

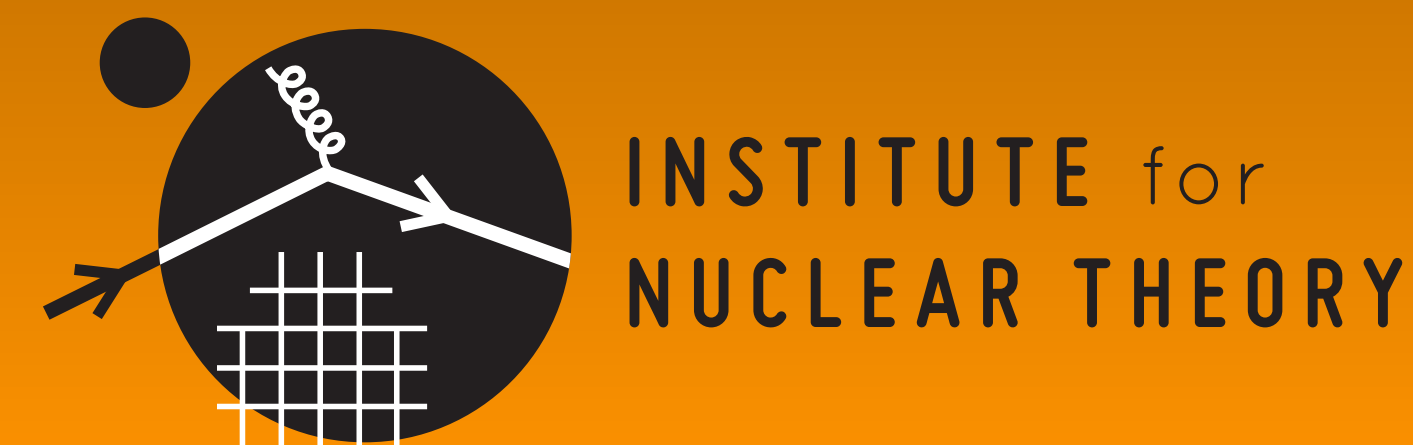
# Neutron Stars in the Multi-Messenger Era

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University of Washington, Seattle

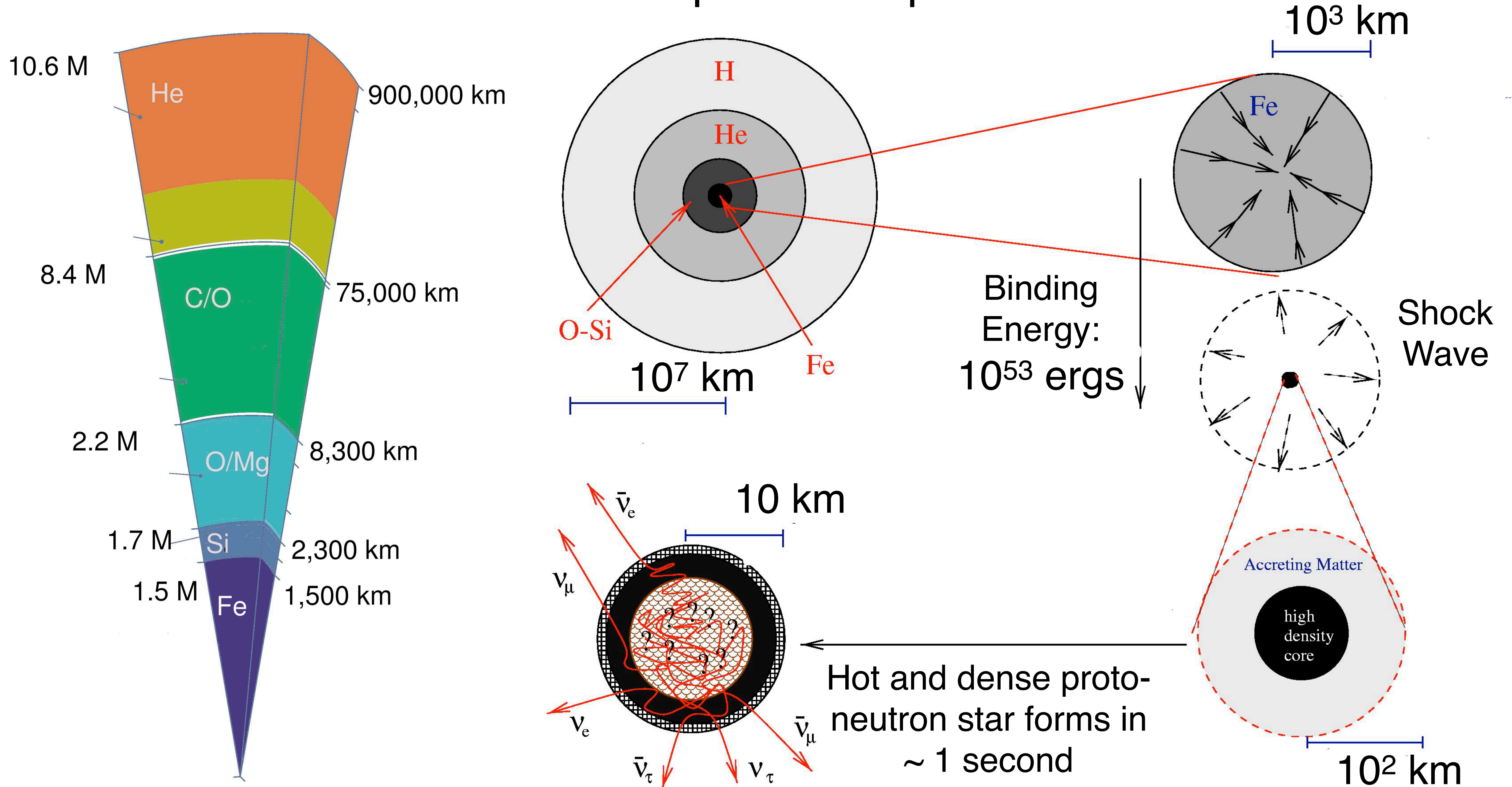
Lecture 1: Structure of Neutron Stars. Mass, radius and tidal deformability. Nuclear interactions and nuclear matter, effective field theory. Phase transitions.

Lecture 2: Neutron star cooling: Proto-neutron star evolution, supernova neutrino emission and detection. Cooling of isolated neutron stars, heating and cooling in accreting neutron stars.

~~Neutron stars as laboratories for particle physics and dark matter.~~



# Core-collapse Supernova

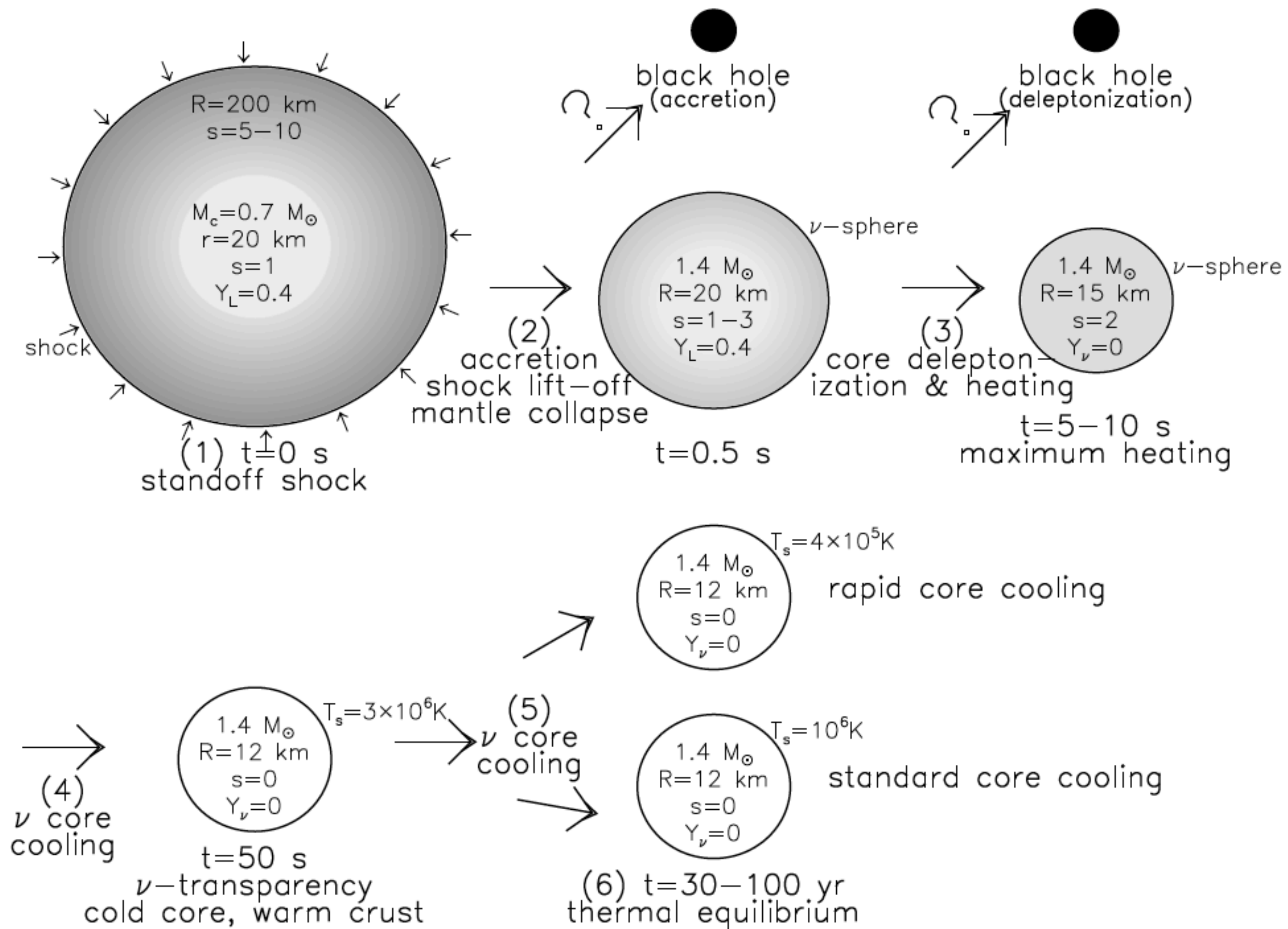


# Thermal Evolution of an Isolated Neutron star

Neutron stars are born hot.

Direct detection of neutrinos is only possible during the first minute from a galactic supernova.

X-ray observations from the surface of a population of neutron stars informs us about late time thermal evolution.

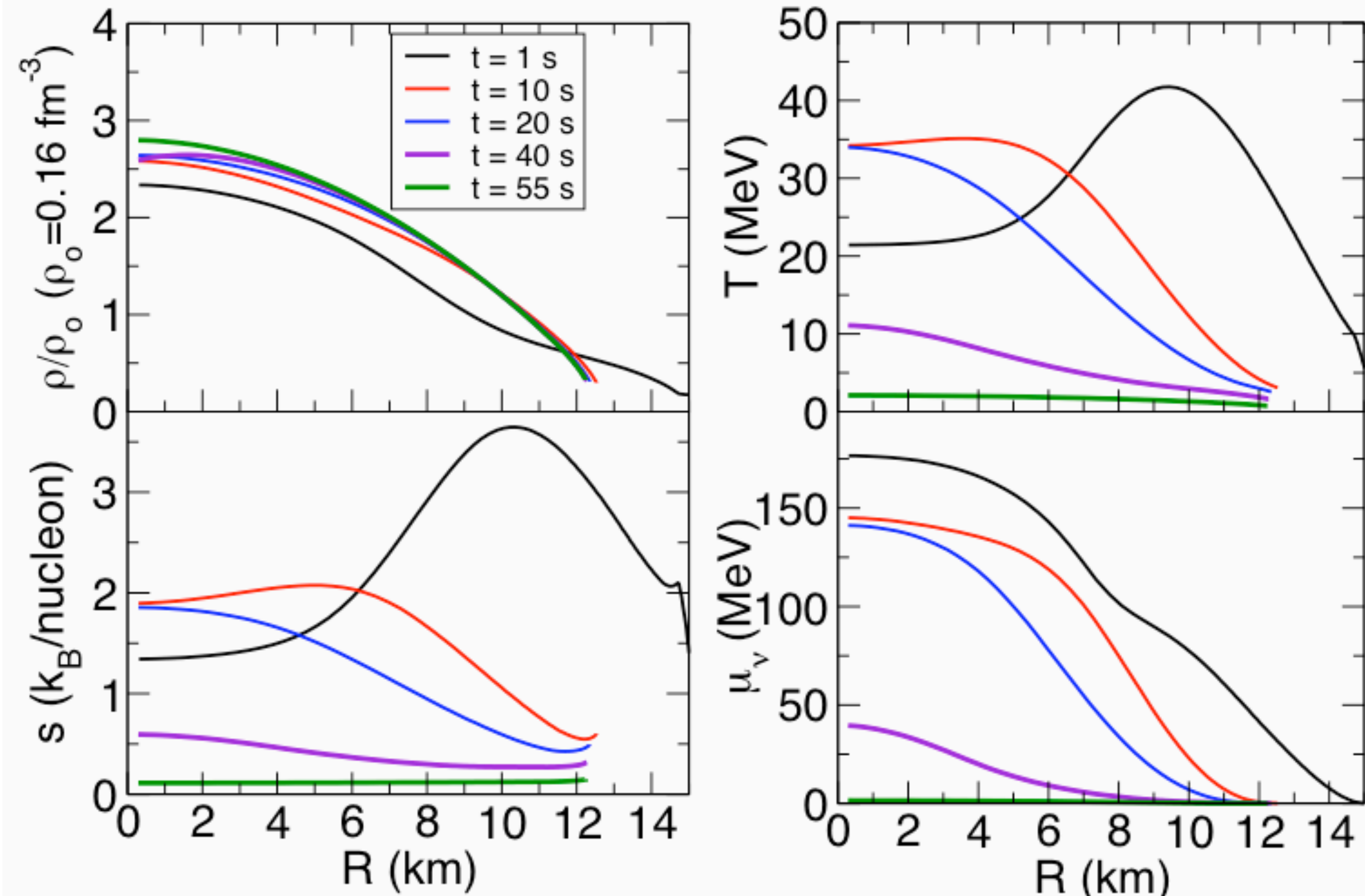


# Proto-Neutron Star Evolution

The proto-neutron star contains a large fraction of the gravitational binding energy - trapped in the form of neutrinos.

Proto-neutron star evolution is largely determined by the diffusion of neutrinos. Convection and accretion can also play a role.

Thermal neutrinos emitted from the surface regions are detectable.

