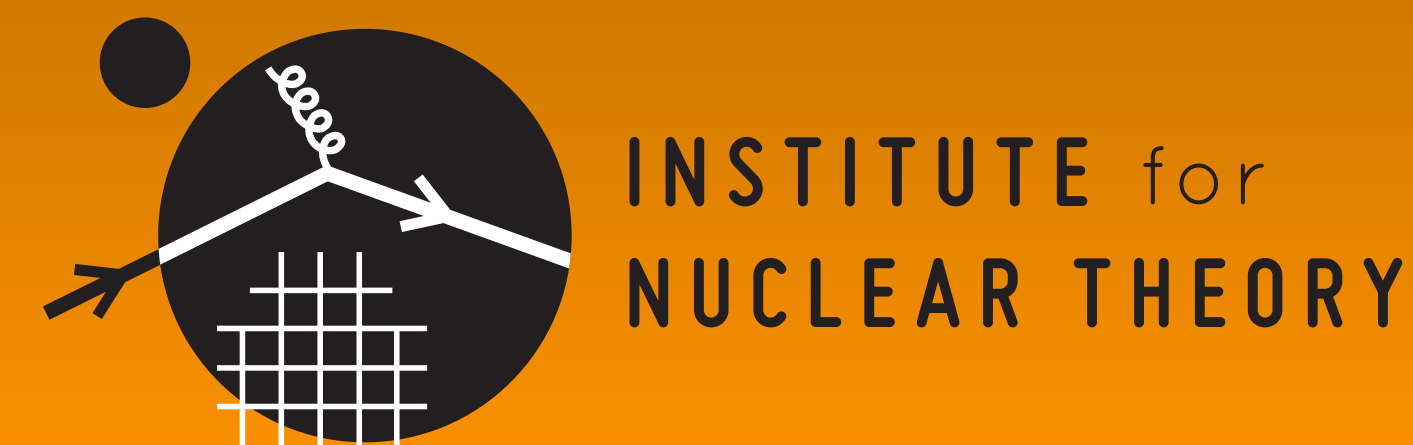


Neutron Stars in the Multi-Messenger Era

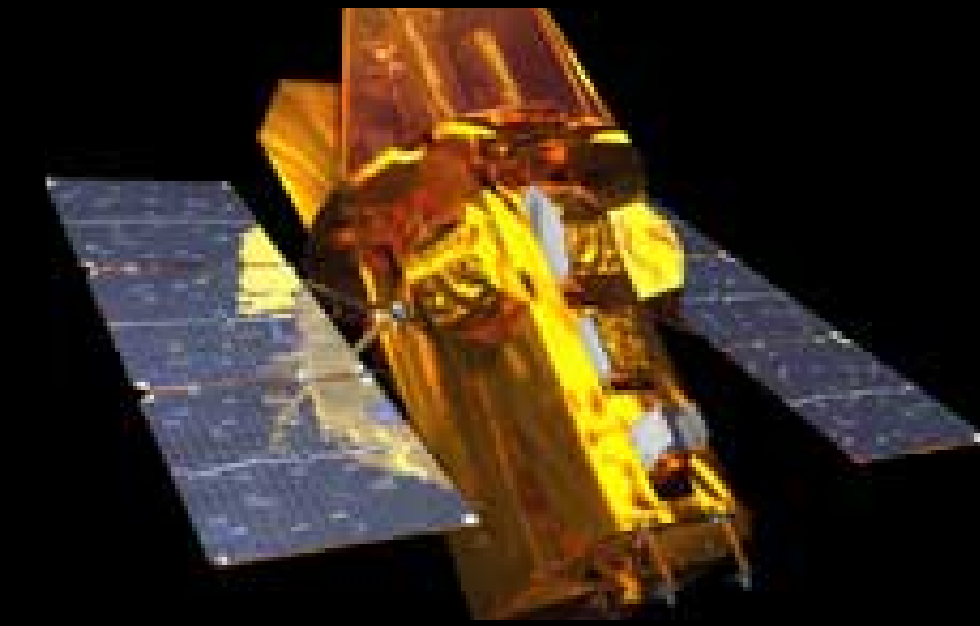
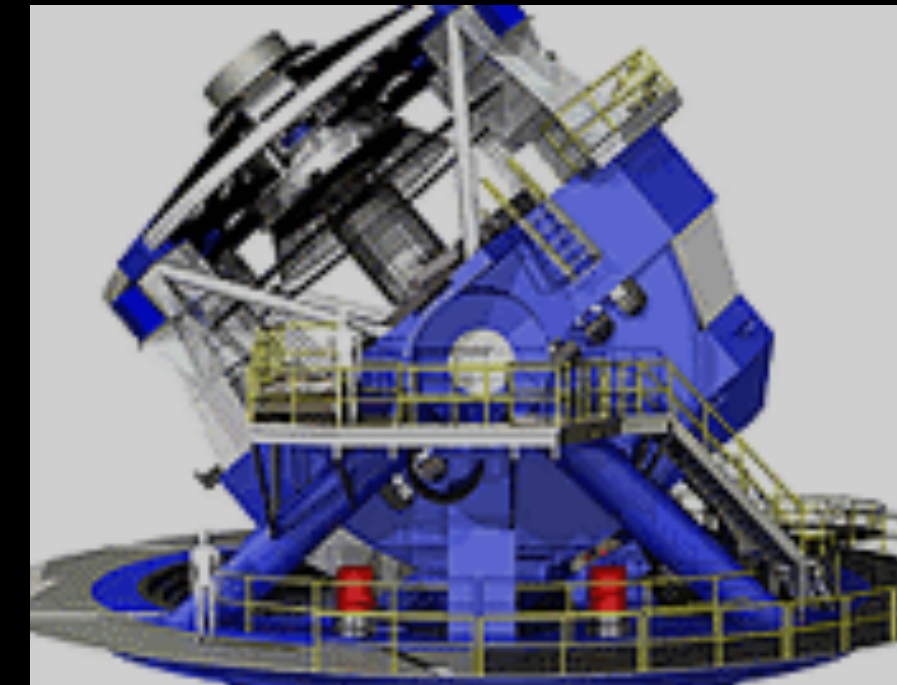
Sanjay Reddy
Institute for Nuclear Theory,
University of Washington, Seattle

Lecture 1: Structure of Neutron Stars. Mass, radius and tidal deformability. Nuclear interactions and nuclear matter, effective field theory. Phase transitions.

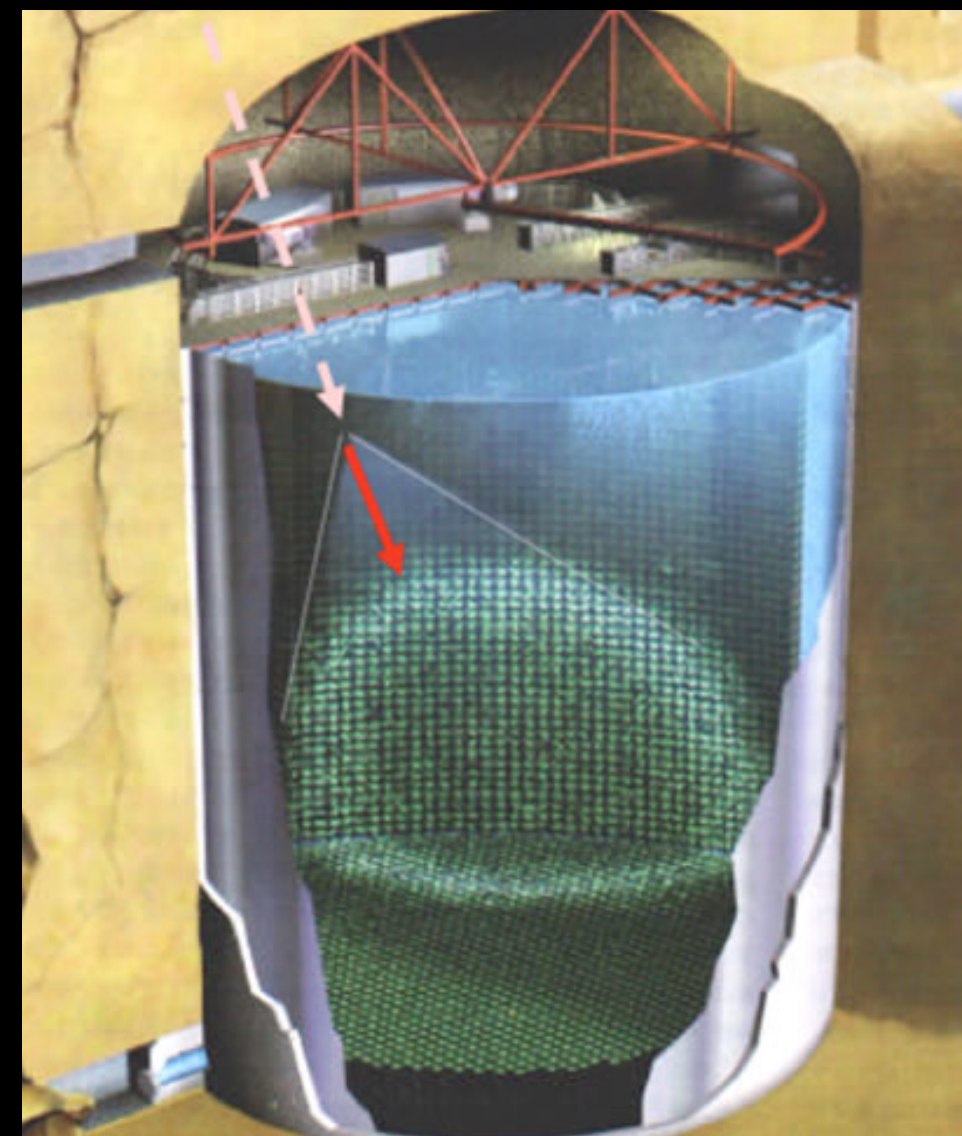
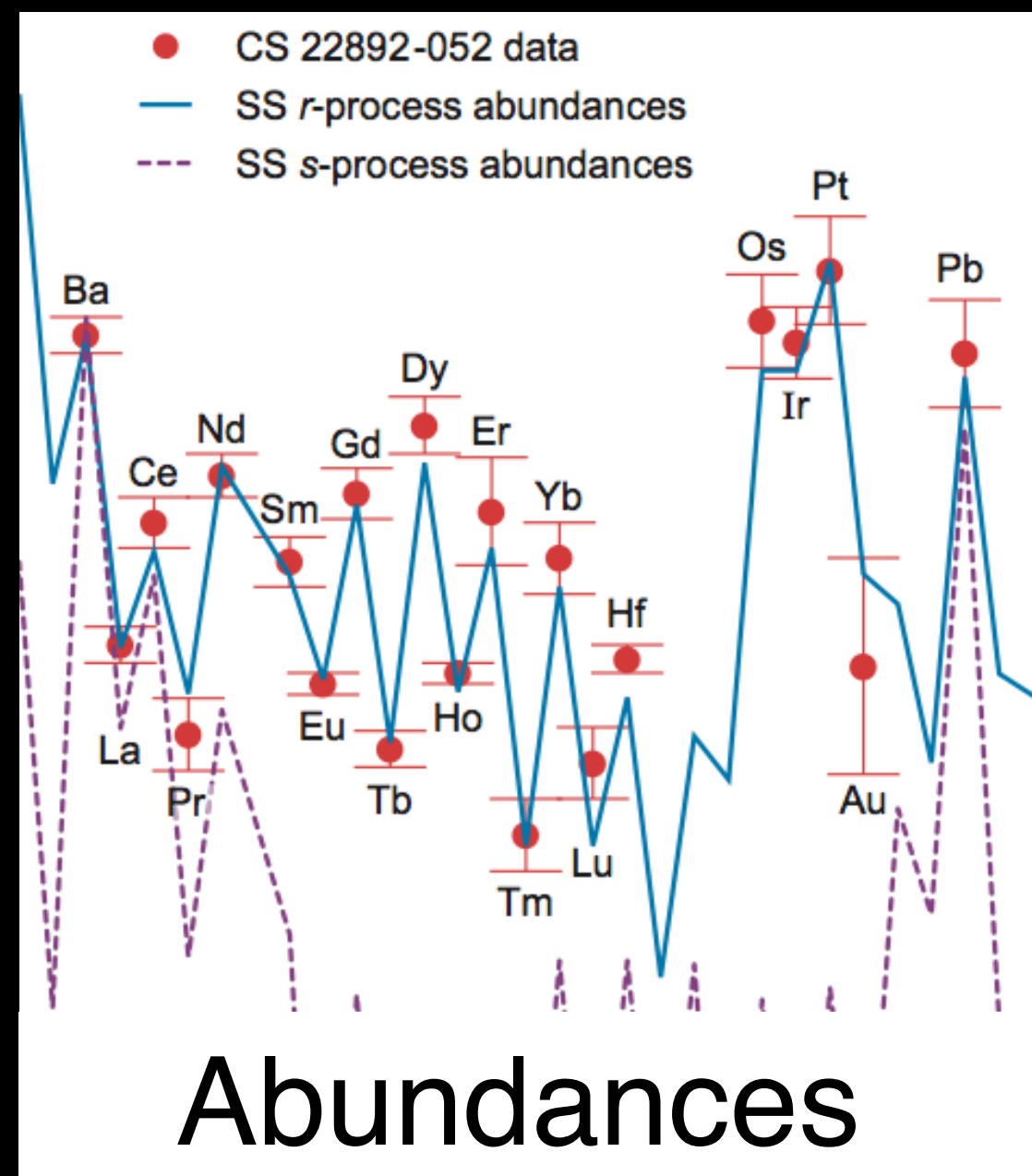
Lecture 2: Neutron star cooling: Proto-neutron star evolution, supernova neutrino emission and detection. Cooling of isolated neutron stars, heating and cooling in accreting neutron stars. Observational constraints. Neutron stars as laboratories for particle physics and dark matter.



Best of Times: The Multi-Messenger Era



Photons



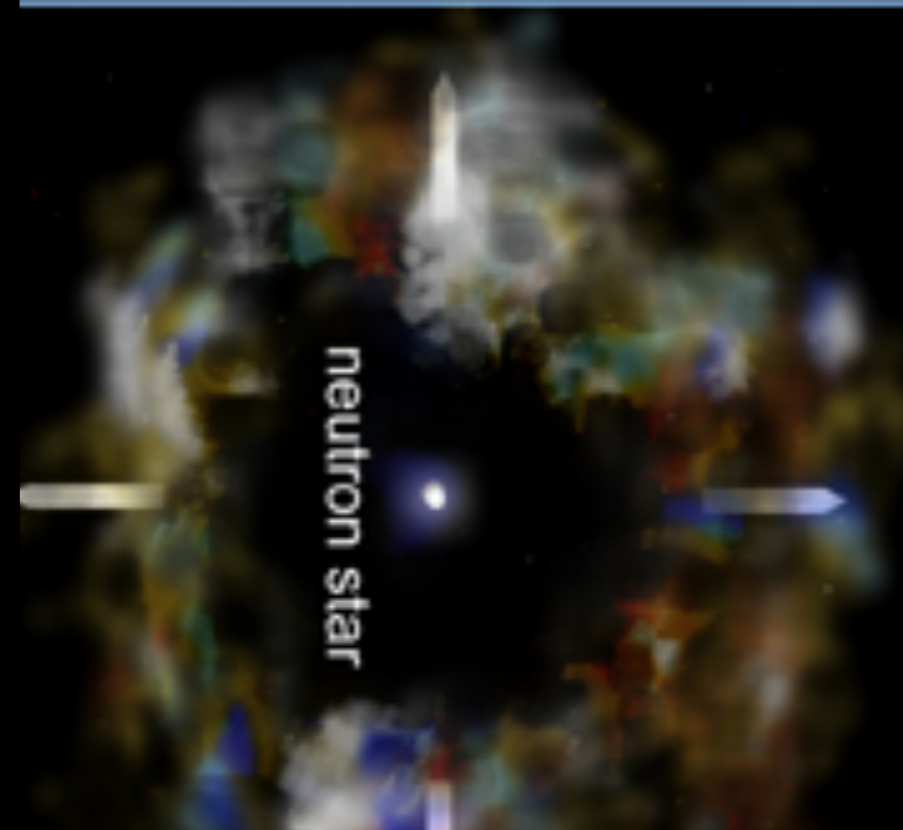
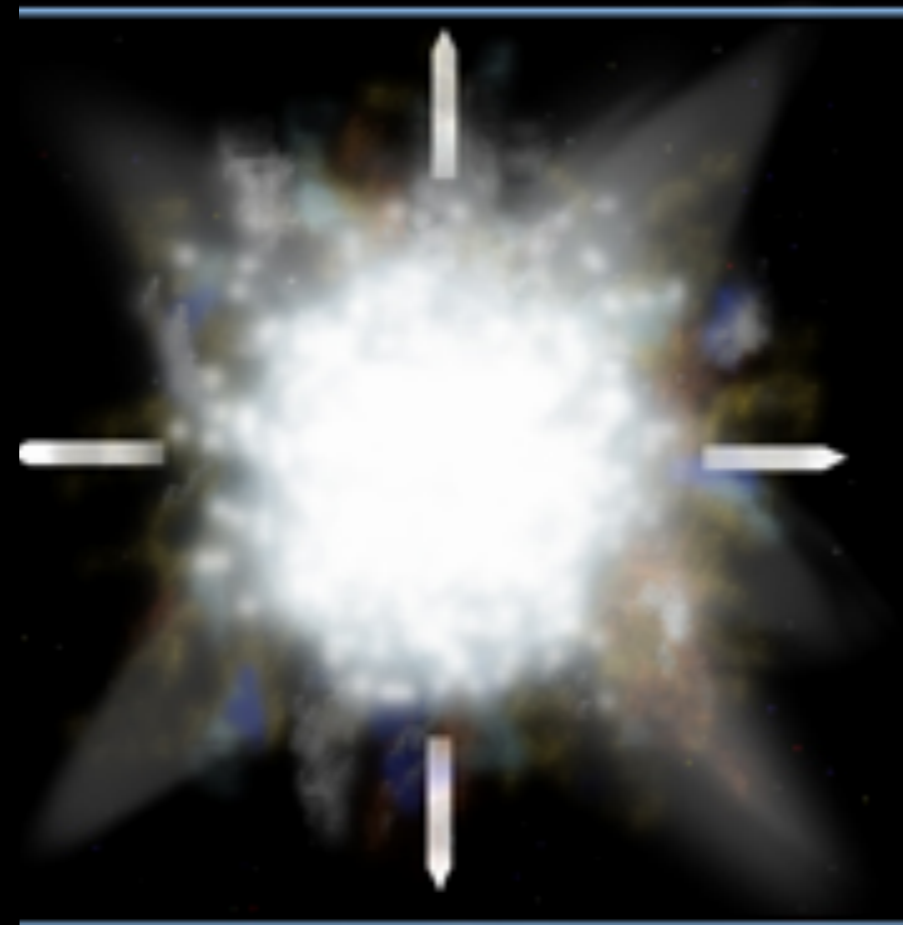
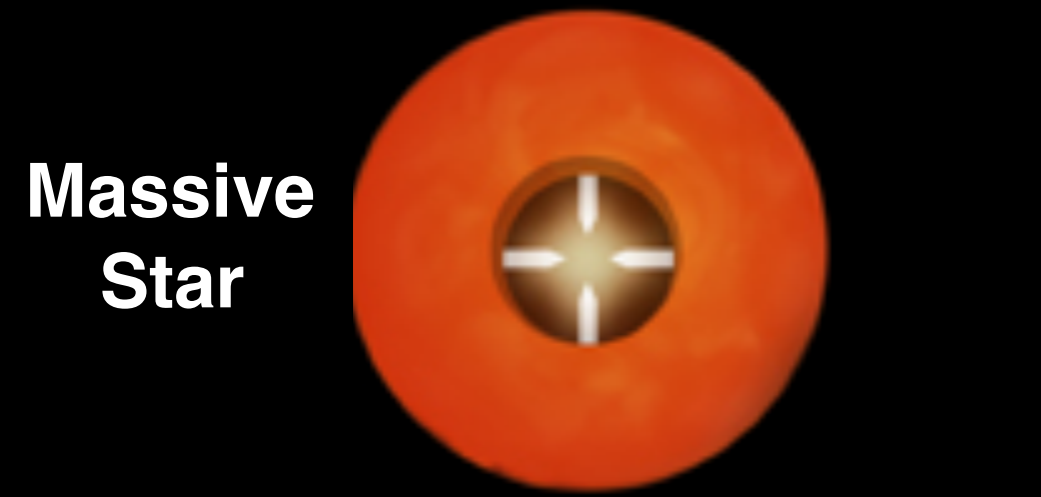
Neutrinos



Gravitational Waves

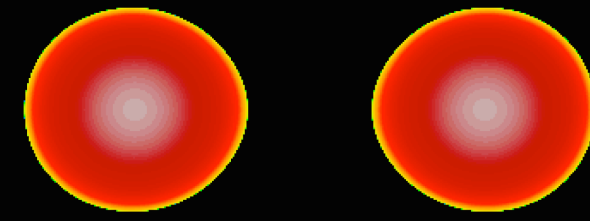
Sources: Neutron Stars are Central

Supernova

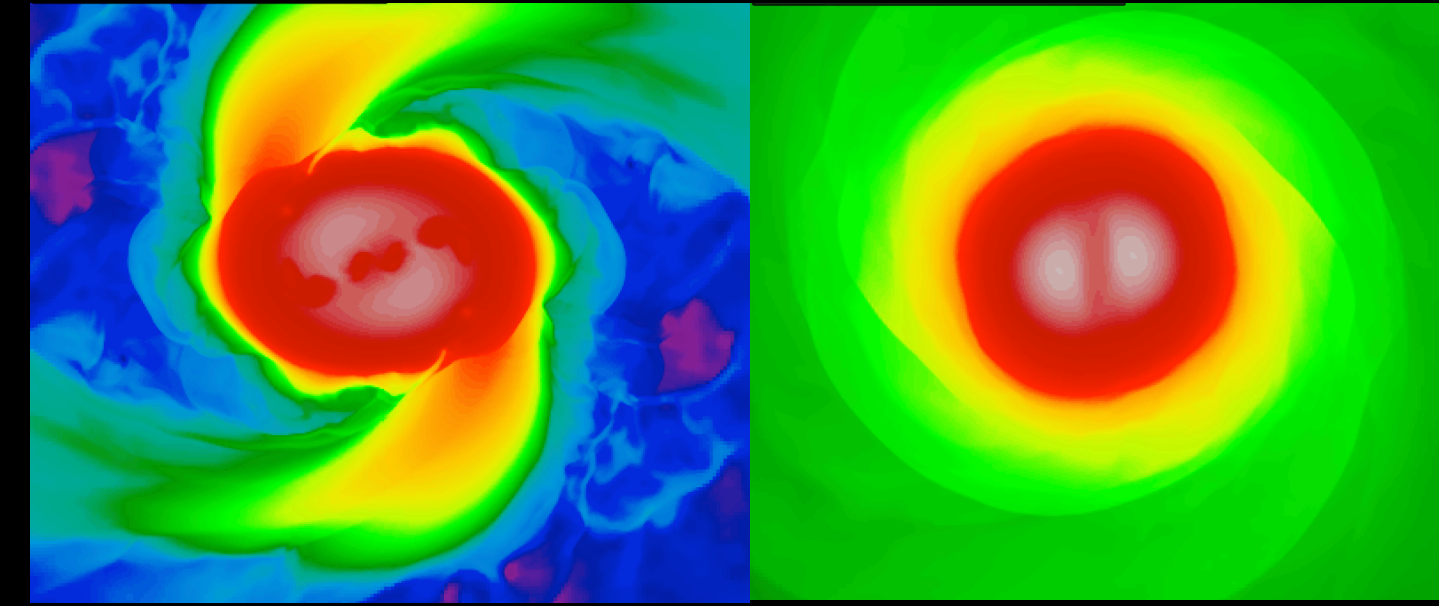


Credit: NASA/CXC/S Lee

Binary Neutron Star Mergers



Credit: Luciano Rezzola

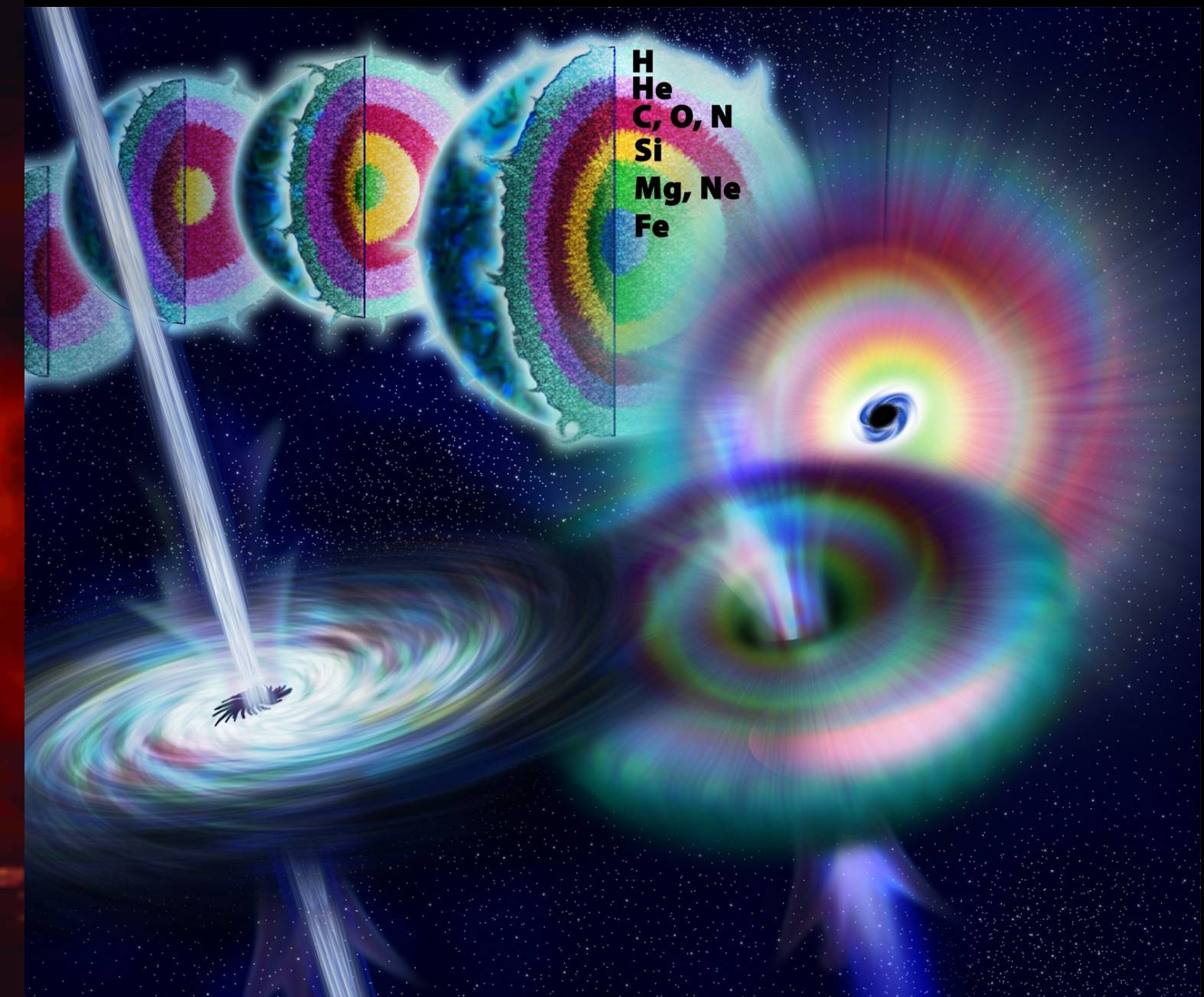


Accreting Neutron Stars



Credit: David Hardy & PPARC

Gamma-Ray Bursts



Credit: Nicolle Rager Fuller/NSF

The Ultimate Collision



The Ultimate Collision



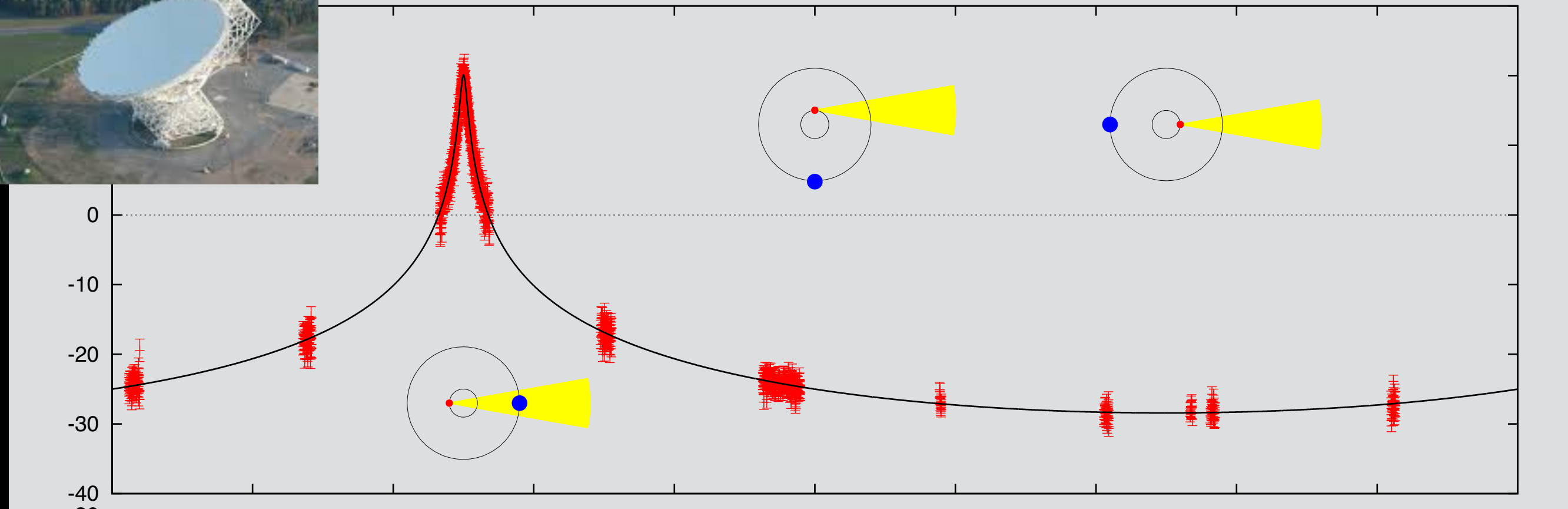
The Ultimate Collision



The gravitational waves, electromagnetic and neutrinos from neutron star mergers are sensitive to the properties of dense matter. Especially to the neutron radius and maximum mass.

Movie Credit: NASA Goddard

Neutron Star Structure: Observations



2 M_{\odot} neutron stars exist.

PSR J1614-2230: $M=1.93(2)$

Demorest et al. (2010)

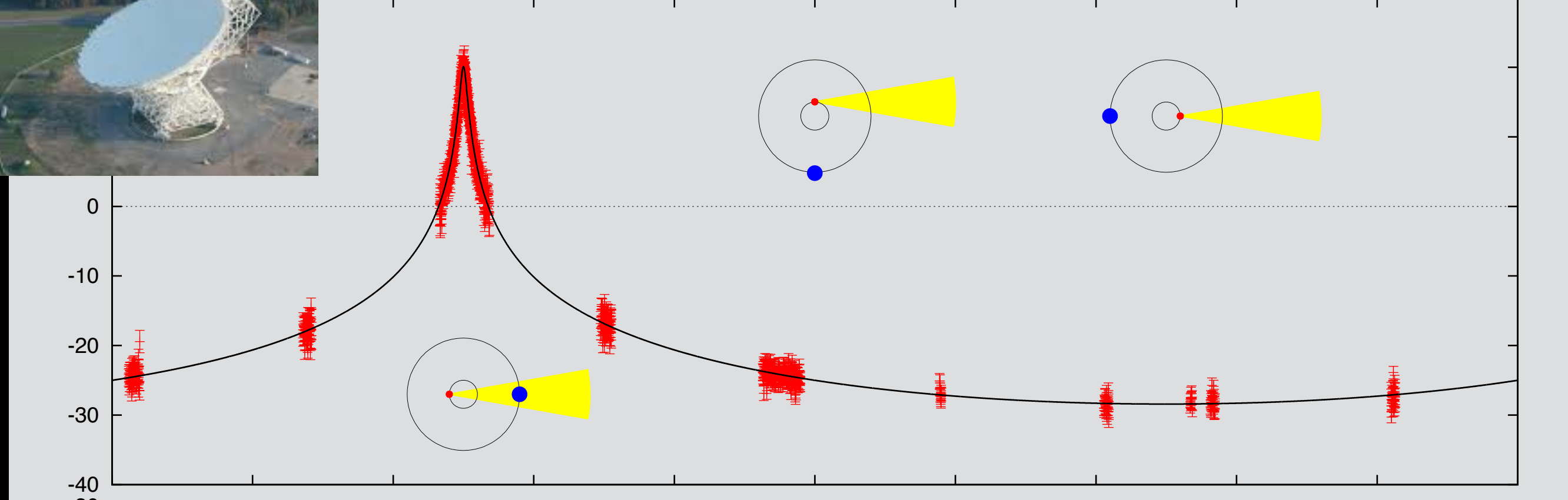
PSR J0348+0432: $M=2.01(4) M_{\odot}$

Anthoniadis et al. (2013)

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Cromartie et al. (2019)

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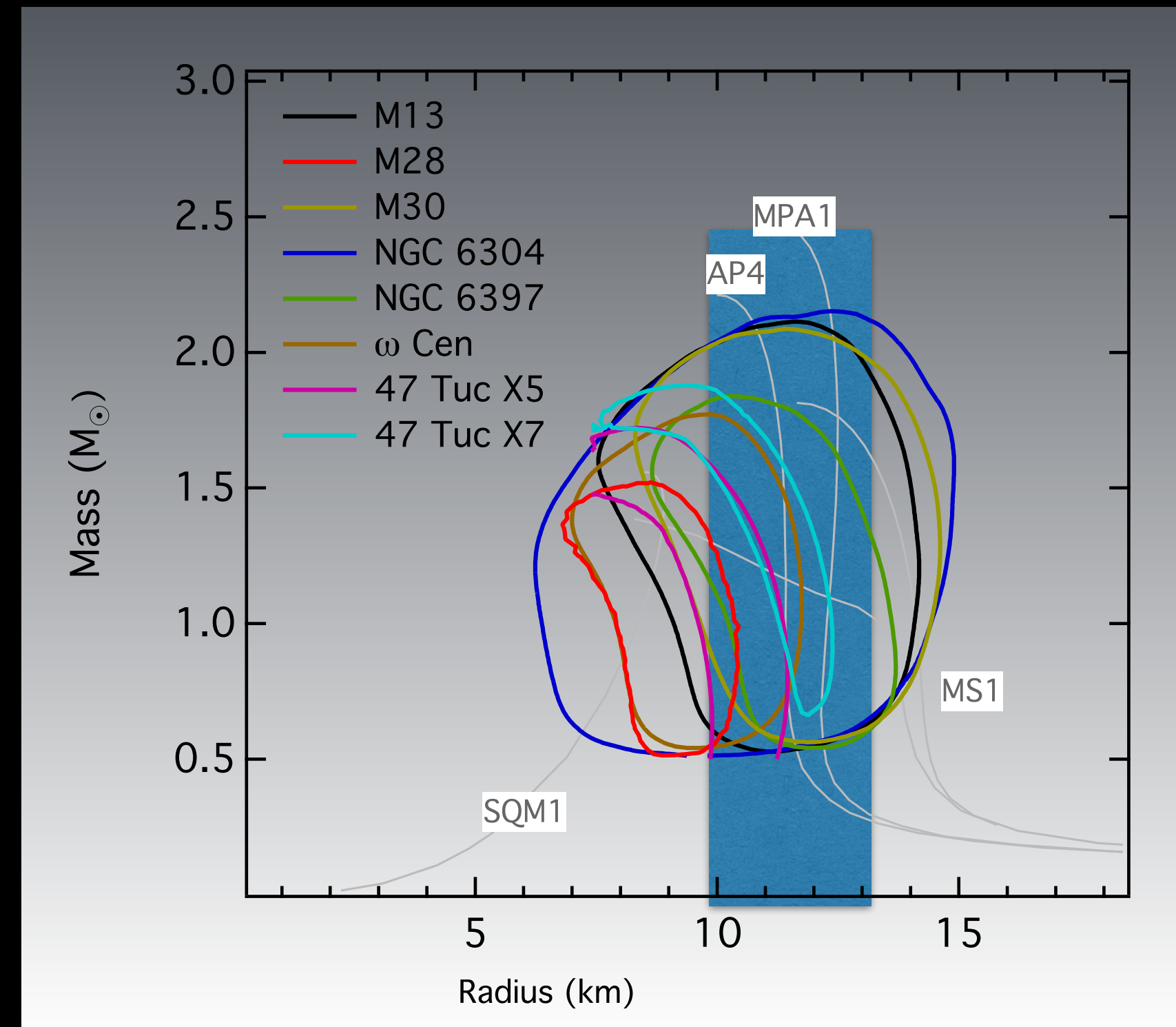
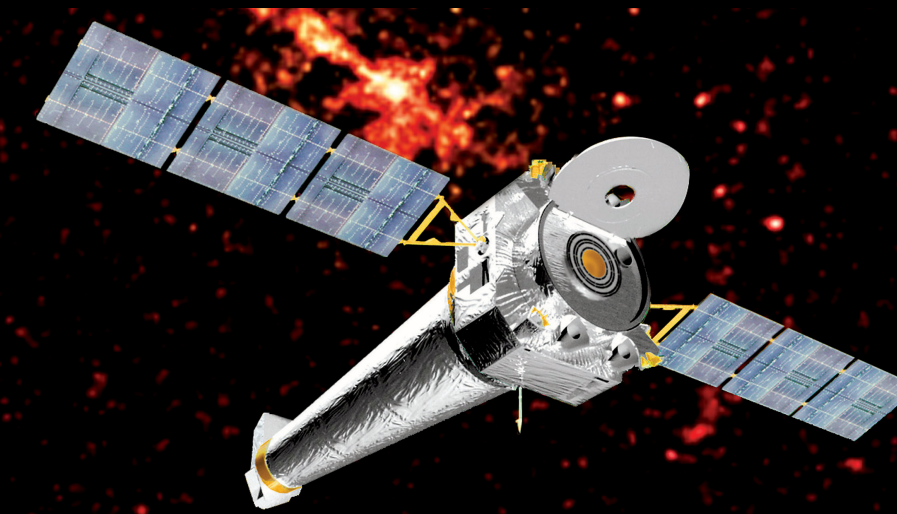
MSP J0740+6620: $M=2.17(10) M_{\odot}$

Cromartie et al. (2019)

Inferred NS radii are small.

Despite poorly understood systematic errors, x-ray observations suggest $R \sim 9-13$ km. Perhaps even preferring a smaller range $R \sim 10-12$ km.

