Probing Quadratic Gravity with Binary Inspirals

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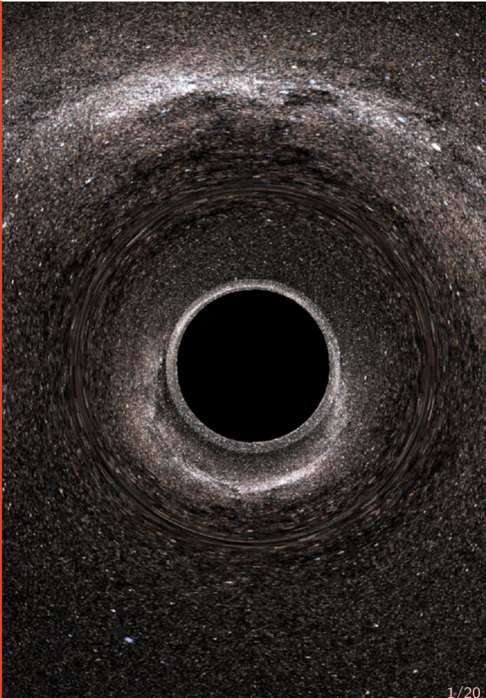


Talk Outline

The Einstein-Hilbert Action

- The Einstein-Hilbert Action gives us the Einstein Field Equations:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_M$$

$$\downarrow \text{(Variation W.R.T } g_{\mu\nu}\text{)}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}$$

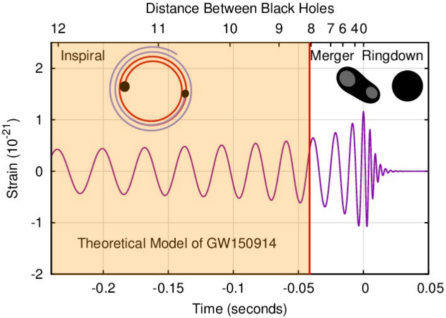
- $\kappa = 8\pi G = 8\pi/M_p^2$
- R is the only independent scalar which we can construct (up to second derivatives) of the metric

Motivating Modifications to General Relativity

- General Relativity (GR) is the *simplest* theory coupling spacetime curvature to matter
- Can consider other theories by adding terms to the Hilbert action, as long as they:
  - Are diffeomorphism invariant, scalar, etc.
  - Limit correctly to GR and Newtonian gravity
- Good reason to look at modified theories
  - Quantum fluctuations, string theory
- **What effect do these modifications have?**
  - Must look at *strong gravity*
  - $\Rightarrow$ Binary Systems are an ideal testing ground

The Post-Newtonian Formalism

- The Post-Newtonian (PN) formalism is an iterative expansion scheme in v/c, for arbitrarily precise solutions to Einstein field equations
  - Requires slow moving, weakly stressed sources (valid for inspiralling binary black holes up to $v/c = .5$)
  - Naturally includes non-linearity and higher multipole characteristics
  - Convention is to just track $1/c^n$, and call those terms "$\frac{n}{2}$PN order"
- 0PN order is called "Newtonian" order



The Quadratic Action

- We will include in our action all independent terms up to *4th* derivatives of the metric:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} + \beta R^2 + \gamma R^{\mu\nu}R_{\mu\nu}\right] + S_M$$

  - These are unavoidable from one-loop renormalisation of matter with semi-classical gravity
  - Non-renormalizability of higher loops means these must be found experimentally
- Consider gravitational waves (GWs) from a compact binary system:

$$S_M = \sum_{a=1}^{2}\int dt\, m_a c\sqrt{(-g_{\mu\nu})_a v_a^\mu v_a^\nu}$$

- By comparing our new gravitational wave solutions to LIGO observations, we can constrain β and γ.

Recasting Corrections as Massive Scalar and Tensor Fields

We can recast both quadratic terms as massive spin-0 and spin-2 fields with some clever manipulation:

$$S = \int d^4x\sqrt{-g}\Big[\frac{\tilde{\tilde{R}}}{2\kappa}-\frac{1}{2}\left(\partial_\mu\pi^{\alpha\beta}\partial^\mu\pi_{\alpha\beta}+m_\pi^2\pi^{\alpha\beta}\pi_{\alpha\beta}\right) -\frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi+m_\phi^2\phi^2\right)\Big]+\tilde{\tilde{S}}_M$$

