Dense Matter and Neutron Stars

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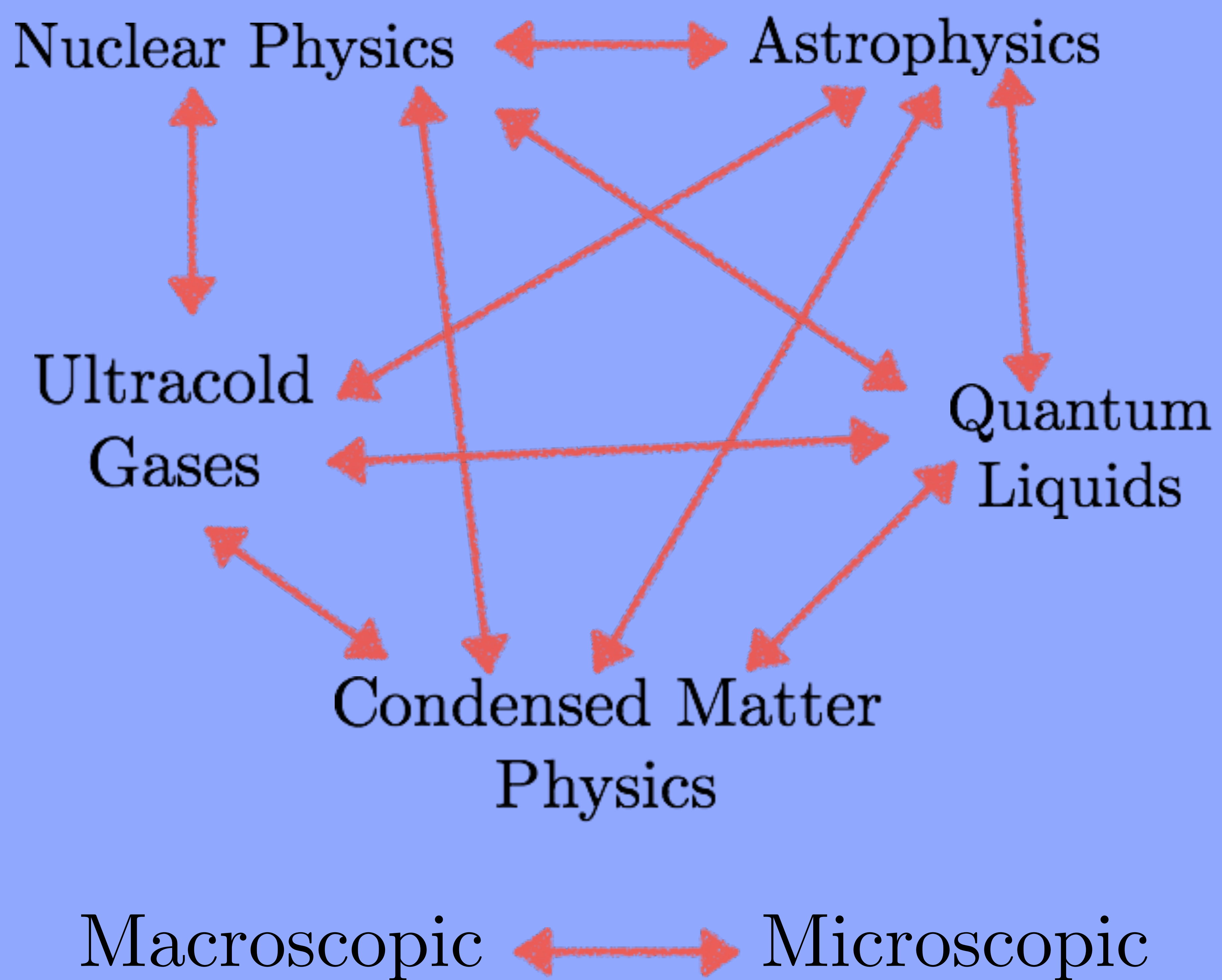
The Niels Bohr International Academy

59th Cracow School of Theoretical Physics
Zakopane, June 14-22, 2019

Bottom lines

- Enormous progress has been made!
- Exciting time for neutron star studies: new data, progress in theory
- Concentrate on physical principles
- Get inspiration from other systems
- Plenty of work to do


Nuclear Physics
Astrophysics
Ultracold Gases
Quantum Liquids
Condensed Matter Physics
Macroscopic
Microscopic

Scale of Neutron Star Masses

- N neutrons in sphere of radius R.
- Neutron separation: $N/R^3 \sim 1/r_n^3$, or $r_n \sim R/N^{1/3}$.
- Fermi energy per particle $\varepsilon_F \sim \hbar v_n/r_n$. Total kinetic energy $\sim N\varepsilon_F$.
- Gravitational binding energy, $E_G \sim G(Nm_n)^2/R$.
- Comparable when

$$N^{2/3} \sim \frac{\hbar v_n}{G m_n^2}.$$

- Neutrons relativistic. $N \sim \alpha_G^{3/2}$, where $\alpha_G = Gm_n^2/\hbar c \sim 10^{-38}$ is the "gravitational fine structure constant".
- Thus $N \sim 10^{57}$, corresponding to a mass of order 10^{33} gm ($1M_\odot$).
- More difficult to explain why sun has a mass $\sim 10^{33}$ gm.