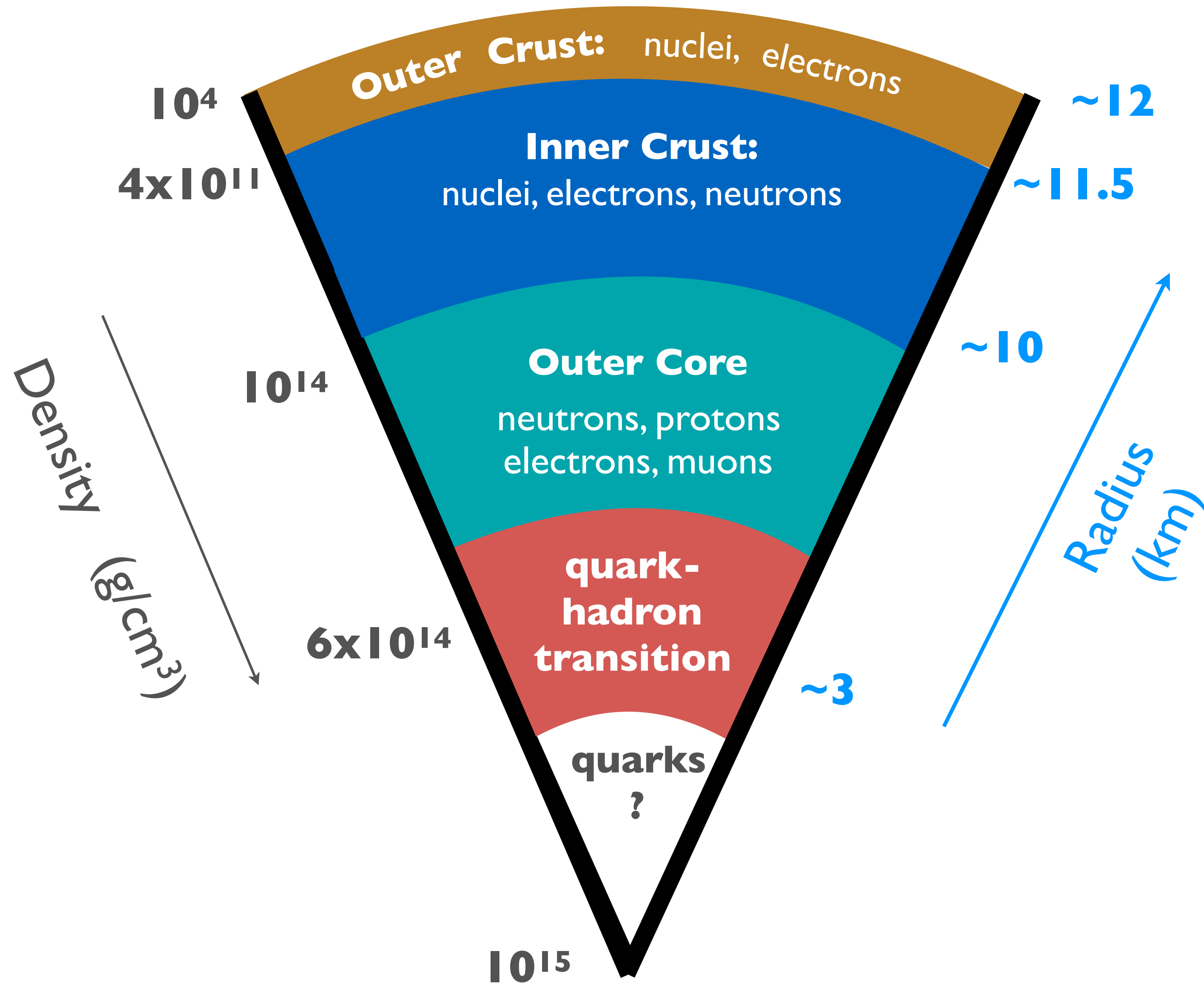
Ozel & Freire (2016)



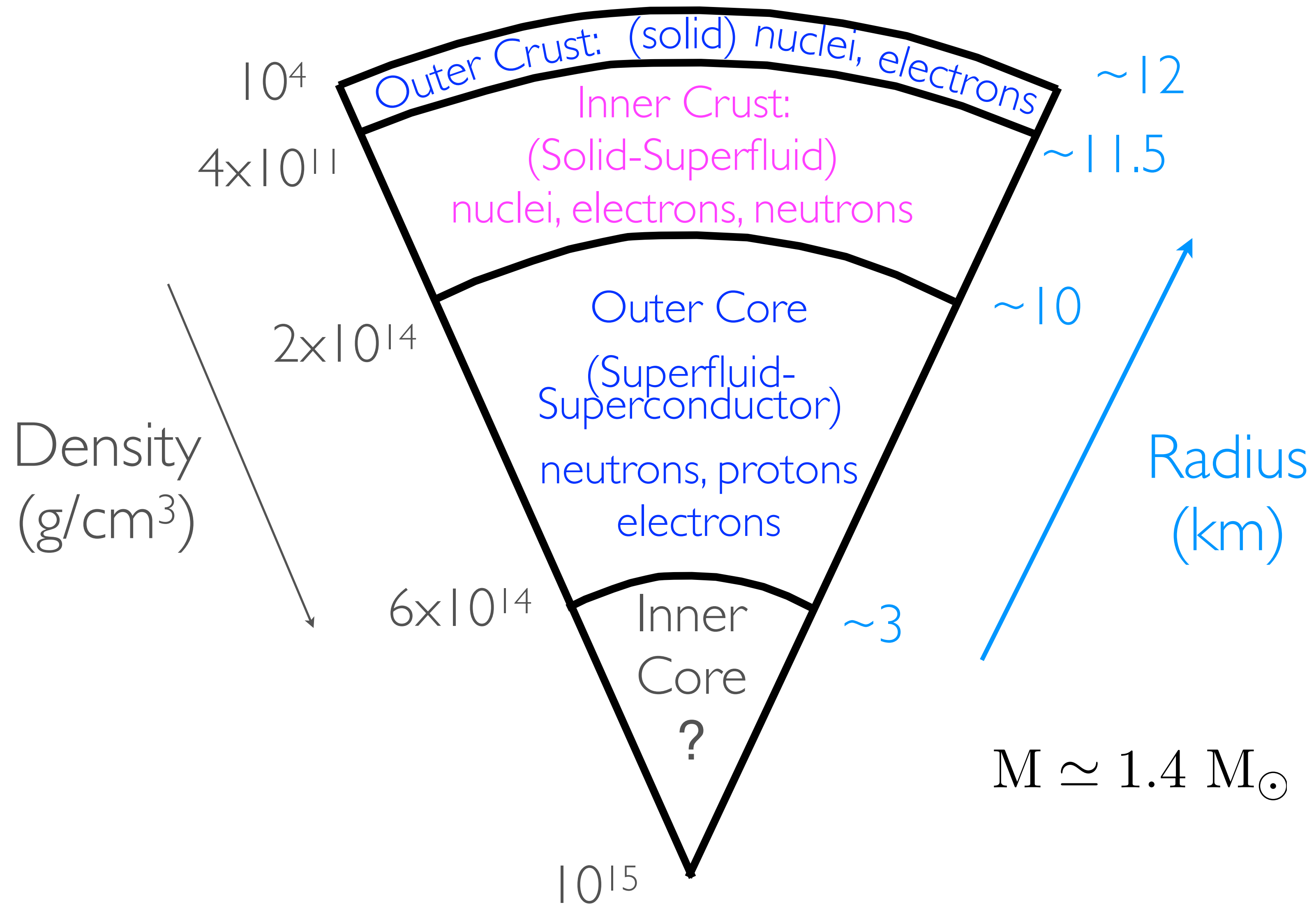
Compressing Matter: A tale of frustration and liberation

| Density | Fermi Energy (Frustration) | Phenomena (Liberation) |
|------------------------------------|--|--|
| $10^3 - 10^6 \text{ g/cm}^3$ | Electron Fermi Energy $\mu_e = 10 \text{ keV} - \text{MeV}$ | Ionization |
| $10^6 - 10^{11} \text{ g/cm}^3$ | Electron Fermi Energy $\mu_e = 1 - 25 \text{ MeV}$ | Neutron-rich Nuclei $e + p \rightarrow n + \nu_e$ |
| $10^{11} - 10^{14} \text{ g/cm}^3$ | Neutron Fermi Energy $\mu_n = 1 - 30 \text{ MeV}$ | Neutron-drip superfluidity |
| $10^{14} - 10^{15} \text{ g/cm}^3$ | Neutron Fermi Energy $\mu_n = 30 - 1000 \text{ MeV}$ | Nuclear matter Quarks ? |

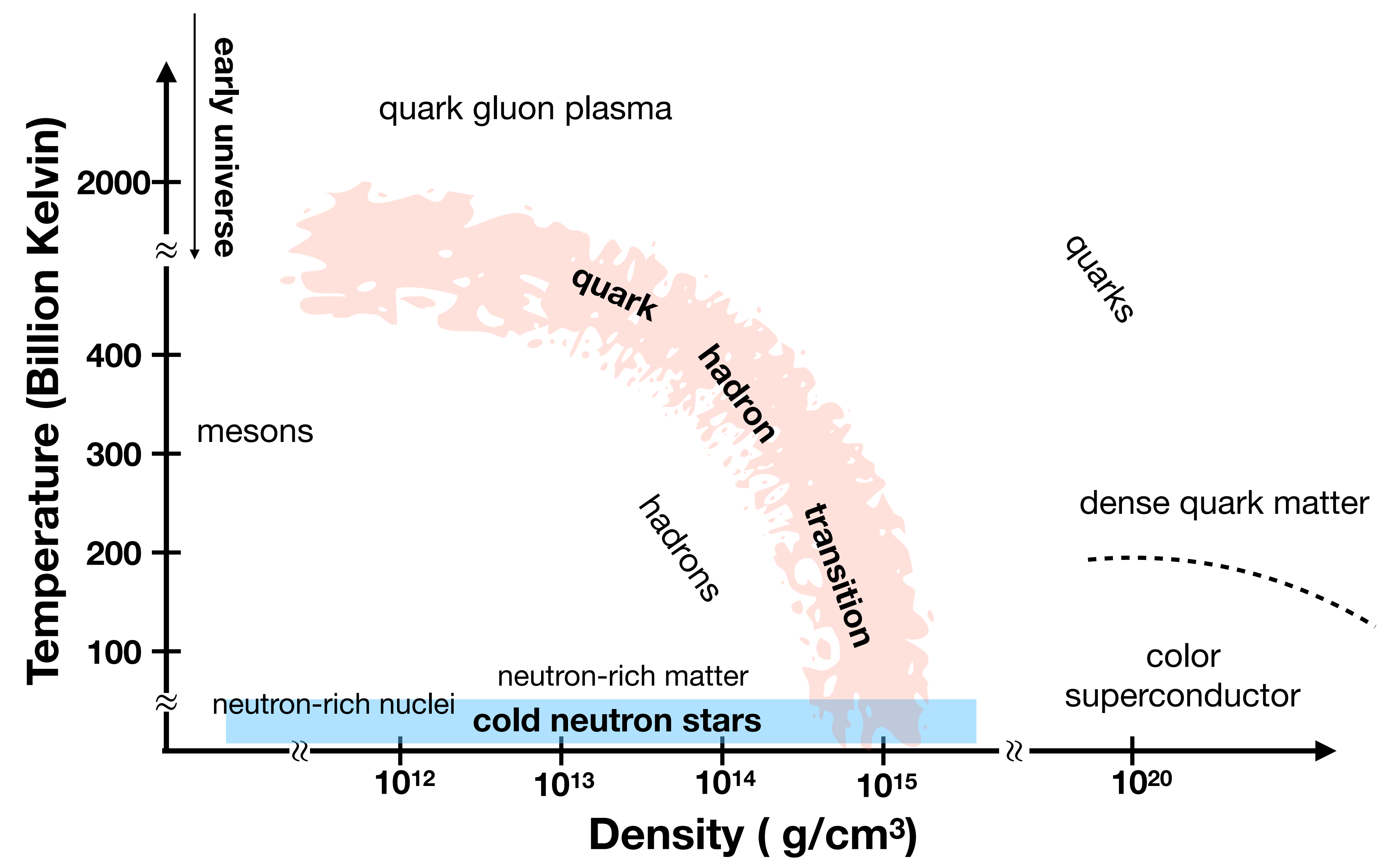
Composition and Phases of Dense Matter in Neutron Stars



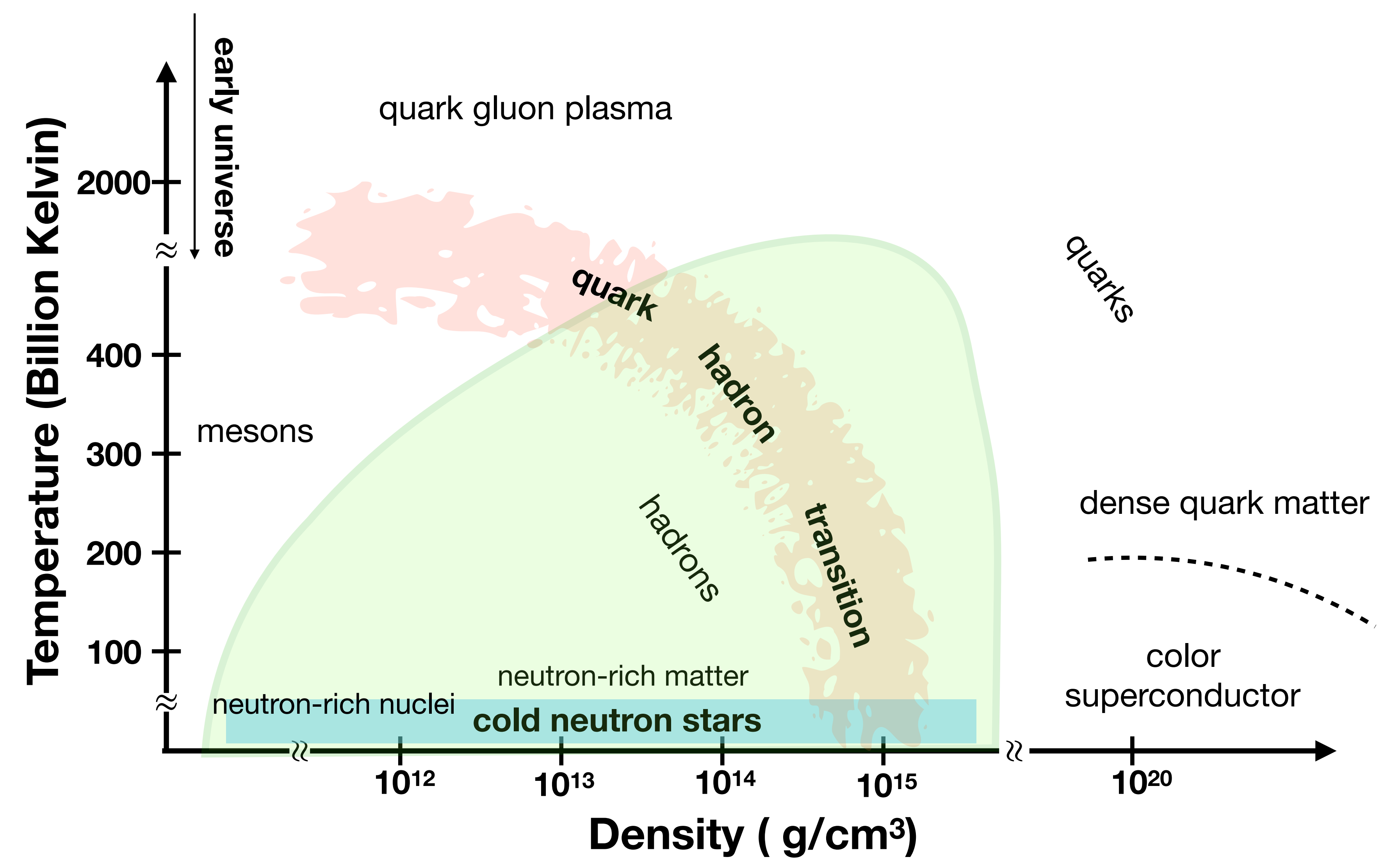
Composition and Phases of Dense Matter in Neutron Stars



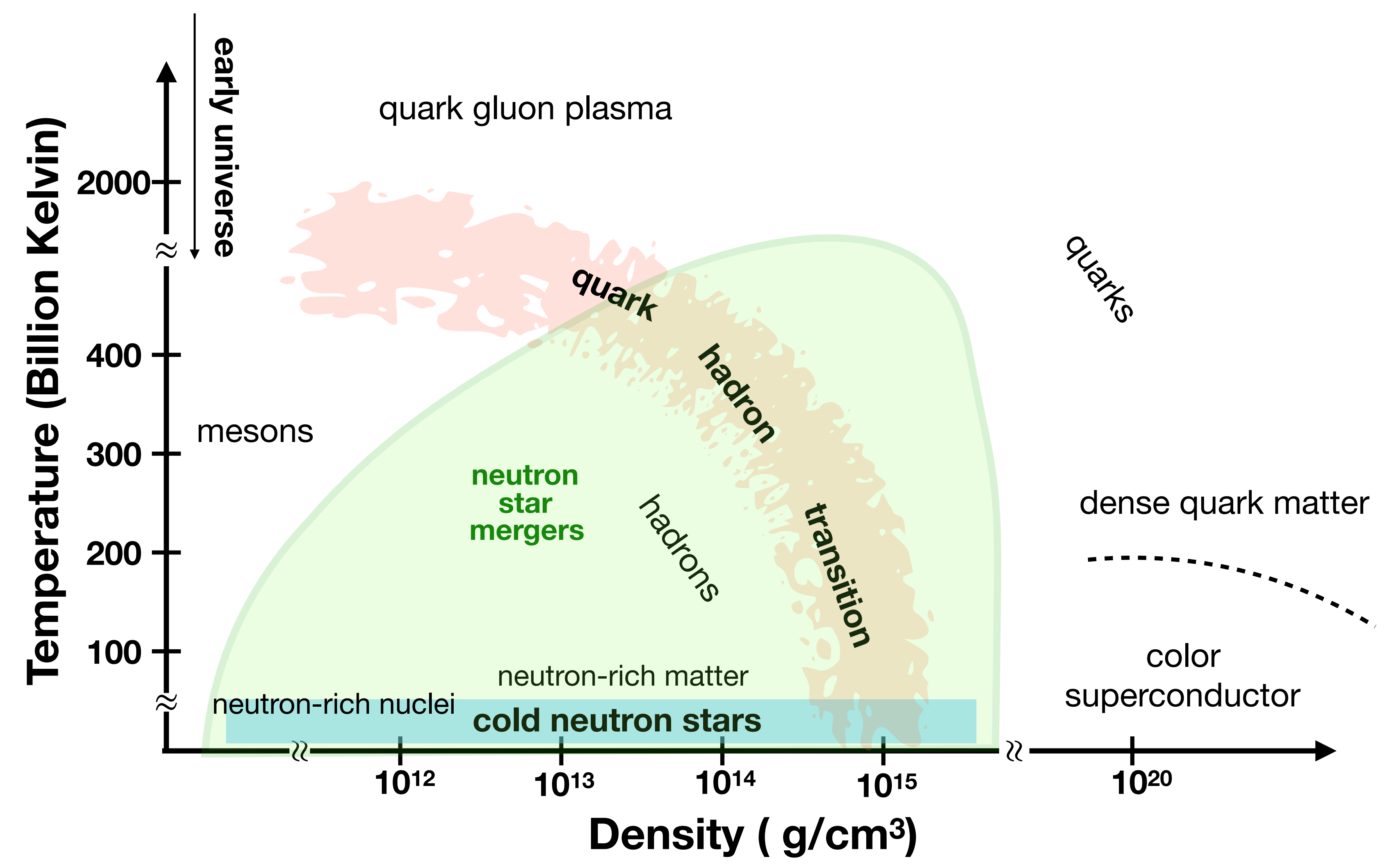
Phase Diagram of Hot & Dense Matter



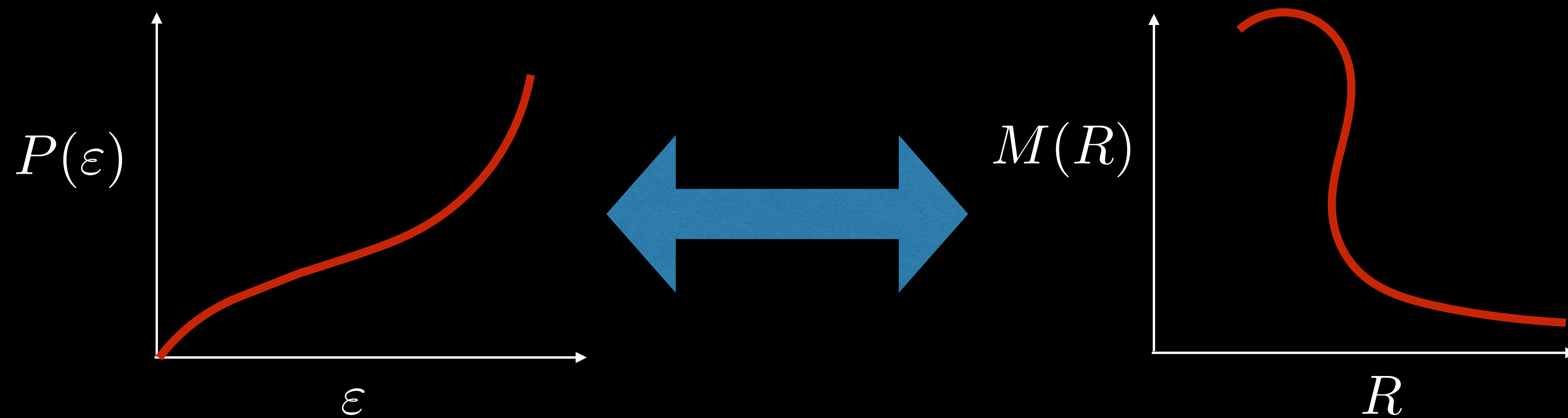
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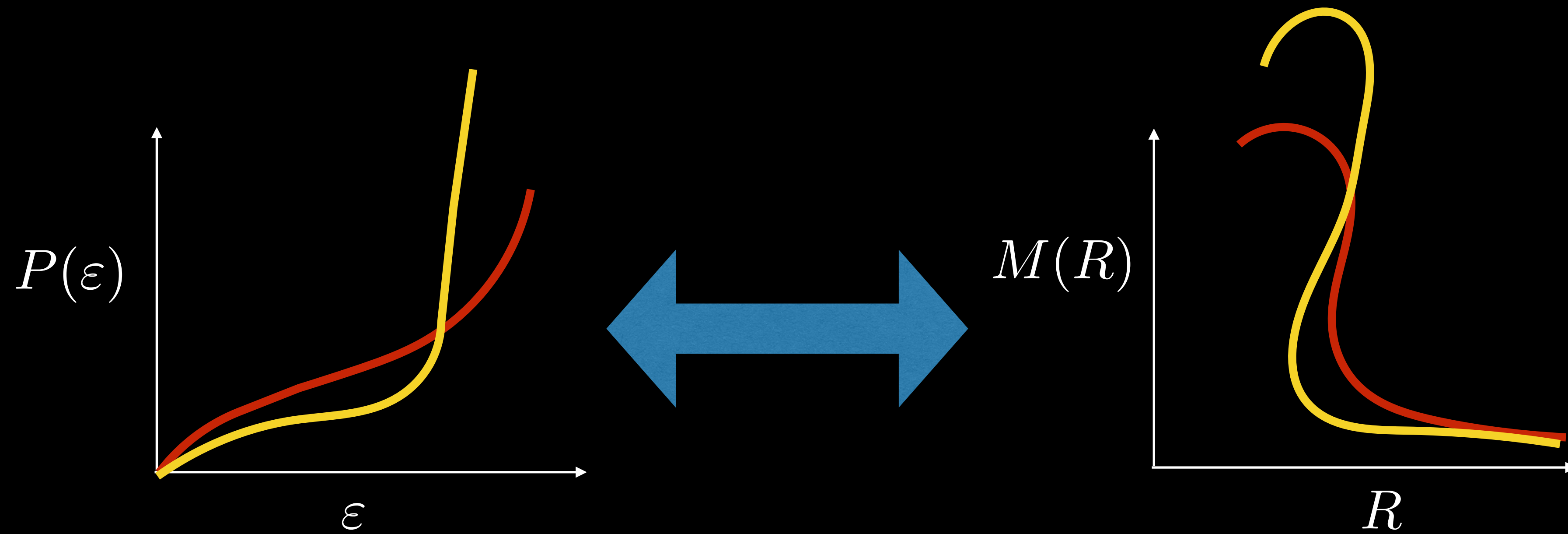


Neutron Star Mass and Radius



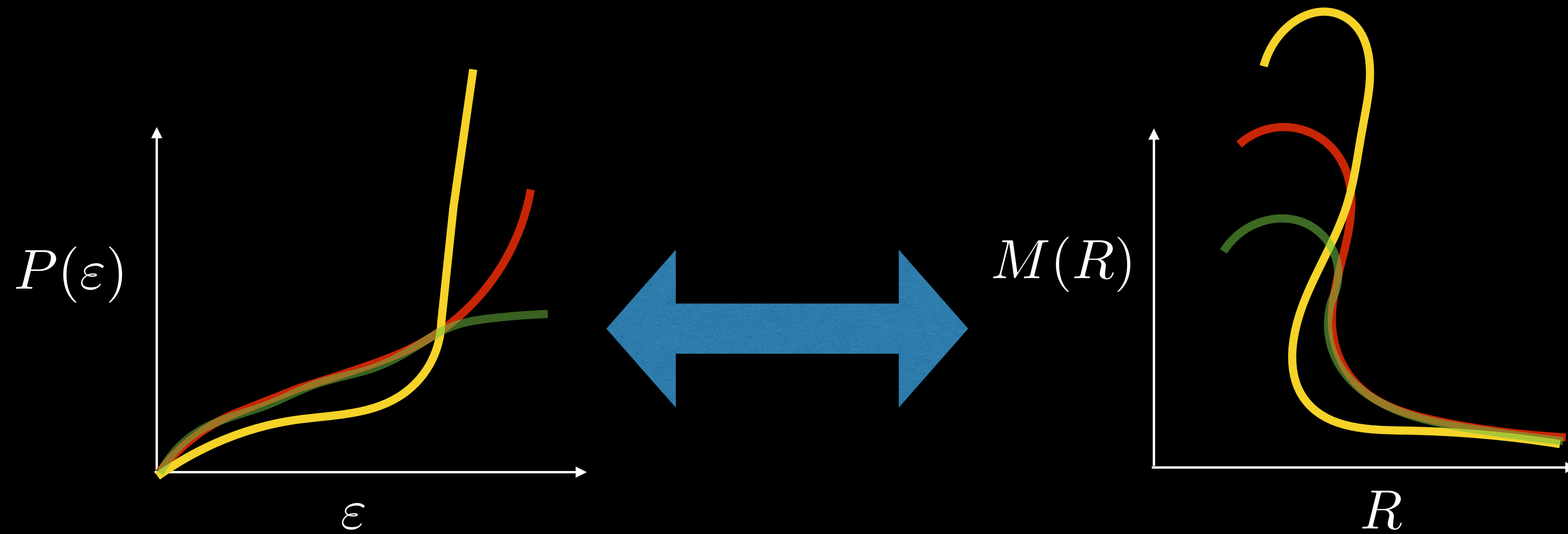
$$P(\epsilon) + \text{Gen.Rel.} = M(R)$$

Neutron Star Mass and Radius



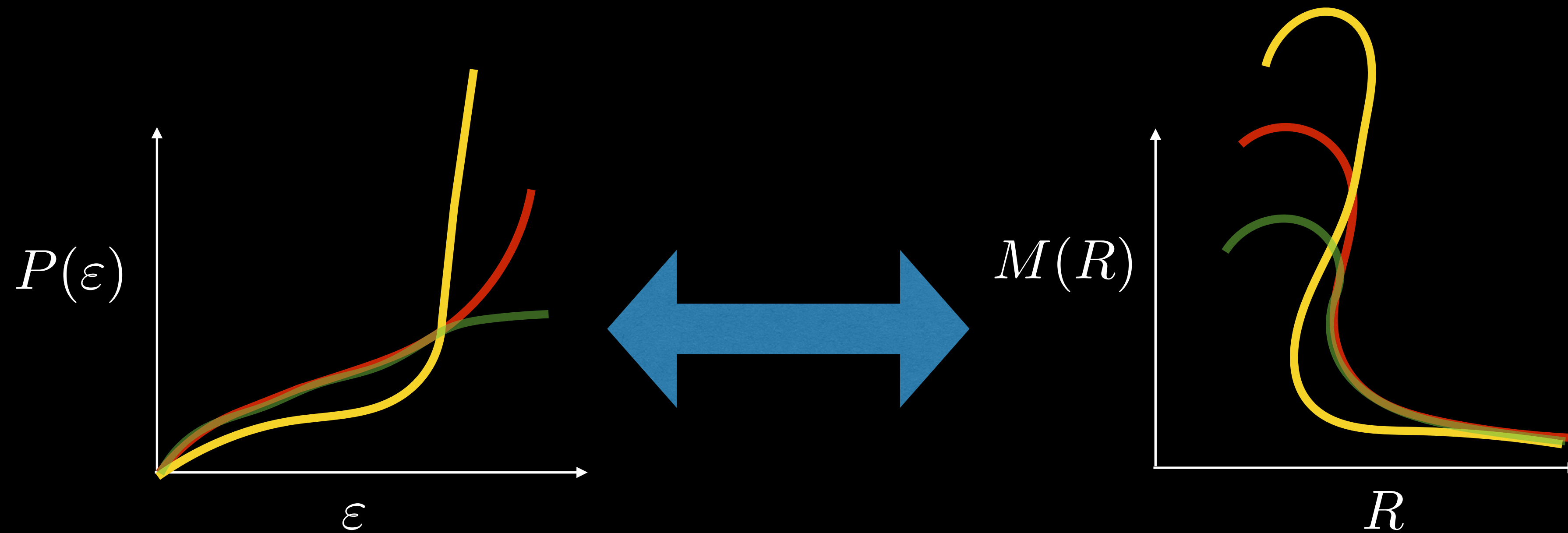
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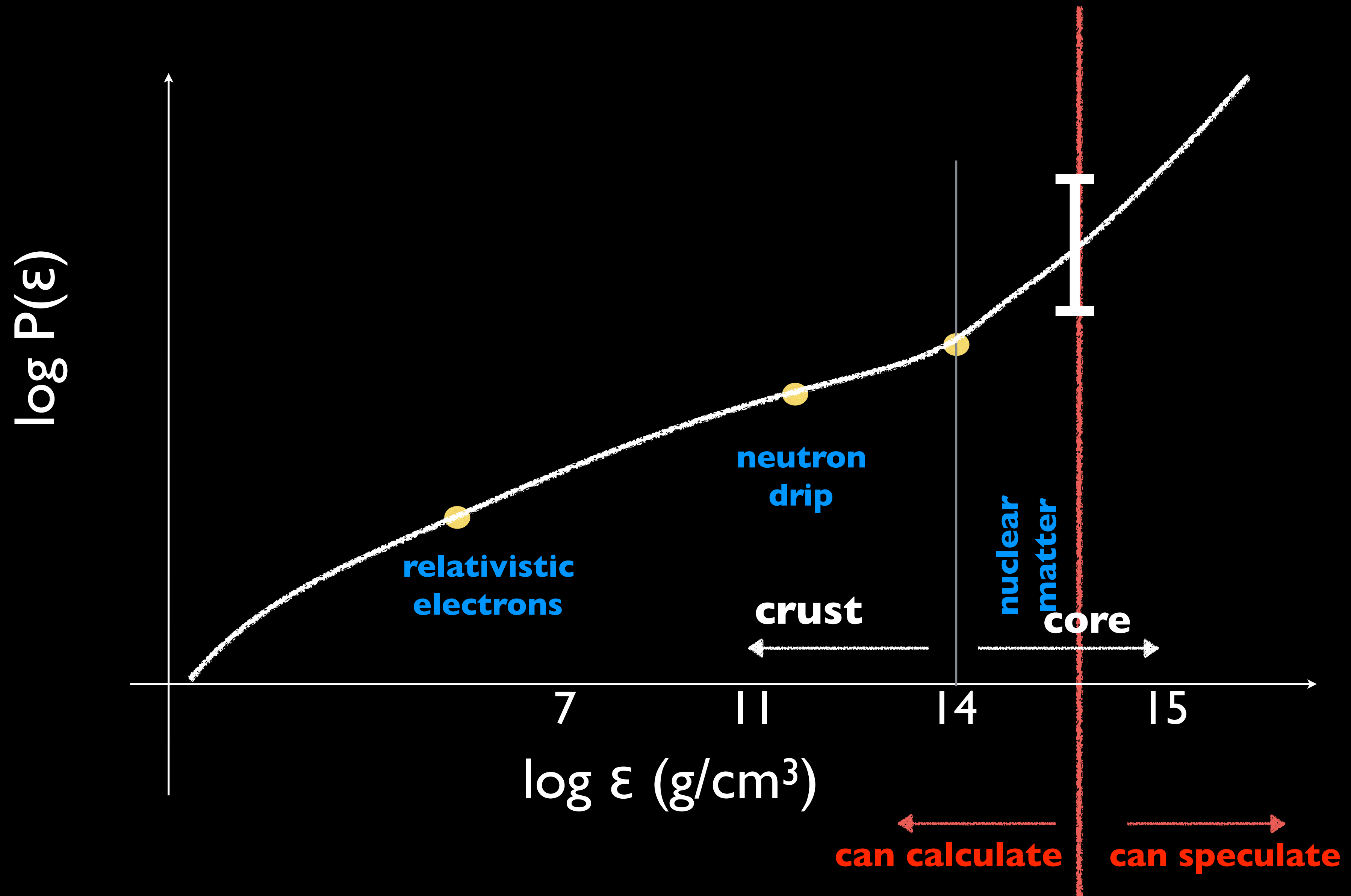
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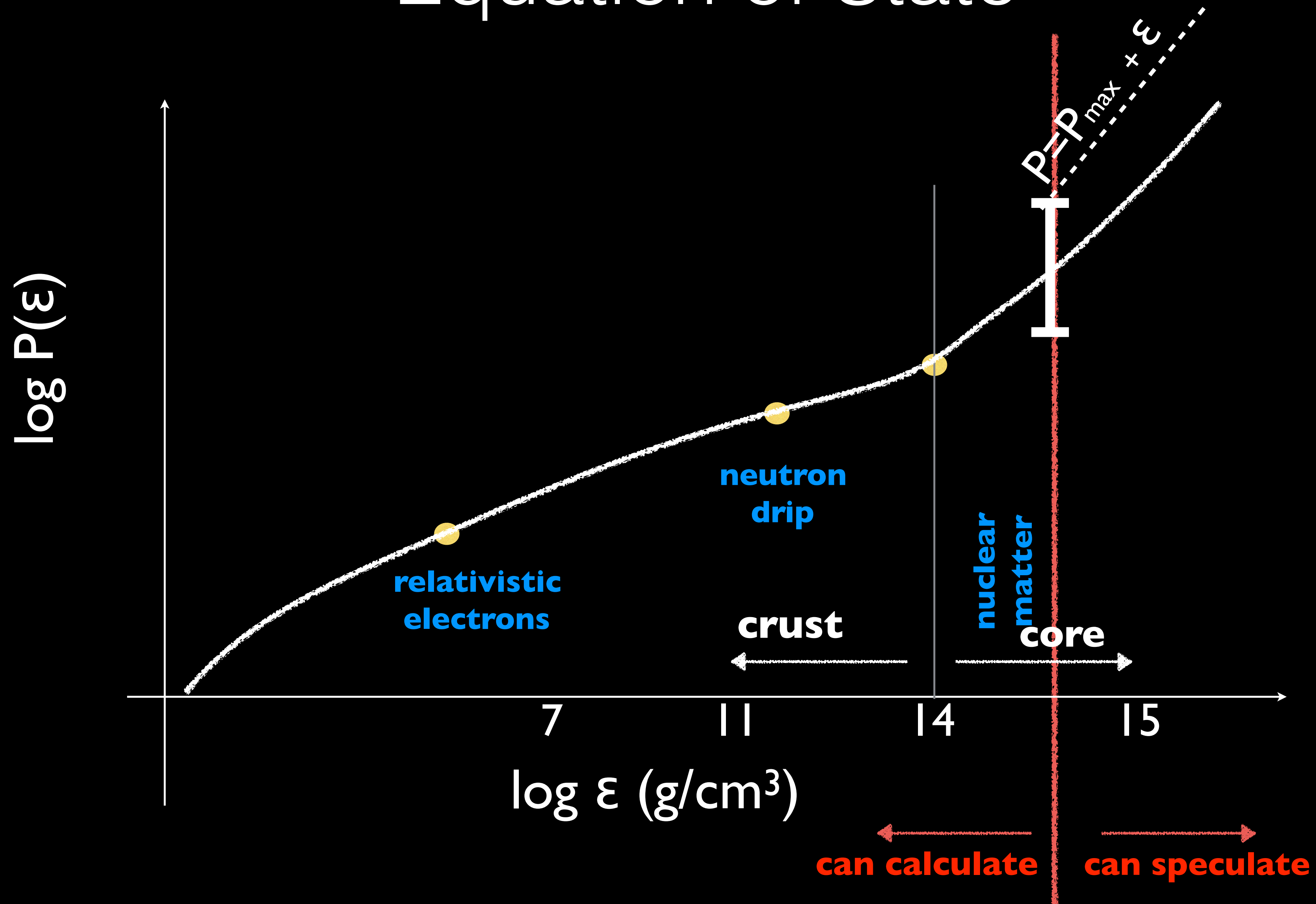
$$P(\epsilon) + \text{Gen.Rel.} = M(R)$$

The relation between pressure and energy density (EoS) uniquely determines the structure of neutron stars.

