Particle Acceleration in Astrophysics

1. General descriptions and Formalisms

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Work based on several PhD theses and collaborations with post doctoral fellows and colleagues

Zakopane-1, 2019

I. Observations of Acceleration In the Universe

1. Direct Observations

A. Galactic Cosmic Rays (Hess 1912)

B. Solar Energetic Particles

2. Observations of Non-thermal Radiation

Long Wave Radio to TeV Gamma-rays

General Observational Features

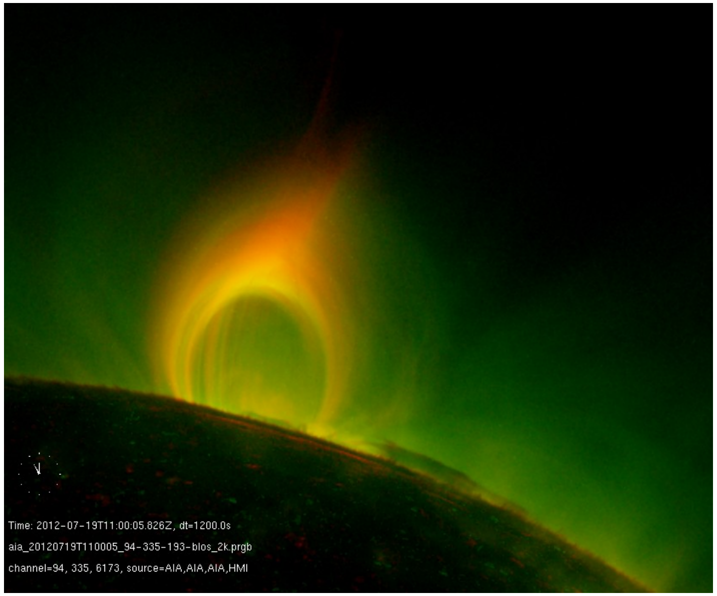
Where: *Planets to Clusters of Galaxies*

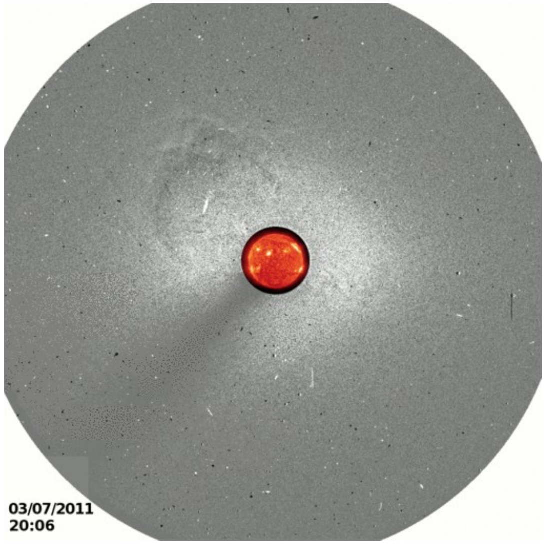
Spatial scales: *10^{8} to 10^{25} cm and beyond*

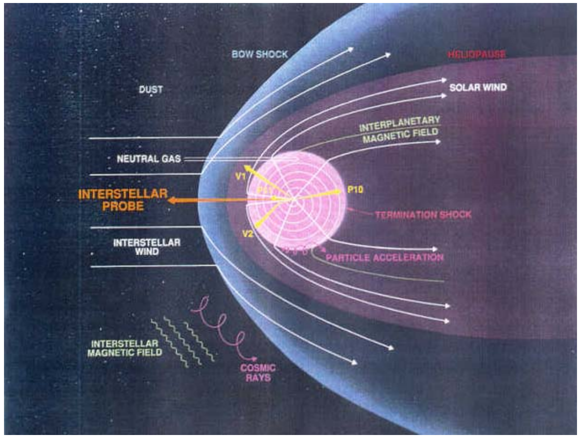
Temporal scales: *Milliseconds to Gigayears*

Energy scales: *10^{3} to 10^{20} eV*

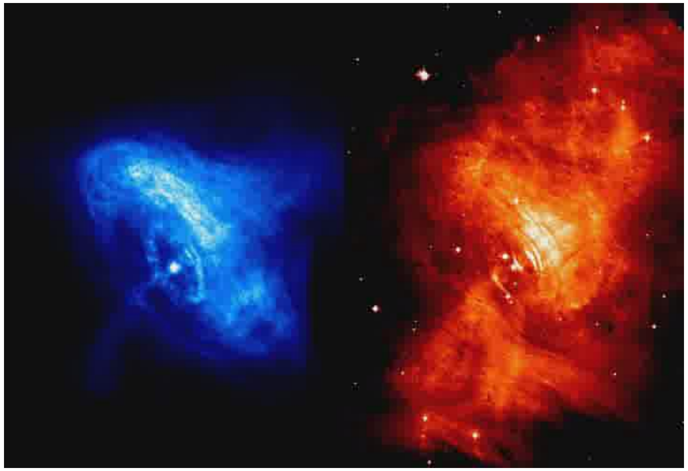
Places: *Solar System*

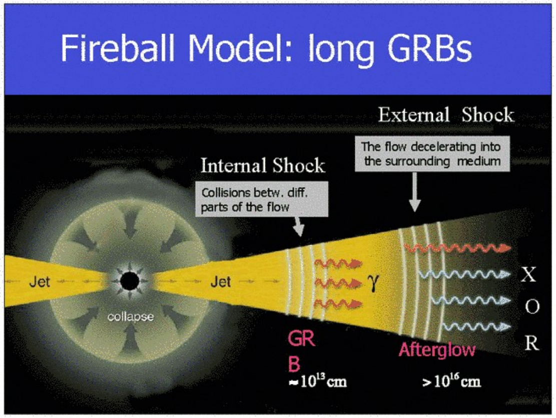


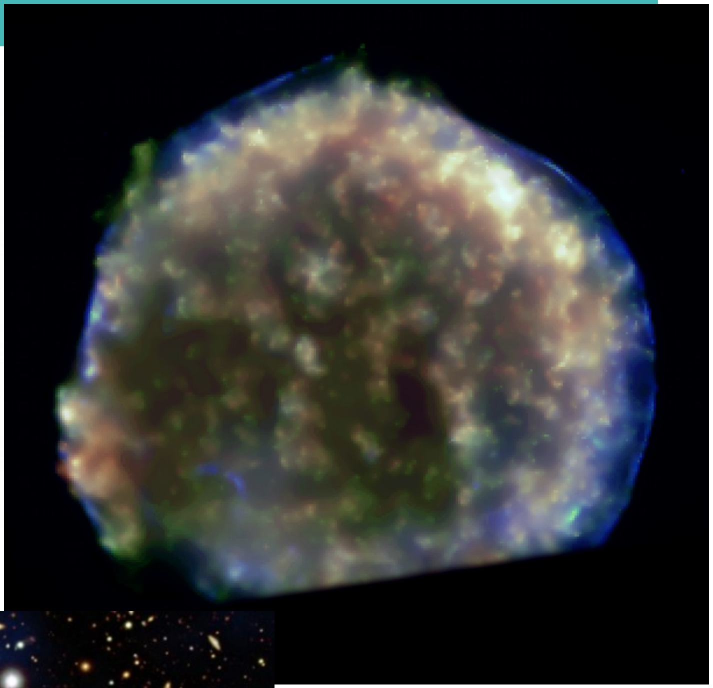


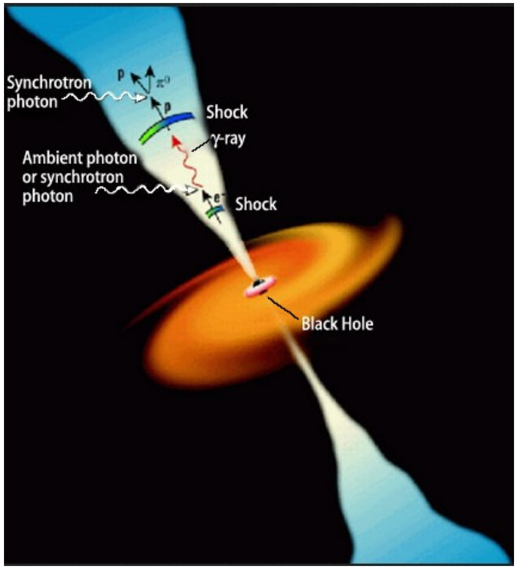


Places: *Galactic and Extragalactic*



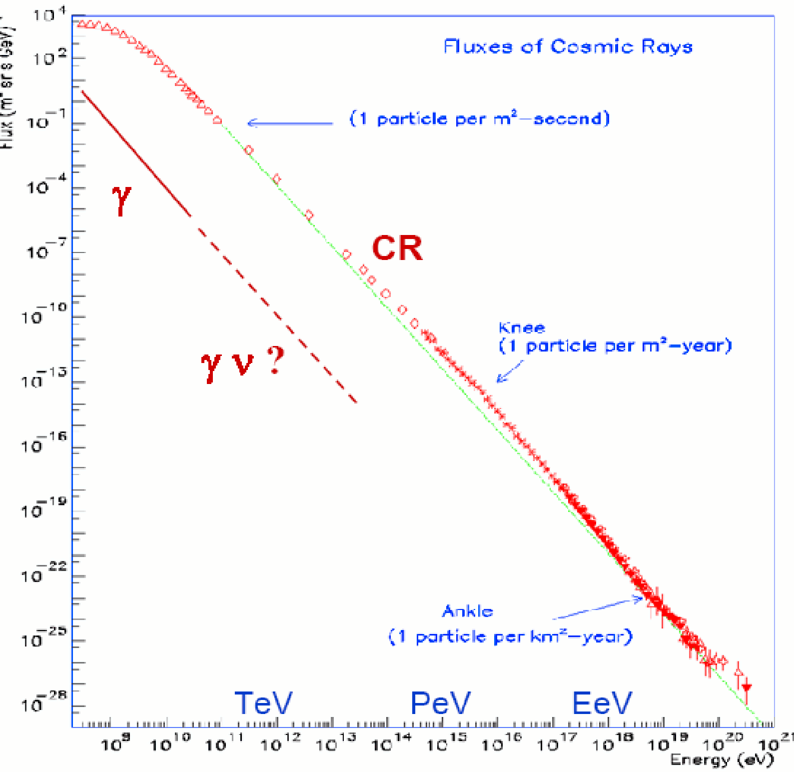


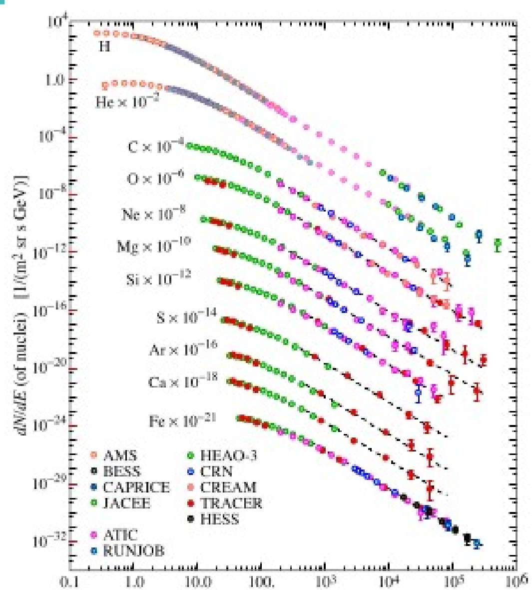


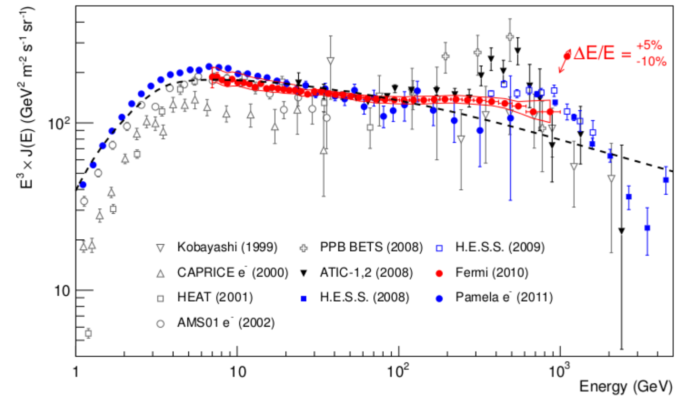


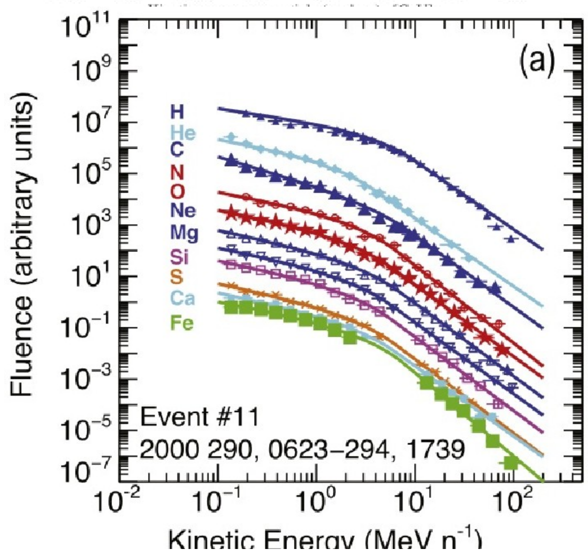
Spectra: *Direct Observations*

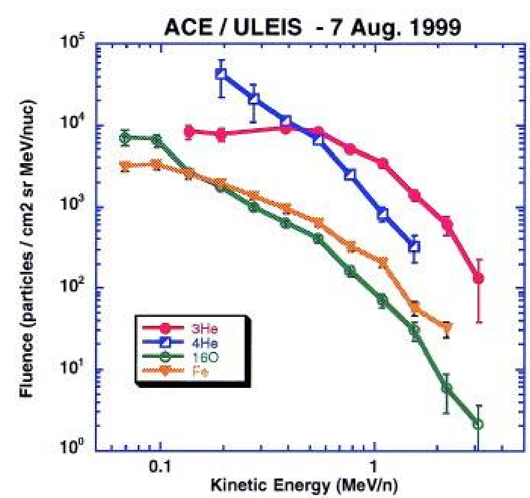
Cosmic Rays and Solar Energetic Particles



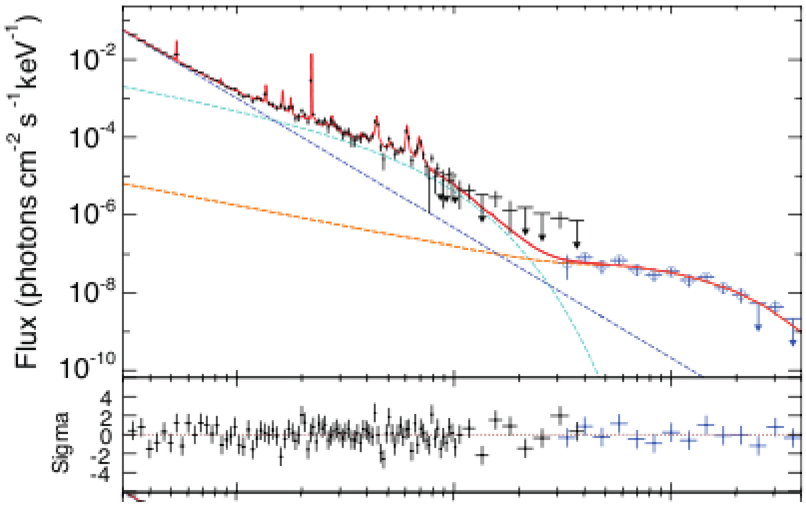


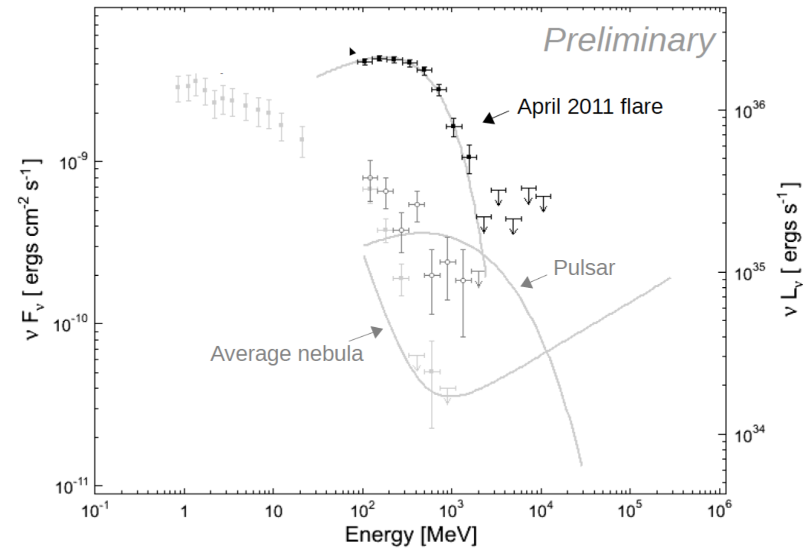




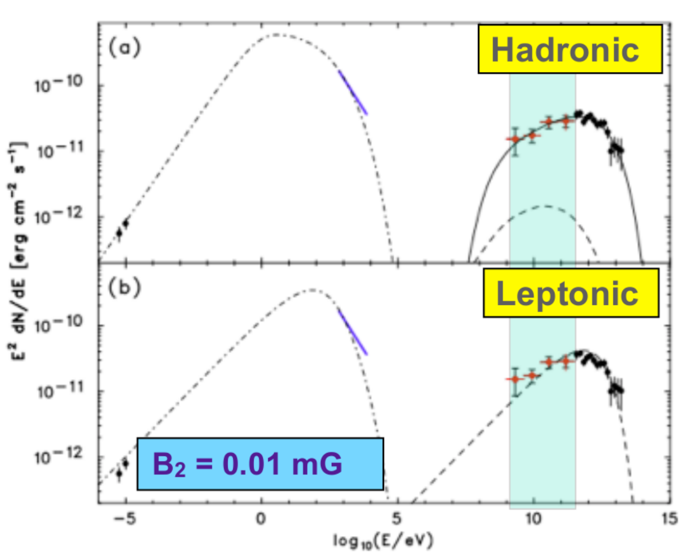


Spectra: *Non-thermal Radiation Producing Particles (RPPs)*





Tanaka+2011



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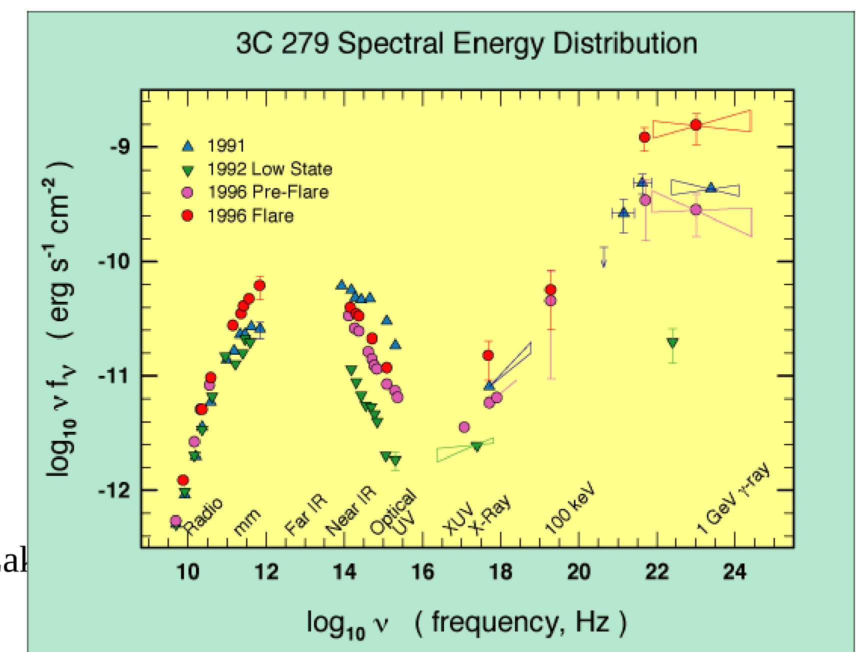


