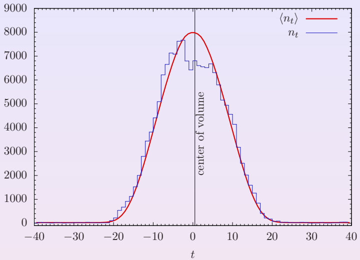
The C_b-C_{dS} transition line has the interpretation as breaking of homogeneity and isotropy of space. A UV fixed point on this line would imply that the short distance physics is related to this symmetry breaking and could have implication for cosmology, but until now we have not found a candidate for such a UV fixed point at the C_b-C_{dS} transition line. Our main interest in the following will be centered at the $A - C_{\mathrm{dS}}$ transition line

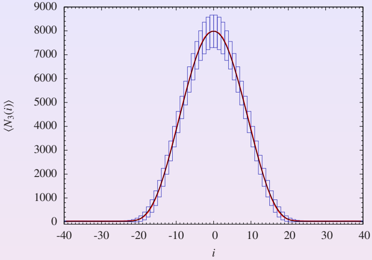
Let us look closer at how the structure of spacetime could change at the phase transition and whether these changes can be used to define a continuum limit of the lattice theory.

For each time-slice t we have a spatial volume $V_3(t) \propto N_3(t)a^3$.



The de Sitter phase





$$\langle N_3(t)\rangle \propto N_4 \frac{1}{\omega(k_0,\Delta)N_4^{1/4}} \cos^3\left(\frac{t}{\omega(k_0,\Delta)N_4^{1/4}}\right),$$

This is exactly the spatial volume profile of a (compressed) four-sphere of volume N_4 if we use a metric

$$d\tau^2 = dt^2 + \ell^2(t)d\Omega_3, \quad V_3(t) \propto \ell^3(t).$$

The fluctuations behave like

$$\Delta N_3(t) = C(k_0,\Delta)\sqrt{N_4}\; F\left(\frac{t}{\omega(k_0,\Delta)N_4^{1/4}}\right), \quad F(0) = 1$$

Thus we have seemingly obtained some of the goals declared in the beginning: obtaining a $\langle g_{\mu\nu}\rangle$ and being able to study the fluctuations around this configuration (at least in the limiting sense of studying the spatial volume).

In fact we can do more: we can obtain the effective minisuperspace action for $\langle N_3(t)\rangle$ from the study of correlation functions $\langle N_3(t)N_3(t')\rangle$ and show that the fluctuations around $\langle N_3(t)\rangle$ are well described expanding this minisuperspace action to quadratic order in the fluctuations.

$$S_{\rm eff} = \frac{1}{\Gamma} \sum_i \left(\frac{\big(N_3(t_{i+1}) - N_3(t_i)\big)^2}{N_3(t_i))} + \delta\, N_3^{1/3}(t_i) \right).$$

$$s_i = \frac{t_i}{N_4^{1/4}}, \quad n_3(s_i) = \frac{N_3(t_i)}{N_4^{3/4}}, \quad \Delta s = \frac{1}{N_4^{1/4}},$$

$$S_{\rm eff} = \frac{\sqrt{N_4}}{\Gamma} \int_{-\pi\omega/2}^{\pi\omega/2} ds \left(\frac{(\dot{n}_3^2(s))}{(n_3(s))} + \delta\, n_3^{1/3}(s) \right), \quad \int ds\, n_3(s) = 1.$$

we now have with high precision:

$$\langle n_3(s) \rangle = \frac{3}{4\omega} \cos^3\left(\frac{s}{\omega}\right), \quad \frac{\delta}{\delta_0} = \left(\frac{\omega_0}{\omega}\right)^{8/3} \quad \omega_0^4 = \frac{3}{8\pi^2}, \; \delta_0 = 9(2\pi^2)^{2/3}$$

$$\Gamma = \Gamma(k_0, \Delta, N_4), \quad \delta = \delta(k_0, \Delta, N_4), \quad \frac{\Delta N_3(t)}{\langle N_3(t) \rangle} \propto \frac{\sqrt{\Gamma(k_0, \Delta, N_4)}}{N_4^{1/4}}$$