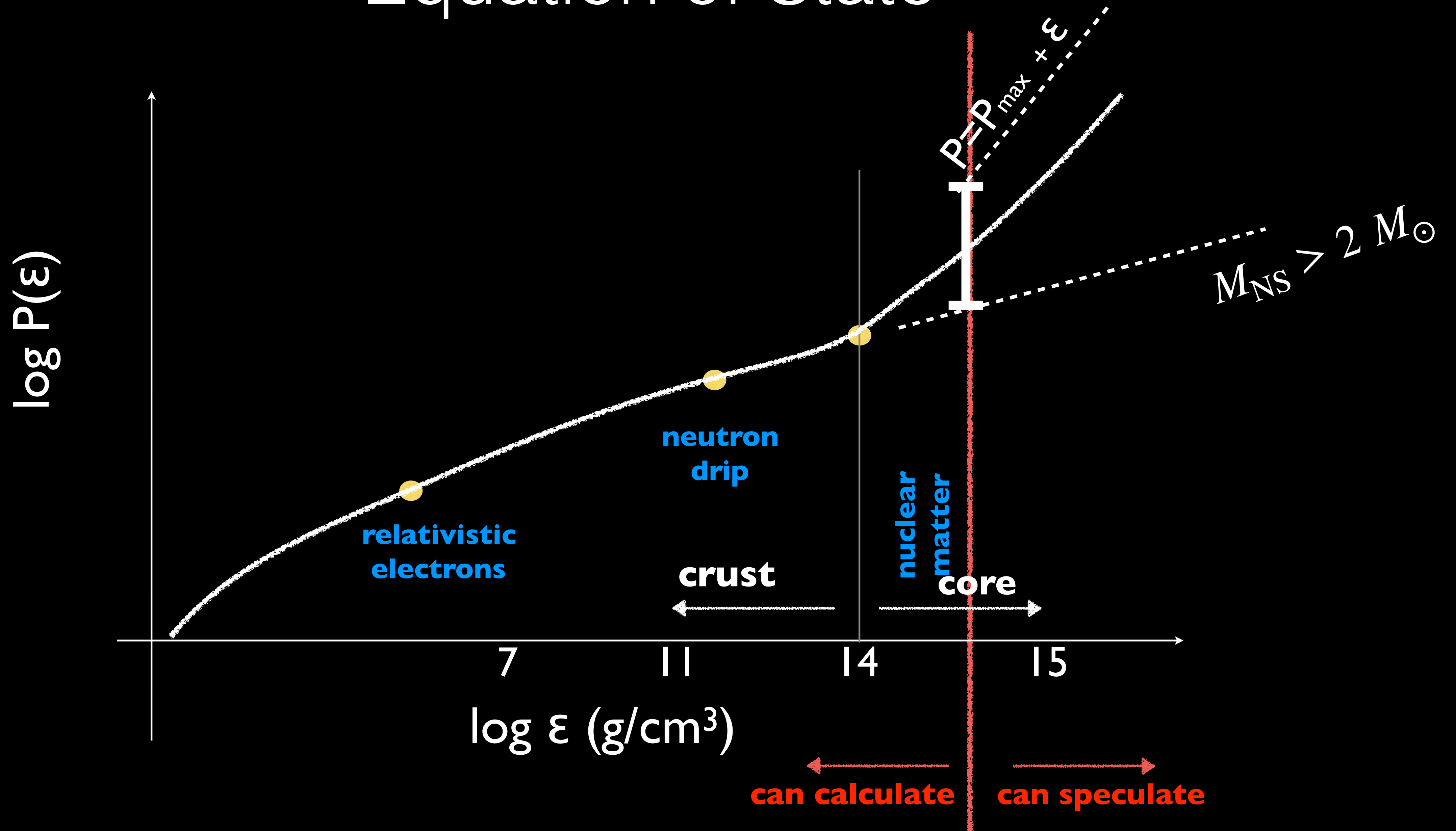
Equation of State



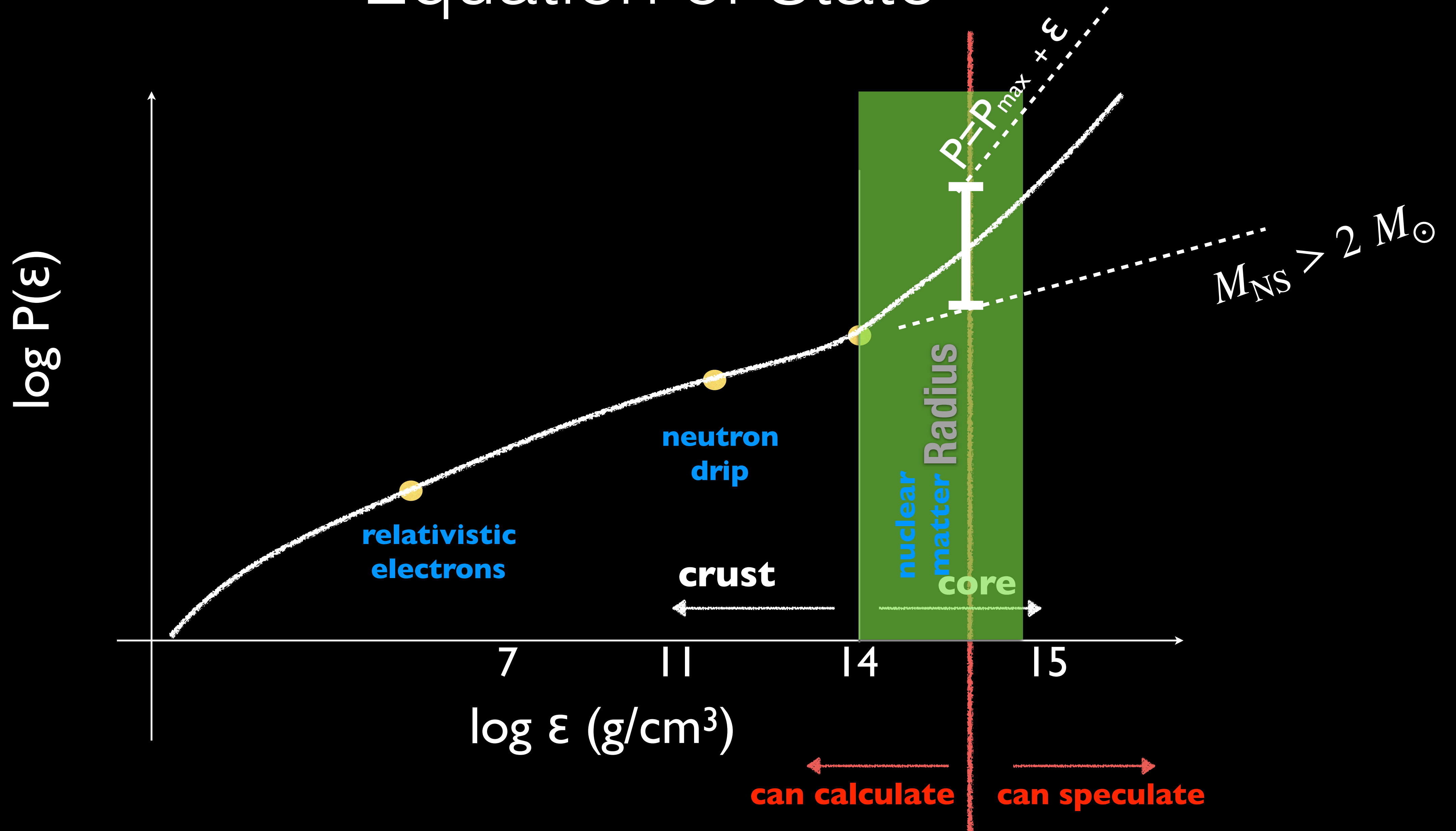
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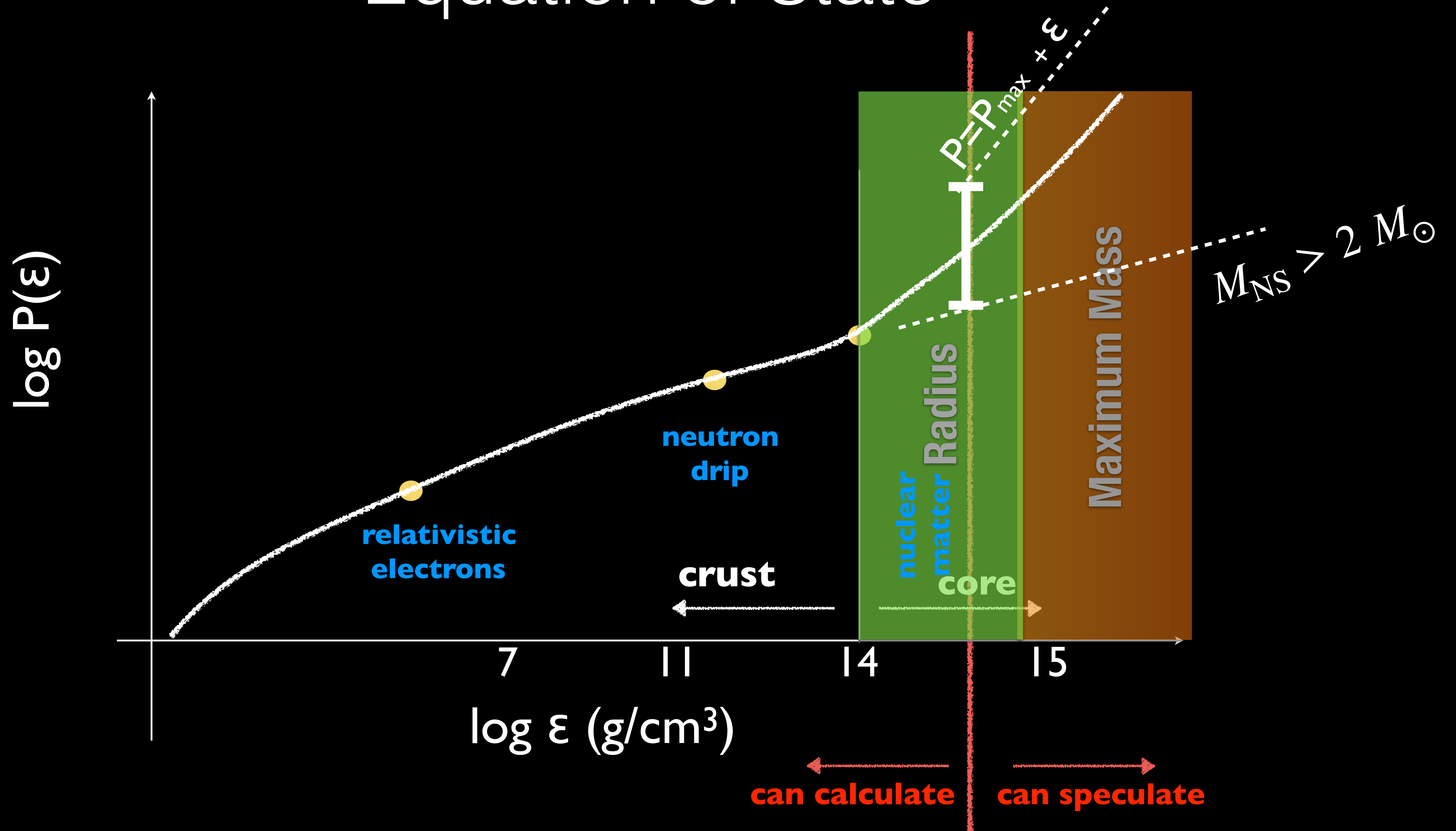
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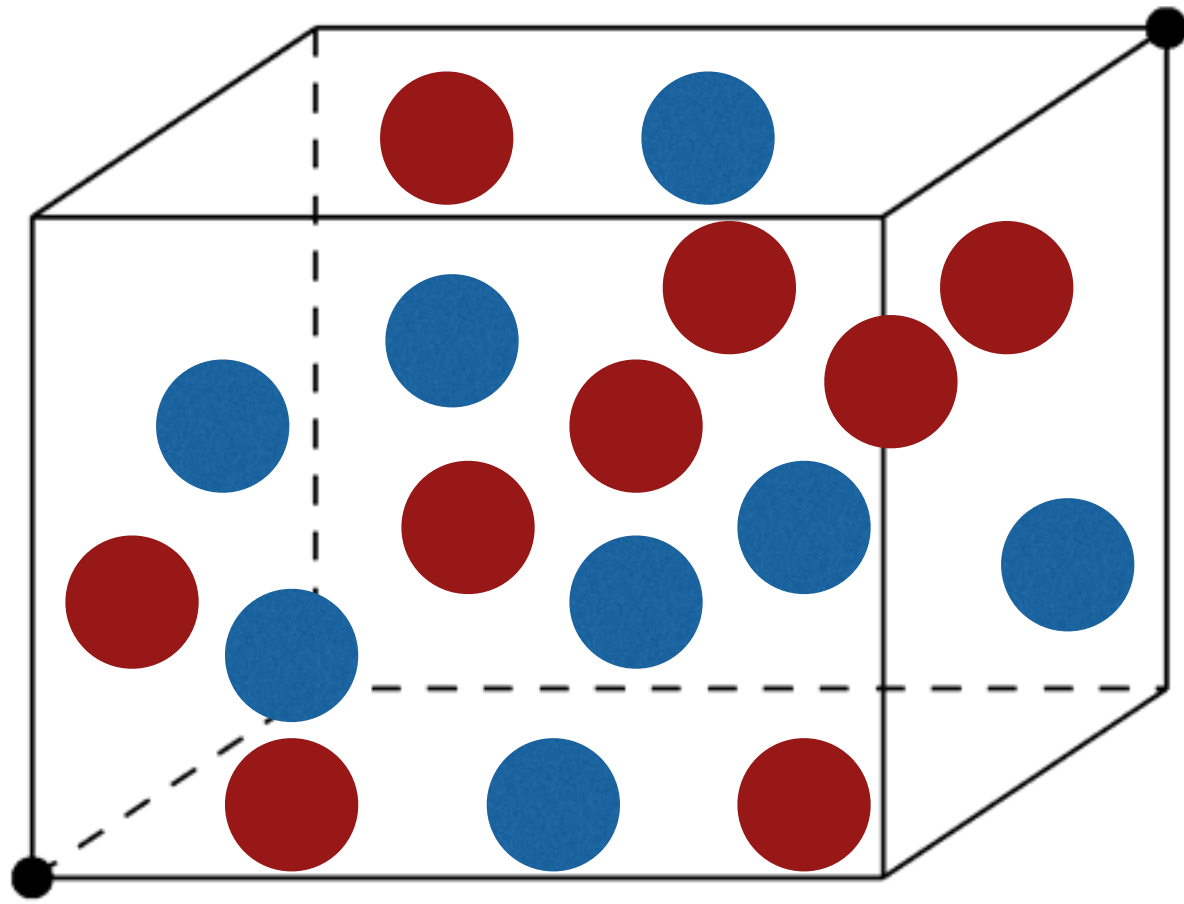
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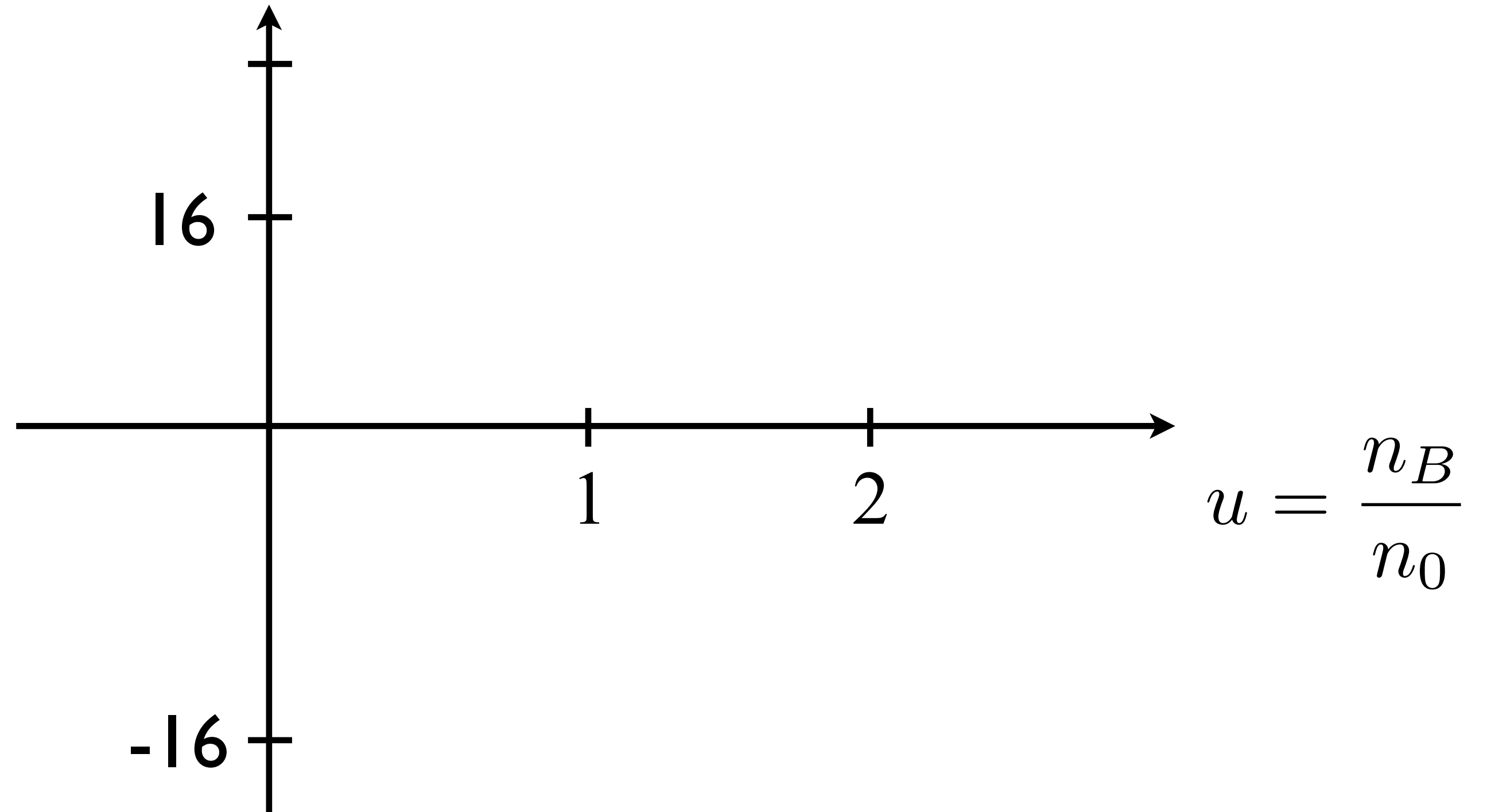


Energy of Uniform Matter: Nucleons in a Large Box



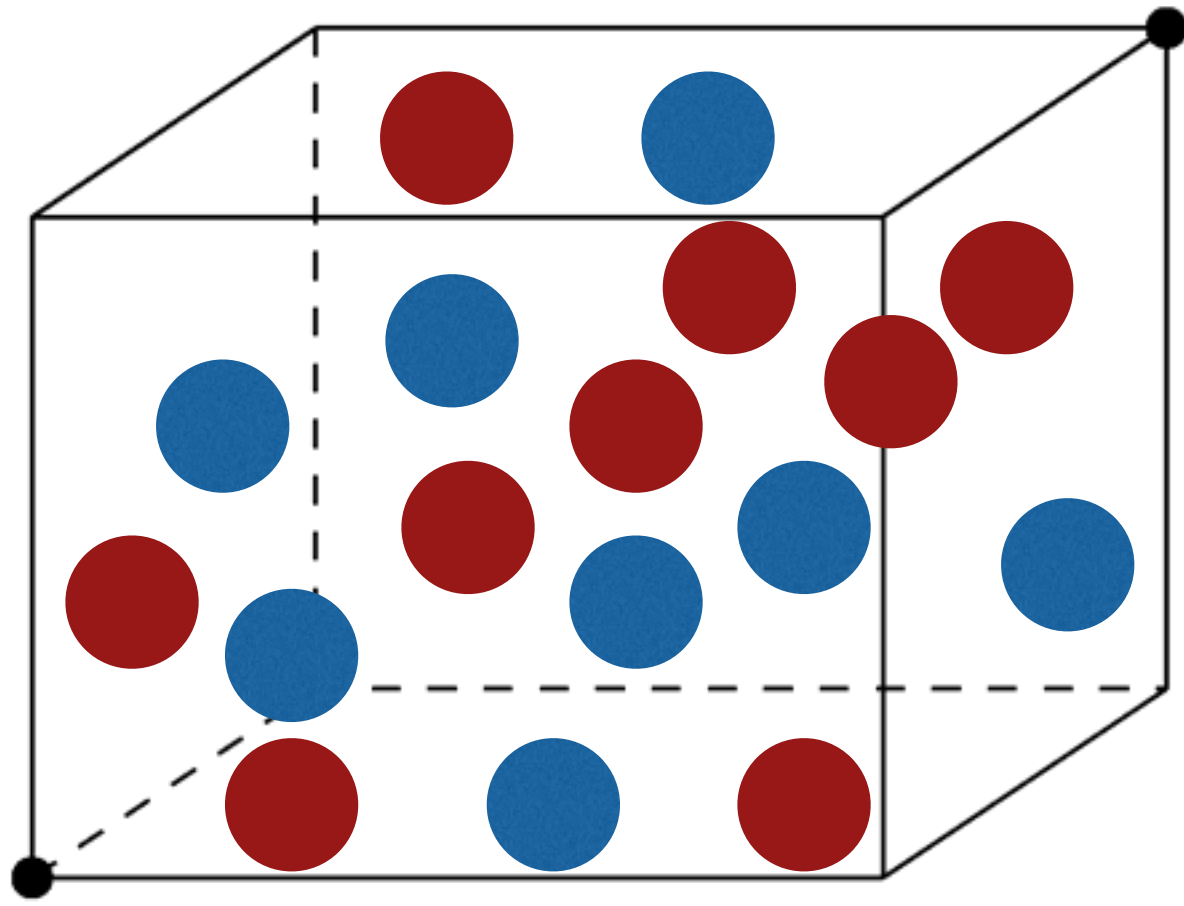
Given a Hamiltonian and a many-body approximation we can calculate the energy of N neutrons + M protons in a box.

$E(\rho_n, \rho_p)$ MeV



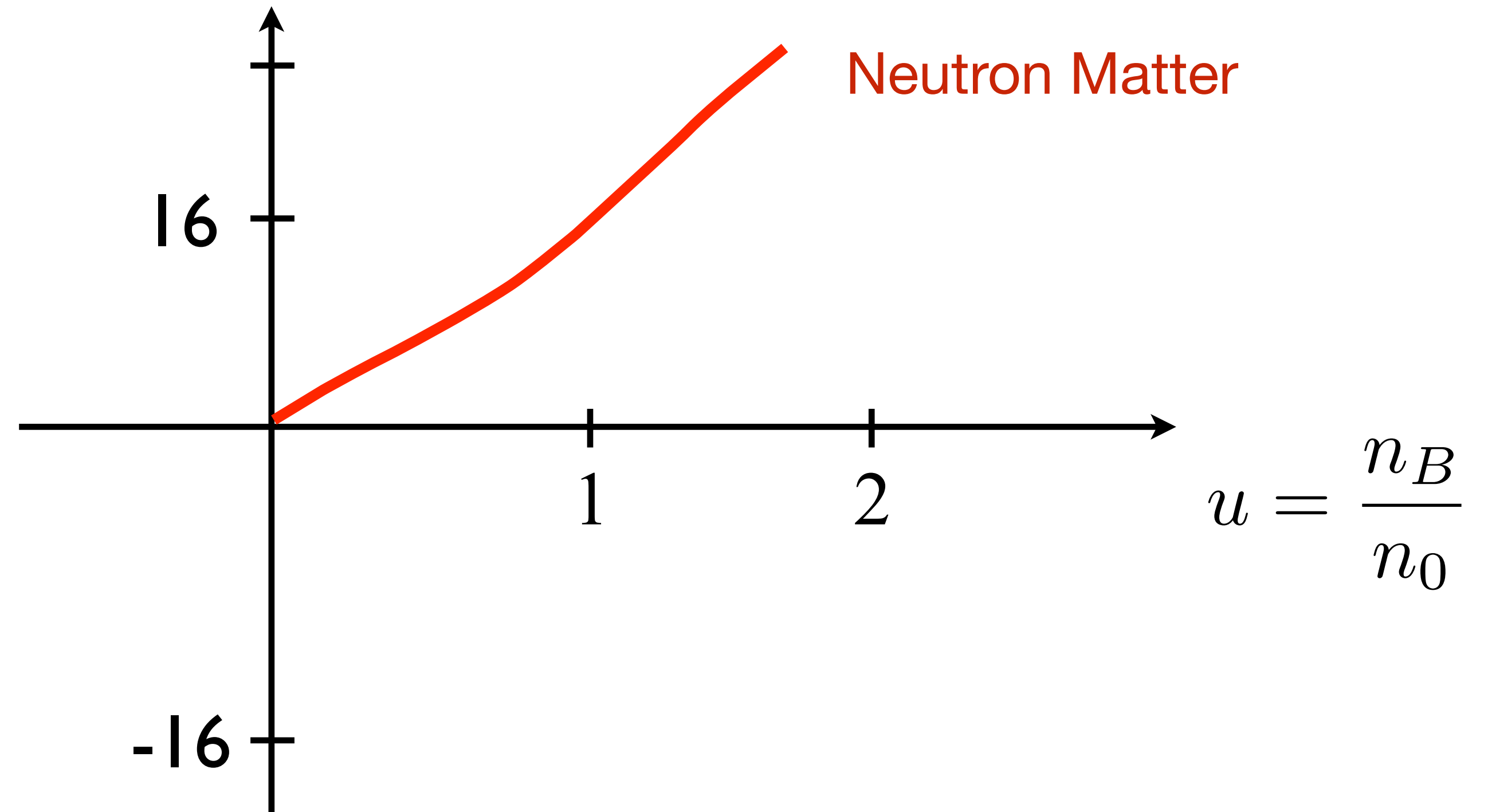
$$P(n) = n^2 \frac{\partial(E/A)}{\partial n}$$

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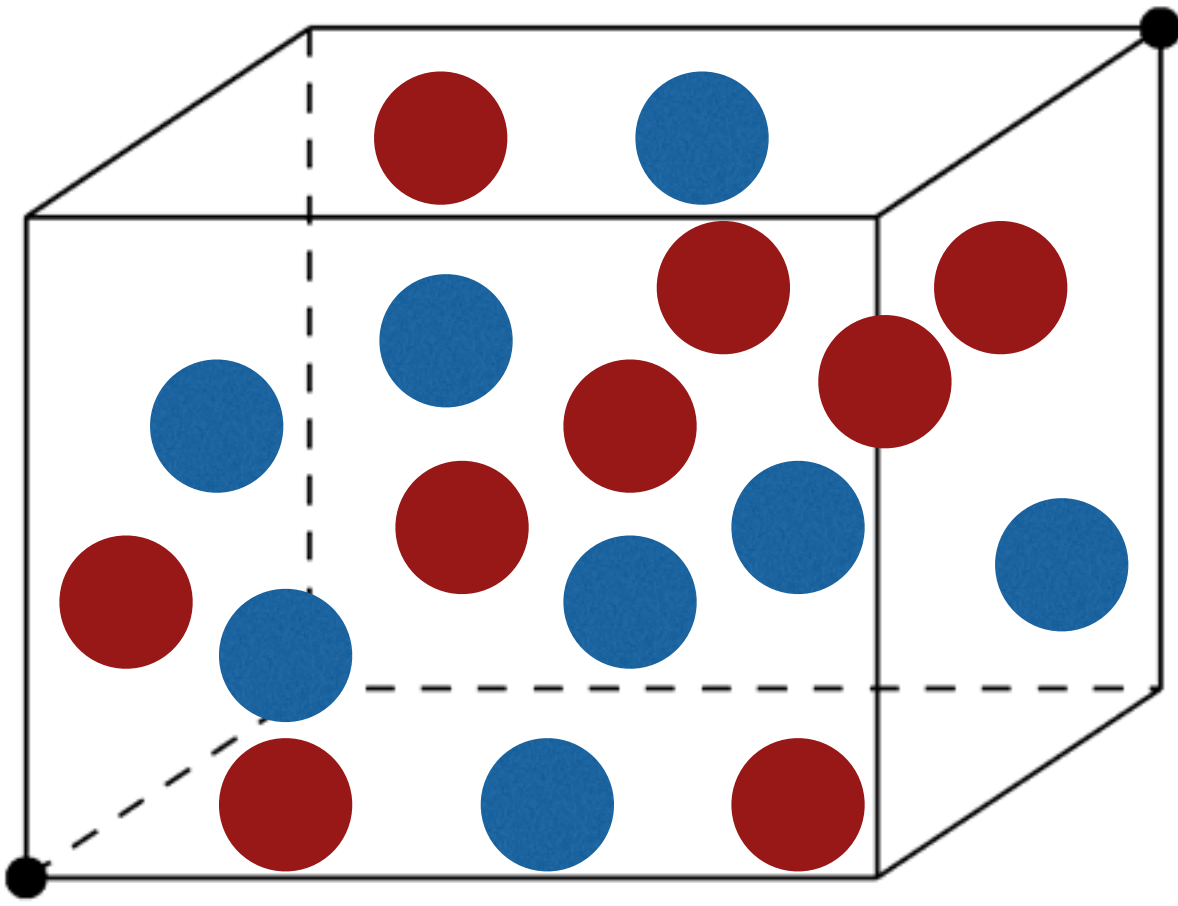
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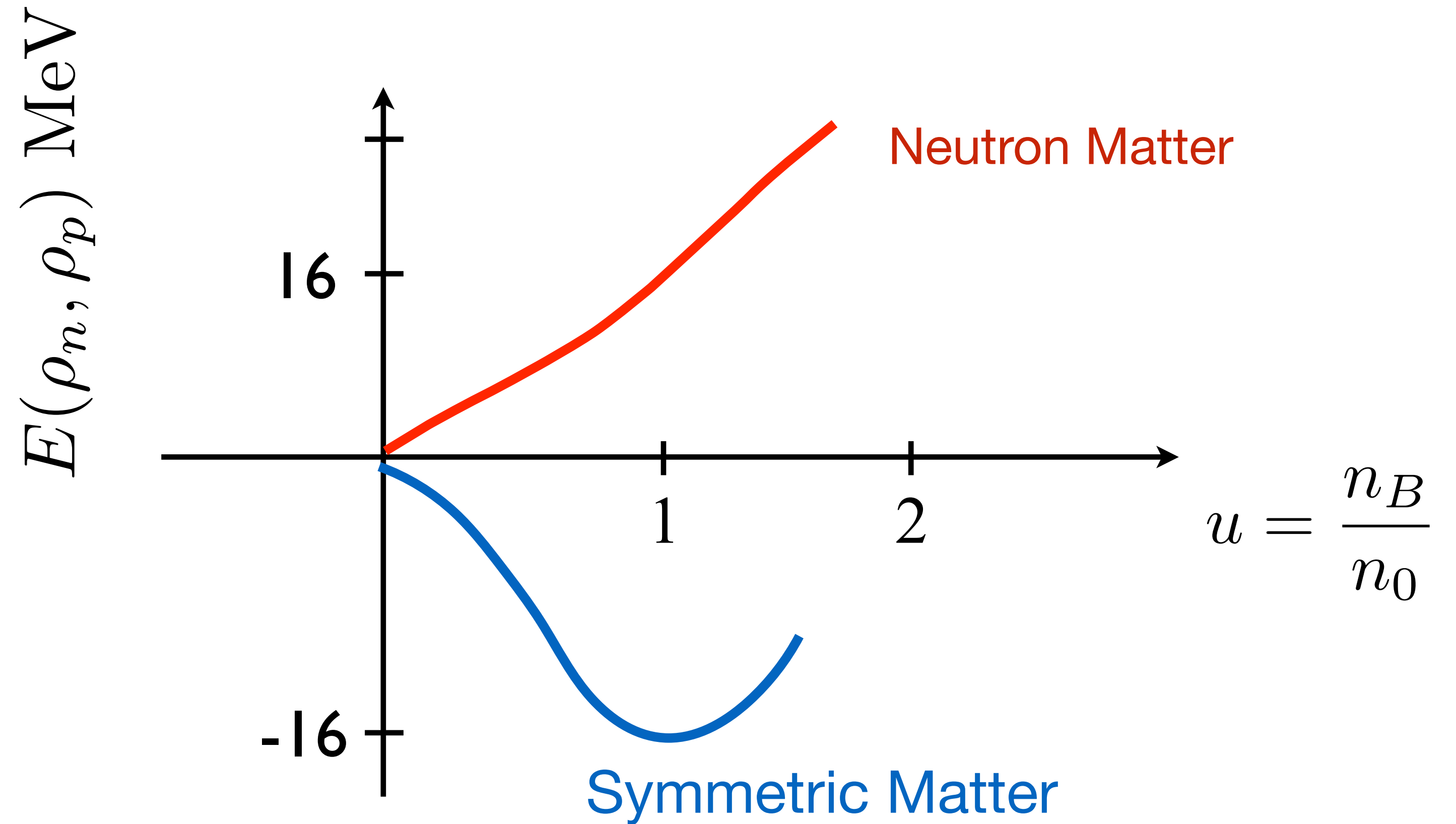


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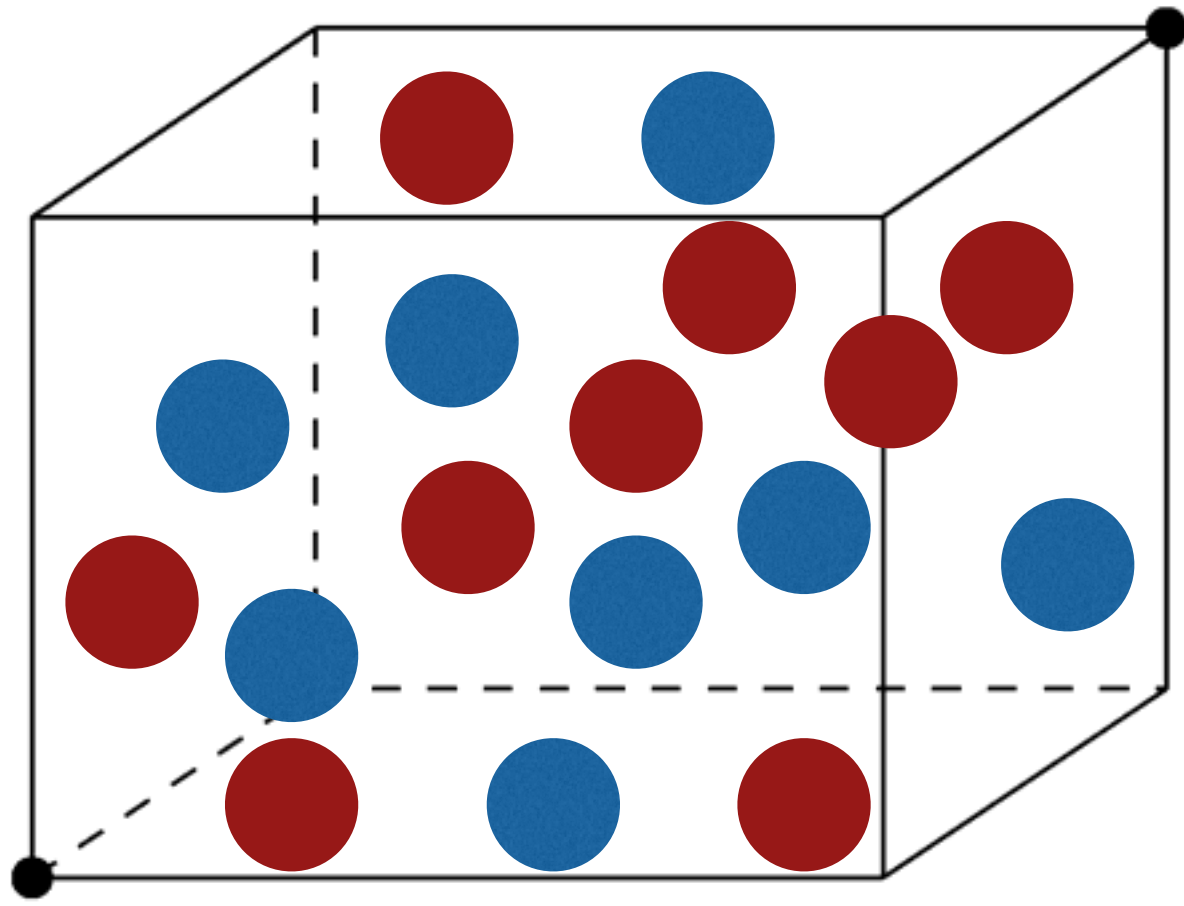


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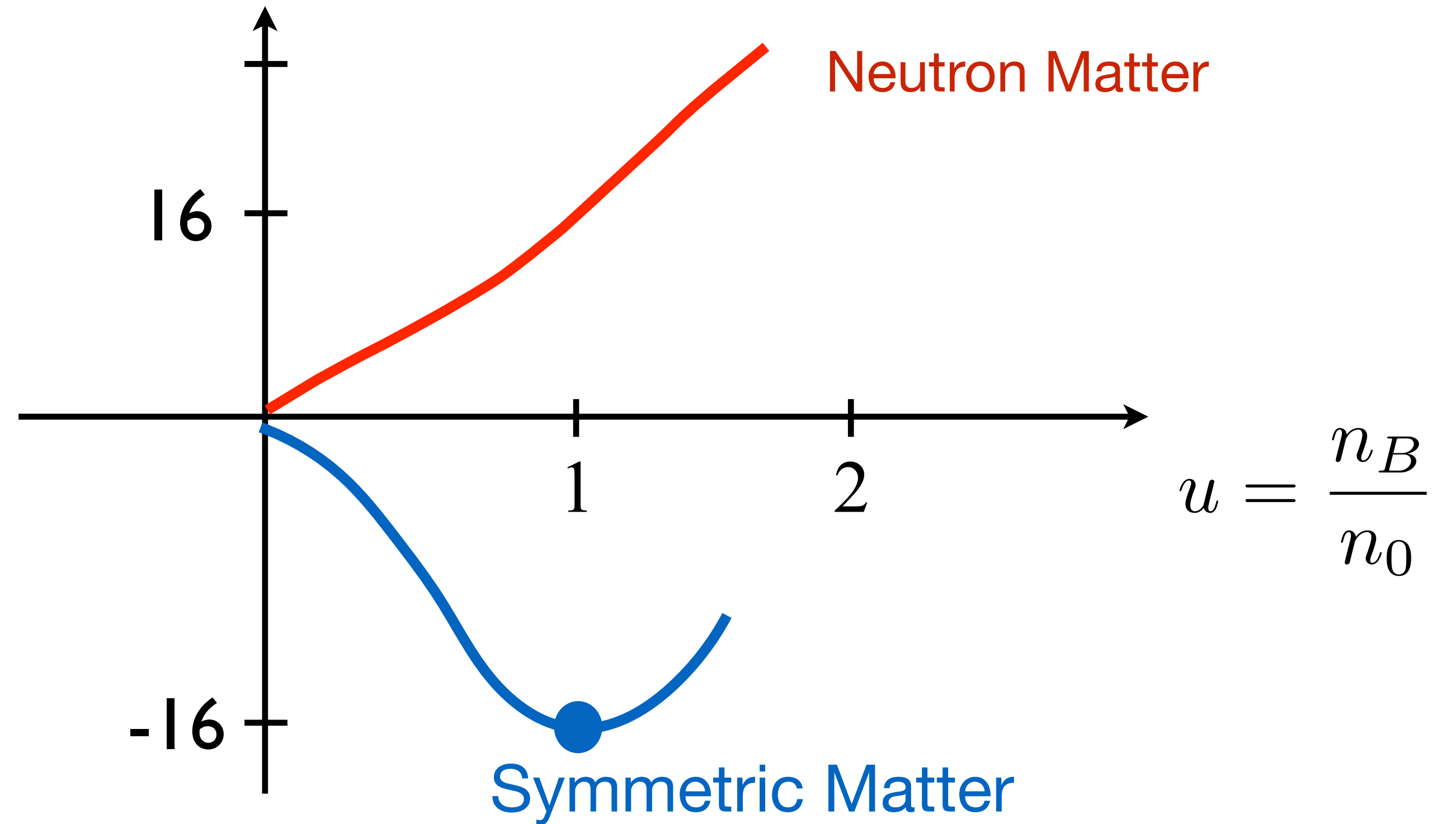
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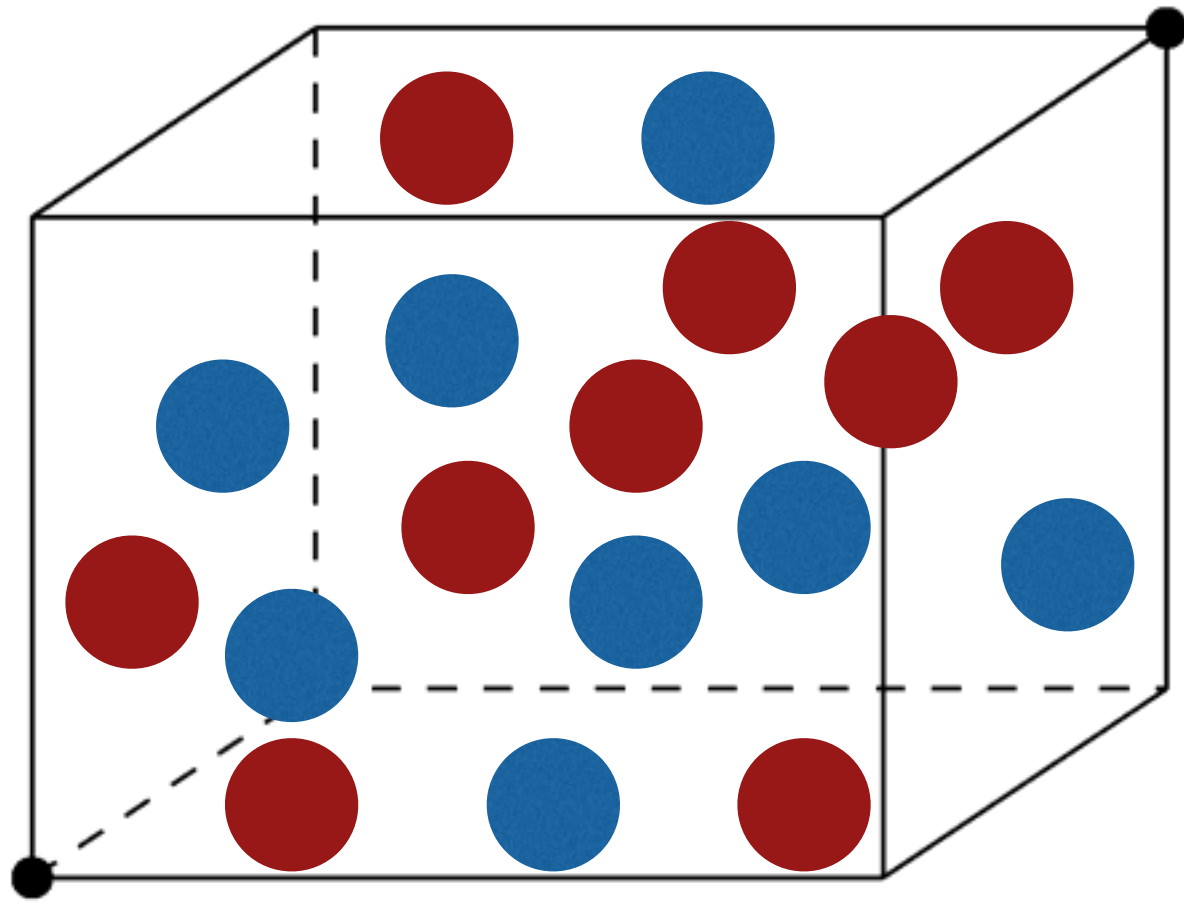
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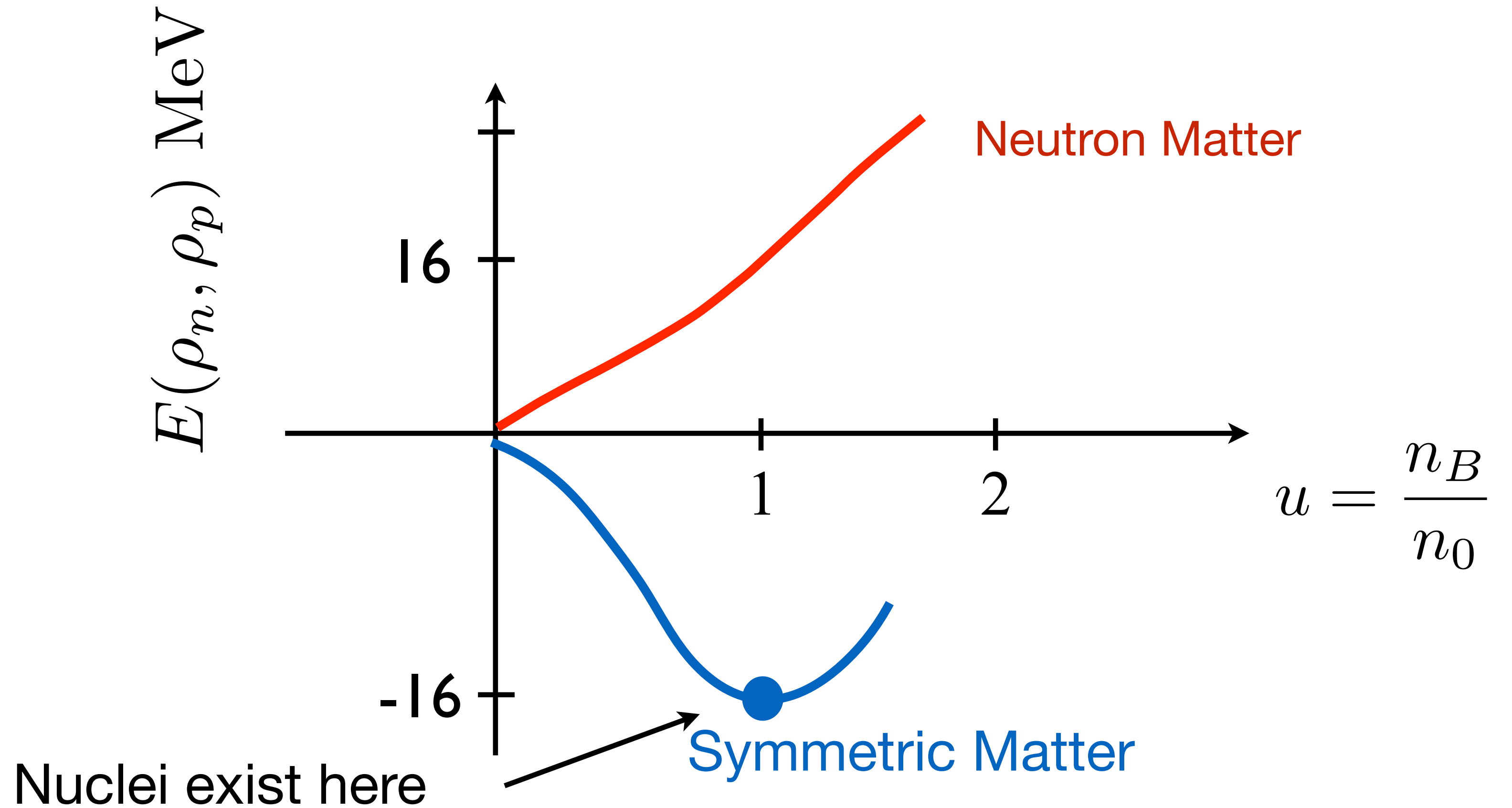


