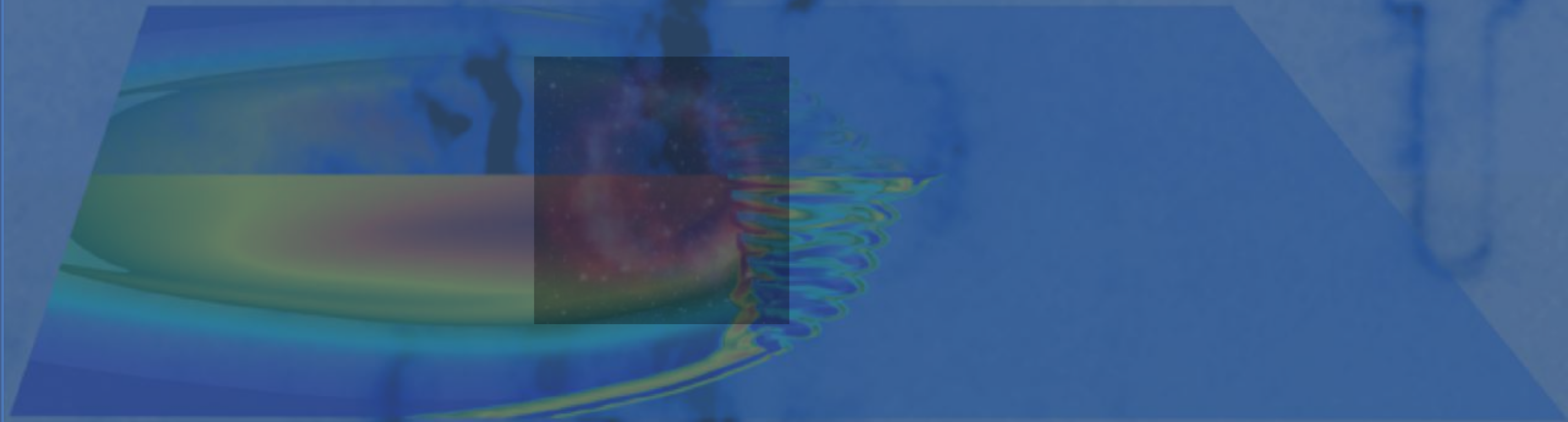
Testing Cosmic particle acceleration in the laboratory



Subir Sarkar

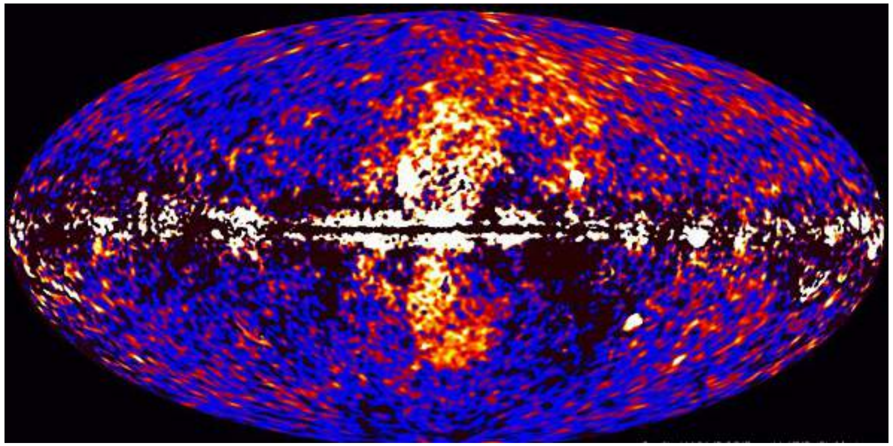
Rudolf Peierls Centre for Theoretical Physics

Cracow School of Theoretical Physics LIX Course, Zakopane, 1422 June 2019

There are many cosmic environments where particles are accelerated to high energies ... probably by MHD turbulence generated by shocks

and emit non-thermal radiation in radio through to γ-rays

The mechanism responsible is likely to be *second*-order Fermi acceleration



A NUMERICAL MODEL OF THE STRUCTURE AND EVOLUTION OF YOUNG SUPERNOVA REMNANTS

Mon. Not. R. astr. Soc. (1973) **161**, 47

S. F. Gull

Mon. Not. R. astr. Soc. (1975) **171**, 263

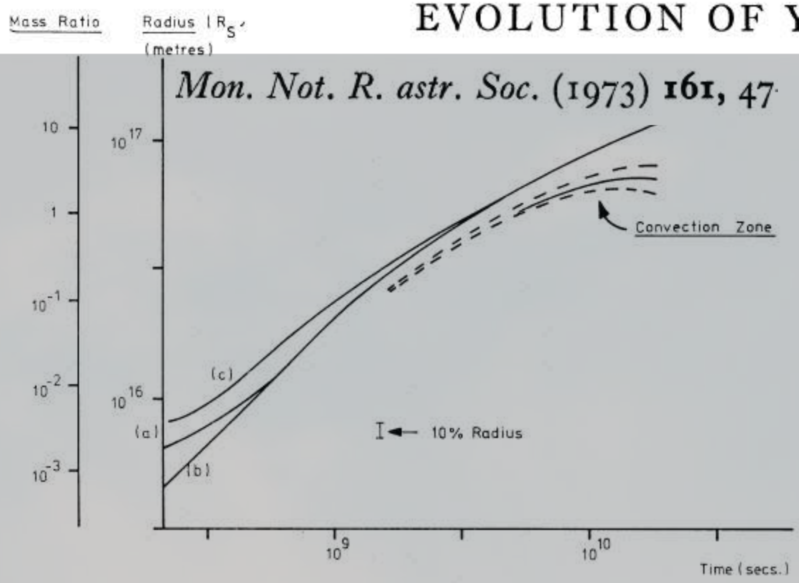


FIG. 8. *Dependence of dynamics on piston model. (i) Adiabatic lapse rate piston, $R_0 = 5\times10^{15}$ m. (ii) Adiabatic lapse rate piston, $R_0 = 5\times10^{14}$ m. (iii) Isothermal piston, $R_0 = 5\times10^{15}$ m.*

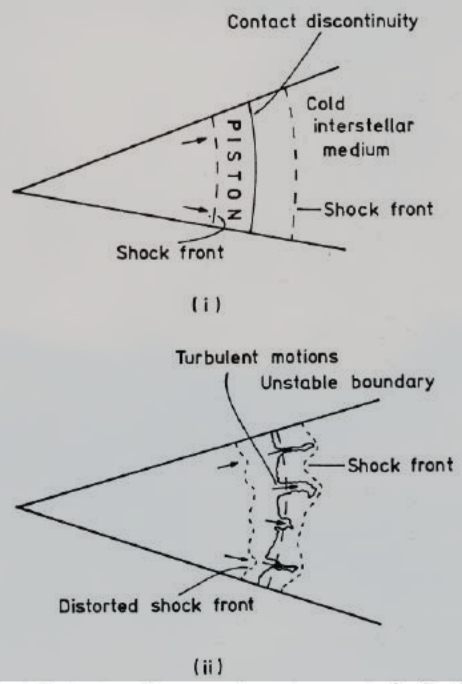


FIG. 3. (1) *Schematic structure of a young supernova remnant, showing the internal shock front.* (2) *Modification of internal structure when the contact discontinuity is distorted by the Rayleigh–Taylor instability. Some fraction of the energy now appears as random motions in the neighbourhood of the filaments.*

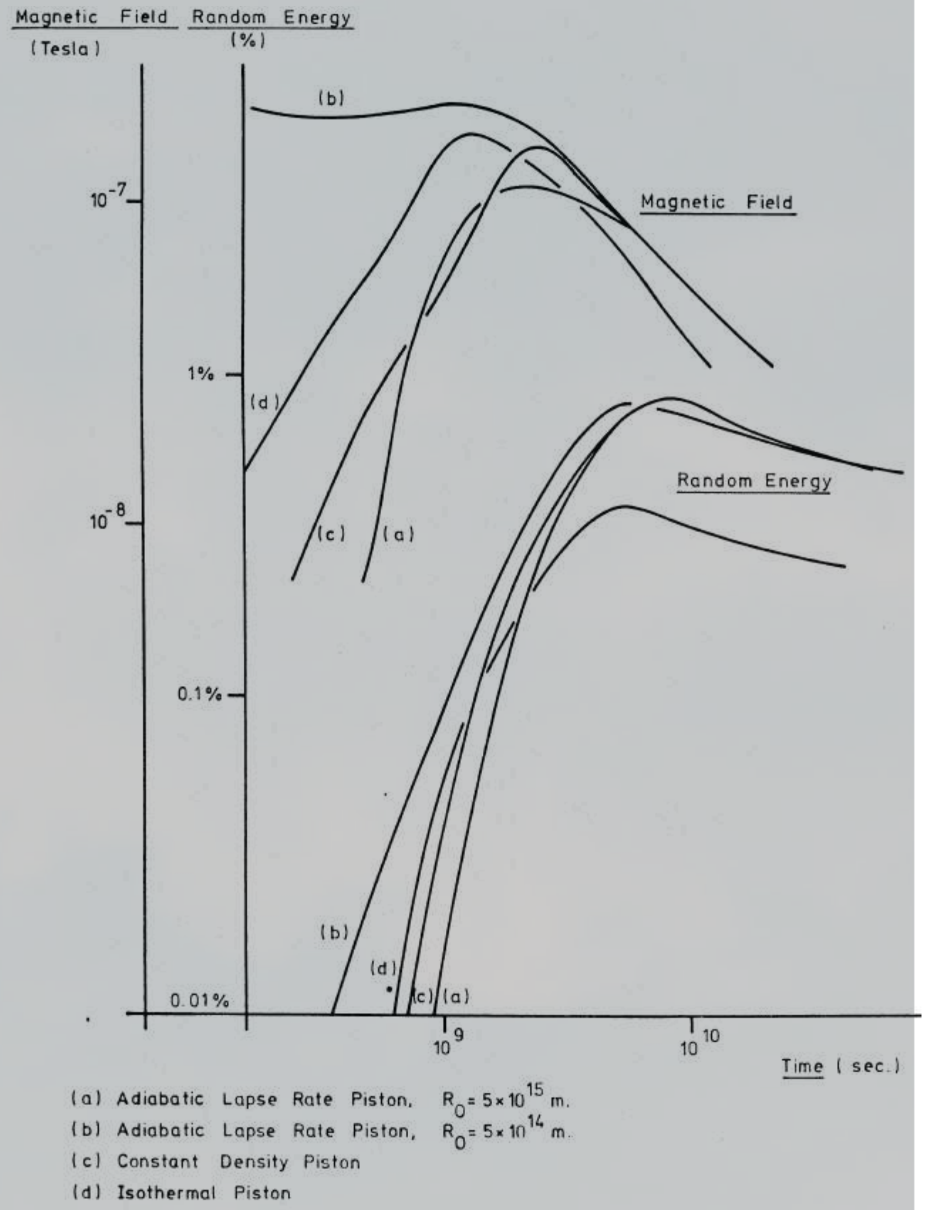
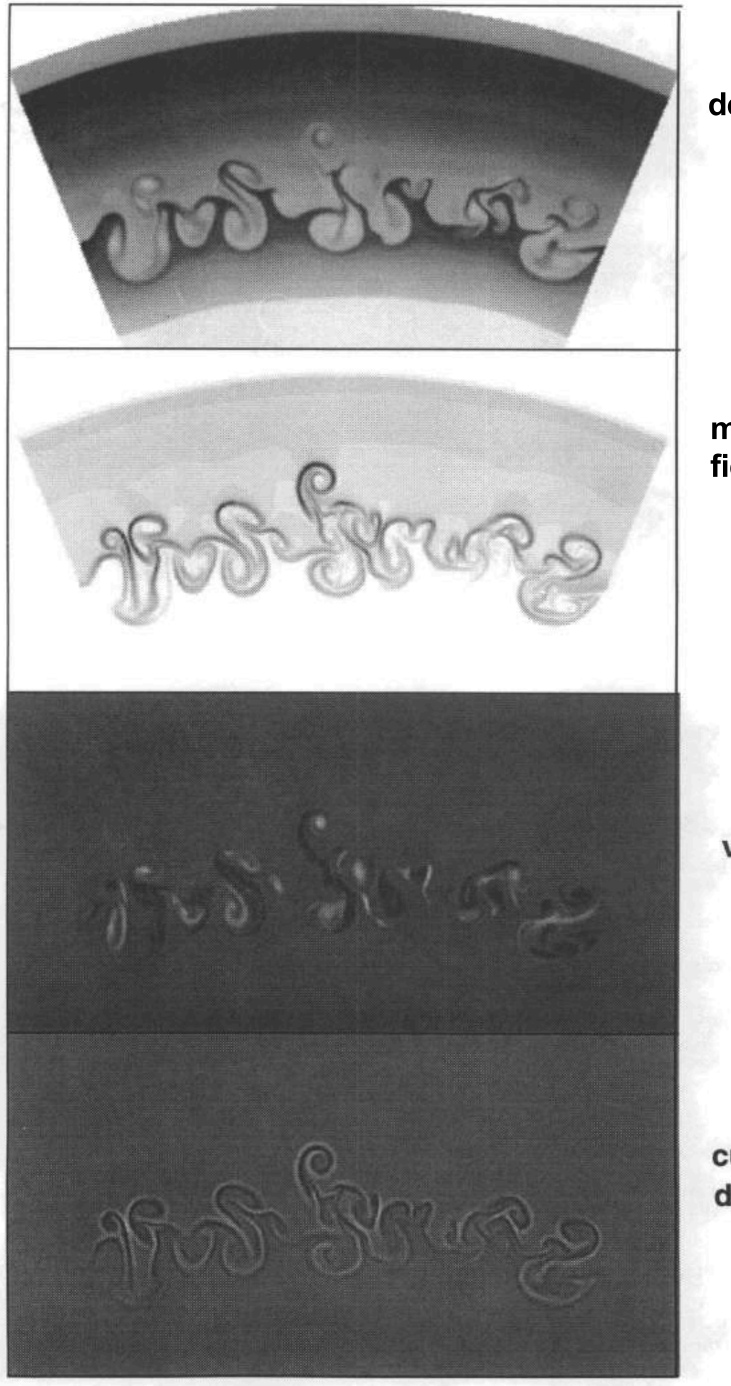
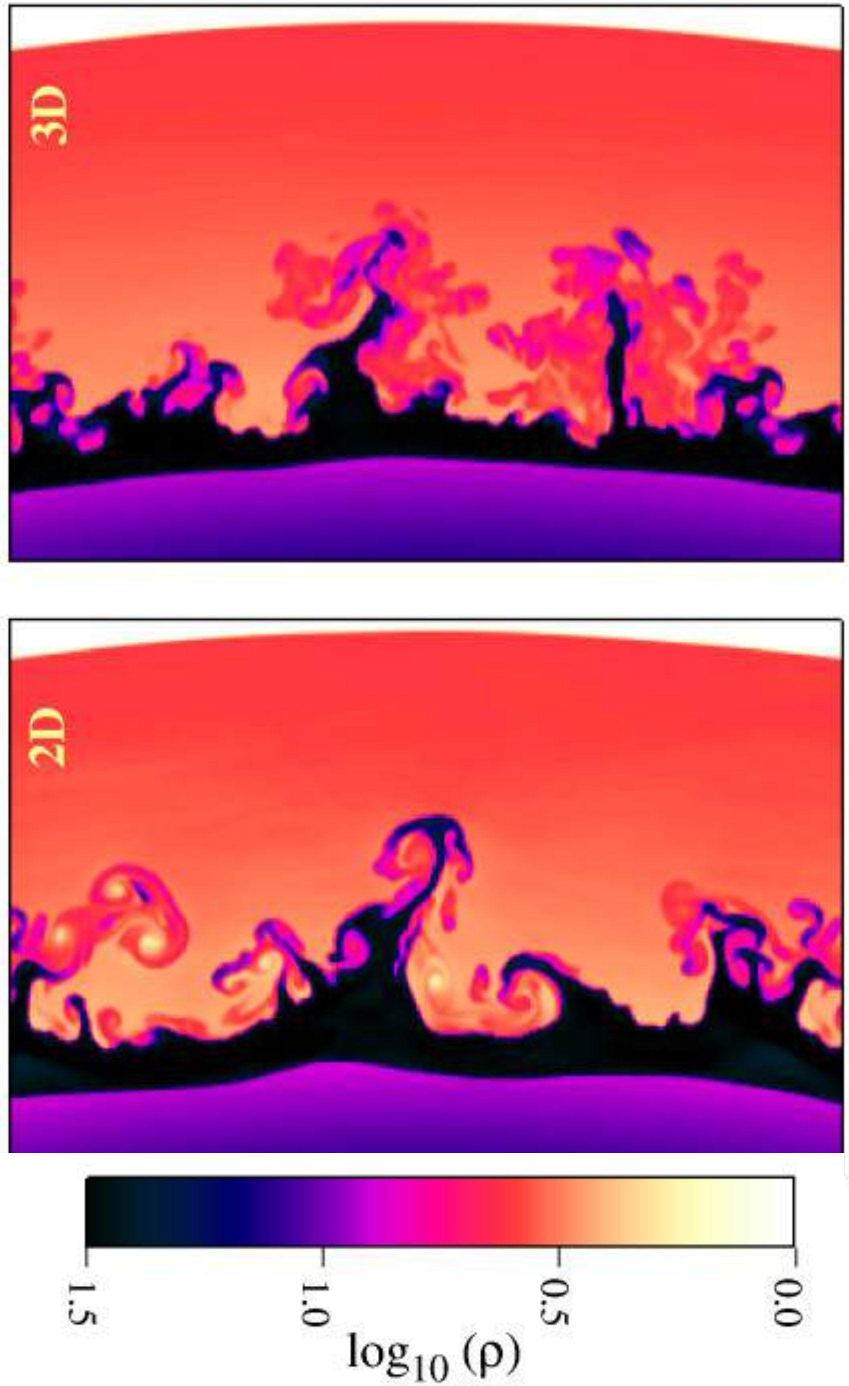