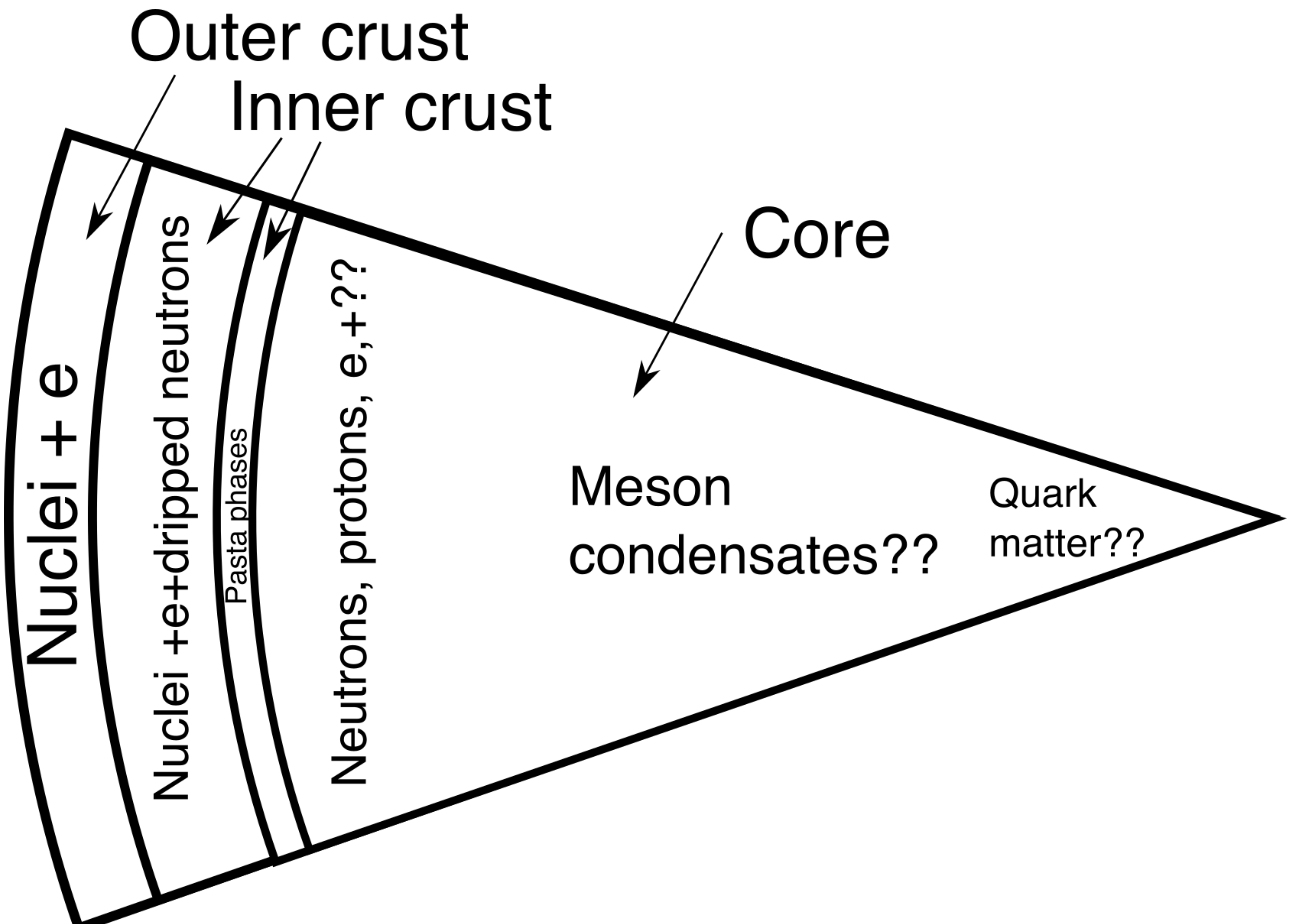
General messages

- No fundamental problems in finding properties of matter below 1-2 times that of nuclear matter.
  Microscopic interactions are well understood from laboratory data.

- Large uncertainties at higher densities due to lack of understanding of basic constituents and interactions.

- Observations, especially mass measurements of neutron stars, provide constraints.

Schematic picture of a neutron star



Electrons simple at high densities

- Weakly interacting except near surface.
  Kinetic (Fermi) energy $\sim (\hbar^2/r_e^2)/2m_e$. Potential energy $\sim e^2/r_e$.
  P.E./K.E $\sim e^2/\hbar v_e$. Electron velocity $v_e \sim \hbar/m_e r_e$.
  Terrestrial matter: $r_e \sim a_0 = \hbar^2/me^2$.
  Higher densities: Interactions less important.

- Pressure ionization. Atom cores overlap. Size of atom in Thomas–Fermi theory $r_{TF} \sim a_0/Z^{1/3}$. Mass density $\rho \approx AZ$ gm/cm^3
  (Coulomb energy $\sim Ze^2/r_{TF}$ and Fermi energy $(\hbar^2/r_e^2)/2m_e \sim (\hbar^2 Z^{2/3}/r_{TF}^2)/2m$
  comparable.)

- Electrons relativistic. $r_e \lesssim \lambda\!\!\!^{-}_e$, $\hbar/m_e c = \alpha a_0$, Compton wavelength.
  Fine structure constant $\alpha = e^2/\hbar c \approx 1/137$.

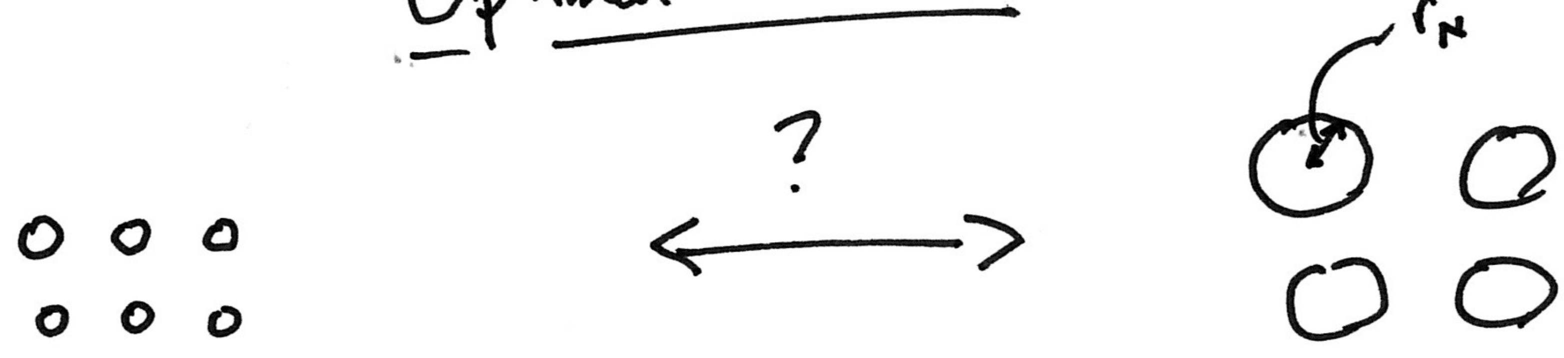
- Screening length $\sim \alpha^{-1/2} r_e \approx 10 r_e$. Screening unimportant for $Z \lesssim 10^3$.

Nuclei

- Matter is cold in neutron stars.
  Nuclear energies $\sim$MeV or more (10^{10}K) than temperature (10^{9}K) or less after one hour.
- Lowest energy nucleus (no electrons for the moment).
  Liquid drop model: bulk, surface and Coulomb energies.

$$E = E_{\mathrm{bulk}} + E_{\mathrm{surf}} + E_{\mathrm{Coul}}$$

Optimal Nucleus



Keep $Z/A = x$ fixed (Z = proton number. A = nucleon (mass) number

$$\frac{E_{surf}}{A} \sim \frac{4\pi r_N^2 \sigma}{A} \sim \frac{1}{r_N}$$

(Area = $4\pi r_N^2$; σ = surface tension; r_N = Nuclear radius)

$$\frac{E_{Coul}}{A} \sim \frac{Z^2 e^2}{r_N A} \sim x^2 r_N^2$$

Minimize energy/A. Bulk term unaffected.

$$\boxed{E_{surf} = 2E_{Coul}}$$

$$A \simeq \frac{12.5}{x^2}$$

(Terrestrial nuclei. Those around $^{56}_{26}Fe$ most stable.)

Nuclei at higher densities

- $\rho \gtrsim 10^6$ g/cm^3, Electrons relativistic and Fermi energy $\gtrsim 1$ MeV.
- Electron capture, $e^- + p \to n + \nu_e$.
- Chemical equilibrium: $\mu_e + \mu_p = \mu_n$, where μ is the chemical potential.
- Matter becomes more neutron rich.
- But perhaps "neutron star" is a misnomer ...

Neutron drip

- Bulk approximation.

$$E = A(-b_{\rm bulk} + b_{\rm symm}\delta^2).$$

  Neutron excess $\delta = (N-Z)/(N+Z) = 1-2x$, where $x = Z/A$ is proton fraction.

- $b_{\rm bulk} \approx 16$ MeV, $b_{\rm symm} \approx 32$ MeV
- Neutron chemical potential. $\mu_n = \partial E/\partial N = -b_{\rm bulk} + 2b_{\rm symm}\delta + \mathcal{O}(\delta^2)$
- Neutron drip: $\mu_n = 0$ gives $\delta_{\rm drip} \approx b_{\rm bulk}/2b_{\rm symm} = 1/4$, or $x_{\rm drip} = (1-\delta_{\rm drip})/2 \approx 3/8$.
- Why is $\delta_{\rm drip}$ so low? Kinetic and potential energy contributions have *same sign* in $b_{\rm symm}$ but *opposite sign* in $b_{\rm bulk}$.
- At drip, $\mu_n - \mu_p = 4b_{\rm symm}\delta_{\rm drip} \approx 2b_{\rm bulk} \approx 32$ MeV.
- Better calculations: $\rho_{\rm drip} \approx 4 \times 10^{11}$ g/cm^3.
- Why so much less that nuclear densities? Electrons relativistic, nucleons non-relativistic + small $\delta_{\rm drip}$.