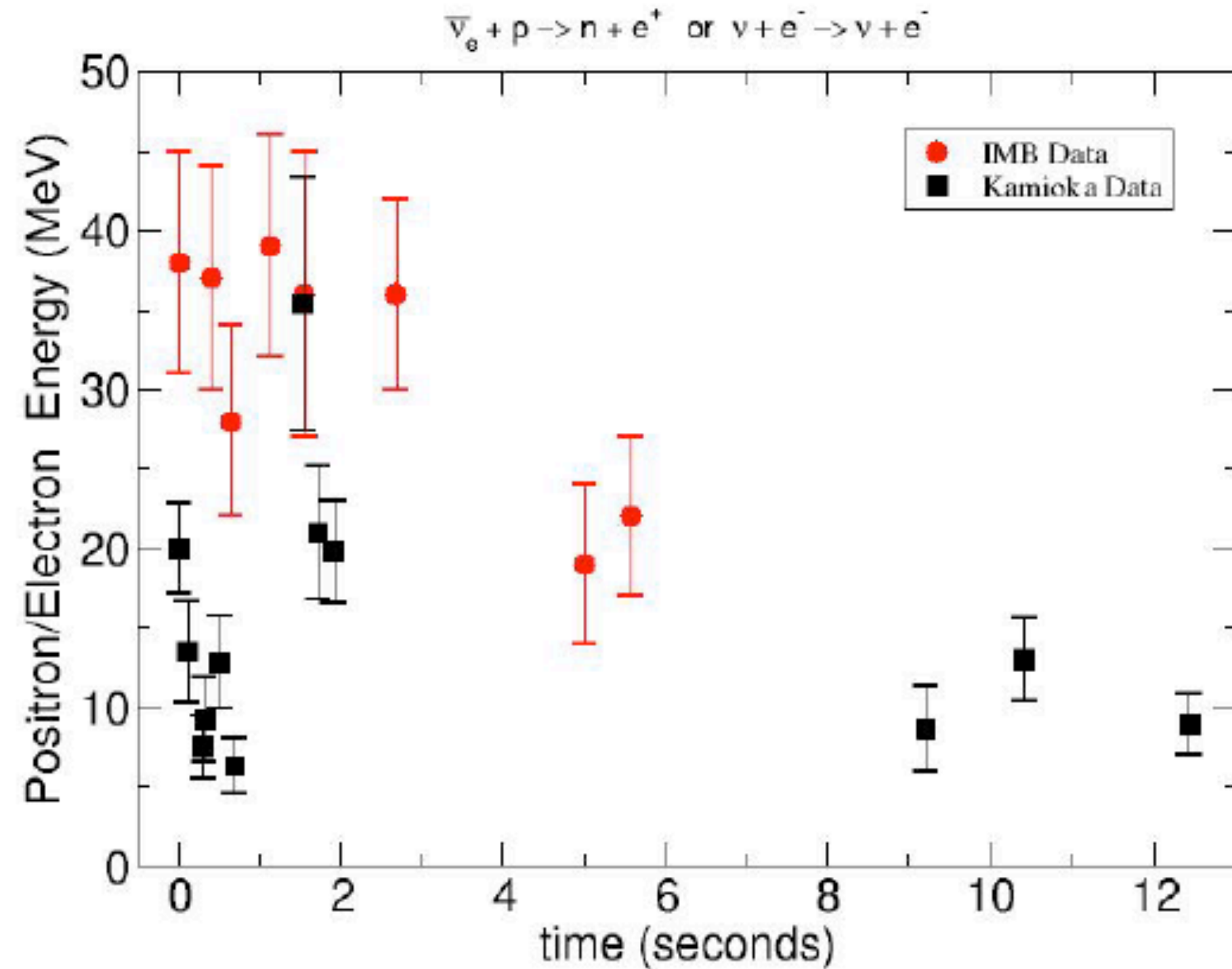


$$T(t) \approx T(0) \left( 1 - \frac{t}{\tau_c} \right)$$

$$\tau_c \approx \frac{C_V R^2}{c \langle \lambda_{\nu_X} \rangle}$$

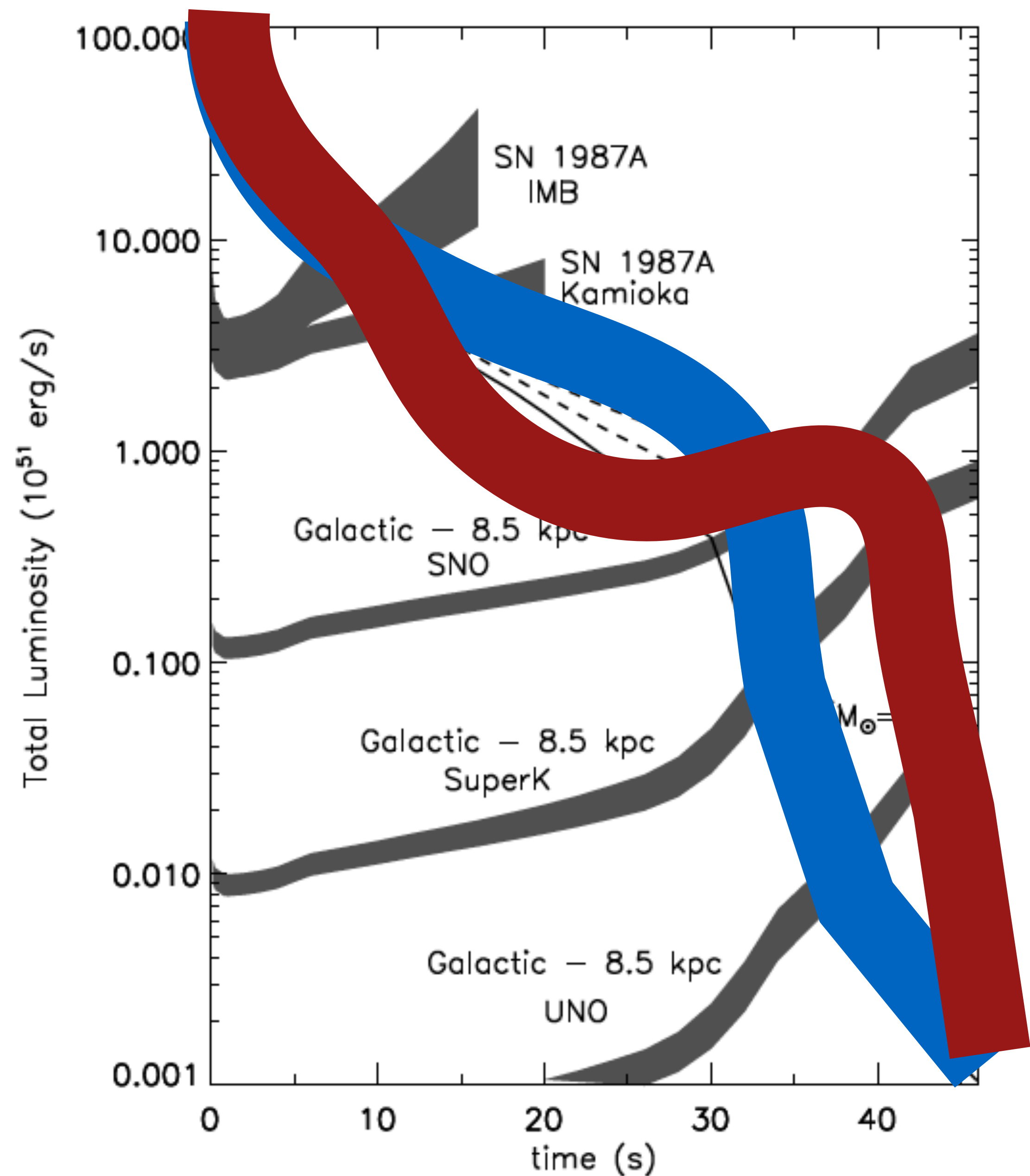
# Supernova Neutrinos

SN 1987a: ~ 20 neutrinos ..in support of supernova theory

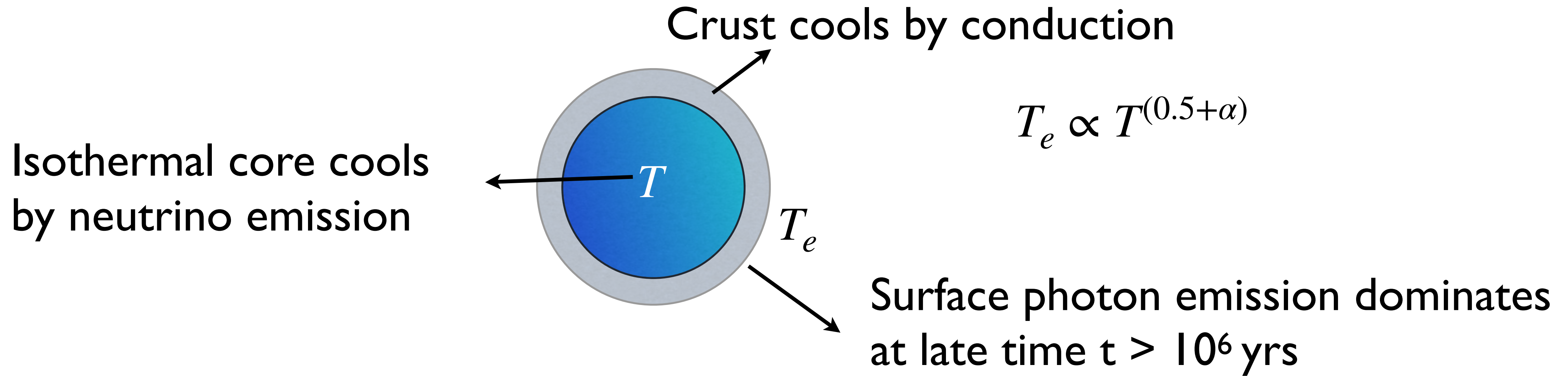


> 10,000 events expected from a future galactic supernovae.

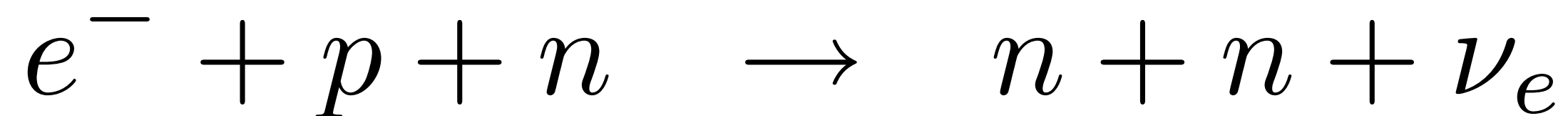
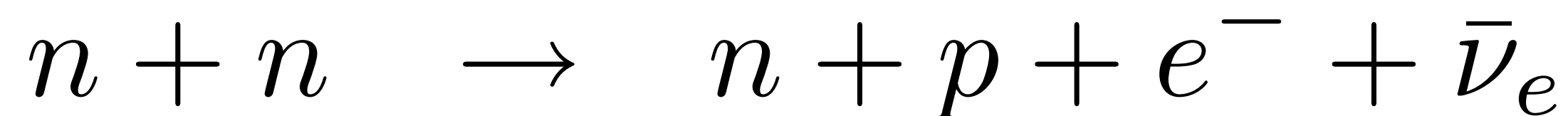
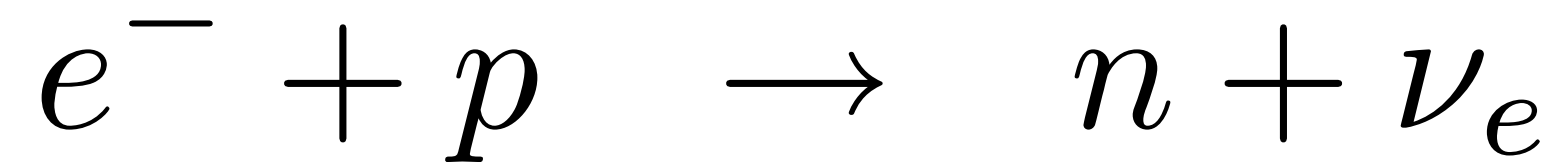
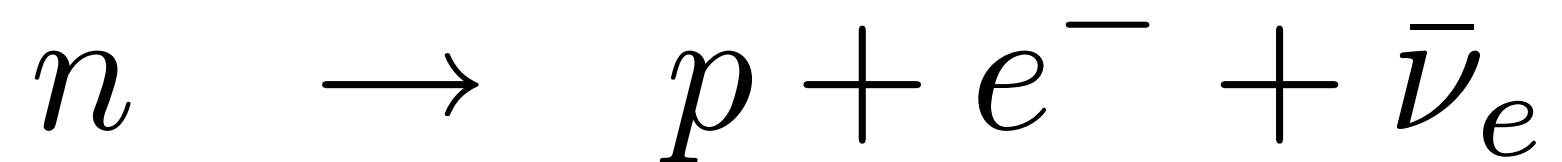
Temporal structure set by neutrino diffusion + convection + fall-back.



# Neutron Star Cooling



Basic neutrino reactions:



$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{25} T_9^6 \frac{\text{ergs}}{\text{cm}^3 \text{ s}}$$

Fast: Direct URCA

$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{22} T_9^8 \frac{\text{ergs}}{\text{cm}^3 \text{ s}}$$

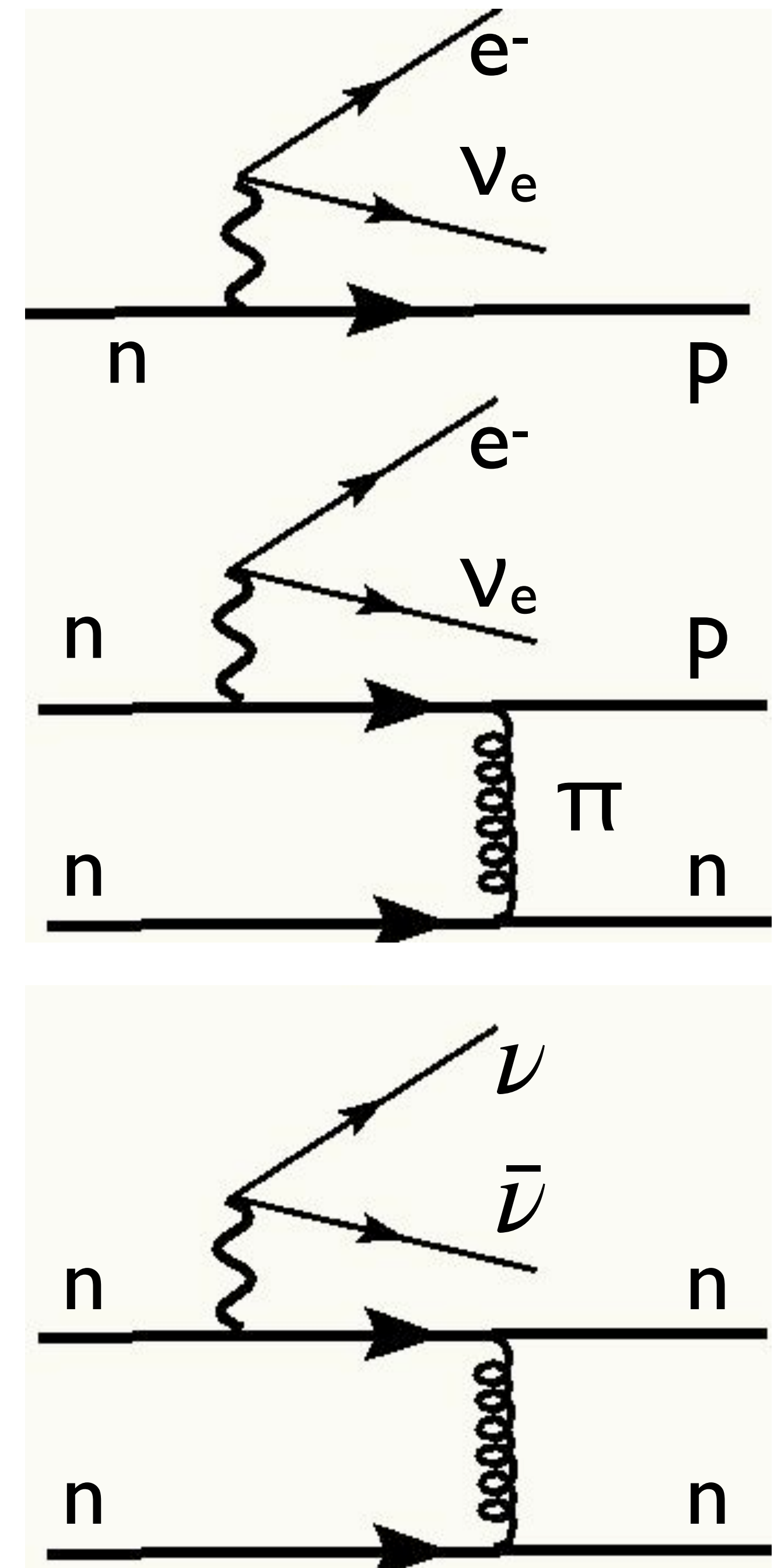
Slow: Modified URCA

# Neutrino Emissivity

Single -particle reactions are fast. Beta decay is the only reaction - “Direct Urca”

Multi-particle reactions are slow. “Modified Urca” can be thought of as beta-decay in the presence of a companion.

Bremsstrahlung reactions are even slower. Because the neutrino momenta are much smaller than that of electrons.

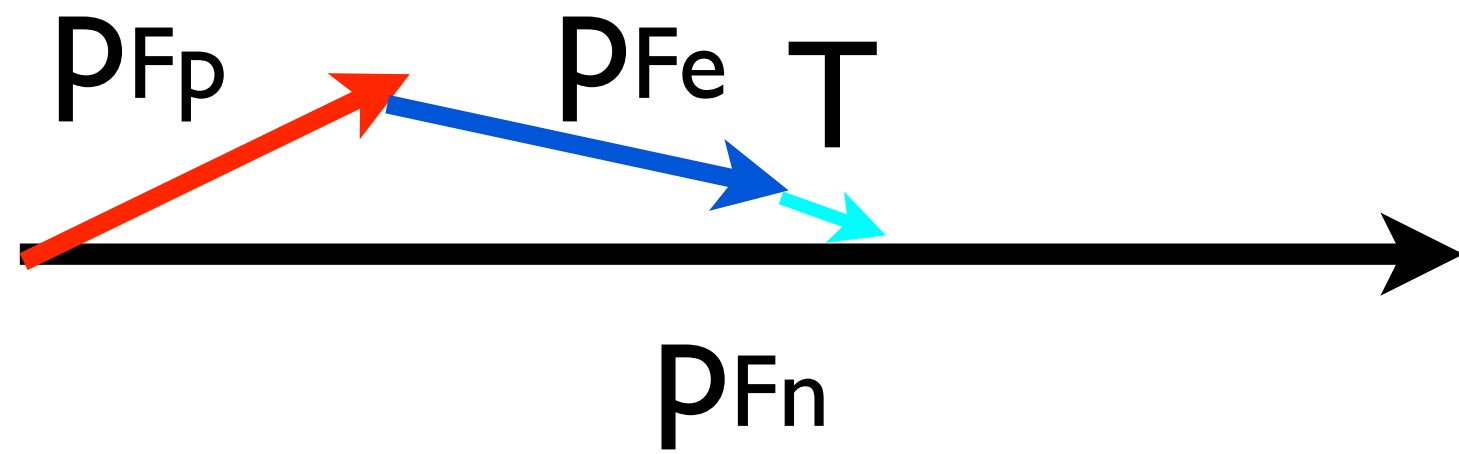


# Neutrino Emission Rates in Normal Nuclear Matter

Name	Process	Emissivity (erg cm <sup>-3</sup> s <sup>-1</sup> )	
Modified Urca cycle (neutron branch)	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$	$\sim 2 \times 10^{21} R T_9^8$	Slow
	$n + p + e^- \rightarrow n + n + \nu_e$		
Modified Urca cycle (proton branch)	$p + n \rightarrow p + p + e^- + \bar{\nu}_e$	$\sim 10^{21} R T_9^8$	Slow
	$p + p + e^- \rightarrow p + n + \nu_e$		
Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$	$\sim 10^{19} R T_9^8$	Slow
	$n + p \rightarrow n + p + \nu + \bar{\nu}$		
	$p + p \rightarrow p + p + \nu + \bar{\nu}$		
Direct Urca cycle	$n \rightarrow p + e^- + \bar{\nu}_e$	$\sim 10^{27} R T_9^6$	Fast
	$p + e^- \rightarrow n + \nu_e$		

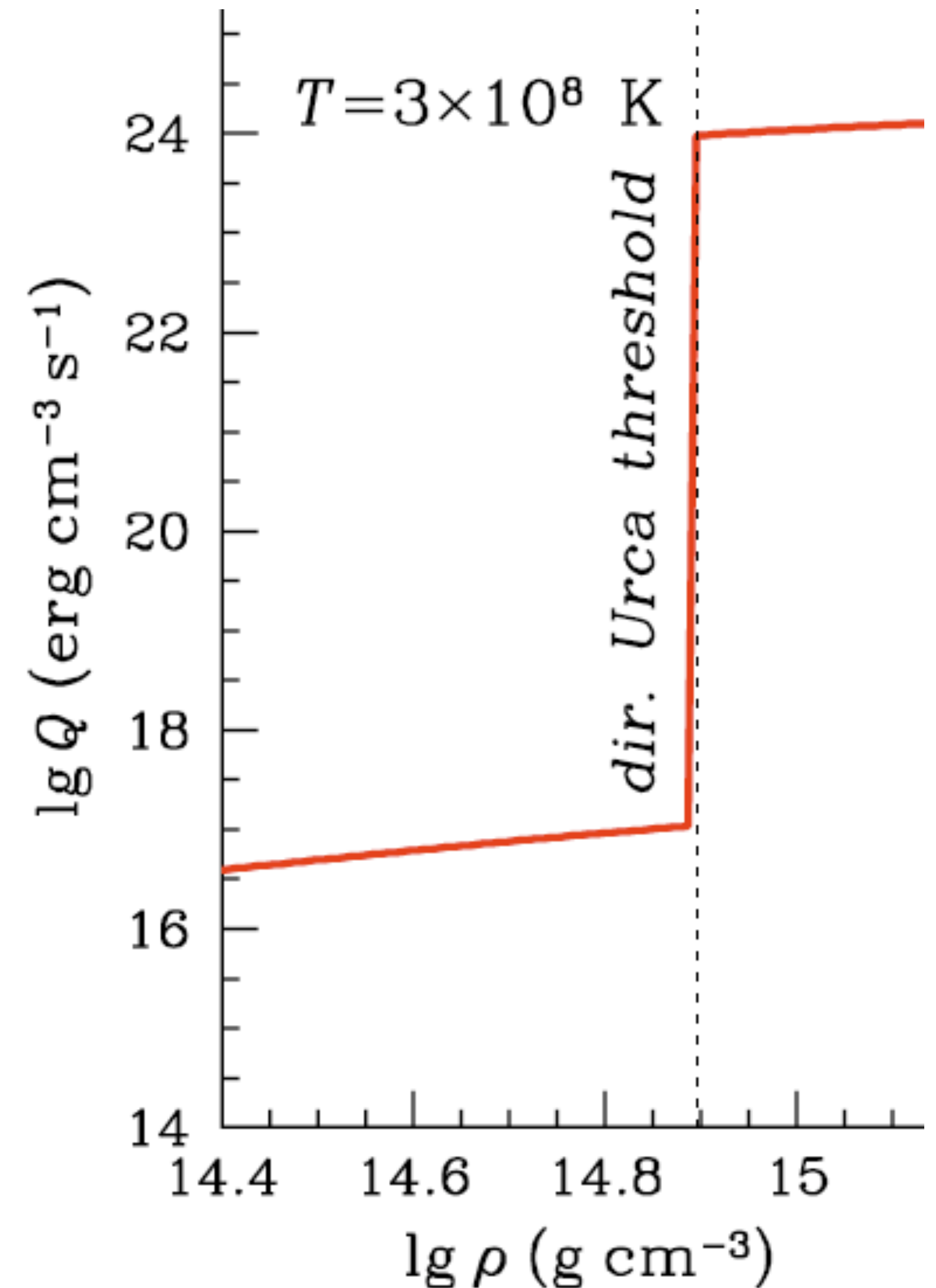


# Proton Fraction & Direct URCA



Neutron decay at the Fermi surface cannot conserve momentum if  $x_p \sim (p_{Fp} / p_{Fn})^3 < 0.1$

Massive stars have larger central density and a larger proton fraction.