Comparison with FRG

The simplest truncation used in FRG is

$$S_k[g_{\mu\nu}] = \frac{1}{16\pi G_k} \int d^4x \sqrt{g(x)} \Big(-R(x) + 2\Lambda_k \Big)$$

where G_k and Λ_k coupling constants running with the scale k. The running coupling constants are believed to have an UV fixed point for $k \to \infty$, where they behave as

$$G_k := g_k/k^2, \quad g_k \to g_*, \qquad \Lambda_k := \lambda_k k^2, \quad \lambda_k \to \lambda_*,$$

where g_k and λ_k are dimensionless coupling constants. In particular we have for the dimensionless combination $G_k \Lambda_k$:

$$G_k \Lambda_k \to g_* \lambda_* \quad \text{for} \quad k \to \infty.$$

The extremum for $S_k[g_{\mu\nu}]$ is a de Sitter universe with cosmological constant Λ_k. We will assume the metric is Euclidean and then the solution is a four-sphere, S^4, with radius $R_k = 3/\sqrt{\Lambda_k}$. One can now study fluctuations around this solution. We will only do that here in the simplest possible way where we use a minisuperspace version of $S_k[g_{\mu\nu}]$ in order to compare with the lattice results. Close to the UV fixed point we then have

$$R_k = \frac{3}{\sqrt{\lambda_k}}\frac{1}{k} \to \frac{3}{\sqrt{\lambda_*}}\frac{1}{k}, \quad V_4(k) = \frac{8\pi^2}{3}R_k^4 = \frac{8\pi^2}{3}\frac{81}{\lambda_k^2}\frac{1}{k^4} \to \frac{8\pi^2}{3}\frac{81}{\lambda_*^2}\frac{1}{k^4}.$$

i.e. the volume $V_4(k)$ of the de Sitter sphere goes to zero when approaching the UV fixed point. Does it make sense to study fluctuations “around” such small universe?

Somewhat surprisingly the answer is affirmative because the coupling constant appearing in the study of fluctuations is $\sqrt{G_k \Lambda_k} = \sqrt{g_k \lambda_k}$ that is never large, even at the UV fixed point where R_k formally is zero.

In order to compare with computer simulations let us consider fluctuations around a de Sitter sphere with fixed volume V_4 rather than fixed Λ. The minisuperspace action written using the metric

$$d\tau^2 = dt^2 + r^2(t) d\Omega_3^2,$$

$$S_k = -\frac{1}{24\pi G_k} \int dt \left(\frac{\dot{V}_3^2}{V_3} + \delta_0 V_3^{1/3} \right), \qquad \int dt V_3(t) = V_4(k),$$

$$V_3(t) = 2\pi^2 r^3(t), \quad \delta_0 = 9(2\pi^2)^{2/3}, \quad \omega_0 = \frac{3}{\sqrt{2}} \frac{1}{\delta_0^{3/8}},$$

$$S_k = -\frac{1}{24\pi} \frac{\sqrt{V_4(k)}}{G_k} \int ds \left(\frac{\dot{v}_3^2}{v_3} + \delta_0 v_3^{1/3} \right), \quad v_3 = \frac{V_3}{V_4^{3/4}}, \ s = \frac{t}{V_4^{1/4}}$$

The fluctuations around

$$v_3^{cl}(s) = \frac{3}{4\omega_0} \cos^3\left(\frac{s}{\omega_0}\right)$$

will for a given k be governed by the effective coupling constant

$$g^2_{\rm eff}(k) = \frac{24\pi\, G_k}{\sqrt{V_4(k)}} = \frac{4}{\sqrt{6}} \Lambda_k G_k = 1.63\, \lambda_k g_k.$$

In the FRG analysis $\lambda_k g_k$ will be running from the present days value for small $k = k_p$ ($\lambda_{k_p} g_{k_p} \approx 10^{-120}$) to the value $\lambda_* g_*$. This UV fixed point value is not really universal, but with a number of different regularizations one finds values like $\lambda_* g_* \leq 0.12$.