Lattice energy

- Electron–nucleus and electron–electron interactions become important as density increases.
- Wigner–Seitz approximation. Replace unit cell by sphere of same volume.

$$\frac{1}{n_{\rm N}} = \frac{4\pi}{3} r_{\rm c}^3$$

Lattice energy

Finite size

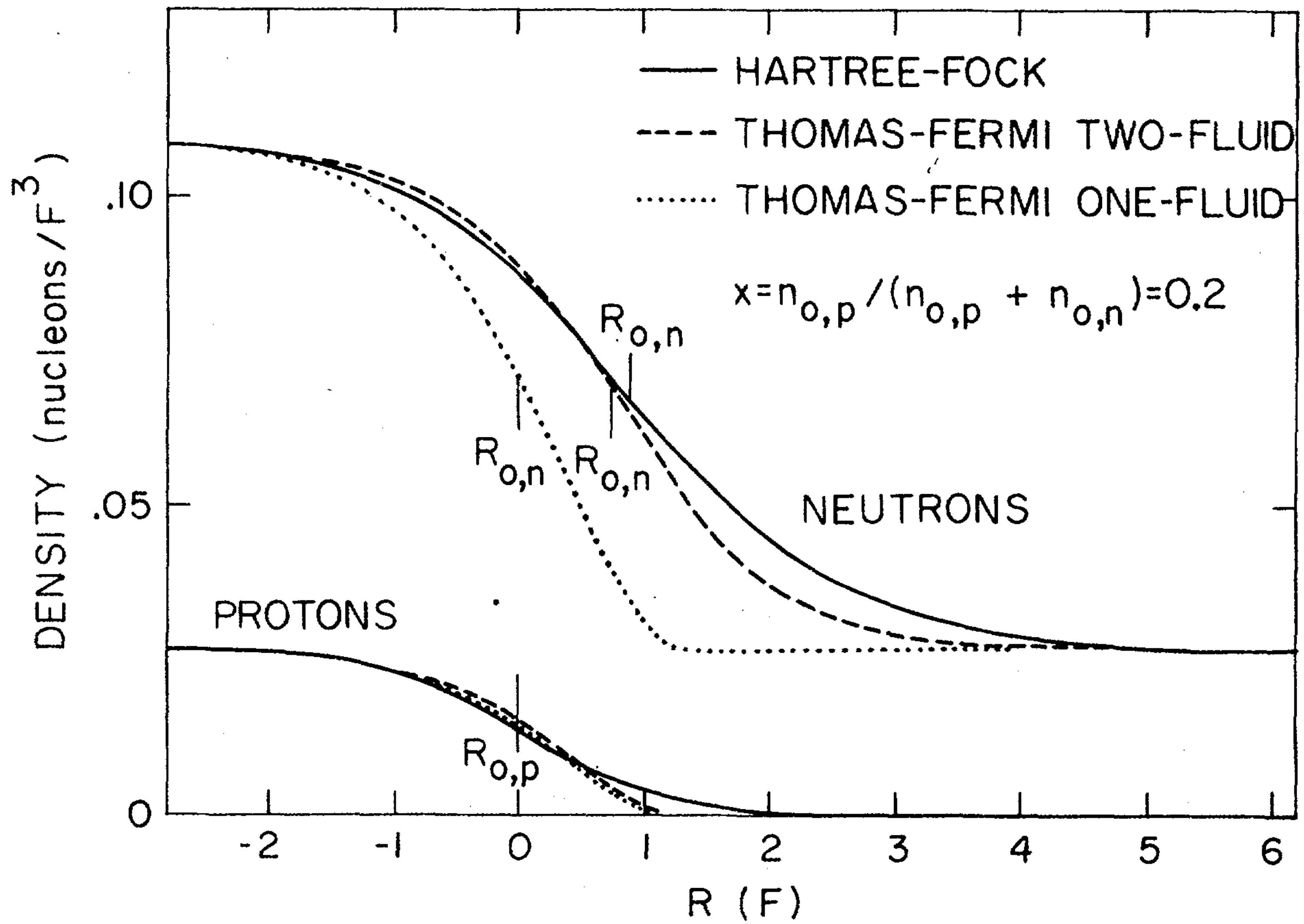
$$E_{\rm Coulomb} = \frac{3}{5}\frac{Z^2e^2}{r_{\rm N}} - \frac{9}{10}\frac{Z^2e^2}{r_{\rm c}} + \frac{3}{10}\frac{Z^2e^2r_{\rm N}^2}{r_{\rm c}^3}$$
$$= \frac{3}{5}\frac{Z^2e^2}{r_{\rm N}}\left[1 - \frac{3}{2}\frac{r_{\rm N}}{r_{\rm c}} + \frac{1}{2}\left(\frac{r_{\rm N}}{r_{\rm c}}\right)^3\right]$$

- Vanishes for $r_{\rm N} = r_{\rm c}$.
- 15% effect at 1/1000 of nuclear density.

Comments on nuclei

- Up to neutron drip density the equilibrium nuclei are known in the lab.
- At higher densities properties must be estimated from theory.
- Shell effects need to be investigated more.
  Spin-orbit interaction becomes weaker.
  Calculations of neutron drops provide information.