# Neutron Star Cooling - Normal Nucleons

Energy Balance Equation:

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

Specific Heat:

$$C_v = \sum_i C_{v i} \quad c_{v i} = N(0) \frac{\pi^2}{3} k_B^2 T \quad \text{with} \quad N(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$
$$C_v = \iiint c_v dV \simeq 10^{38} - 10^{39} \times T_9 \text{ erg K}^{-1} \equiv CT$$

Neutrino Luminosity  
(Modified URCA)

$$L_\nu = \iiint \epsilon_\nu dV \simeq 10^{38} - 10^{40} \times T_9^8 \text{ erg K}^{-1} \equiv NT^8$$

Photon Luminosity  
(surface)

$$L_\gamma = 4\pi R^2 \sigma T_e^4 = ST^{2+4\alpha}$$

# Analytic Model for Neutron Star Cooling

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu$$

$$C_v = CT \quad L_\nu = NT^8 \quad L_\gamma = ST^{2+4\alpha}$$

- Neutrino Cooling Era:  $L_\nu \gg L_\gamma$

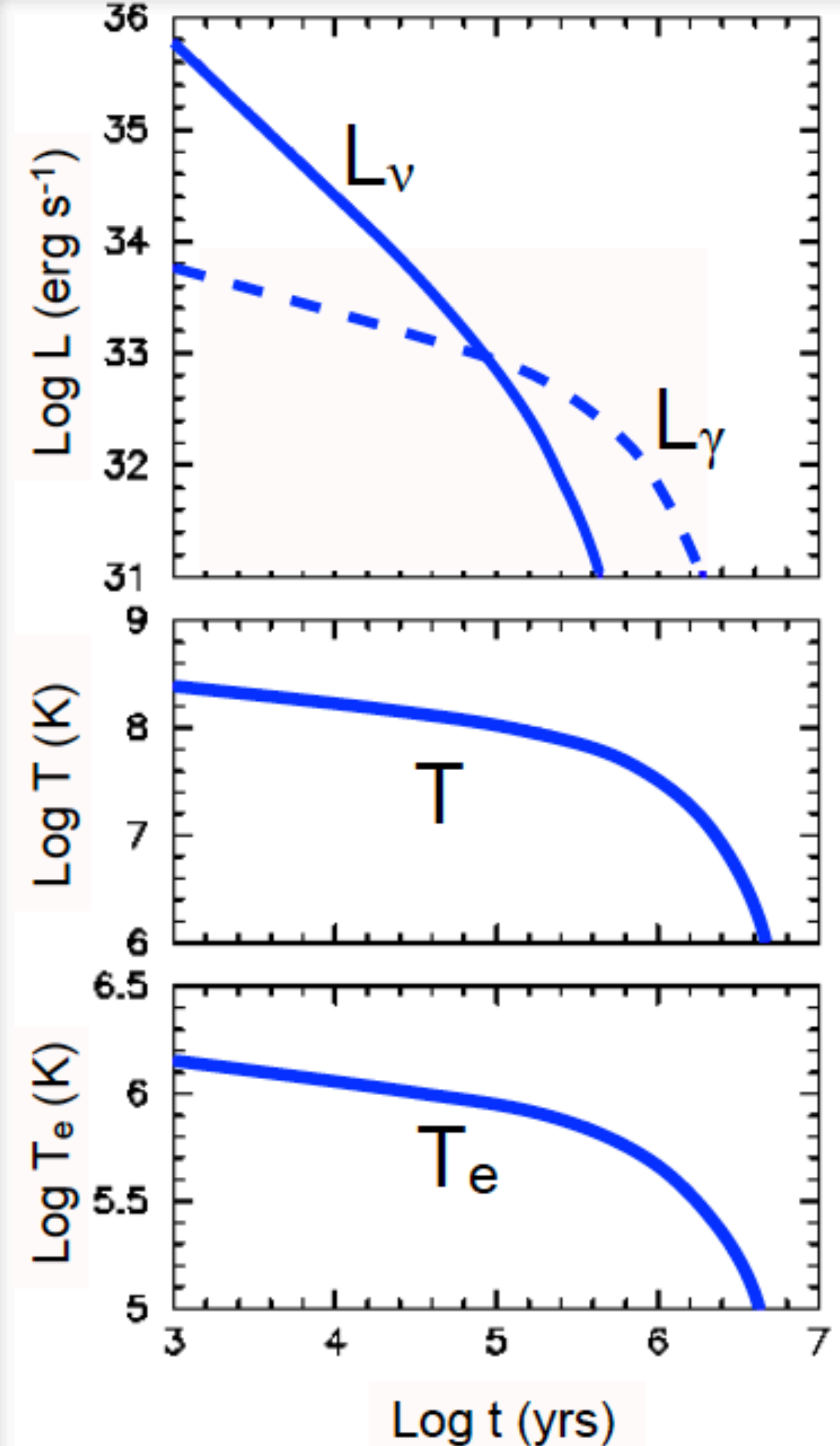
$$\frac{dT}{dt} = -\frac{N}{C}T^7 \Rightarrow t - t_0 = A \left[ \frac{1}{T^6} - \frac{1}{T_0^6} \right]$$

$$T \propto t^{-1/6} \quad \text{and} \quad T_e \propto t^{-1/12}$$

- Photon Cooling Era:  $L_\gamma \gg L_\nu$

$$\frac{dT}{dt} = -\frac{N}{S}T^{1+\alpha} \Rightarrow t - t_0 = A \left[ \frac{1}{T^\alpha} - \frac{1}{T_0^\alpha} \right]$$

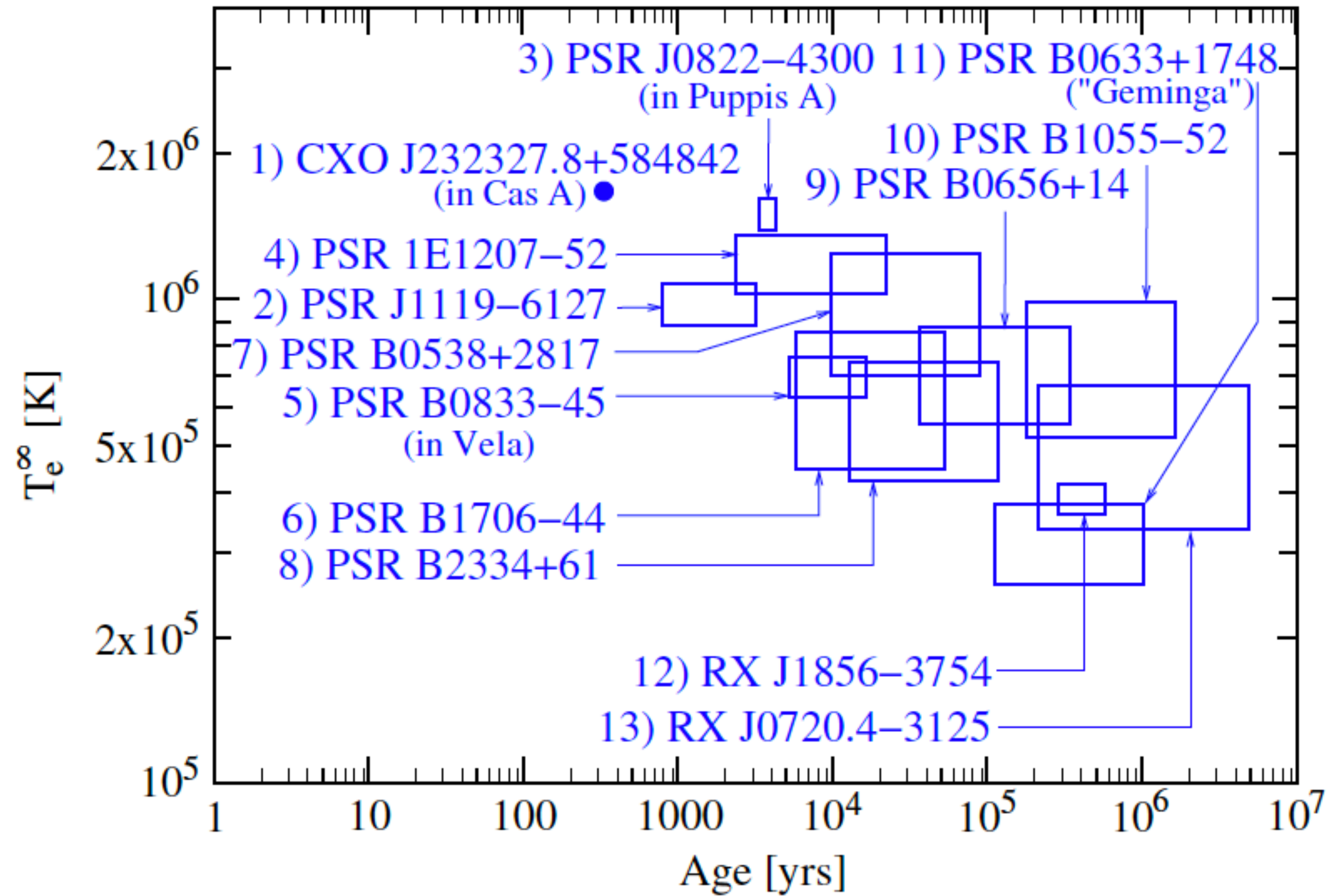
$$T \propto t^{-1/\alpha} \quad \text{and} \quad T_e \propto t^{-1/2\alpha}$$



# Neutron Star Cooling Data: Isolated Neutron Stars with Thermal Emission.

Ages are estimated either from pulsar spin-down or by association with a supernova. Age uncertainties are estimated.

Uncertainty in the temperature due to atmosphere and magnetic fields are estimated.

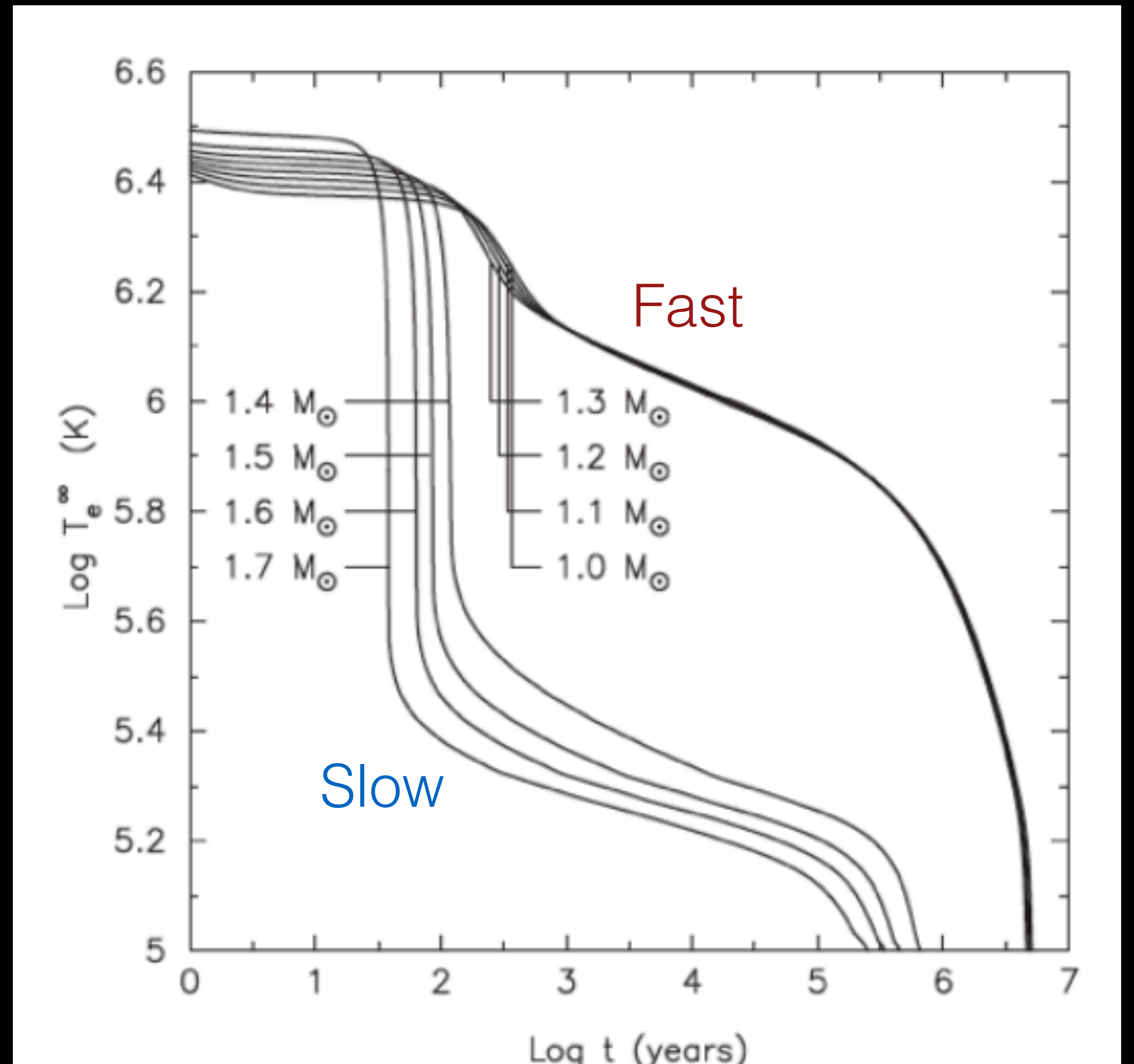


# Cooling Models with only Normal Neutrons is Inadequate

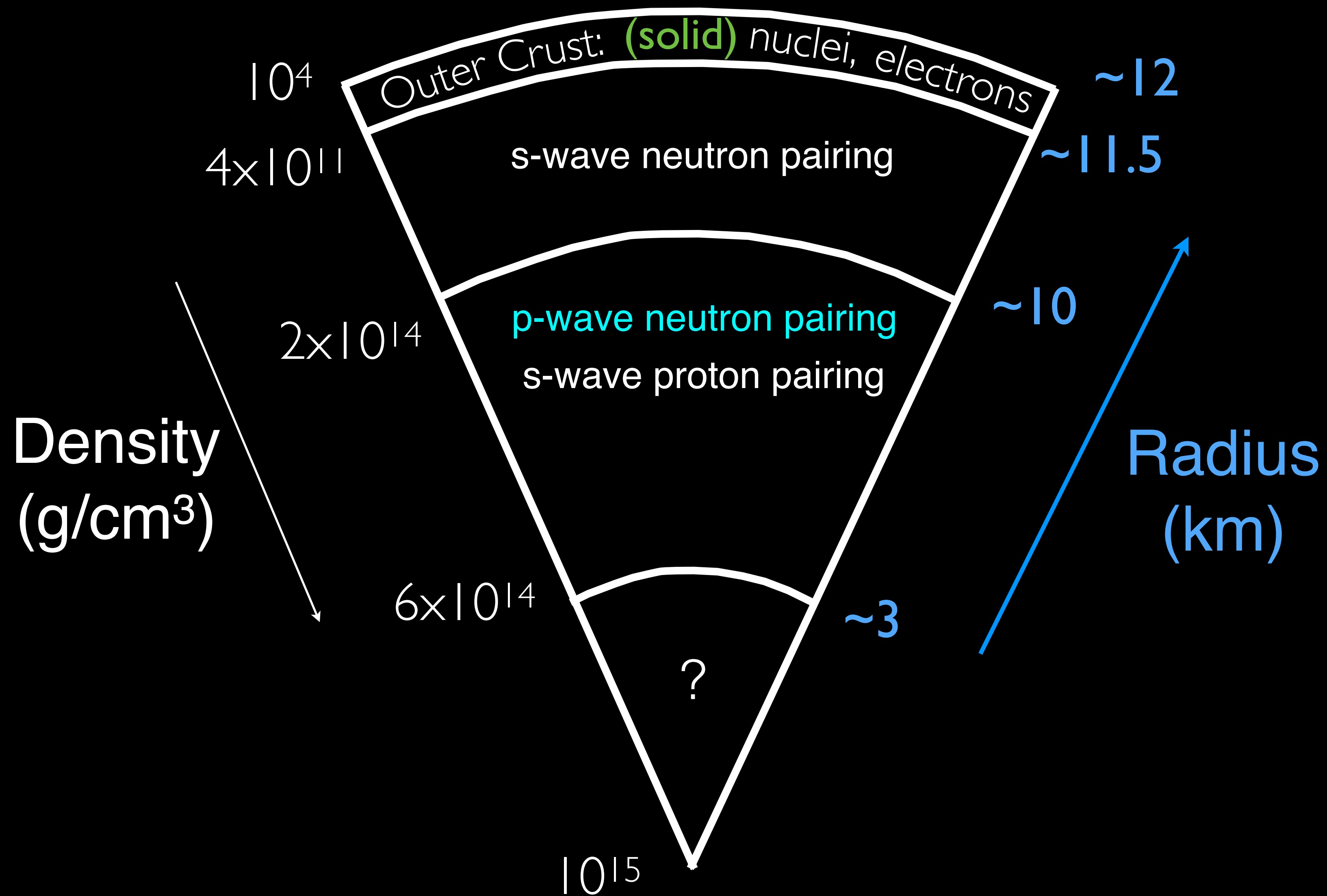
Even a small fraction of the normal core with DURCA can lead to very rapid cooling.

This is incompatible with the observed trends in the neutron stars population.

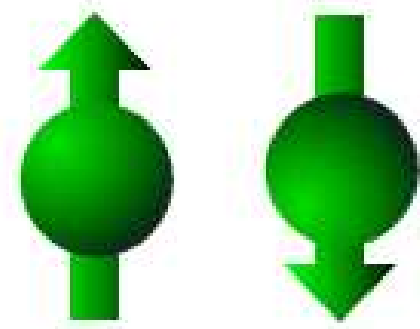
There is a need for an intermediate process between DURCA and MURCA.



# Phases of Cold Dense Matter in Neutron Stars

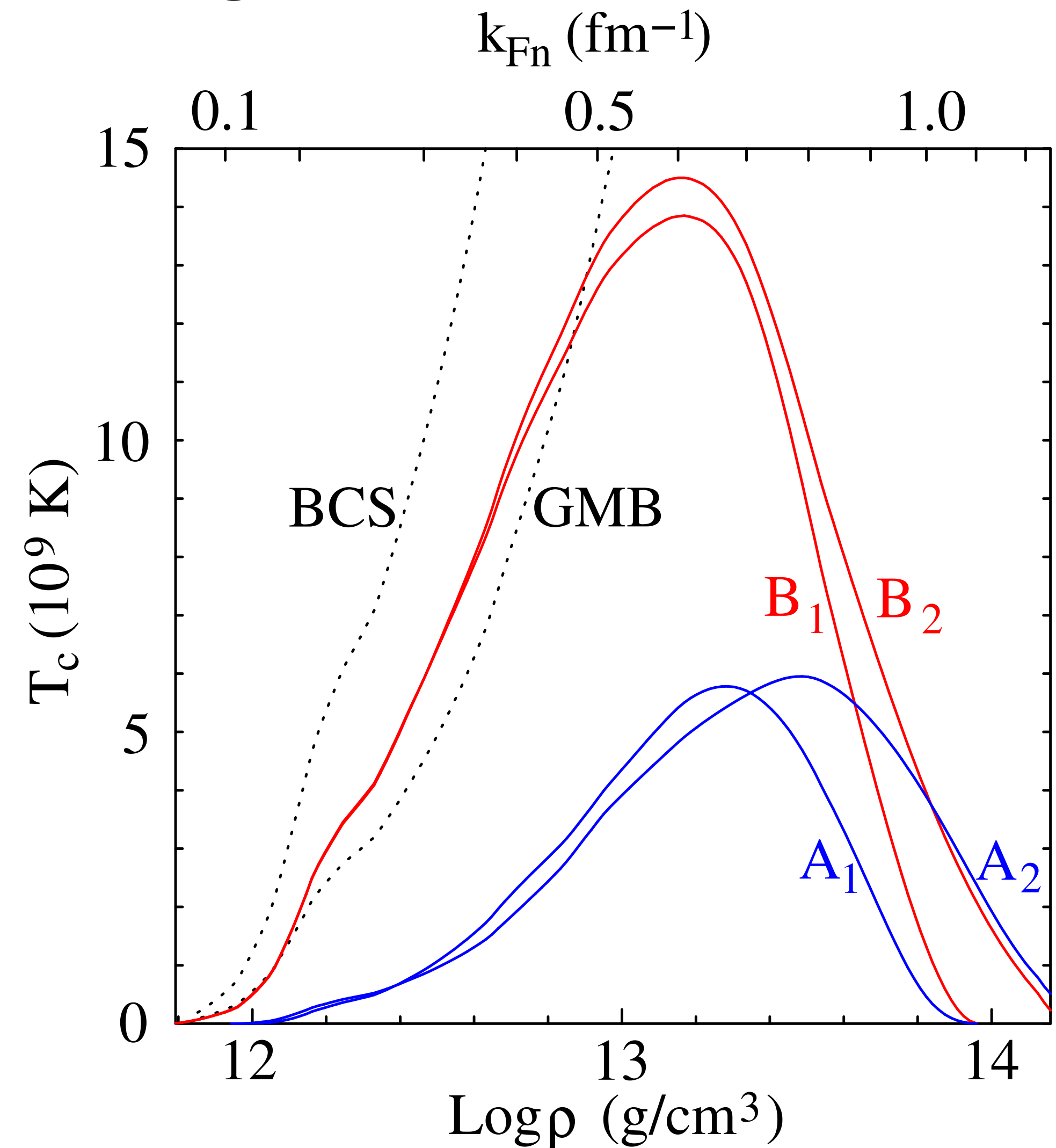


# S-wave pairing



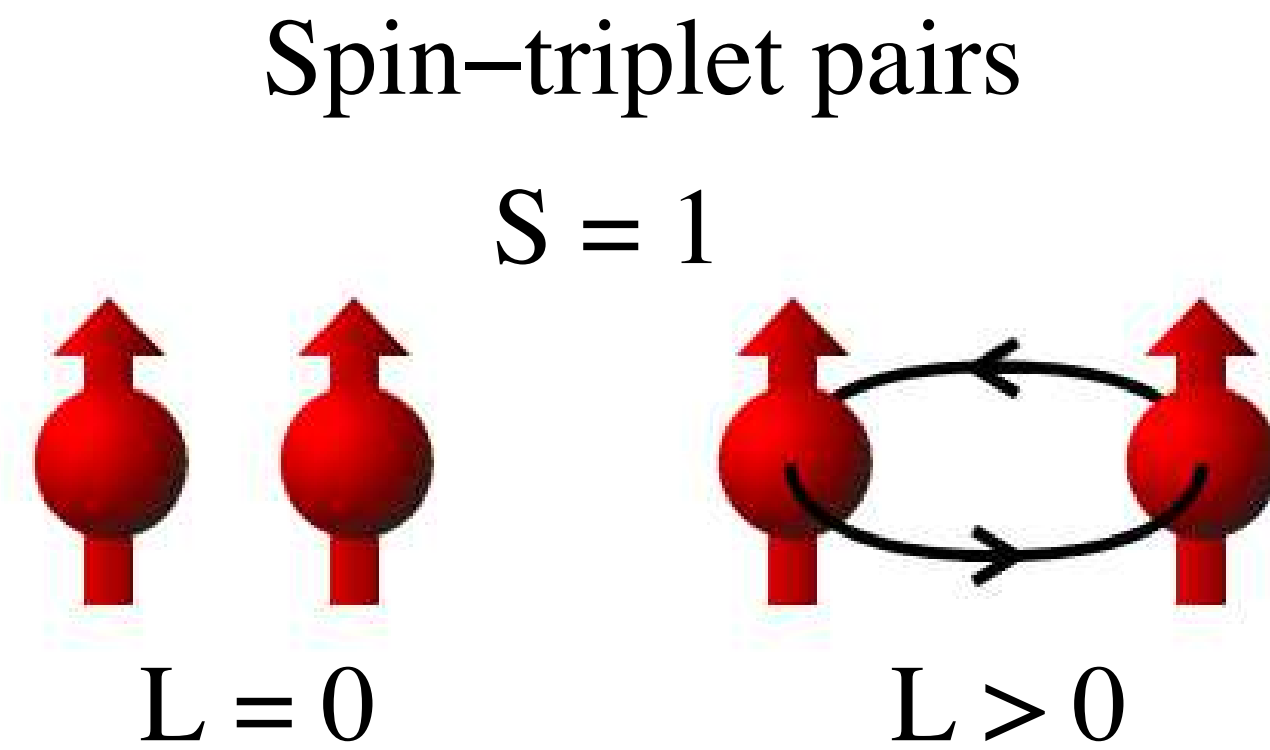
$L = 0$

- The nucleon-nucleon interaction is attractive at low energy.
- Perturbation theory fails, but Quantum Monte Carlo and lattice methods may be reliable.
- Best estimates for the critical temperature suggest that a large fraction of neutrons in the crust will be superfluid in the crust.