Outline

I. Acceleration Mechanisms: *General Remarks*

II. Turbulence: *General Remarks*

III. Kinetic Equations of Transport and Acceleration

Different forms of the Fokker-Planck equation

IV. Transport and Acceleration Coefficients

Energy losses and gains; Scatterings and diffusion

V. Some Solutions: *Analytic and Numerical*

I. Acceleration Mechanisms

Electric Fields and Turbulence

"1st" and 2nd Order Fermi

Magnetized Plasmas

A. ELECTRIC FIELDS: $\mathcal{E}$ (parallel to **B** field)

Acceleration Rate: $dp/dt = e\mathcal{E}$

Astrophysical Plasmas Highly Conductive: $\mathcal{E} \longrightarrow 0$

Dricer Field: $\mathcal{E}_D = kT/(e\lambda_{\rm Coul})$

$\mathcal{E} < \mathcal{E}_D$: Energy Gain $\Delta E < kT(L/\lambda_{\rm Coul})$

$\mathcal{E} > \mathcal{E}_D$: Runaway Unstable Distribution Leads to

PLASMA TURBULENCE

1. Double Layers (DLs) in Earth's Magnetosphere

Multiple DLs: Difussive Process like

PLASMA TURBULENCE

2. Unipolar Induction in High B field of Neutron Stars
Extreme Relativistic Energies: Pair Cascade

B. FERMI ACCELERATION

Random scattering by moving scattering centers.

Diffusive Process: Why Acceleration?

More headon than trailing scatterings

Phase space availibility

$$\frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{pp}\frac{\partial f}{\partial p}) \rightarrow \frac{\partial}{\partial E}(D(E)\frac{\partial N}{\partial E}) - \frac{\partial}{\partial E}(A(E)N) \quad (1)$$

tions?

Fermi Acceleration Mechanisms

General Remarks

1. Second order or Stochastic Acceleration *(Fermi 1949)*

   Second order Fermi: Scattering by TURBULENCE $D_{\mu\mu}, D_{pp}$

   Energy Gain rate: $\dot{E}_G = A_{\rm SA} = ED_{pp}/p^2 = ED_{\mu\mu}(v_A/v)^2$

[First order accel. in contracting magnetic bottle: *Fermi 1953*]

2. Acceleration in converging flows: *For example Shocks*

   Momentum Change First Order $\delta p/p \sim u_{sh}/v$

   But need repeated passages across the shock

   Most likely scattering agent is TURBULENCE

Energy Gain rate $\dot{E}_{gain} = \delta p/\delta t_c$ $\delta t_c \sim \zeta(\lambda_s/u_{sh}) \sim \zeta(v/u_{sh})D_{\mu\mu}^{-1}$

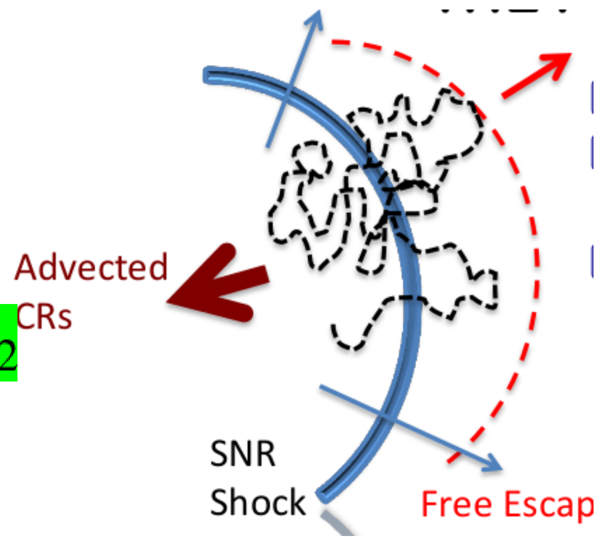
$$\dot{E}_G \equiv A_{\rm sh} = ED_{\mu\mu}(u_{\rm sh}/v)^2$$

Acceleration Rate By Shocks

- *Cartoon of a Shock*
- Down stream

Upstream

$$u_2 = u_{\rm sh}/r,\ n_2 = r n_{\rm bg}$$
$$B_2 = r\eta_{\rm amp} B_{\rm bg},\ \kappa_{s2} \sim v^2 \tau_{\rm sc2}$$



$$u_1 = u_{\rm sh},\ n_1 = n_{\rm bg}$$
$$B_1 = B_{\rm bg},\ \kappa_{s1} \sim v^2 \tau_{\rm sc1}$$

$$\tau_{\rm acc}(p) \equiv \frac{E}{A_{\rm sh}} = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \left(\frac{\kappa_{s1}}{u_1} + \frac{\kappa_{s2}}{u_2} \right) \frac{dp}{p}$$

Krymsky 1979; Drury 1983; Lagage & Cesardky 1983

$$A_{\rm sh} = \zeta E (u_{\rm sh}/v)^2 \tau_{\rm sc1}^{-1}$$

Comparison of Stochastic and Shock Acceleration Rates

Define $R_1 = (D_{pp}/p^2)/D_{\mu\mu} = \tau_{sc}/\tau_{ac}$

Rate Ratio $A_{\rm SA}/A_{\rm sh} \sim R_1 (v/u_{\rm sh})^2$

At relativistic energies $R_1 = (v_A/v)^2 \ll 1$

so that $A_{SA}/A_{sh} \sim (v_A/u_{\rm sh})^2 = \mathcal{M}_A^{-2} \ll 1$

But at High Fields and Low energies $R_1 \gg 1$ and

$$A_{SA}/A_{sh} \sim R_1 (v/u_{\rm sh})^2 \gg 1$$

(Pryadko and Petrosian 1997)

II. Turbulence

Required for all Acceleration Models

II. PLASMA TURBULENCE

1. Turbulence Generation
2. Turbulence Cascade
3. Turbulence Damping
4. Interactions with Particles
5. Spectrum of the Accelerated Particles

1. TURBULENCE GENERATION

Turbulence is Very Common in Astrophysics

Hydrodynamic: Ordinary Reynolds number

$$R_e = Lv/\nu \gg 1, \quad \eta = \mathrm{Viscousity}$$

In MHD: Magnetic Reynolds number

$$R_m = Lv/\eta \gg 1, \quad \eta = \mathrm{Mag.\,Diff.\,Coeff.}$$

Thus most flows or fluctuations lead to generation of turbulence on scales around L (or waves with k-vector k_{min}= 1/L)

2. TURBULENCE CASCADE

HD: Large eddies breaking into small ones

Eddy turnover or *cascade* time

$$\tau_{\rm cas} \sim 1/[kv(k)] < L/u_{\rm sound}$$

MHD: Nonlinear wave-wave interactions

$$\omega(k_1) = \omega(k_2) + \omega(k_3); \quad k_1 = k_2 + k_3$$

$$\tau_{\rm cas}/V_{Alfven}$$

Dispersion Relation: *(Low Beta Plasma,* $V_{Alfven} >> V_{Sound}$*)*

$$\omega(k) = k_{||}V_{Alfven},\ kV_{Alfven},\ k_{||}V_{Sound}$$

For Alfven, Fast and Slow Modes

3. TURBULENCE DAMPING

Viscous or Collisional Damping: $k^{-1} \gg \lambda_{\rm coll}$

Collisonless Damping: $k^{-1} \ll \lambda_{\rm coll}$

Thermal: *Heating of Plasma*

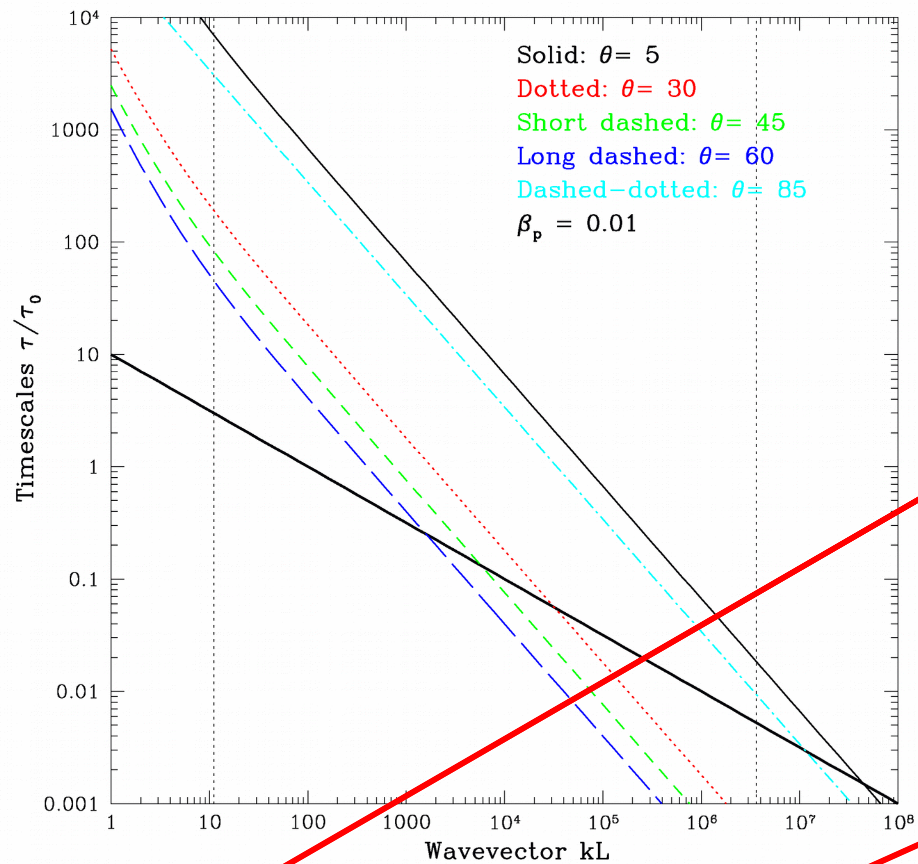
Nonthermal*: Particle Acceleration*

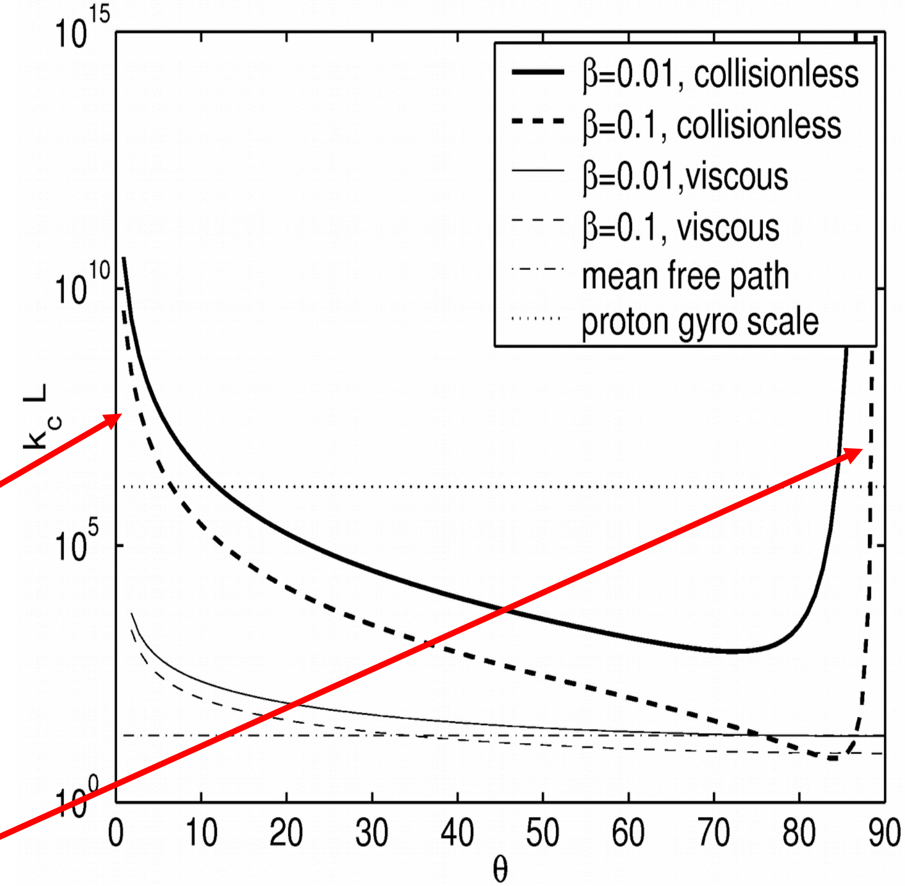
$$\text{Turbulence is damped} \quad \text{for} \quad k > k_{\max}$$

$$\text{where} \quad \tau_{damp}(\propto k^{-1}) = \tau_{cas}(\propto k^{-1/2})$$

$$\textit{Inertial Range} \quad k_{\min} < k < k_{\max}$$

3. Turbulence Damping





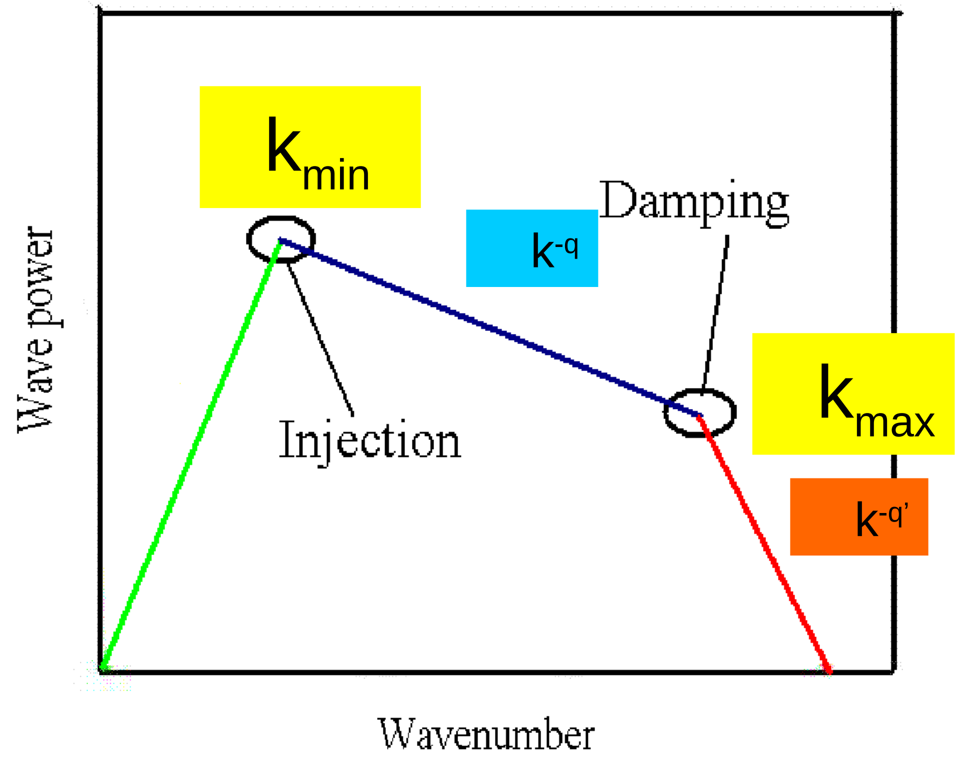
Parallel (and perpendicular) waves are not damped

Turbulence Spectrum

$W(k) = ?$

General Features:

- Injection scale: $k_{\rm min}$
- Cascade and index q
- Damping scale or $k_{\rm max}$



Kinetic Equation:

$$\frac{\partial W(\mathbf{k},t)}{\partial t} = \dot{Q}_p(\mathbf{k},t) - \gamma(\mathbf{k})W(\mathbf{k},t) + \nabla_i\left[D_{ij}\nabla_j W(\mathbf{k},t)\right] - \frac{W(\mathbf{k},t)}{T^W_{\rm esc}(\mathbf{k})}$$

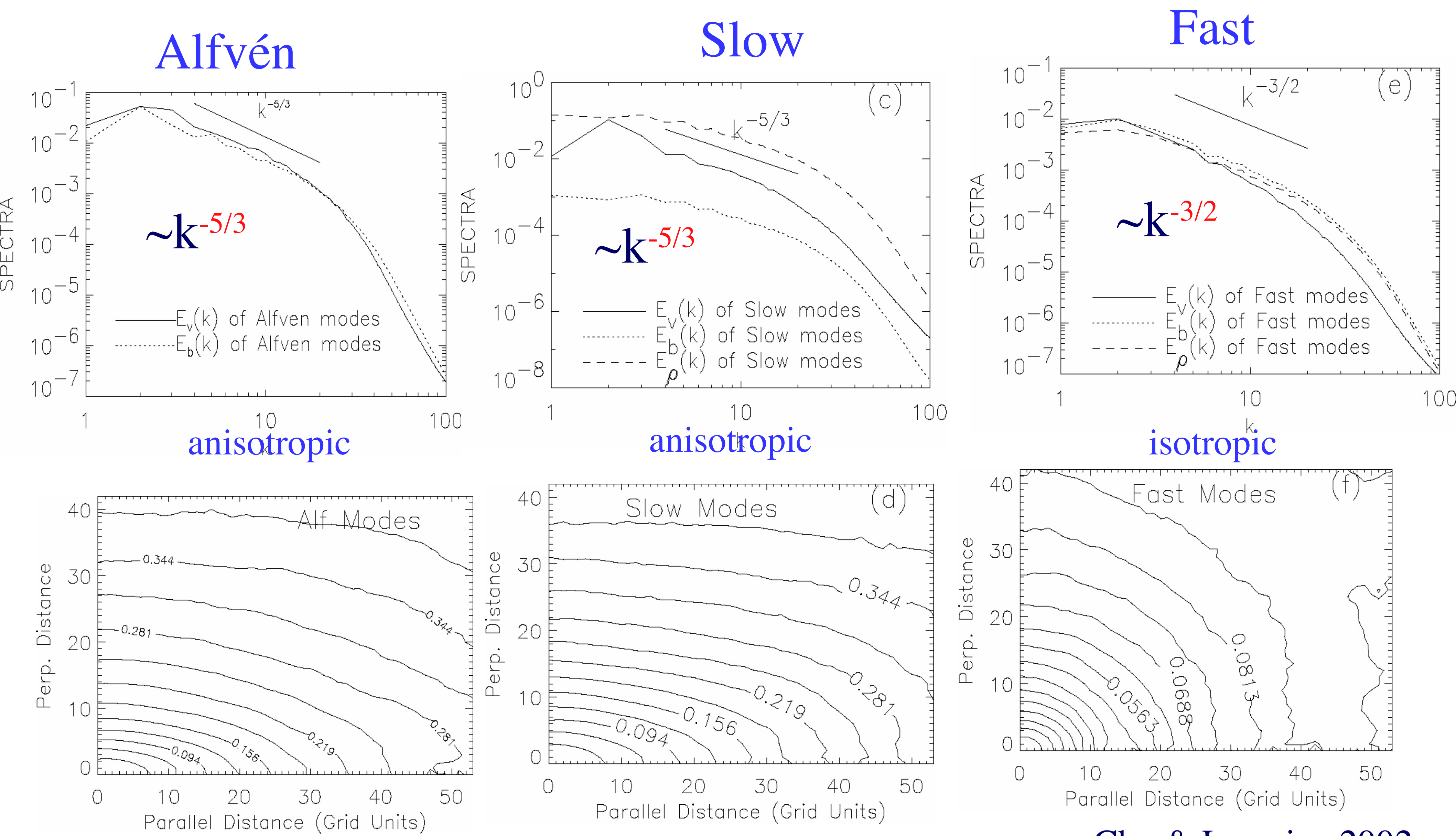
$\dot{Q}_p(\mathbf{k})$: Rate of wave generation.