Reduction of surface (interface) tension

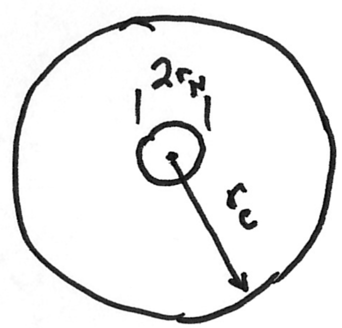


Matter inside and outside nuclei become more similar

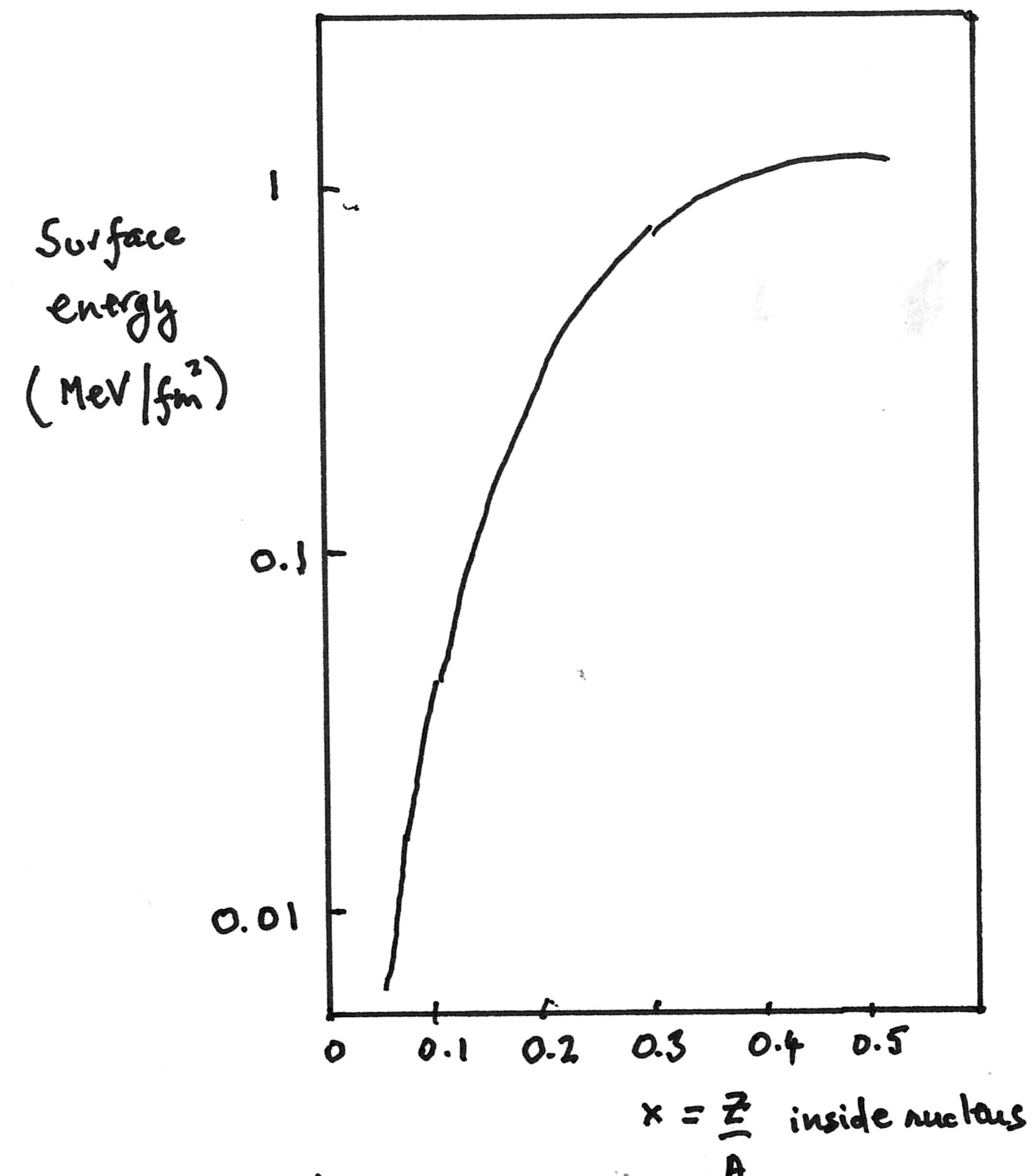
Effects of higher density

- Surface tension reduced as nuclear matter becomes neutron rich.

- Solid state effects become significant



r_c – radius of sphere containing Z electrons



(Ravenhall, Bennett, CJP PRL 28, 978 (1972))

- Net result: Z remains roughly 40.

Melting

- Classical plasma of point nuclei in a uniform background charge. Dimensionless parameter
$$\Gamma = \frac{Z^2 e^2}{r_\mathrm{c} k_\mathrm{B} T}$$
- At melting $\Gamma_\mathrm{M} \approx 175$.
- Coulomb energy differs little for different crystal structures
fcc $U = -0.895929 \ldots Z^2 e^2 / r_\mathrm{c}$
bcc $U = -0.895873 \ldots Z^2 e^2 / r_\mathrm{c}$

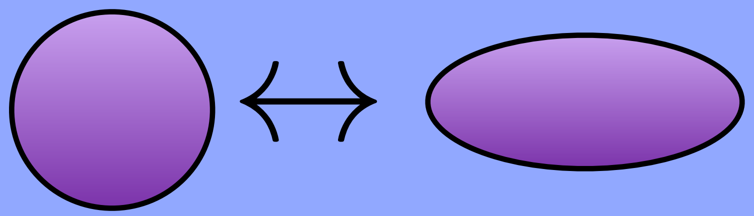
$$k_\mathrm{B} T_\mathrm{M} = \frac{Z^2 e^2}{\Gamma_\mathrm{M} r_\mathrm{c}} \approx 0.013 Z^{5/3} (nx)^{1/3}\ \mathrm{MeV}$$

Equilibrium nucleus

- Virial relation still holds, but with the total Coulomb energy, including the lattice contribution.
- Coulomb energy reduced, equilibrium A increases.

Fission instability

- Bohr and Wheeler (1938). Nucleus unstable to quadrupolar distortion if



$$E_{\rm surface} < \frac{1}{2} E^0_{\rm Coulomb}$$

$E^0_{\rm Coulomb}$ is the Coulomb energy of an *isolated* nucleus
(Rather insensitive to medium effects.)