The mass terms are:

$$m_\phi^2 = \frac{1}{12\kappa(\beta+\gamma/4)} \qquad m_\pi^2 = \frac{1}{2\kappa\gamma}$$

To *linear* order, the transformation to this frame is:

$$\tilde{\tilde{g}}_{\mu\nu} \approx g_{\mu\nu}+\sqrt{2\kappa}\eta_{\mu\nu}\phi+\sqrt{4\kappa}\pi_{\mu\nu}$$

Quadratic Gravity as an Effective Field Theory

- We cutoff our Lagrangian at quadratic order to avoid non-renormalizability at the 2-loop level
- Stelle[1] noted the negative norm states of the massive spin-2 field
  - We must interpret this as an *effective* field theory
- Quick and dirty calculation to show realm of validity:

$$M_p^2\ R > \alpha R^{\text{quad}} \Rightarrow M_p^2\ p^2 > \alpha p^4 \quad \text{(In momentum space)}$$
$$\Rightarrow M_p^2/r^2 > \alpha/r^4$$
$$m_{\phi,\pi} \approx M_p^2/\alpha \Rightarrow m_{\phi,\pi} r > 1$$

- We can then see that far-field plane waves $e^{-i(\omega t - \vec{k}\vec{x})}$ are suppressed:

$$v^2 \approx GM/r < 1 < m_{\phi,\pi} r \quad \Rightarrow \quad m_{\phi,\pi} > \Omega^2 \approx \omega^2$$
$$\Rightarrow \quad k^2 = \omega^2 - m_{\phi,\pi}^2 < 0$$

[1] K. S. Stelle (1978). “Classical Gravity with Higher Derivatives”. In: *General Relativity and Gravitation*.

Existing Constraints and Work

- Current constraints on deviations from gravity:
  - Torsion-balance experiments (excluded in $10^{-5}eV \lesssim m_\phi \lesssim 10^{-3}eV$)
  - Lunar ranging, satellite, and solar system tests (eg. Stelle's estimate: $m_\phi \gtrsim 10^{-16}eV$)
  - Astrophysical distance measurements constrain $f(R)$ gravities ($m_\phi \gtrsim 10^{-30}eV$)
- To lowest order, these deviations look like Yukawa potentials $\alpha G e^{-r/\lambda}$
  - Usually constrain on the coupling strength α for fixed range λ
  - We set $\alpha \approx 1$ and the instead constrain the mass $m_{\phi,\pi} \approx 1/\lambda$
- We follow the well-studied PN methodology for finding GWs, specifically the formalism of Blanchet's detailed review.[2]

[2] Luc Blanchet (2014). "Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries". In: *Living Reviews in Relativity*.

Linearized Equations of Motion

- We can find the linearized field equations for ϕ and $\pi_{\mu\nu}$:
  - We also cut off the source terms at lowest PN order

$$\Box\phi - m_\phi^2\phi = -\sum_{a=1}^{2}\sqrt{\frac{\kappa}{2}}m_a c\,\delta^3(\vec{x}-\vec{y}_a(t))$$

$$\Box\pi_{\mu\nu} - m_\pi^2\pi_{\mu\nu} = \sum_{a=1}^{2}\sqrt{\kappa}m_a\left(v_{\mu a}v_{\nu a}+\frac{c^2}{4}\eta_{\mu\nu}\right)\delta^3(\vec{x}-\vec{y}_a(t))$$

- Which have Yukawa-like solutions:

$$\phi(x) = \sum_{a=1}^{2}\sqrt{\frac{G}{4\pi}}m_a c\frac{e^{-m_\phi c|\vec{x}-\vec{y}_a(t_r)|}}{|\vec{x}-\vec{y}_a(t_r)|}$$

$$\pi_{\mu\nu}(x) = -\sum_{a=1}^{2}\sqrt{\frac{G}{2\pi}}\frac{m_a}{c}\left(v_{\mu a}v_{\nu a}+\frac{c^2}{4}\eta_{\mu\nu}\right)\frac{e^{-m_\pi c|\vec{x}-\vec{y}_a(t_r)|}}{|\vec{x}-\vec{y}_a(t_r)|}$$