$T^W_{\rm esc}$: Wave leakage timescale.

$\gamma(k) = \gamma_e + \gamma_p$: The damping coefficients.

D_{ij}: Wave diffusion tensor.

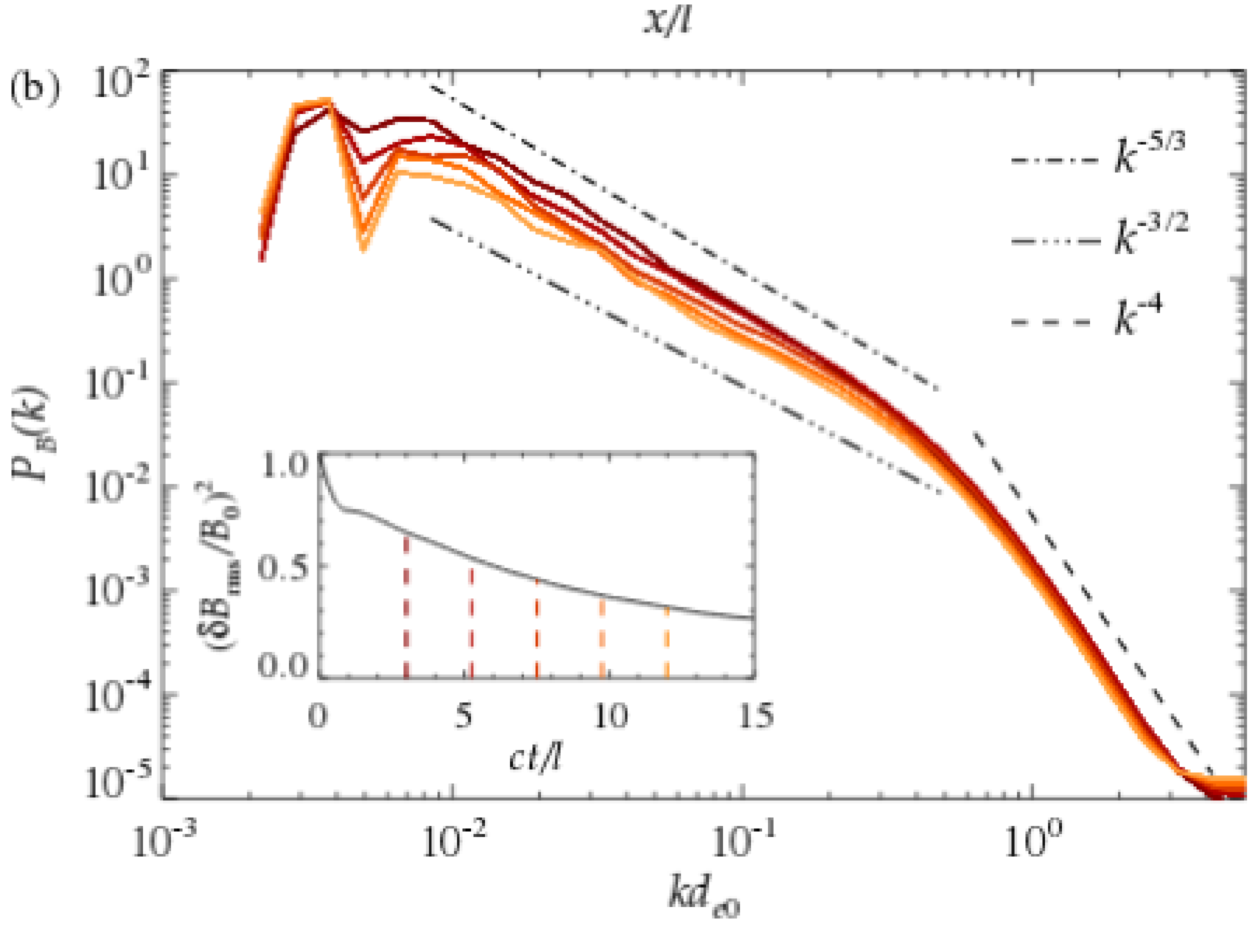
2. Cascade of MHD Turbulence



Cho & Lazarian 2002

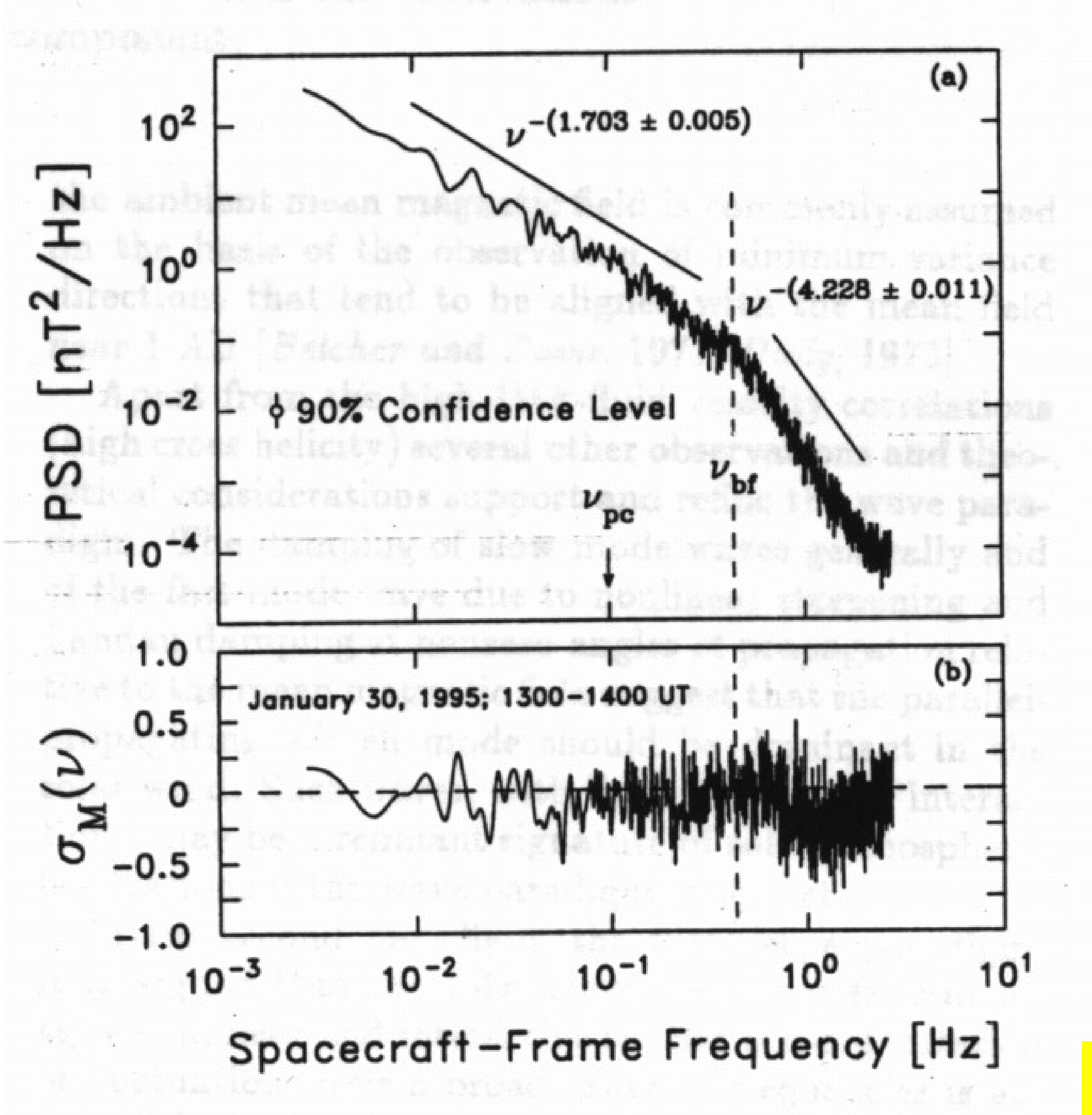
PIC Simulations of MHD Turbulence

2D simulation



Comisso and Sironi 2018; arXiv:1809.01168

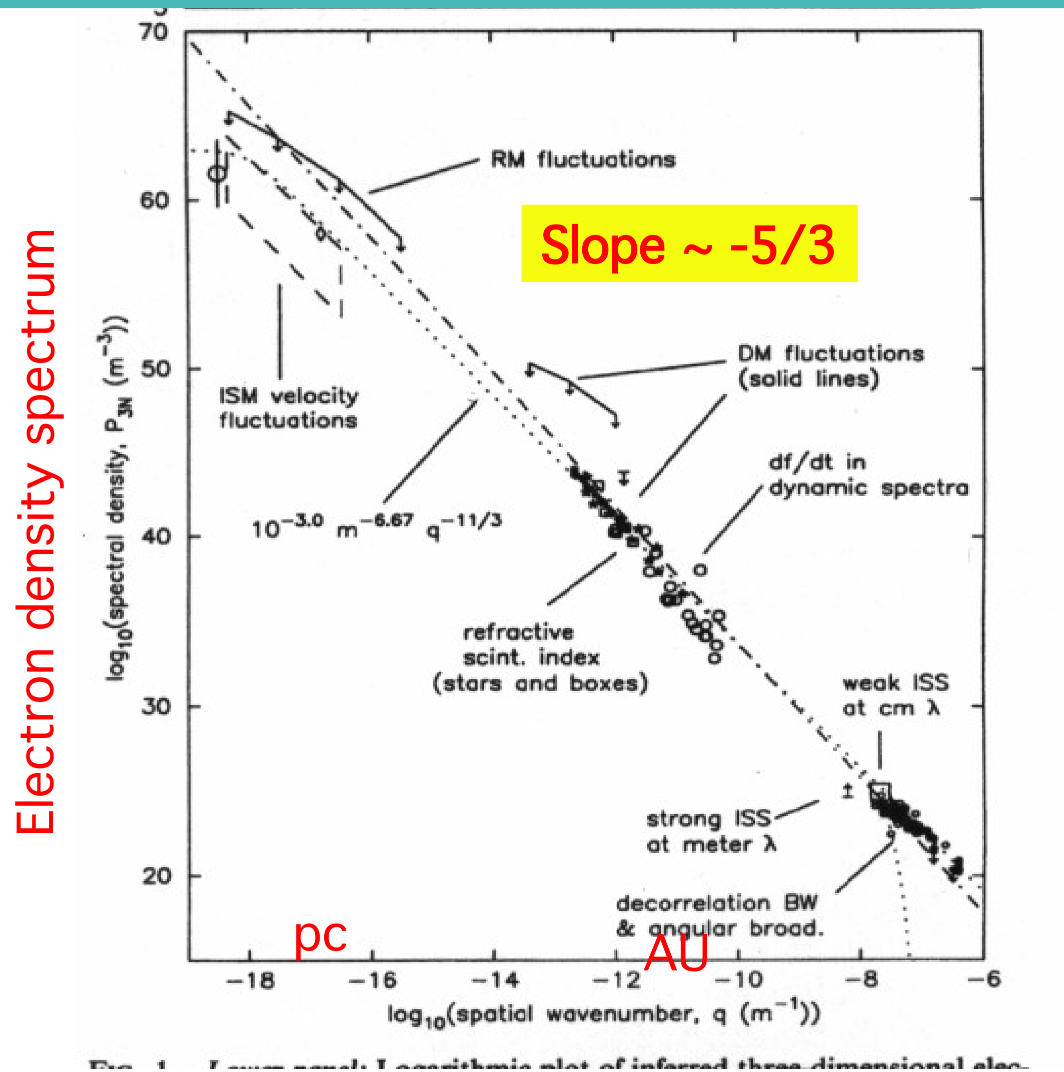
Magnetic fluctuations in Solar wind



Magnetic fluctuations in Solar wind

Leamon et al (1998)

Turbulence Spectrum in the ISM



4. Interactions with Particles: *Heating and Acceleration*

Resonant Wave-Particle Interactions

Interaction Rates

Dispersion Relations

Particle Kinetic Equation

Wave-Particle Interaction Rates

Dominated by Resonant Interactions

$$D_{ij} = \pi e^2 \sum_{n=-\infty}^{+\infty} \int d^3k \langle d_{ij} \rangle \delta\left(\boldsymbol{k} \cdot \boldsymbol{v} - \omega + \frac{n\eta_0}{\gamma} \Omega_0 \right),$$

Lower energy particles interacting with higher wavevectors or frequencies

Wave-Particle Interaction

$$
\begin{aligned}
D_{\mu\mu} = \frac{\langle \Delta\mu \Delta\mu^* \rangle}{2t} = \frac{\pi \Omega^2 (1-\mu^2)}{B_0^2} \sum_j \sum_{n=-\infty}^{\infty} \int \mathrm{d}^3k\, \delta(k_\parallel v_\parallel - \omega_j + n\Omega) \Bigg\{ \frac{c^2}{v^2}(1-\mu^2) \\
\times J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right) R^j_{\parallel\parallel}(\boldsymbol{k}) + \tfrac{1}{2} J^2_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg(P^j_{\mathrm{RR}}(\boldsymbol{k}) + \mu^2 \frac{c^2}{v^2} R^j_{\mathrm{RR}}(\boldsymbol{k}) \\
+ \mathrm{i}\mu \frac{c}{v}[T^j_{\mathrm{RR}}(\boldsymbol{k}) - Q^j_{\mathrm{RR}}(\boldsymbol{k})]\Bigg) + \tfrac{1}{2} J^2_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg(P^j_{\mathrm{LL}}(\boldsymbol{k}) + \mu^2 \frac{c^2}{v^2} R^j_{\mathrm{LL}}(\boldsymbol{k}) \\
+ \mathrm{i}\mu \frac{c}{v}[Q^j_{\mathrm{LL}}(\boldsymbol{k}) - T^j_{\mathrm{LL}}(\boldsymbol{k})]\Bigg) - \tfrac{1}{2} J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) \\
\times \Bigg[\mathrm{e}^{2\mathrm{i}\psi}\Bigg(P^j_{\mathrm{RL}}(\boldsymbol{k}) + \mathrm{i}\mu \frac{c}{v}[Q^j_{\mathrm{RL}}(\boldsymbol{k}) + T^j_{\mathrm{RL}}(\boldsymbol{k})] - \mu^2 \frac{c^2}{v^2} R^j_{\mathrm{RL}}(\boldsymbol{k})\Bigg) \\
+ \mathrm{e}^{-2\mathrm{i}\psi}\Bigg(P^j_{\mathrm{LR}}(\boldsymbol{k}) - \mathrm{i}\mu \frac{c}{v}[Q^j_{\mathrm{LR}}(\boldsymbol{k}) + T^j_{\mathrm{LR}}(\boldsymbol{k})] - \mu^2 \frac{c^2}{v^2} R^j_{\mathrm{LR}}(\boldsymbol{k})\Bigg)\Bigg] \\
+ \frac{\mathrm{i}c}{\sqrt{2}v}(1-\mu^2)^{1/2} J_n\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg[J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) \\
\times \Bigg(Q^j_{\mathrm{R}\parallel}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\psi} - T^j_{\parallel\mathrm{R}}(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\psi} + \mathrm{i}\mu \frac{c}{v}(R^j_{\mathrm{R}\parallel}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\psi} + R^j_{\parallel\mathrm{R}}(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\psi})\Bigg) \\
+ J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg(Q^j_{\mathrm{L}\parallel}(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\psi} + T^j_{\parallel\mathrm{L}}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\psi} \\
+ \mathrm{i}\mu \frac{c}{v}(R^j_{\parallel\mathrm{L}}(\boldsymbol{k})\mathrm{e}^{\mathrm{i}\psi} - R^j_{\mathrm{L}\parallel}(\boldsymbol{k})\mathrm{e}^{-\mathrm{i}\psi})\Bigg)\Bigg]\Bigg\}. \qquad (31)
\end{aligned}
$$