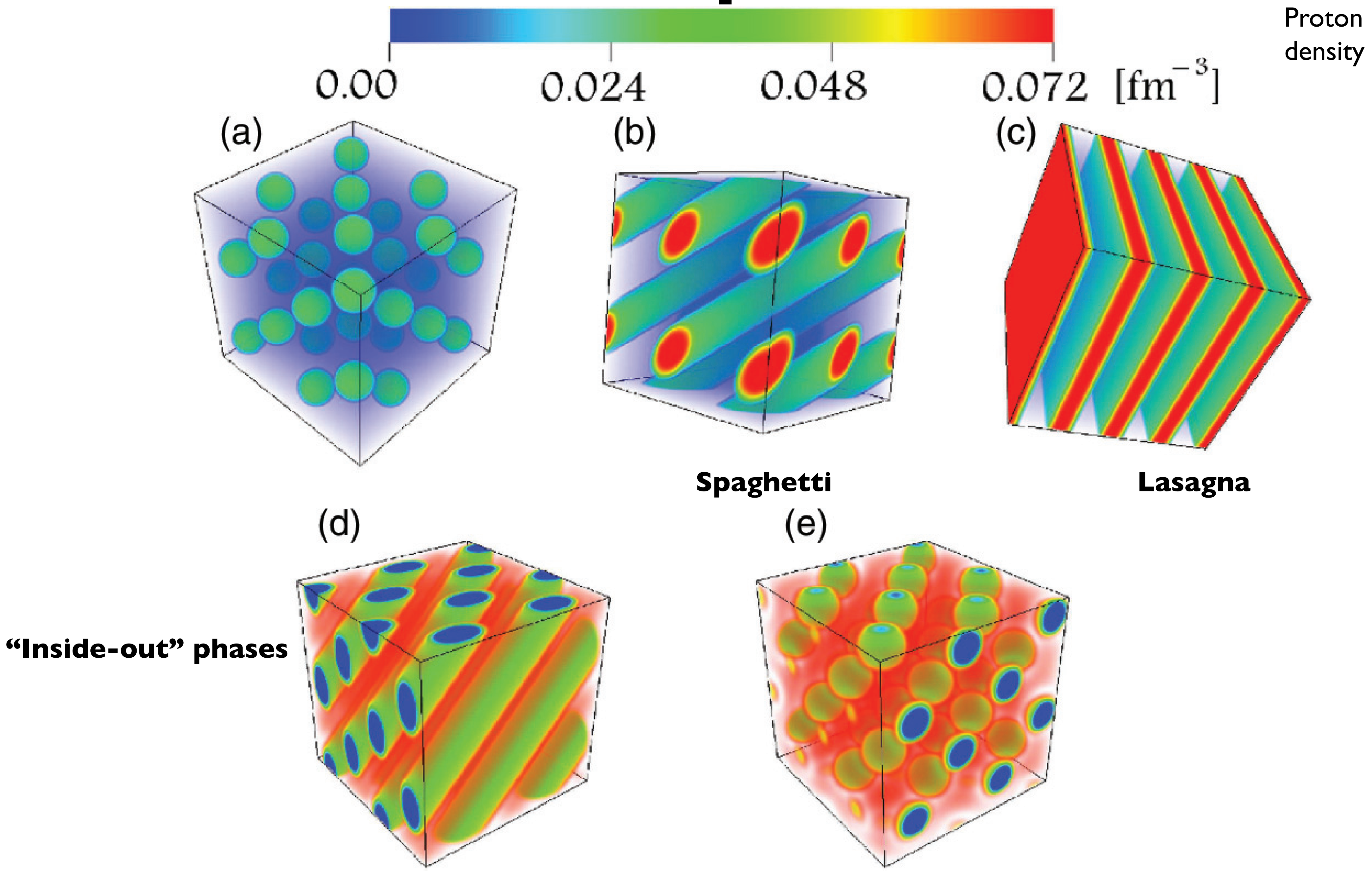
- Equilibrium nucleus unstable if

$$E_{\rm surface} = 2E_{\rm Coulomb} = 2E^0_{\rm Coulomb}\left[1 - \frac{3}{2}\frac{r_{\rm N}}{r_{\rm c}} + \frac{1}{2}\left(\frac{r_{\rm N}}{r_{\rm c}}\right)^3\right] = \frac{1}{2}E^0_{\rm Coulomb}$$

or

$$\frac{r_{\rm N}}{r_{\rm c}} \gtrsim \frac{1}{2}$$

Pasta phases



(Image from Okamoto, Minoru et al., Phys.Rev. C **88**, 025801 (2013))

Where does the crust end?

- Start with uniform phase of neutrons, protons and electrons at nuclear density.
- Proton fraction is $\sim 5\%$.
- Reduce density until matter is unstable to creation of density wave. $E = E_0 1/2 V_q \delta n_q^2$. (Actually there are two densities, neutron and proton.)
- Coulomb interaction (and low compressibility of electrons) favors small wavelengths.
- Terms in energy $\propto (\nabla n)^2$ favor large wavelengths.
- Instability density gives upper bound on density at which structure appears. Transition has to be 2nd order on general grounds. ($(\delta n)^3$ term in energy!)
- Include 3rd and 4th order terms. As density is reduced, the most stable state goes through the sequence of pasta phases found from liquid drop ideas.
- Rather general for a number of systems (block copolymers)

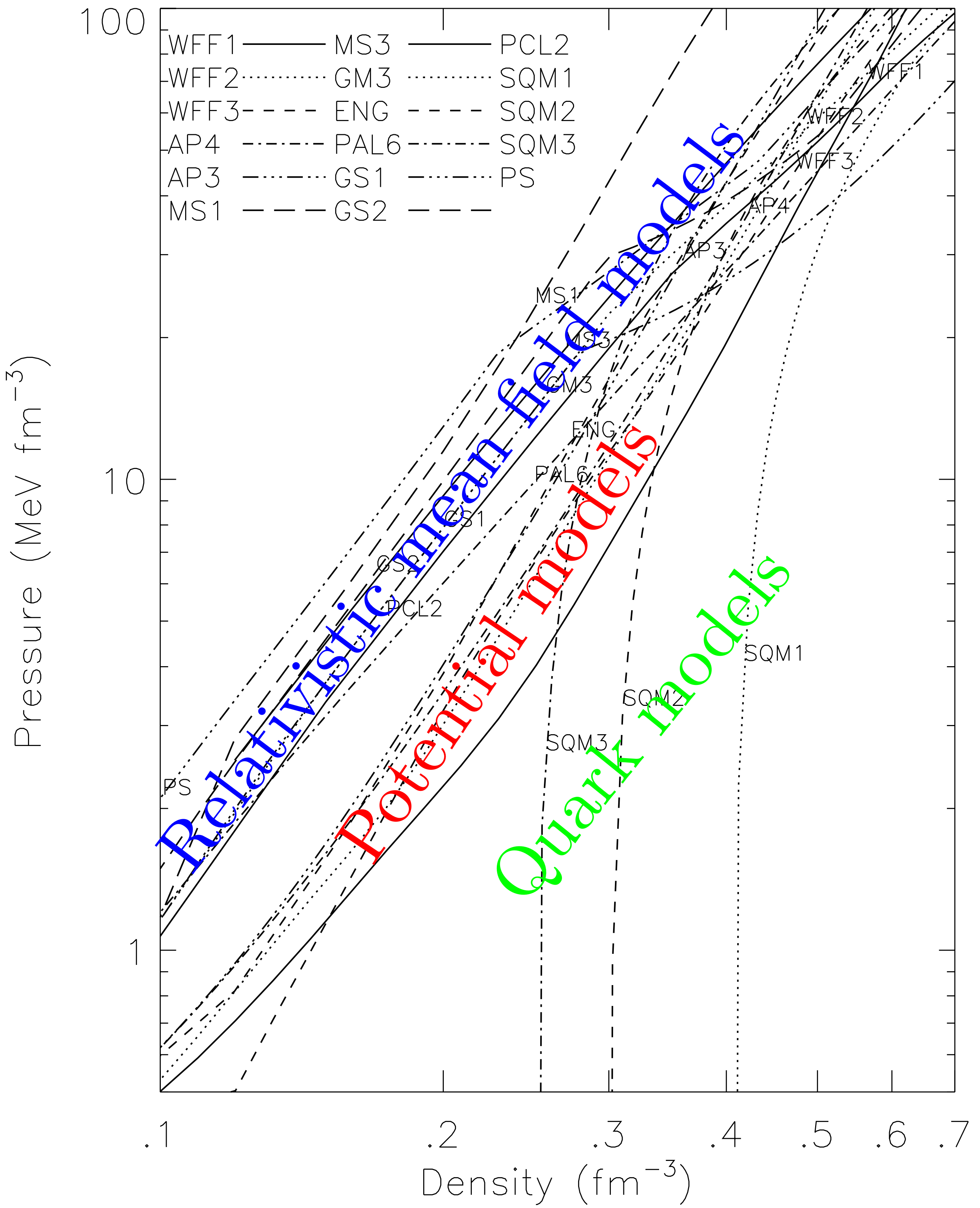
Properties of uniform nuclear and neutron matter

- Solve many-body problem for a specific nucleon-nucleon interaction. Great progress over past few decades due to development of a family of Monte-Carlo methods.

- Interaction obtained by direct fit to N-N scattering data, supplemented by phenomenological 3-body interaction or an effective field theory approach in which one expands the effective interaction between nucleons in powers of the momentum (Weinberg).

- Compare with other models.