FIG. 9. *Turbulent energy and magnetic field in the convection zone. Note that, whilst the individual piston models show great differences in the early part of the evolution (particularly for small R_0) the predicted turbulent energies and magnetic fields agree to within a factor of 2 when the mass ratio is greater than 0·1 ($t > 10^9$ s).*

... confirmed by subsequent 2-D and 3-D simulations

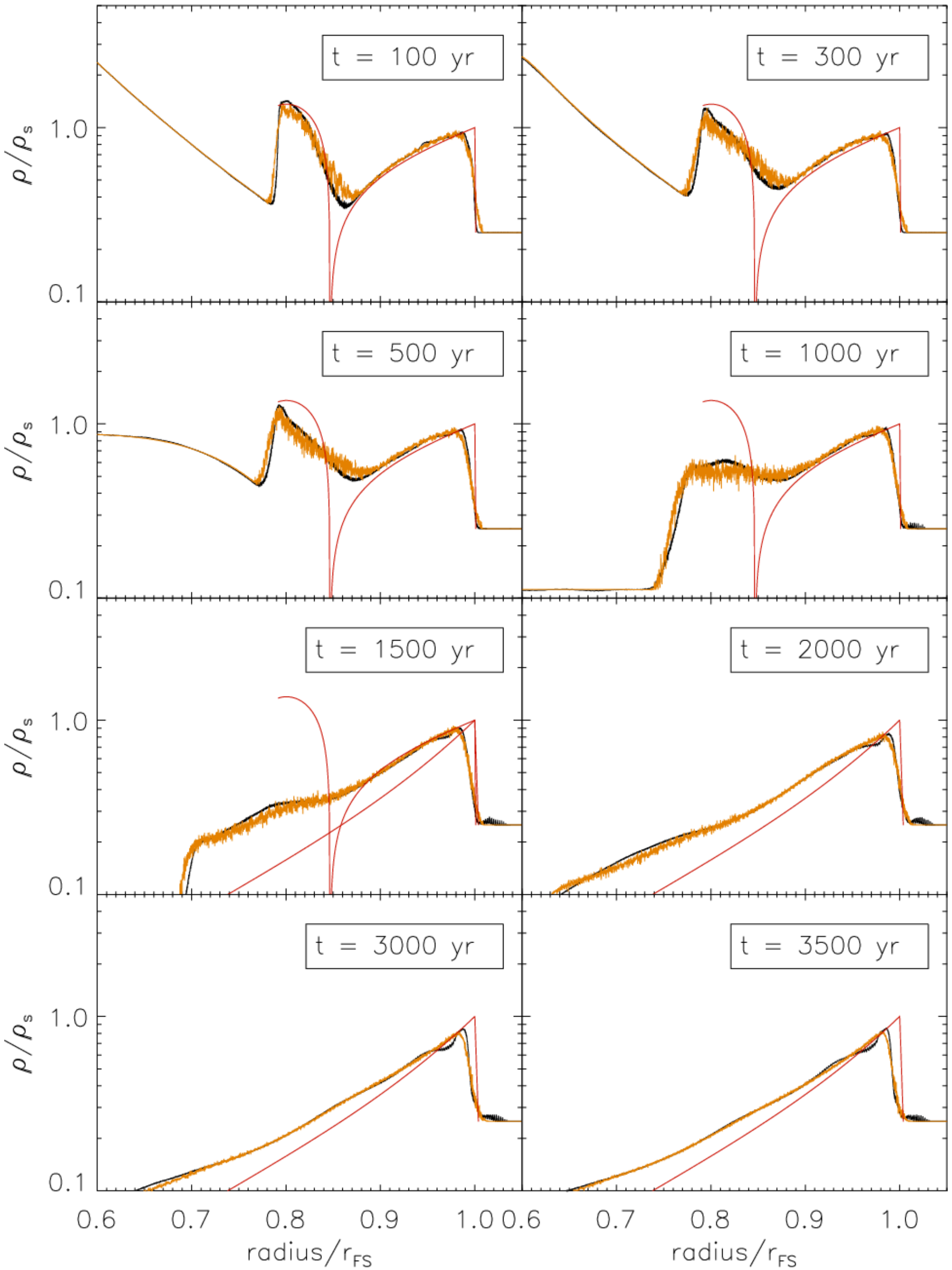
Jun & Norman, ApJ **465**:800, 1996

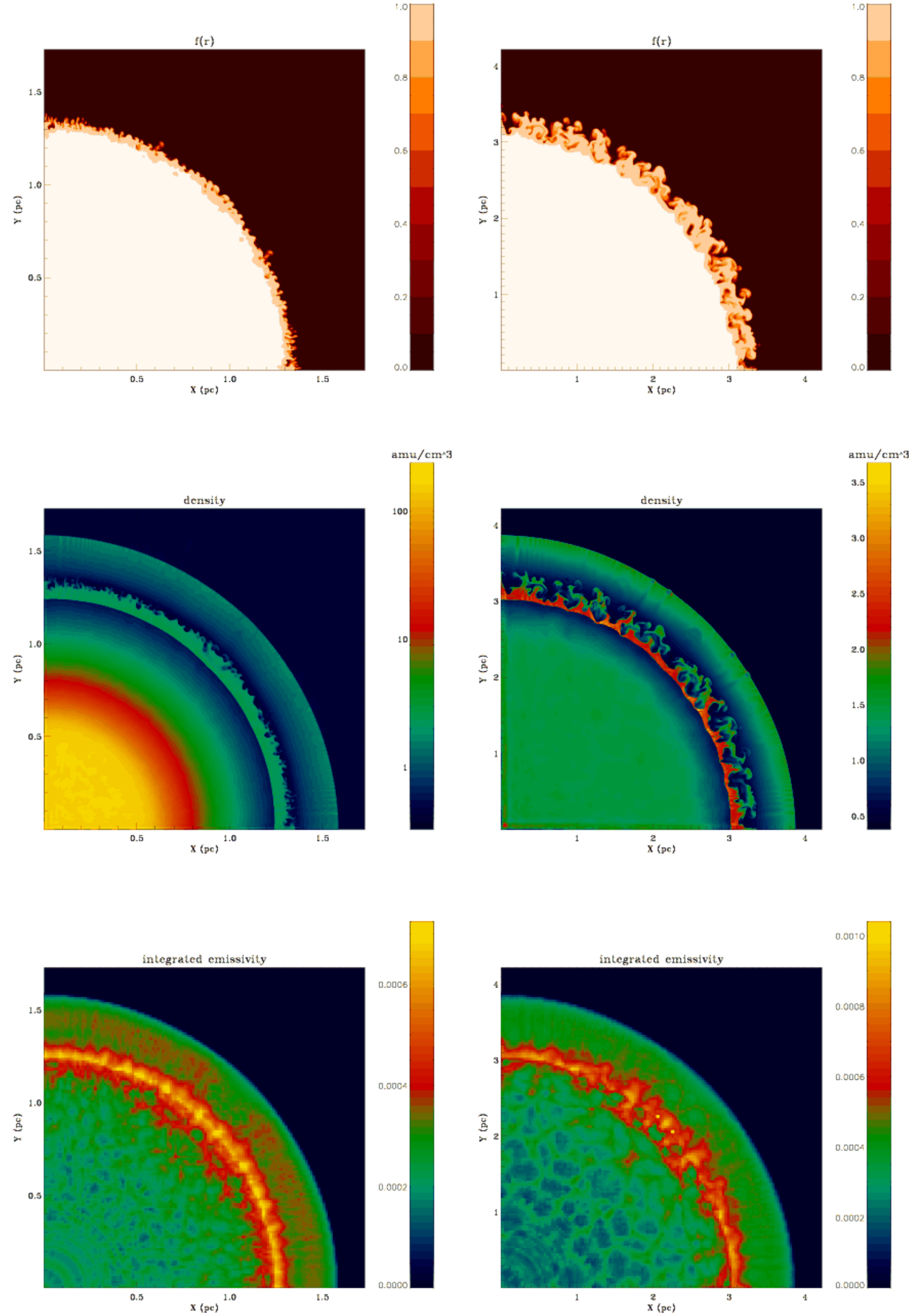




Blondin & Ellison, ApJ **560**:244, 2001

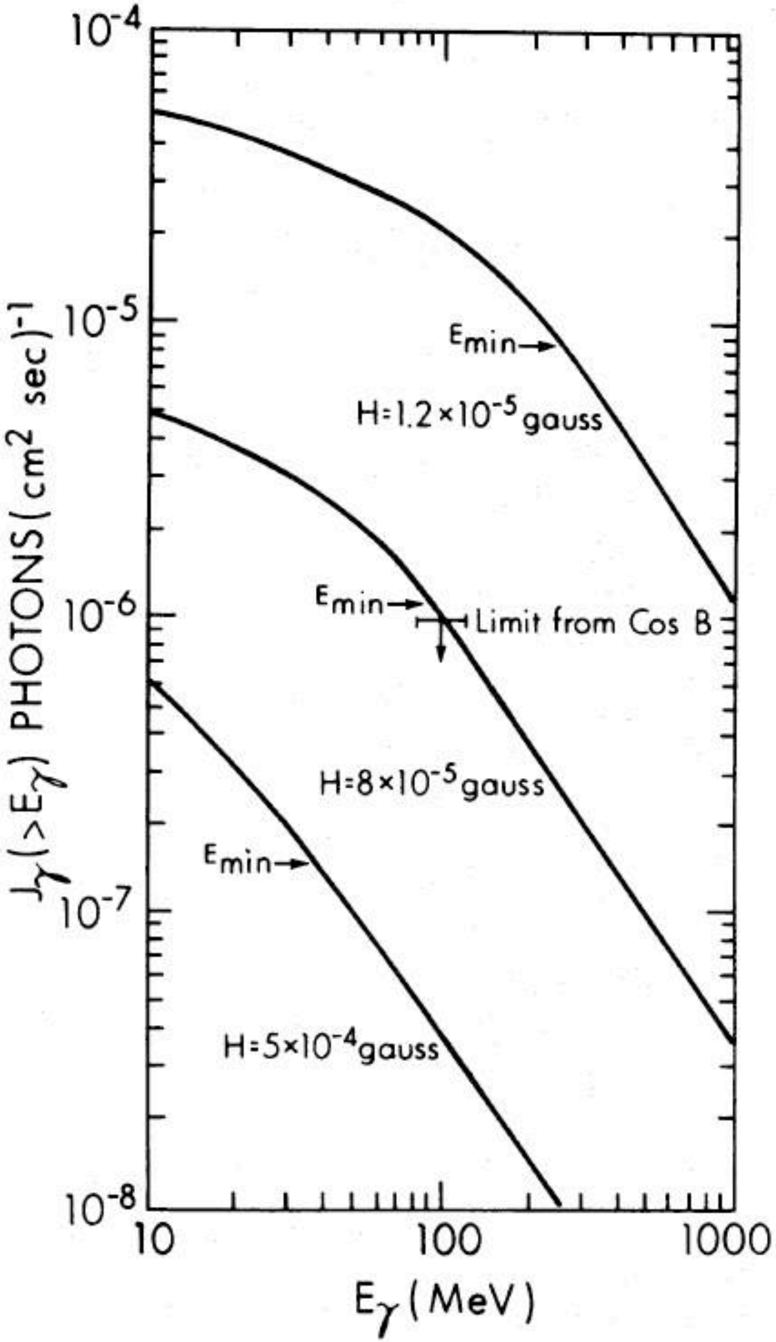
3-D Simulation of the growth of the Rayleigh-Taylor instability in SNRs





Fraschetti, Teyssier, Ballet, Decourchelle, A&A **515**:A104, 2010

Turbulent amplification of magnetic fields *behind* SNR shocks



(Cowsik & Sarkar, MNRAS **191**:855,1980)

Upper limit on the γ-ray flux from Cas A (generated by *non*-thermal electron **bremsstrahlung**) implies *amplification* of the magnetic field in the radio shell well above the compressed interstellar field … just as was predicted by Gull

Relativistic electrons ⊗ magnetic field → radio
" ⊗ X-ray emitting plasma → γ-rays
∴ radio ⊕ X-rays ⊕ γ-rays ⇒ magnetic field

Recently both MAGIC & Fermi *detected* γ-rays from Cas A ⇒ *minimum B*-field of ~100 μG

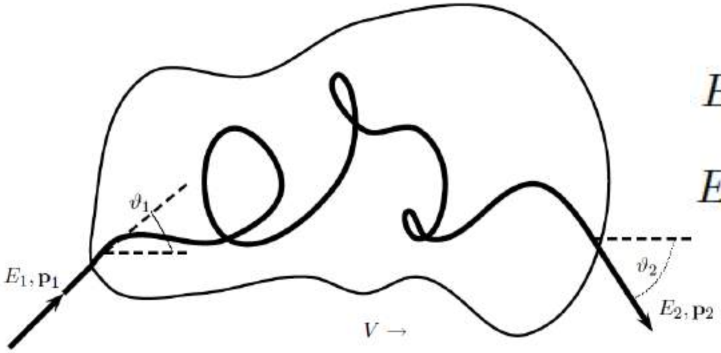
(Abdo et al, ApJ **710**:L92,2018)

(Emission mechanism probably π^0 decay or inverse-Compton scattering … so limit set is *conservative*)

… also suggested by the observed thinness of X-ray synchrotron emitting filaments

(Vink & Laming, ApJ **584**:758,2003)

2ND-ORDER FERMI ACCELERATION



$$E_1' = \gamma E_1(1 - \beta\cos\vartheta_1) \quad \text{where} \quad \beta = V/c \quad \text{and} \quad \gamma = 1/\sqrt{1-\beta^2}$$

$$E_2 = \gamma E_2'(1 + \beta\cos\vartheta_2')$$

$$\langle\cos\vartheta_2'\rangle = 0 \qquad \langle\cos\vartheta_1\rangle = \int \cos\vartheta_1 \frac{\mathrm{d}n}{\mathrm{d}\Omega_1}\,\mathrm{d}\Omega_1 \Big/ \int \frac{\mathrm{d}n}{\mathrm{d}\Omega_1}\,\mathrm{d}\Omega_1 = -\frac{\beta}{3}$$

$$\xi = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\cos\vartheta_1 + \beta\cos\vartheta_2' - \beta^2\cos\vartheta_1\cos\vartheta_2'}{1-\beta^2} - 1 \quad \Rightarrow \quad \langle\xi\rangle = \frac{1+\beta^2/3}{1-\beta^2} - 1 \simeq \frac{4}{3}\beta^2$$