Jaekel & Schlickeiser

Wave-Particle Interaction

$$
\begin{aligned}
D_{\mu p} &= \frac{\langle \Delta\mu \Delta p^* \rangle}{2t} \\
&= \frac{\pi \mathrm{i} \Omega^2}{B_0^2}(1-\mu^2)^{1/2}\frac{pc}{v}\sum_j \sum_{n=-\infty}^{\infty} \int \mathrm{d}^3k\, \delta(k_\parallel v_\parallel - \omega_j + n\Omega)\Bigg[\!\!\Bigg[-\mathrm{i}\frac{c}{v}\mu(1-\mu^2)^{1/2} \\
&\times J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right) R^j_{\parallel\parallel}(\boldsymbol{k}) + \frac{(1-\mu^2)^{1/2}}{2}\Bigg\{ J_{n+1}^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg(Q^j_{\mathrm{RR}}(\boldsymbol{k}) \\
&+ \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{RR}}(\boldsymbol{k})\Bigg) - J_{n-1}^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg(Q^j_{\mathrm{LL}}(\boldsymbol{k}) - \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{LL}}(\boldsymbol{k})\Bigg) \\
&+ J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg[\mathrm{e}^{2\mathrm{i}\psi}\Bigg(Q^j_{\mathrm{RL}}(\boldsymbol{k}) + \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{RL}}(\boldsymbol{k})\Bigg) \\
&- \mathrm{e}^{-2\mathrm{i}\psi}\Bigg(Q^j_{\mathrm{LR}}(\boldsymbol{k}) - \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{LR}}(\boldsymbol{k})\Bigg)\Bigg]\Bigg\} + \frac{1}{\sqrt{2}} J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) \\
&\times \Bigg[\mu \mathrm{e}^{-\mathrm{i}\psi}\Bigg(-Q^j_{\mathrm{L}\parallel}(\boldsymbol{k}) + \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{R}\parallel}(\boldsymbol{k})\Bigg) - \mathrm{i}\frac{c}{v}(1-\mu^2)\mathrm{e}^{\mathrm{i}\psi} R^j_{\parallel\mathrm{L}}(\boldsymbol{k})\Bigg] \\
&+ \frac{1}{\sqrt{2}} J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right)\Bigg[\mu \mathrm{e}^{\mathrm{i}\psi}\Bigg(Q^j_{\mathrm{R}\parallel}(\boldsymbol{k}) + \mathrm{i}\mu\frac{c}{v} R^j_{\mathrm{R}\parallel}(\boldsymbol{k})\Bigg) \\
&- \mathrm{i}\frac{c}{v}(1-\mu^2)\mathrm{e}^{-\mathrm{i}\psi} R^j_{\parallel\mathrm{R}}(\boldsymbol{k})\Bigg]\Bigg]\!\!\Bigg].
\end{aligned}
$$

Jaekel & Schlickeiser

Wave-Particle Interaction action

$$D_{pp} = \frac{\langle \Delta p \Delta p^* \rangle}{2t}$$

$$= \frac{\pi \Omega^2 p^2 c^2}{B_0^2 v^2} \sum_j \sum_{n=-\infty}^{\infty} \int d^3k\, \delta(k_\| v_\| - \omega_j + n\Omega) \Bigg\{ \mu^2 J_n^2 \left(\frac{k_\perp v_\perp}{\Omega}\right) R^j_{\|\|}(k)$$

$$+ \frac{1-\mu^2}{2} \left[J_{n-1}^2 \left(\frac{k_\perp v_\perp}{\Omega}\right) R^j_{LL}(k) + J_{n+1}^2 \left(\frac{k_\perp v_\perp}{\Omega}\right) R^j_{RR}(k)\right.$$

$$\left. + J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) [R^j_{LR}(k)e^{-2i\psi} + R^j_{RL}(k)e^{2i\psi}] \right]$$

$$+ \frac{\mu(1-\mu^2)^{1/2}}{\sqrt{2}} \left[J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) [R^j_{\|L}(k)e^{i\psi} + R^j_{L\|} \quad e^{-i\psi}] \right.$$

$$\left. + J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) [R^j_{\|R}(k)e^{-i\psi} + R^j_{R\|}(\quad] \right] \Bigg\}. \quad (33)$$

Jaekel & Schlickeiser

Cold plasma dispersion relation

(Propagating Along Field Lines)

$$(ck)^2 = \omega^2 \left[1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega - q_i/|q_i|\Omega_i)} \right].$$

Plasma Parameter:

$$\alpha = \frac{\omega_{pe}}{\Omega_e} = 1.0 \left(\frac{n}{10^9 \mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{B_0}{100\mathrm{G}} \right)^{-1}$$

Abundances: Electrons, protons and alpha particles

Wave-Particle Interaction

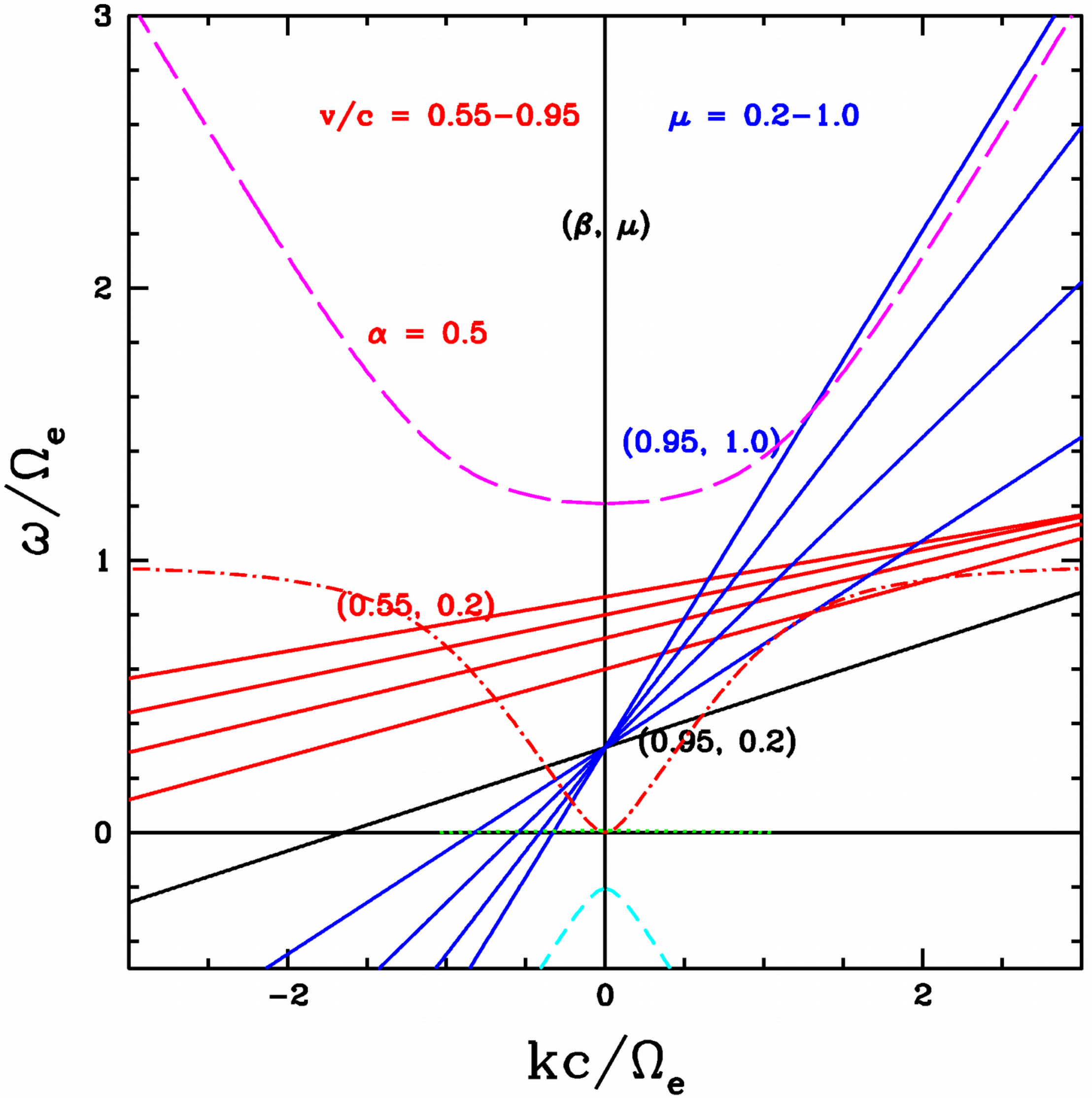
Parallel Propagating Waves

$$D_{ab} = \frac{(\mu^{-2} - 1)}{\tau_{\mathrm{p}i}\gamma^2} \sum_{j=1}^{N} \chi(k_j) \begin{cases} \mu\mu(1 - x_j)^2, & \text{for } ab = \mu\mu; \\ \mu p x_j(1 - x_j), & \text{for } ab = \mu p; \\ p^2 x_j^2, & \text{for } ab = pp, \end{cases}$$

$$\chi(k_j) = \frac{|k_j|^{-q}}{|\beta\mu - \beta_{\mathrm{g}}(k_j)|} \quad \text{and} \quad x_j = \mu\omega_j/\beta k_j\,.$$

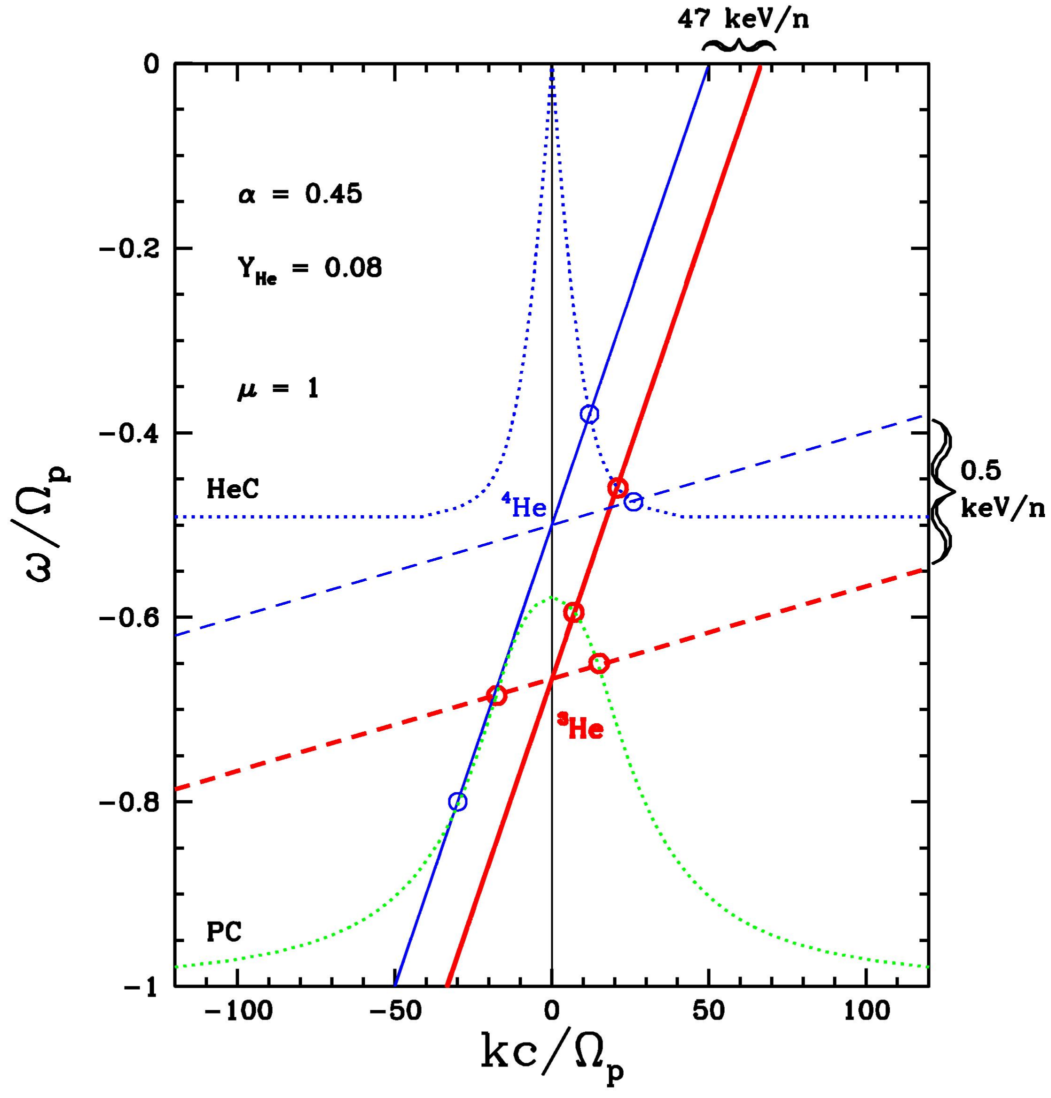
Resonant Interaction *electrons*

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$

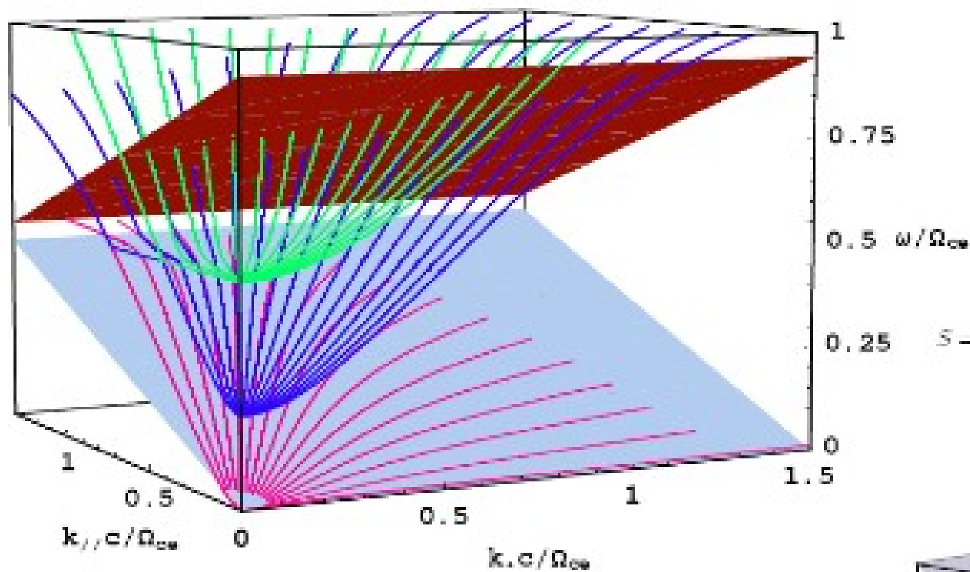


Resonant Wave-Particle Interactions
4He and 3He

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$



2D Dispersion realation



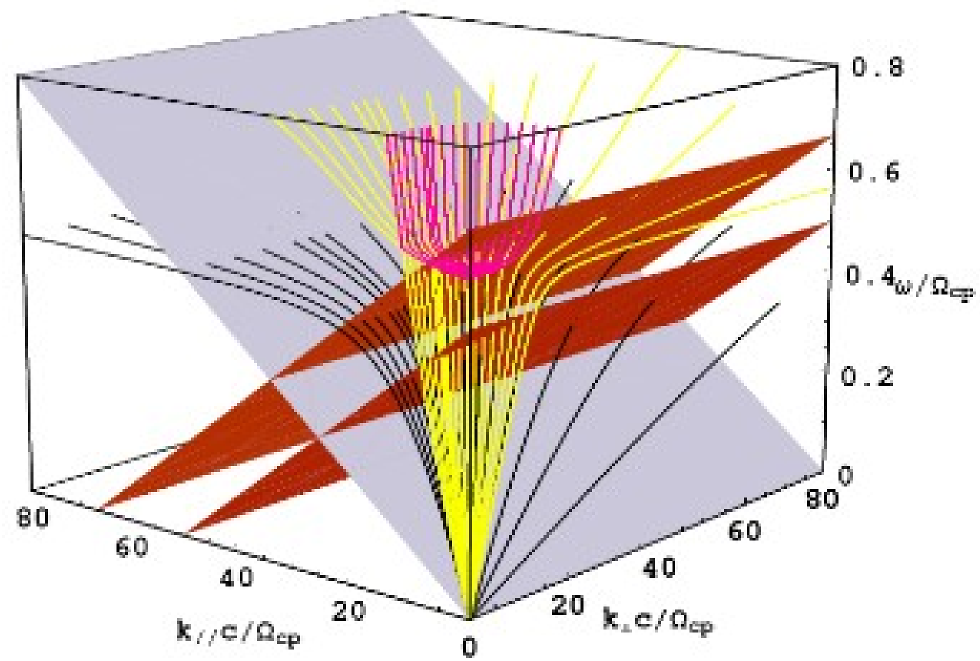
$$\tan^2\theta = \frac{-P(n^2 - R)(n^2 - L)}{(Sn^2 - RL)(n^2 - P)}$$

$$S = \frac{1}{2}(R + L), \quad D = \frac{1}{2}(R - L), \quad P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}, \quad R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\left(\frac{\omega}{\omega + \epsilon_s\Omega_s}\right),$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\left(\frac{\omega}{\omega - \epsilon_s\Omega_s}\right), \quad \omega_{ps}^2 = \frac{4\pi n_s q_s^2}{m_s}, \quad \Omega_s = \frac{|q_s|B}{m_s c}, \quad \epsilon_s = \frac{q_s}{|q_s|}$$