Representative equations of state



Lattimer and Prakash, Ap. J. **550**, 426 (2001)

TABLE 1

EQUATIONS OF STATE

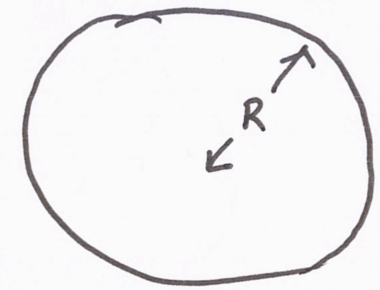
Symbol	Reference	Approach	Composition
FP	Friedman & Pandharipande (1981)	Variational	*np*
PS	Pandharipande & Smith (1975)	Potential	$n\pi^0$
WFF(1–3)	Wiringa, Fiks & Fabrocine (1988)	Variational	*np*
AP(1–4)	Akmal & Pandharipande (1997)	Variational	*np*
MS(1–3)	Müller & Serot (1996)	Field theoretical	*np*
MPA(1–2)......	Müther, Prakash, & Ainsworth (1987)	Dirac-Brueckner HF	*np*
ENG	Engvik et al. (1996)	Dirac-Brueckner HF	*np*
PAL(1–6)	Prakash et al. (1988)	Schematic potential	*np*
GM(1–3)	Glendenning & Moszkowski (1991)	Field theoretical	*npH*
GS(1–2)	Glendenning & Schaffner-Bielich (1999)	Field theoretical	*npK*
PCL(1–2)	Prakash, Cooke, & Lattimer (1995)	Field theoretical	*np*HQ
SQM(1–3)......	Prakash et al. (1995)	Quark matter	Q (*u*, *d*, *s*)

NOTE.—"Approach" refers to the underlying theoretical technique. "Composition" refers to strongly interacting components (*n* = neutron, *p* = proton, H = hyperon, K = kaon, Q = quark); all models include leptonic contributions.

Stellar Stability

Non relativistically: $E = E_{grav} + E_{matt}$, with $E_{grav} \sim -\frac{G M^2}{R}$

$V = \frac{4\pi}{3} R^3$



Equilibrium $\frac{\partial E}{\partial V} \Rightarrow P_{matt} + P_{grav} = 0$

[Virial theorem]

$$P_{grav} = \frac{1}{3}\frac{E_{grav}}{V} \sim \frac{1}{V^{4/3}}$$

Like gas with adiabatic index $\Gamma = \frac{4}{3}$. $(P \sim \rho^{\Gamma})$

stability $\Gamma_{matt} > \frac{4}{3}$

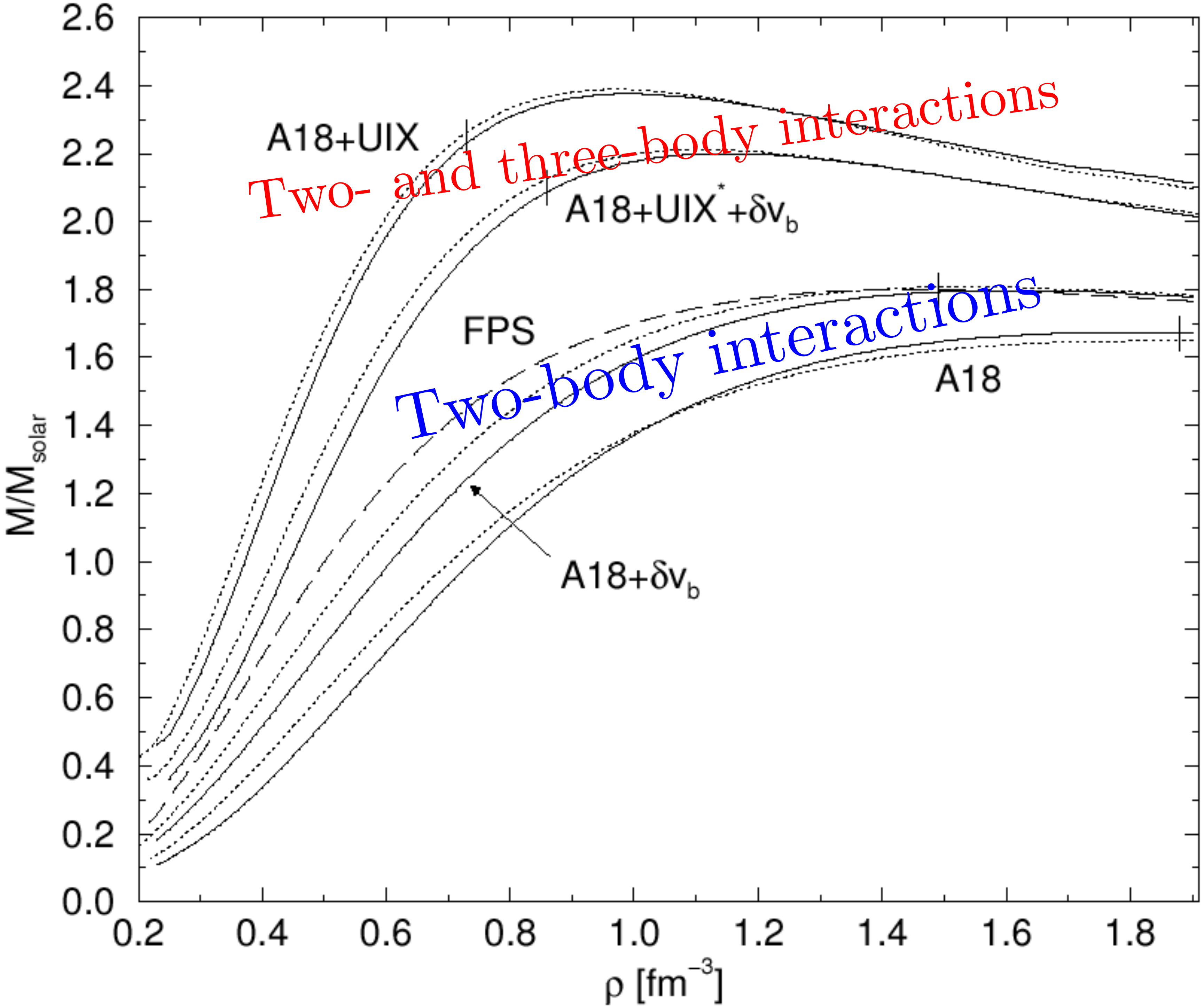
Examples Degenerate, non-relativistic particles. $\Gamma = 5/3$ O.K.

Degenerate relativistic electrons + nuclei. $\Gamma = 4/3$ Touchy!
[white dwarfs] [Saved by electron mass.]

General relativity Gravity stronger. Total energy density and pressure contribute to grav. force, not just rest mass.