Fast particles collide with moving magnetised clouds (Fermi, 1949) … particles can gain *or* lose energy, but head-on collisions (⇒ gain) are more probable, hence energy increases on average proportionally to the velocity-*squared*

It was subsequently realised that MHD turbulence or plasma waves can also act as scattering centres (Sturrock 1966, Kulsrud and Ferrari 1971)

⇒ Diffusion in momentum described by Fokker-Planck equation for phase-space density

$$\frac{\partial f}{\partial t} = -\frac{1}{p^2}\frac{\partial}{\partial p}\left(-p^2 D_{pp}\frac{\partial f}{\partial p}\right) - \frac{f}{\tau_{esc}} + \frac{I_0\,\delta(p-p_0)\,\delta(t-t_0)}{4\pi p^2}$$

Kaplan 1956; Hall & Sturrock 1967; Tverskoi 1967; Ostrowski & Siemieniec-Oziebło 1997

TRANSPORT EQUATION ⟹ INJECTION + DIFFUSION + CONVECTION + LOSS

E.g. in an expanding flux tube in the turbulent region in a young SNR:

$$\frac{\partial n}{\partial t} = \frac{n}{\tau_e} - \left[2K_F + \frac{1}{3}\left(\frac{d\ln B_r}{dt} - \frac{d\ln L}{dt}\right)\right]E\frac{\partial n}{\partial E} + K_F E^2 \frac{\partial^2 n}{\partial E^2} + I(\epsilon, t),$$

Escape loss — Betatron acceleration — Adiabatic expansion — Convection — Diffusion — Injection

By making the following integral transforms …

$$n = n' \exp\left[-\int_{t_0}^{t} \frac{dt'}{\tau_e(t')}\right],$$

$$x = E \exp\left[-\int_{t_0}^{t} \left\{2K_F(t') + \frac{1}{3}\left[\frac{d\ln B_r(t')}{dt'} - \frac{d\ln L(t')}{dt'}\right]\right\} dt'\right],$$

$$y = \exp\left[\int_{t_0}^{t} K_F(t')\, dt'\right].$$

The Green's function is: $G' = \frac{1}{\sqrt{4\pi y}} \exp\left[-\left(\ln\frac{x}{x_0} - y\right)^2 / 4y\right]$ *Log*-normal distribution

So the energy spectrum is: $n(\epsilon, t) = \int_{t_0}^{t} dt'_0 \int_{-\infty}^{\infty} d\epsilon'_0\, \tilde{G}(\epsilon, \epsilon'_0, t, t'_0)\, I(\epsilon'_0, t'_0).$

Cowsik & Sarkar, MNRAS **207**:745,1984

THE SOLUTION TO THE TRANSPORT EQUATION IS AN *APPROXIMATE* POWER-LAW SPECTRUM AT LATE TIMES, WITH *CONVEX* CURVATURE

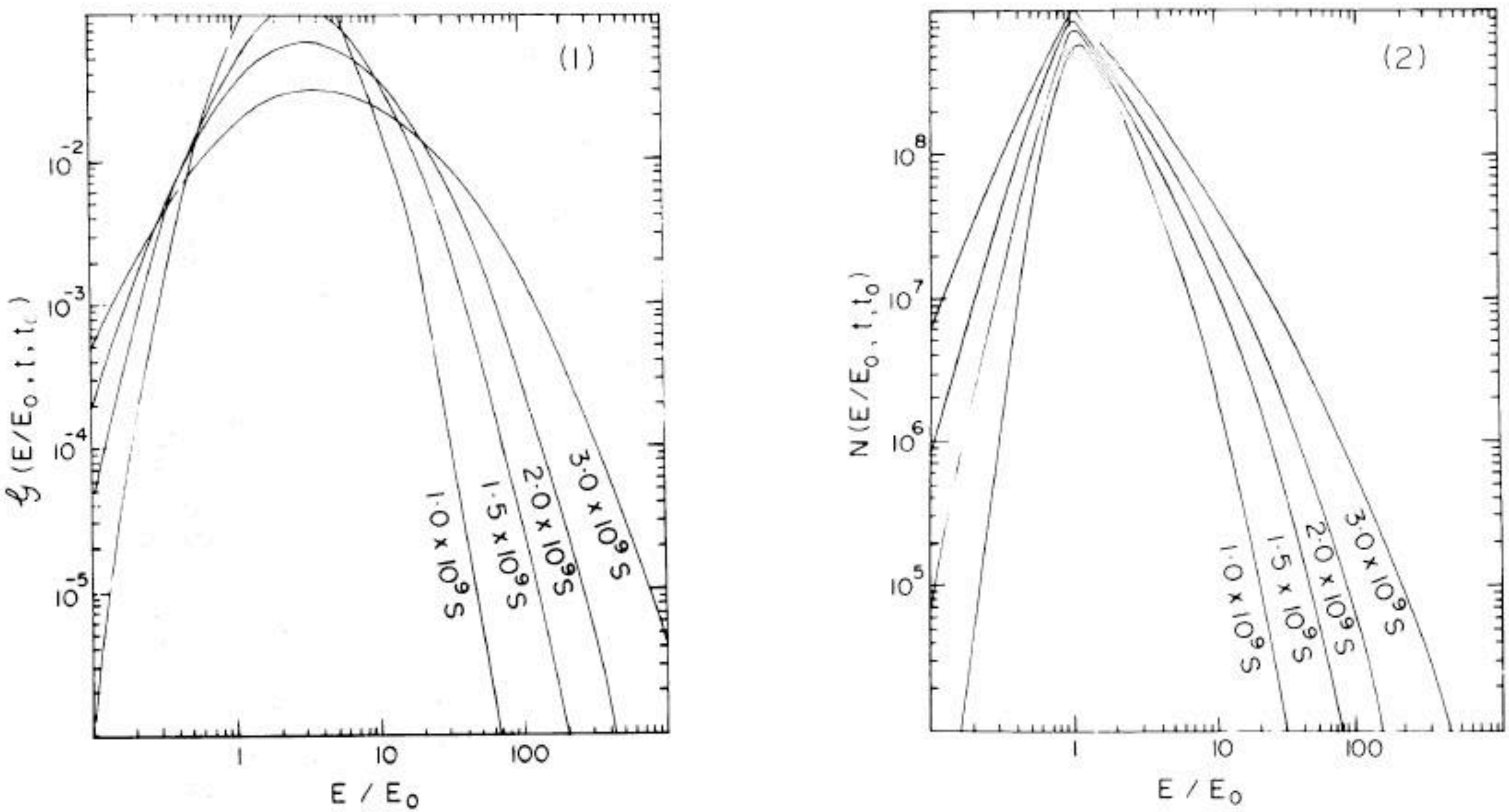


Figure 3. Evolution of the energy spectrum of particles corresponding to (1) Impulsive injection (of 1 particle) and (2) continuous injection (of 1 particle s^{-1}), for a constant rate of stochastic acceleration, $K_0 = 10^{-2}$ yr^{-1}. [Piston model (a); $t_0 = 10$ yr; $\tau_e \gg t$.]