Cold atom experiments help validate many-body theory of strong short-range interactions.

# P-wave Triplet Pairing

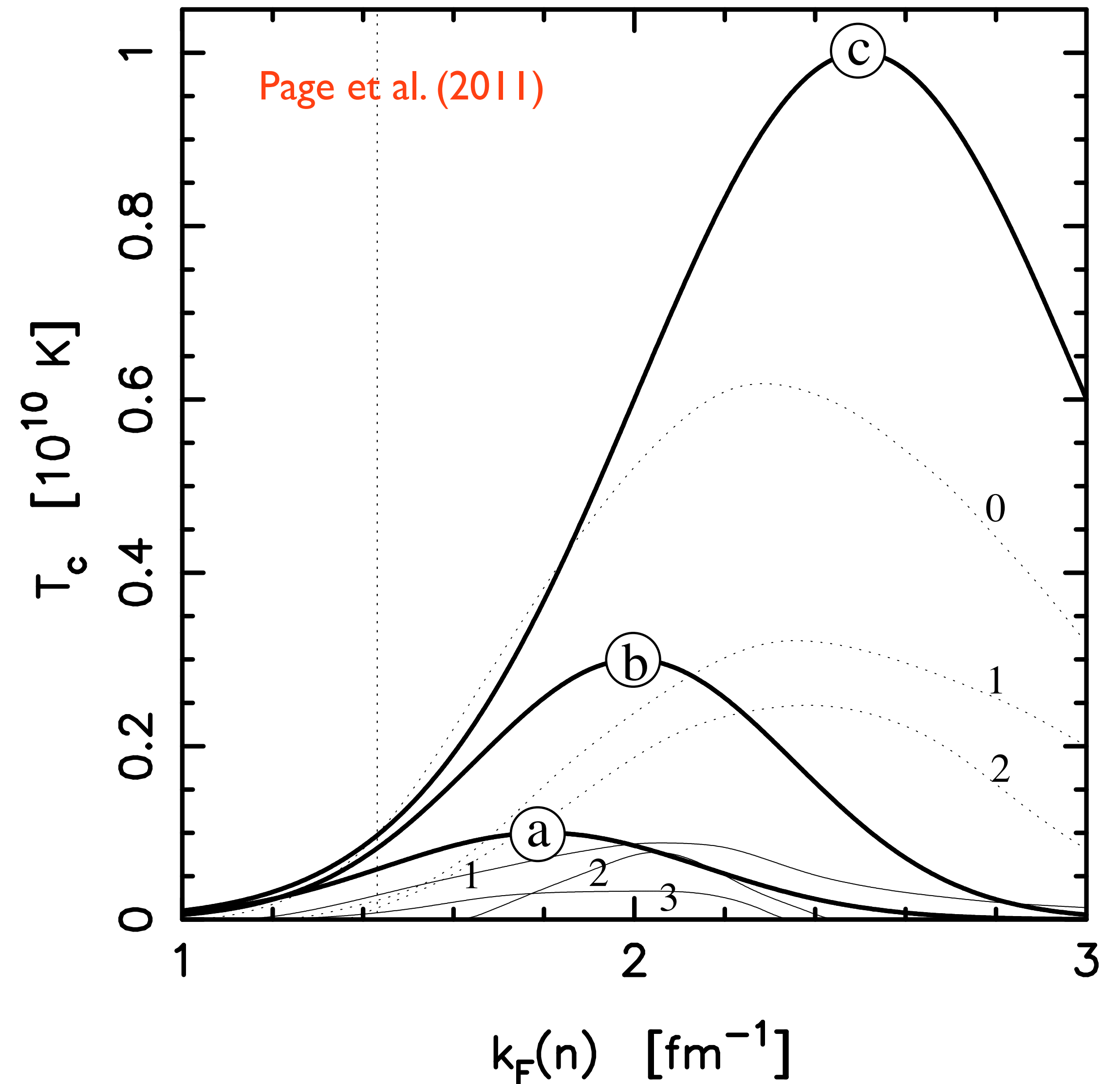


S-wave interaction is repulsive at high density.

Attraction is in spin-1 channel due to P-wave interaction.

Neutron pairing in the core has a large effect on neutron star cooling. Magnitude of the pairing gap and critical temperature is very uncertain .. large many-body corrections.

Neutron  ${}^3P_2 - {}^3F_2$





# Neutrino Emission in Superfluid Neutron Matter

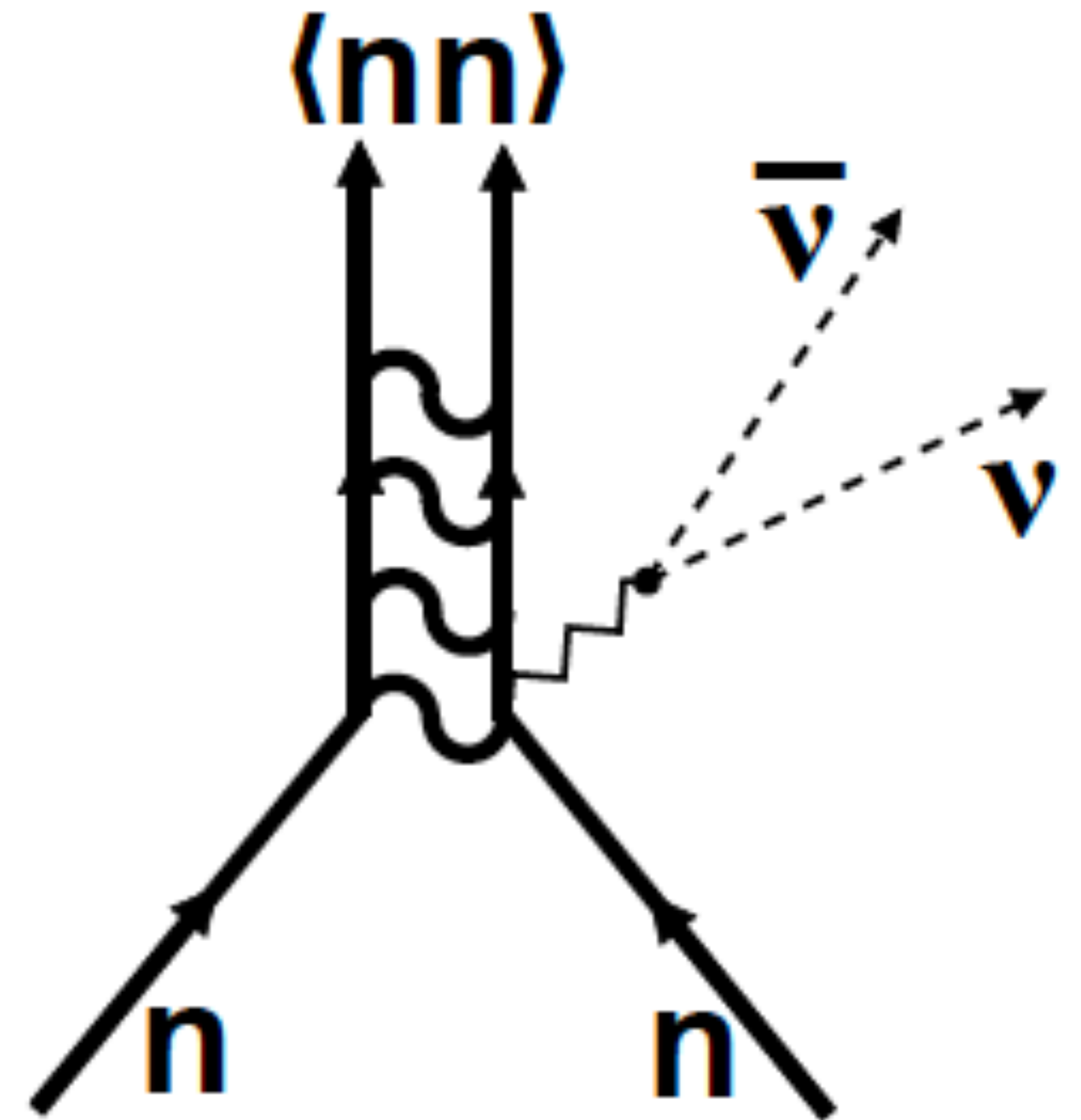
Near the critical temperature Cooper pairs form and dissociate due to thermal fluctuations.

Two neutron quasi-particles combine to form a Cooper pair, the binding energy is radiated as neutrino- anti-neutrino pairs.

This process is called the PBF process - Pair Breaking and Formation.

$$\epsilon_{\nu}^{nn} = \frac{G_F^2}{60 \pi^4} \int_0^{\infty} d\omega \omega^6 S_{\sigma}(\omega)$$

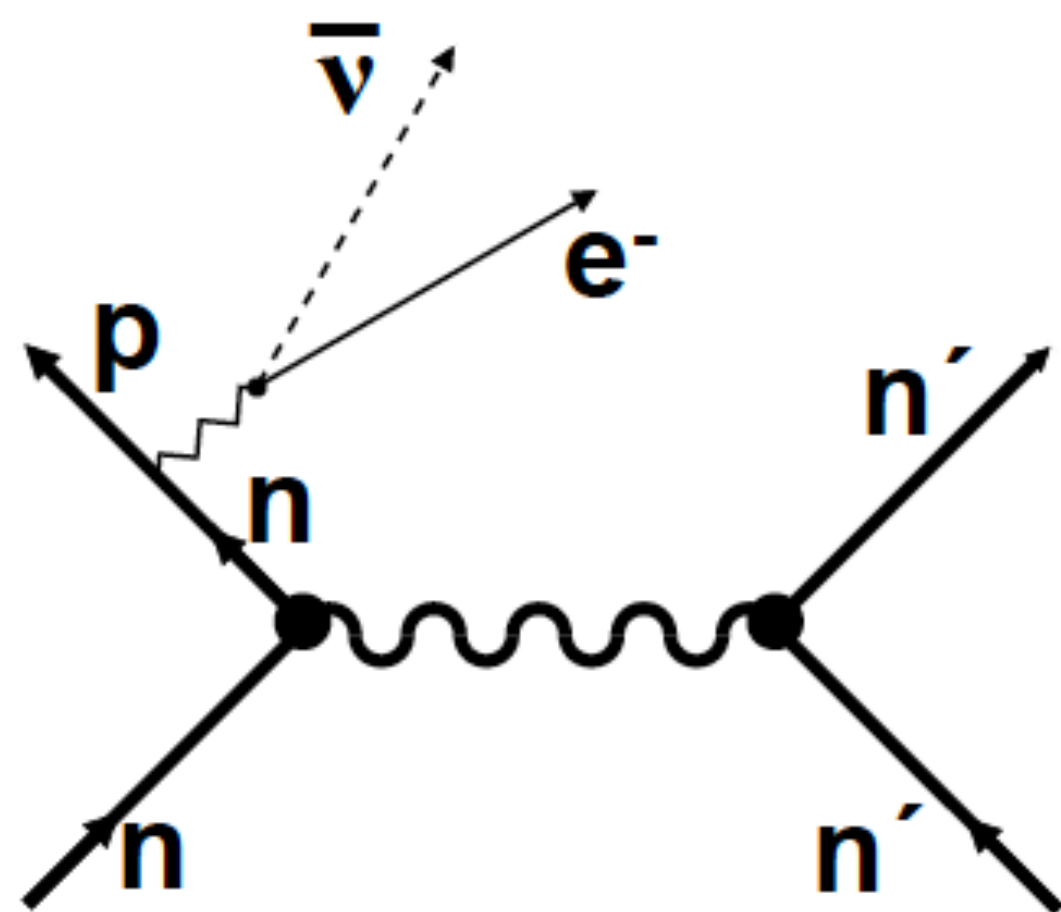
Spin-spin correlation function in superfluid neutron matter.



Cooper pair  
formation

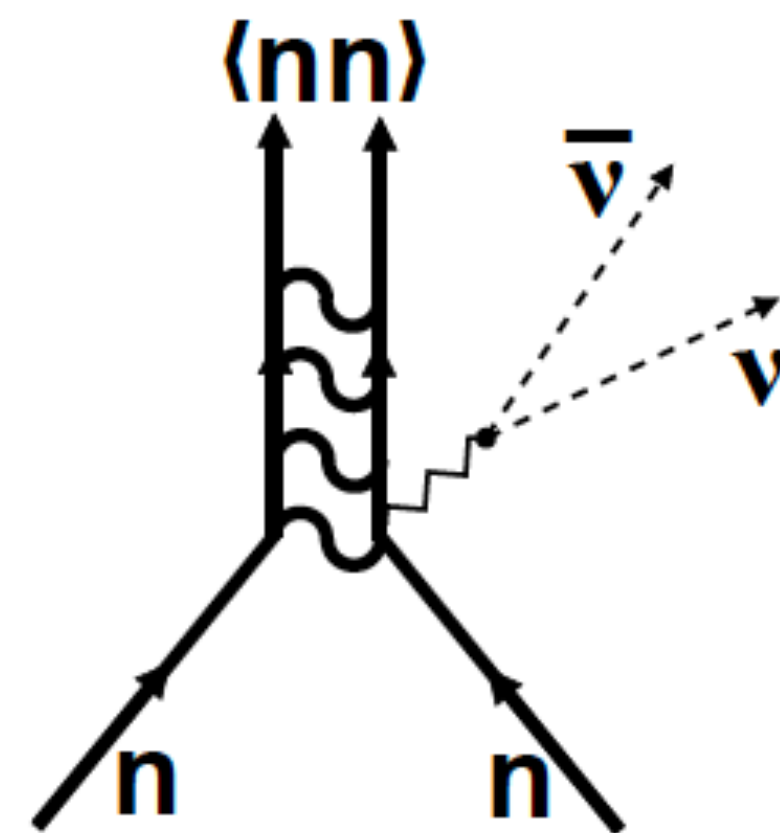
# Neutrino Emission in Superfluid Neutron Matter

**SLOW**



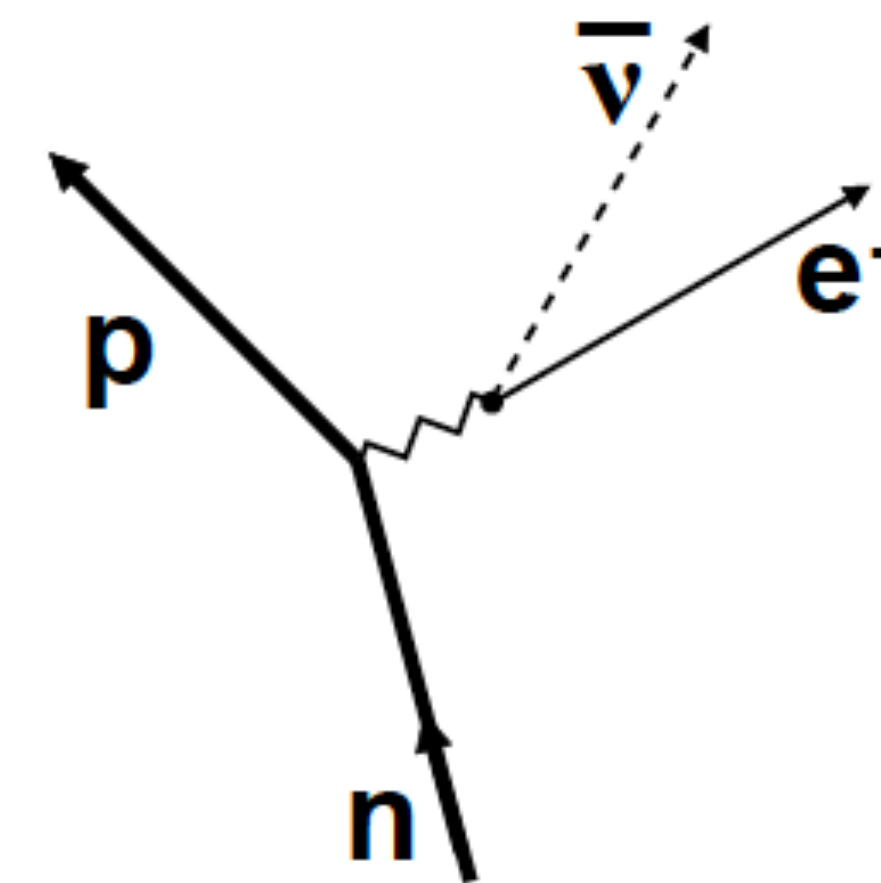
Modified Urca

**MEDIUM**



Cooper pair  
formation  
("PBF")

**FAST**



Direct Urca

# Neutrino Emission Rates in Normal Nuclear Matter

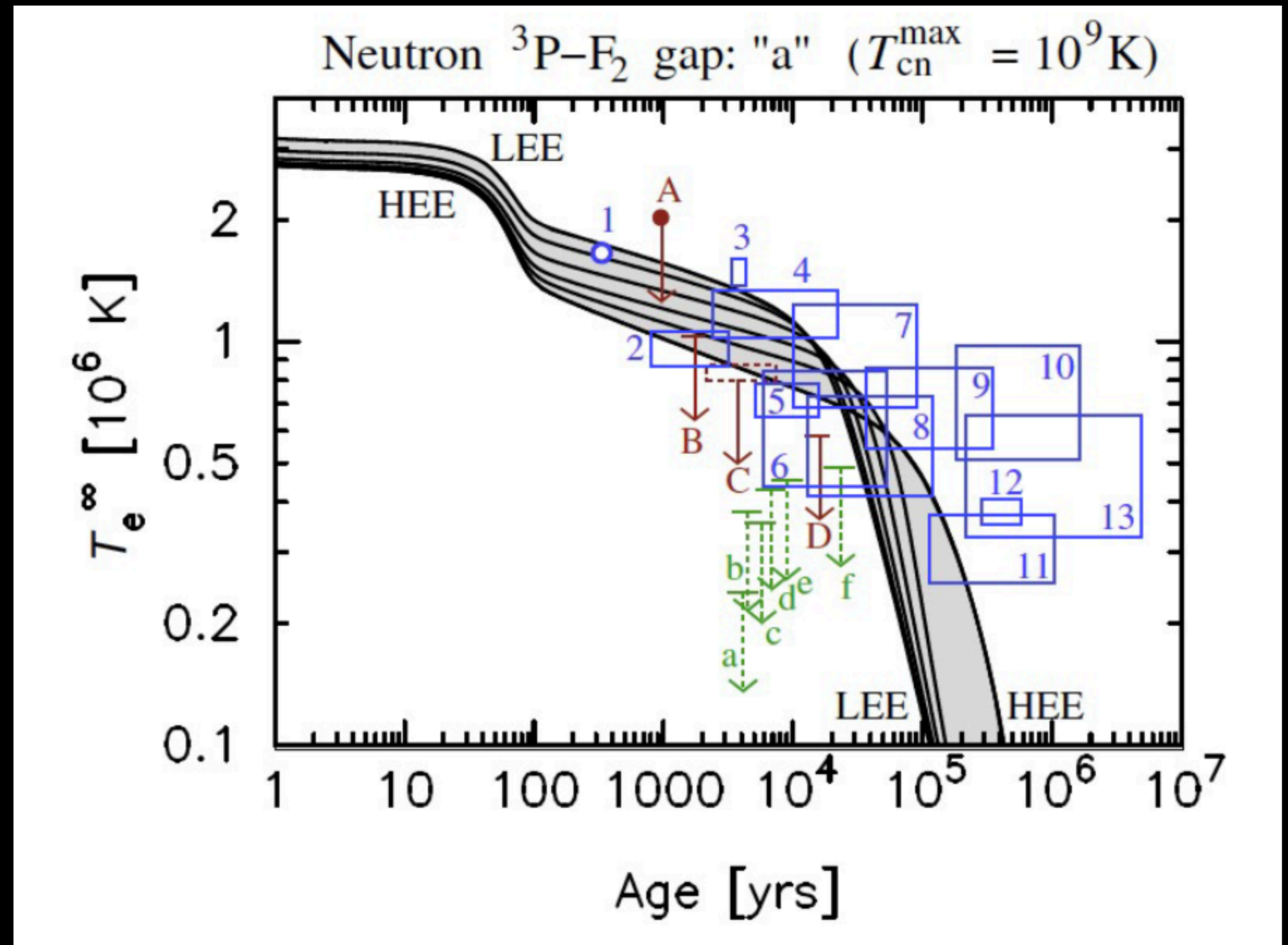
Name	Process	Emissivity ( $\text{erg cm}^{-3} \text{s}^{-1}$ )	
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Modified Urca cycle (proton branch)	$p + n \rightarrow p + p + e^- + \bar{\nu}_e$	$\sim 10^{21} R T_9^8$	Slow
	$p + p + e^- \rightarrow p + n + \nu_e$		
Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$	$\sim 10^{19} R T_9^8$	Slow
	$n + p \rightarrow n + p + \nu + \bar{\nu}$		
	$p + p \rightarrow p + p + \nu + \bar{\nu}$		
Direct Urca cycle	$n \rightarrow p + e^- + \bar{\nu}_e$	$\sim 10^{27} R T_9^6$	Fast
	$p + e^- \rightarrow n + \nu_e$		
Cooper pair formations	$n + n \rightarrow [nn] + \nu + \bar{\nu}$	$\sim 5 \times 10^{21} R T_9^7$	Medium
	$p + p \rightarrow [pp] + \nu + \bar{\nu}$	$\sim 5 \times 10^{19} R T_9^7$	