Modified Binary Dynamics

- We can compute corrections to the geodesic equation by transforming the conservation equation $\tilde{\tilde{\nabla}}_\mu \tilde{\tilde{T}}^{\mu\nu} = 0$ back into our original $g_{\mu\nu}$ coordinates.
- Then we can calculate the *relative* acceleration to Newtonian order, as well as the angular frequency:

$$a^i = -\frac{G(m_1+m_2)}{r^2}\hat{n}\left(1+2e^{-m_\phi r}\left(m_\phi r+1\right)-3e^{-m_\pi r}\left(m_\pi r+1\right)\right)$$
$$\Omega^2 = \frac{G(m_1+m_2)}{r^3}\left(1+2e^{-m_\phi r}\left(m_\phi r+1\right)-3e^{-m_\pi r}\left(m_\pi r+1\right)\right)$$

- where $r = |\vec{y}_1 - \vec{y}_2|$, and $\hat{n} = (\vec{y}_1 - \vec{y}_2)/r$

Energy-Balance Equations

- From the acceleration, we can find an effective Lagrangian for the binary and therefore the energy:

$$E = -\frac{Gm_1m_2}{r}\left(\frac{1}{2} + 2e^{-m_\phi r} - 3e^{-m_\pi r}\right) \tag{1}$$

- The far-field flux will be highly suppressed for the massive fields, and so we can use the usual GR flux
- Identifying flux and energy loss gives us a convenient way to calculate the change in phase, without needing high PN terms:

$$\frac{dE}{dt} = -\mathcal{F} \tag{2}$$

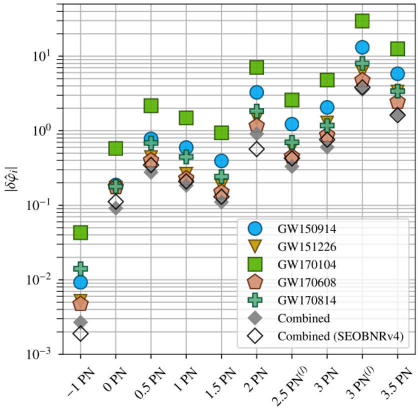
Corrections to the Binary Phase

- It is possible to carefully subsitute (in a PN-sense) our angular frequency Ω into the energy-balance equation
  - Then use the definition of phase $\frac{d\varphi}{dt} = \Omega$ to solve an ODE in φ and r.

$$\varphi = -\frac{r^{5/2}}{32 m_1 m_2 (G(m_1+m_2))^{3/2}} \left[1 + e^{-m_\phi r}\left(\frac{5}{2} - \frac{5}{3} m_\phi r\right) - e^{-m_\pi r}\left(\frac{15}{4} - \frac{5}{2} m_\pi r\right) + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

- There are both 0PN and -1PN terms
  - We can get multipole moments lower than quadrupole from the massive fields

Using Observations to Constrain Corrections



- 90% upper bounds on the GR violating parameter $\delta\hat{\varphi}$

- GW observations allow us constrain possible deviations of phase from GR at each PN order
- Then we can constrain our spin-0 and spin-2 masses:

$$m_\phi \geqslant 2.3 \times 10^{-11} eV$$
$$m_\pi \geqslant 3.2 \times 10^{-11} eV$$

(LIGO/Virgo Collaborations, 2019)

Conclusions

- Recast quadratic gravity as a massive spin-0 and spin-2 field alongside the usual graviton, and derived linear, lowest order field equations
  - To Newtonian order, they respectively act as attractive and repulsive Yukawa potentials modifying gravity
- Found -1PN and 0PN corrections to GW phase of an inspiralling binary system in quadratic gravity
- Placed constraints on quadratic gravity from real GW observations from LIGO and Virgo Collaborations

Thanks to the 59th Cracow School of Theoretical Physics for inviting me to give this seminar!

Extras: Recasting the Lagrangian

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} + \beta R^2 + \gamma R^{\mu\nu}R_{\mu\nu}\right] + S_M$$

Setting $S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R$ and $\alpha = \beta + \frac{\gamma}{4}$,

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa} + \alpha R^2 + \gamma S^{\mu\nu}S_{\mu\nu}\right] + S_M$$

Using Lagrange multipliers, and the following conformal transformation,

$$\widetilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = (1 + \sqrt{2\kappa}\phi)$$

$$S = \int d^4x\sqrt{-g}\left[\frac{\widetilde{R}}{2\kappa} + \pi^{\mu\nu}\widetilde{S}_{\mu\nu} - \frac{1}{4\gamma}\pi^{\mu\nu}\pi_{\mu\nu} - \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + m_\phi^2\phi^2\right)\right] + \widetilde{S}_M \tag{3}$$

Separating $\pi^{\mu\nu}$ from $\widetilde{h}^{\mu\nu}$ we obtain the final result.