Cowsik & Sarkar, MNRAS **207**:745,1984

(Park & Petrosian, ApJ **446**:699,1995; Becker, Le & Dermer, ApJ **647**:539,2006 … generalised for *any* momentum- and time-dependence of diffusion co-efficient by: Mertsch, JCAP **12**:010,2010)

THE SYNCHOTRON RADIATION SPECTRUM DEPENDS ON THE ELECTRON ACCELERATION TIME-SCALE ... AND *HARDENS* WITH TIME

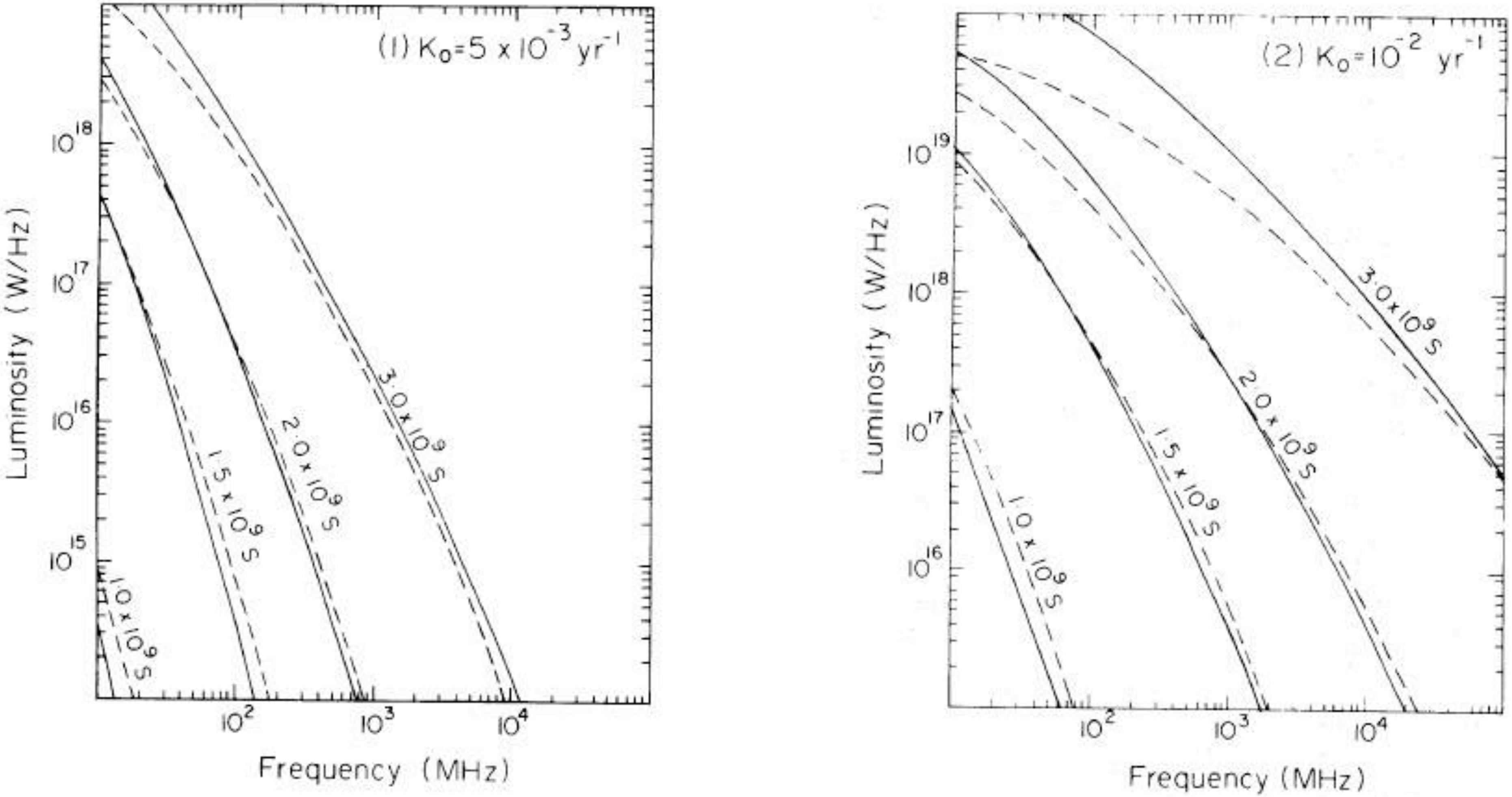
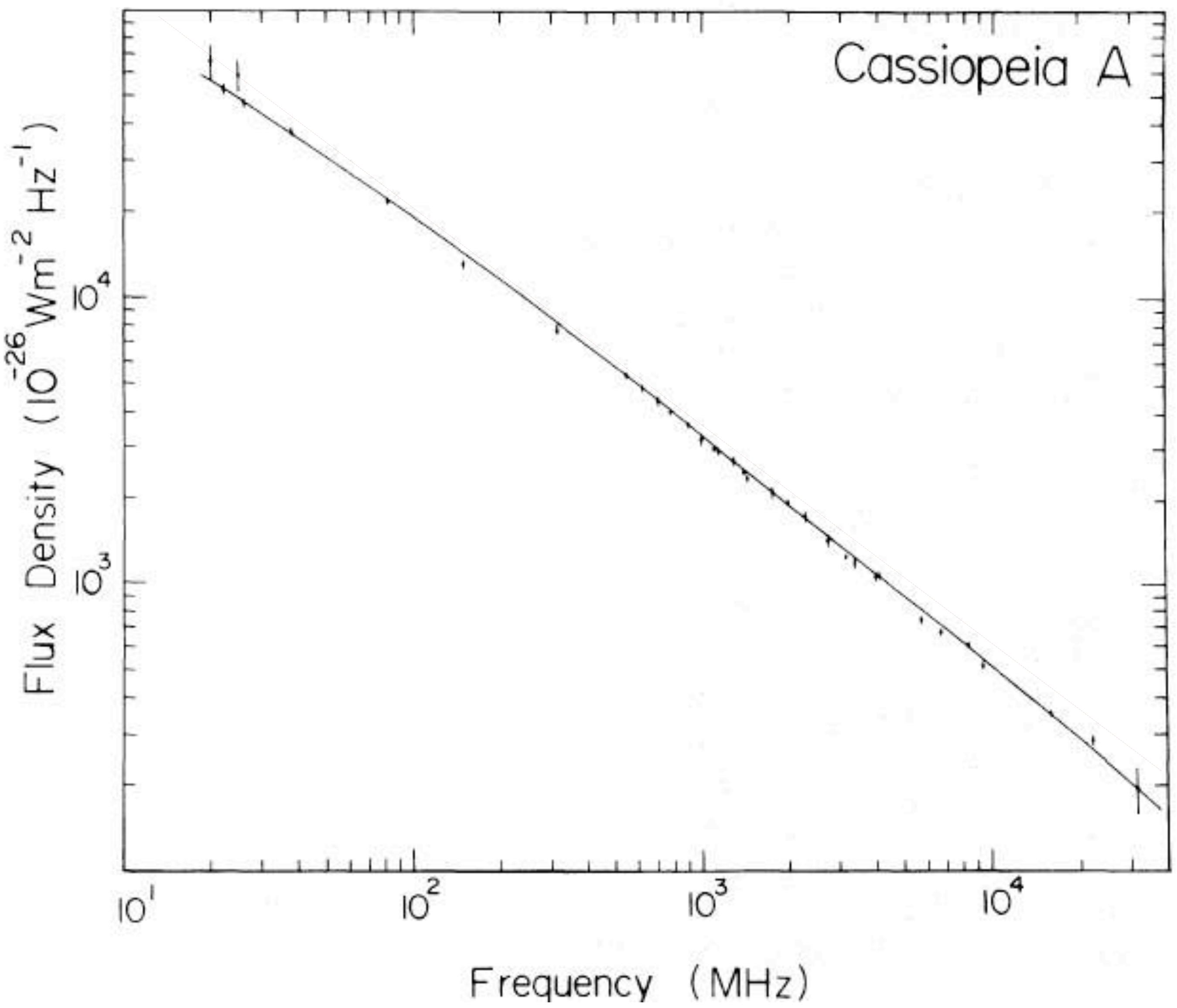


Figure 4. Evolution of the synchrotron spectrum corresponding to impulsive injection (dashed line, $E_{inj} = 10^{46}$ erg) and continuous injection at a constant rate (solid line, $\dot{E}_{inj} = 10^{38}$ erg s^{-1}), for various values of the (constant) stochastic acceleration rate, K_0. [Piston model (a): $t_0 = 10$ yr; $E_0 = 1$ MeV, $\tau_e \gg t$.]

Cowsik & Sarkar, MNRAS **207**:745,1984

The radio spectrum of Cassiopea A is indeed a *convex* power-law

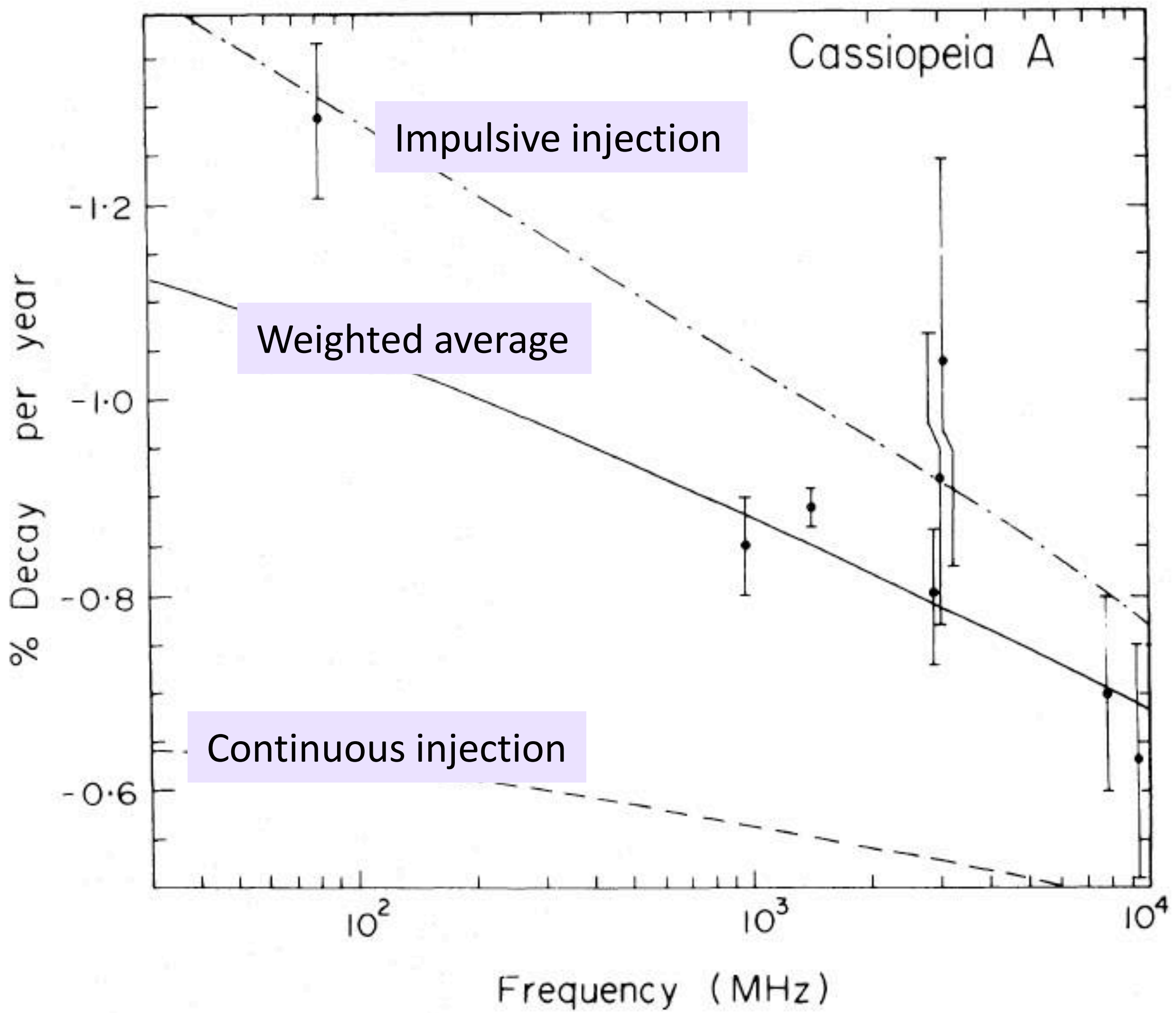


Cowsik & Sarkar, MNRAS **207**:745,1984

… *perfectly* fitted by the log-normal spectrum expected from *2nd order* Fermi acceleration by MHD turbulence due to plasma instabilities *behind* the shock

(Efficient 1st-order 'Diffusive Shock Acceleration' should yield a *concave* spectrum)

... AND FITS THE OBSERVED *FLATTENING* OF THE SPECTRUM WITH TIME



Cowsik & Sarkar, MNRAS **207**:745,1984

Figure 7. The expected decay rate of the Cas A spectrum as a function of frequency is shown separately for the cases with (dashed line) and without (dot-dashed line) continuing injection of low-energy electrons, assuming identical expansion rates. The solid line is the weighted average of the two curves (in the ratio 1: 2) and represents the decay rate of the total flux, assuming a third of it to arise from regions where low-energy particles are injected (see text for details). Observational data are from Baars *et al*. (1977).

Can we simulate 2nd-order Fermi acceleration in the laboratory Using lasers to create a turbulent plasma?