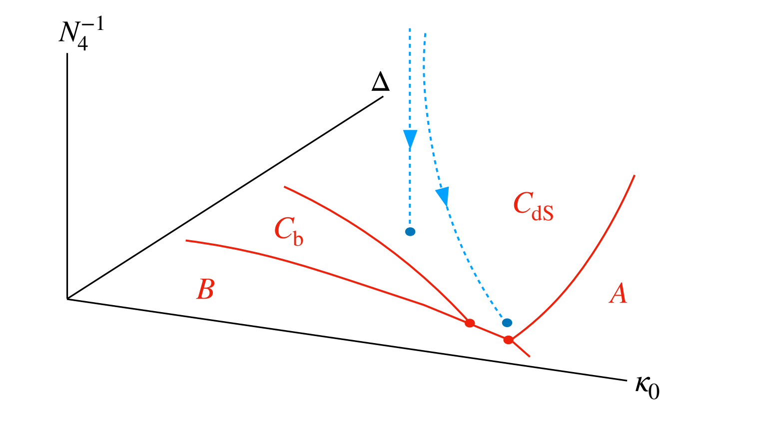
The infrared limit

Ignore for a moment that $\delta \neq \delta_0$. Then it is natural to identify

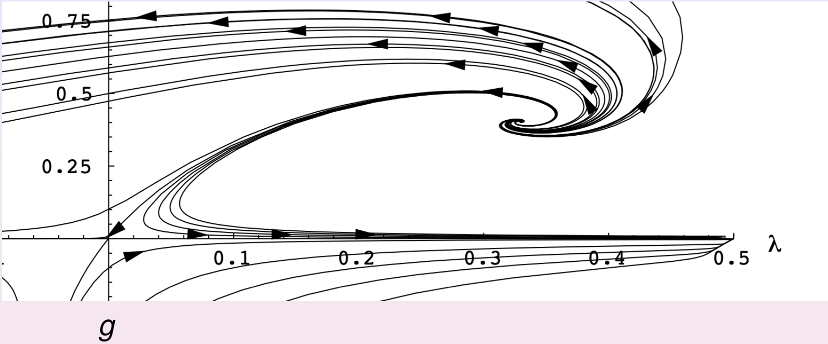
$$\frac{\sqrt{N_4}}{\Gamma(k_0,\Delta,N_4)} = \frac{\sqrt{V_4(k)}}{24\pi G_k} = \frac{1}{1.63\,\lambda_k g_k} = \frac{1}{1.63\,\Lambda_k G_k}.$$

For fixed (k_0, Δ) $\Gamma(k_0, \Delta, N_4)$ will be independent of N_4 for sufficiently large N_4. Thus, for $N_4 \to \infty$ we see that $\lambda_k g_k \to 0$. In the FRG context this implies that we either approach the so-called Gaussian fixed point or an IR fixed point where $k \to 0$. These fixed points can now be identified with the $N_4 \to \infty$ limit of CDT for fixed k_0, Δ. Since we observe perfect finite size scaling in CDT for the fluctuations it is natural to identify the linear size of the lattice, i.e. $N_4^{1/4}$ with the correlation length of the system. Thus the $N_4 = \infty$ surface becomes the critical surface of infinite correlation length and:

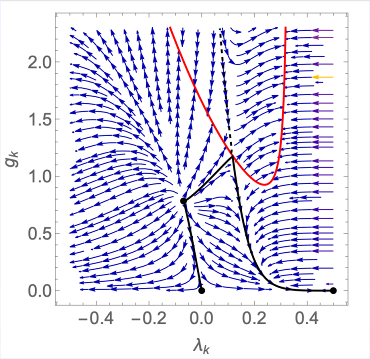
$$\xi = N_4^{1/4}$$



The interior of phase C_{dS} for $N_4 = \infty$ becomes an critical IR surface (compare to critical line for ϕ^4 diagram)



Flow (λ_k, g_k) from the UV fixed point towards the Gaussian fixed point and (0,0) and towards an IR fixed point at (0.5,0), with decreasing k from infinity at the UV fixed point. (Reuter and Saueressing, 2002)



Similar flow in a slightly modified FRG model (Saueressig and Wang (2023)). In this model one has both a Gaussian fixed point and an IF fixed point and for both

$$k\frac{d(\lambda_k g_k)}{dk} = 4(\lambda_k g_k), \quad k \to 0, \quad \lambda_k g_k \approx c\, k^4.$$

$$\text{Gaussian FP}: \quad \lambda_k, g_k \propto k^2, \qquad IR\ FP: \quad g_k \propto k^4, \ \ \lambda_k = \frac{1}{2}.$$

For fixed k_0, Δ in phase C_{dS} we have for large N_4

$$\frac{\sqrt{N_4}}{\Gamma(k_0,\Delta)} = \frac{\sqrt{V_4(k)}}{24\pi G_k}, \quad \xi = N_4^{1/4}, \quad N_4 a^4 \propto V_4(k).$$

Thus

$$a^2 \propto \frac{G_k}{\Gamma(k_0,\Delta)}$$

For the Gaussian FP we have $G_k = g_k/k^2 \approx G_0 = \ell_p^2$ for small k, while for the IR FP $G_k \propto k^2$. Thus:

$$a \propto \frac{\ell_p}{\sqrt{\Gamma(k_0,\Delta)}} \quad (\mathrm{G-FP}), \qquad a \propto \frac{k}{\sqrt{\Gamma(k_0,\Delta)}} \frac{\sqrt{g_{\tilde{k}}}}{\tilde{k}^2} \quad (\mathrm{IR-FP}).$$

$$\xi \propto \frac{\sqrt{\Gamma(k_0,\Delta)}}{G_0 k^2} \quad (\mathrm{G-FP}), \qquad \xi \propto \frac{\sqrt{\Gamma(k_0,\Delta)}}{\sqrt{g_{\tilde{k}}}} \frac{\tilde{k}^2}{k^2} \quad (\mathrm{IR-FP}).$$