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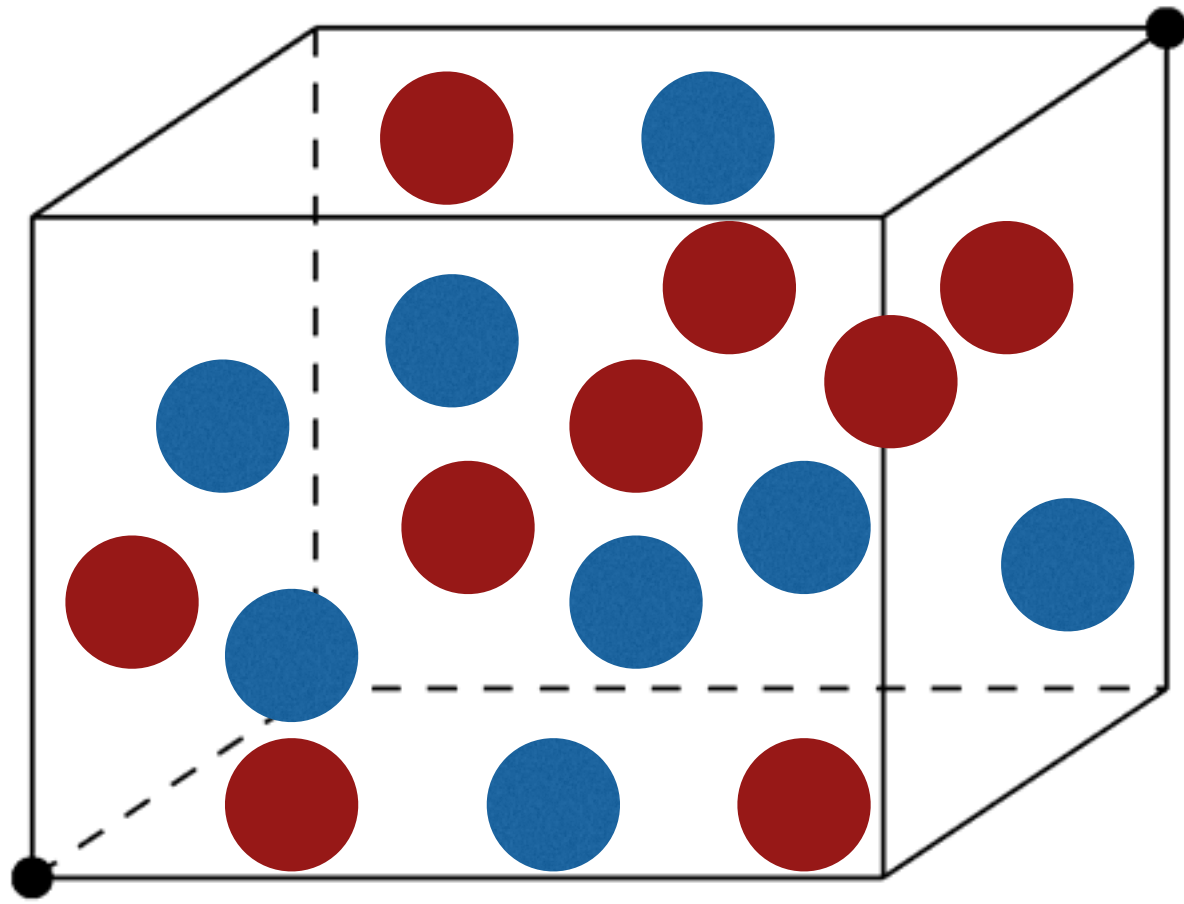


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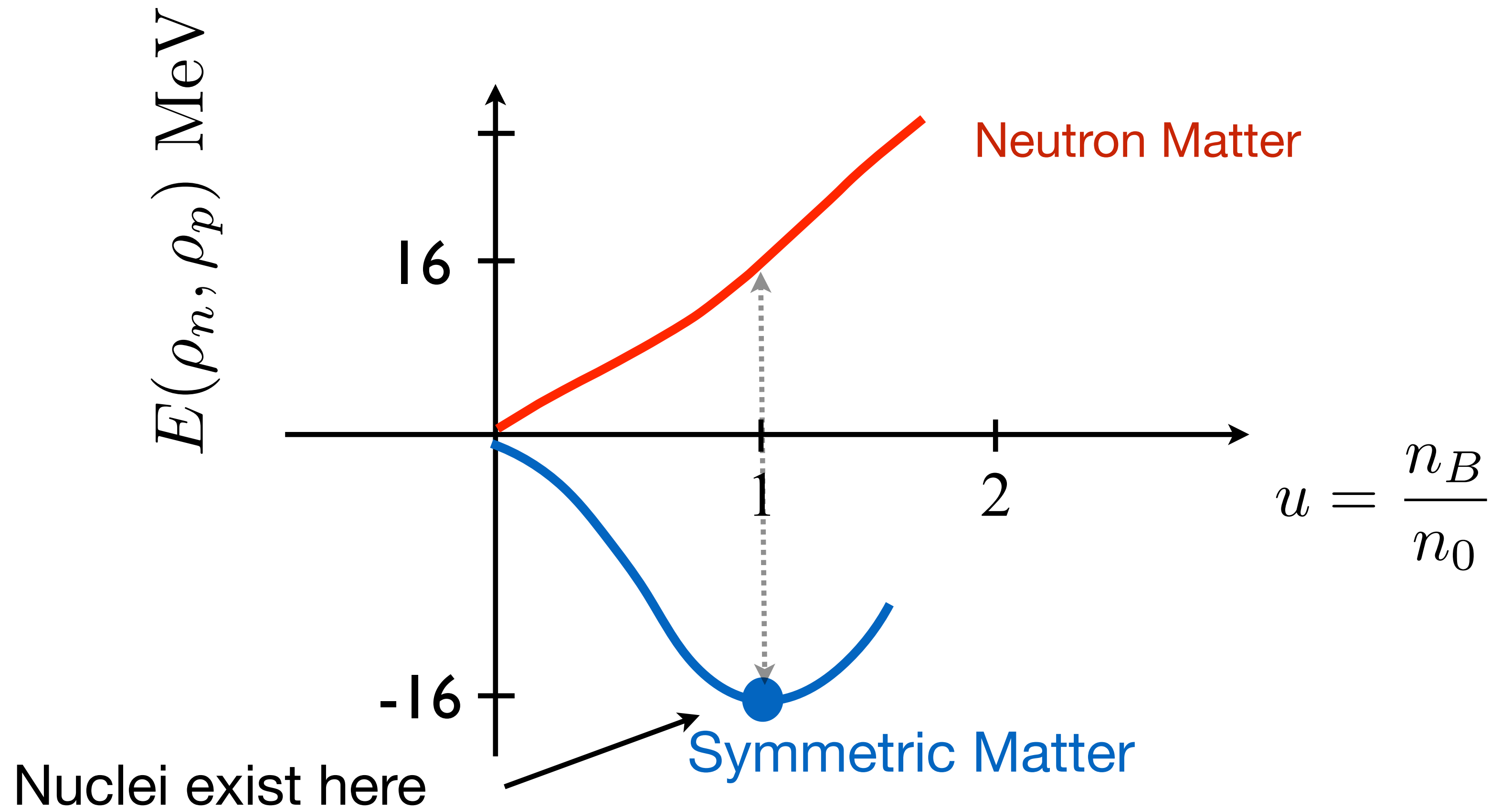


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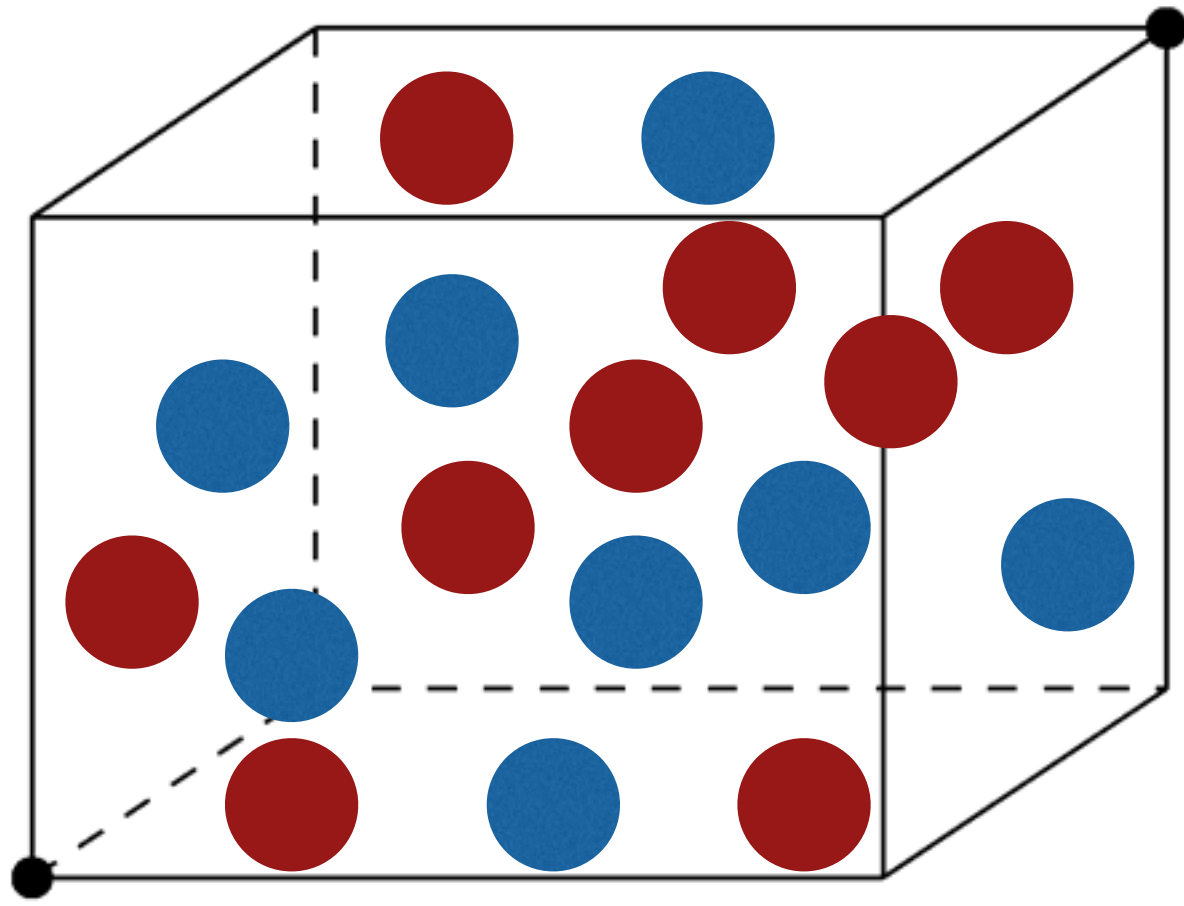


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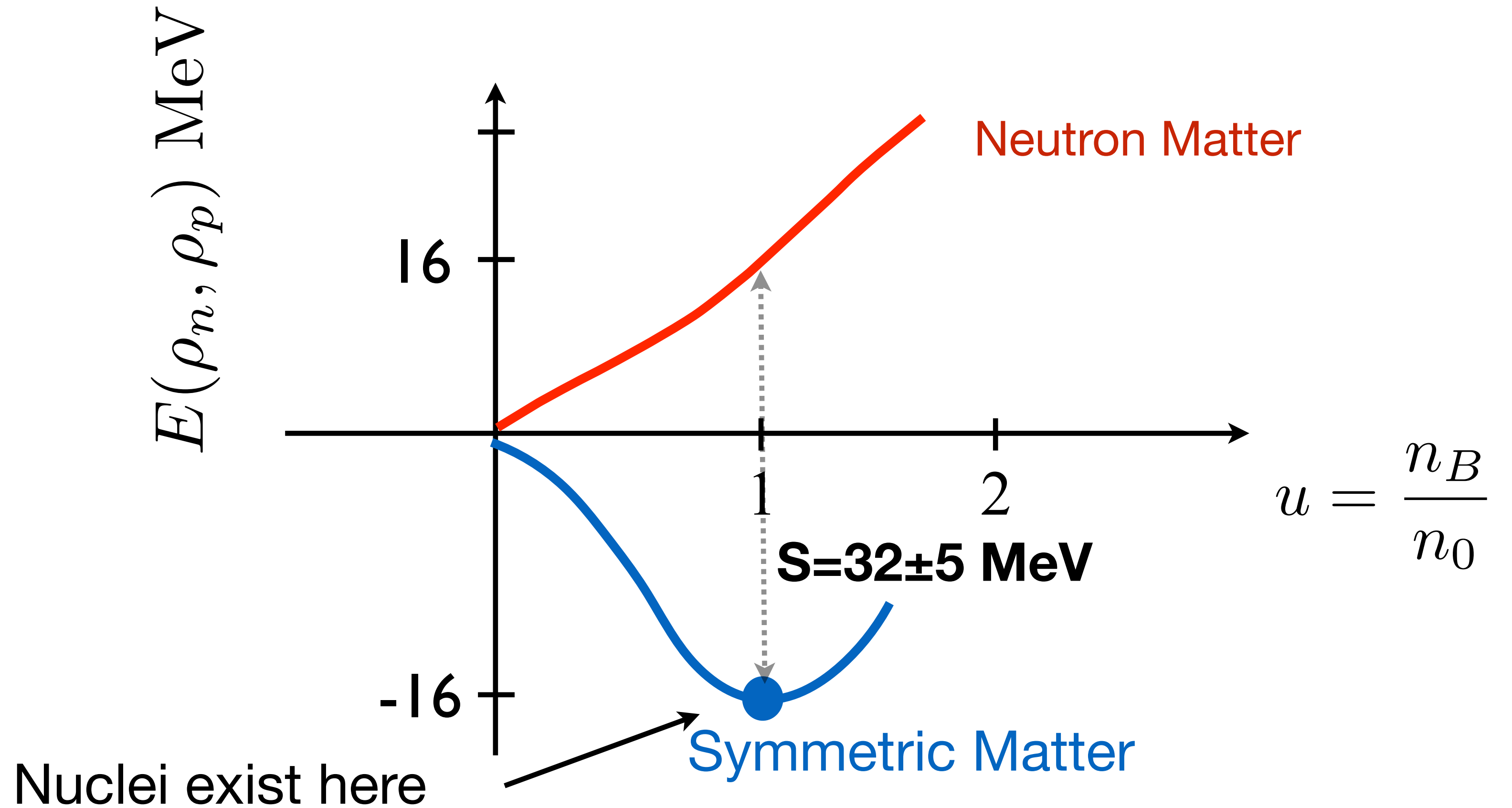


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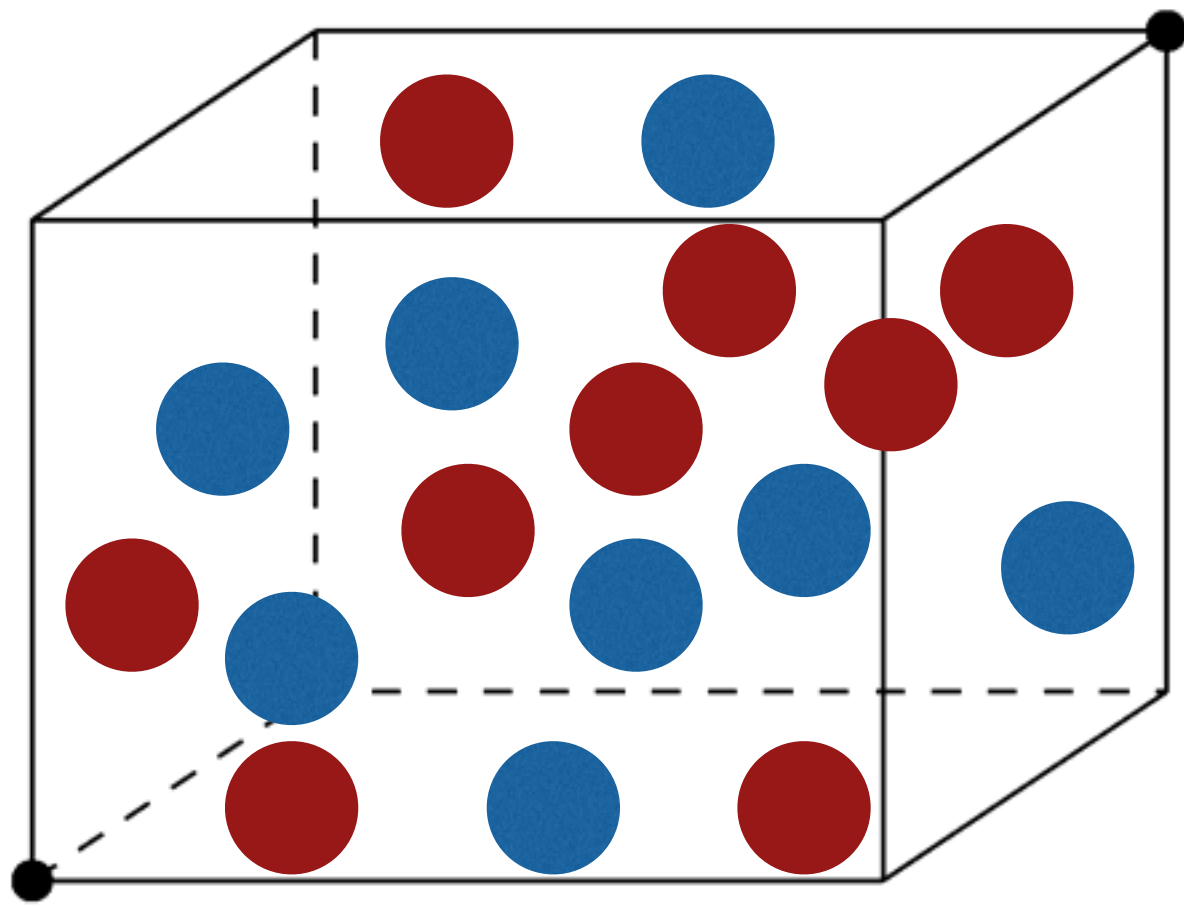


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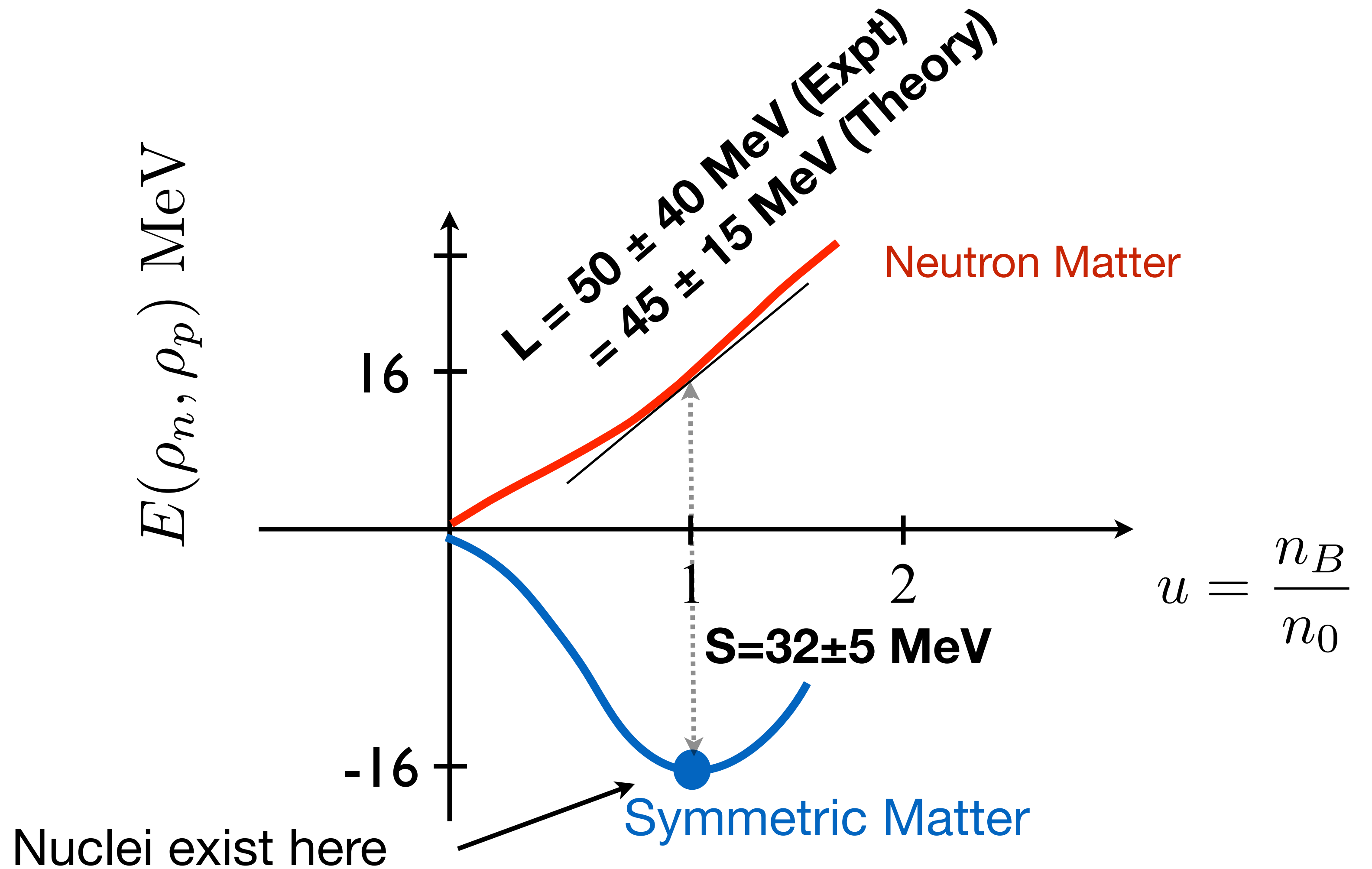


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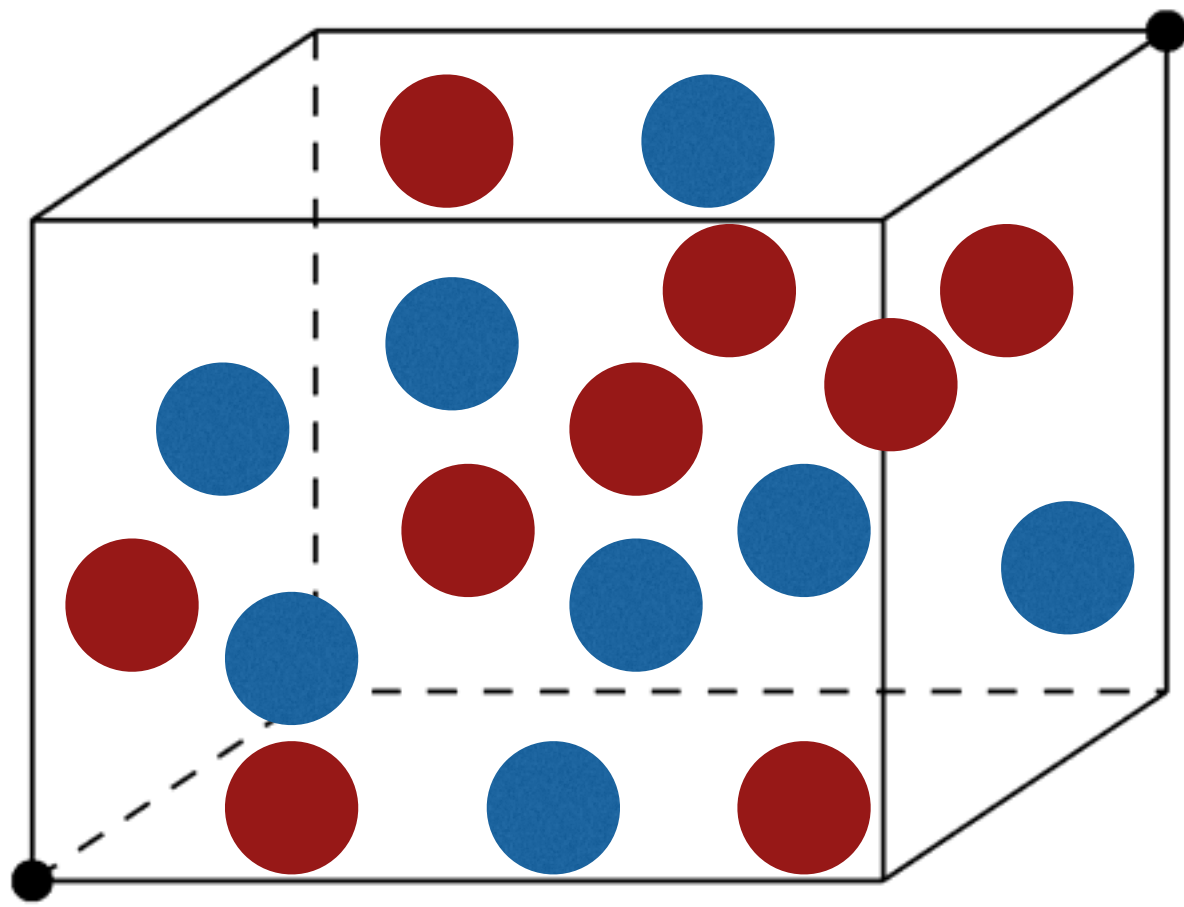


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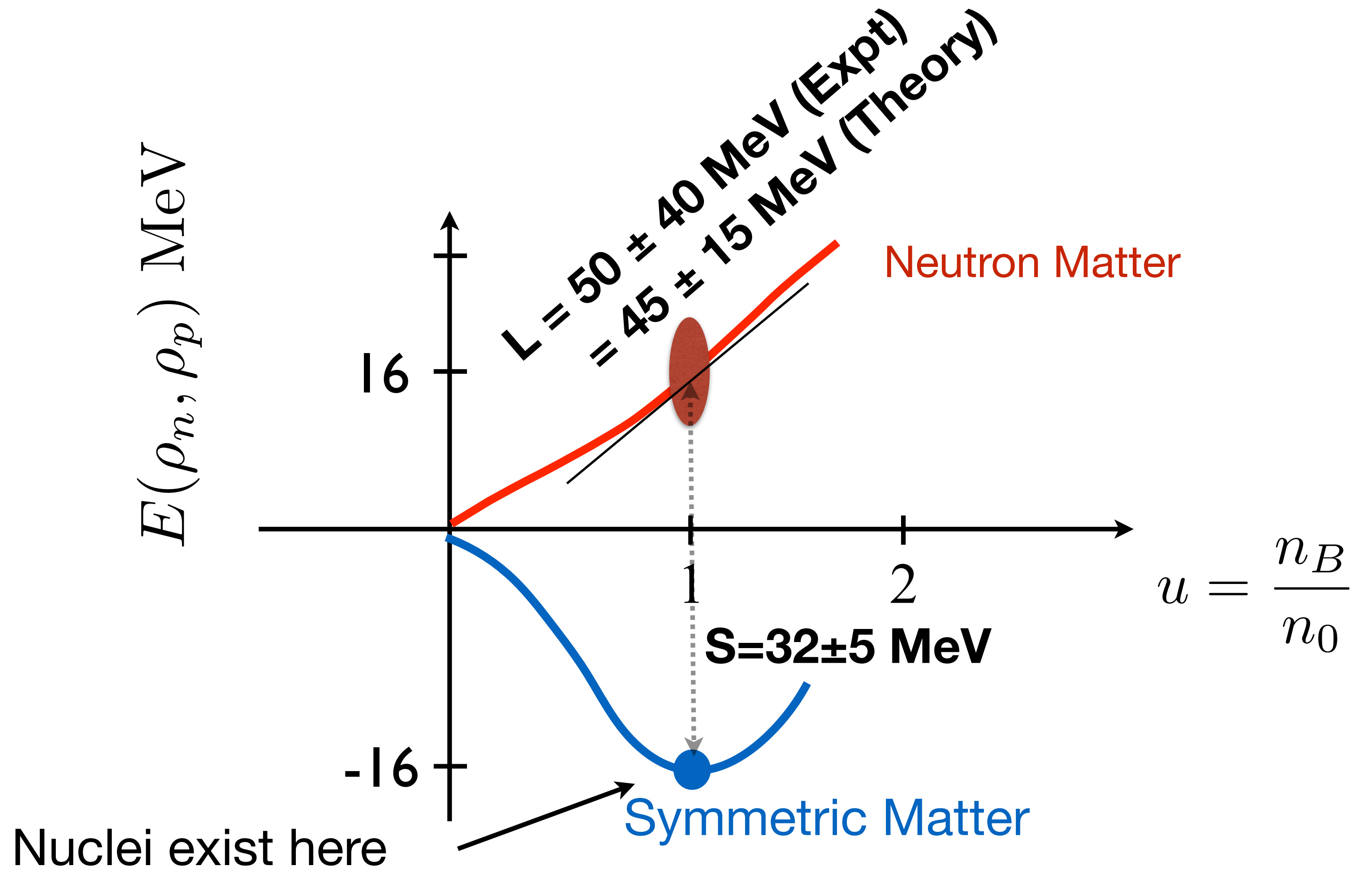


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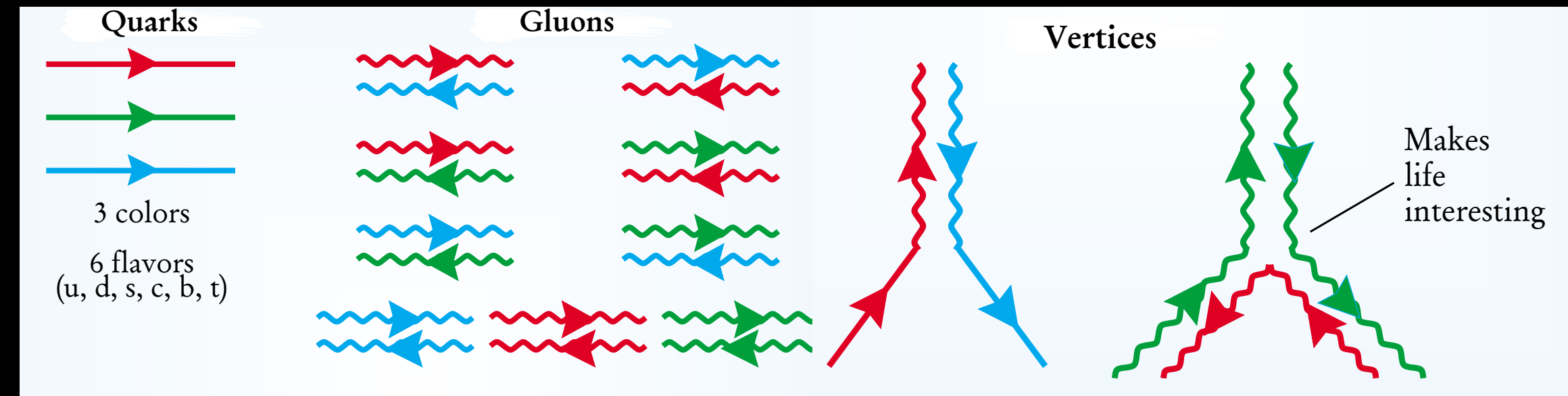


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Nuclear Interactions

QCD (Lagrangian) is simple is write down

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

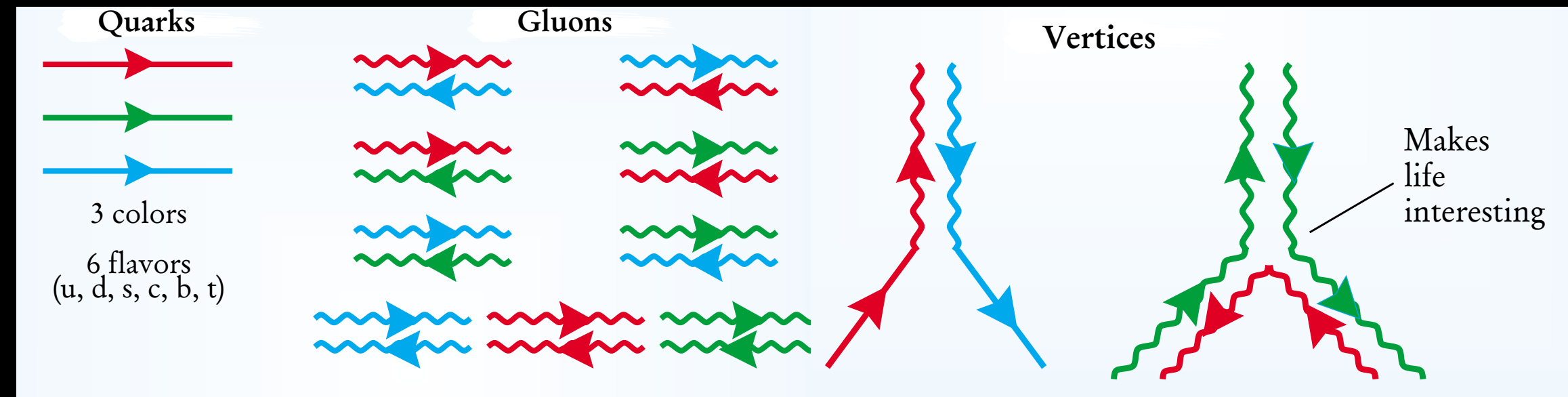


F. Wilczek, Physics Today (2000)

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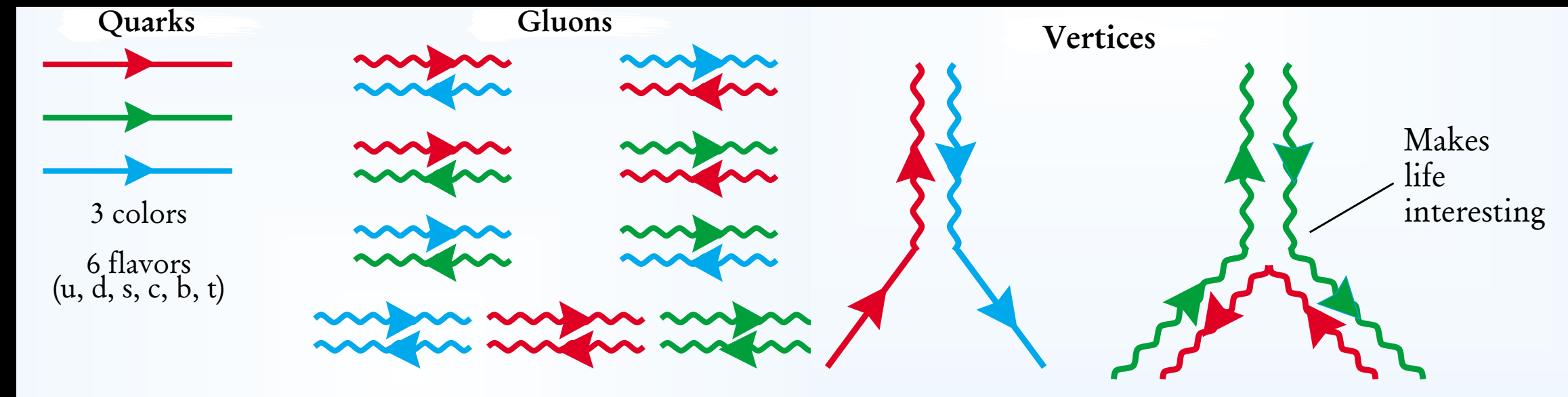


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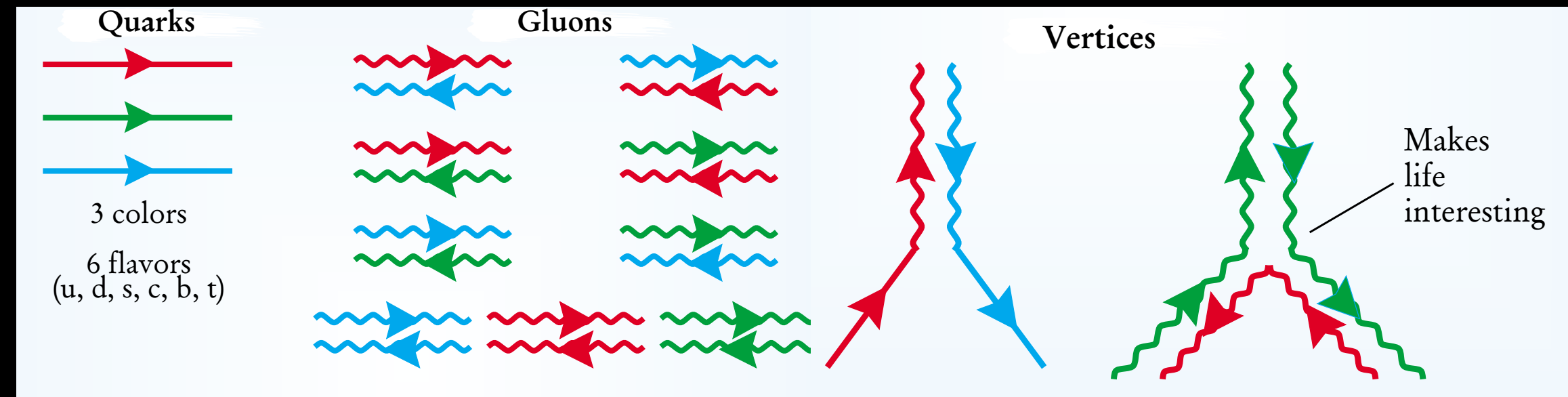