# A Finely Tuned Model with Neutron Pairing

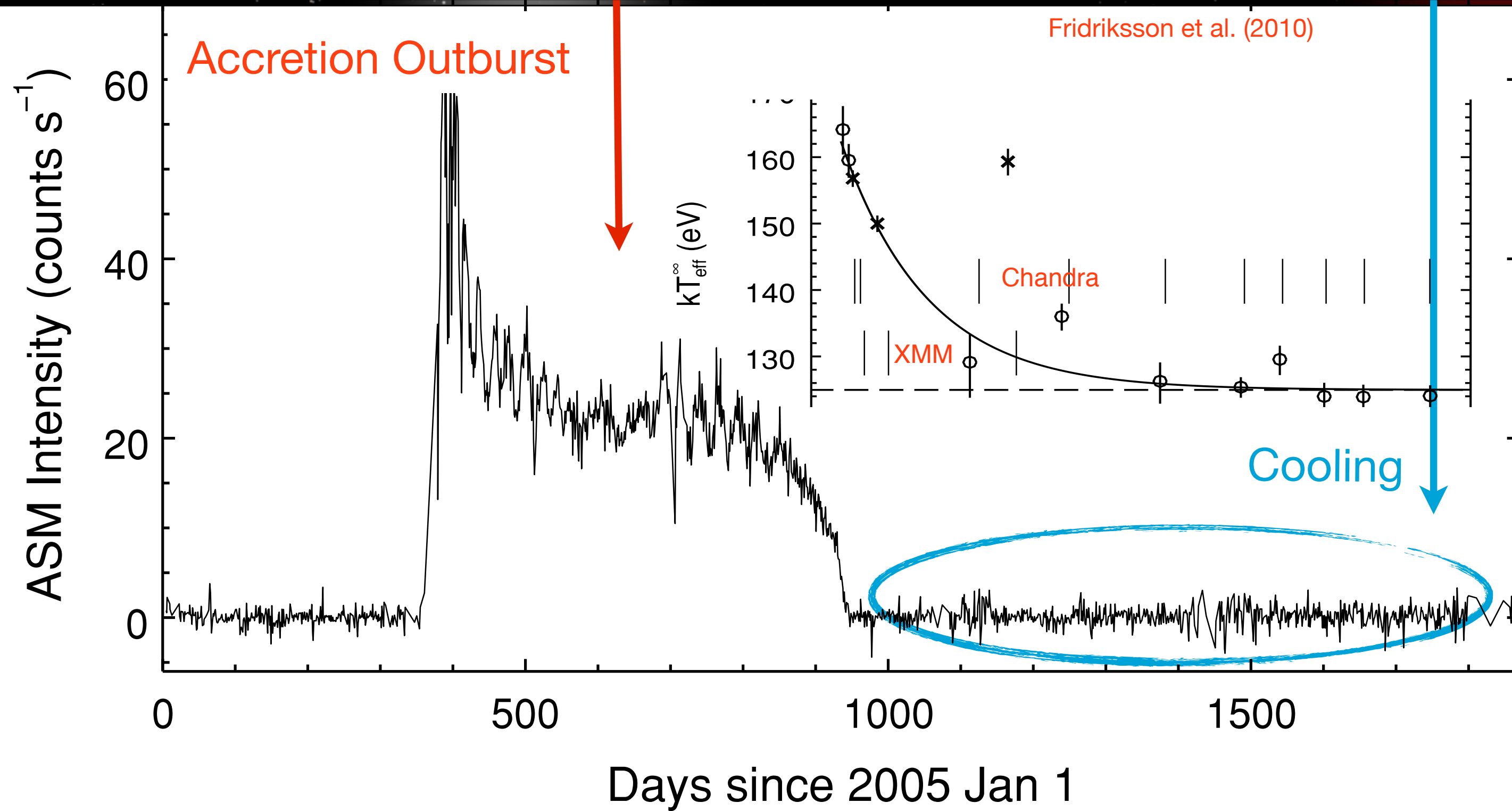
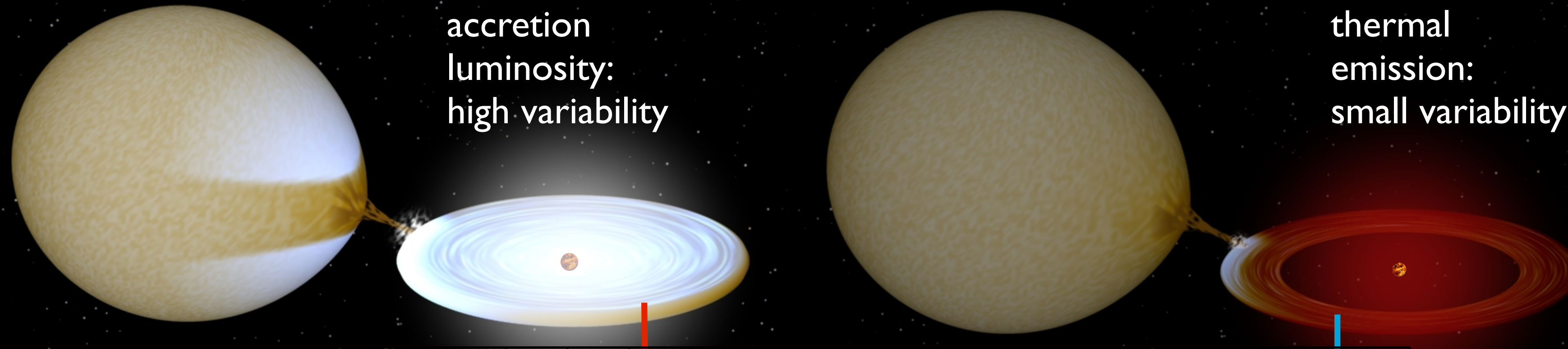
Minimal Cooling Paradigm D. Page et al. (2011)

Neutron stars in their mid-life are roughly compatible with a finely tuned model of neutrinos cooling with  ${}^3P_2$  neutron superfluidity.

The situation (in my opinion) suggests that there is something missing.

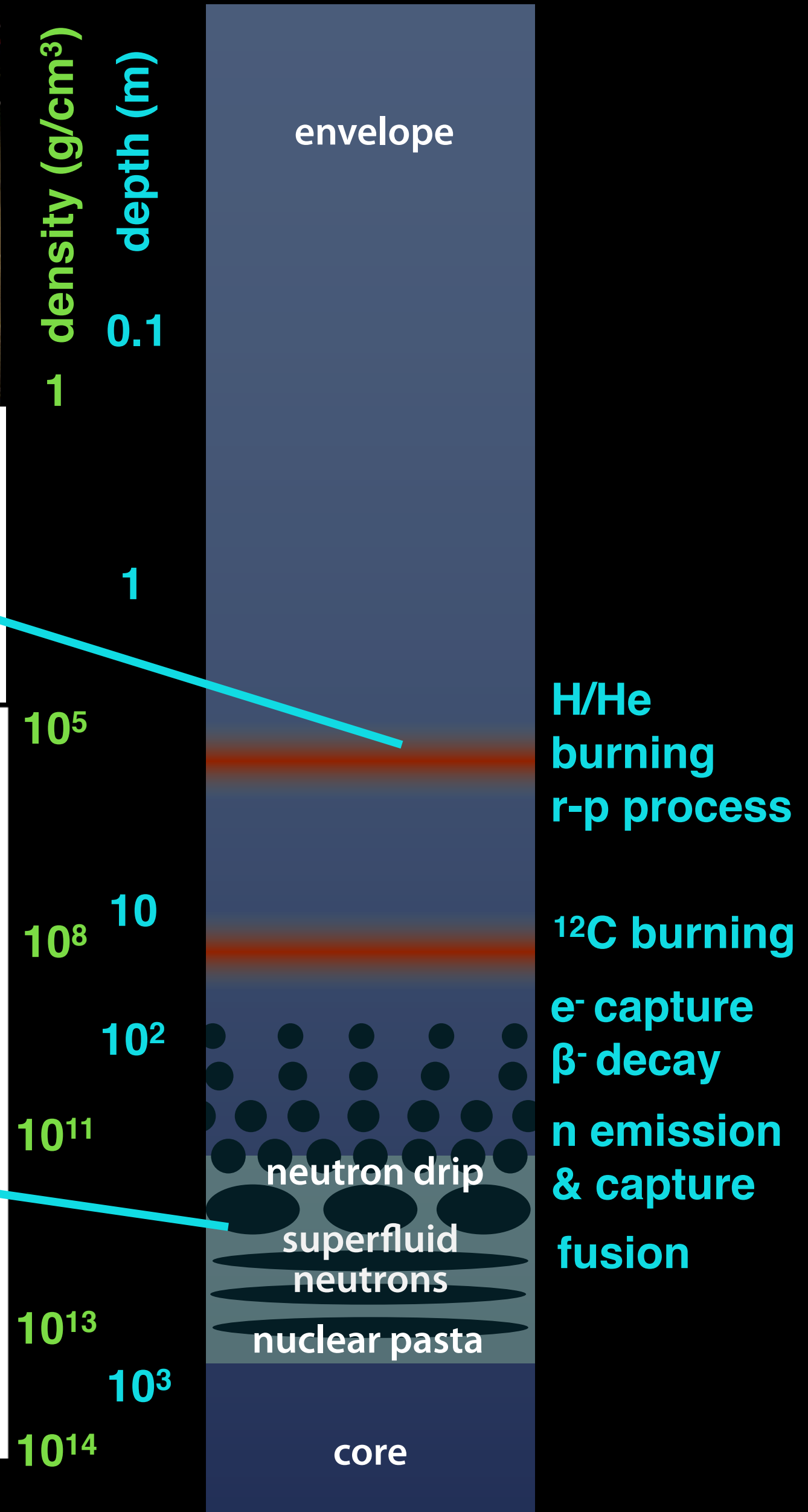
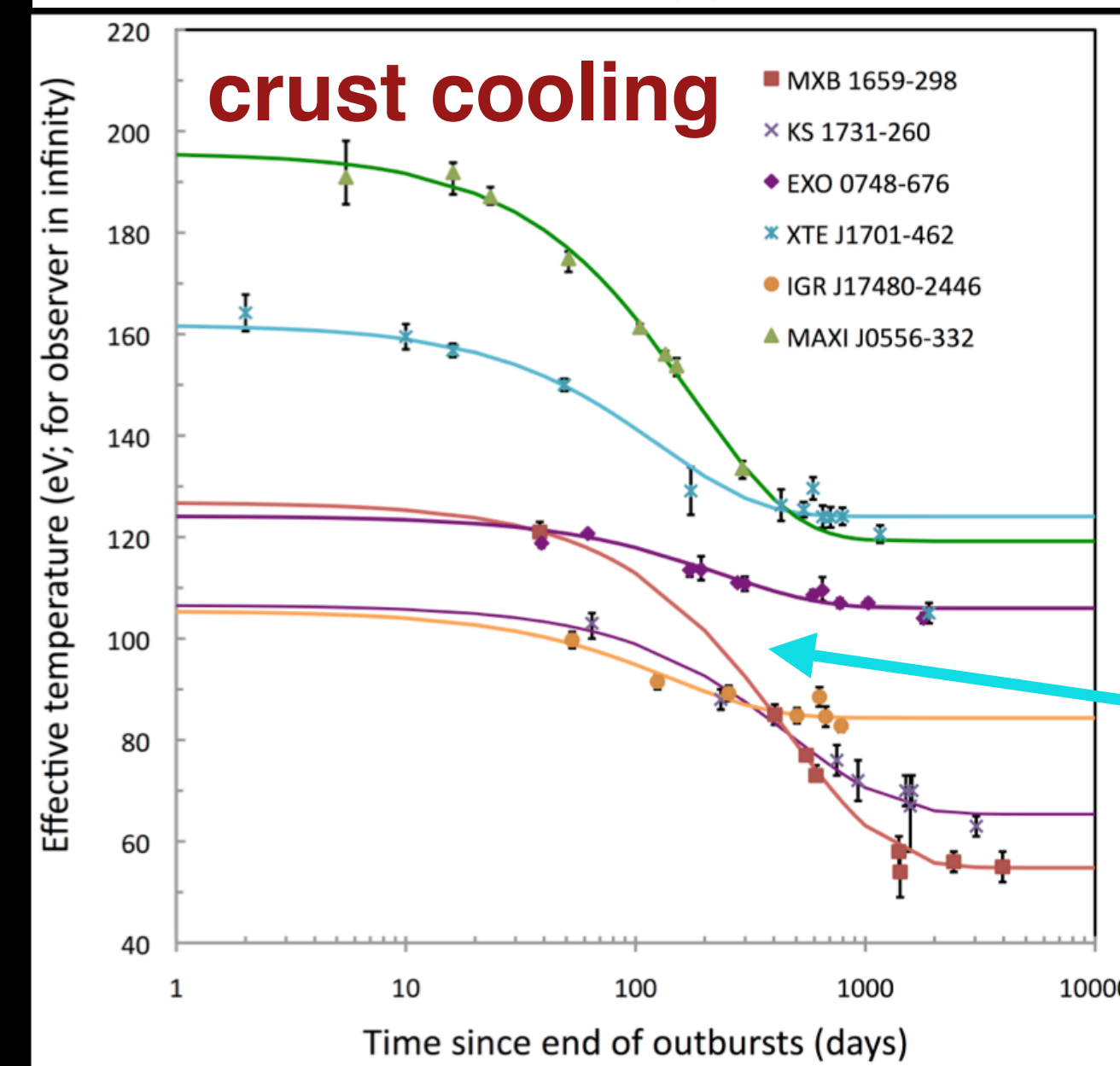
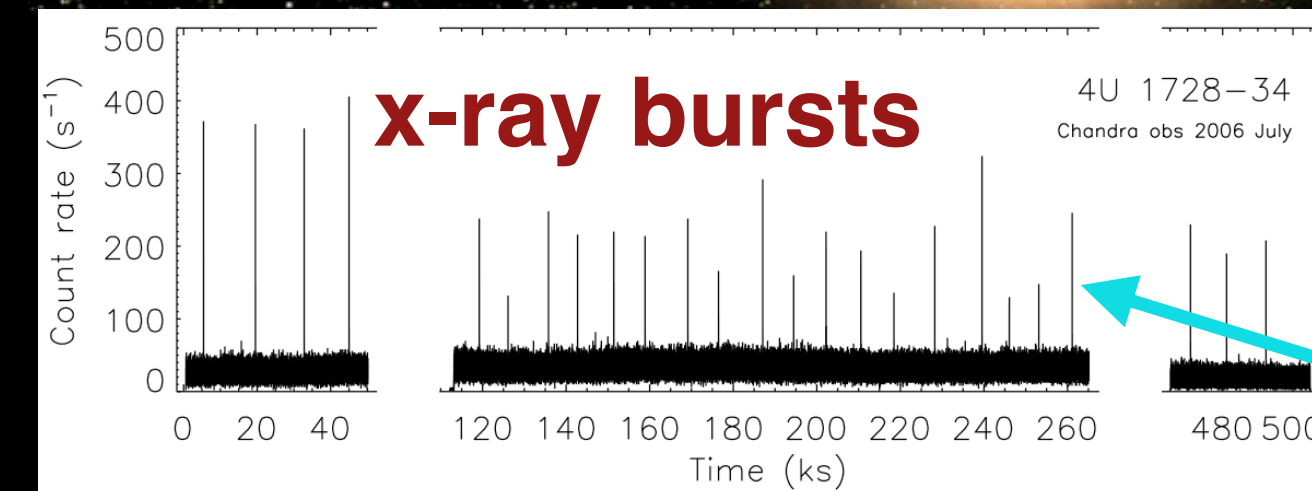
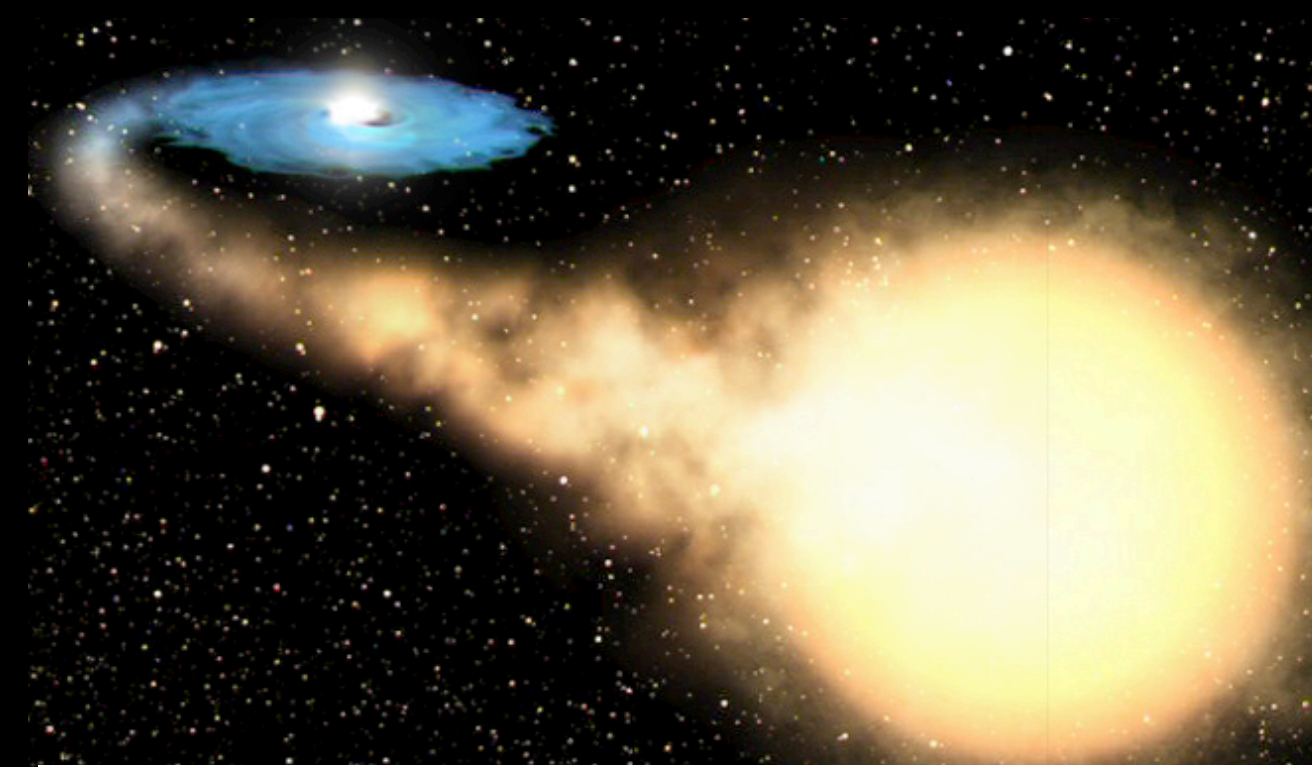