The Utraviolet limit

To identity a lattice UV fixed point on the critical surface we should keep the renormalized, continuum coupling fixed while changing the lattice couplings in such a way that the correlation length goes to infinity. The renormalized coupling constant can take any value between its UV fixed point value and the nearest IR fixed point value. We apply this philosophy to the dimensionless coupling constant $G\Lambda = g\lambda$. Thus we keep $g_k \lambda_k$ fixed, and from

$$\frac{\sqrt{N_4}}{\Gamma(k_0, \Delta, N_4)} = \frac{\sqrt{V_4(k)}}{24\pi G_k} = \frac{1}{1.63\, \lambda_k g_k}.$$

and from the fact that the critical surface is at $N_4 \to \infty$, it is seen that we can only approach a possible UV fixed point if we follow a path $(k_0(N_4), \Delta(N_4))$ in the lattice coupling constant space such that $\Gamma(k_0(N_4), \Delta(N_4), N_4) \to \infty$.

The only region in the $C_{\rm dS}$ phase where $\Gamma(k_0, \Delta, N_4)$ goes to infinity is when we approach the $A-C_{\rm dS}$ transition line (and $N_4 \to \infty$). However, at this line we also have that δ in our effective action increases dramatically and we can no longer ignore the fact that $\delta \neq \delta_0$ when we compare our lattice results with the FRG calculations.

On the lattice the "deformed" spheres arise because there are "too many" N_3 compared to the time extension. We can compensate for that by decreasing the spatial lattice spacing, while keeping the temporal lattice spacing unchanged

$$a_s \;(= a) \;\to \tilde{a}_s = \Big(\frac{\omega}{\omega_0}\Big)^{4/3} a, \qquad a_t \;(= a) \;\to \tilde{a}_t = a.$$

$$V_3(t) \to \tilde{V}_3(t) = \Big(\frac{\omega}{\omega_0}\Big)^4 V_3(t), \qquad V_4 \to \tilde{V}_4 \approx \Big(\frac{\omega}{\omega_0}\Big)^4 V_4.$$

$$S = \frac{1}{24\pi \tilde{G}} \int dt \left(\frac{\dot{\tilde{V}}_3^2(t)}{\tilde{V}_3(t)} + \tilde{\delta} \cdot \tilde{V}_3^{1/3} \right), \qquad \int dt \;\tilde{V}_3(t) = \tilde{V}_4,$$

$$\tilde{G} = \frac{\omega^4}{\omega_0^4} G, \qquad \tilde{\delta} = \frac{\omega^{8/3}}{\omega_0^{8/3}} \delta = \delta_0.$$

So given computer data N_4, ω, Γ we can associate a corresponding continuum, round S^4 via:

$$(N_4, \omega, \Gamma) \to (V_4, \omega, G) \to (\tilde{V}_4, \omega_0, \tilde{G}),$$

$$\frac{\sqrt{N_4}}{\Gamma} = \frac{\sqrt{V_4}}{24\pi G} = \frac{\omega^2}{\omega_0^2} \frac{\sqrt{\tilde{V}_4}}{24\pi \tilde{G}} = \frac{\omega^2}{\omega_0^2} \frac{1}{1.63\, \lambda_k g_k}$$

$$\boxed{\omega^2(k_0, \Delta, N_4)\, \Gamma\big(k_0, \Delta, N_4\big) = 1.63\, \lambda_k g_k\, \omega_0^2 \sqrt{N_4}}$$

How to observe this in the MC simulations?