Extras: Geodesic Equations

Taking the spatial component of $\tilde{\tilde{\nabla}}_\mu \tilde{\tilde{T}}^{\mu\nu} = 0$, we can find the geodesic equations in our *original* frame:

$$\frac{dP^i_{GR}}{dt} = F^i_{GR} + \sqrt{16\pi G}\,\partial_i \phi - \sqrt{32\pi G}\,\partial_i \pi_{\mu\nu} v^\mu v^\nu + \mathcal{O}\left(\frac{1}{c^2}\right) \quad (4)$$

Here the linear momentum density P^i_{GR} and the force density F^i_{GR} are given by

$$P^i_{GR} = c\frac{g^{GR}_{\mu i} v^\mu}{\sqrt{-g^{GR}_{\rho\sigma} v^\rho v^\sigma}}$$

$$F^i_{GR} = \frac{c}{2}\frac{\partial_i g^{GR}_{\mu\nu} v^\mu v^\nu}{\sqrt{-g^{GR}_{\rho\sigma} v^\rho v^\sigma}} \quad (5)$$

Extras: Details on Binary Phase Calculations 1

Freq. parameter x, with $M = m_1 + m_2, \mu = m_1 m_1/M, \nu = \mu/M$:

$$x \equiv \left(\frac{GM\Omega}{c^3}\right)^{\frac{2}{3}}$$

$$\Rightarrow r = \frac{GM}{xc^2}\left(1 + 2e^{-m_\phi \frac{GM}{xc^2}}\left(m_\phi \frac{GM}{xc^2} + 1\right) - 3e^{-m_\pi \frac{GM}{xc^2}}\left(m_\pi \frac{GM}{xc^2} + 1\right)\right)$$

Then its possible to solve

$$\frac{dE}{dx} = -\mu c^2 \Bigg[\frac{1}{2} + \frac{1}{3}e^{-m_\phi \frac{GM}{xc^2}}\left(5 + 5m_\phi \frac{GM}{xc^2} - \left(m_\phi \frac{GM}{xc^2}\right)^2\right) - \frac{1}{2}e^{-m_\pi \frac{GM}{xc^2}}\left(5 + 5m_\pi \frac{GM}{xc^2} - \left(m_\pi \frac{GM}{xc^2}\right)^2\right) + \mathcal{O}\left(\frac{1}{c^2}\right)\Bigg]$$

And we also have the usual GR flux in terms of x:

$$\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \left[1 + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

Extras: Details on Binary Phase Calculations 2

Then we introduce a dimensionless time parameter, where t_c is the binary collision time:

$$\Theta \equiv \frac{\nu c^3}{5GM}\left(t_c - t\right)$$

$$d\varphi/dt = \Omega \quad \Rightarrow \quad \frac{d\varphi}{d\Theta} = -\frac{5}{\nu}x^{3/2}$$

Then our energy-balance equation becomes

$$\frac{dE}{dx}\frac{dx}{d\varphi}\frac{x^{3/2}c^3}{GM} = -\mathcal{F} \tag{6}$$

Then we can write down the full differential equation for the phase:

$$\frac{d\varphi}{dx} = \frac{5x^{-7/2}}{32\nu}\left[\frac{1}{2} + \frac{1}{3}e^{-m_\phi \frac{GM}{xc^2}}\left(5 + 5m_\phi\frac{GM}{xc^2} - \left(m_\phi\frac{GM}{xc^2}\right)^2\right)\right.$$
$$\left. - \frac{1}{2}e^{-m_\pi \frac{GM}{xc^2}}\left(5 + 5m_\pi\frac{GM}{xc^2} - \left(m_\pi\frac{GM}{xc^2}\right)^2\right) + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

Extras: Constraints on β and γ

$$m_\phi \geqslant 2.3 \times 10^{-11} eV$$
$$m_\pi \geqslant 3.2 \times 10^{-11} eV$$

This corresponds to:

$$\beta / M_p^2 \lesssim 10^{19} eV^{-2}$$
$$\gamma / M_p^2 \lesssim 10^{20} eV^{-2}$$

Although these may seem like 'big' numbers, we are in the weakly stressed regime so curvature is small and we are still within the realm of validity for our EFT: $m_{\phi,\pi} r \gtrsim 1$