# Transiently Accreting Neutron Stars



# Physical Processes in Accreting Neutron Stars

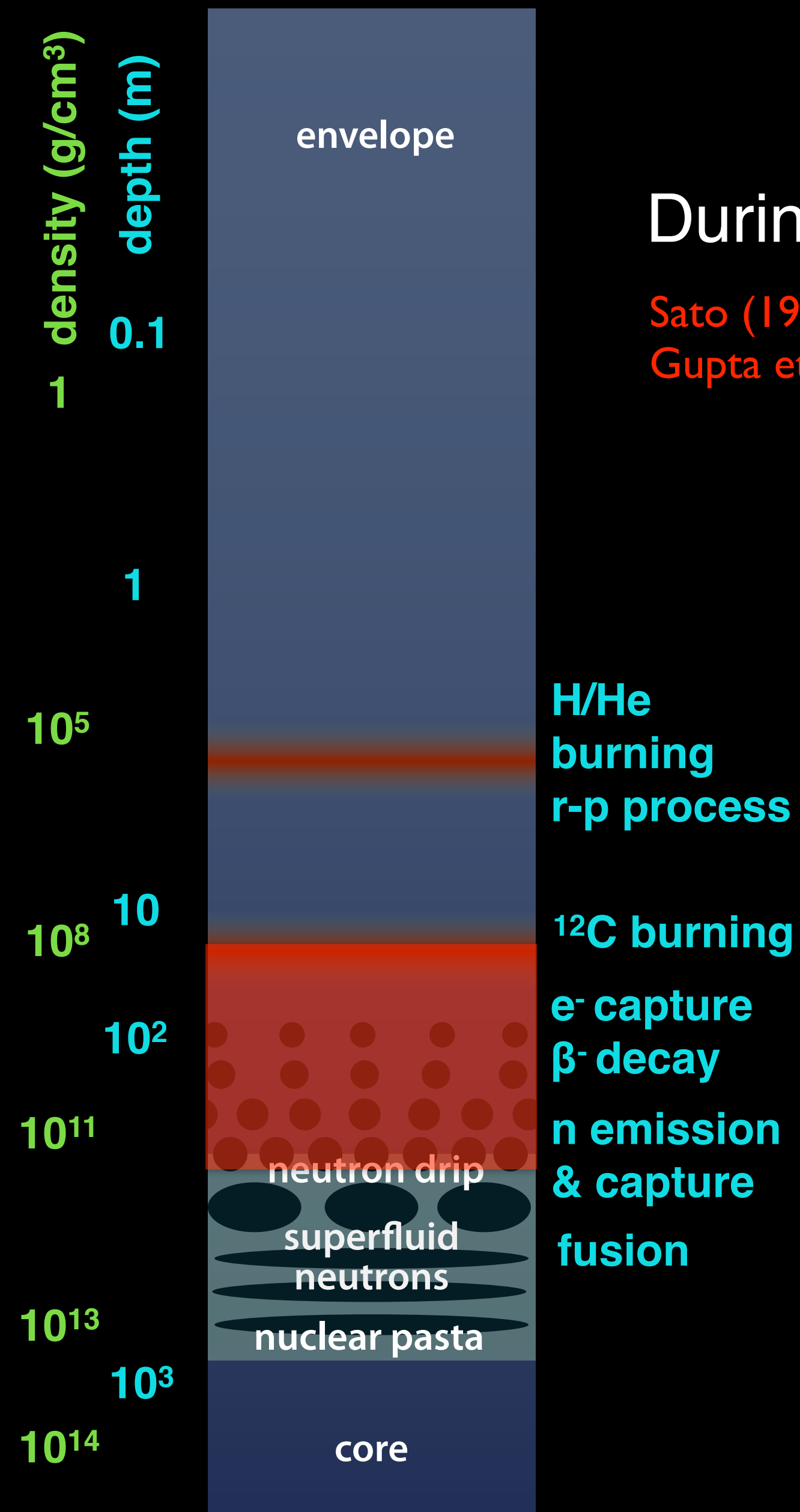
- Accreting neutron stars host phenomena that uniquely probe the physics of its ultra dense interior.
- It is a data driven field.
- Interpreting this data requires a coordinated effort that combines theory, experiment and observations. JINA-CEE has played a key role.



# Deep Crustal Heating

During accretion nuclear reactions release:  $\sim 2-4$  MeV / nucleon

Sato (1974), Haensel & Zdunik (1990), Brown, Bildsten Rutledge (1998)  
Gupta et al (2007,2011).



Warms up old neutron stars

# Cooling Post Accretion

All known Quasi-persistent sources show cooling after accretion

- After a period of intense accretion the neutron star surface cools on a time scale of  $\sim 1000$  days.
- This relaxation was first discovered in 2001 and 6 sources have been studied to date.
- Expected rate of detecting new sources  $\sim 1/\text{year}$ .

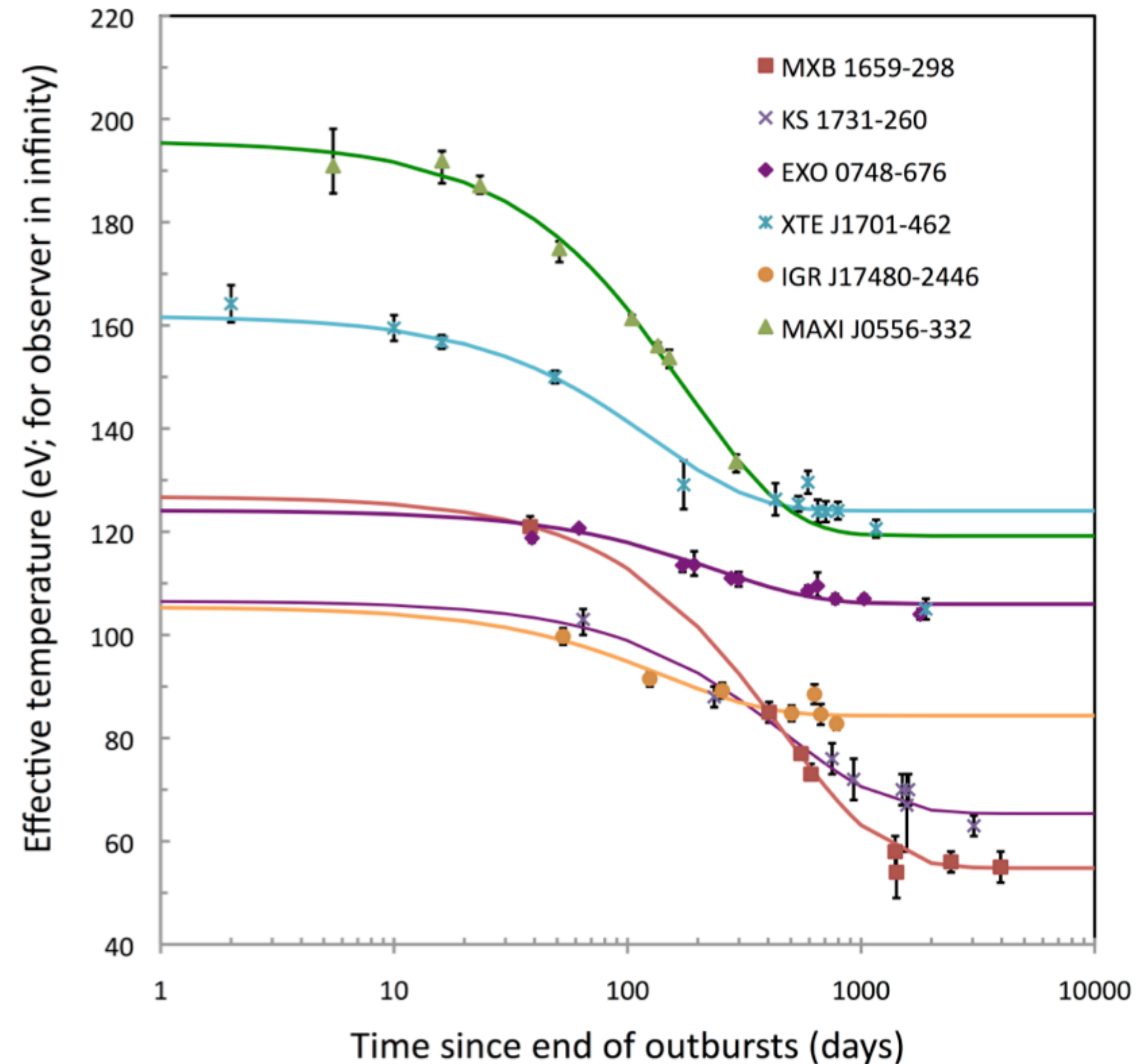


Figure from Rudy Wijnands (2013)

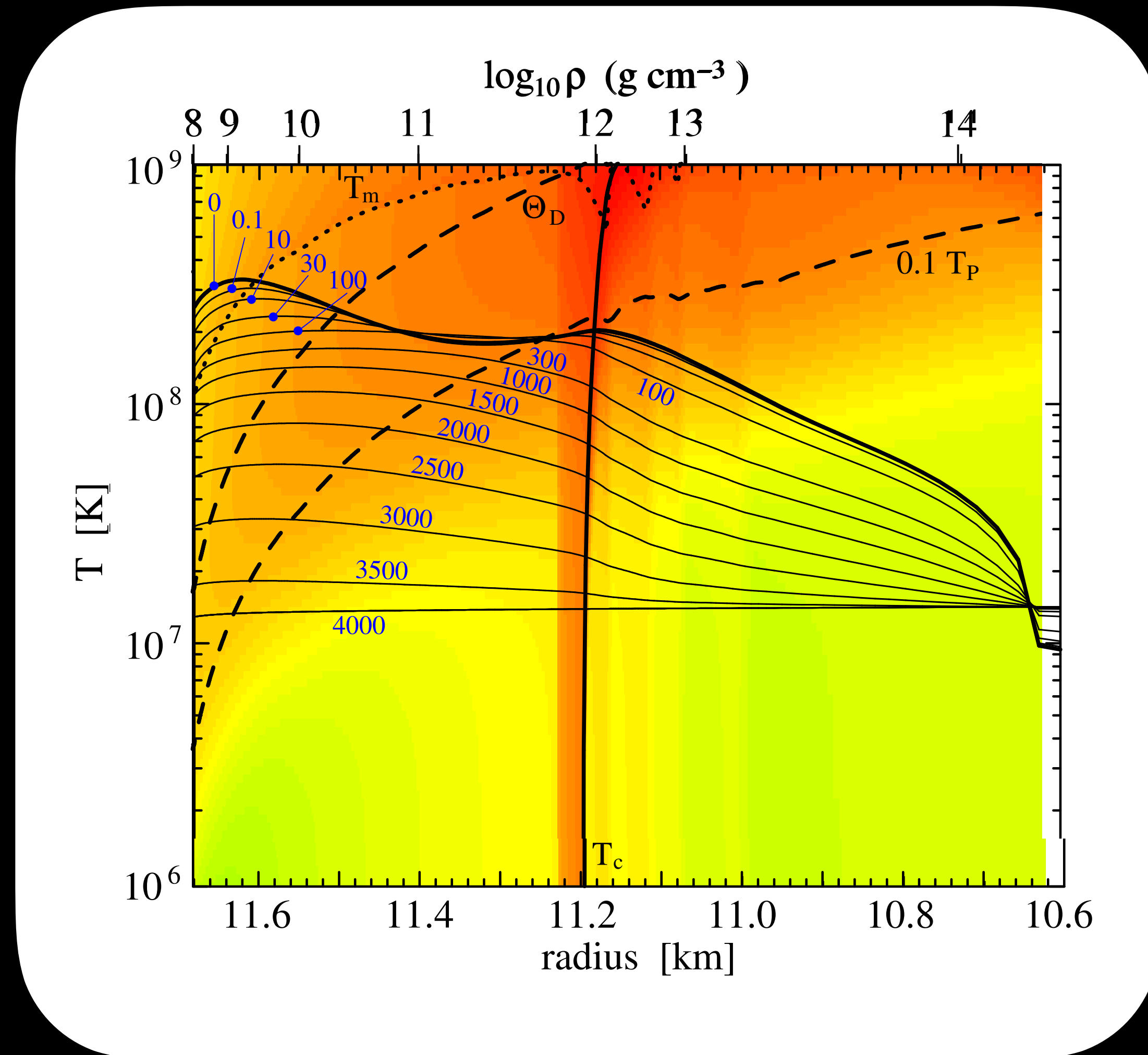


# Thermal Evolution of the Crust

Temperature profile in the crust depends on the duration of the accretion phase.

When accretion ends heat flows into the core and is radiated away as neutrinos.

Timescale for cooling is set by the heat diffusion time.



# Connecting to Crust Microphysics

Crustal Specific Heat

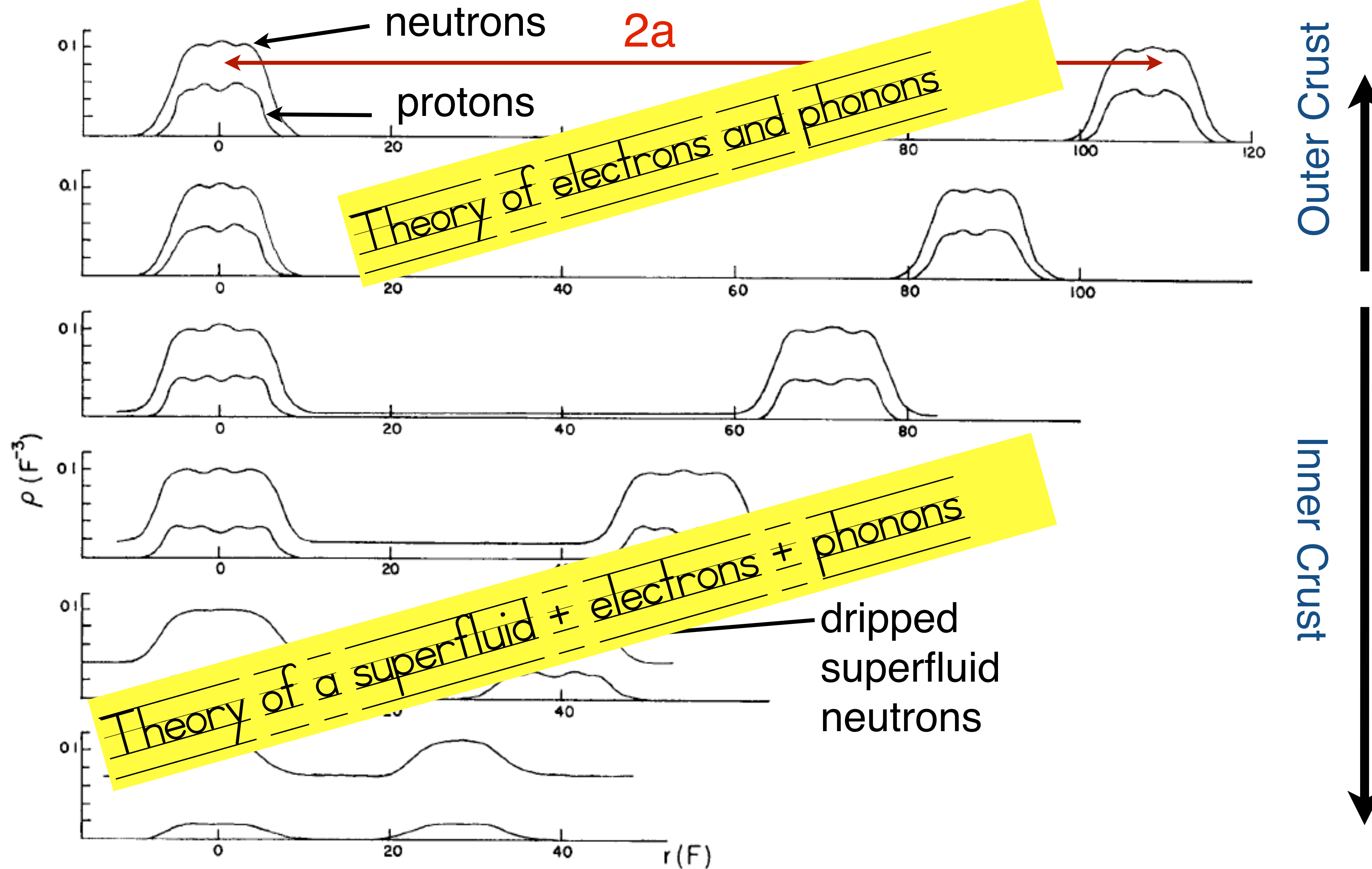
Crust Thickness

$$\tau_C \simeq \frac{C_V}{\kappa} \Delta R^2$$

Thermal Conductivity

- Observed timescales are short.
- Requires small specific heat and large thermal conductivity.
- Favors a solid (with small impurity fraction) and superfluid inner crust.

# Low Energy Excitations in the Crust



Baym Pethick & Sutherland (1971)

Negele & Vautherin (1973)

# Phonons in the Inner Crust



Proton (clusters) move collectively on lattice sites.  
Displacement is a good collective coordinate.

Neutron superfluid: Goldstone excitation is the  
fluctuation of the phase of the condensate.

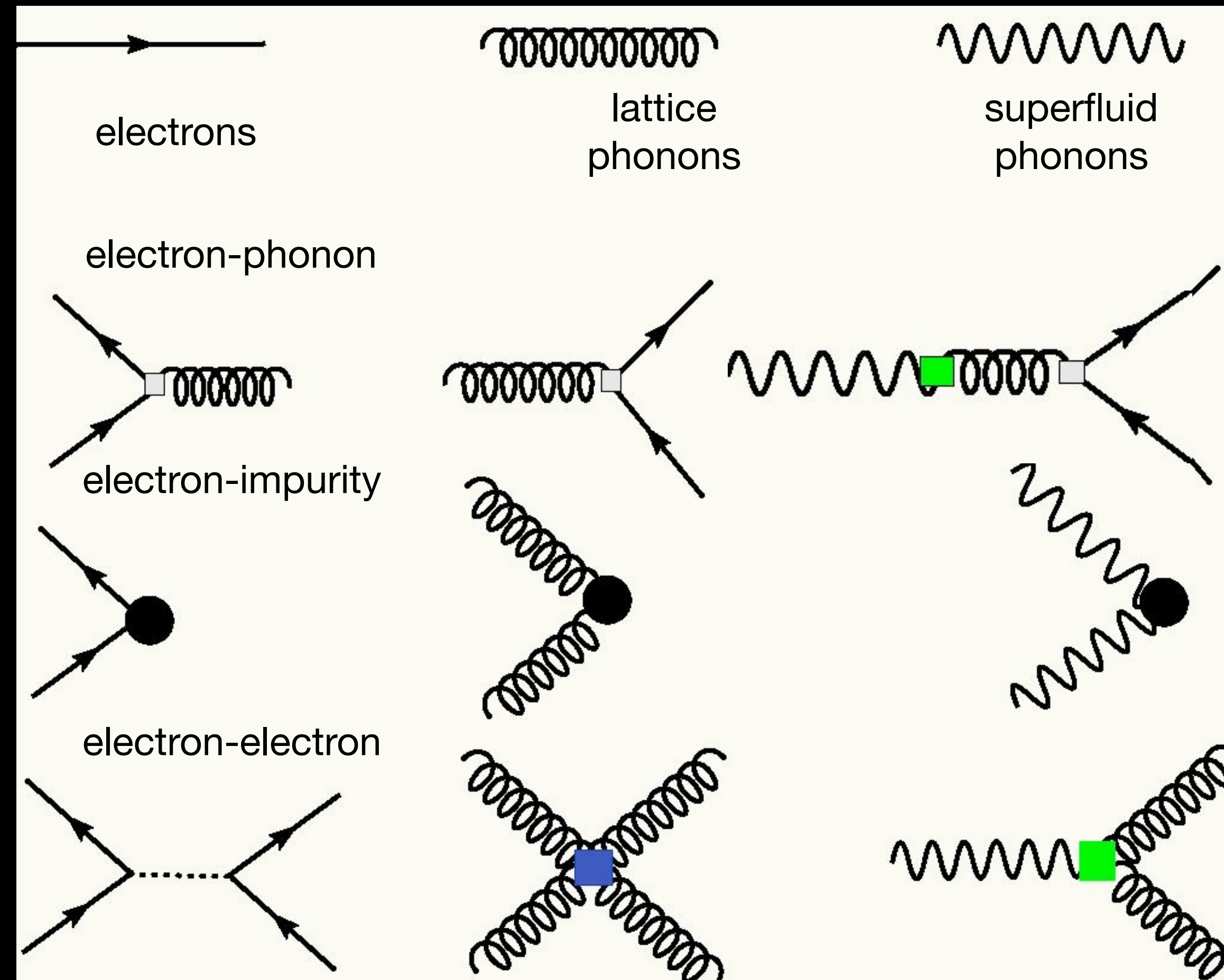
Vector Field:  $\xi_i(r, t)$   
Scalar Field:  $\phi(r, t)$