The laser bay at the National Ignition Facility, Lawrence Livermore National Laboratory consists of 192 laser beams delivering 2 MJ of laser energy in 20 ns pulses

Laboratory experiments *can* test and validate astrophysical models

$$\left.\begin{array}{l} \ell, u, \rho \\ \tau = \ell / u \\ p = \rho u^2 \end{array}\right\} \xrightarrow[\text{transform}]{\text{self-similar}} \left\{\begin{array}{l} \ell', u', \rho' \\ \tau' = \dfrac{\ell'/\ell}{u'/u}\tau \\ p' = \dfrac{\rho'}{\rho}\left(\dfrac{u'}{u}\right)^2 p \end{array}\right.$$

→ Equations of ideal MHD have *no* intrinsic scale, hence similarity relations exist

→ This requires that Reynolds number, magnetic Reynolds number, *etc* are all large – in both the astrophysical and analogue laboratory systems

$$\frac{\partial \rho'}{\partial t'} + \nabla' \cdot (\rho' \mathbf{u}') = 0$$

The difficulty, so far, remains in achieving these to be large enough for the dynamo to be operative

$$\rho'\left(\frac{\partial \mathbf{u}'}{\partial t'} + \mathbf{u}' \cdot \nabla' \mathbf{u}'\right) = -\nabla' P' + \frac{1}{R_e}\nabla' \cdot \boldsymbol{\sigma}' + \mathbf{F}'_{EM}$$

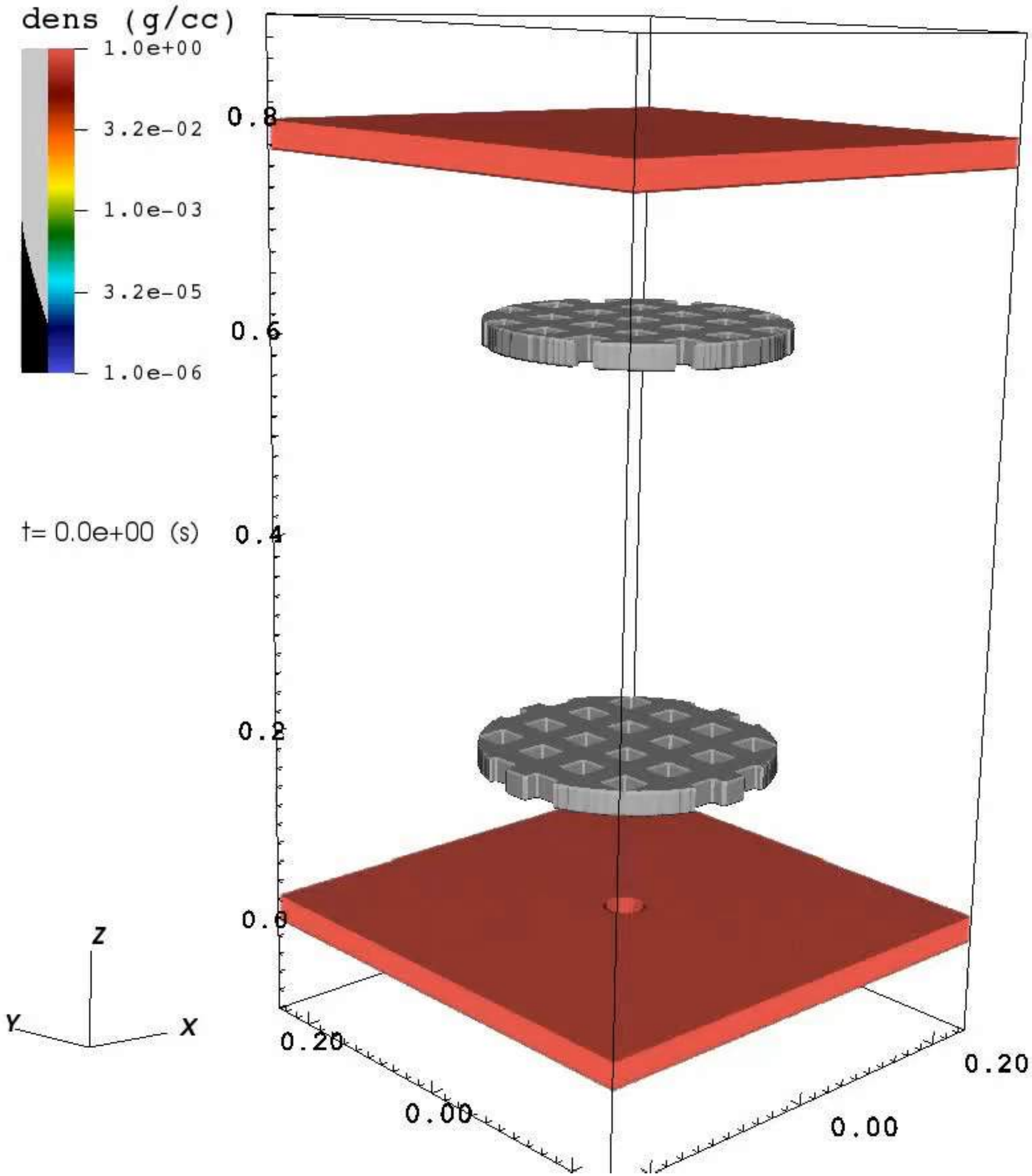
Reynolds number

$$\frac{\partial}{\partial t'}\left(\rho' \varepsilon' + \frac{\rho' \mathbf{u}'^2}{2}\right) + \nabla' \cdot \left(\rho' \mathbf{u}'\left(\varepsilon' + \frac{\mathbf{u}'^2}{2}\right) + P' \mathbf{u}'\right) = \frac{1}{R_e}\nabla' \cdot (\boldsymbol{\sigma}' \cdot \mathbf{u}') - \mathbf{J}' \cdot \mathbf{E}'$$

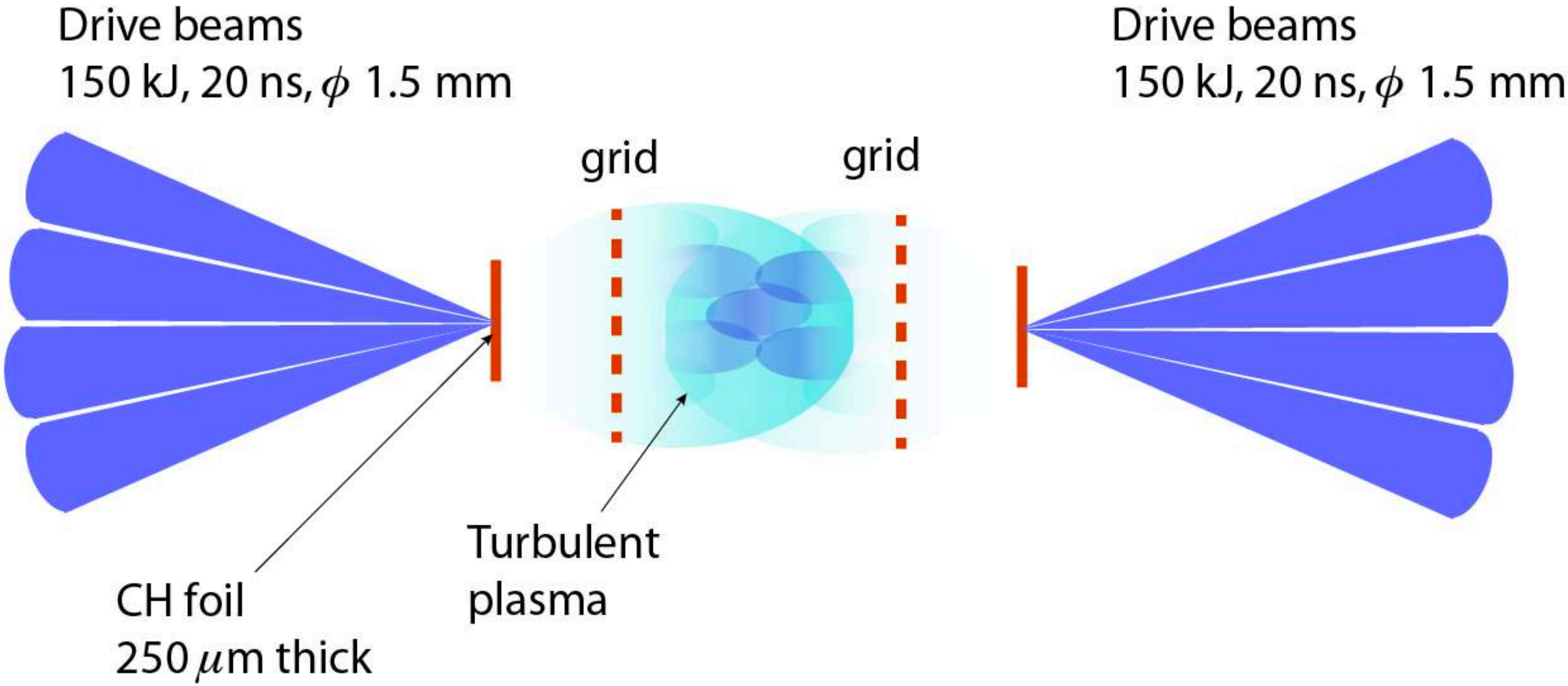
$$\frac{\partial \mathbf{B}'}{\partial t'} = \nabla' \times (\mathbf{u}' \times \mathbf{B}') + \frac{1}{R_m}\nabla'^2 \mathbf{B}'$$

Magnetic Reynolds number

FLASH simulation of laser generated MHD turbulence



Courtesy: Petros Tzeferacos
University of Chicago

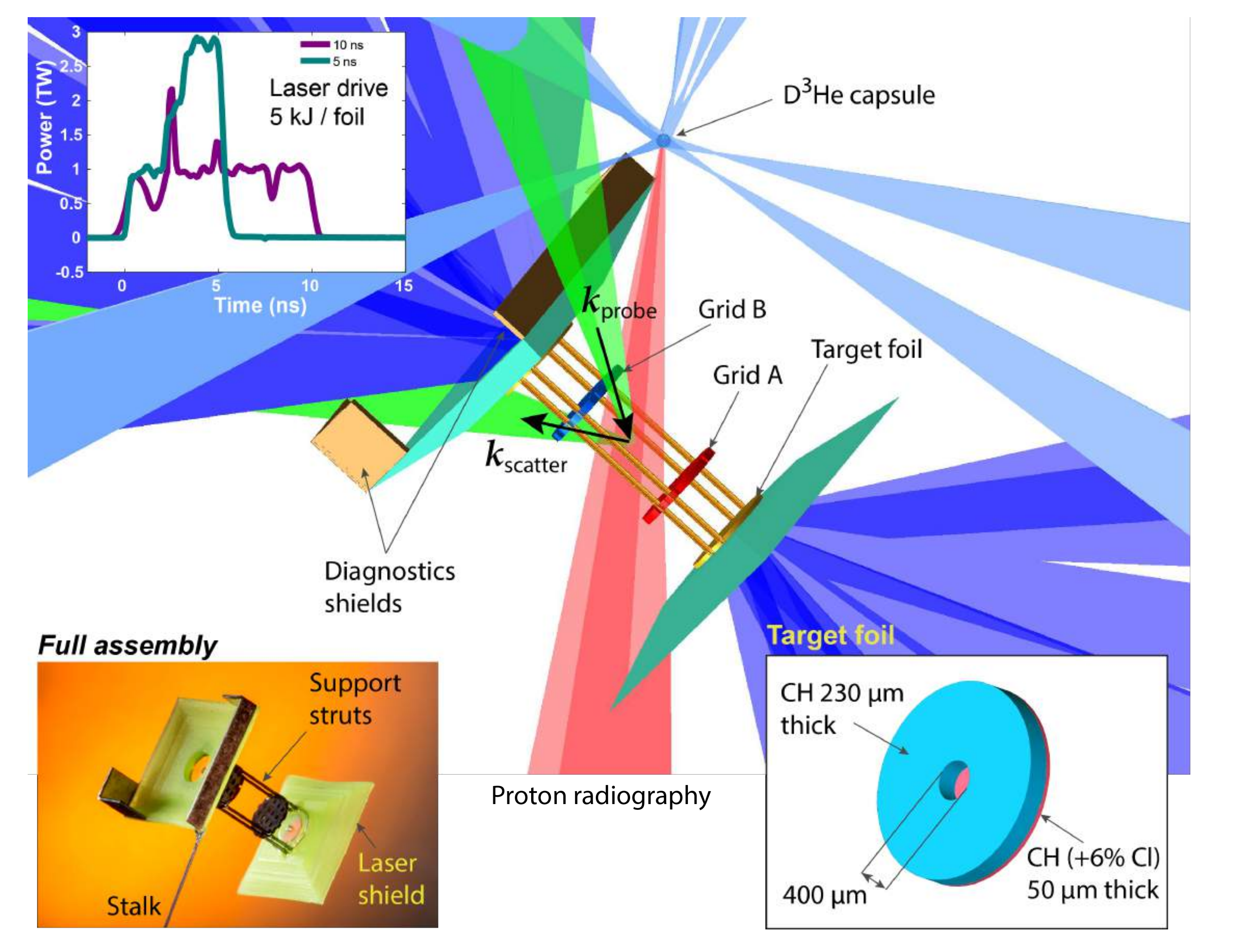


RMS magnetic field	$B \sim 1.2\,\mathrm{MG}$
Mean turbulent velocity	$u = 6 \times 10^7$ cm/s
Scale of the turbulence cells	$\ell \sim 0.06\,\mathrm{cm}$
Plasma size	$L = 0.4\,\mathrm{cm}$
Initial proton momentum	$p_0 = 0.002\, m_\mathrm{p} c$
Temperature	$T = 700\,\mathrm{eV}$
Electron density	$n = 7 \times 10^{20}\ 1/\mathrm{cm}^3$
Density relation	$\nabla n/n \sim O(1)$
Plasma beta	$\beta = 13.7\ (1.2\,\mathrm{MG}/B)^2$
Alfvénic Mach number	$M_\mathrm{a} = u/v_\mathrm{a} = 6$
Reynolds number	$Re = 1200$
Magnetic Reynolds number	$R_\mathrm{m} = 25000$

Table 1: The expected plasma parameters for the proposed experiment at the NIF

Beyer et al, J. Plasma Phys. **84**:905840608,2018

Use colliding flows & grids to create strong turbulence



Tzeferacos et al. Nature Comm. **9**:591,2018

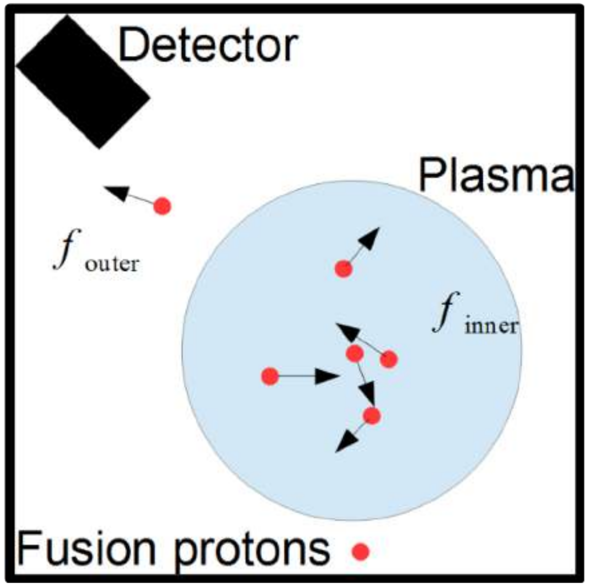
The colliding flows contain D and ~3 MeV protons are produced via D+D → T + p reactions