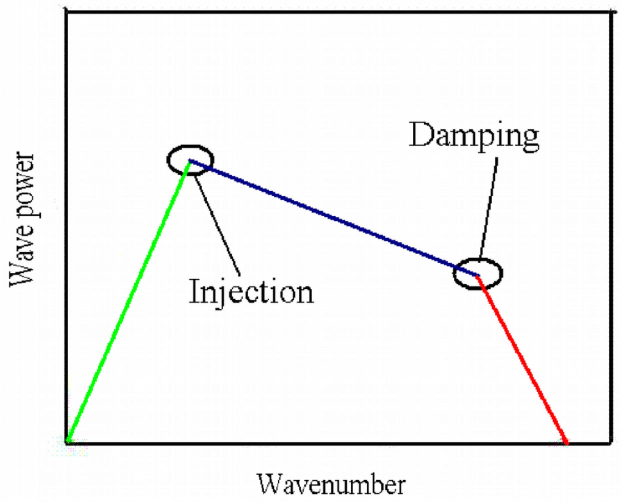
Resonance Condition

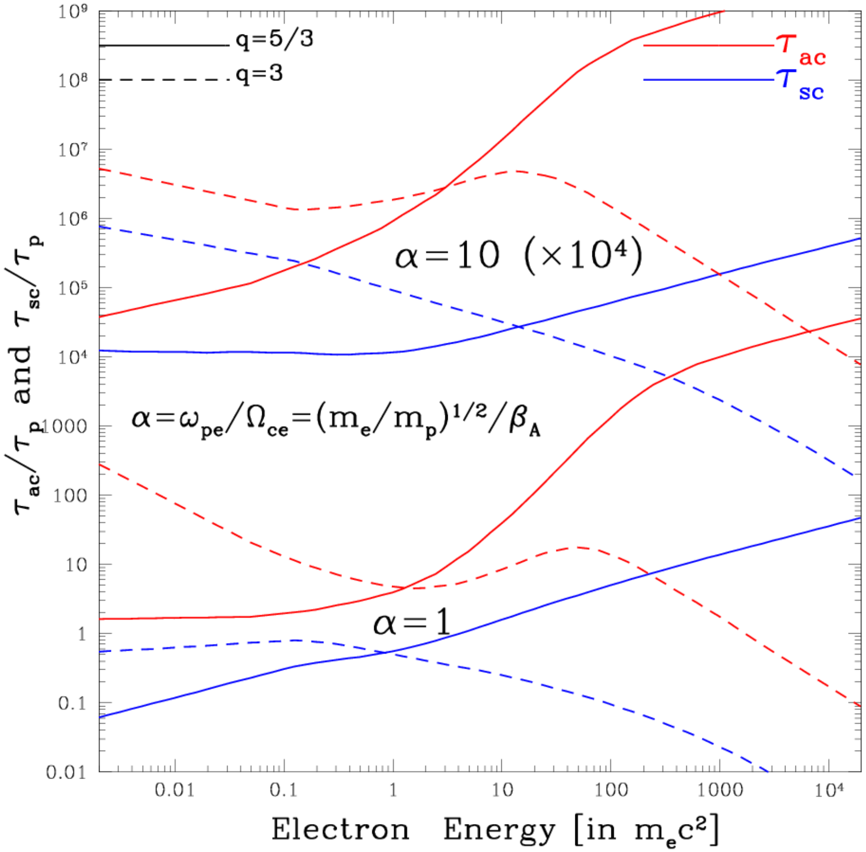
$$\omega = k_{\|} v\mu + n\Omega_i/\gamma$$



Model Scattering and Acceleration Times

Pryadko and Petrosian 1997

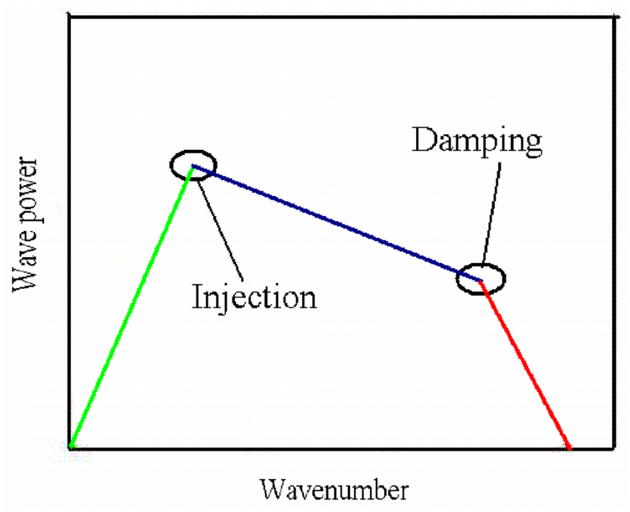


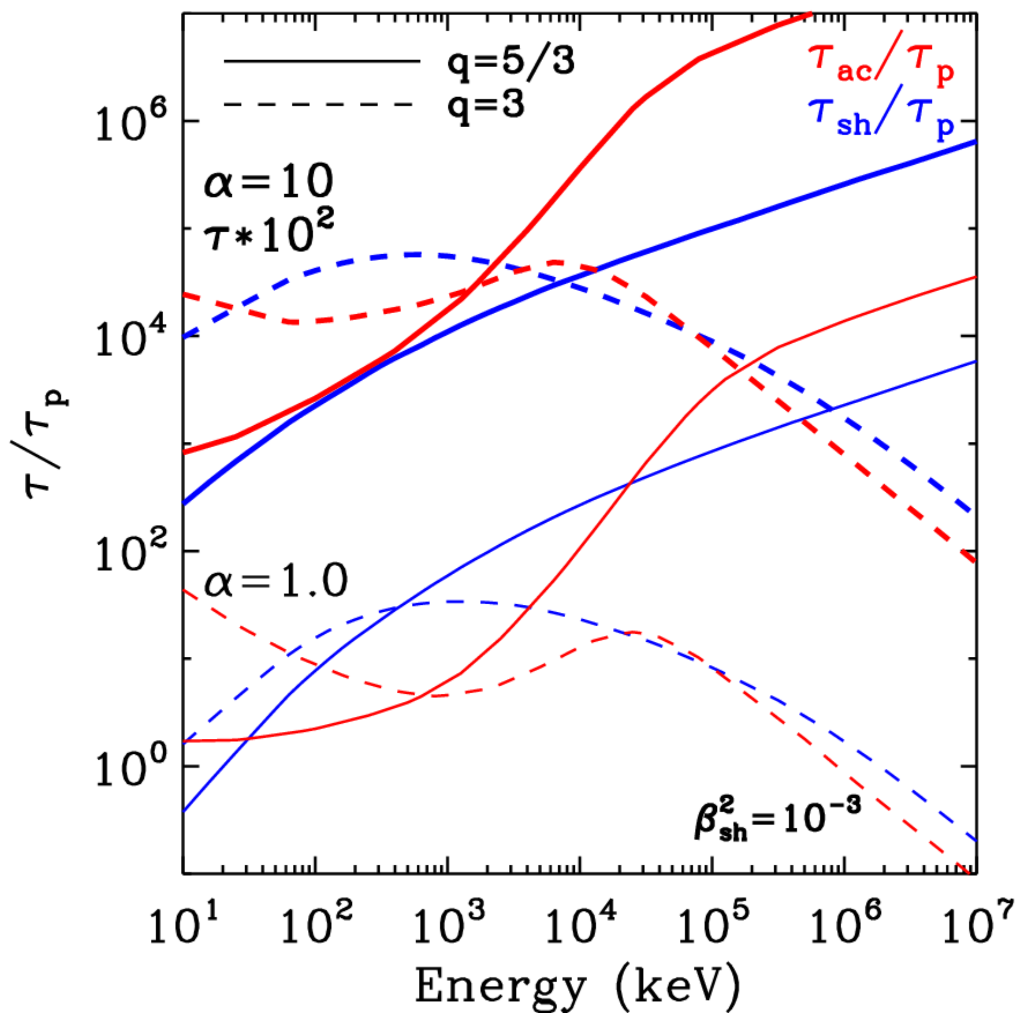


$$\alpha = \left(\frac{\omega_{pe}}{\Omega_e}\right) \propto \left(\frac{\sqrt{n}}{B}\right) \text{ and } u_{turb} \sim 8\pi\delta B^2 \sim \rho v_{turb}^2$$

Shock and Stochastic Acceleration Times

Pryadko and Petrosian 1997





Comparison of Stochastic and Shock Acceleration Rates

Define $R_1 = (D_{pp}/p^2)/D_{\mu\mu} = \tau_{sc}/\tau_{ac}$

Rate Ratio $A_{\rm SA}/A_{\rm sh} \sim R_1 (v/u_{\rm sh})^2$

At relativistic energies $R_1 = (v_A/v)^2 \ll 1$

so that $A_{SA}/A_{sh} \sim (v_A/u_{\rm sh})^2 = \mathcal{M}_A^{-2} \ll 1$

But at High Fields and $R_1 \gg 1$

Low energies $A_{SA}/A_{sh} \sim R_1 (v/u_{\rm sh})^2 \gg 1$ and

we have an Hybrid Mechanism

(Petrosian 2012)

III. Kinetic Equation for Acceleration and Transport in Magnetized Plasmas

Many Faces of the Fokker-Planck Equation

Particle Acceleration and Transport

The Kinetic Equation

Fokker-Planck Equation for Gyrophase Average Dist. $f(t,s,\mu,p)$

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial s} = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p} + D_{p\mu}\frac{\partial f}{\partial \mu}\right] + \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu} + D_{\mu p}\frac{\partial f}{\partial p}\right] - \frac{1}{p^2}\frac{\partial}{\partial p}(p^2\dot{p}f) + \dot{\mathcal{S}}$$

1. ISOTROPIC if $\tau_{\rm sc} \sim 1/D_{\mu\mu} \ll \tau_{\rm cross} = L/v \;\; (\text{and if} \;\; D_{\mu\mu} \gg D_{pp}/p^2)$

Can Define

$$F(t,s,p) = \frac{1}{2}\int_{-1}^{1} d\mu f(t,s,p,\mu), \qquad \dot{S}(t,s,p) = \frac{1}{2}\int_{-1}^{1} d\mu \dot{\mathcal{S}}(t,s,p,\mu)$$

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial s}\kappa_{ss}\frac{\partial F}{\partial s} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\kappa_{pp}\frac{\partial F}{\partial p} - p^2\langle\dot{p}\rangle F\right) + p\frac{\partial \kappa_{sp}}{\partial s}\frac{\partial F}{\partial p} - \left(\frac{1}{p^2}\frac{\partial F}{\partial s}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right) + \dot{S}(s,t,p)$$

$$\kappa_{ss} = \frac{v^2}{8}\int_{-1}^{1} d\frac{(u1-\mu^2)^2}{D_{\mu\mu}}, \quad \kappa_{sp} = \frac{v}{4p}\int_{-1}^{1} d\mu(1-\mu^2)\frac{D_{\mu p}}{D_{\mu\mu}}, \quad \kappa_{pp} = \frac{1}{2p^2}\int_{-1}^{1} d\mu(D_{pp} - D_{\mu p}^2/D_{\mu\mu})$$

Particle Acceleration and Transport

Shock acceleration

Fokker-Planck Equation for Gyrophase Average Dist. $f(t,s,\mu,p)$

$$\frac{\partial f}{\partial t}+v\mu\frac{\partial f}{\partial s}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p}+D_{p\mu}\frac{\partial f}{\partial \mu}\right]+\frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}+D_{\mu p}\frac{\partial f}{\partial p}\right]-\frac{1}{p^2}\frac{\partial}{\partial p}(p^2\dot{p}f)+\dot{\mathcal{S}}$$

If there is flow convergence (e.g. SHOCK) $\frac{1}{3}\frac{\partial u}{\partial s}\frac{1}{p^2}\frac{\partial}{\partial p}(p^3 f)-u\frac{\partial f}{\partial s}$

1. ISOTROPIC if $\tau_{\rm sc}\sim 1/D_{\mu\mu}\ll\tau_{\rm cross}=L/v\ \ (\text{and if}\ \ D_{\mu\mu}\gg D_{pp}/p^2)$

Can Define $F(t,s,p)=\frac{1}{2}\int_{-1}^{1}d\mu f(t,s,p,\mu),\quad \dot{S}(t,s,p)=\frac{1}{2}\int_{-1}^{1}d\mu\dot{\mathcal{S}}(t,s,p,\mu)$

$$\frac{\partial F}{\partial t}=\frac{\partial}{\partial s}\kappa_{ss}\frac{\partial F}{\partial s}+\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\kappa_{pp}\frac{\partial F}{\partial p}-p^2\langle\dot{p}\rangle F\right)+p\frac{\partial\kappa_{sp}}{\partial s}\frac{\partial F}{\partial p}-\left(\frac{1}{p^2}\frac{\partial F}{\partial s}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)+\dot{S}(s,t,p)$$

$A_{\rm shock}$

$$\kappa_{ss}=\frac{v^2}{8}\int_{-1}^{1}d\frac{(u1-\mu^2)^2}{D_{\mu\mu}},\quad \kappa_{sp}=\frac{v}{4p}\int_{-1}^{1}d\mu(1-\mu^2)\frac{D_{\mu p}}{D_{\mu\mu}},\quad \kappa_{pp}=\frac{1}{2p^2}\int_{-1}^{1}d\mu(D_{pp}-D_{\mu p}^2/D_{\mu\mu})$$

Particle Acceleration and Transport

Magnetic Field Variation

Fokker-Planck Equation for Gyrophase Average Dist. $f(t,s,\mu,p)$

$$\frac{\partial f}{\partial t}+v\mu\frac{\partial f}{\partial s}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p}+D_{p\mu}\frac{\partial f}{\partial \mu}\right]+\frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}+D_{\mu p}\frac{\partial f}{\partial p}\right]-\frac{1}{p^2}\frac{\partial}{\partial p}(p^2\dot{p}f)+\dot{\mathcal{S}}$$

$$\frac{v\partial \ln B}{2\partial s}\frac{\partial}{\partial \mu}\left[(1-\mu^2)f\right]$$

$\tau_{\rm sc}\sim 1/D_{\mu\mu}\ll\tau_{\rm cross}=L/v\ \ (\text{and if}\ \ D_{\mu\mu}\gg D_{pp}/p^2)$

Can Define $F(t,s,p)=\frac{1}{2}\int_{-1}^{1}d\mu f(t,s,p,\mu),\quad \dot{S}(t,s,p)=\frac{1}{2}\int_{-1}^{1}d\mu\dot{\mathcal{S}}(t,s,p,\mu)$

$$\frac{\partial F}{\partial t}=\frac{\partial}{\partial s}\kappa_{ss}\frac{\partial F}{\partial s}+\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\kappa_{pp}\frac{\partial F}{\partial p}-p^2\langle\dot{p}\rangle F\right)+p\frac{\partial\kappa_{sp}}{\partial s}\frac{\partial F}{\partial p}-\left(\frac{1}{p^2}\frac{\partial F}{\partial s}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)+\dot{S}(s,t,p)$$

$$\kappa_{ss}=\frac{v^2}{8}\int_{-1}^{1}d\frac{(u1-\mu^2)^2}{D_{\mu\mu}},\quad \kappa_{sp}=\frac{v}{4p}\int_{-1}^{1}d\mu(1-\mu^2)\frac{D_{\mu p}}{D_{\mu\mu}},\quad \kappa_{pp}=\frac{1}{2p^2}\int_{-1}^{1}d\mu(D_{pp}-D_{\mu p}^2/D_{\mu\mu})$$

Particle Acceleration and Transport

2. If Unresolved or Homogeneous *Use spatially integrated eq.*

Define $N(t,E)dE = \int dV[4\pi p^2 F(t,s,p)dp]$ and $\dot{Q}(t,E)dE = \int dV[4\pi p^2 \dot{S}(t,s,p)dp]$

$$\frac{N(t,E)}{T_{\rm esc}(E)} = -\frac{4\pi p^2}{\beta}\int dV \frac{\partial}{\partial s}\left(\kappa_{ss}\frac{\partial F}{\partial s} - uF - F\frac{1}{p^2}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Diffusion Accel. Loss Escape Source

Known as the Leaky Box Model

The Source Terms

2. If Unresolved or Homogeneous *Use spatially integrated eq.*

$$N(t,E)dE = \int dV[4\pi p^2 F(t,s,p)dp] \quad \text{and} \quad \dot{Q}(t,E)dE = \int dV[4\pi p^2 \dot{S}(t,s,p)dp]$$

D

$$\frac{N(t,E)}{T_{\rm esc}(E)} = -\frac{4\pi p^2}{\beta}\int dV \frac{\partial}{\partial s}\left(\kappa_{ss}\frac{\partial F}{\partial s} - uF - F\frac{1}{p^2}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

$$\dot{Q}(E) \propto n(p^2/v)e^{-(E/kT)}$$

Need density *n and* temperature *T: Assume we know these*

Energy Loss Terms

2. If Unresolved or Homogeneous *Use spatially integrated eq.*

$$N(t,E)dE = \int dV[4\pi p^2 F(t,s,p)dp] \quad \text{and} \quad \dot{Q}(t,E)dE = \int dV[4\pi p^2 \dot{S}(t,s,p)dp]$$

D $$\frac{N(t,E)}{T_{\rm esc}(E)} = -\frac{4\pi p^2}{\beta}\int dV \frac{\partial}{\partial s}\left(\kappa_{ss}\frac{\partial F}{\partial s} - uF - F\frac{1}{p^2}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

$$\dot{E}_L = 4\pi r_0^2 \ln\Lambda mc^3 n/\beta + (4/9) r_0^2 c \beta^2 \gamma^2 B^2 \qquad \tau_L = E/\dot{E}_L$$

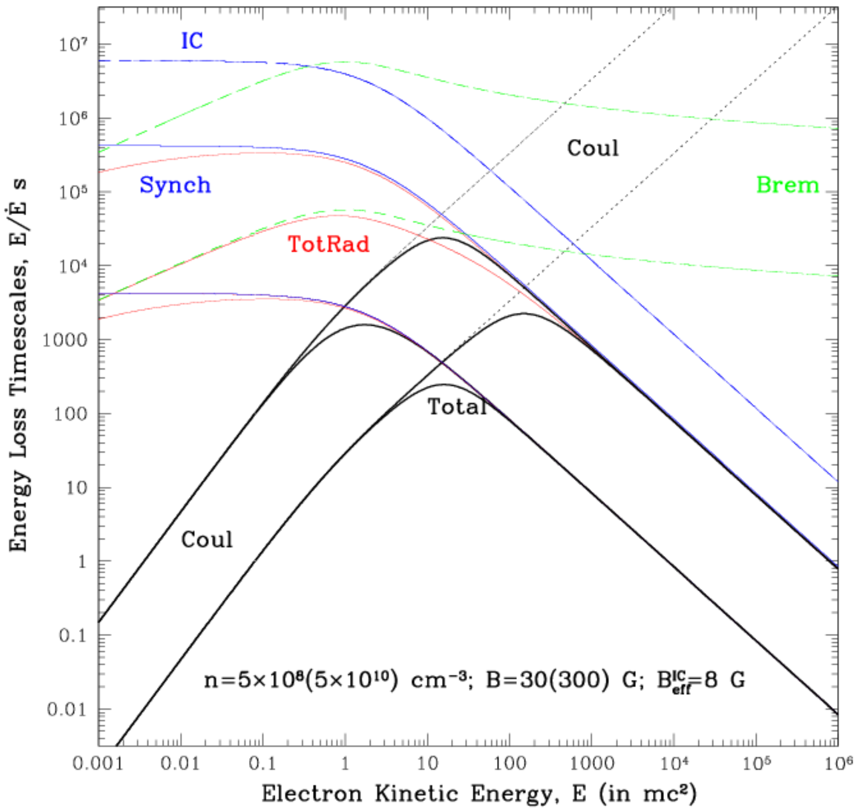
Need density *n*, temperature *T* and magnetic field *B* (+soft photon) energy densities: *Assume we know these.*