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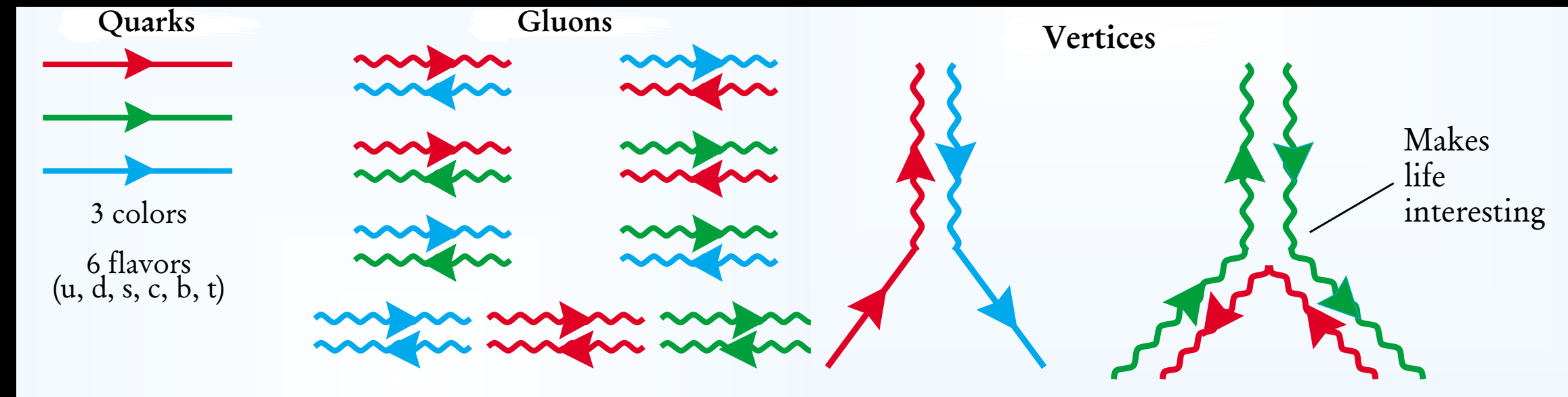
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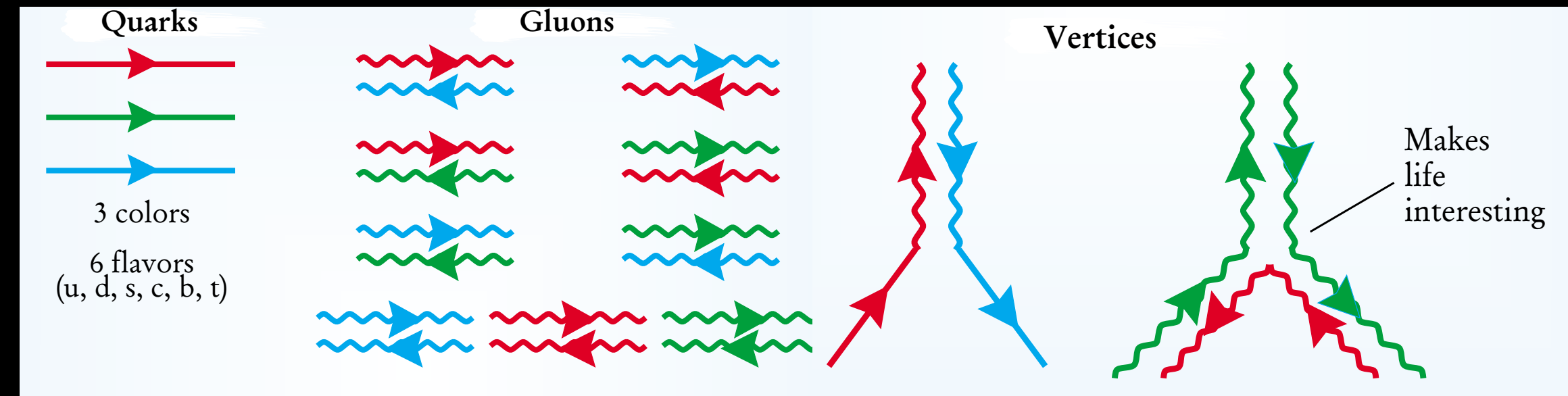
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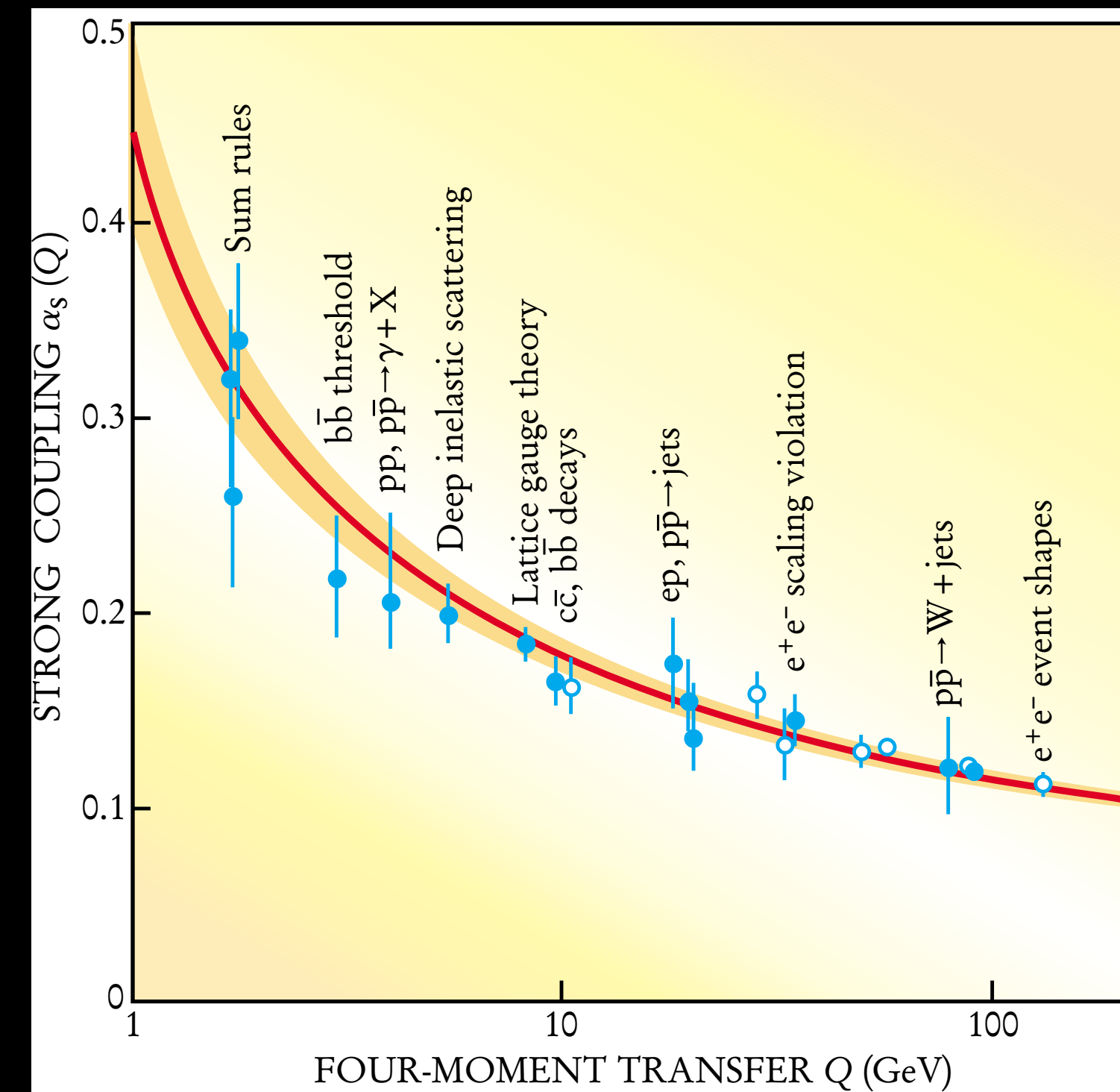
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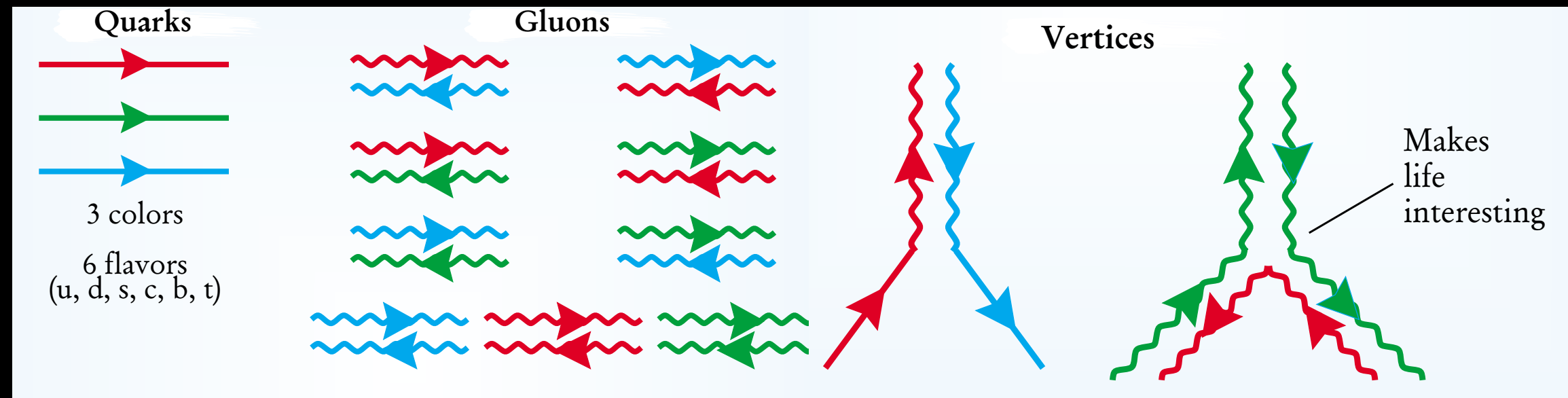
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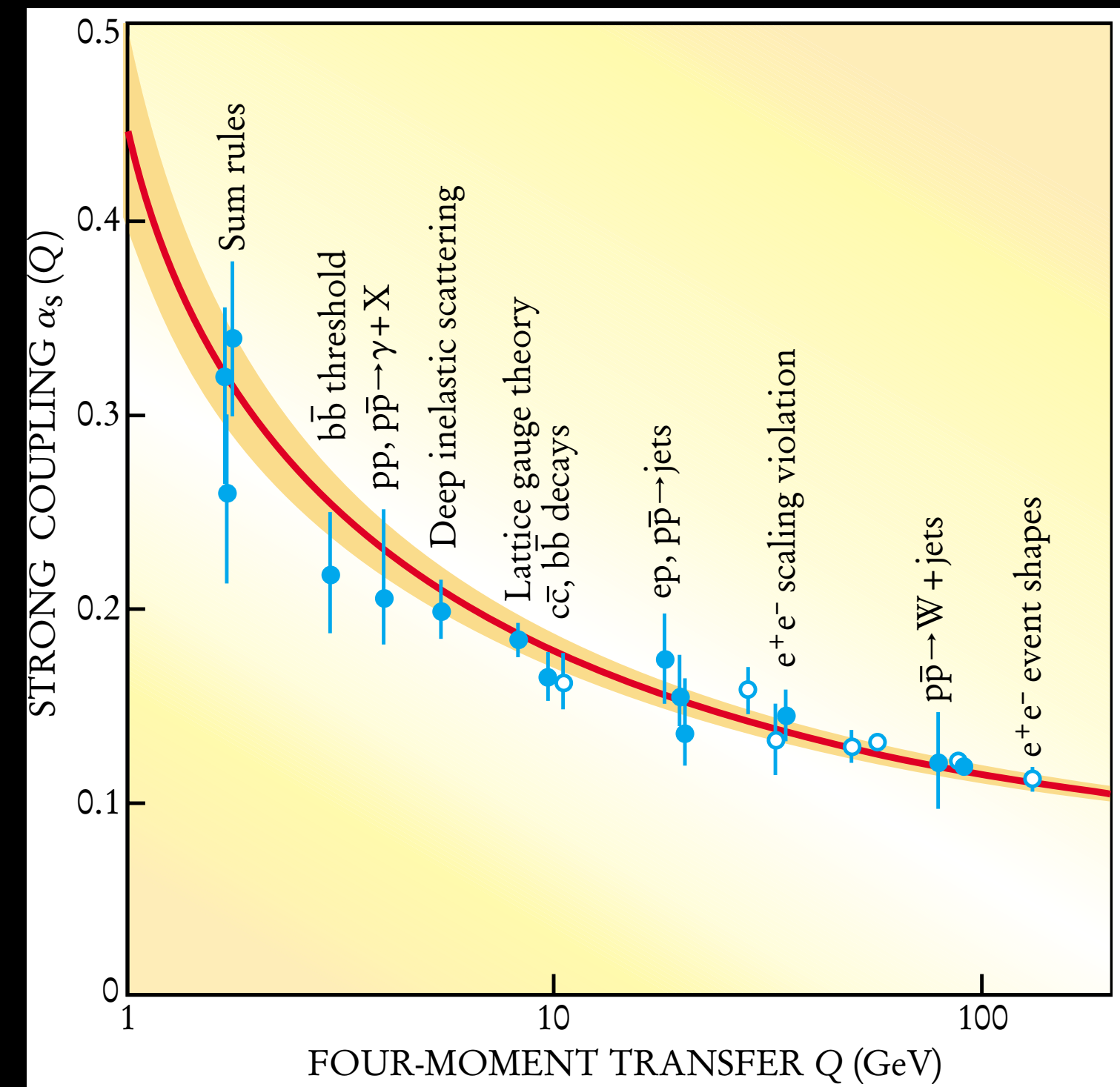
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The low energy QCD vacuum is non-perturbative:

- It confines quarks to color singlet states.
- Spontaneously breaks chiral symmetry.

F. Wilczek, Physics Today (2000)



Nuclear Interactions

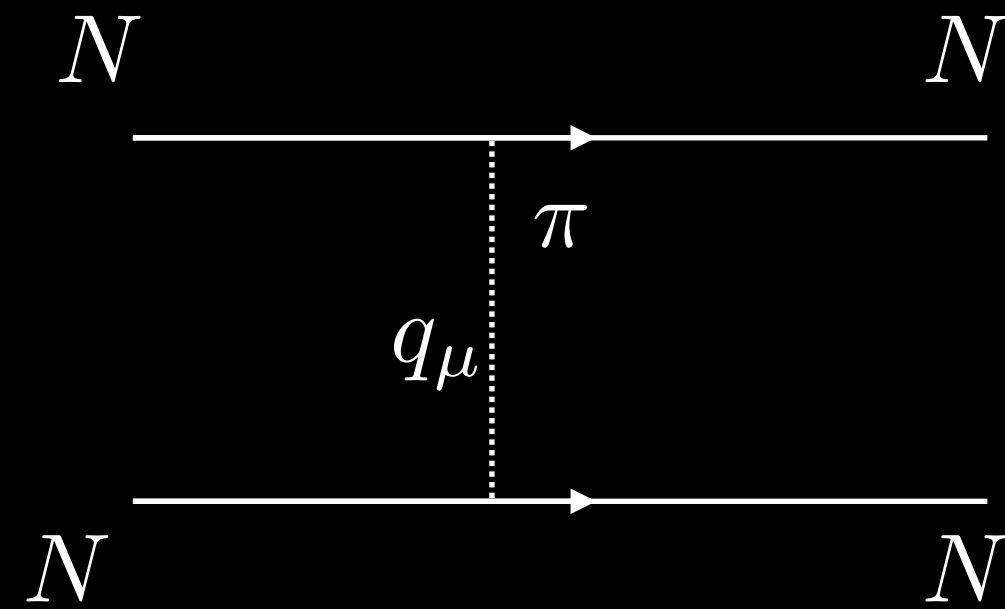
- Baryons and mesons are the relevant low energy degrees of freedom at low energy. Interactions between them are strong, complex, and short-range.
- Pions are special. They are the Goldstone bosons associated with chiral symmetry breaking and provide the longest range force between nucleons.
- Other mesons are significantly heavier. It is not very useful to single them out as mediators of the strong interaction between composite color singlet states.
- How then can we write down a theory of strong interactions between nucleons at low energy ?



Nucleon-Nucleon Potentials

One-pion
exchange:

$$\mathcal{L}_{NN\pi} = \psi_N^\dagger \left(i\partial_t - \frac{\nabla^2}{2M_N} \right) \psi_N - \frac{g_a}{f_\pi} \psi_N^\dagger \tau^a \sigma \cdot \nabla \pi^a \psi_N$$



$$V_\pi(q) = - \left(\frac{g_A}{\sqrt{2}f_\pi} \right)^2 \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2}$$

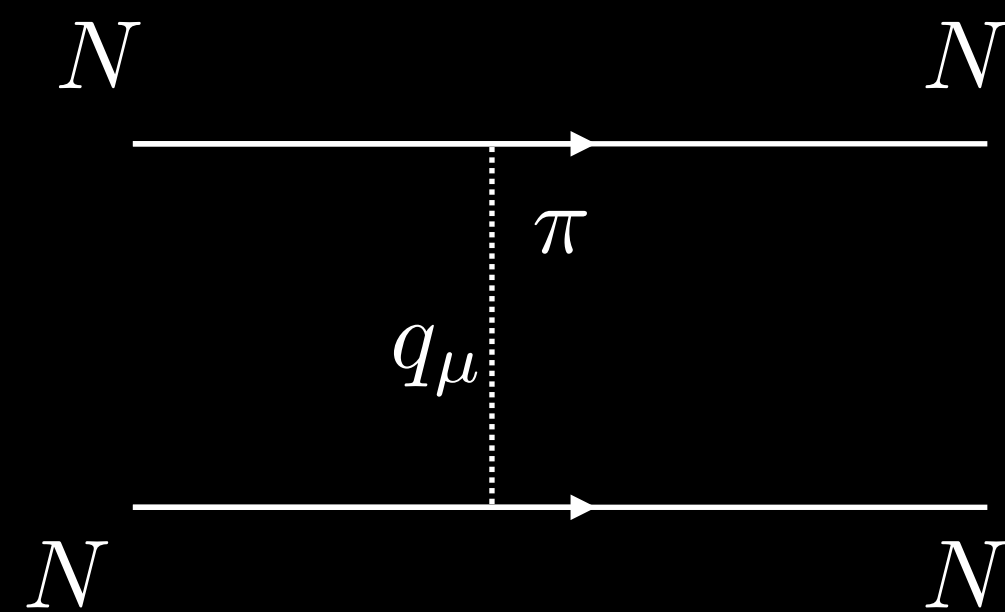
In coordinate space the potential is

$$V_\pi(q) = - \left(\frac{g_A}{\sqrt{2}f_\pi} \right)^2 \tau_1 \cdot \tau_2 \left[S_{12} \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{3} \sigma_1 \cdot \sigma_2 \delta^3(r) \right]$$

Nucleon-Nucleon Potentials

One-pion
exchange:

$$\mathcal{L}_{NN\pi} = \psi_N^\dagger \left(i\partial_t - \frac{\nabla^2}{2M_N} \right) \psi_N - \frac{g_a}{f_\pi} \psi_N^\dagger \tau^a \sigma \cdot \nabla \pi^a \psi_N$$



$$V_\pi(q) = - \left(\frac{g_A}{\sqrt{2}f_\pi} \right)^2 \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2}$$

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Potential depends on spin and iso-spin.

It has a tensor component: $S_{12} = 3(\sigma_1 \cdot \hat{r}_1) (\sigma_2 \cdot \hat{r}_2) - \sigma_1 \cdot \sigma_2$

It is singular: $V(r \rightarrow 0) \approx \frac{1}{r^3}$

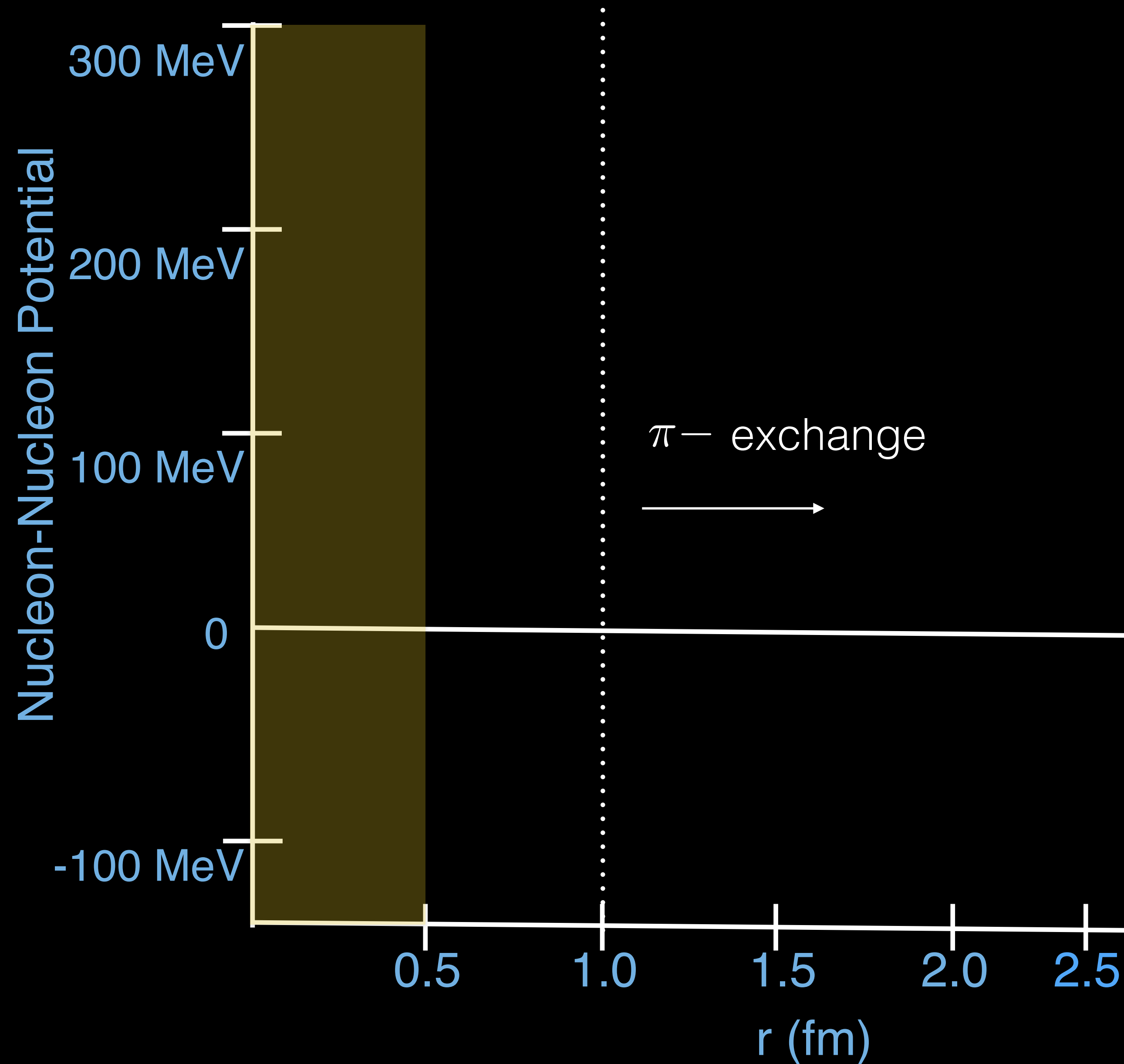
Nuclear Forces at Short Distances

They are essential even at low energy.

Are constrained by nucleon-nucleon scattering data (phase shifts).

Models favor strong repulsion. (hard-core)

Range of these forces is comparable to the intrinsic size of the nucleon.



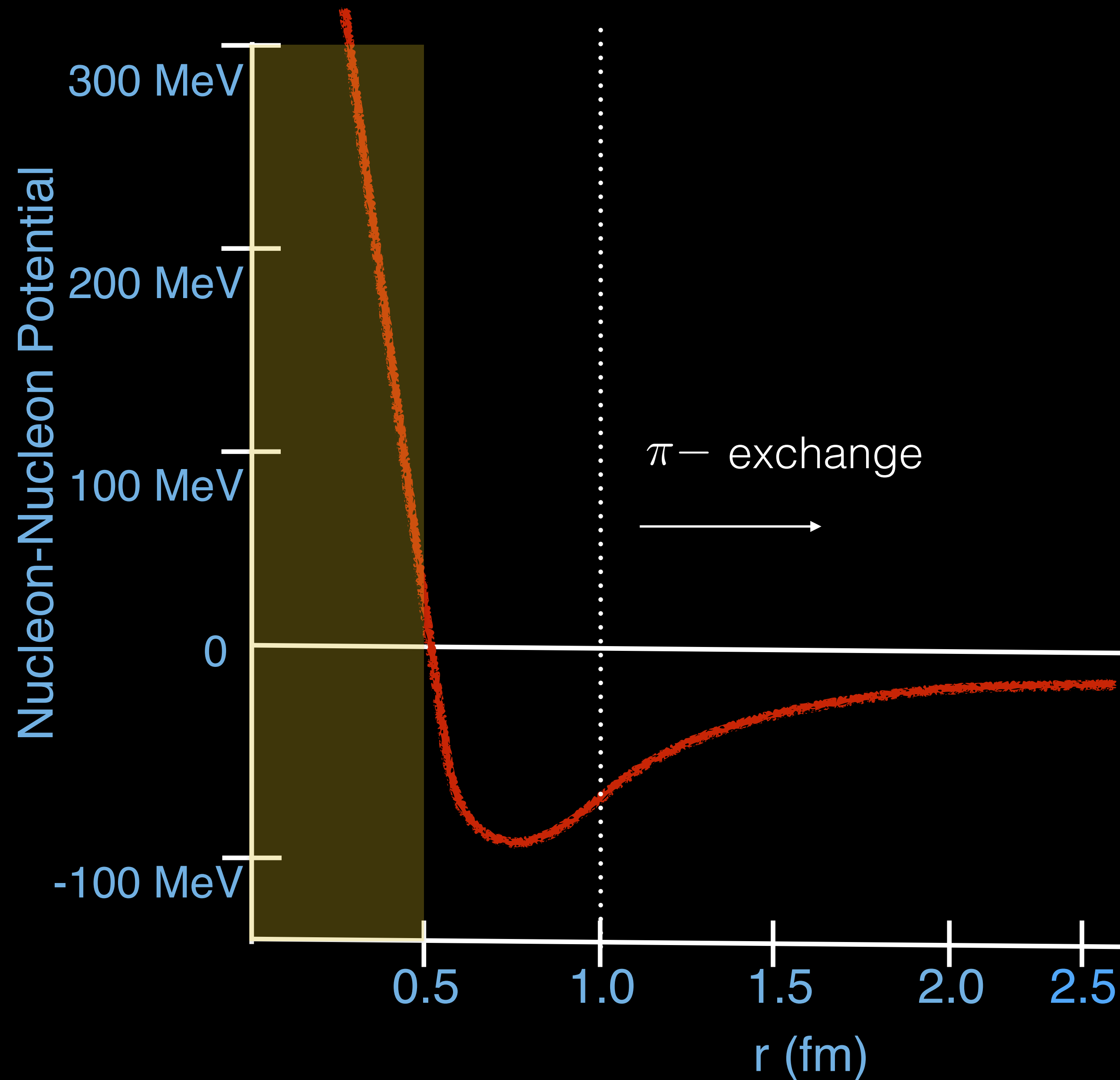
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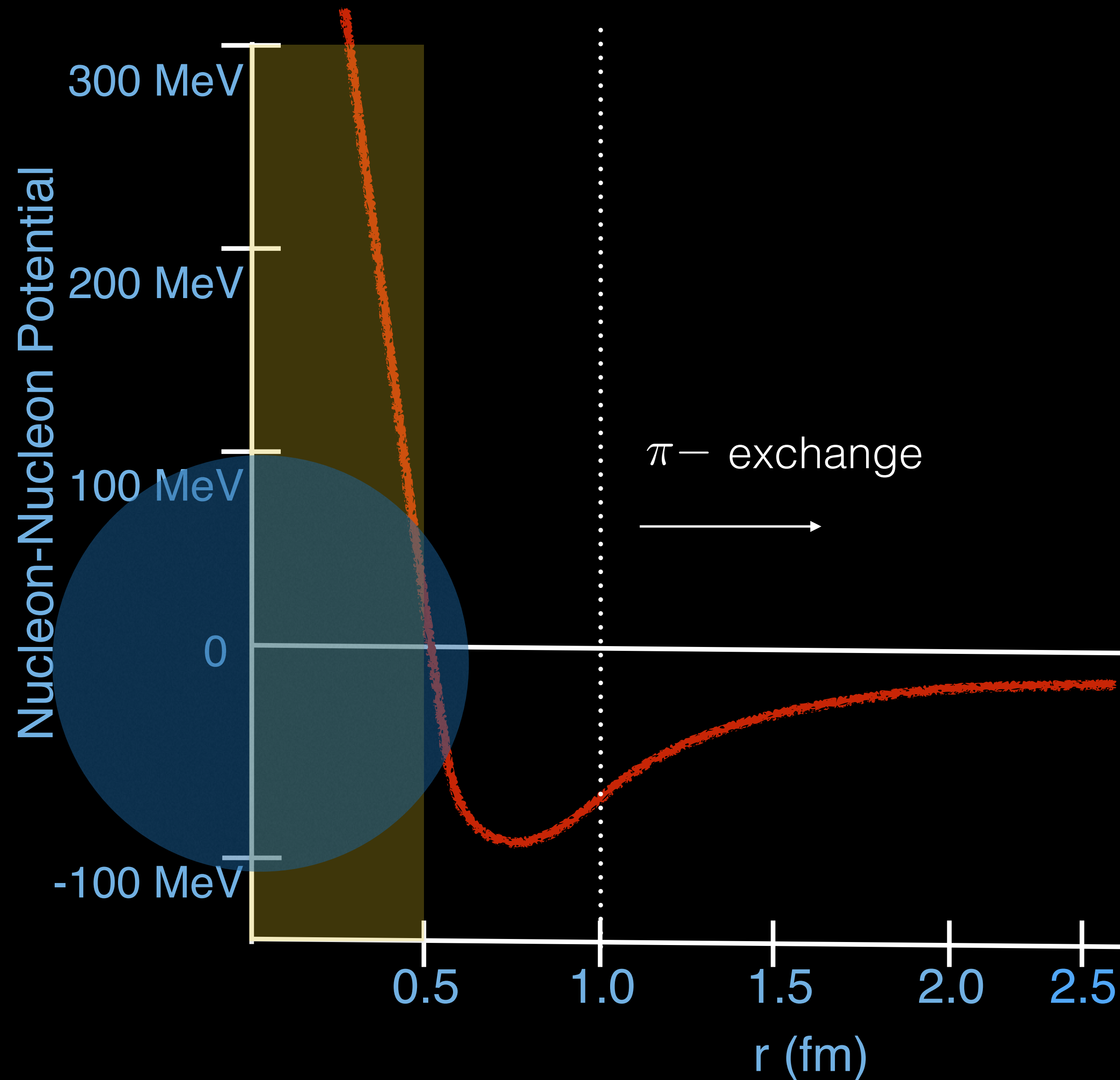
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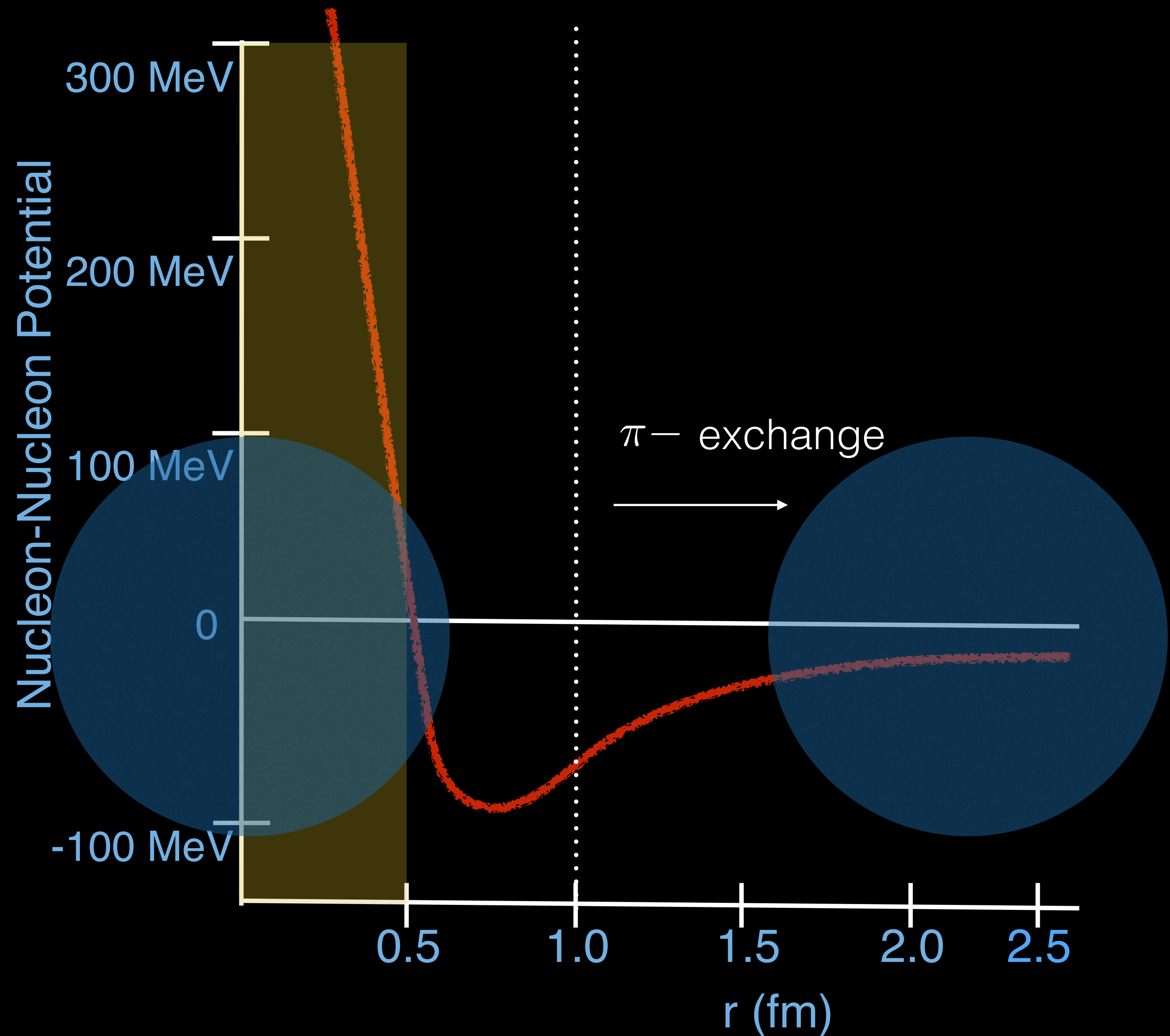
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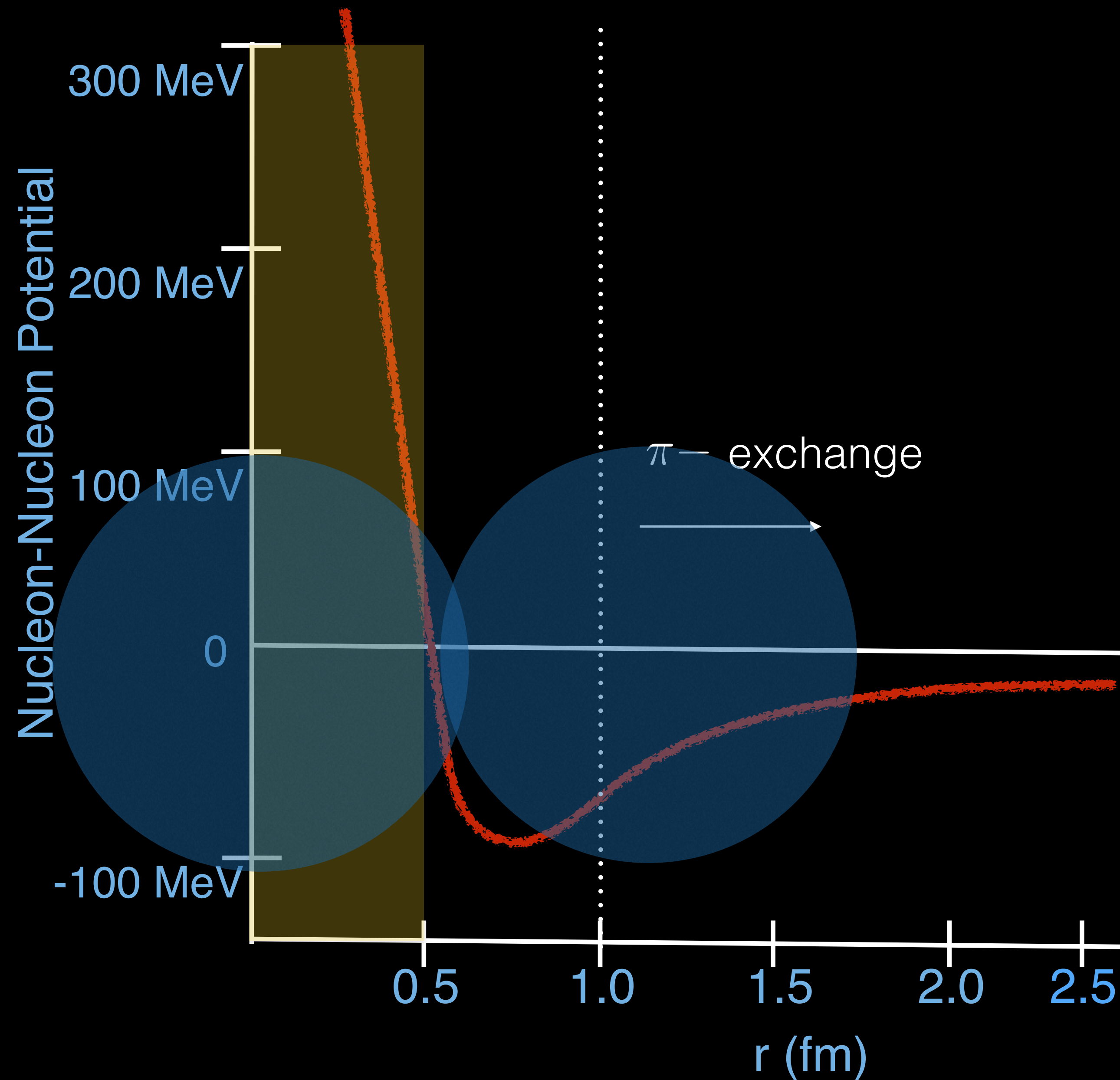
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A Realistic Potential Model

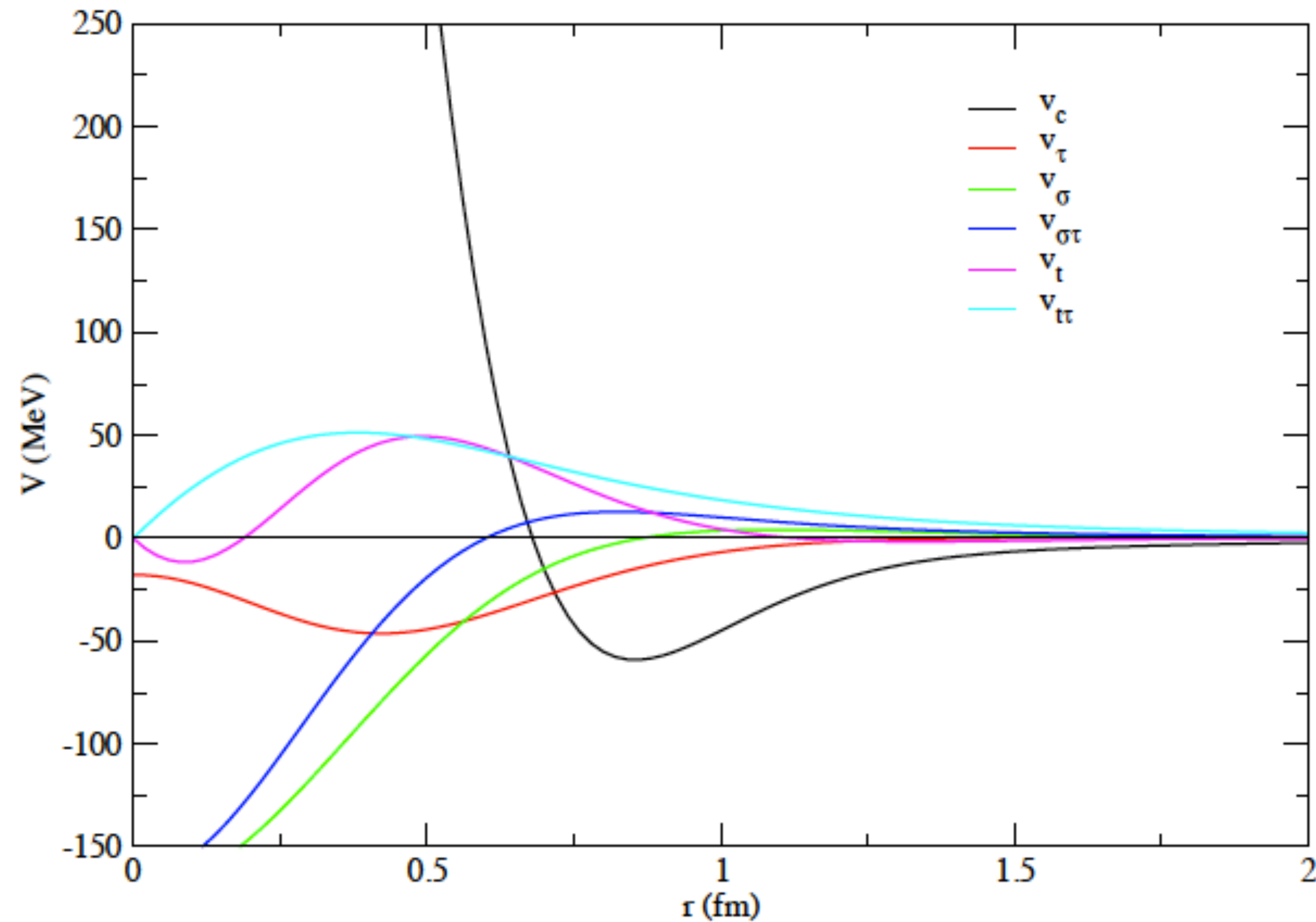
$$V_{ij} = \sum_p v_p(r_{ij}) O_{ij}^p$$

Intricate spin, isospin and tensor structure.

$$\begin{aligned} O_{ij}^p = & [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L} \cdot \mathbf{S})^2] \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L} \cdot \mathbf{S})^2] \otimes \tau_i \cdot \tau_j \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}] \otimes T_{ij} \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}] \otimes (\tau_i + \tau_j)_z \end{aligned}$$

$$S_{ij} = 3\sigma_i \cdot \hat{r}_{ij}\sigma_j \cdot \hat{r}_{ij} - \sigma_i \cdot \sigma_j \quad T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i \cdot \tau_j$$

Argonne v_{18}



Potential is Neither Unique Nor Observable (in QM)

Potential Models: Relies on a set of (reasonable) assumptions about the short distance behavior to solve the Schrödinger equation and fit observables.

Effective Field Theory: Relies on a separation of scales to Taylor expand potential in powers of momenta or inverse radial separation. Coefficients of the expansion are determined by fitting to observables.

Potential is Neither Unique Nor Observable (in QM)

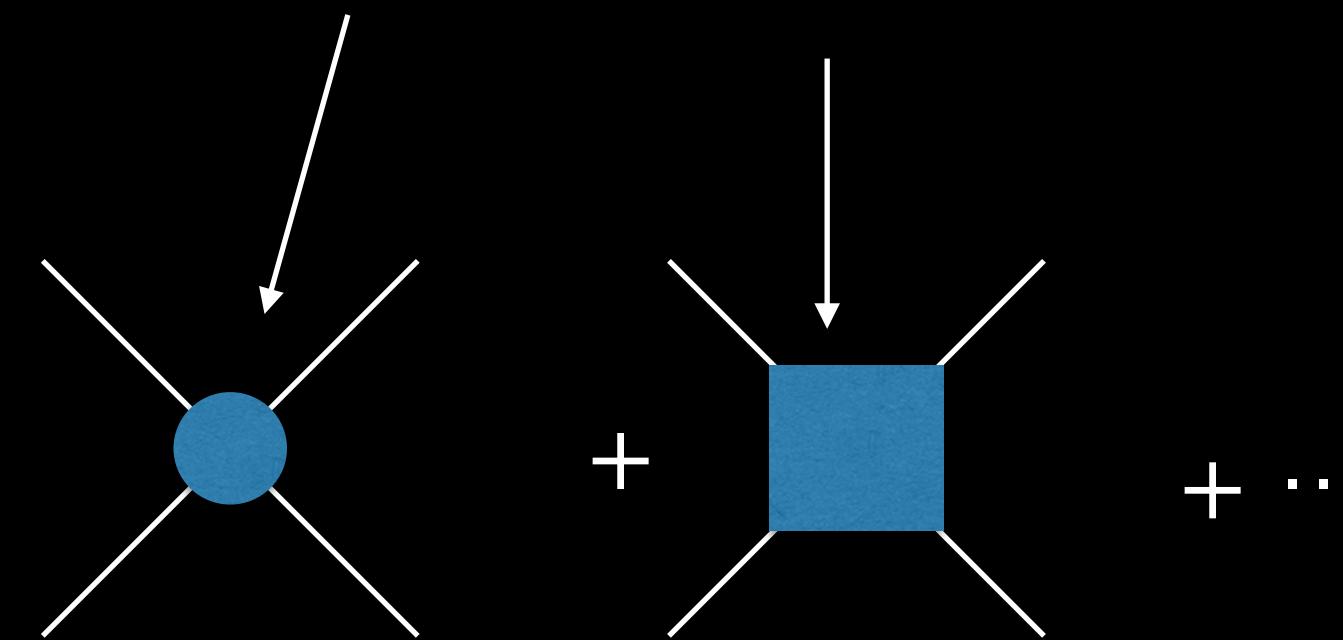
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A simple (heuristic) EFT example:

$$V_\omega(q) = \frac{g_\omega^2}{q^2 + m_\omega^2} \approx \frac{g_\omega^2}{m_\omega^2} - \frac{g_\omega^2}{m_\omega^2} \frac{q^2}{m_\omega^2} + \dots$$

Exchange of heavy bosons at low energy cannot be resolved.