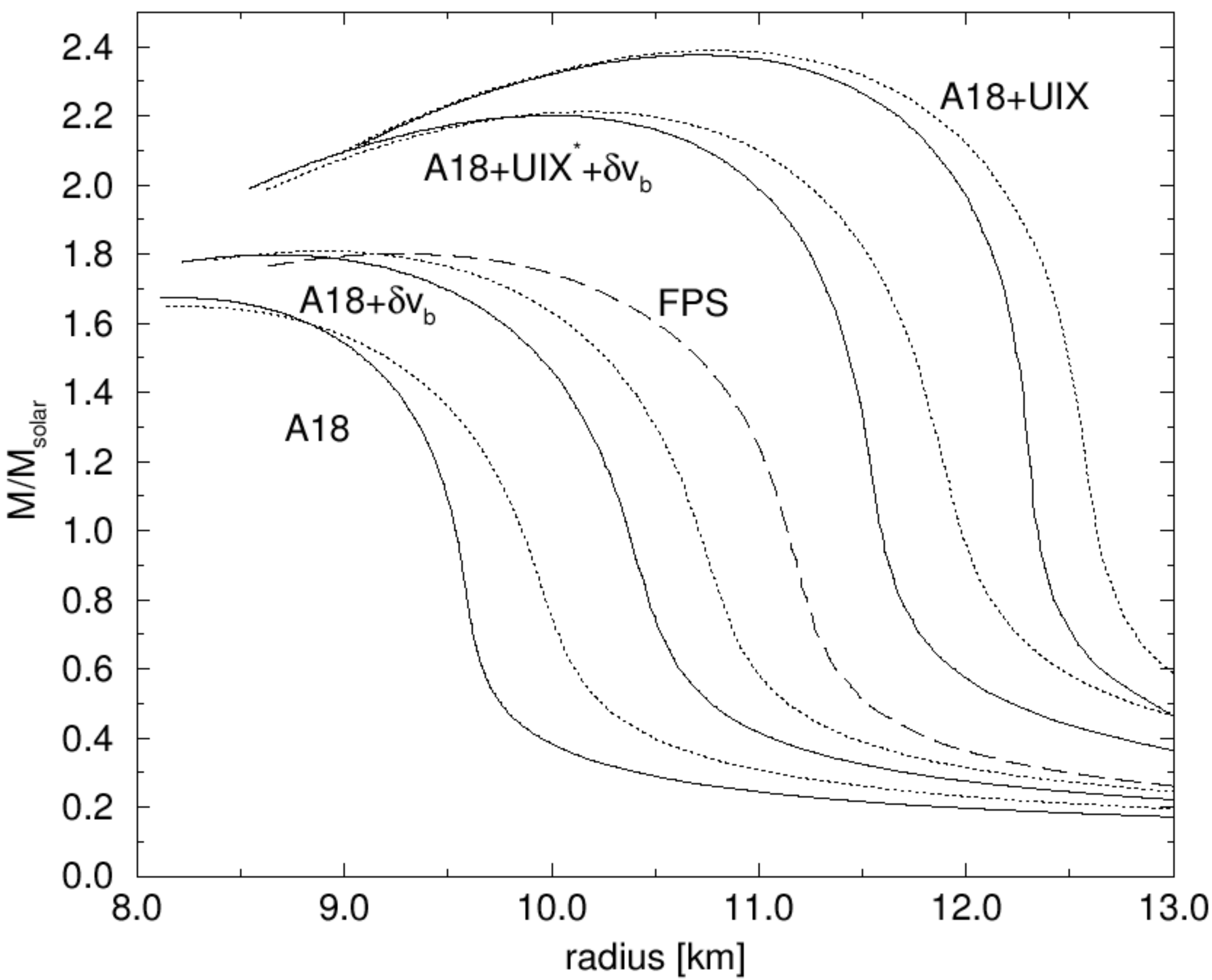
Free massless quarks: $P \sim n_q^{4/3}$. Very touchy!

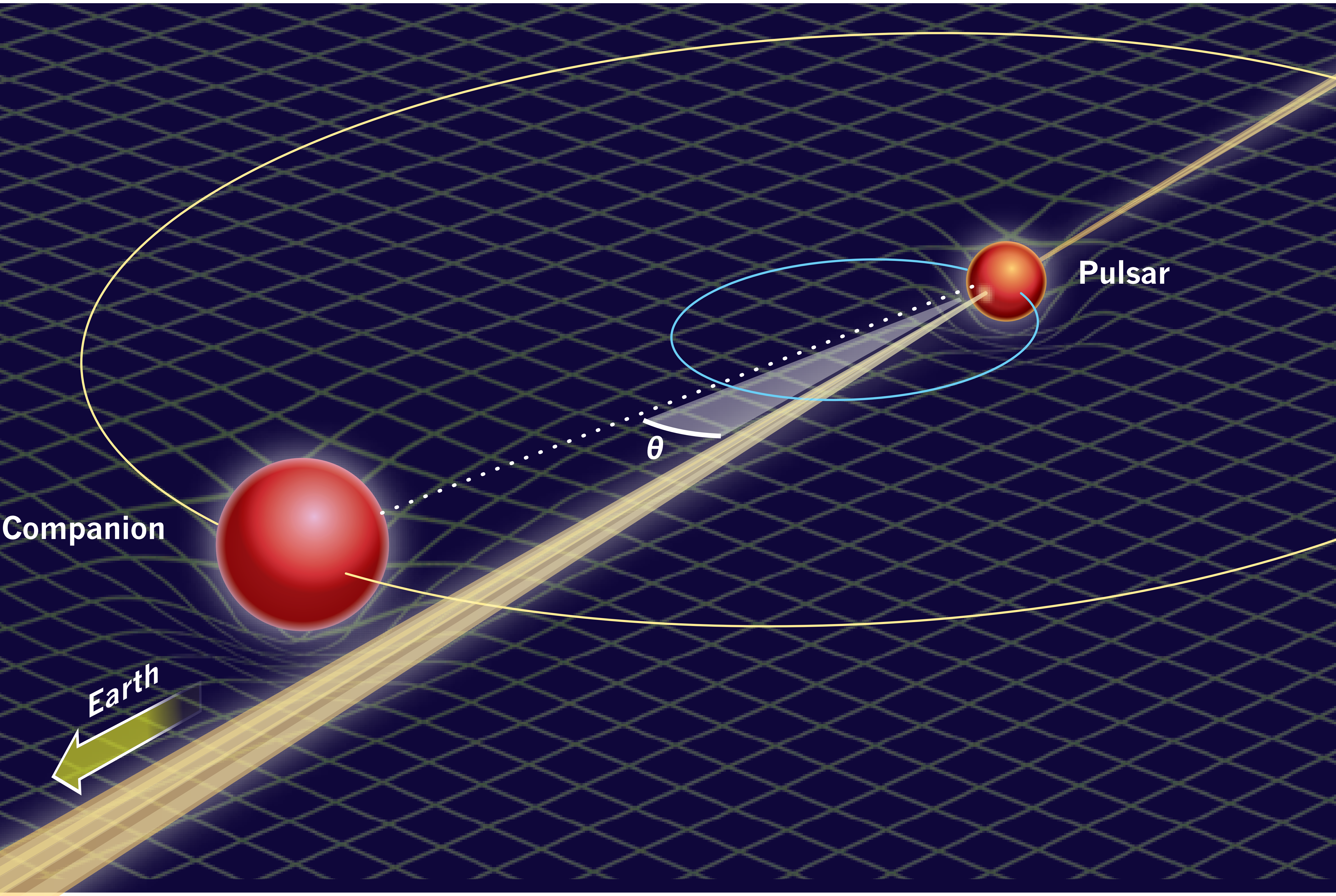
Stellar mass versus central density



Akmal, Pandharipande and Ravenhall, Phys. Rev. C **58**, 1804 (1998).

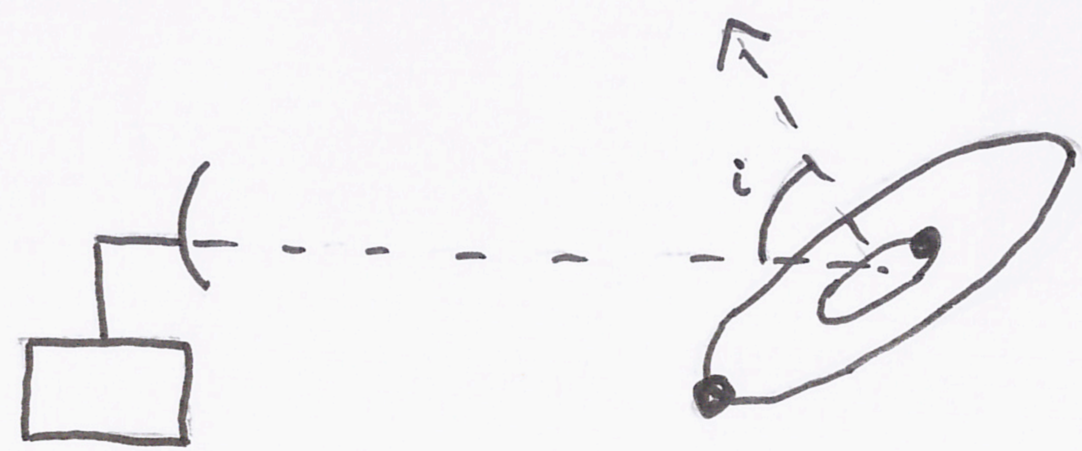
Mass versus stellar radius





Courtesy M. C. Miller

J 1614 −2230



- Binary stars
- Newtonian gravity.