Energy Loss Terms: *Electrons*

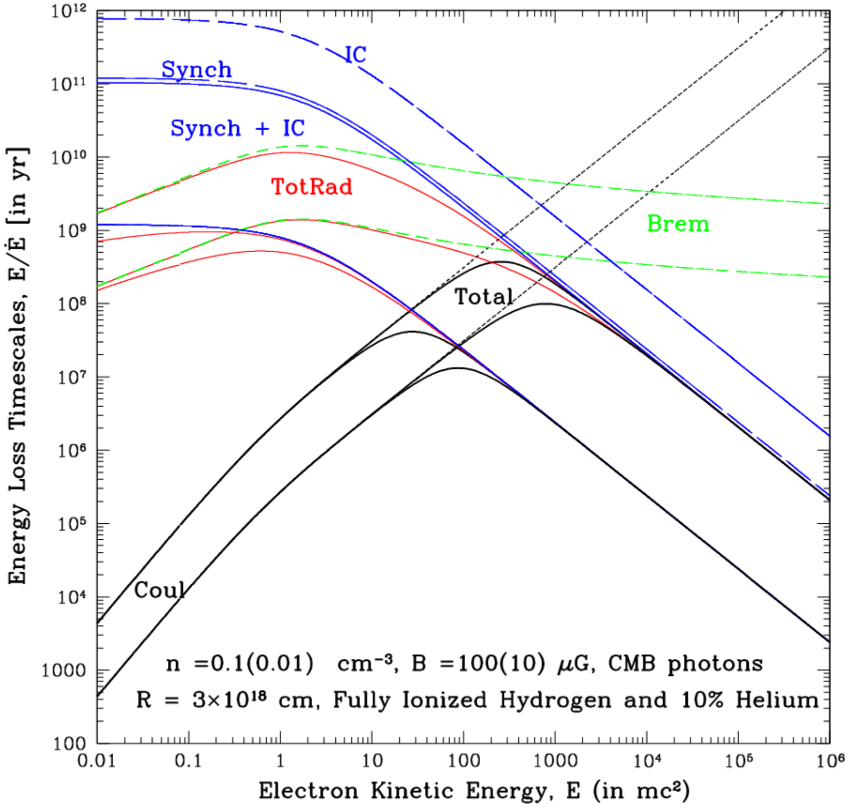
Cold Target; E>kT

Coulomb, Bremsstrahlung, Synchrotron and Inverse Compton

Solar Plasma



Supernova Remnant



Energy Loss Terms: *Electrons*

Cold Target; E>kT

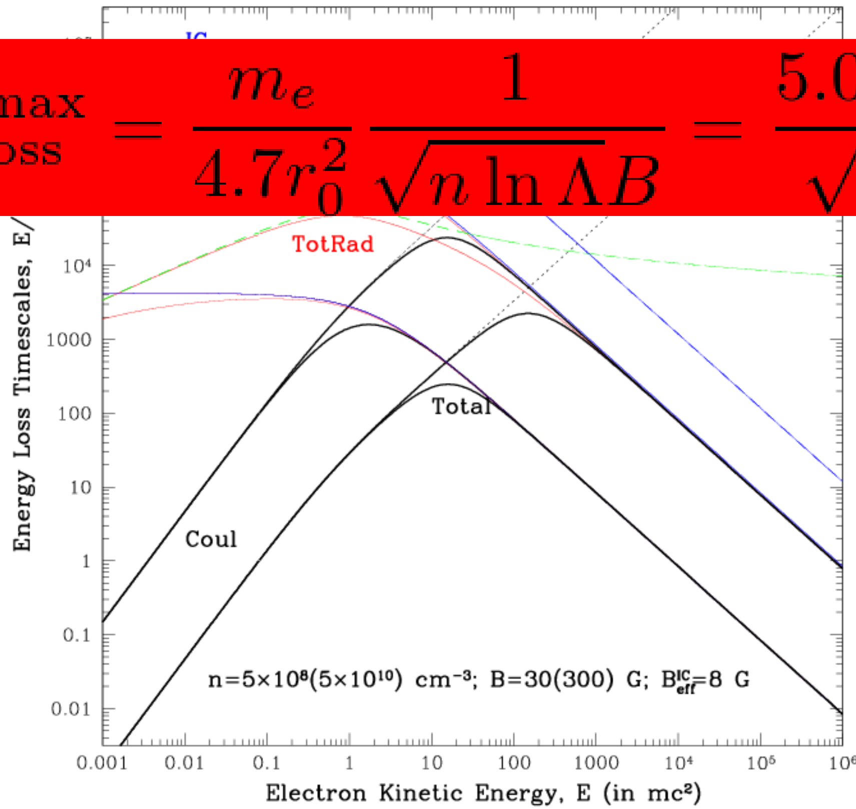
Coulomb, Brem … erse Compton

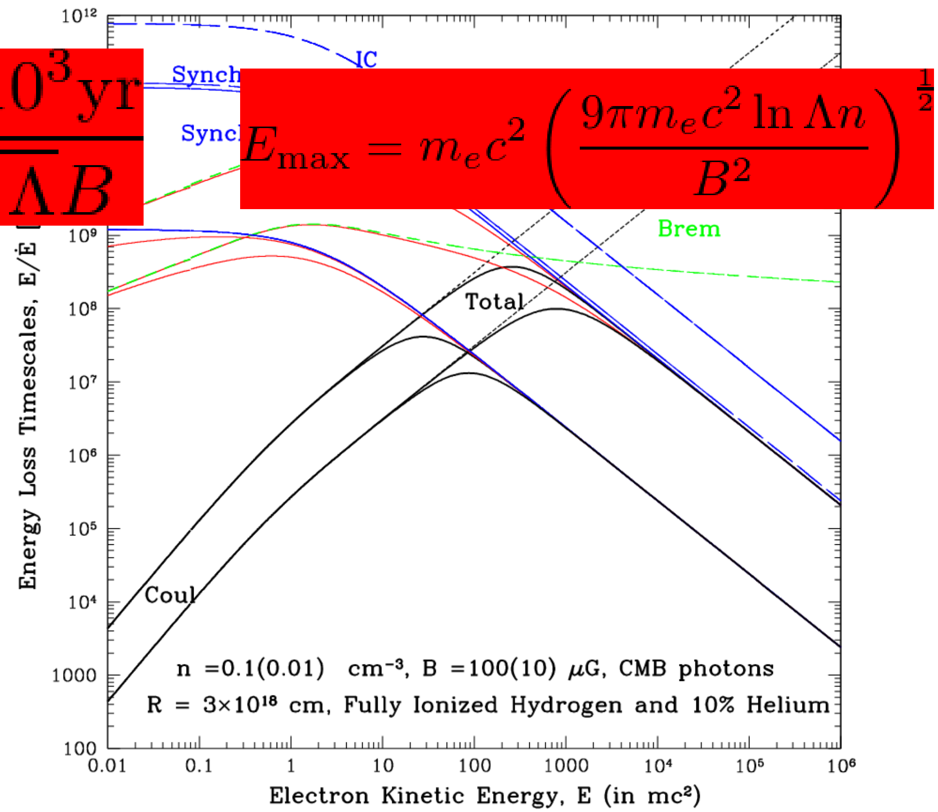
$$\tau_{\rm loss} = 2\tau_{\rm loss}^{\rm max}\frac{E/E_{\rm max}}{1+(E/E_{\rm max})^2}$$

Solar Plas … a Remnant

$$\tau_{\rm loss}^{\rm max} = \frac{m_e}{4.7 r_0^2}\frac{1}{\sqrt{n\ln\Lambda}B} = \frac{5.0\times10^3{\rm yr}}{\sqrt{n\ln\Lambda}B}$$

$$E_{\rm max} = m_e c^2\left(\frac{9\pi m_e c^2\ln\Lambda n}{B^2}\right)^{\frac{1}{2}}$$



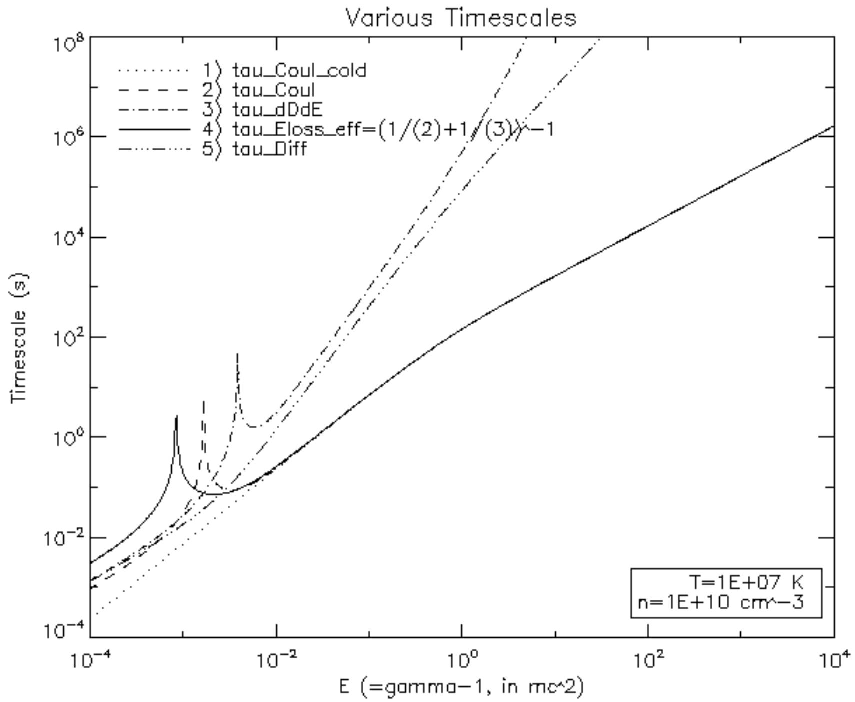


Energy Loss Terms: *Electrons*

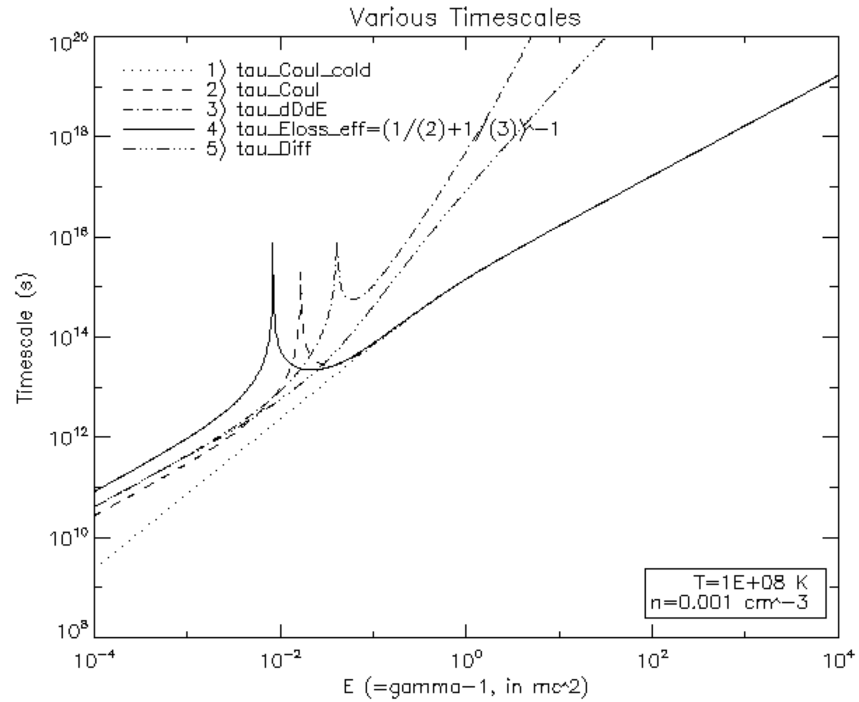
Warm Target; E~kT

Coulomb Loss and Diffusion rates

Solar Plasma



Cluster of Galaxies



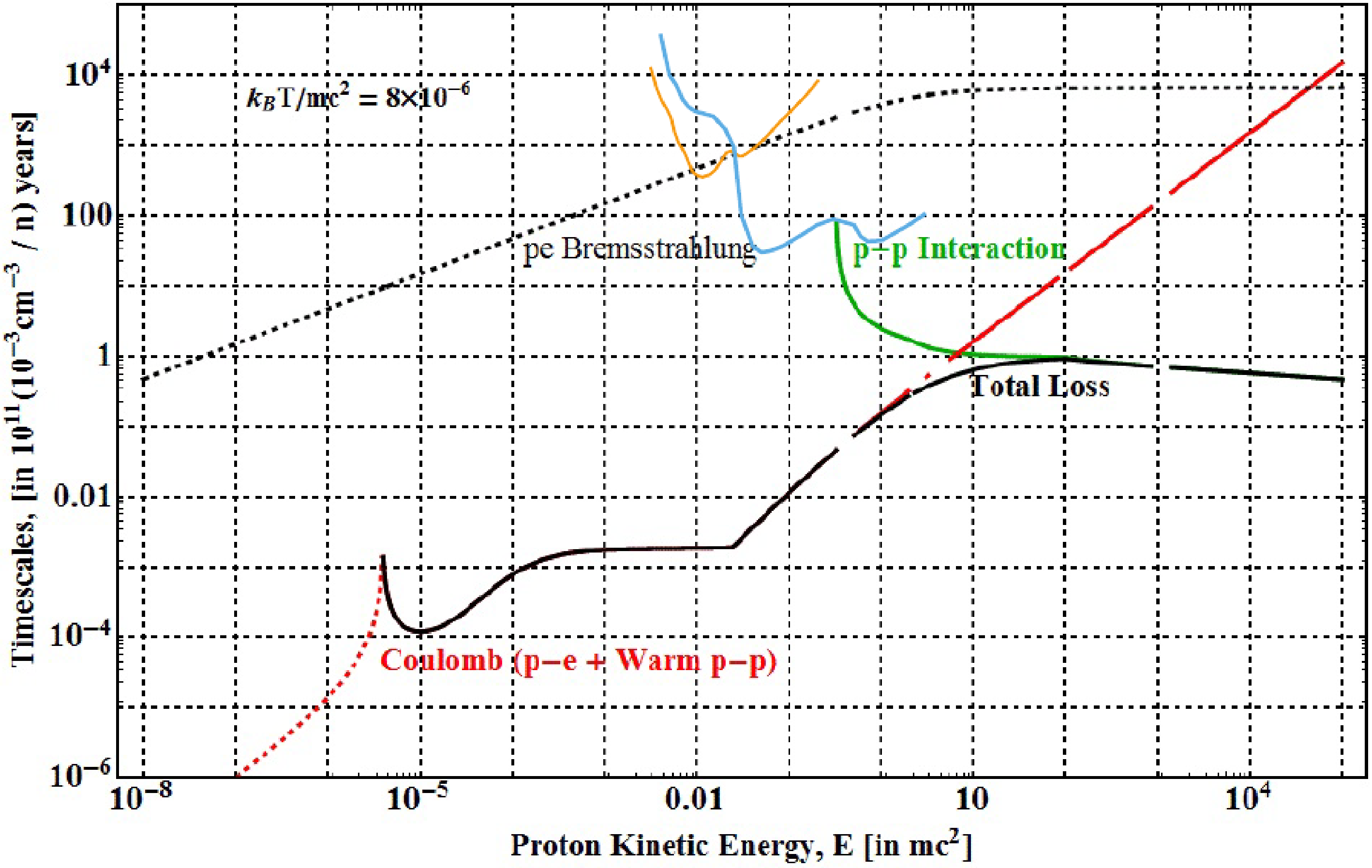
$$\dot{E}^{\rm hot}_{\rm Coul} = \dot{E}^{\rm cold}_{\rm Coul}\left[{\rm erf}(\sqrt{x}) - 4\sqrt{\frac{x}{\pi}}e^{-x}\right],$$

$$D_{\rm Coul}(E) = \dot{E}^{\rm cold}_{\rm Coul}\left(\frac{kT}{m_e c^2}\right)\left[{\rm erf}(\sqrt{x}) - 2\sqrt{\frac{x}{\pi}}e^{-x}\right], \quad {\rm with} \quad x \equiv \frac{E m_e c^2}{kT}.$$