Potential is Neither Unique Nor Observable (in QM)

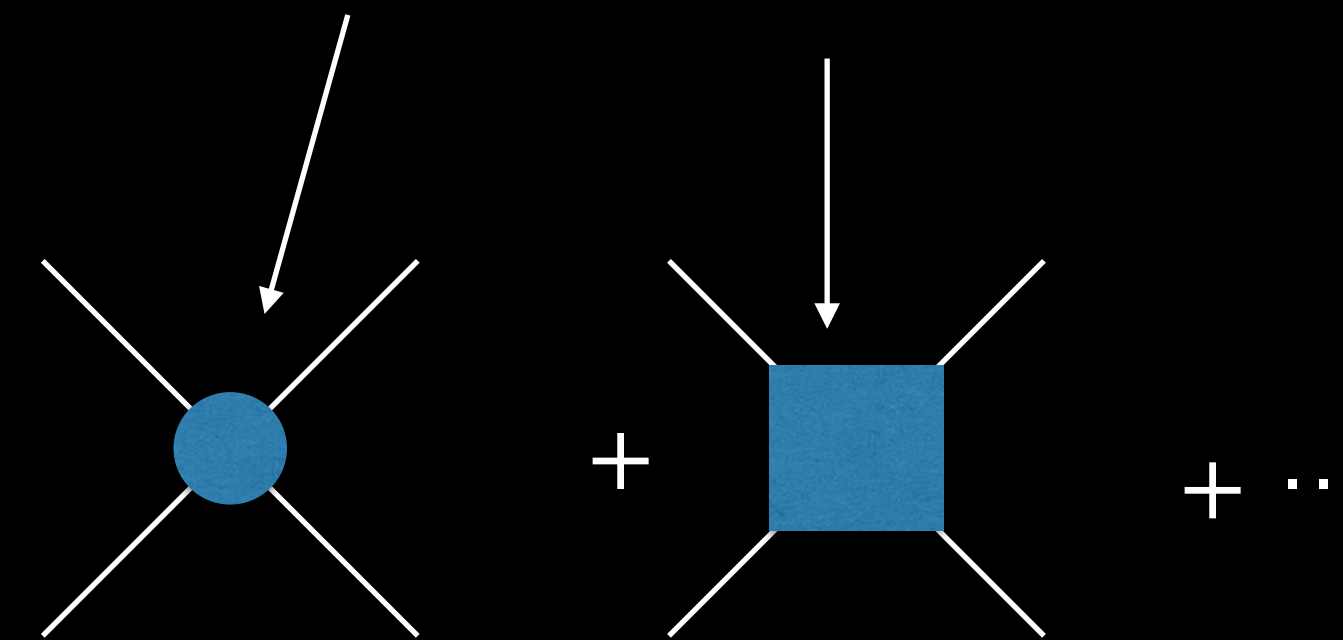
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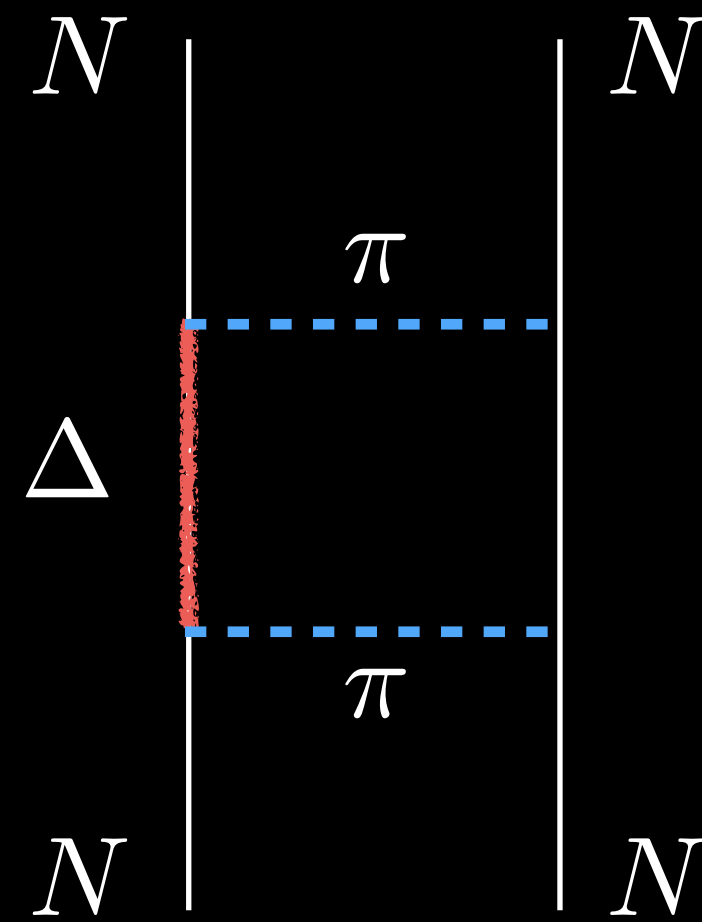
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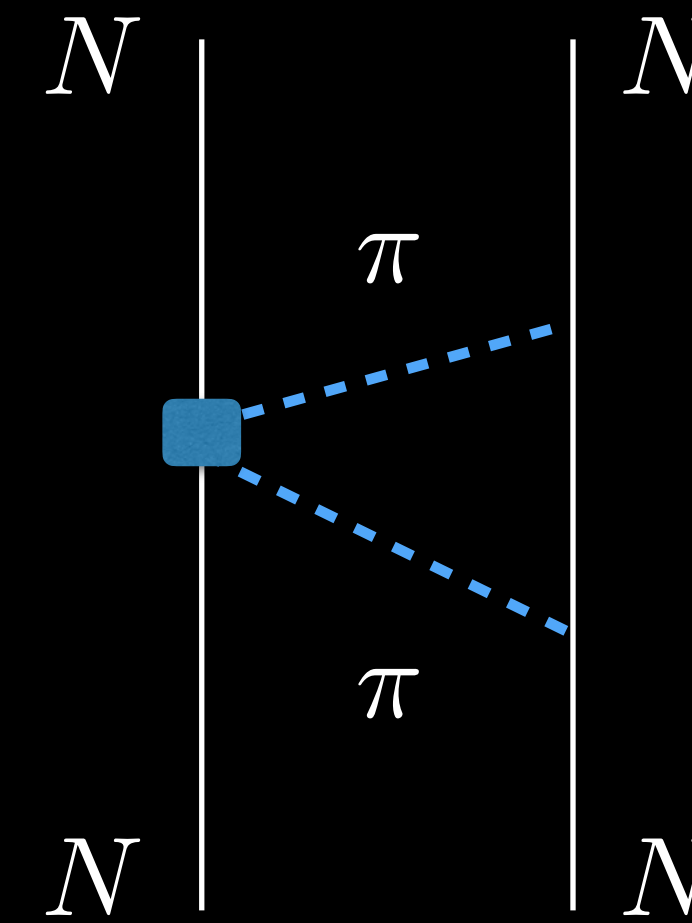
When several heavy particles may be exchanged, or when the underlying mechanism is unknown, the general expansion is

$$V_{\text{short}}(q) = C_0 + C_2 \frac{q^2}{\Lambda^2} + \dots$$

Nucleons are composite with internal excitations

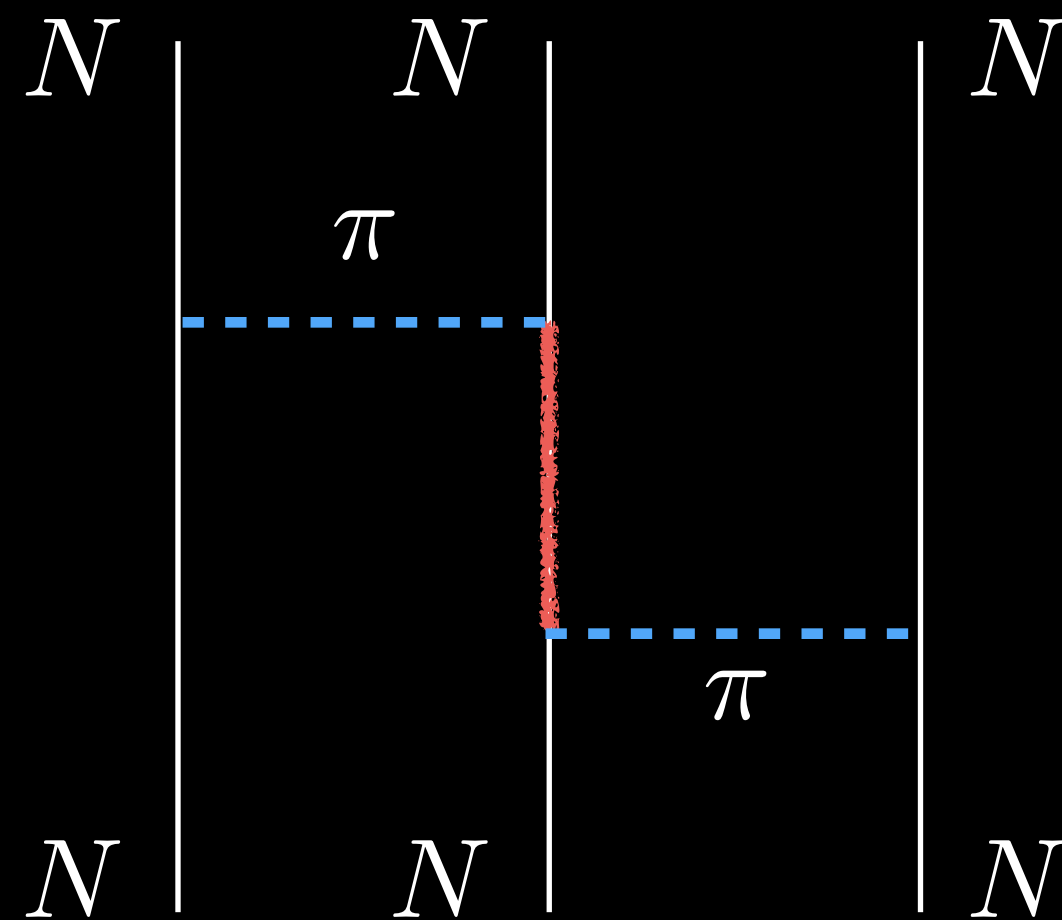


At low energy

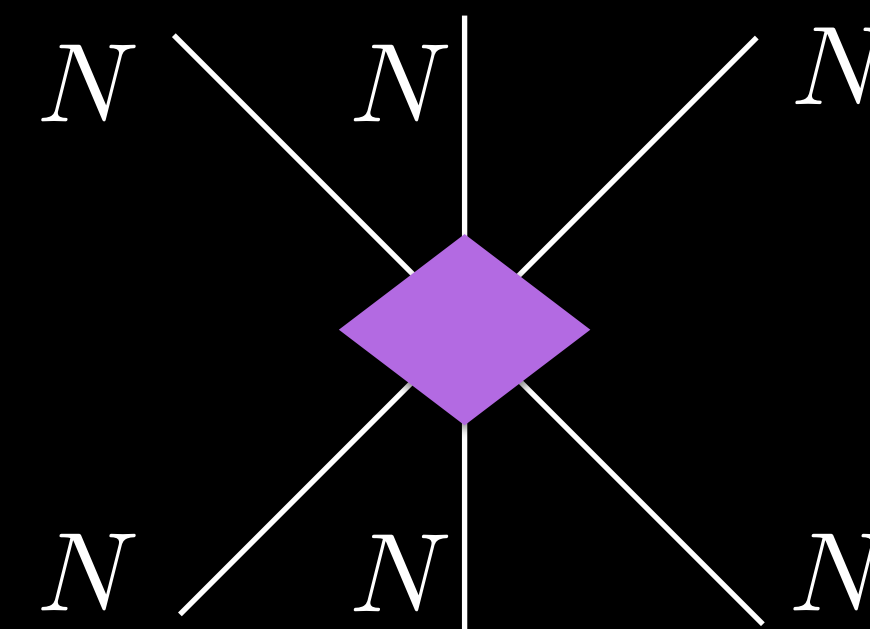
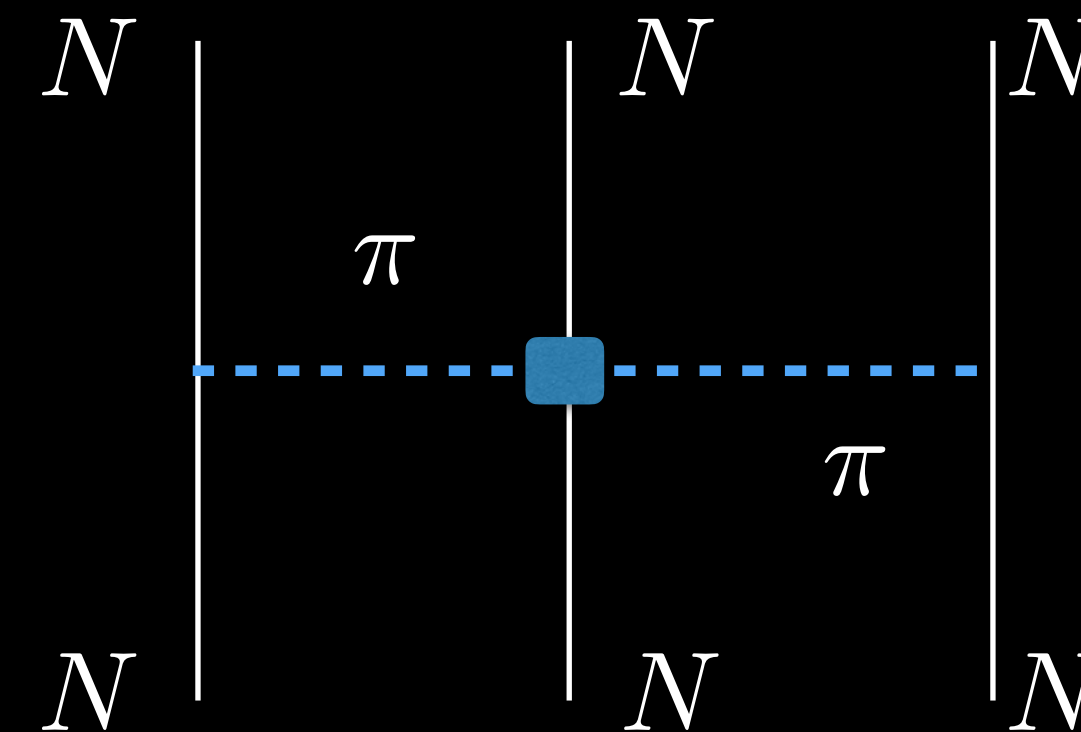


$$m_{\Delta} - m_N \simeq 1232 - 939 \text{ MeV} \approx 300 \text{ MeV}$$

There are three and many-body forces:



At low energy

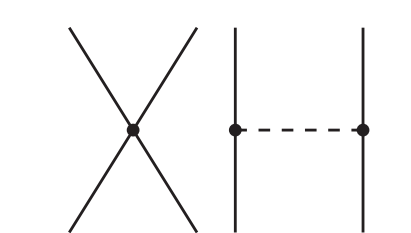
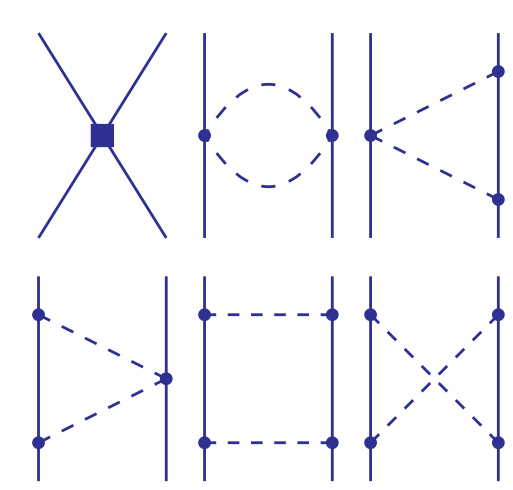
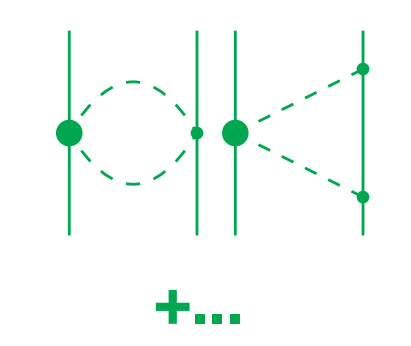
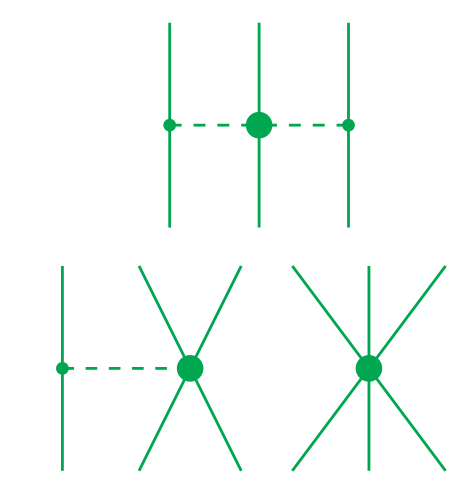
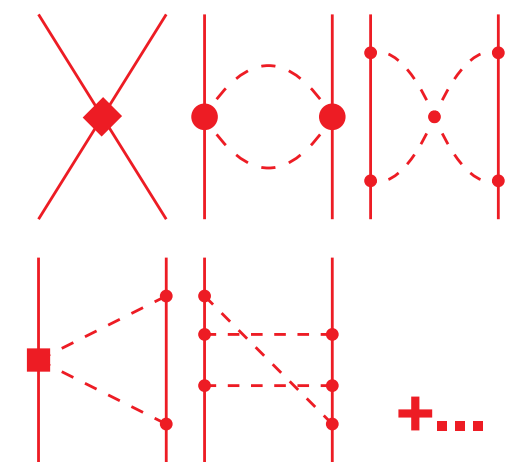
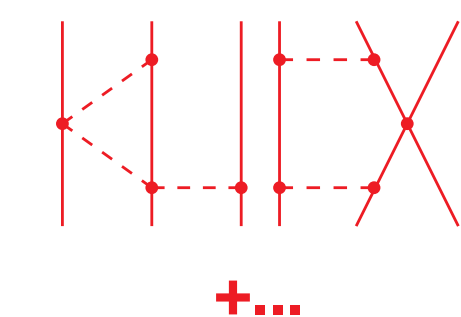
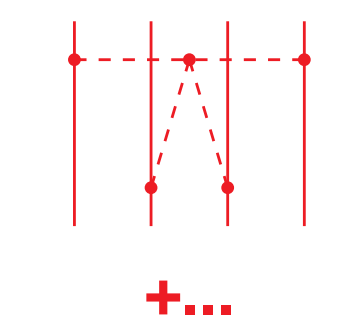


Chiral EFT

Systematic approach to low energy nuclear interactions.

Expectation is that the expansion will remain valid up to 1-2 times nuclear saturation density.

Consistent treatment of two, three and many-body forces.

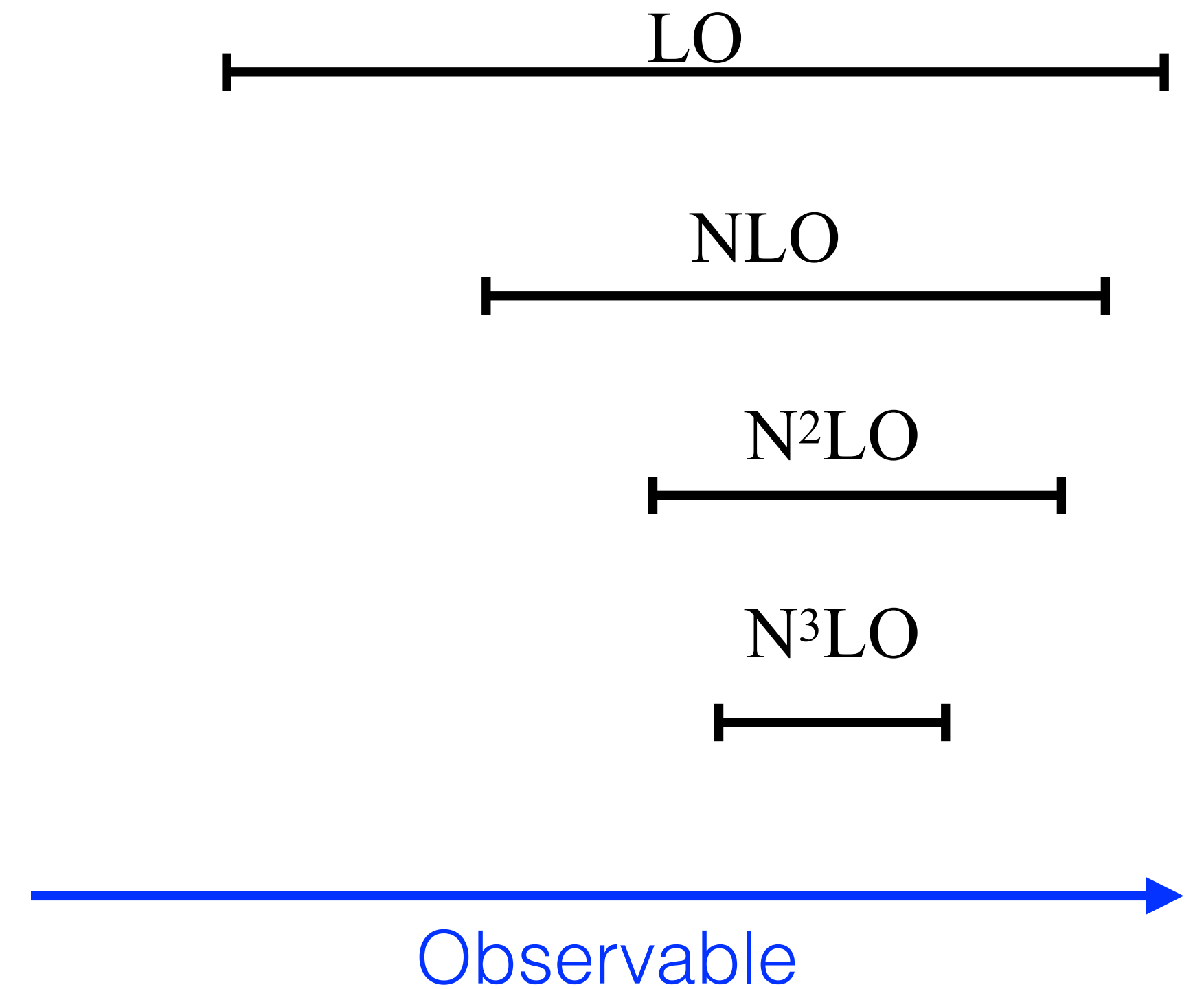
| | 2N Force | 3N Force | 4N Force |
|--|---|---|---|
| LO $(Q/\Lambda_\chi)^0$ |  | | |
| NLO $(Q/\Lambda_\chi)^2$ |  | | |
| NNLO $(Q/\Lambda_\chi)^3$ |  |  | |
| N³LO $(Q/\Lambda_\chi)^4$ |  |  |  |

Effective Field Theory: Error Estimation

Organizes the nuclear Hamiltonian in powers of the momentum:

$$\frac{Q}{\Lambda_B}$$

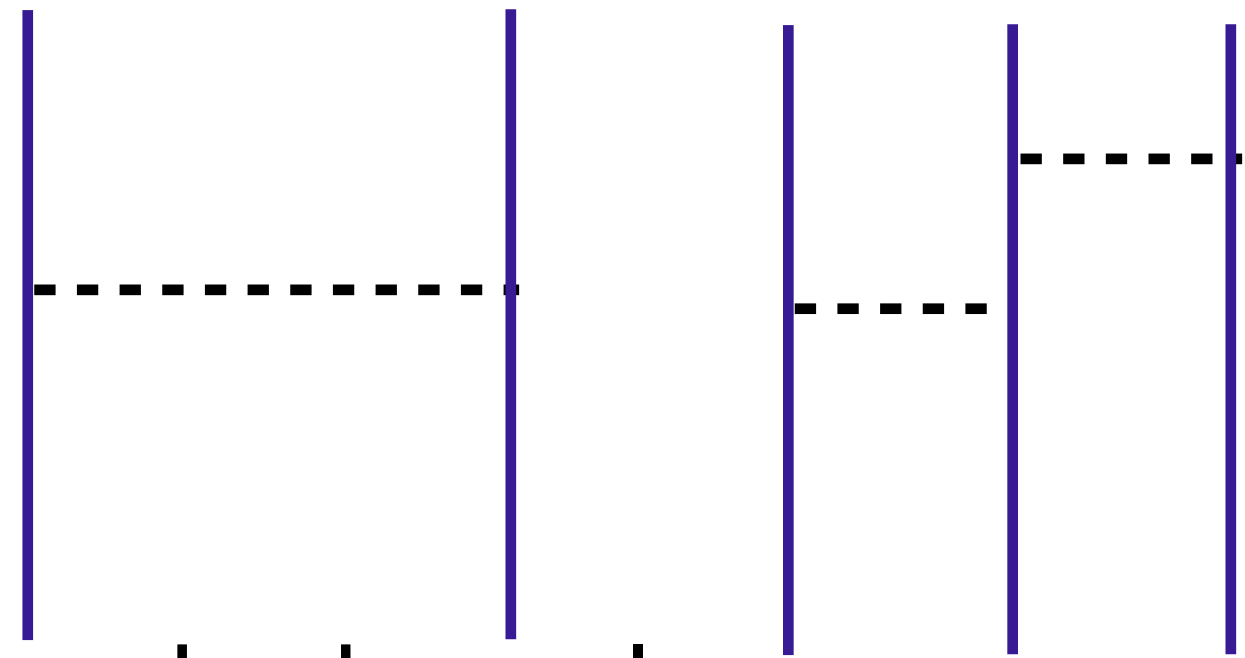
| | 2N force | 3N force | 4N force |
|-------------------|----------|----------|----------|
| LO | | | |
| NLO | | | |
| N ² LO | | | |
| N ³ LO | | | |



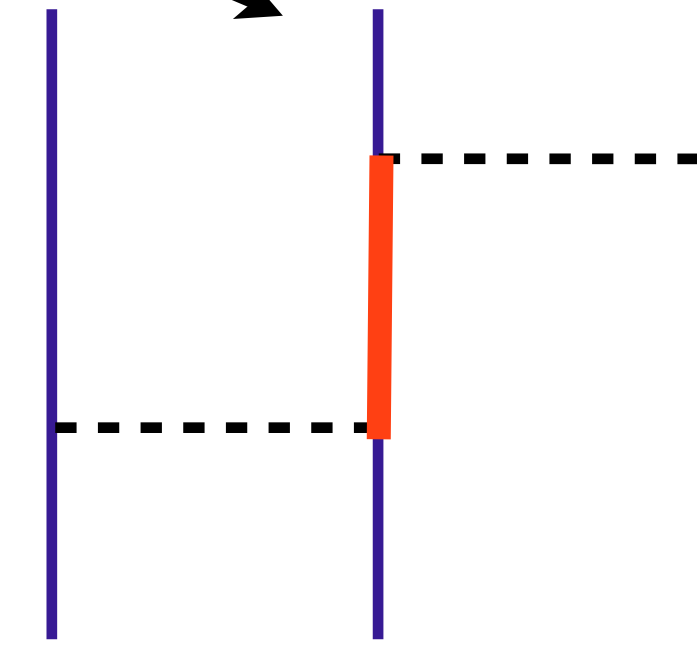
Allows for error estimation. Provides guidance for the structure of three and many-body forces.

Ground State Energy

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \dots$$



two-body nucleon-nucleon potential is well constrained by scattering data.

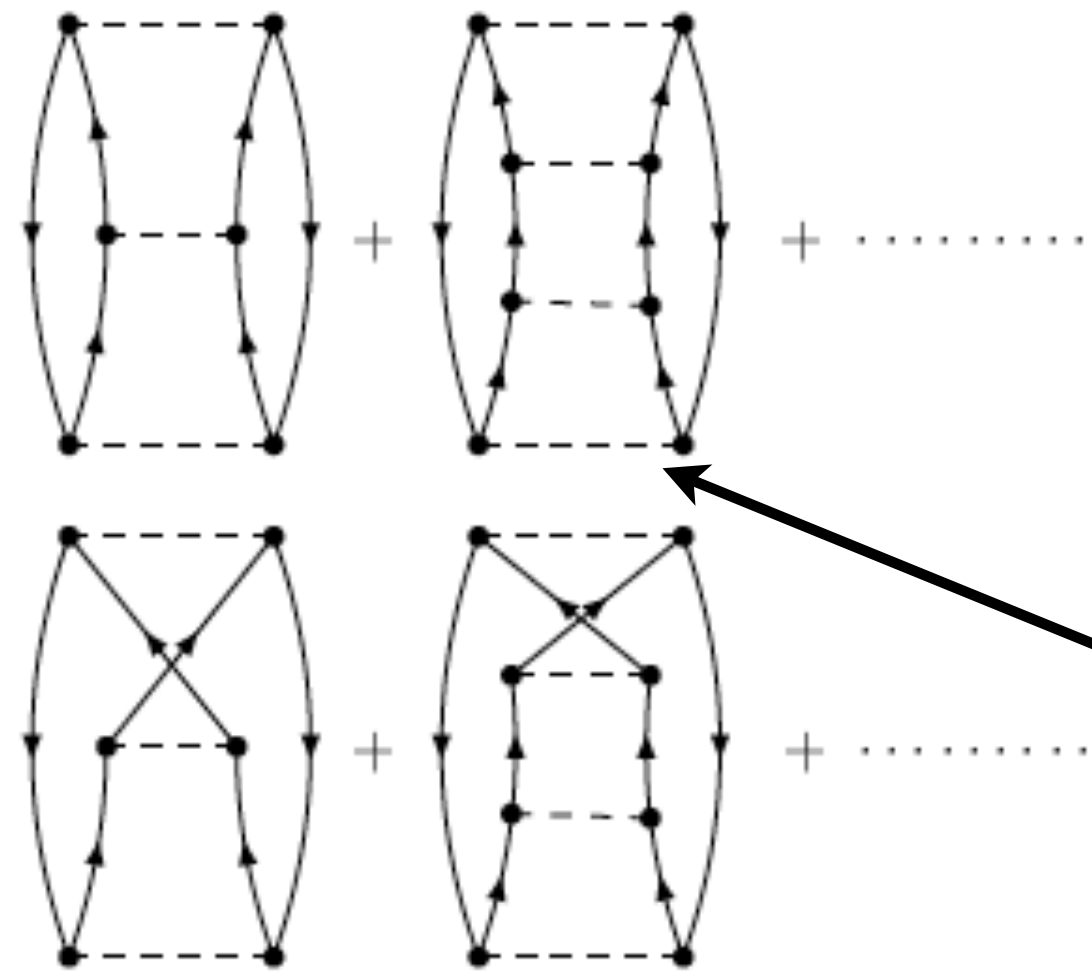


three-neutron potential is constrained by light nuclei.

Quantum Many-Body Theory:
Quantum Monte Carlo
Diagrammatic Methods
(perturbation theory)

$E(\rho_n, \rho_p)$: Energy per particle

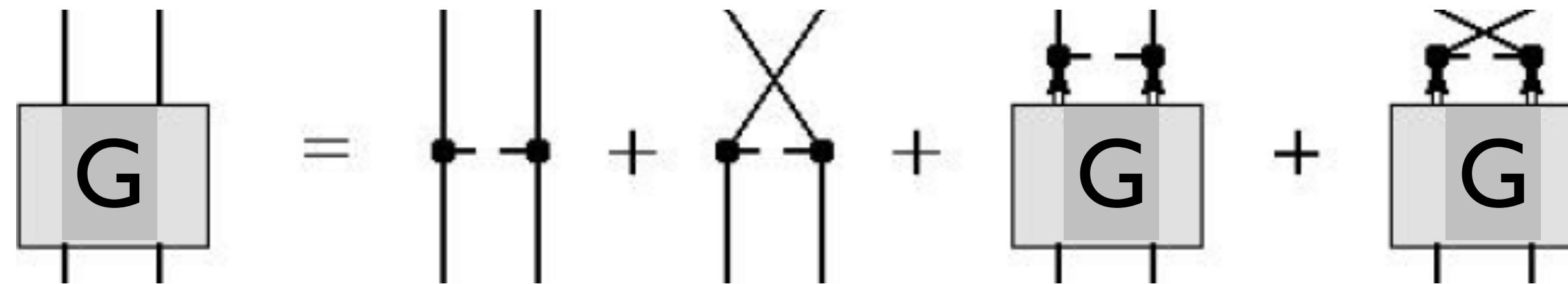
Diagrammatic Methods



Sum certain classes of Feynman diagrams to capture non-perturbative aspects.

nucleon-nucleon interaction

Eg. Bruckner or G-matrix Theory:



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | v | k_3 k_4 \rangle +$$

$$+ \sum_{k'_3 k'_4} \langle k_1 k_2 | v | k'_3 k'_4 \rangle \frac{(1 - \theta_F(k'_3))(1 - \theta_F(k'_4))}{\omega - e_{k'_3} - e_{k'_4}} \langle k'_3 k'_4 | G(\omega) | k_3 k_4 \rangle$$

Quantum Monte Carlo

Variational Monte Carlo:
$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Greens Function Monte Carlo:

$$\begin{aligned} \Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V &= \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n \\ \Psi(\tau \rightarrow \infty) &= a_0\psi_0 \end{aligned}$$

- Evolve particle coordinates.
- MC kinetic terms.
- Explicitly compute potential.

$$\Psi(\mathbf{R}_n, \tau) = \int G(\mathbf{R}_n, \mathbf{R}_{n-1}) \cdots G(\mathbf{R}_1, \mathbf{R}_0) \Psi_V(\mathbf{R}_0) d\mathbf{R}_{n-1} \cdots d\mathbf{R}_0$$

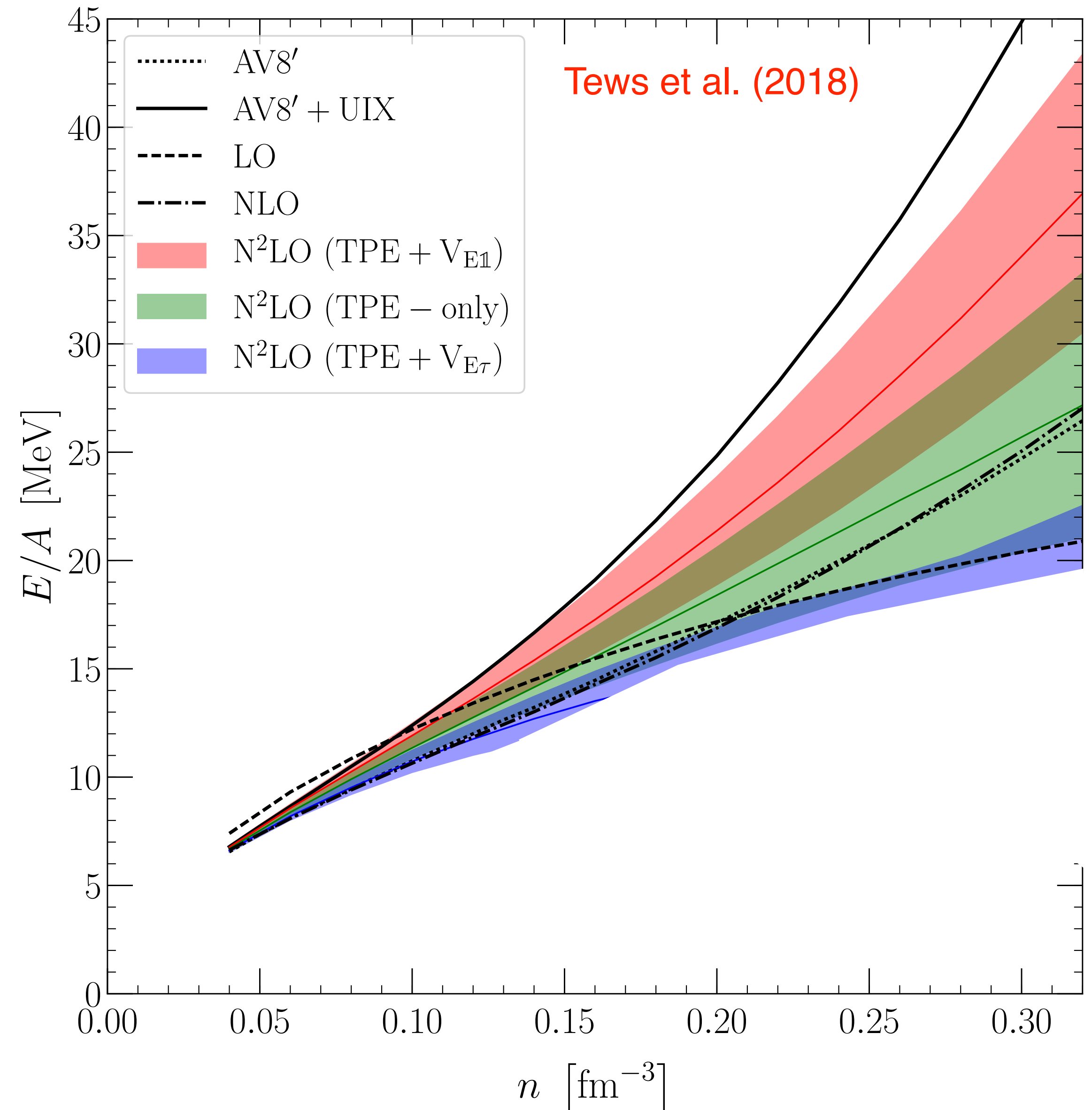
Fermion sign problem - limits GFMC

Equation of State of Neutron Matter

Reliable calculations of neutron matter are now possible using QMC and EFT inspired Hamiltonians.