# Excitations and Interactions in the Inner Crust

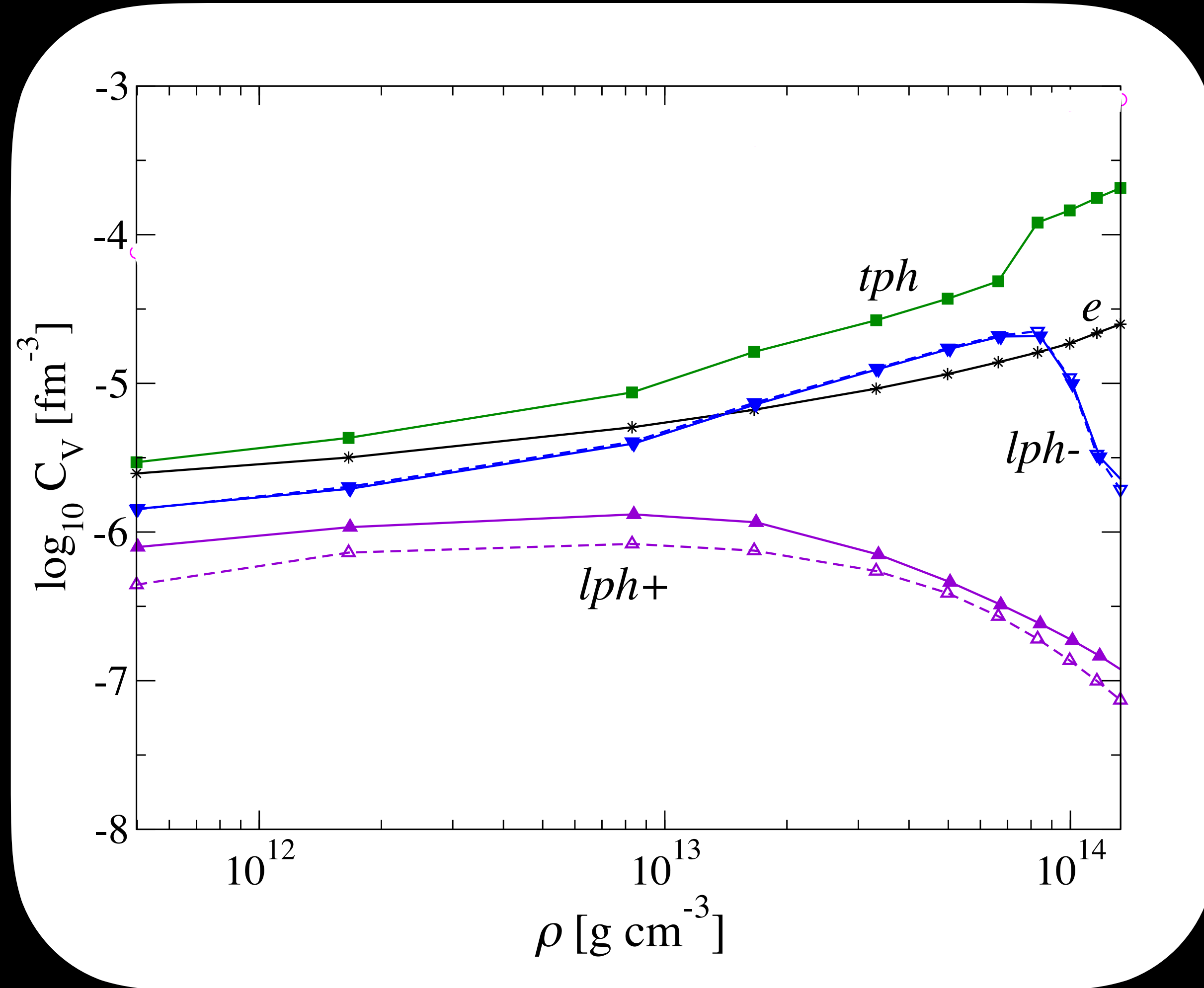
Electrons and 2 longitudinal and 2 transverse phonons are the relevant excitations.

Thermal and transport properties of the solid and superfluid crust can be calculated using an effective field theory.

Mixing between phonons leads to strong Landau damping. Phonon conduction is highly suppressed.



# Crustal Specific Heat



Electrons:

$$C_V^e \simeq \mu_e^2 T$$

Phonons:

$$C_V^i \simeq \frac{T^3}{v_i^3}$$

If neutrons were normal

$$C_V^n \simeq M k_{Fn} T$$

their contribution would overwhelm.

# Thermal Conduction

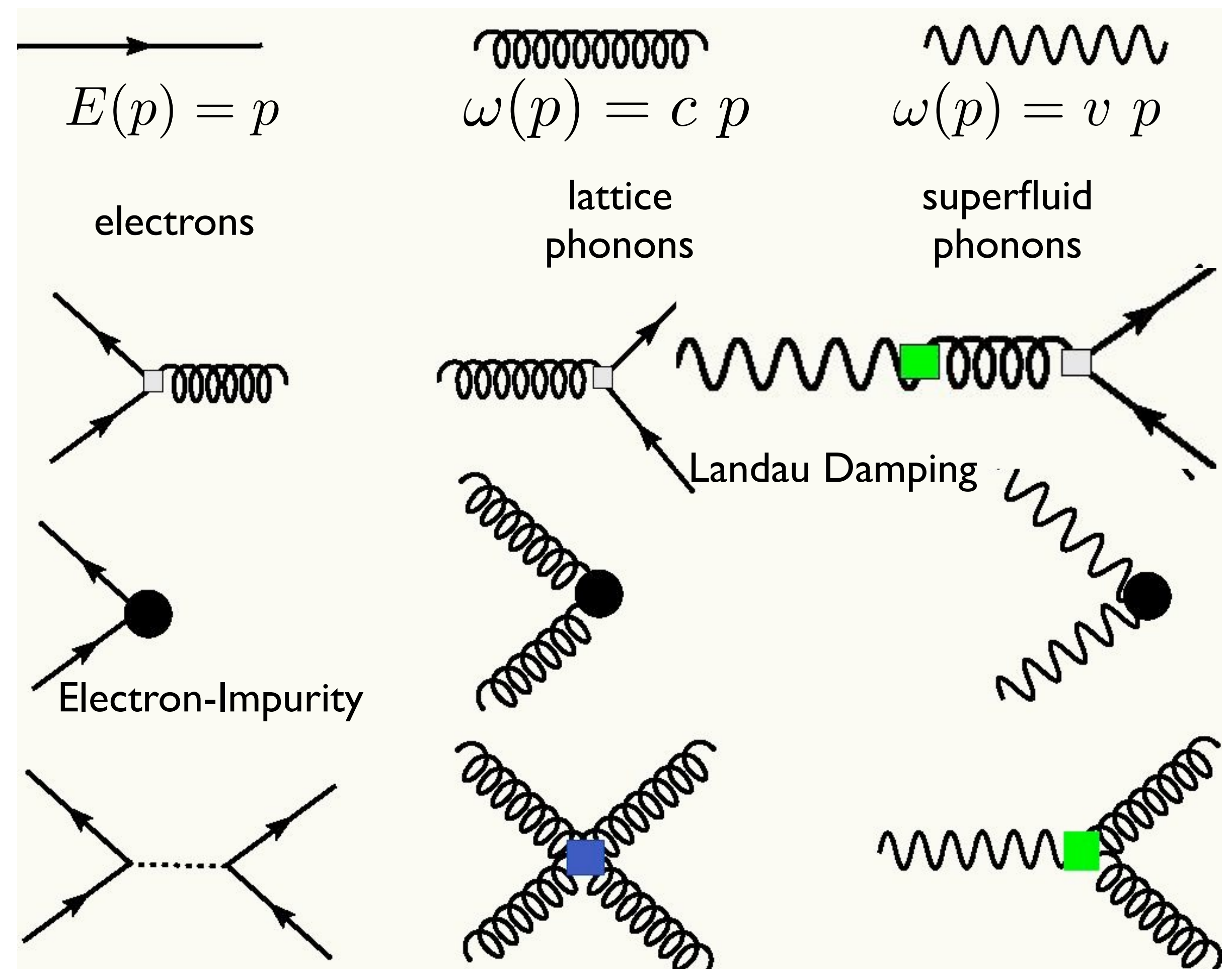
$$\kappa = \frac{1}{3} C_v \times v \times \lambda$$

- Dissipative processes:

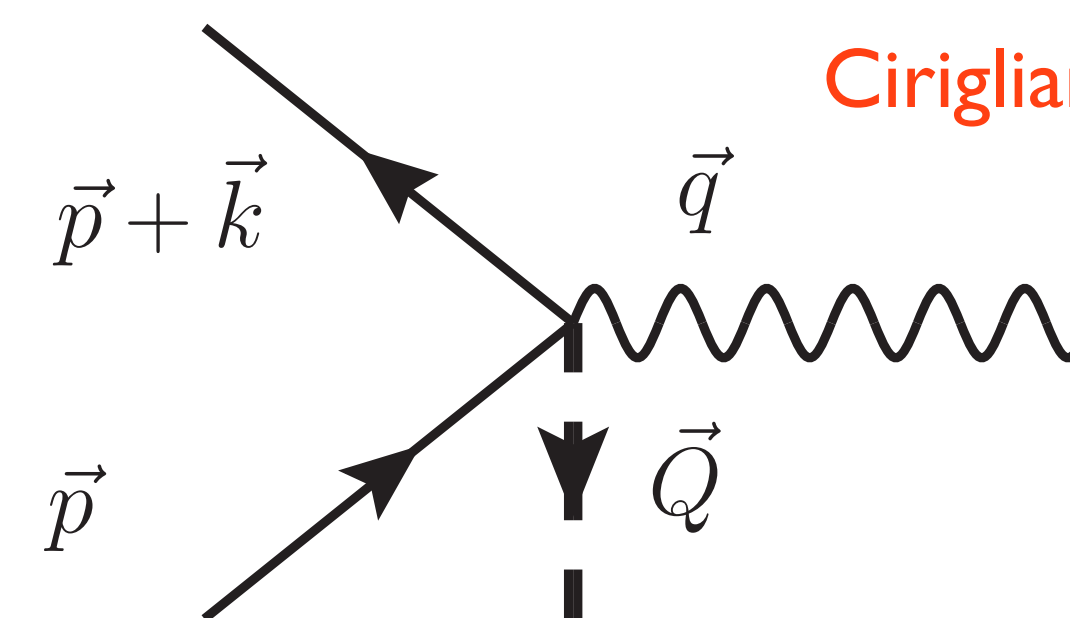
- Umklapp is important:

$$\frac{k_{\text{Fe}}}{q_{\text{D}}} = \left(\frac{Z}{2}\right)^{1/3} > 1$$

Electron Bragg scatters and emits a transverse phonon.



Cirigliano, Reddy & Sharma (2011)



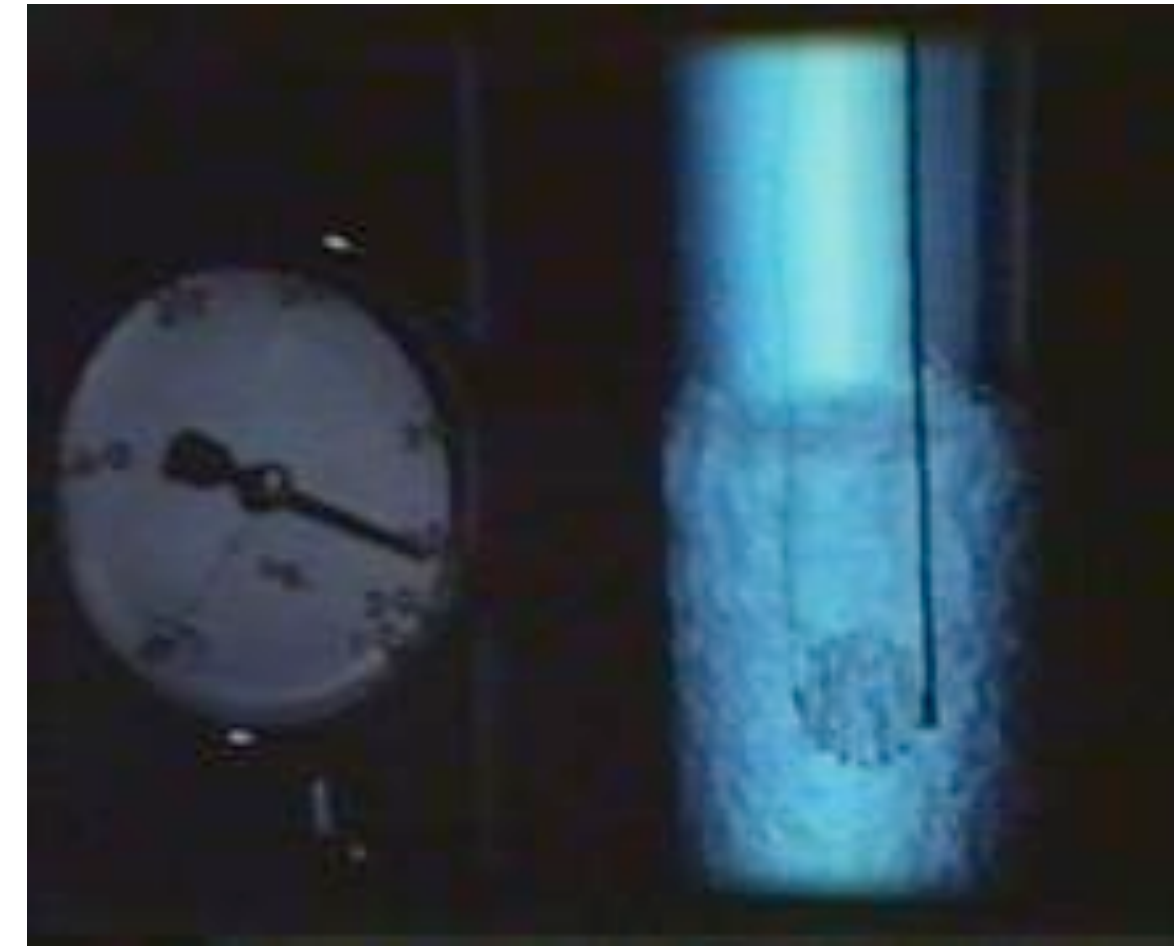
Flowers & Itoh (1976)

# Superfluid Heat Conduction

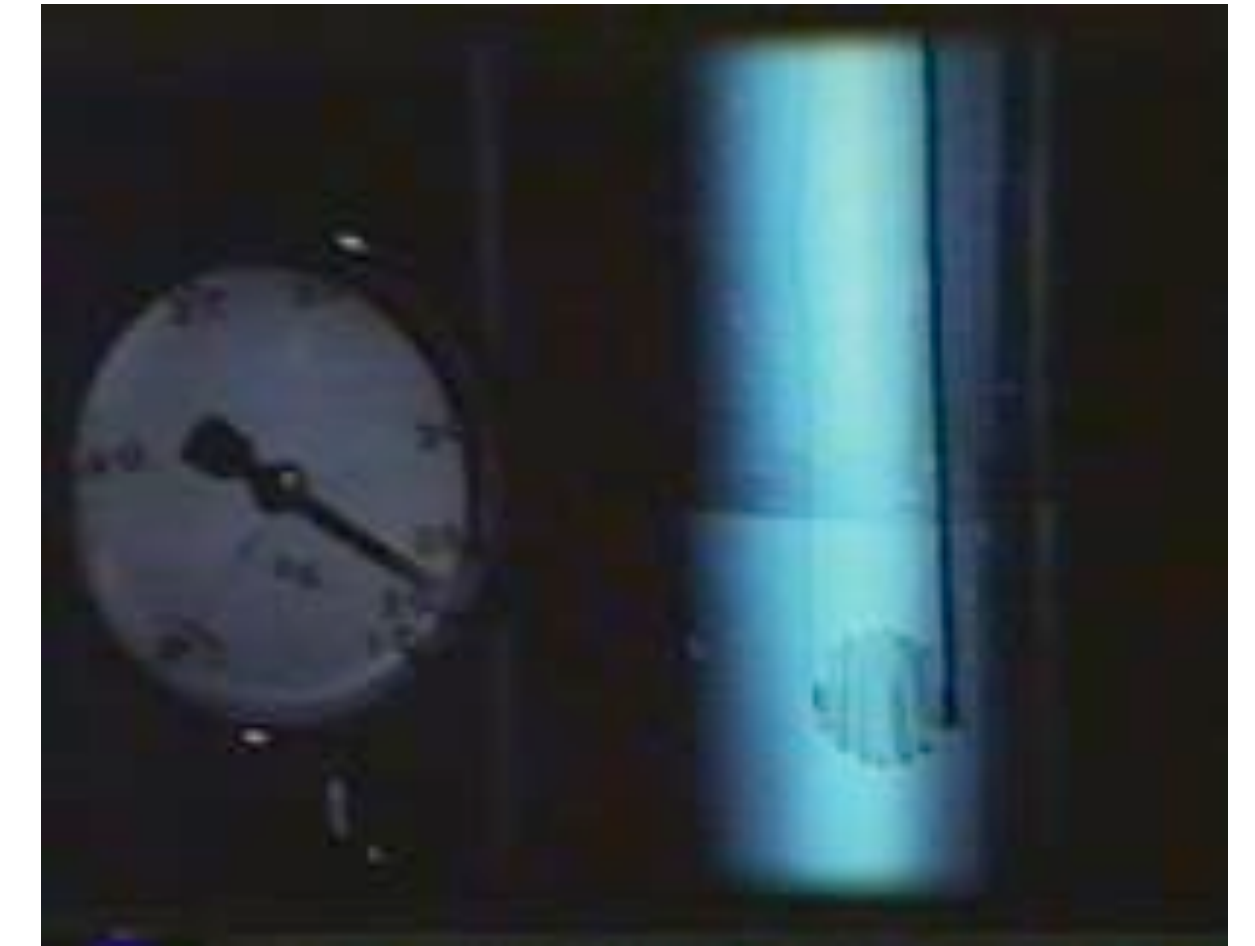
Photographs: JF Allen and JMG Armitage (St Andrews University 1982).

Its impossible to sustain a temperature gradient in bulk superfluid helium !

$$\vec{Q} = S^{(\text{sPh})} T \vec{v}_n$$
$$S^{(\text{sPh})} = \frac{1}{3} C_v^{(\text{sPh})} = \frac{2\pi^2}{15 c_s^3} T^3$$



$T > T_c$



$T < T_c$

Two fluid model: Counter-flow transports heat.  
(Its the superfluid phonon fluid)

The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or (ii) superfluid turbulence.

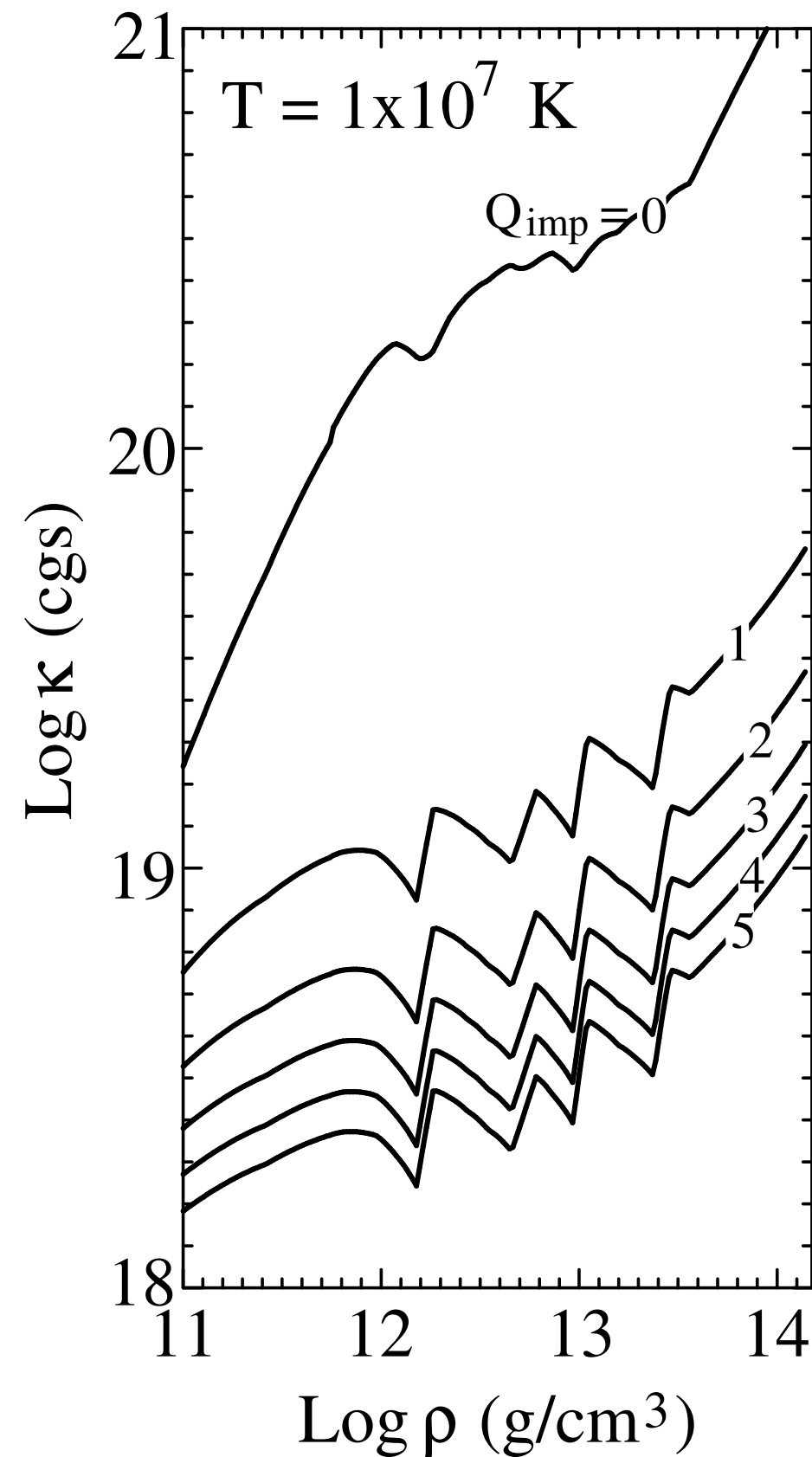
Why does this not occur in neutron stars ?