Fokker-Plank diffusion coefficients

- Diffusion coefficient $D_\varepsilon = \frac{\langle(\Delta\varepsilon)^2\rangle}{\Delta t} = \frac{p^2}{m_p^2} D_p$

- Ohm's law

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} - \beta\frac{\delta_i}{l}\nabla P_e + \frac{\delta_i}{l}\mathbf{j}\times\mathbf{B} + \frac{1}{R_m}\mathbf{j} + \left(\frac{\delta_e}{l}\right)^2\frac{\partial\mathbf{j}}{\partial t}$$

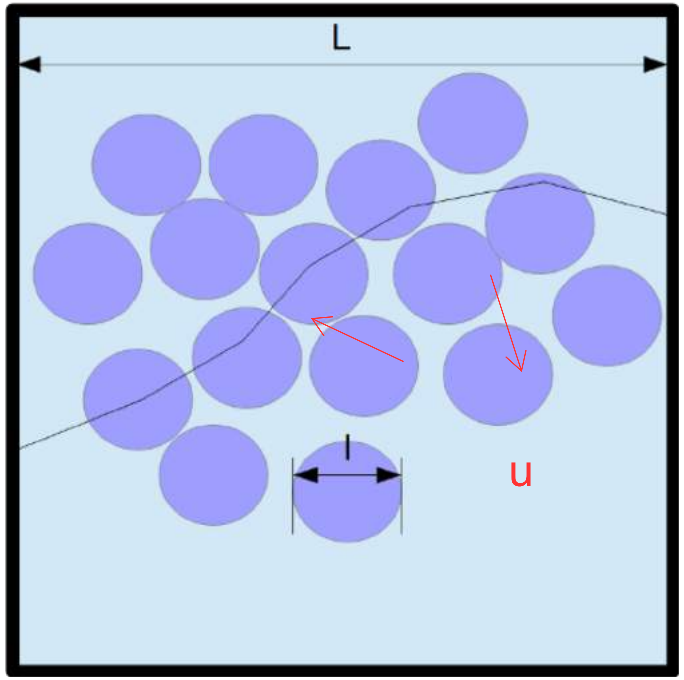


Taking the fields and flows to be uncorrelated over one cell size, the momentum diffusion coefficient is:

$$D_p = \frac{l}{c}\left(\frac{4e^2B^2}{3}\frac{u^2}{c^2} + e^2T^2\left(\frac{\nabla n}{n}\right)^2\right)\frac{m_p c}{p}$$

… and the spatial diffusion coefficient is:

$$D_x = \frac{m_p^2c^5}{3q^2lB^2}\left(\frac{p}{m_p c}\right)^3 \qquad \tau_{esc} = \frac{L^2}{D_x}$$



So $D_p\, D_x \propto p^2$ … i.e. solution applies to non-rel. case too

Beyer *et al*, J. Plasma Phys. **84**:905840608,2018

Relevant time scales

- Streaming time $\tau_{cross} = 1.7\times10^{-10}s$
- Scattering time $\tau_{90} = 1.5\times10^{-10}s\left(\frac{B}{1.2MG}\right)^{-2}\left(\frac{l}{0.1cm}\right)^{-1}$
- Escape time $\tau_{esc} = 5.5\times10^{-10}s\left(\frac{B}{1.2MG}\right)^{2}\left(\frac{l}{0.1cm}\right)$

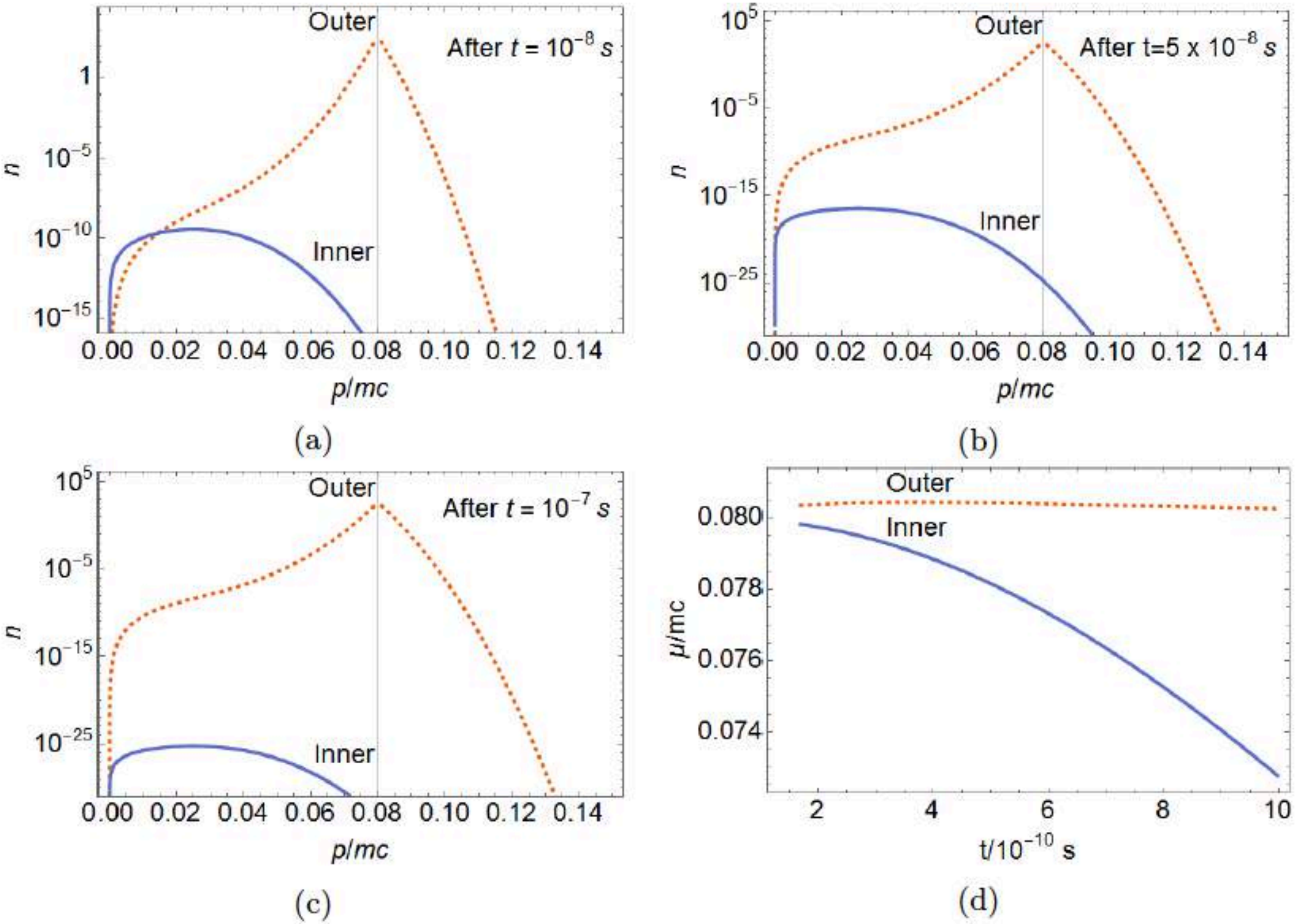
To ensure diffusion, the scattering time must be *smaller* than the escape time

However the inferred parameters are on the edge between **ballistic escape** and **diffusion** … so need *higher* magnetic field to ensure diffusion

Parameter	Omega facility	Scaled NIF value
RMS magnetic field	0.12 MG	1.2 – 4 MG
Correlation length	~0.1cm	~0.05cm
Temperature	450 eV	700 eV
Electron/Ion density	$\sim10^{20}$/cm^3	$\sim7\text{x}10^{20}$/cm^3
Mean turbulence velocity	150 km/s	600 km/s
Plasma beta	125	13.7
Reynolds number	370	~1200
Magnetic Reynolds number	870	~20000

Beyer *et al*, J. Plasma Phys. **84**:905840608,2018

Analytic solution to the Fokker-Planck equation



Beyer *et al*, J. Plasma Phys. **84**:905840608,2018

$$n_{\text{inner}} = \frac{2\hat{p}^2\sqrt{\Psi}}{\sqrt{k\tau}\,(1-\Psi)} e^{-\frac{\left(\hat{p}^3+\hat{p}_0^3\right)(1+\Psi)}{3\sqrt{k\tau}(1-\Psi)}} \mathrm{I}_0\left[\frac{4\left(\hat{p}\hat{p}_0\right)^{\frac{3}{2}}\sqrt{\Psi}}{3\sqrt{k\tau}\,(1-\Psi)}\right]$$

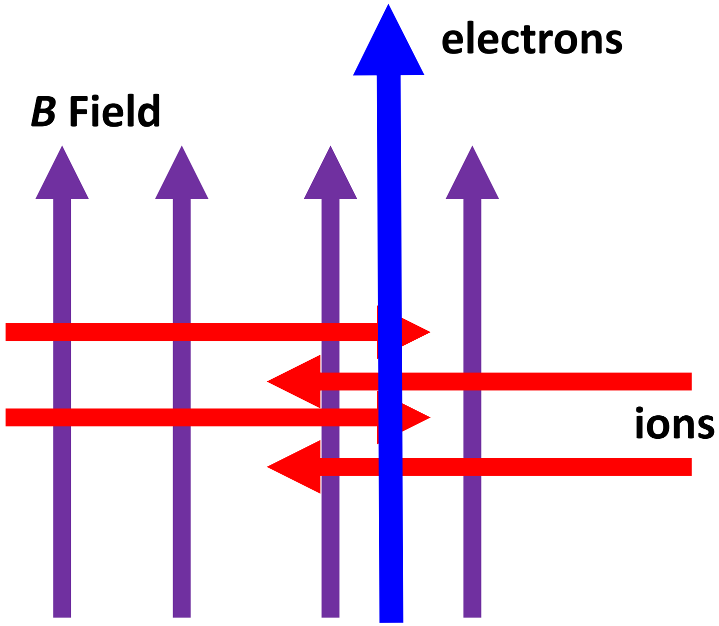
$$n_{\text{outer}}(p, p_0, t, t_0) = \int_0^t \frac{n_{\text{inner}}(p, p_0, t', t_0)}{\tau_{\text{esc}}}\,\mathrm{d}t'$$

… holds even for non-relativistic particles - as long as $D_p D_x \propto p^2$ (Mertsch, JCAP **12**:10,2011)

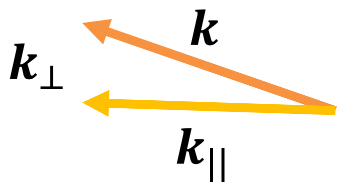
Expect mean energy to increase by 10-200 keV and FWHM by 0.24-1.2 MeV – ***detectable***!

Particle acceleration relies on there being a injection mechanism

- ➜ For diffusive shock acceleration to work, the particles must cross the shock *many* times i.e. their Larmor radius must exceed the shock thickness
- ➜ There must *already* be a population of energetic particles in order for the Fermi process to operate …. this is the 'injection problem'
- ➜ This pre-acceleration mechanism can be provided by wave-plasma instabilities, such as the modified two-stream instability



Lower-hybrid waves
(at perpendicular shocks)

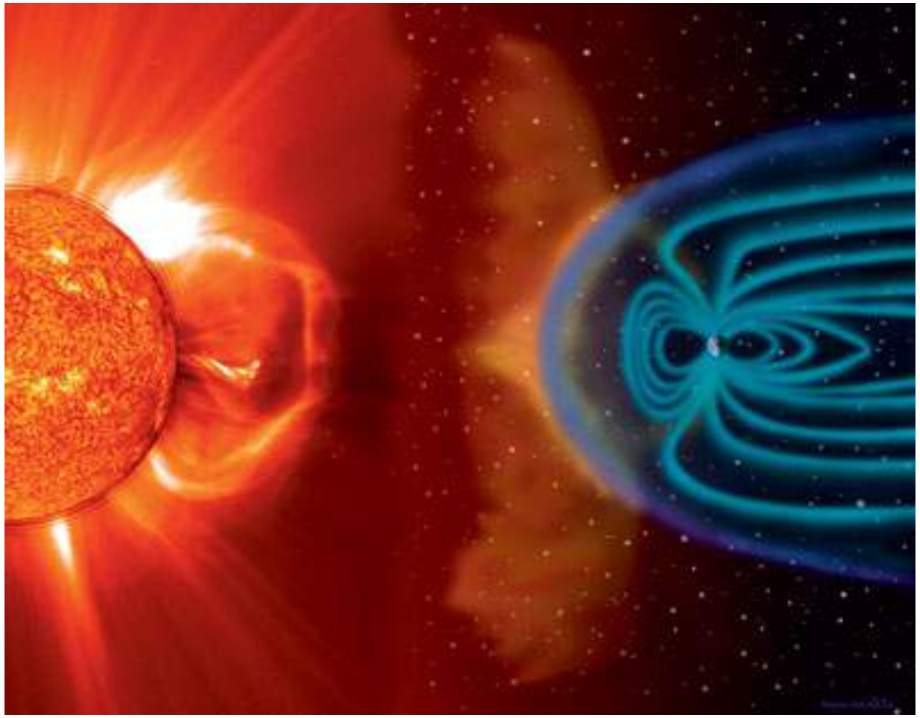


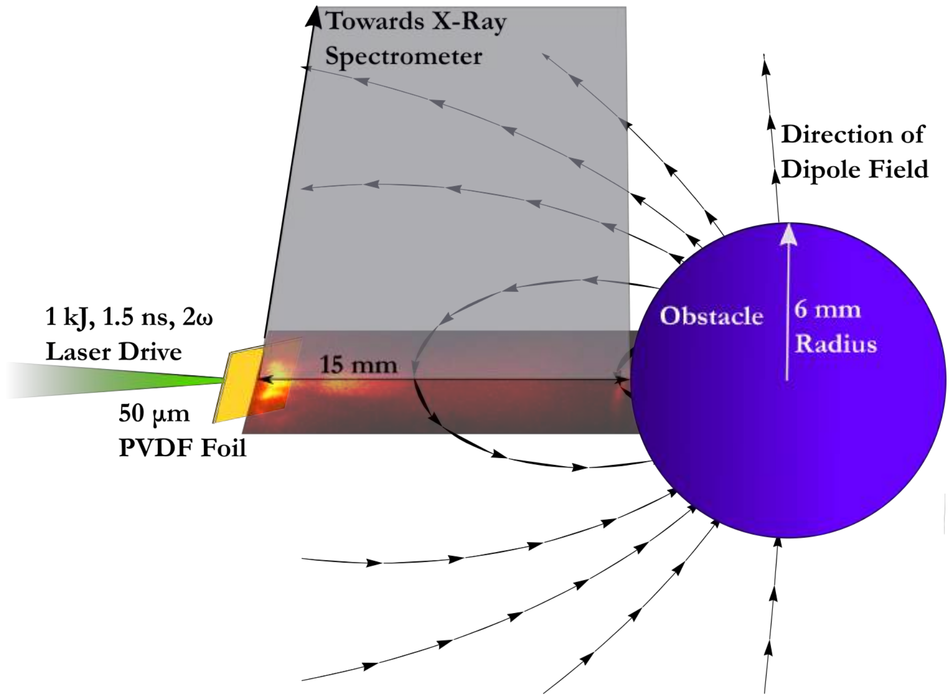
$$\omega = \boldsymbol{k}_{||} \cdot \boldsymbol{v}_i \approx \boldsymbol{k}_{\perp} \cdot \boldsymbol{v}_e$$