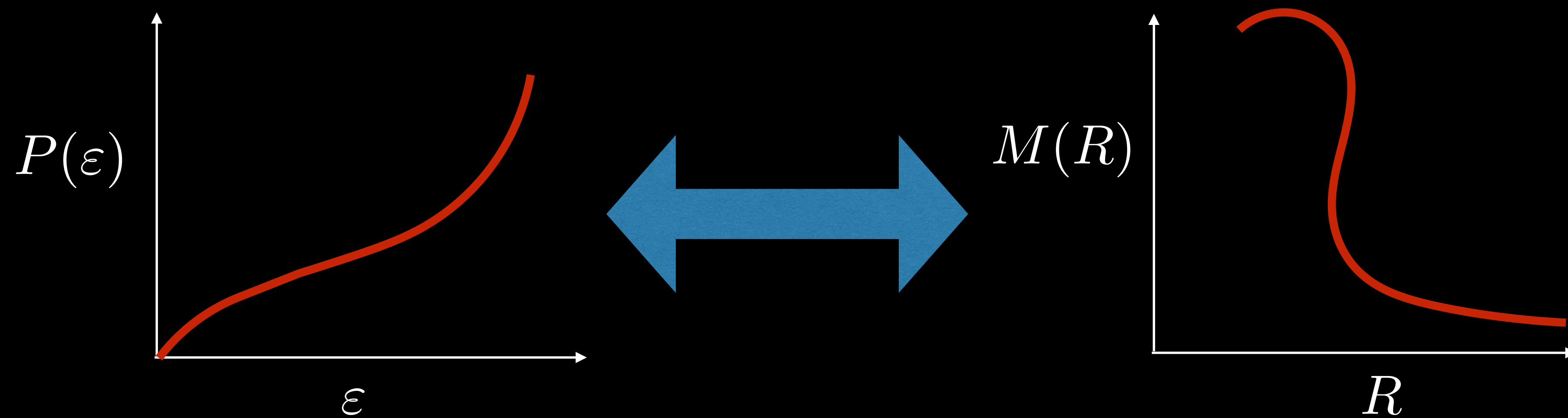
Order-by-order convergence is good at $n=0.16 \text{ fm}^{-3}$ and reasonable at $n=0.32 \text{ fm}^{-3}$.

| | $n=0.16 \text{ fm}^{-3}$ | $n=0.32 \text{ fm}^{-3}$ |
|-----------------------------------|--------------------------|--------------------------|
| Energy (MeV) | 15 ± 3 | 30 ± 15 |
| Pressure (MeV/ fm^{-3}) | 2.5 ± 1 | 13 ± 5 |



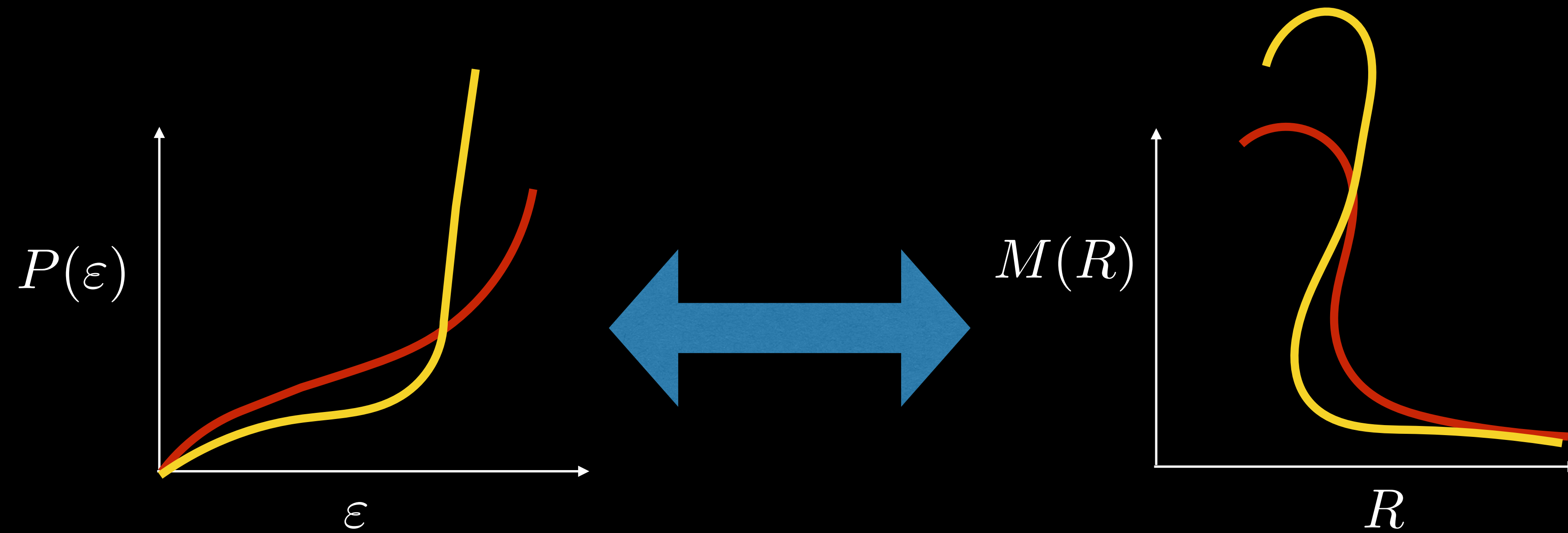
Akmal & Pandharipande 1998, Hebeler and Schwenk 2009, Gandolfi, Carlson, Reddy 2010, Tews, Kruger, Hebeler, Schwenk (2013), Holt Kaiser, Weise (2013), Roggero, Mukherjee, Pederiva (2014), Wlazlowski, Holt, Moroz, Bulgac, Roche (2014), Tews et al. (2018)

Equation of State and Neutron Star Structure



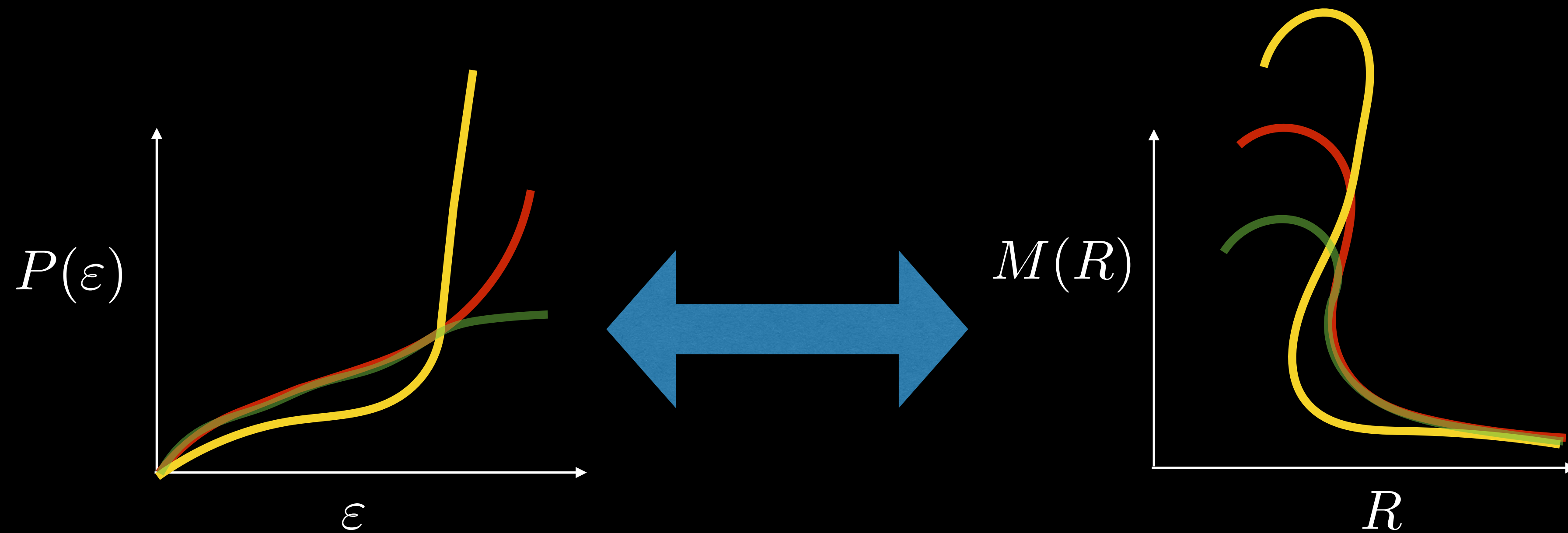
$$P(\varepsilon) + \text{Gen.Rel.} = M(R)$$

Equation of State and Neutron Star Structure



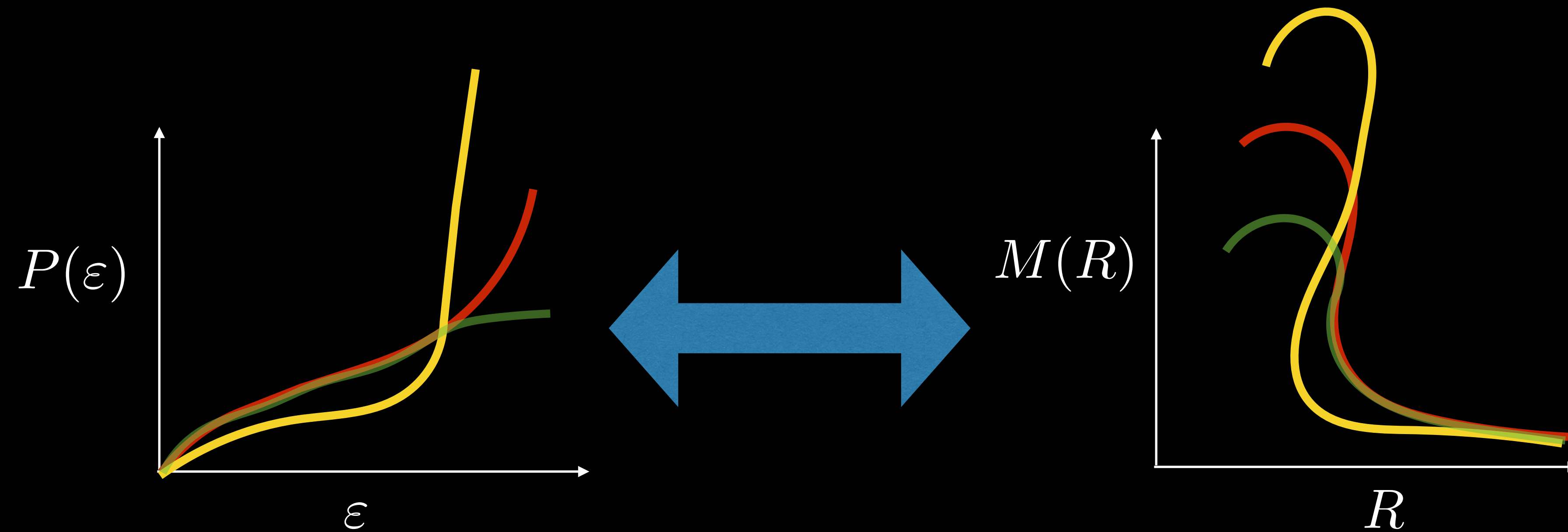
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Equation of State and Neutron Star Structure



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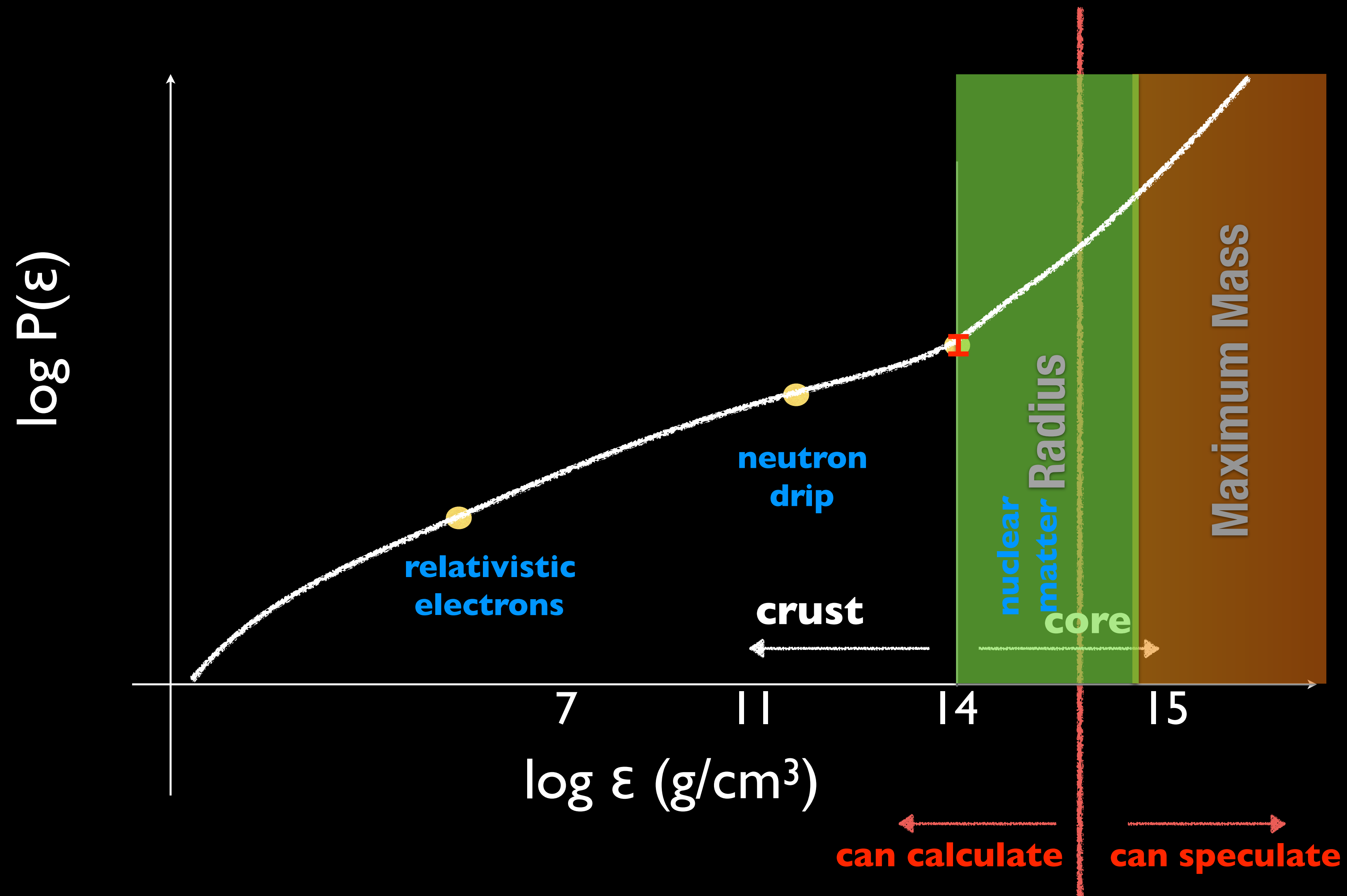
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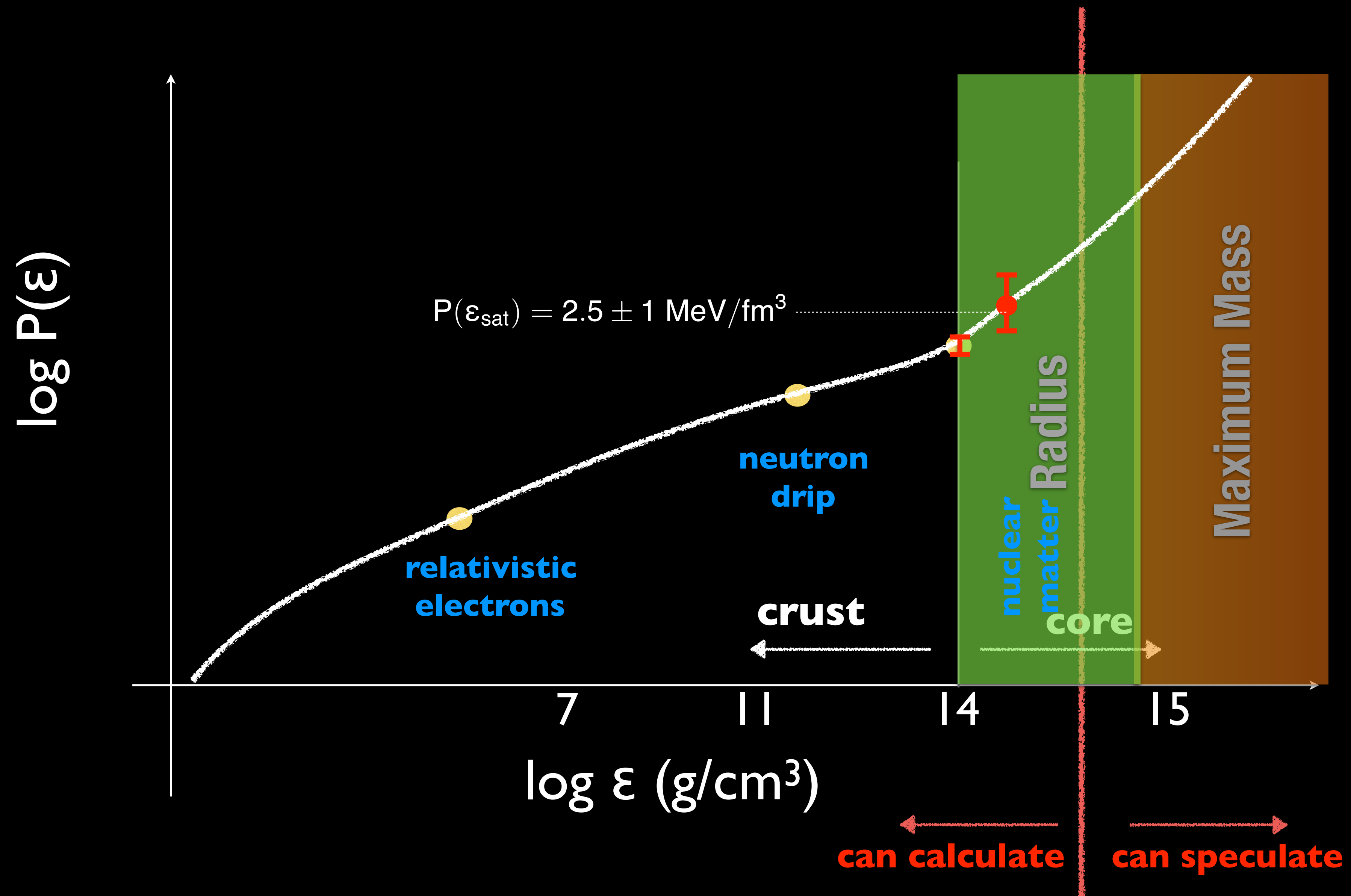
$$P(\epsilon) + \text{Gen.Rel.} = M(R)$$

A small radius and large maximum mass implies a rapid transition from low pressure to high pressure with density.

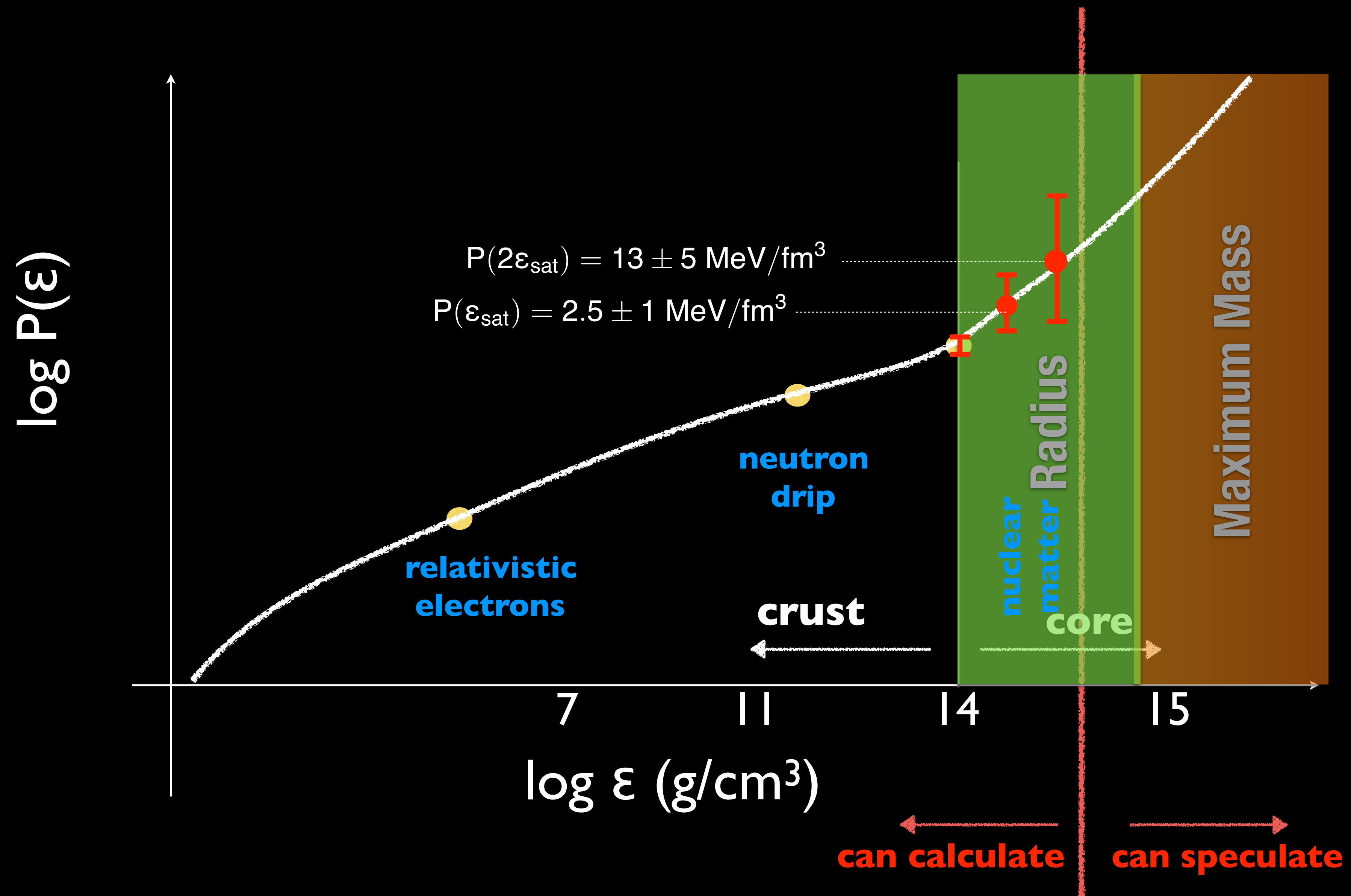
Constraints on the Equation of State



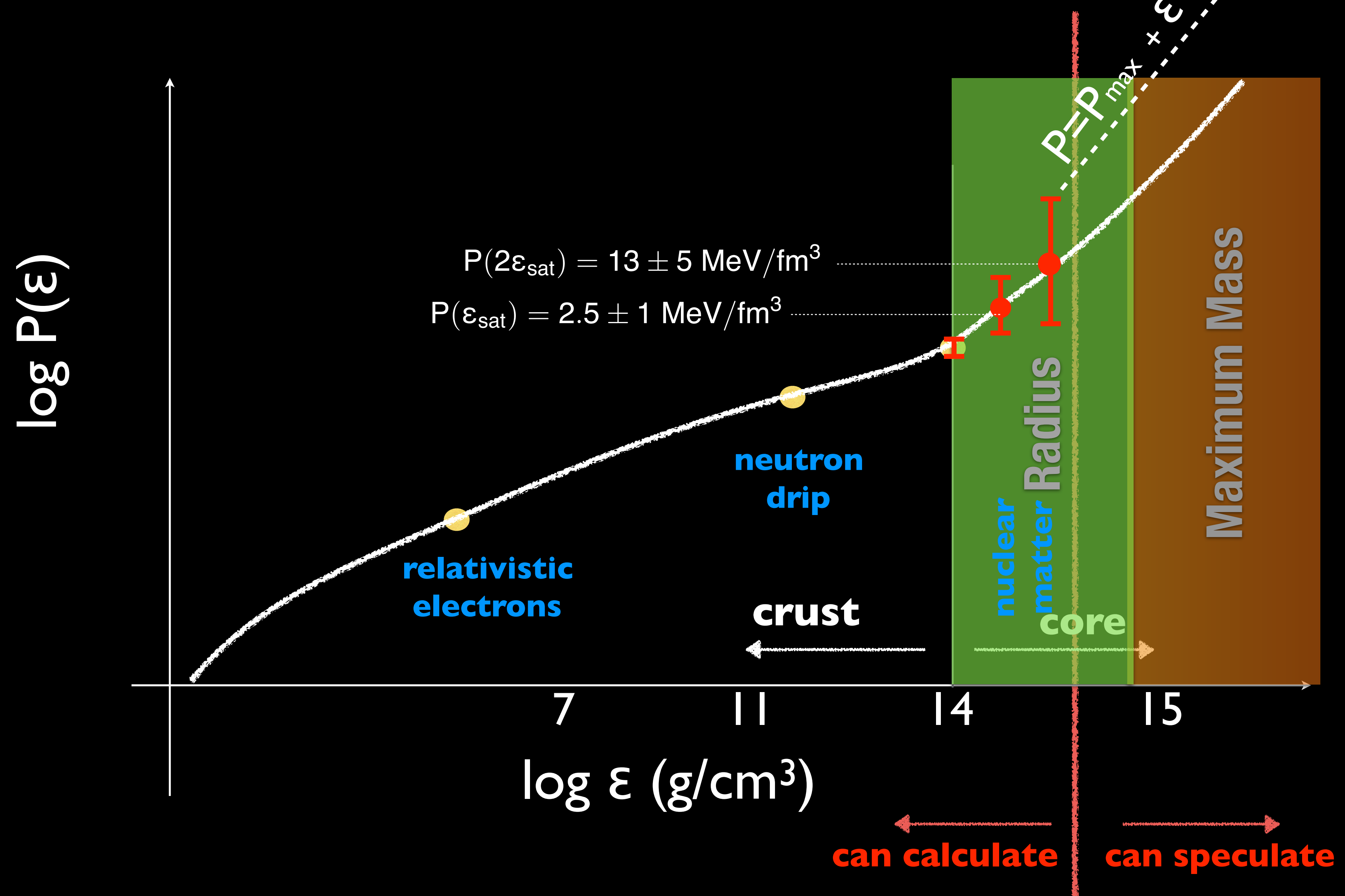
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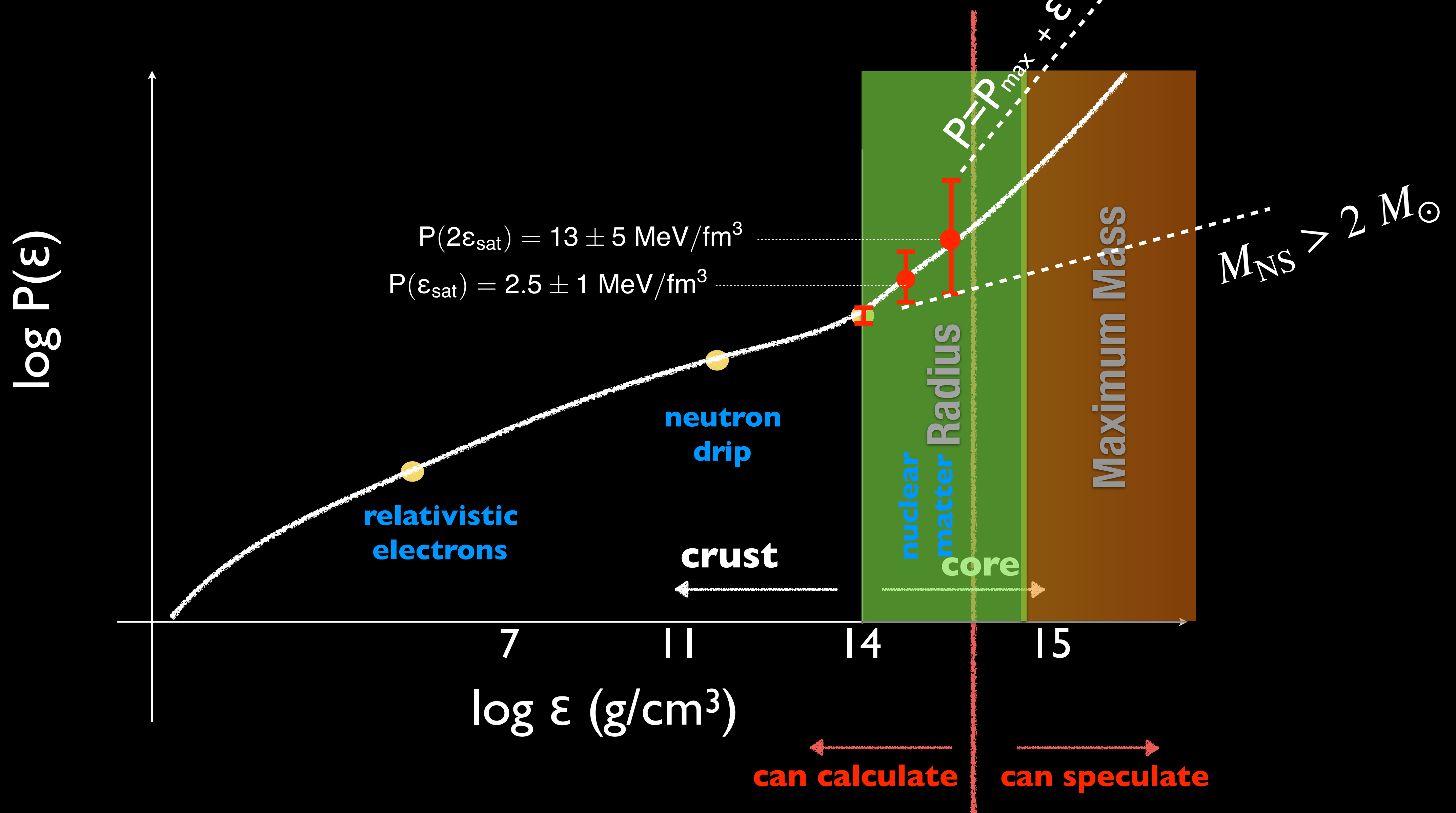
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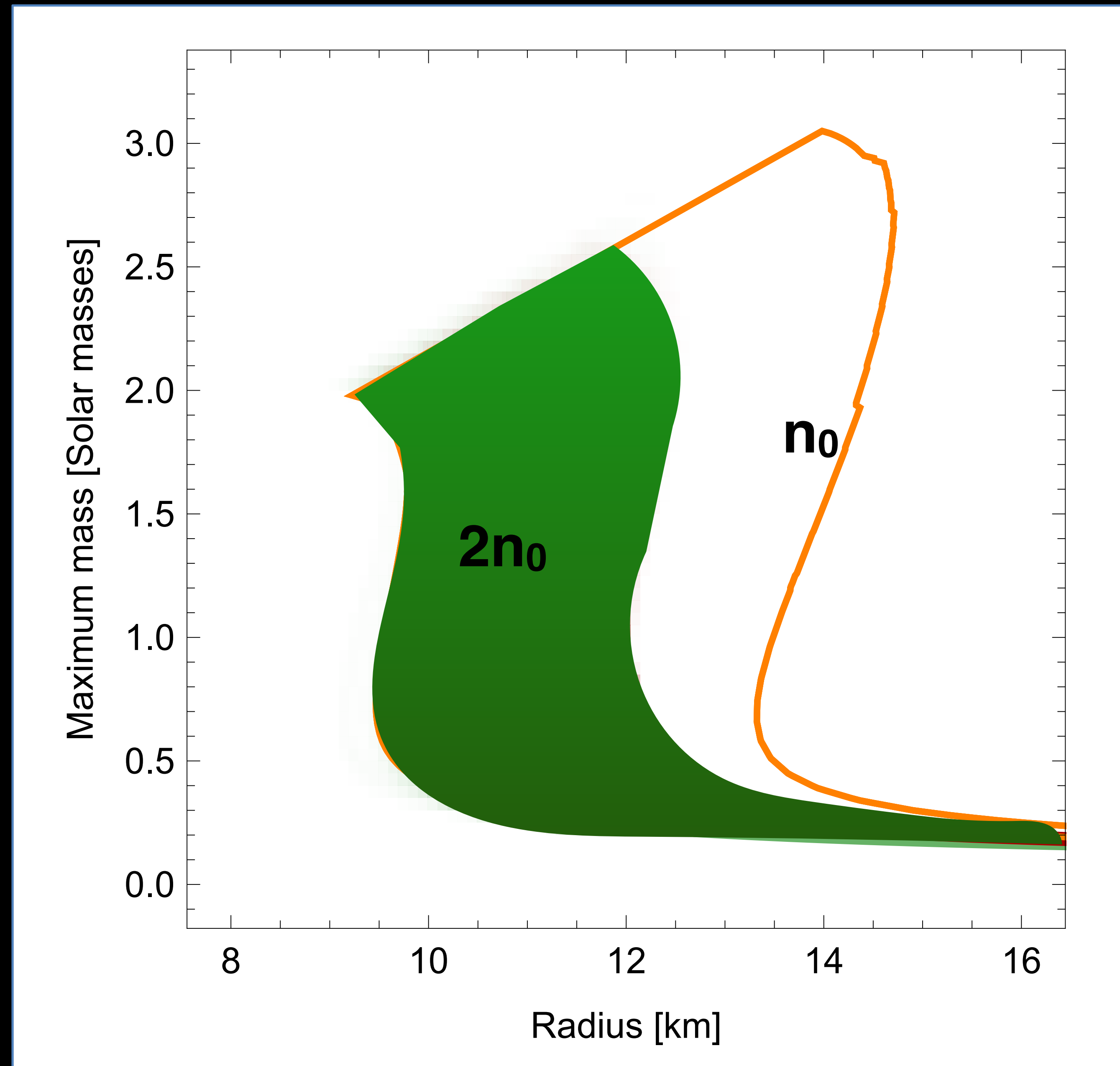


Dense matter EOS and NS structure

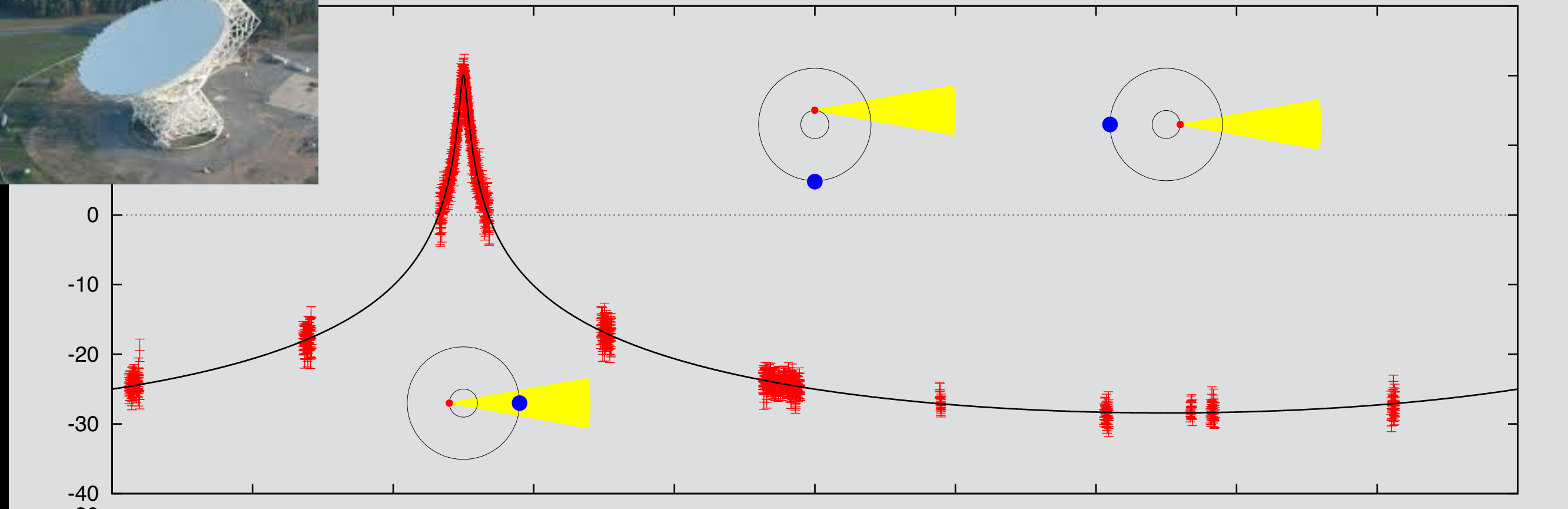
Neutron matter calculations and a sound speed at higher density constrained by 2 solar mass NS and causality provide useful constraints on the NS properties.

$$R_{1.4} = 9.5 - 12.5 \text{ km}$$

$$M_{\text{max}} = 2.0 - 2.5 M_{\text{solar}}$$



Neutron Star Structure: Observations



2 M_{\odot} neutron stars exist.

PSR J1614-2230: $M=1.93(2)$

Demorest et al. (2010)

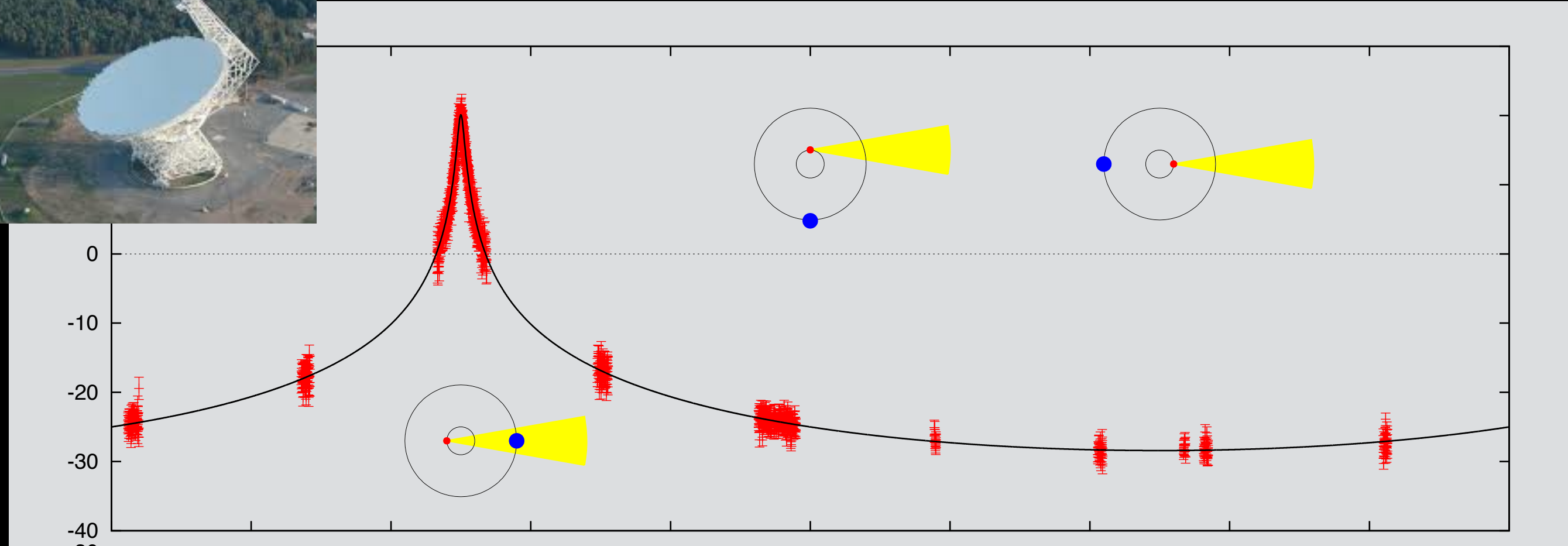
PSR J0348+0432: $M=2.01(4) M_{\odot}$

Anthoniadis et al. (2013)

MSP J0740+6620: $M=2.17(10) M_{\odot}$

Cromartie et al. (2019)

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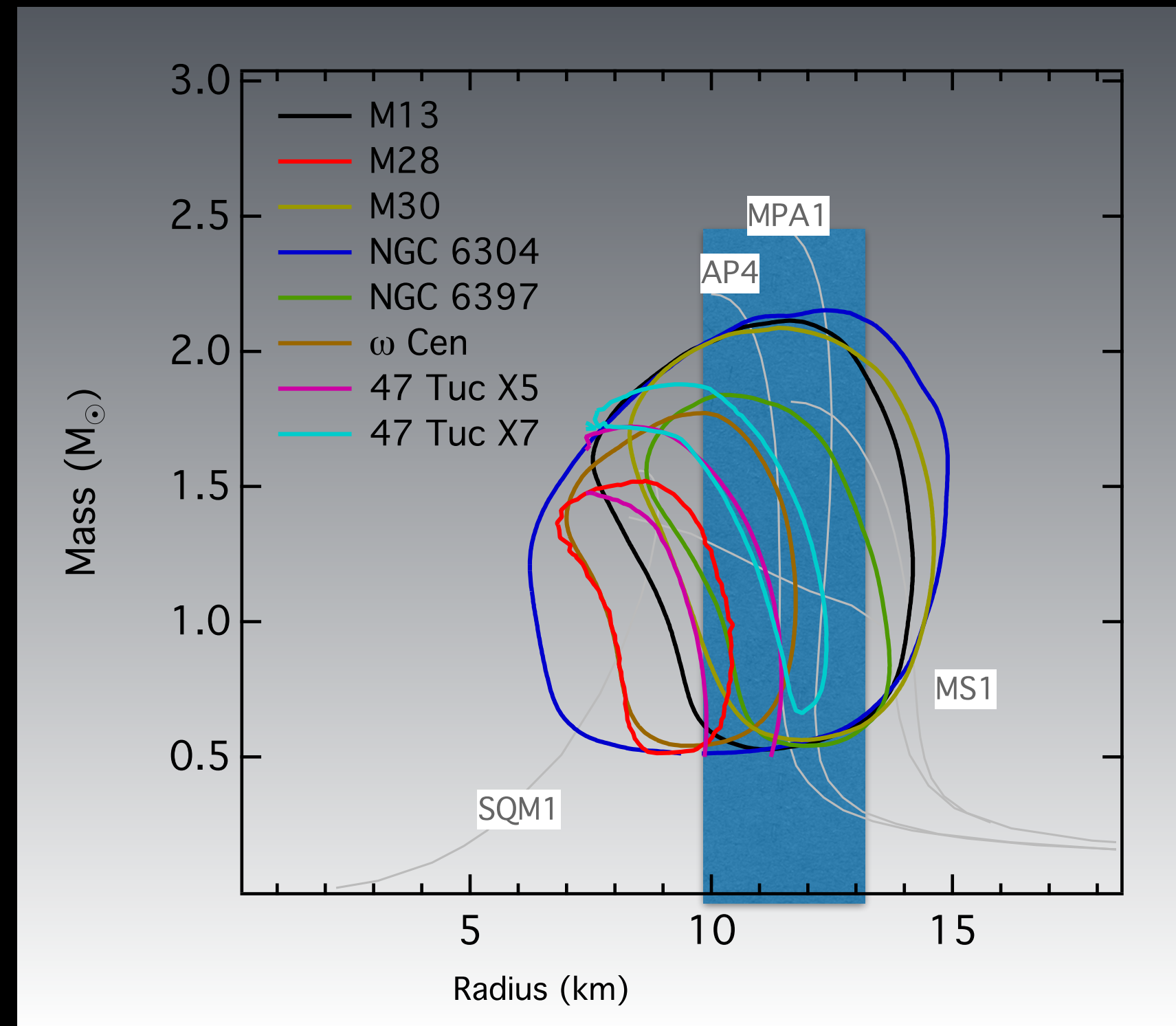
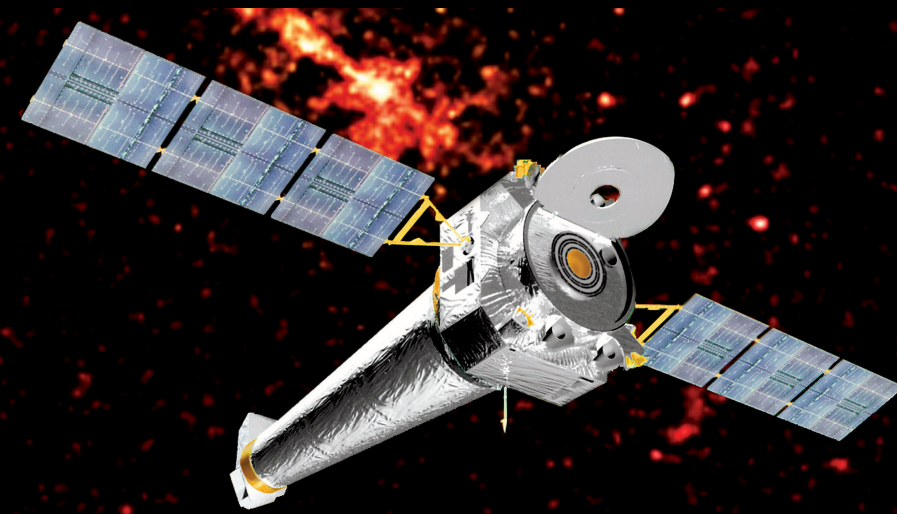
MSP J0740+6620: $M=2.17(10) M_{\odot}$

Cromartie et al. (2019)

Inferred NS radii are small.

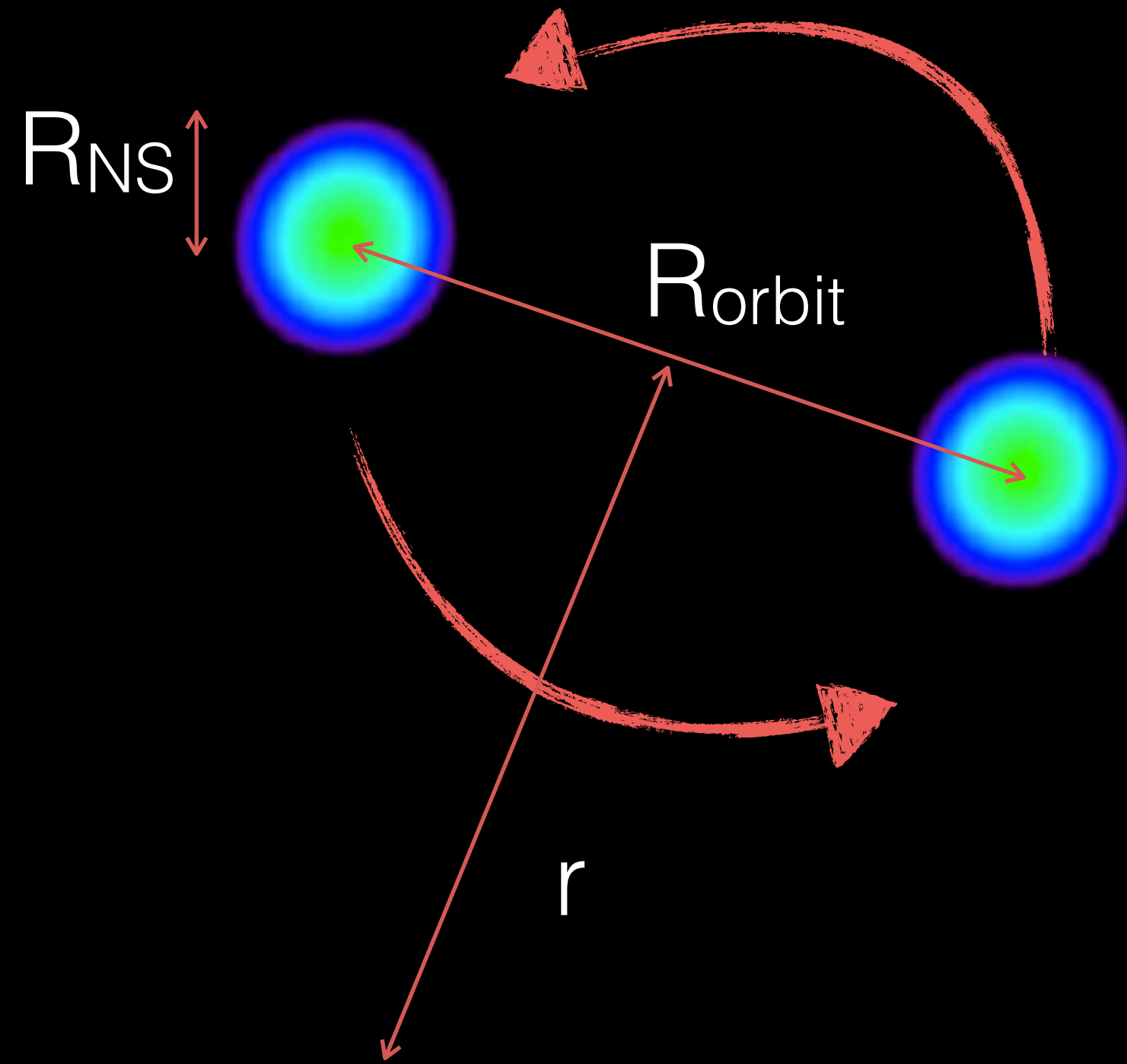
Despite poorly understood systematic errors, x-ray observations suggest $R \sim 9-13$ km. Perhaps even preferring a smaller range $R \sim 10-12$ km.