Answer: Fluid motion is damped by electrons.



# Electron Conduction

$$\kappa_e = \frac{1}{9} \mu_e^2 T \lambda_e$$



## Electron-phonon:

$$\begin{cases} \lambda_e^{\text{ph}} \propto v_t^3 / T^2 & T \geq T_{\text{um}} \\ \lambda_e^{\text{ph}} \propto v_t^4 / T^3 & T \ll T_{\text{um}} \end{cases}$$

$$T_{\text{um}} = (4e^3 / 9\pi) v_t k_{\text{Fe}}$$

## Electron-impurity:

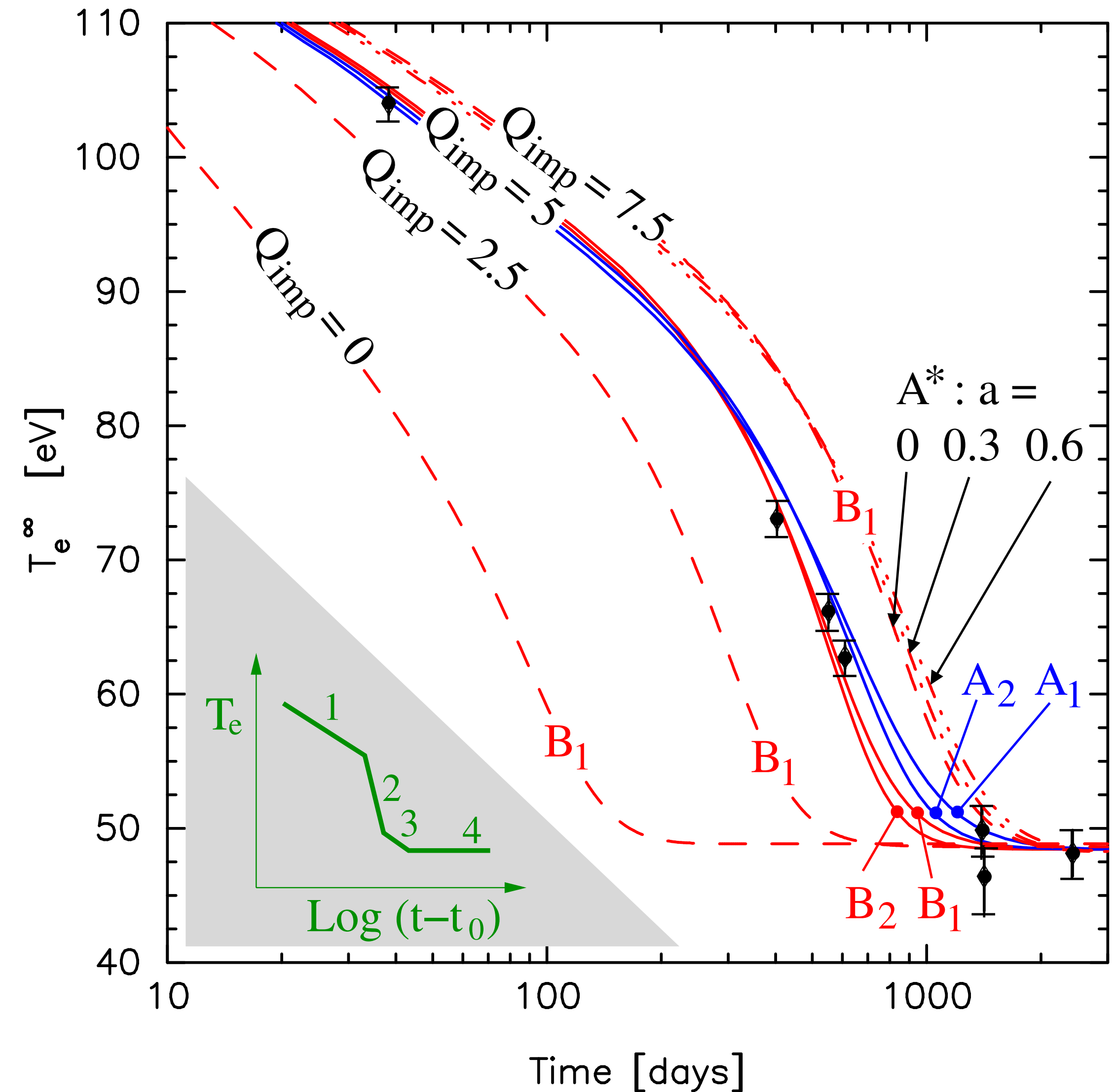
$$\lambda_e^{\text{imp}} = \frac{3\pi \langle Z \rangle}{4e^4 Q_{\text{imp}} k_{\text{Fe}}} \Lambda^{-1}$$

$$Q_{\text{imp}} = \frac{1}{n_{\text{ion}}} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

Impurity scattering is important at low temperature.

# Unraveling Thermal Relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times.
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible !
- If neutrons were unpaired the cooling time scale would be too large.



# Measuring the Heat Capacity of the Core

Heat the star, allow it to relax, and observe the change in temperature:

$$C_{NS} dT = dQ$$

When  $C_{NS} = \alpha T$ :  $\frac{\alpha}{2} (T_f^2 - T_i^2) = \Delta Q$

Lower limit:  $C_{NS}(T_f) > 2 \frac{\Delta Q}{T_f}$

$$\Delta Q = \dot{H} \times t_H - L_\nu \times (t_H + t_{obs})$$

heating  
rate

duration  
of heating

neutrino  
cooling rate

time of observation  
(after heating ceases)



# Observations of KS 1731-260

Quiescent Surface Temperature (post relaxation):  $T_s = 63.1$  eV

Accretion Phase: 12 yrs at  $dM/dt \approx 10^{17}$  g/s

Thermal Relaxation:  $t \approx 8$  yrs

Wijnands et al. (2002) Cackett et al. (2010)

## Inferred Core Temperature:

Insulating envelope supports a temperature gradient near the surface.

Heavy element envelope:  $T_c^\infty = 7.0 \times 10^7$  K  $\left( \frac{T_s^\infty}{63.1 \text{ eV}} \right)^{1.82}$

Light element envelope:  $T_c^\infty = 3.1 \times 10^7$  K  $\left( \frac{T_s^\infty}{63.1 \text{ eV}} \right)^{1.65}$

## Inferred Energy Deposition:

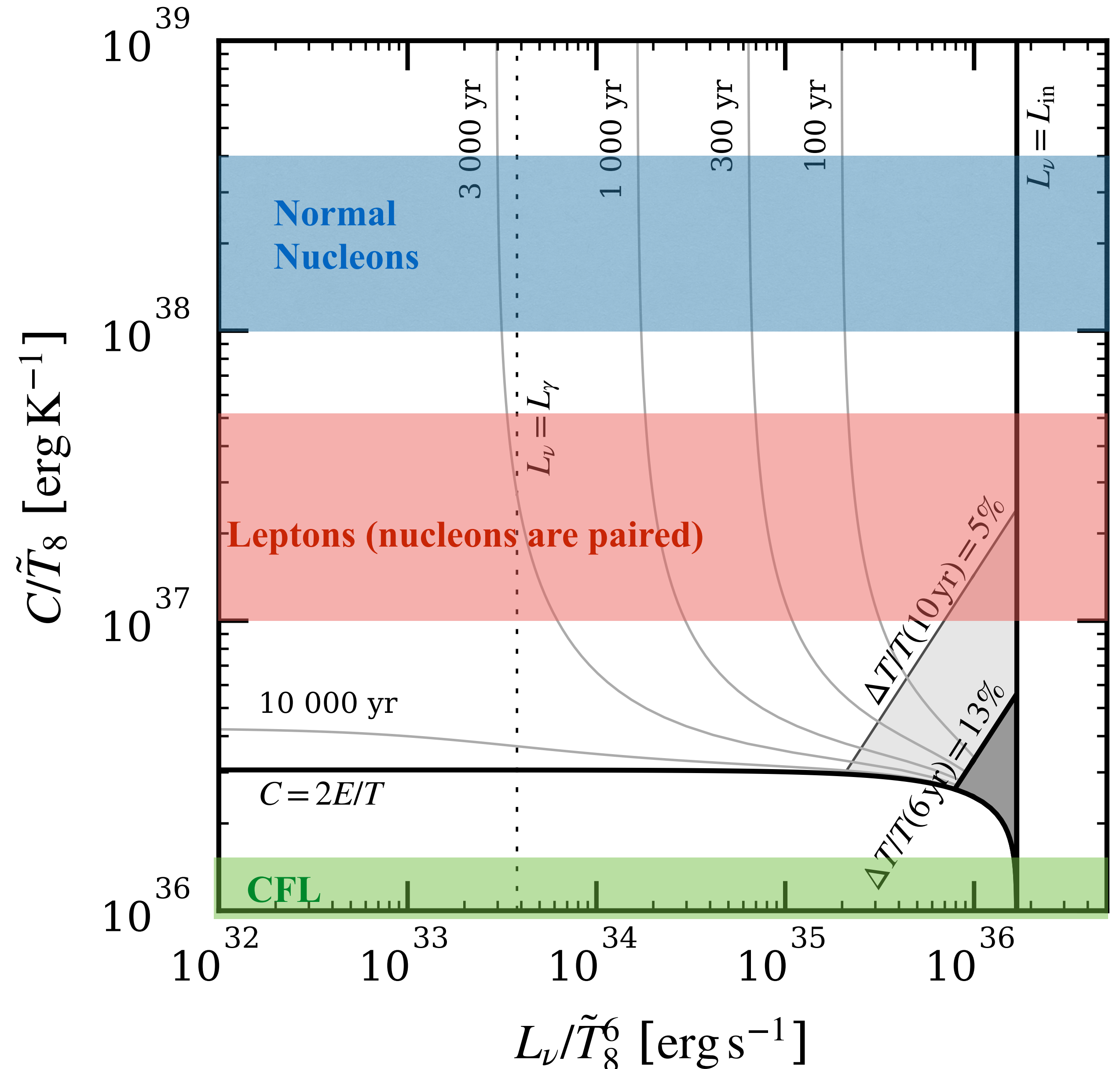
$$\Delta Q = \dot{H} \times t_H = 6 \times 10^{43} \text{ ergs} \left( \frac{Q_{nuc}}{2 \text{ MeV}} \right) \left( \frac{\dot{M}}{10^{17} \text{ g/s}} \right) \left( \frac{t_H}{10 \text{ yrs}} \right)$$

# Lower Limit on the Core Specific Heat: Current & Future

The limit is compatible with most models of dense matter.

One exception is a neutron star core made entirely of CFL quark matter.

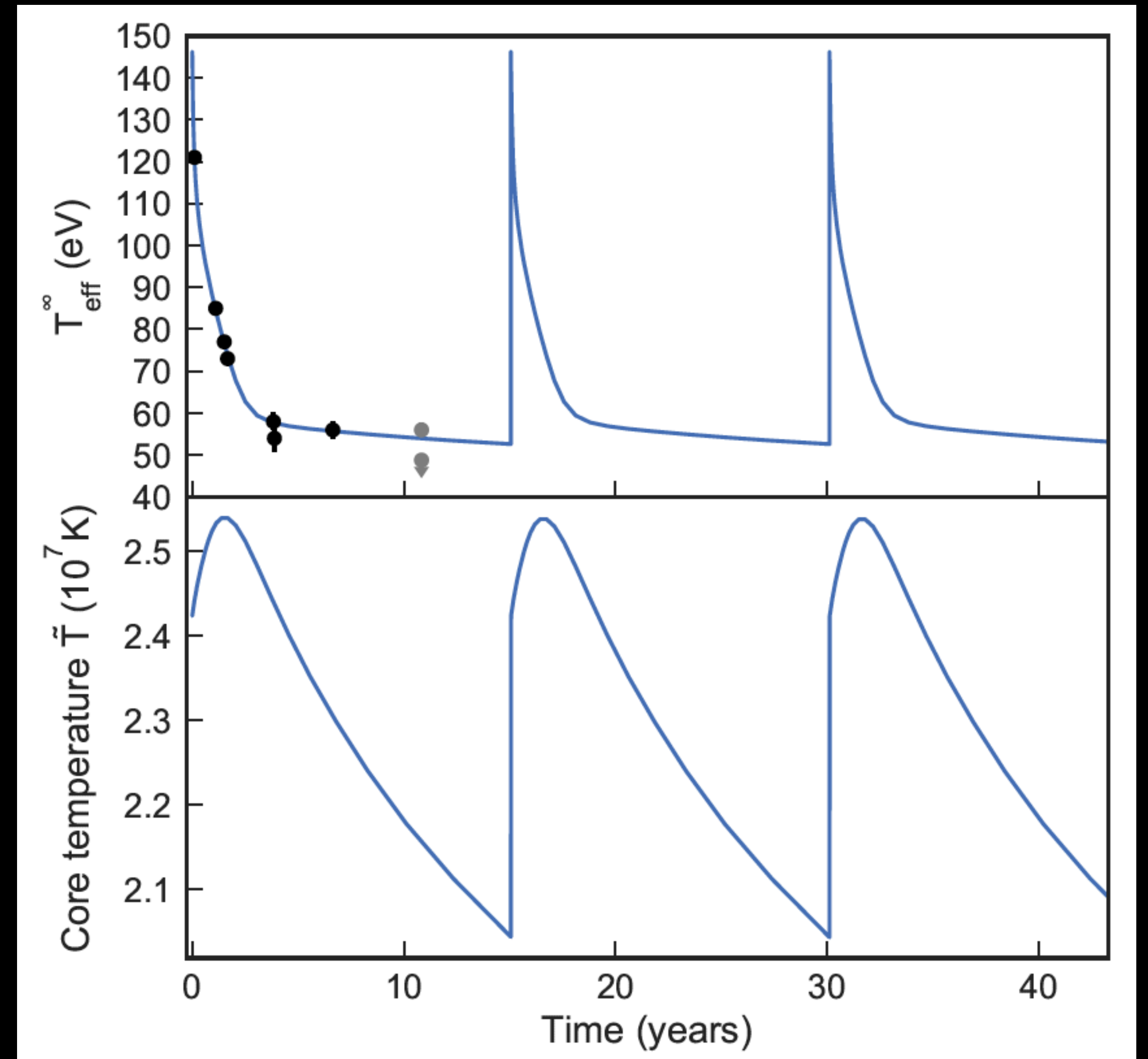
If temperature variation is observed on a 10 year time scale, it would imply some form of exotic matter in which most baryons are frozen!



# Long Term Evolution of Accreting Neutron Stars

Balance between neutrino luminosity and crustal heating sets the average core temperature.

If we know the heating and accretion rate on average then a measurement of the neutron star surface temperature provides a constraint on the core neutrino luminosity!



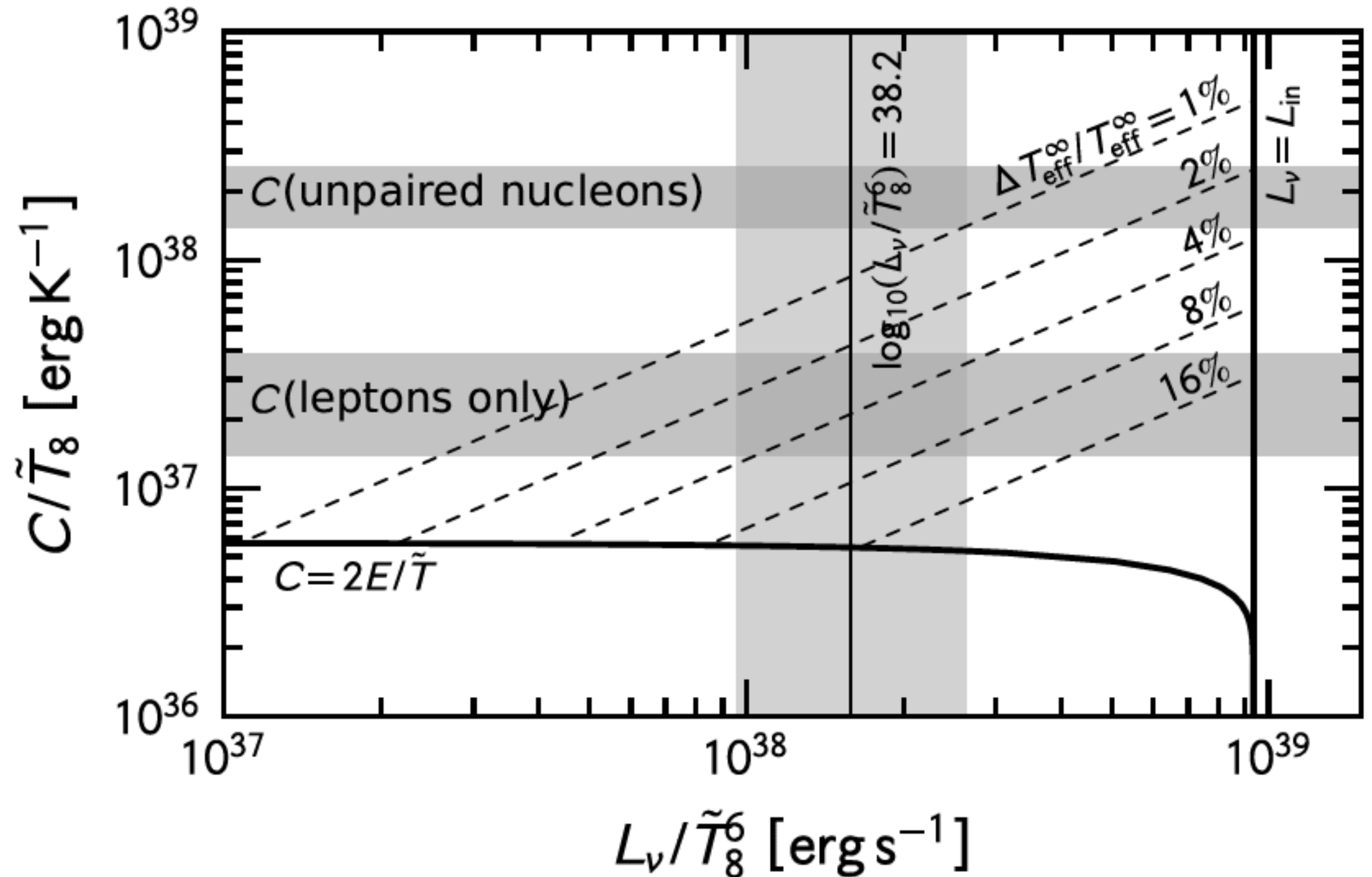
# Rapid neutrino cooling in the neutron star MXB 1659-29

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Evidence for diversity. Not all neutron star cores are the same!

If we observe cooling on a 10 year timescale we can obtain an upper bound on the core specific heat as well!



# Conclusions and Outlook

Theory and observations have revealed much about the crust and the outer core during recent years. Nuclear physics provides a consistent interpretation of diverse data.

Small observed radii and large maximum mass of neutron stars suggest a rapid increase in the speed of sound in the inner cores of neutron stars. Favor a complex yet smooth transition to quark degrees of freedom.

Evolution of accreting neutron stars provided new insights about the thermal and transport properties of the inner crust. Interpretation requires a superfluid state.

Observations of GWs and EM signals from neutron star mergers will offer many exciting opportunities for research in dense matter theory. The interpretation of the multi-messenger signals relies on it.

We need a few more GW170817's and a core-collapse supernova in our own galaxy is now well overdue.