Energy Loss Terms: *Protons*

Warm Target; E~kT

Coulomb, Bremsstrahlung; Pion, neutron and line production

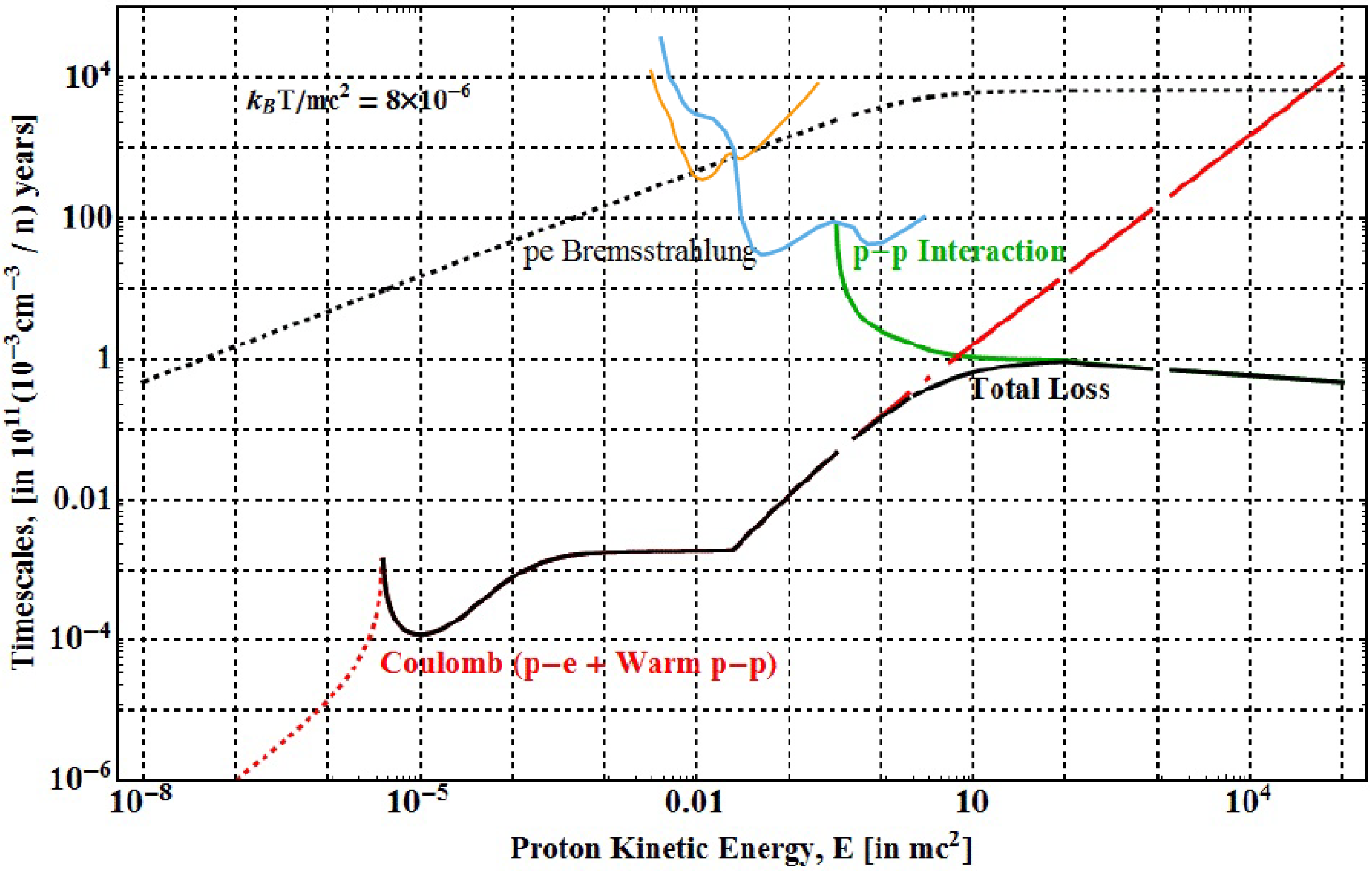


Zakopane-1, 2019

Energy Loss Terms: *Protons*

Warm Target; E~kT

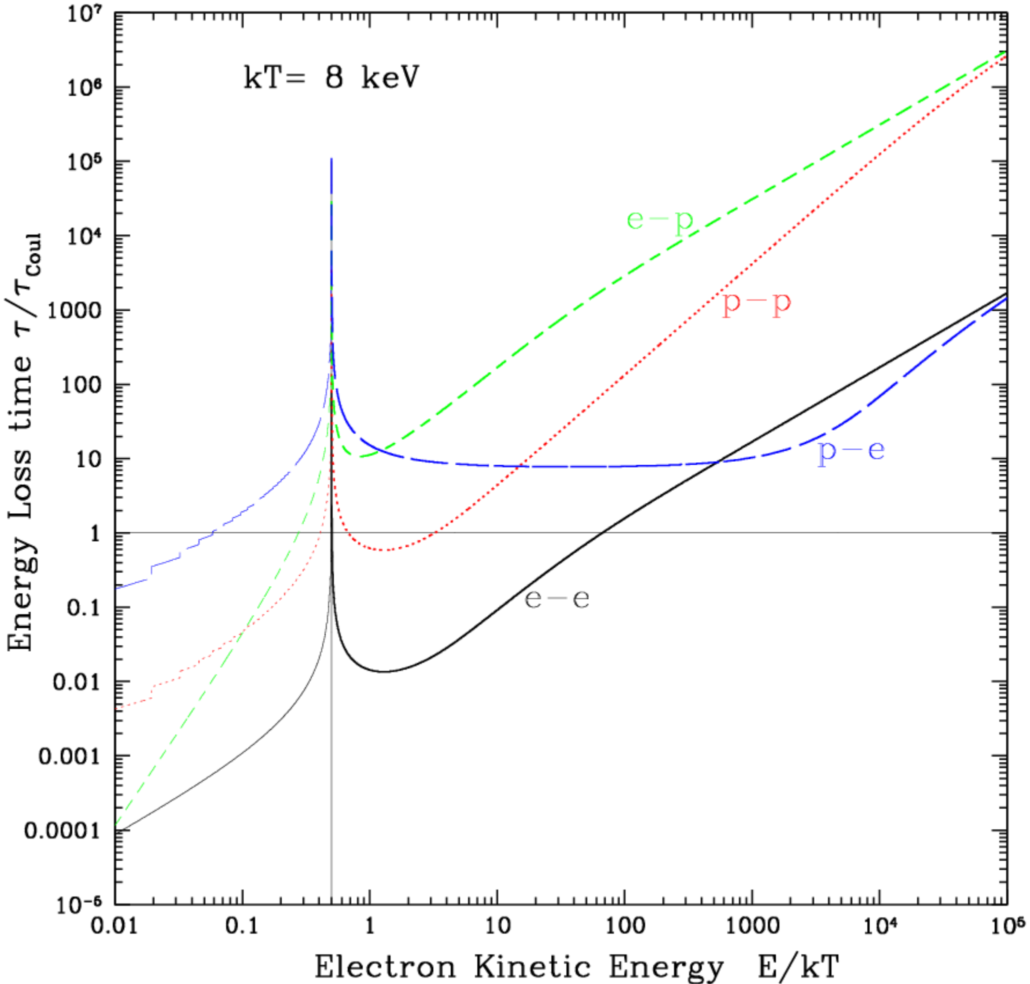
Coulomb, Bremsstrahlung; Pion, neutron and line production



Zakopane-1, 2019

Energy Loss Terms: *Protons and Electrons*
Warm Target

Coulomb collisions only



Acceleration Coefficients

Pitch angle averaged; Spatially integrated

The Leaky Box Model

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Diffusion

$$D_{EE} = v^2 p^2 \bar{\kappa}_{pp}$$

Acceleration.

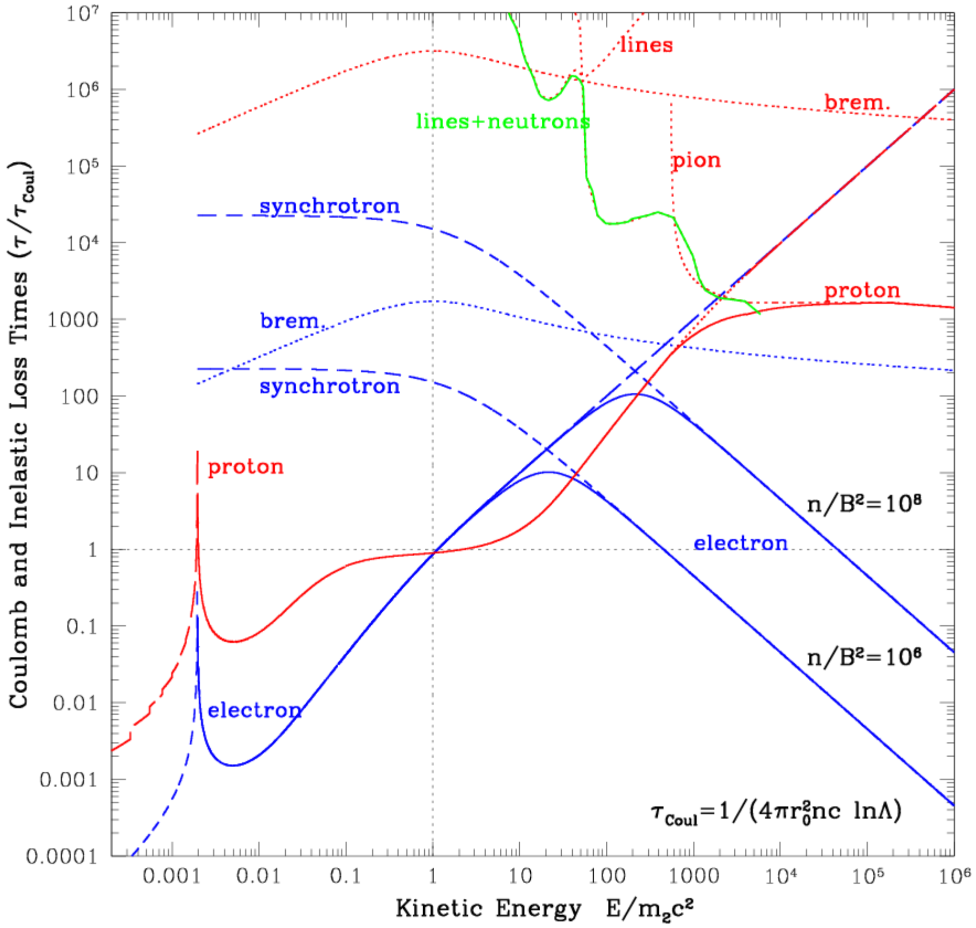
$$A = A_{\rm SA} + A_{\rm sh}$$

$$A_{\rm SA} = \frac{2\gamma^2 - 1}{\gamma^2 + \gamma}\left(\frac{D_{EE}}{E}\right), \quad A_{\rm sh} = \zeta E\left(\frac{u_{\rm sh}^2}{\kappa_{ss}}\right) = 8\zeta E\left(\frac{u_{\rm sh}}{v}\right)^2\left(\langle\frac{(1-\mu^2)^2}{\bar{D}_{\mu\mu}}\rangle\right)^{-1}$$

$$\tau_{\rm diff} = \frac{D_{EE}}{E^2}, \ \tau_{\rm ac,SA}(E) = \frac{1}{\bar{\kappa}_{pp}} = \frac{p^2}{\langle \bar{D}_{pp} - \bar{D}_{p\mu}^2/\bar{D}_{\mu\mu}\rangle}, \ \tau_{\rm ac,sh} = \left(\frac{v}{u_{\rm sh}}\right)^2\left(\frac{3}{\zeta}\right)\tau_{\rm sc}, \ \tau_{\rm sc}(E) = 3\frac{\bar{\kappa}_{ss}}{v^2} = \frac{3}{4}\langle\frac{(1-\mu^2)^2}{\bar{D}_{\mu\mu}}\rangle, \ \tau_L = \frac{E}{\dot{E}_L}$$

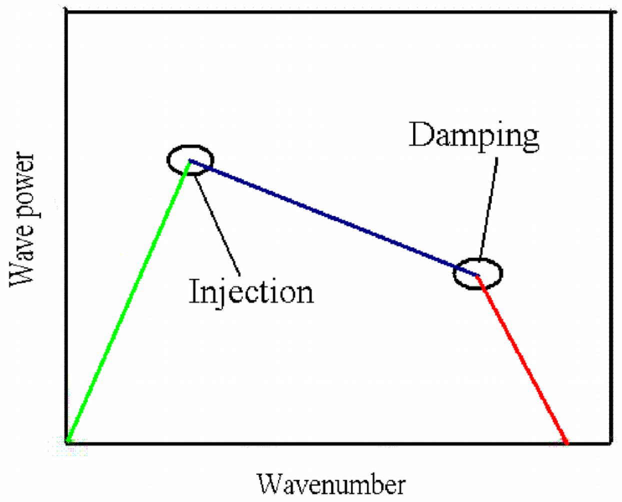
Energy Loss Terms: *Protons and Electrons*
Warm Target

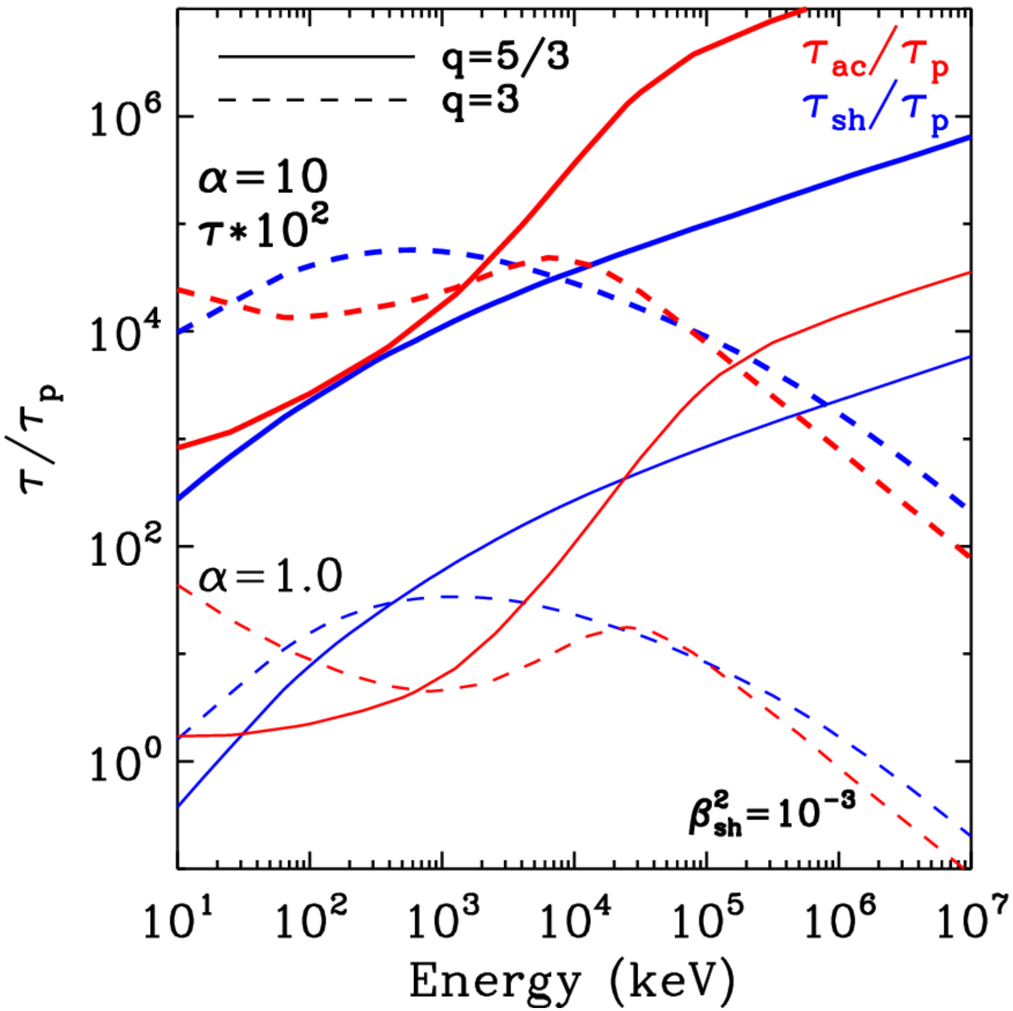
Coulomb, Bremsstrahlung and Synchrotron (IC)



Shock and Stochastic Acceleration Times

Pryadko and Petrosian 1997





The Escape Time

Strong Diffusion limit $\tau_{sc} \ll \tau_{\rm cross}$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

$$\frac{N(t,E)}{T_{\rm esc}(E)} = -\frac{4\pi p^2}{\beta}\int dV \frac{\partial}{\partial s}\left(\kappa_{ss}\frac{\partial F}{\partial s} - uF - F\frac{1}{p^2}\frac{\partial}{\partial p}(p^3\kappa_{sp})\right)$$

$$\frac{T_{\rm esc}}{\tau_{\rm cross}} \simeq \left(\frac{\tau_{\rm sc}}{\tau_{\rm cross}} + \beta_{sh} + \xi\beta_A\right)^{-1}$$

$$\tau_{\rm cross} \equiv L/v$$

The Escape Time: *Field Convergence*

Weak Diffusion limit $\tau_{\rm sc} \gg \tau_{\rm cross}$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Free escape; uniform *B* field $T_{\rm esc} \propto \tau_{\rm cross}$

Converging B Field lines $T_{\rm esc} \propto \tau_{sc}$

The Escape Time

Combined equation

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Strong diffusion $T_{\rm esc} \sim \tau_{\rm cross}^2/\tau_{\rm sc}$

Weak diffusion $T_{\rm esc} \sim \tau_{\rm cross}$

Converging B-field $T_{\rm esc} \propto \tau_{\rm sc}$

Combined equation (Malyshkin and Kulsrud 2001)

$$T_{\rm esc} = \tau_{\rm cross}\left(\eta + \frac{\tau_{\rm cross}}{\tau_{\rm sc}} + \ln\eta\frac{\tau_{\rm sc}}{\tau_{\rm cross}}\right)$$

The Escape Times

Numerical Simulations

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Strong diffusion $T_{\rm esc} \sim \tau_{\rm cross}^2/\tau_{\rm sc}$

Weak diffusion $T_{\rm esc} \sim \tau_{\rm cross}$

Converging B-field $T_{\rm esc} \propto \tau_{\rm sc}$

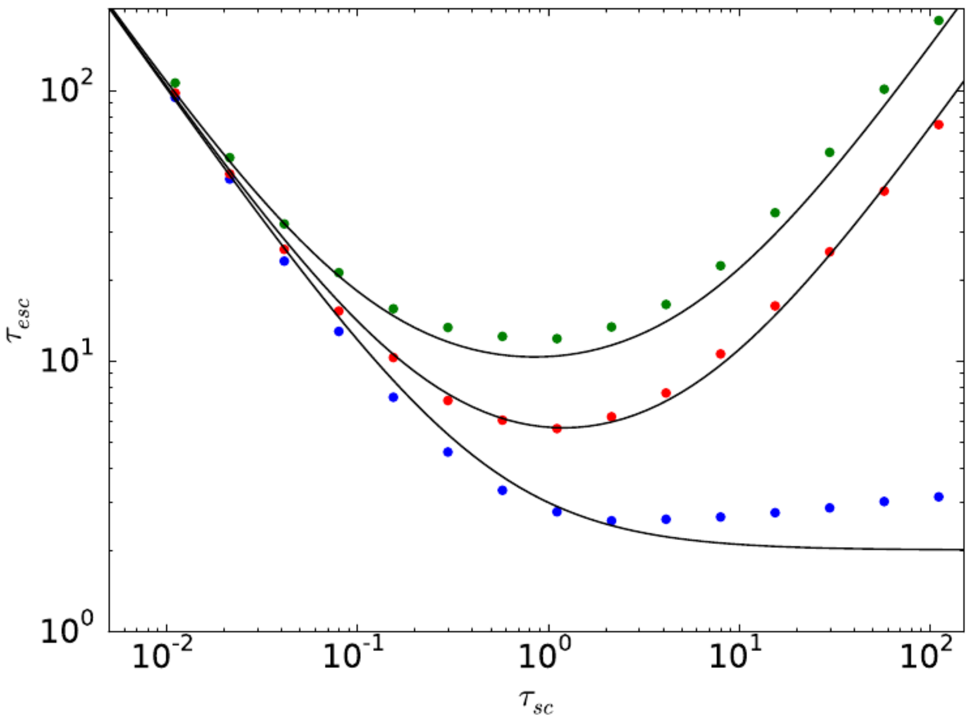
Combined equation (Malyshkin and Kulsrud ApJ 2001)

$$T_{\rm esc} = \tau_{\rm cross}\left(\eta + \frac{\tau_{\rm cross}}{\tau_{\rm sc}} + \ln\eta\,\frac{\tau_{\rm sc}}{\tau_{\rm cross}}\right)$$

Agrees with simulations

Points, from Effenberger and Petrosian)

Astrophysical Journal Letters,868:L28



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The Parameters and Solutions