Let $N_4 = \infty$ and assume

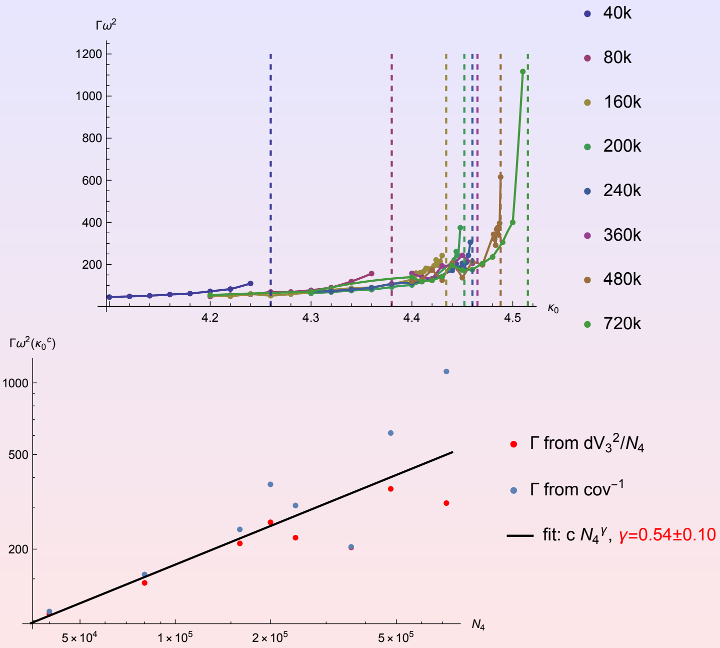
$$\omega^2(k_0,\Delta)\Gamma(k_0,\Delta) = \frac{C(\Delta)}{|k_0(\Delta)^c - k_0|^\alpha}, \quad k_0 \to k_0(\Delta)^c.$$

Weak Δ dependence and we will ignore it in the following. For finite N_4 we do not have a critical k_0^c, but $\omega^2(k_0,N_4)\,\Gamma(k_0,N_4)$ will have a maximum for a so-called pseudo-critical point $k_0^c(N_4)$ that will approach k_0^c for $N_4 \to \infty$:

$$k_0^c(N_4) = k_0^c - \frac{c}{N_4^\beta}$$

$$\omega^2(k_0^c(N_4),N_4)\,\Gamma(k_0^c(N_4),N_4) \propto \frac{1}{|k_0^c - k_0^c(N_4)|^\alpha} \propto N_4^{\alpha\beta}$$

Thus we want to find $k_0^c(N_4)$ and measure $\gamma = \alpha\beta$.



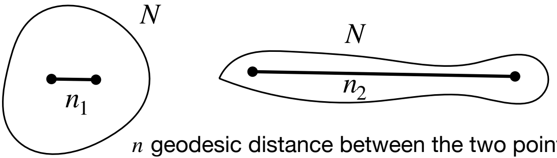
Γω²
1200
1000
800
600
400
200
4.2
4.3
4.4
4.5
κ₀
40k
80k
160k
200k
240k
360k
480k
720k
Γω²(κ₀ᶜ)
1000
500
200
5 × 10⁴
1 × 10⁵
2 × 10⁵
5 × 10⁵
N₄
Γ from dV₃²/N₄
Γ from cov⁻¹
fit: c N₄^γ, γ=0.54±0.10

Conclusion: our MC data can be made consistent with the FRG picture.

However: this consistence was obtained by insisting on the interpretation $\xi = N_4^{1/4}$. Such an identification is unproblematic if we for $N_4 = \infty$ first can define a correlation length related to an observable in a standard way and then study the phase diagram where $\xi \to \infty$. Here one can ask: what is correlated with a correlating length ξ?

In 2d quantum gravity, where one can solve the models analytically, the answer is points in spacetime!.

Points are not correlated in an interesting way for a fixed geometry. However, when one averages over all geometries this changes completely.



n geodesic distance between the two points

$$P_N(n) = \frac{1}{N^{1/d_h}} \, F\left(\frac{n}{N^{1/d_h}}\right), \qquad \int_0^\infty dx\, F(x) = 1$$

Assume no fixed N, but a fixed lattice cosmological constant μ $(\mu_c = 0)$

$$\langle N \rangle \propto \frac{1}{\mu}, \qquad \tilde{P}_\mu(n) = \sum_N e^{-\mu N} P_N(n) \propto \exp\left[-\frac{n}{\langle N \rangle^{1/d_h}}\right], \quad n > \langle N \rangle^{1/d_h}$$

In 2d one can calculate the correlator between two points separated a geodesic distance n and they have a lattice correlation length $\xi \propto N_2^{1/d_h}$, where d_h is the Hausdorff dimension of quantum spacetime.

Then: if the correlation length is between spacetime points (which is in some sense the most obvious manifistation of “quantum geometry”), the nature of the phase transitions (rearrangments of “geometries”, where some of these geometries might not have a “continuum” interpretation) might be different from the “usual” Landau-like phase transitions characterized by local order parameters. In fact there has always been some non-standard aspects of the CDT phase transitions, even if we have until now tried to stick to the standard classifications. The attempts to fit the data to FRG calculations have highlighted that one should maybe think differently about the concept of critical phenomena in theories of quantum geometries.

At the moment we are trying to apply this new perspective to the $A - C_{\rm dS}$ transition, which we until now had viewed as a first order transition.