Ozel & Freire (2016)



Binary Inspiral and Gravitational Waves

GWs are produced by fluctuating quadrupoles.



$$h_{\mu\nu}(r, t) = \frac{2G}{r} \ddot{I}_{ij}(t_R)$$

$$\ddot{I}_{ij}(t) \approx M R_{\text{orbit}}^2 f^2 \approx M^{5/3} f^{2/3}$$

$$h \approx 10^{-23} \left(\frac{M_{\text{NS}}}{M_{\odot}} \right)^{5/3} \left(\frac{f}{200 \text{ Hz}} \right)^{2/3} \left(\frac{100 \text{ Mpc}}{r} \right)$$



- Advanced LIGO can detect GWs from binary neutron stars out to about 200 Mpc at design sensitivity. Detection rate $\sim 1-50$ per year.

GW170817: Gravitational Waves from Neutron Stars!

PRL 119, 161101 (2017)

Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

week ending
20 OCTOBER 2017



GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

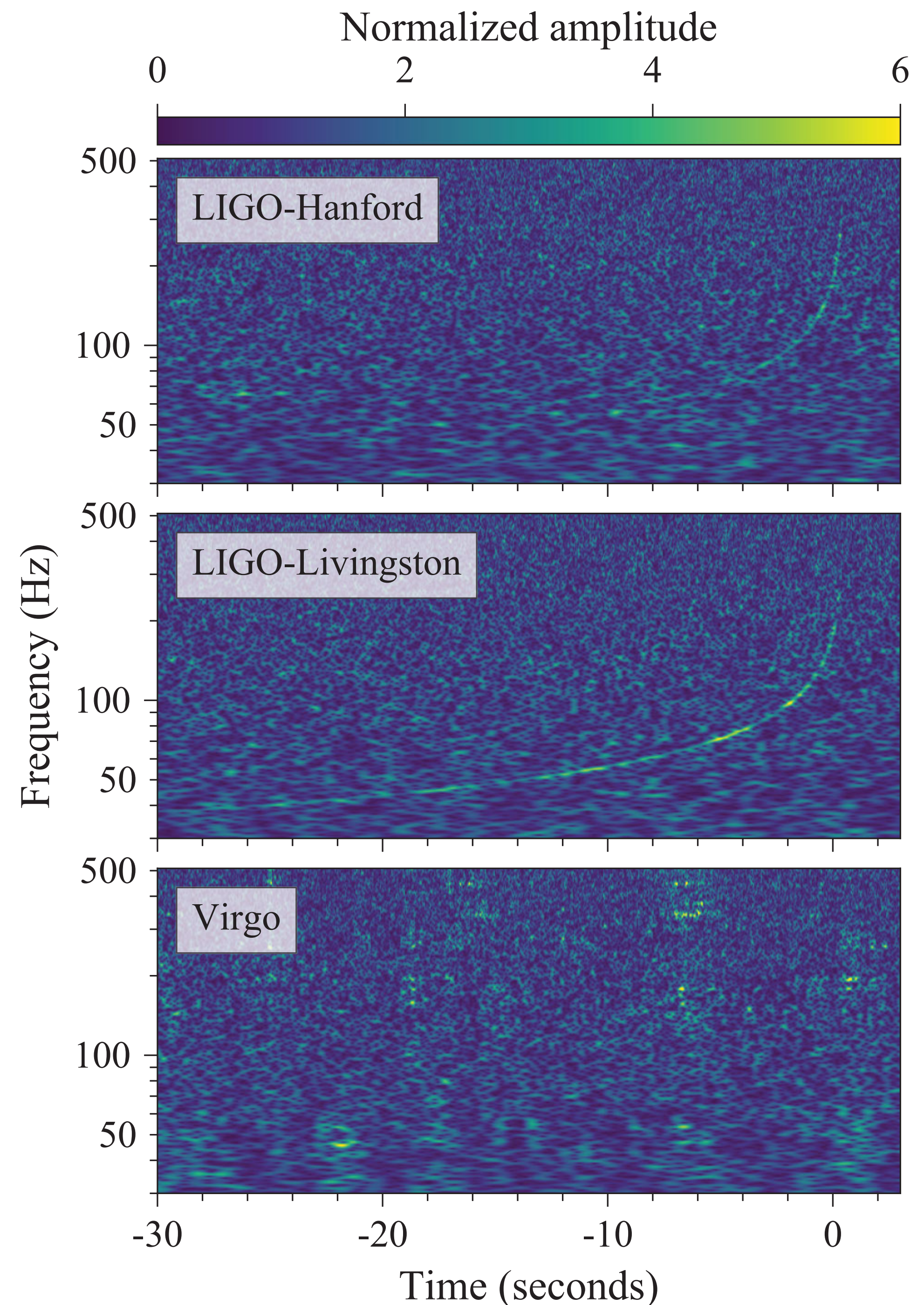
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

Component masses:

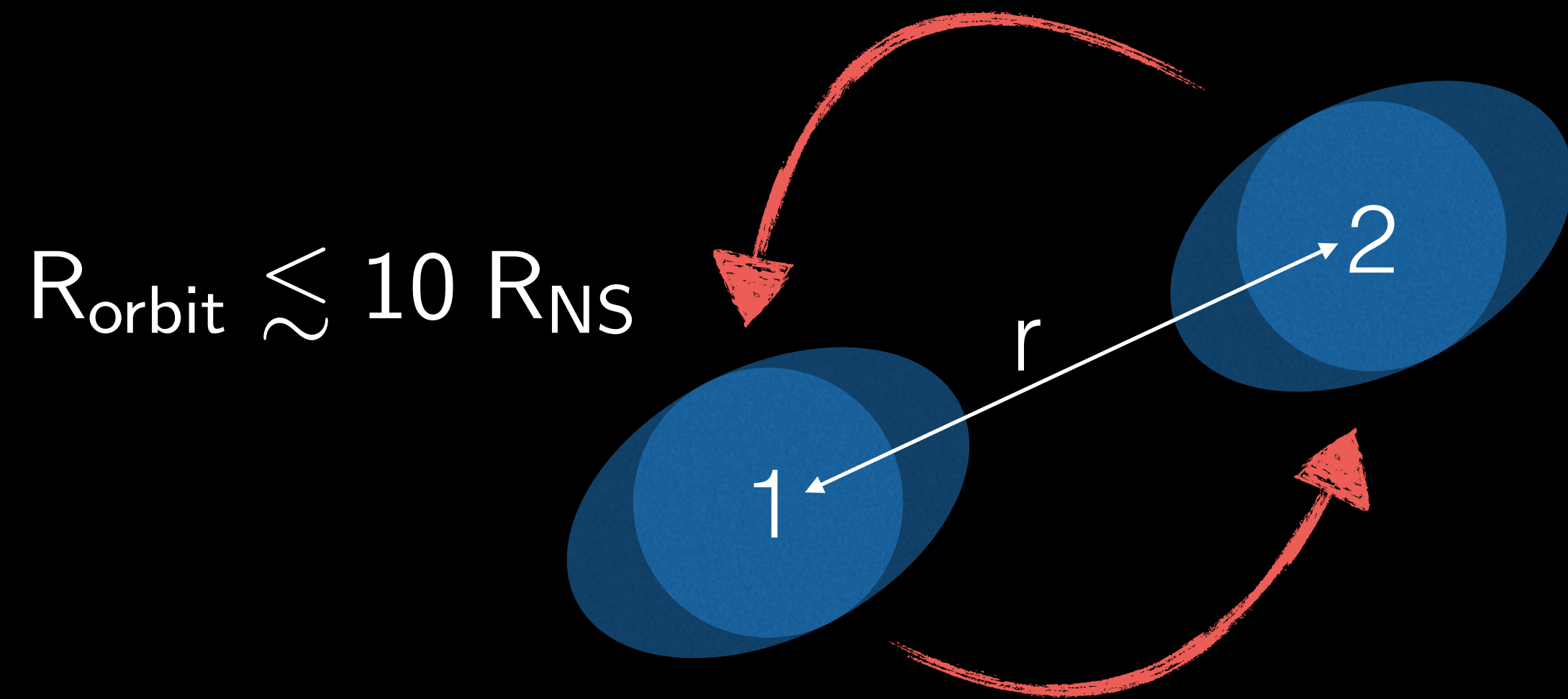
$$m_1 = 1.47 \pm 0.13 M_\odot$$
$$m_2 = 1.17 \pm 0.09 M_\odot$$

Chirp Mass: $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = 1.188^{+0.004}_{-0.002} M_\odot$

Total Mass: $M = m_1 + m_2 = 2.74^{+0.04}_{-0.01} M_\odot$



Tidal Deformation: Measuring the Neutron Star Radius



Tidal forces deform neutron stars.
Induces a quadrupole moment.

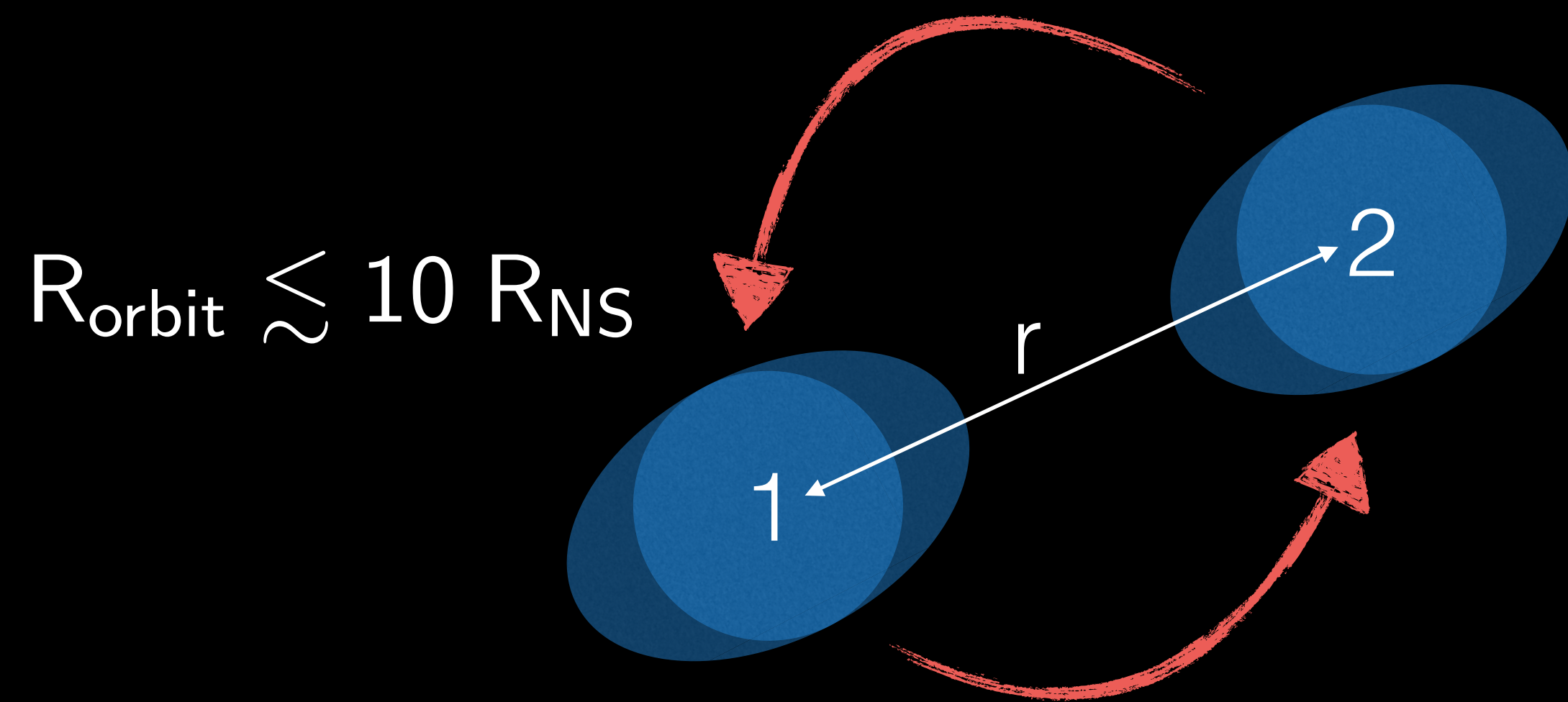
$$Q_{xy} = \lambda E_{xy}$$

↑
tidal deformability

$$E_{xy} = -\frac{\partial^2 V_G}{\partial x \partial y}$$

↑
external field

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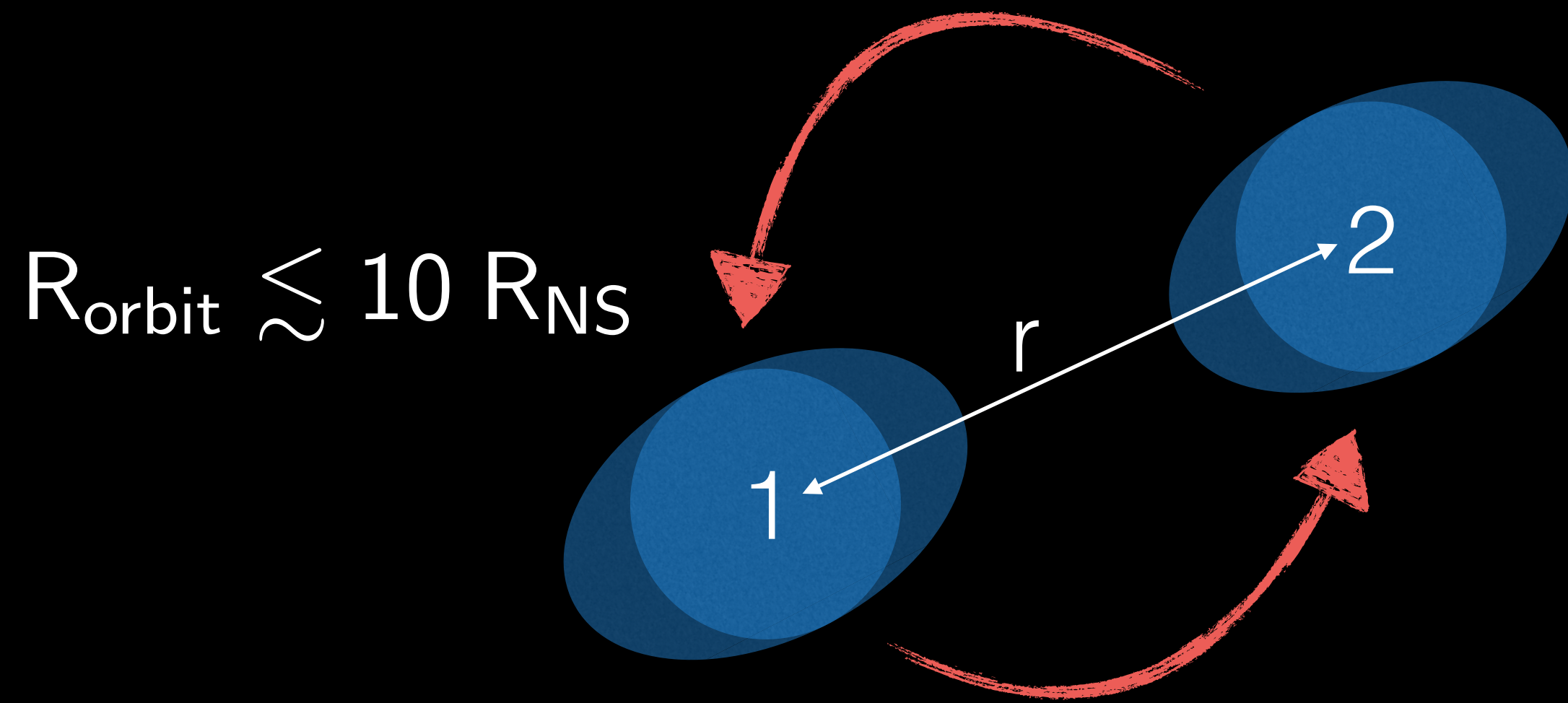
↑
tidal deformability

$$E_{xy} = -\frac{\partial^2 V_G}{\partial x \partial y}$$

↑
external field

Tidal interactions change the rotational phase: $\delta\Phi = -\frac{117}{256} v^5 \frac{M}{\mu} \tilde{\Lambda}$

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↑
tidal deformability

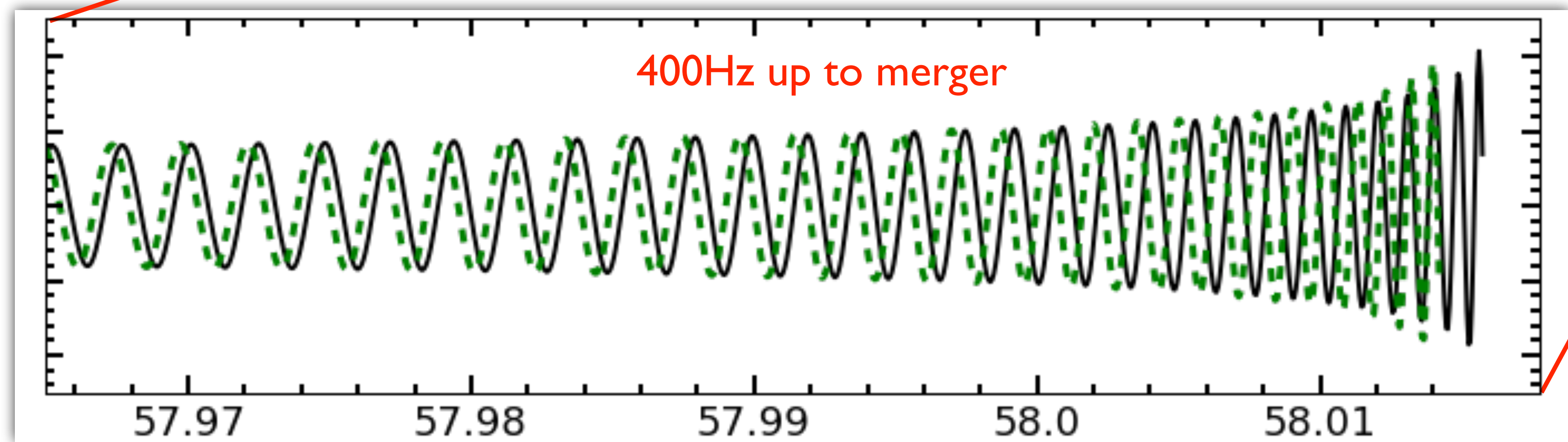
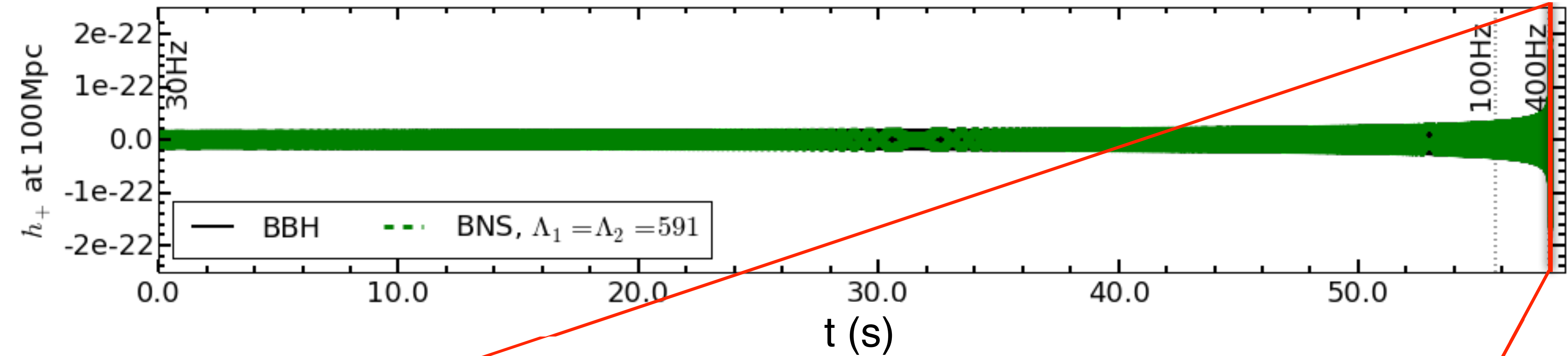
$$E_{xy} = -\frac{\partial^2 V_G}{\partial x \partial y}$$

↑
external field

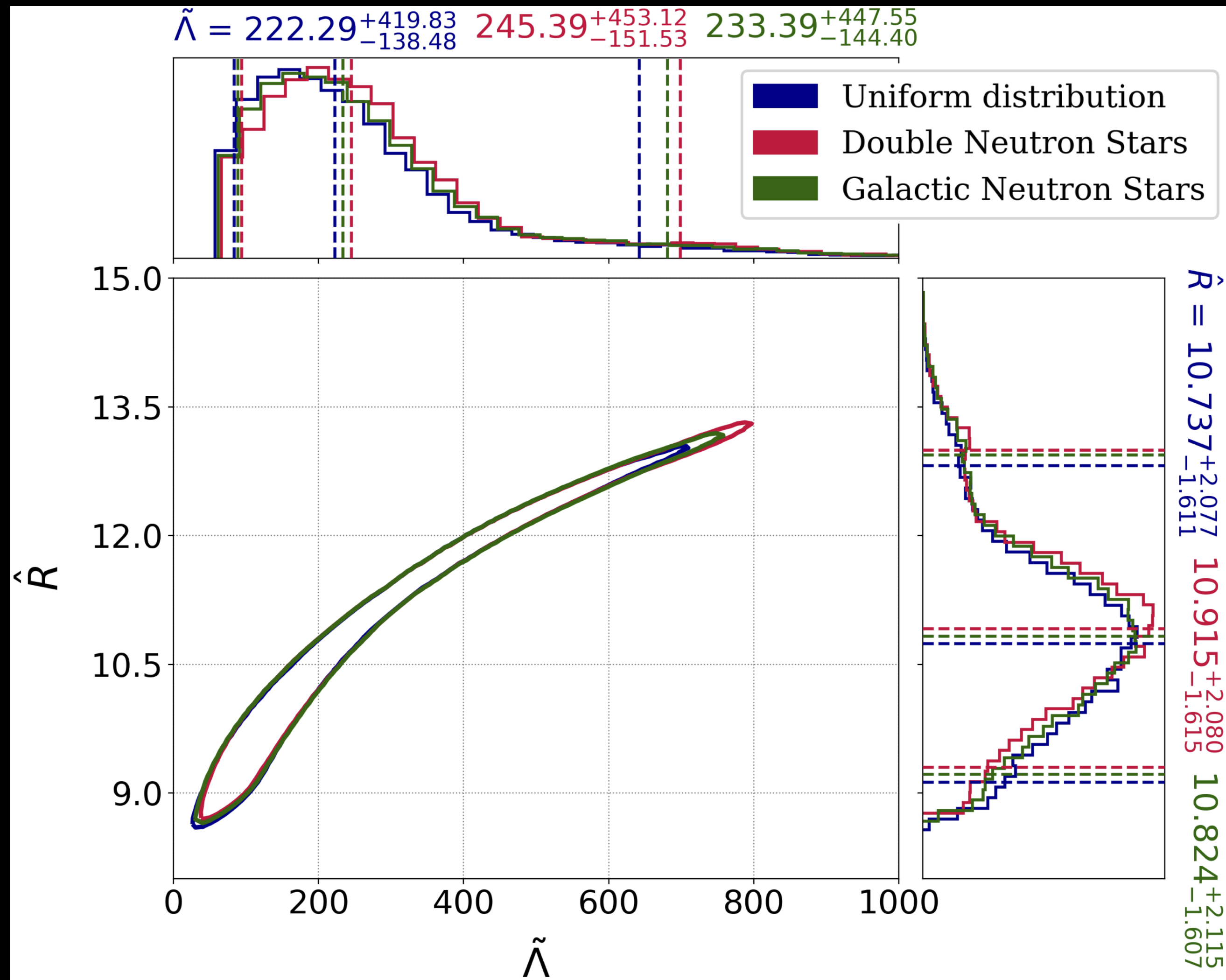
Tidal interactions change the rotational phase: $\delta\Phi = -\frac{117}{256} v^5 \frac{M}{\mu} \tilde{\Lambda}$

Dimensionless binary tidal deformability: $\tilde{\Lambda} = \frac{16}{13} \left(\left(\frac{M_1}{M} \right)^5 \left(1 + \frac{M_2}{M_1} \right) \Lambda_1 + 1 \leftrightarrow 2 \right)$

Tidal Effects at Late Times



Neutron Stars are Small



Tidal deformations observed in GW170817 are small and suggests that the NS radius:

$$R < 13 \text{ km}$$

Requiring a maximum mass greater than $2 M_{\text{sun}}$ implies:

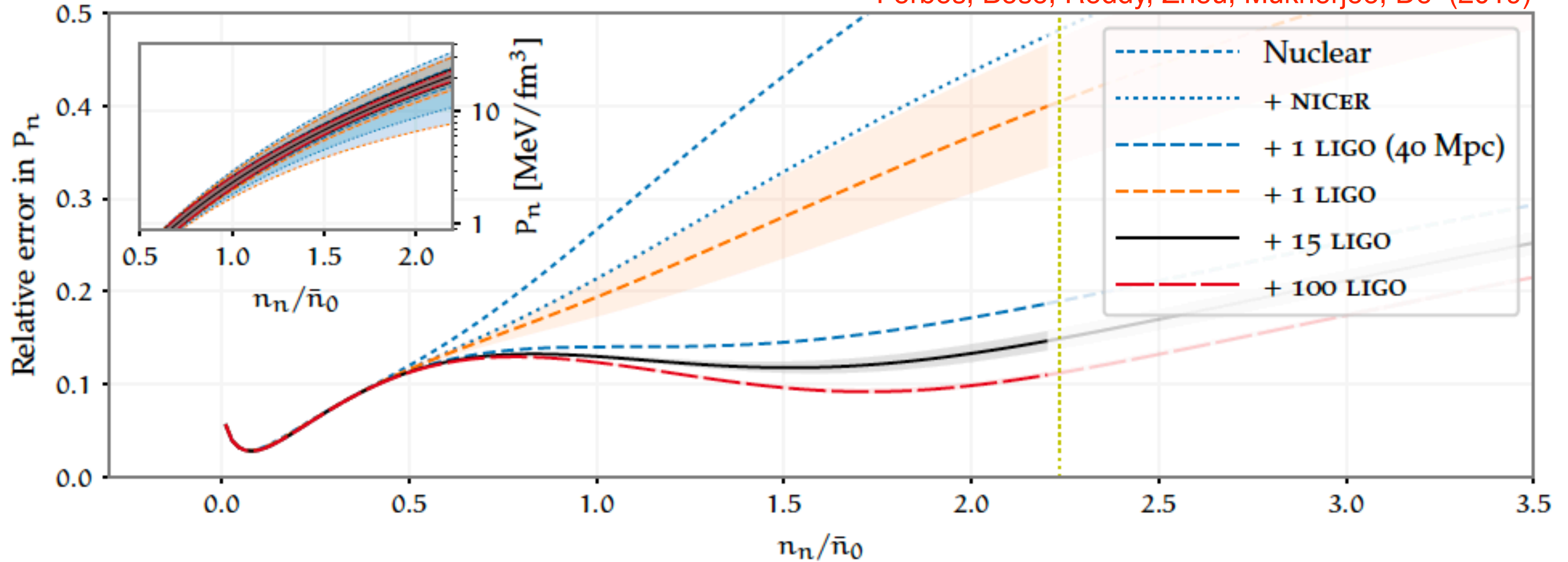
$$R > 9 \text{ km}$$

De et al. PRL (2018)

See also LIGO and Virgo Scientific Collaboration arXiv:1805.11581v1

Future Constraints from aLIGO-VIRGO Observations

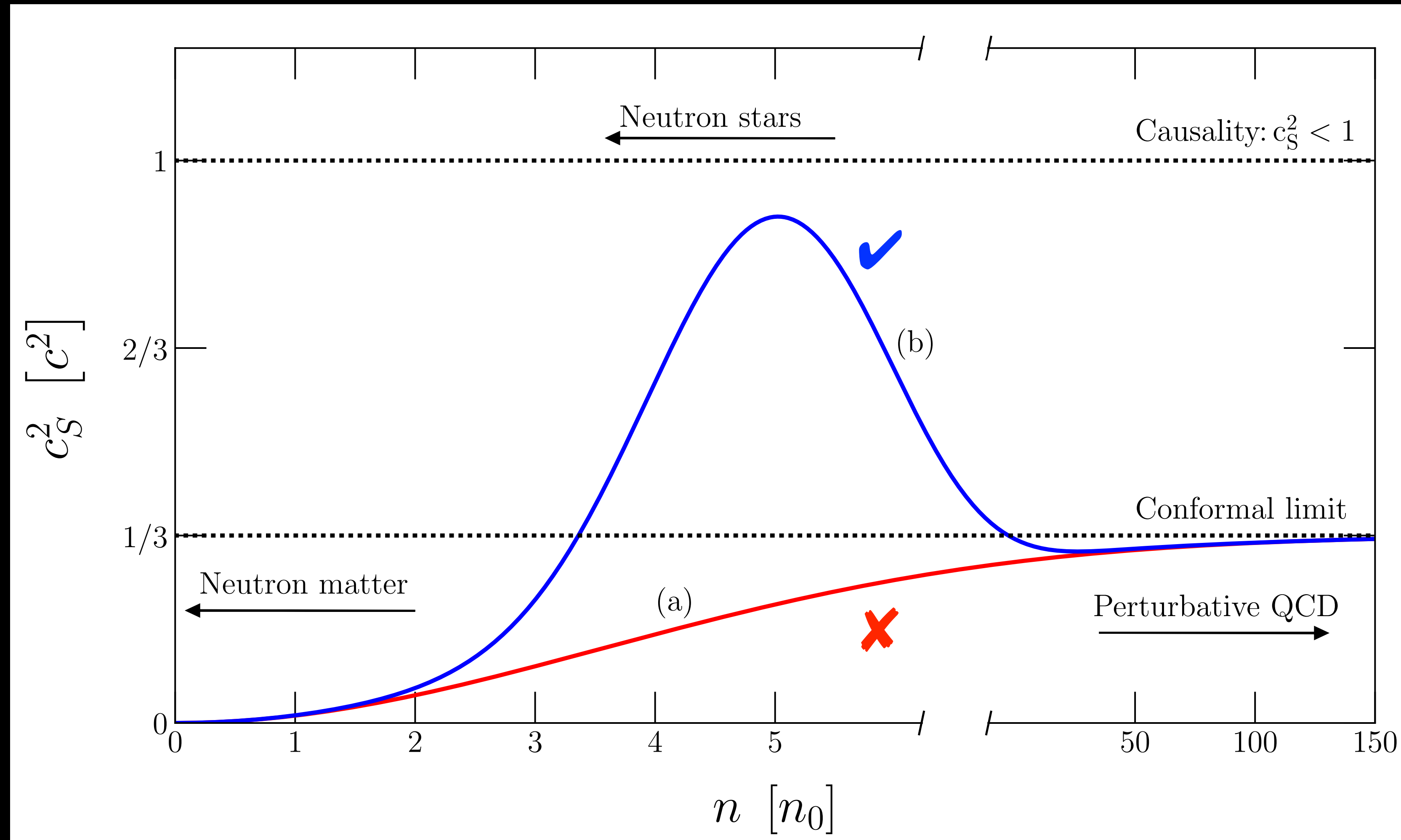
Forbes, Bose, Reddy, Zhou, Mukherjee, De (2019)



At aLIGO design sensitivity we should be able to constrain the EOS of dense matter in the outer core at the 20% level with 15-20 observations.

Speed of Sound in Dense Matter

Large observed maximum mass combined with small radius and neutron matter calculations suggests a rapid increase in pressure in the neutron star core. Implies a large and non-monotonic sound speed in dense QCD matter.



Composition of the Inner Core

Many possibilities have been considered in past which include:

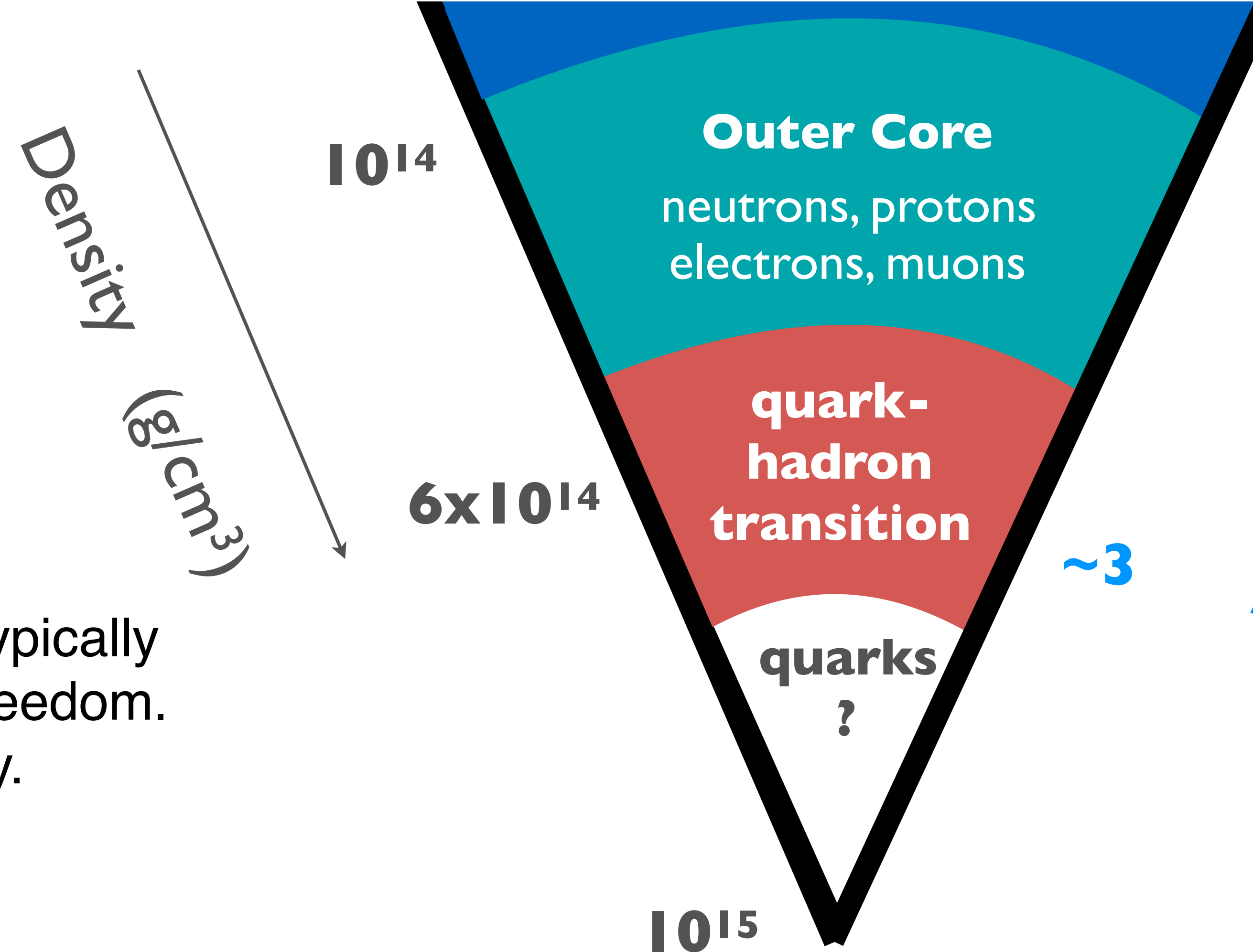
- Hyperons
- Pion and Kaon Condensates
- Mixed phases of nuclear and quark matter
- Pure quark matter.

In all of these scenarios the equation of state is typically softened by the appearance of new degrees of freedom. Supporting 2 solar mass neutron stars is not easy.

The Alternate:

- Quark-hadron transition is smooth.
- May not even be a phase transition - cross-over.
- In the core, nucleons could persist as correlated states at the quark Fermi surface.

McLerran and Reddy (2019)



A Model for Cold Dense Quarkyonic Matter

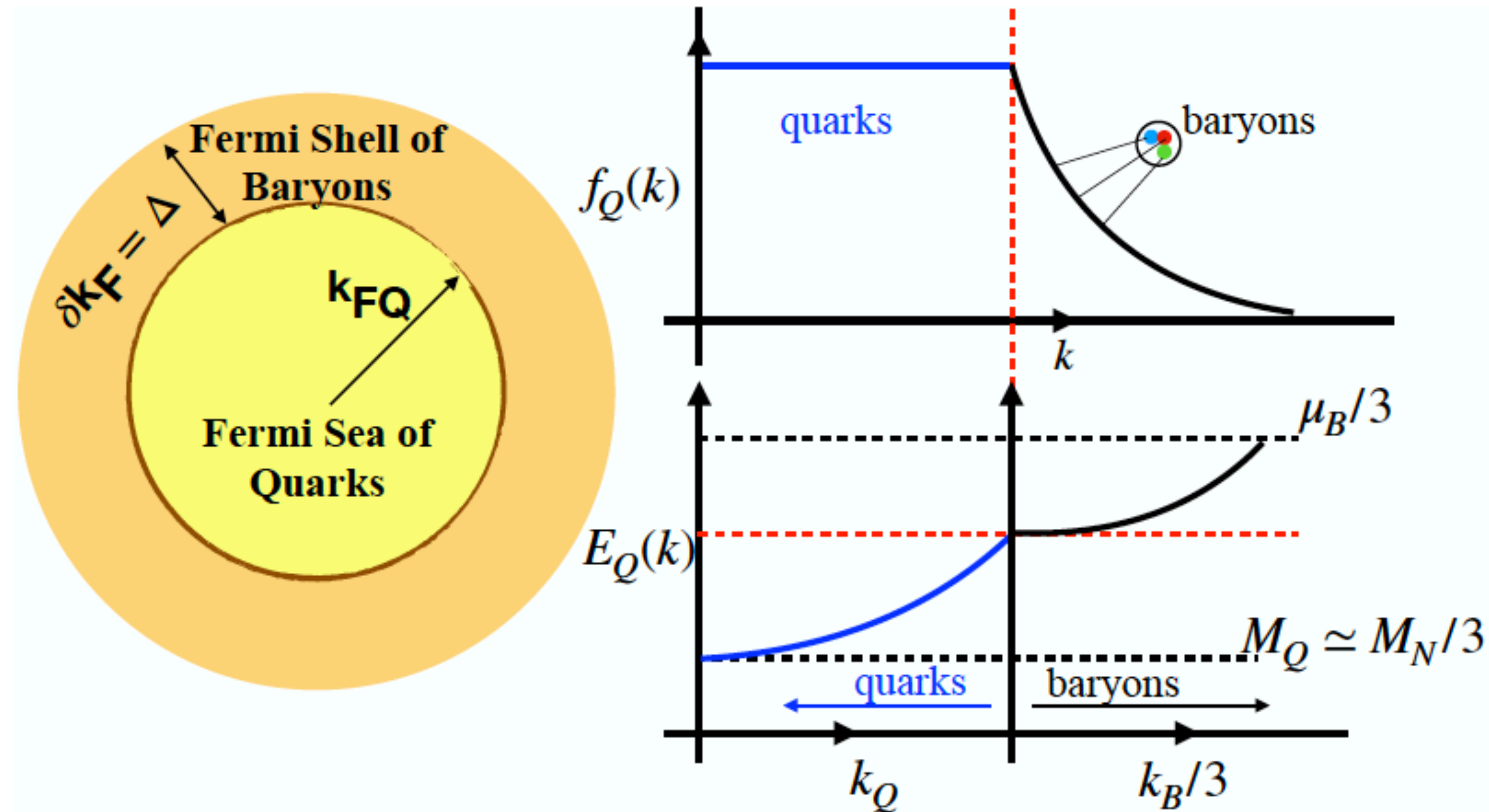
At intermediate density nucleons are confined to a shell.

The shell has a width of the order of

$$\delta k_F = \Lambda_{\text{QCD}}$$

Quarks inside the Fermi surface are weakly interacting due to Pauli blocking of intermediate states.

Density of nucleons saturates due to strong repulsive forces.



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