Characteristics of the plasma and turbulence

The required parameters of the Problem

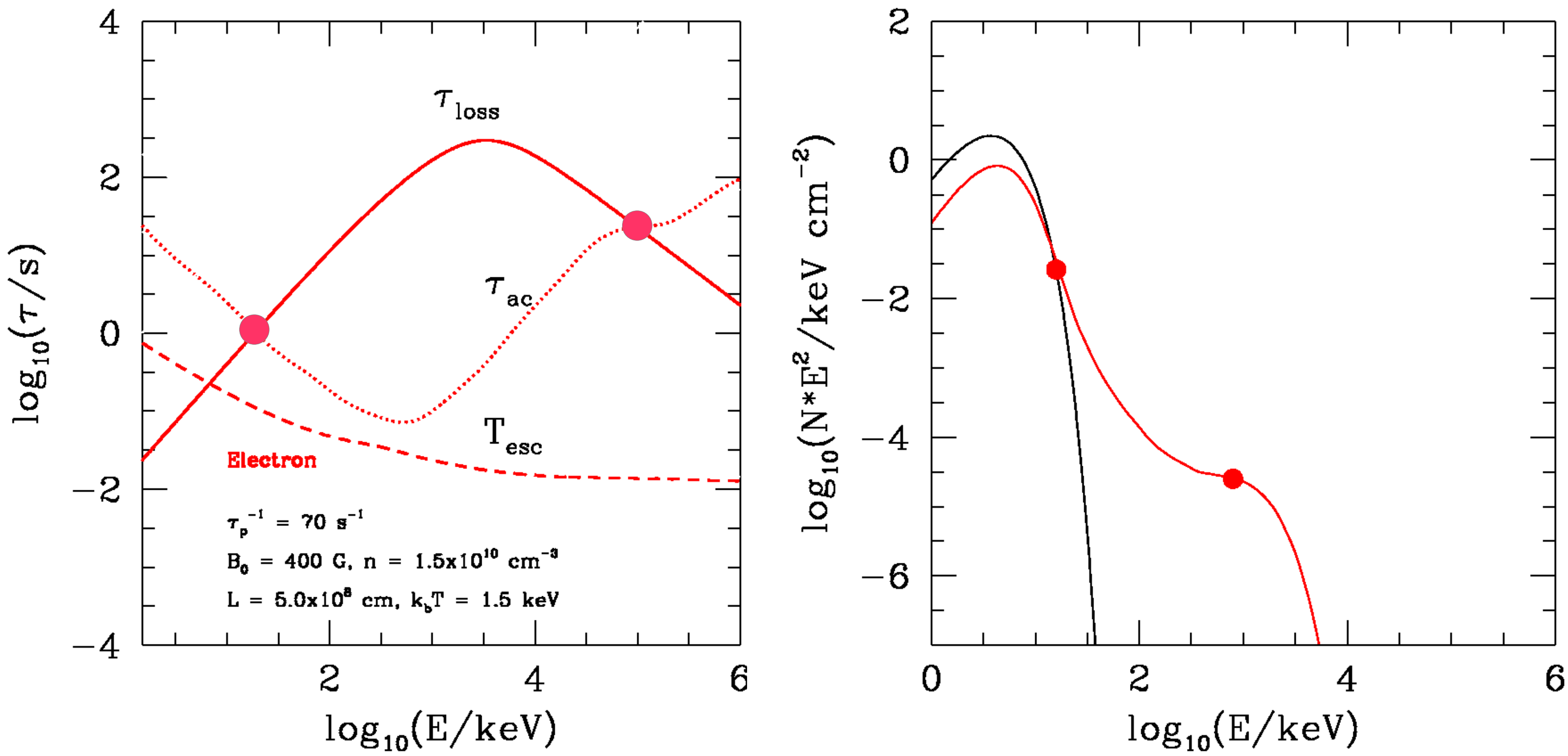
Density, Temperature, Magnetic Field (soft photons)

Turbulence energy density, spectral index and kmax

Some Numerical Solutions

Assumed turbulence spectrum

Including acceleration energy loss and escape



An important distinction

between Accelerated and Escaping Spectra

Particles in the acceleration site; *N(E)*

Particles in radiating or observing sites; $\dot{Q}(E) = N(E)/T_{\rm esc}(E)$

A. Closed; no escape $T_{\rm esc} = \infty, \;\; Q(E) = 0$

more heating than acceleration

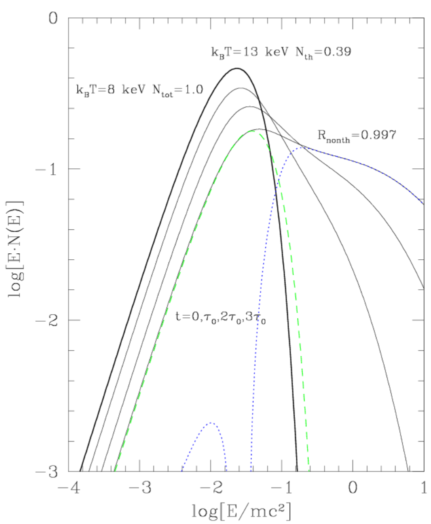
VP, East, ApJ, 2008 ; VP, Kang, ApJ, 2015

B. Open with escape

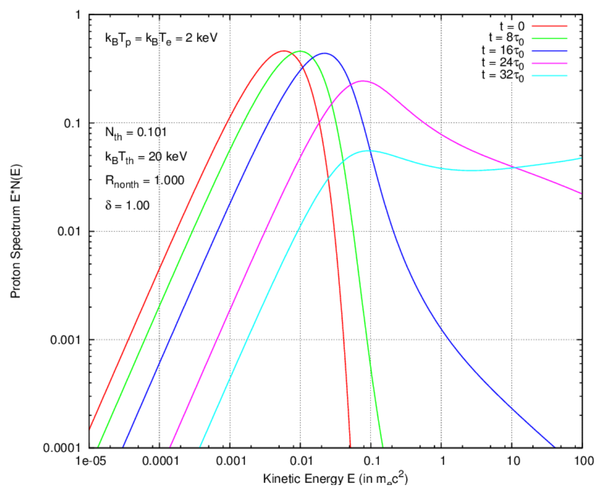
harder or softer escaping spectra

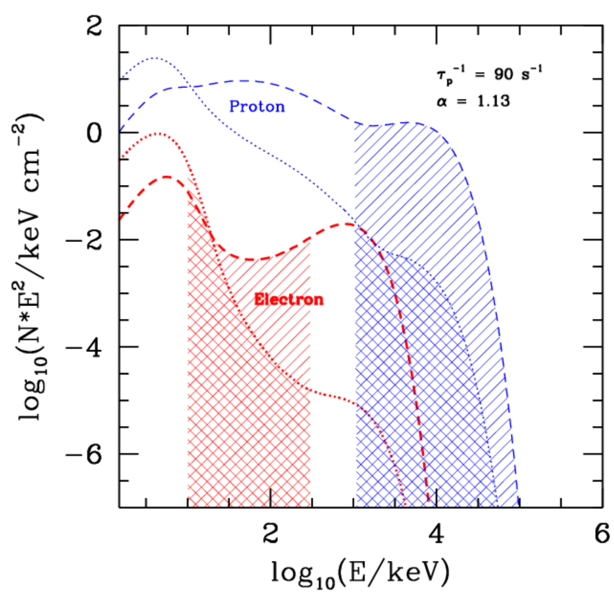
VP, Liu, ApJ, 2004

Electrons



Protons





The Final unknown

Magnetic Turbulence Diffusion Approximation

Damping

$$\frac{\partial \mathcal{W}(\mathbf{k},t)}{\partial t} = \dot{Q}_{\mathcal{W}}(\mathbf{k},t) + \frac{\partial}{\partial k_i}\left[D_{ij}\frac{\partial}{\partial k_j}\mathcal{W}(\mathbf{k},t)\right] - \Gamma(\mathbf{k})\mathcal{W}(\mathbf{k},t) - \frac{\mathcal{W}(\mathbf{k},t)}{T_{\text{esc}}^{\mathcal{W}}(\mathbf{k})}$$

$$D_{ij} = \delta_{ij}\frac{C}{4\pi}k^2\frac{\tau_{NL}^{-2}}{\tau_{NL}^{-1} + \tau_A^{-1}} = \delta_{ij}\frac{C}{4\pi}\frac{\mathcal{W}k^7}{(\mathcal{W}k^3)^{1/2}k + \omega(\mathbf{k})}$$

Suppression of turbulence cascade by waves

Coupled turbulence and particle kinetic equation

Toward a Complete Treatment Stochastic Acceleration by Turbulence

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k_i}\left[D_{ij}\frac{\partial}{\partial k_j}W\right] - \Gamma(\mathbf{k})W - \frac{W}{T^W_{\rm esc}(\mathbf{k})} + \dot{Q}^W,$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left[D_{EE}\frac{\partial N}{\partial E} - (A - \dot{E}_L)N\right] - \frac{N}{T^p_{\rm esc}} + \dot{Q}^p.$$

Jiang et al. 2008

SUMMERY-1

Acceleration happens everywhere and all scales

Turbulence is the main ingredient of acceleration

For complete treatment of Acceleration and transport of high energy particles we need to include all particle-particle, particle-field, wave-particle and wave-wave interactions

A critical role is played by the escape time

Some Analytic solutions

D. SOME STEADY STATE SOLUTIONS

$$\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2}[D(E)N] - \frac{\partial}{\partial E}\{[A(E) - \dot{E}_L]N\} - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Green's Functions: $\dot{Q} = Q_0\delta(E - E_0)$

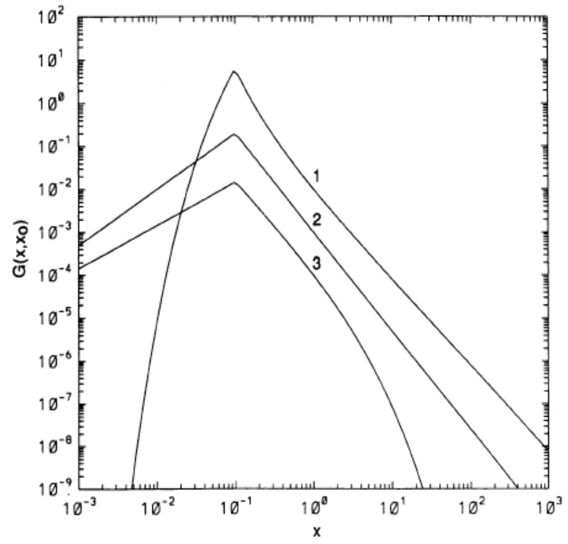
1. Constant Coefficients no Losses: $\dot{E}_L = 0$

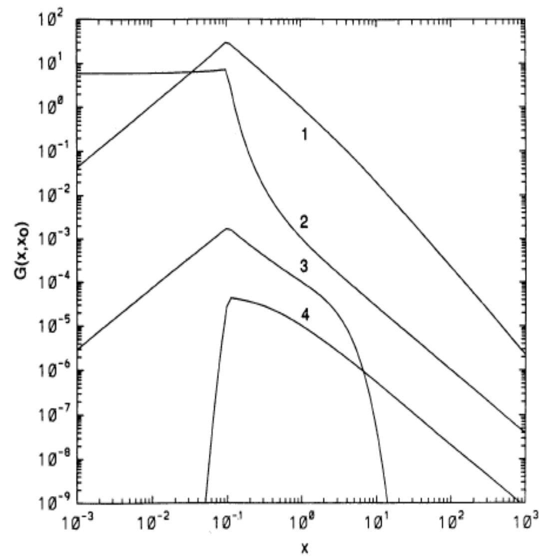
(Direct acceleration. *e.g.* shock acceleration $D = 0$)

$$N(E) \propto Q_0 E^{-(1+E/AT_{\rm esc})}$$

2. Simple Coefficients no Losses: $D \propto E^q, A \propto E^{q-1}, T_{\rm esc} \propto E^s$

3. Effects of Energy Losses: $\dot{E}_L \propto E^\gamma$



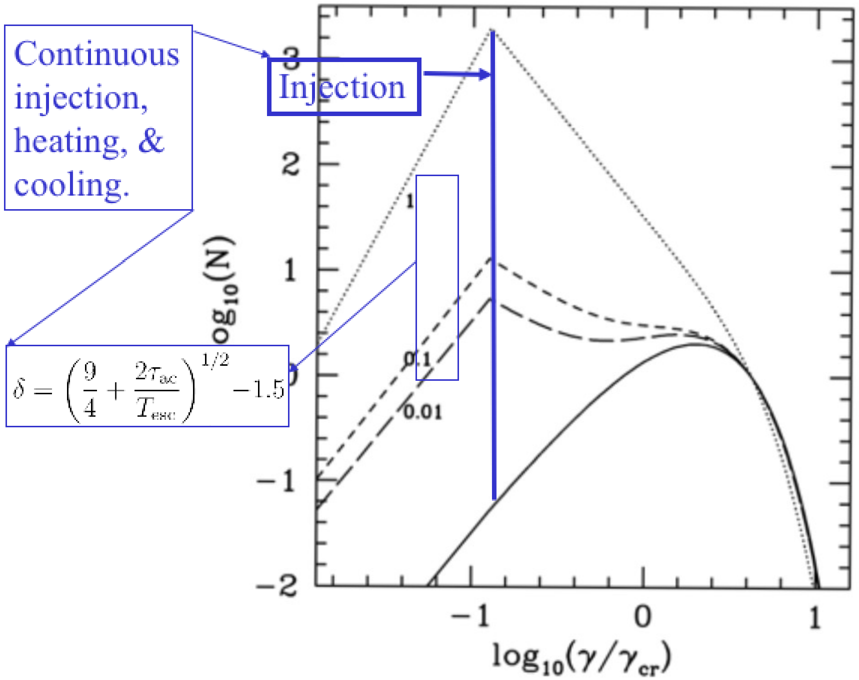


Another example: Rel. Acc. + Synch loss

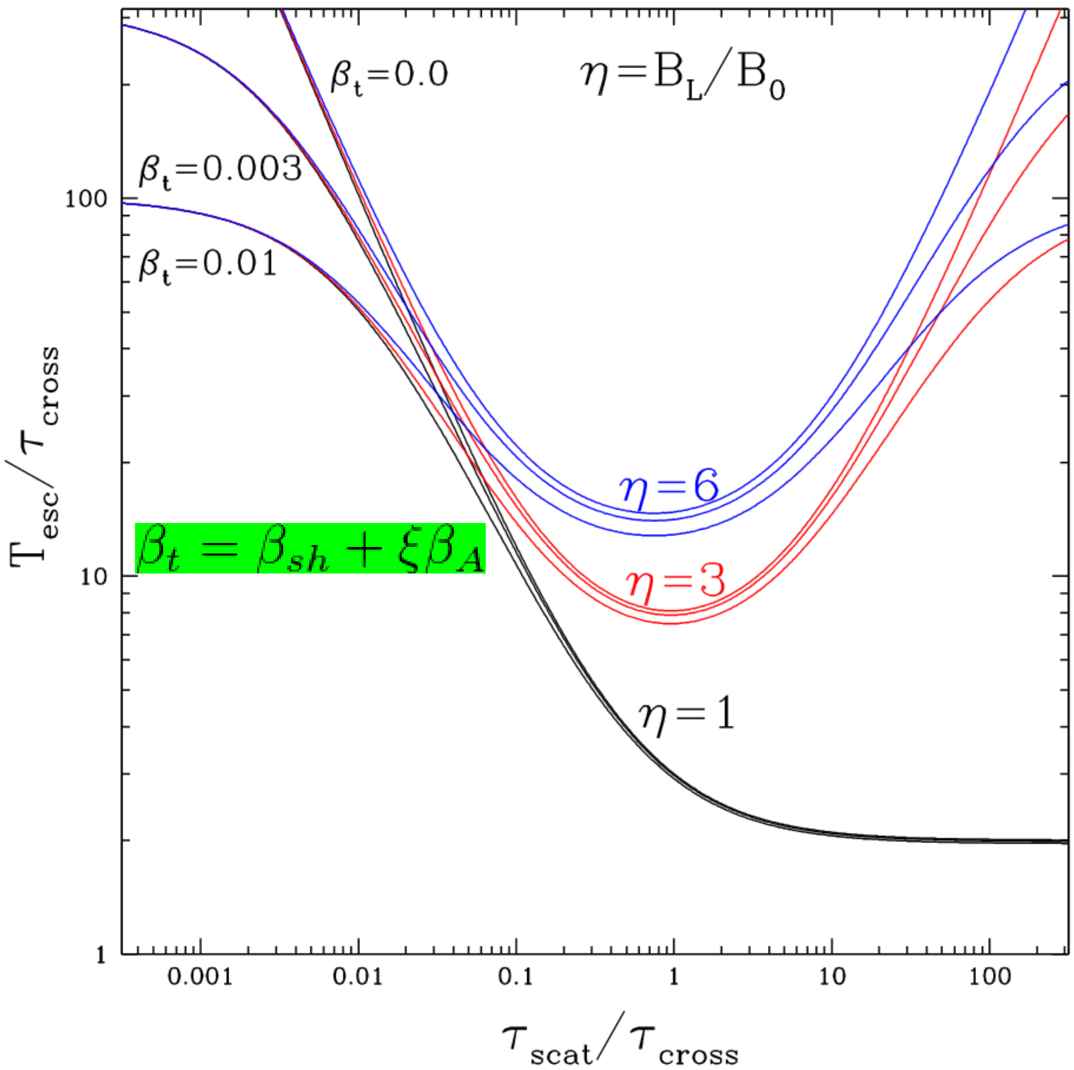
$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial \gamma}\left[\frac{\partial \gamma^2 N}{\partial \gamma} - \left(4\gamma - \frac{4\gamma^2 \tau_{\rm ac}}{\tau_0}\right) N\right] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

$$\tau_{\rm ac} = \frac{C_1}{f_{\rm turb}} \frac{cR}{v_{\rm A}^2}$$

$$\tau_{\rm syn}(\gamma) = 9m_e^3 c^5/4e^4 B^2\gamma = \tau_0/\gamma$$



The Escape Time



Zakopane-1, 2019

The Escape Times

Numerical Simulations

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial N}{\partial E}\right) - \frac{\partial}{\partial E}[(A - \dot{E}_L)N] - \frac{N}{T_{\rm esc}} + \dot{Q}$$

Strong diffusion $T_{\rm esc} \sim \tau_{\rm cross}^2/\tau_{\rm sc}$

Weak diffusion $T_{\rm esc} \sim \tau_{\rm cross}$

Converging B-field $T_{\rm esc} \propto \tau_{\rm sc}$

Combined equation (Malyshkin and Kulsrud ApJ 2001)

$$T_{\rm esc} = \tau_{\rm cross}\left(\eta + \frac{\tau_{\rm cross}}{\tau_{\rm sc}} + \ln\eta\,\frac{\tau_{\rm sc}}{\tau_{\rm cross}}\right)$$