Waves in *simultaneous* Cherenkov resonance with ions and electrons

$$E_e \sim \alpha^{2/5} \left(\frac{m_e}{m_i}\right)^{1/5} m_i u^2$$

Laboratory experiment to investigate particle injection at shocks



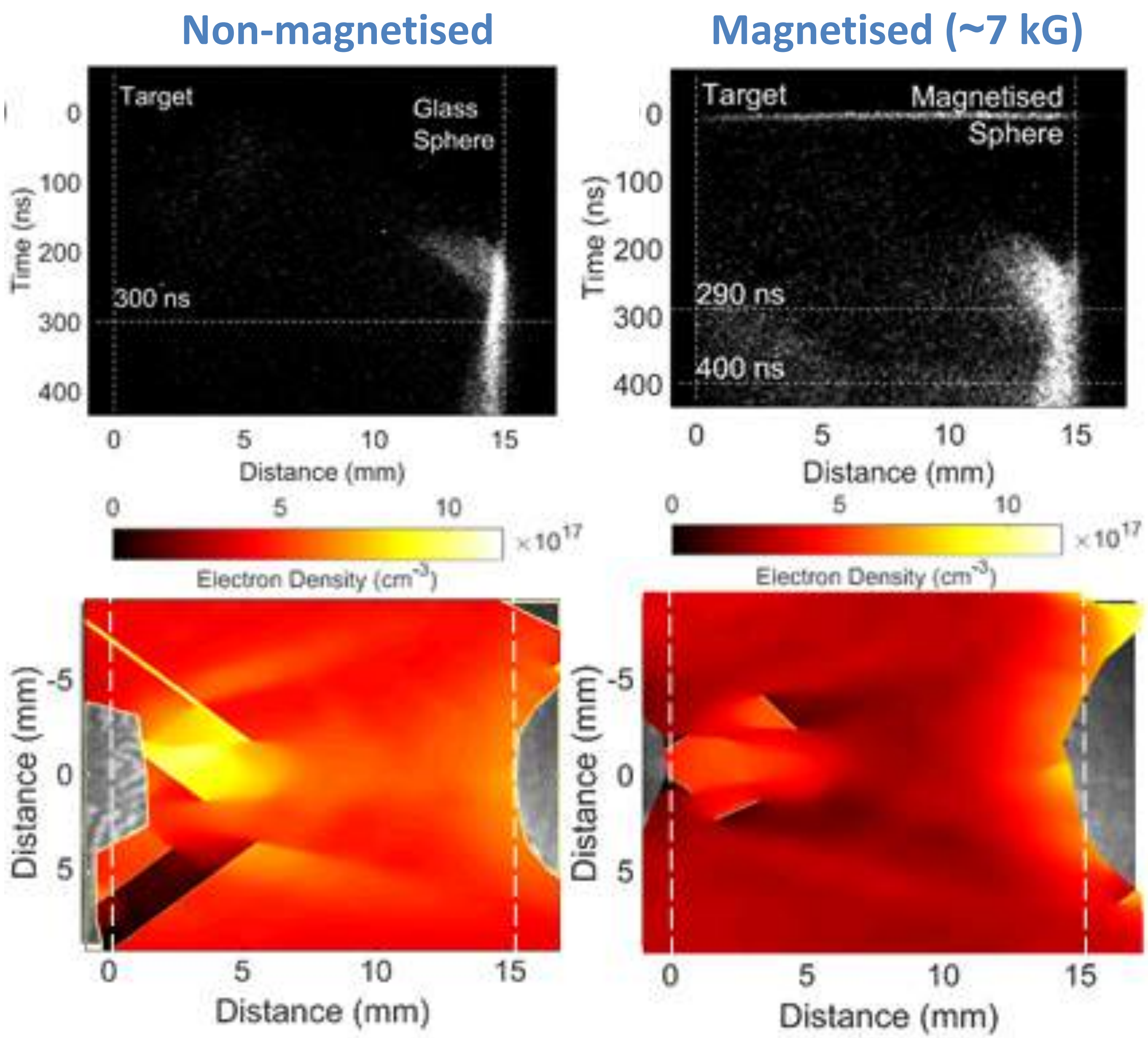


- ➜ Lower-hybrid acceleration provides a possible mechanism to pre-heat electrons above the thermal background

- ➜ This instability has been suggested to explain observed X-ray excess in cometary knots (Bingham *et al.* 2004)

- ➜ We have performed an experiment at LULI, Paris to study this process

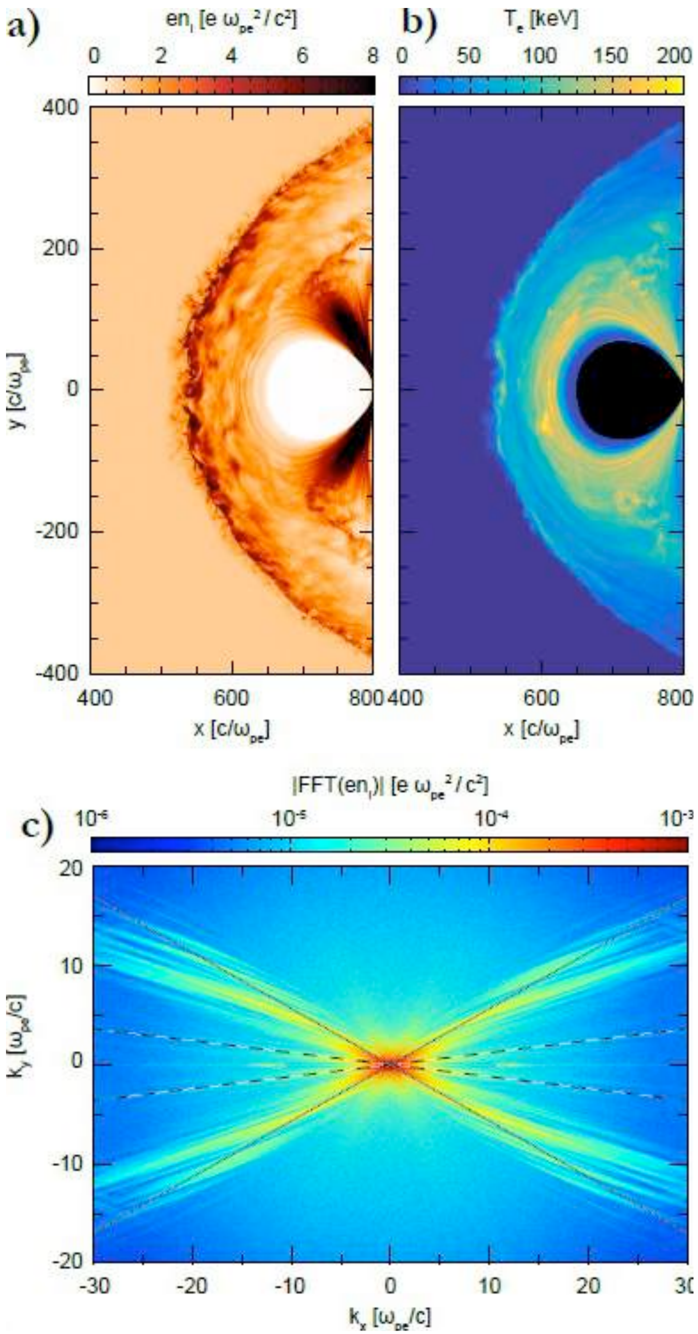
Laboratory experiment to investigate particle injection at shocks



Rigby *et al*. Nature Physics **14**:475,2018

- ➜ Incoming plasma with velocity $\sim$70 km/s
- ➜ Data shows formation of a shock when magnetic field is present
- ➜ Reflected ions have mean free path of a few mm (larger than their Larmor radius)
- ➜ Plasma $\beta \sim 0.2$ for quasi-perpendicular shock, hence magnetised two stream instability can be excited

PIC simulations show lower-hybrid heating of electrons near shock



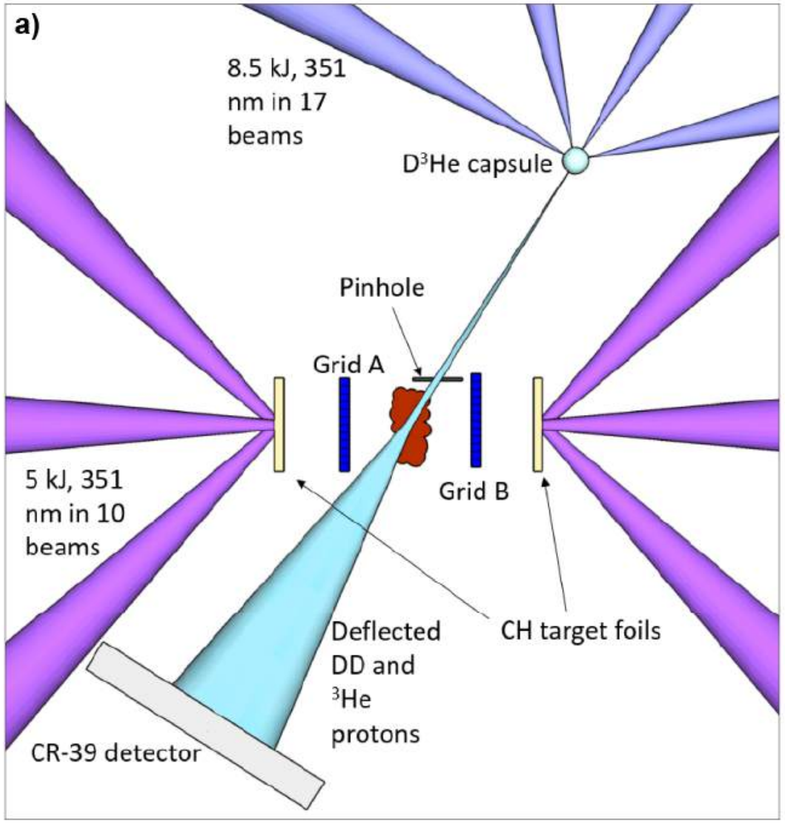
OSIRIS PIC simulations

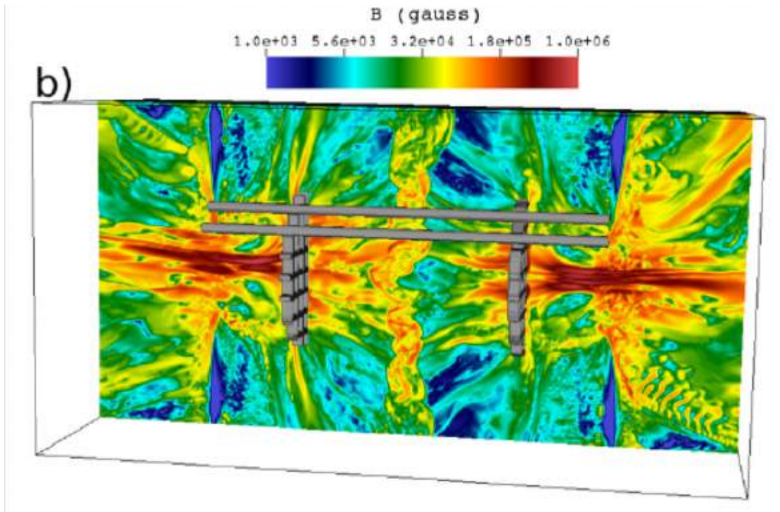
- → We have performed 2D PIC using the massively parallel code OSIRIS
- → Simulations are performed with a reduced mass ratio and higher flow velocity, but Alfvenic Mach number is kept the same (scale invariance)

- → Shock is formed with electron heating along B-field lines
- → Turbulent wave spectrum is formed with dispersion relation consistent with LH waves

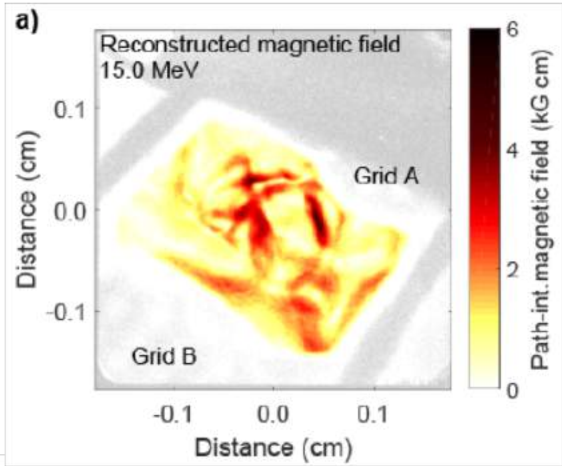
Measurement of 'cosmic ray' diffusion

- An experiment was undertaken to measure the diffusion coefficient in the plasma at the Omega facility, University of Rochester.
- A pinhole was inserted to collimate the proton flux from an imploding D3He capsule.
- Without magnetic fields, the pinhole imprints a sharp image of the pinhole onto the detector.
- Random magnetic fields will induce perpendicular velocities to the protons resulting in smearing of the pinhole imprint.





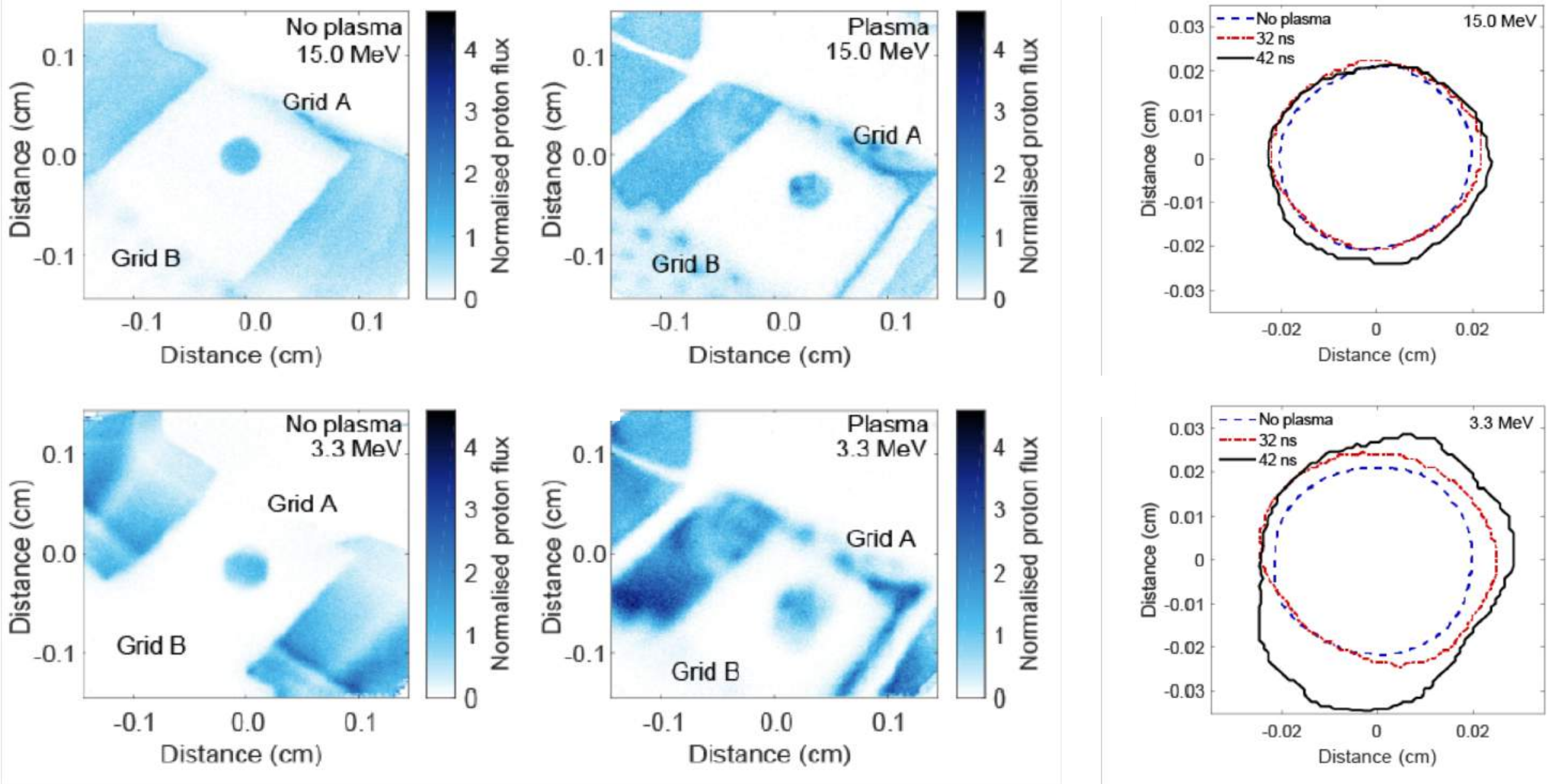
Chen *et al.* (2018), to appear



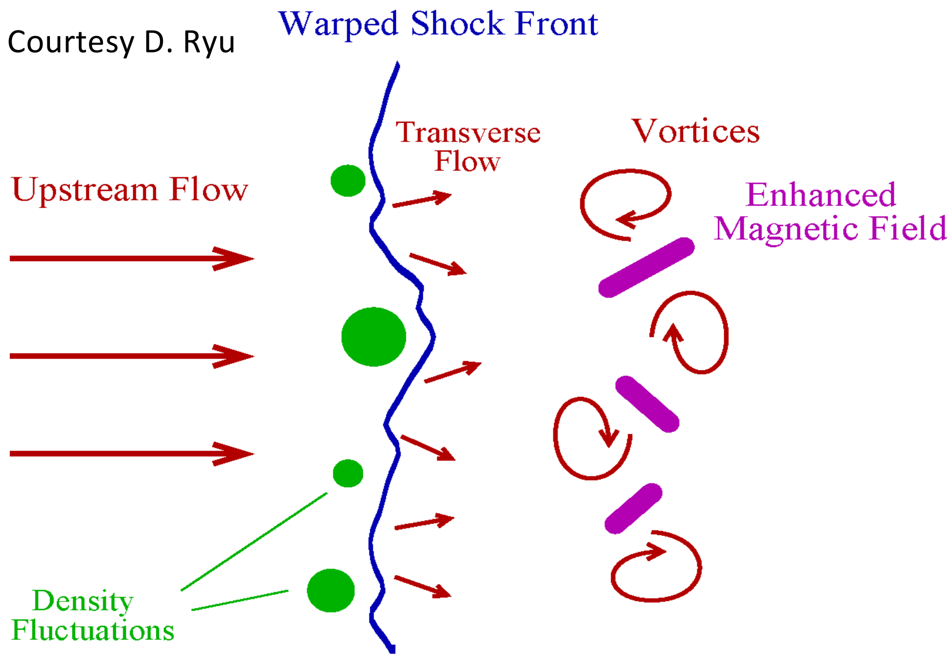
Observe *smearing* of the edges of the pinhole imprint

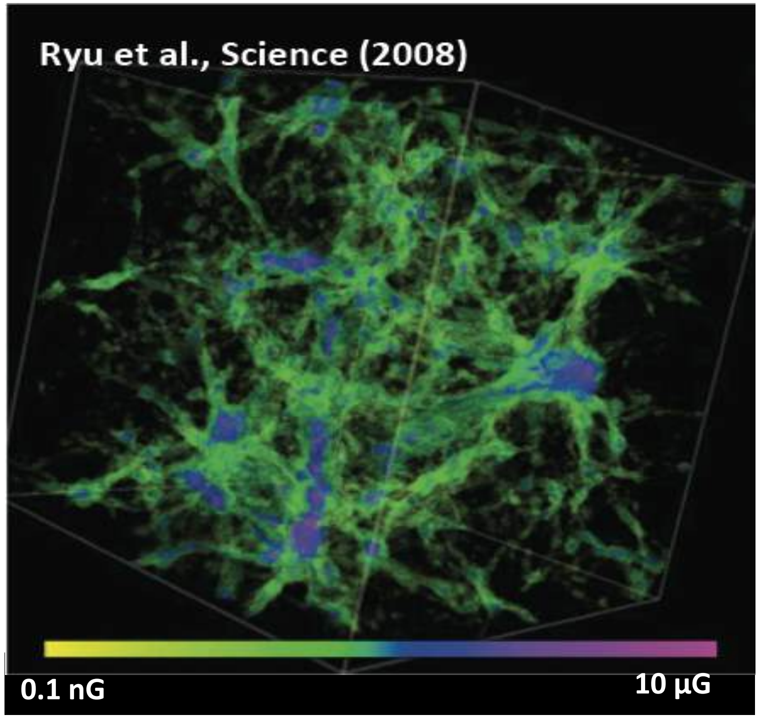
…. Could in principle be caused by multiple effects (turbulent fluid motions, plasma instabilities, etc) … but all can be shown to be *negligible* in practice

→ Ascribed to stochastic magnetic fields



Cosmic generation of magnetic fields invokes MHD turbulence





100 Mpc

→Assume there are tiny magnetic fields generated before structure formation

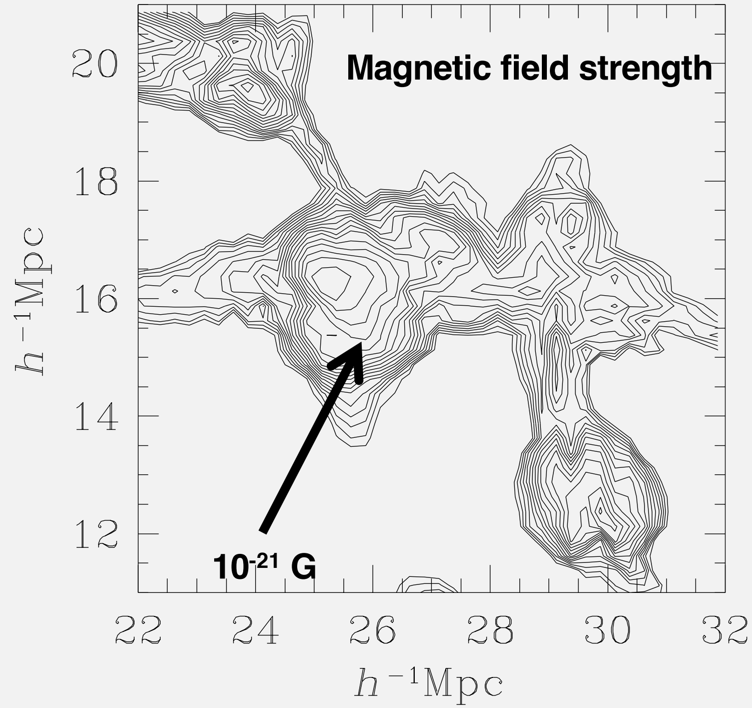
→Magnetic field are then amplified to dynamical strength and coherence length by turbulent motions

Biermann's battery mechanism occurs at curved shocks

Magnetic field is produced by misaligned T_e and n_e gradients

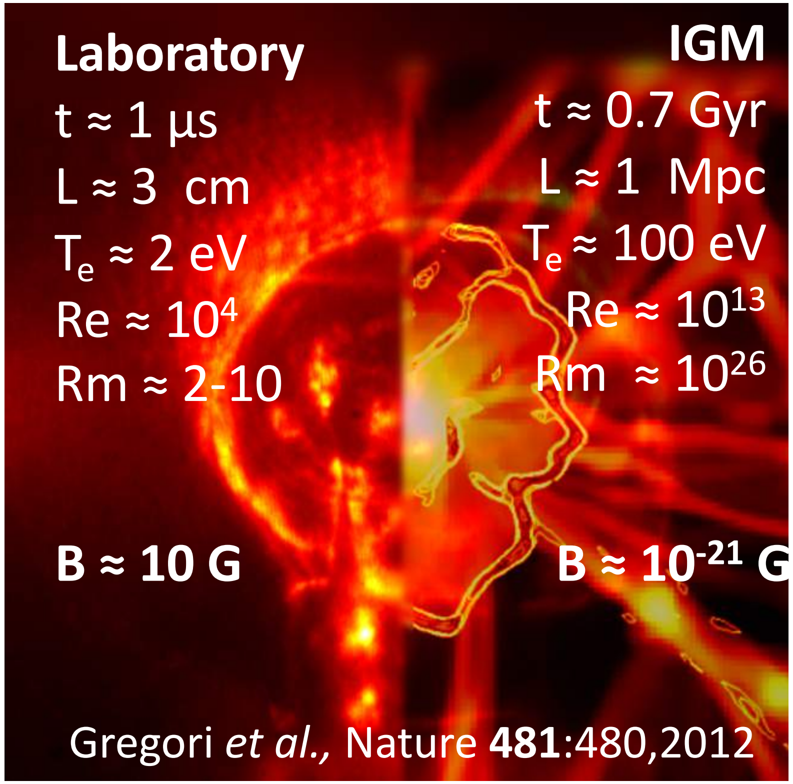
→ Develops on scales set by shocks in the interstellar/intergalactic medium

→ Structure formation simulations show that a tiny magnetic field is produced near shocks



Kulsrud *et al*. ApJ (1997)

Laser plasma experiments can also generate magnetic fields at shocks



→ Magnetic fields scales with vorticity: $B \sim \omega \sim 1/t$

→ Scaled laboratory values are in agreement with simulations of structure formation

Summary

Plasmas of astrophysical relevance can be investigated in the laboratory because of the *scale invariance* of the governing MHD equations

- E.g. cosmic magnetic fields can be produced by the 'Biermann Battery' and subsequently amplified by turbulent dynamo action
- Fusion protons can be produced inside the colliding streams and their momentum space diffusion rate can be measured
- Stochastic 2^{nd}-order Fermi acceleration will *soon* be tested

We cannot yet make an universe in the laboratory …
but we can (nearly) make a supernova!

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