Need angle of inclination i.

- General relativity. More quantities to measure (precession of orbit, time delay, ...)
- J 1614 −2230. LUCKY! EDGE ON! LARGE TIME DELAY
- Scale of delay: $\frac{\text{schwarzschild radius}}{c} = \frac{2GM}{c^3} \sim \frac{3\,\text{km}}{3\times10^5\,\text{km/s}} \left(\frac{M}{M_\odot}\right)$

(Helped by logarithmic term.)

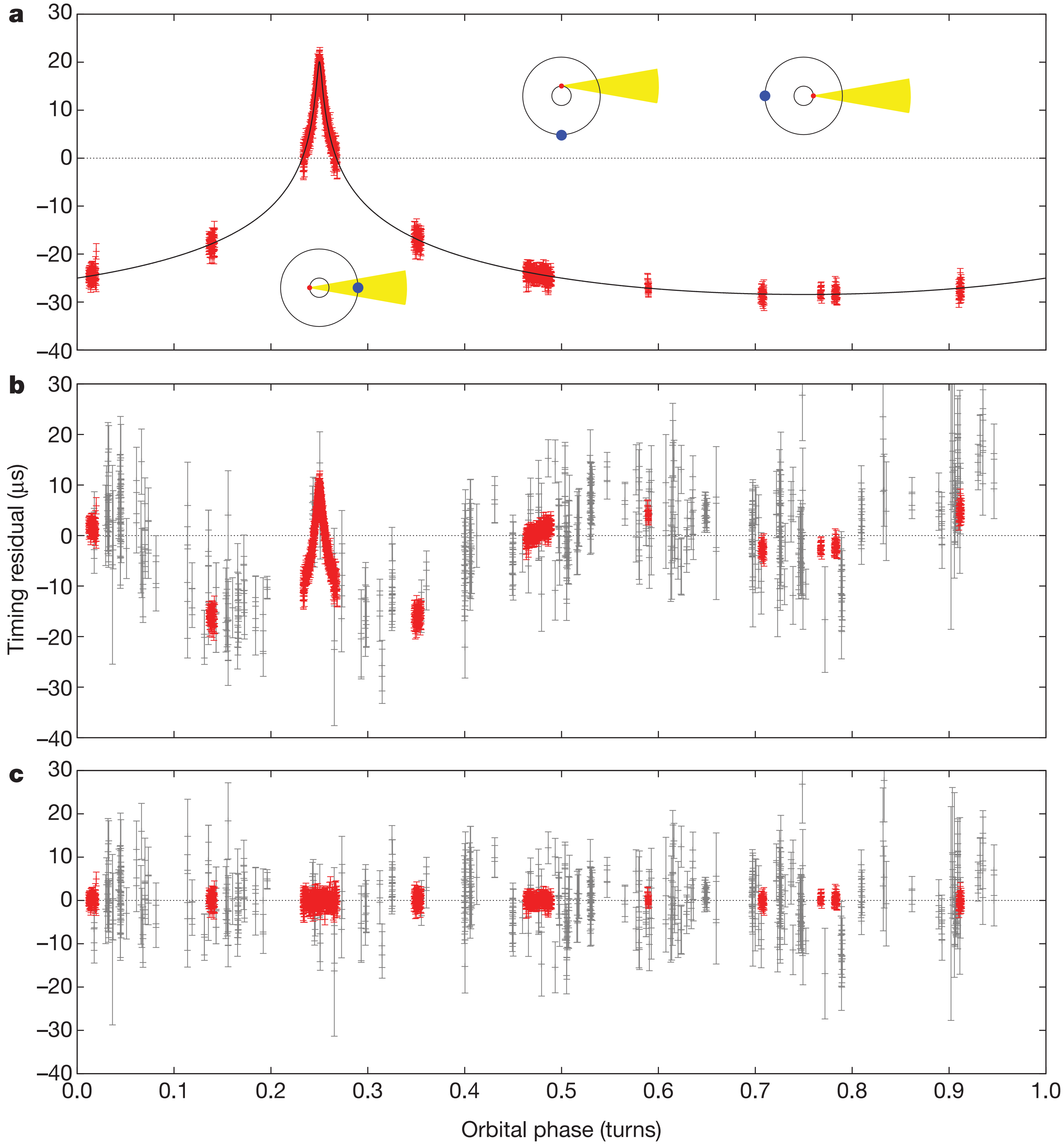
$$\sim 10\,\mu s \left(\frac{M}{M_\odot}\right)$$

Shapiro delay for J1614-2230 General relativity important!

Shapiro delay for best-fit model

Best-fit model without general relativity

Deviations from fit in uppermost plot



Implications of a $2M_\odot$ neutron star

- Many equations of state ruled out
- Models
  - Ones based on nucleons, either microscopic or schematic.
  - Ones based on quark degrees of freedom.
- Observed masses consistent with conventional models, but that does not mean that they are right.

Heavy Neutron Stars

Neutron-star–white-dwarf binary systems

- J1614-2230. $M = 1.97 \pm 0.04 M_\odot$
  Demorest et al., Nature **467**, 1081 (2010).
  $M = 1.928 \pm 0.017 M_\odot$
  Fonseca et al., Ap. J. **832**:167 (2016).

- J0348+0432. $M = 2.01 \pm 0.04 M_\odot$
  Antoniadis et al., Science **330**, 448 (2013).

- J0740+6620. $M = 2.17^{+0.11}_{-0.10} M_\odot$
  Cromartie et al., arXiv 1904.06759.

Neutron Star Interior Composition Explorer

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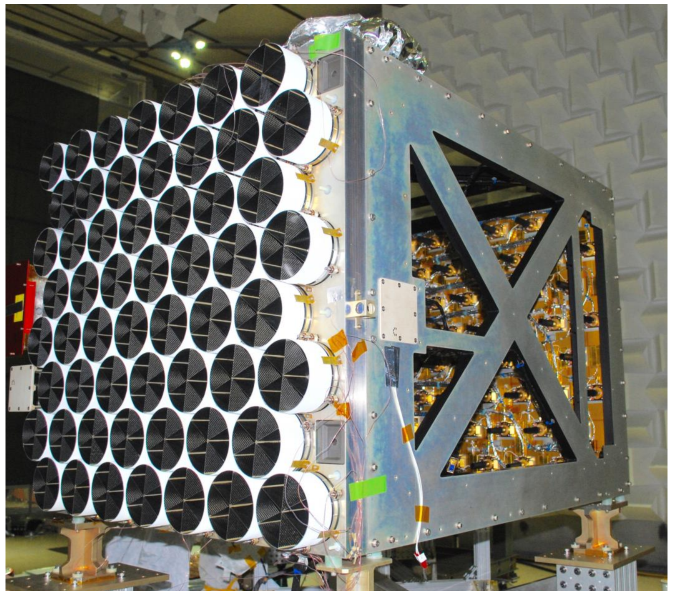
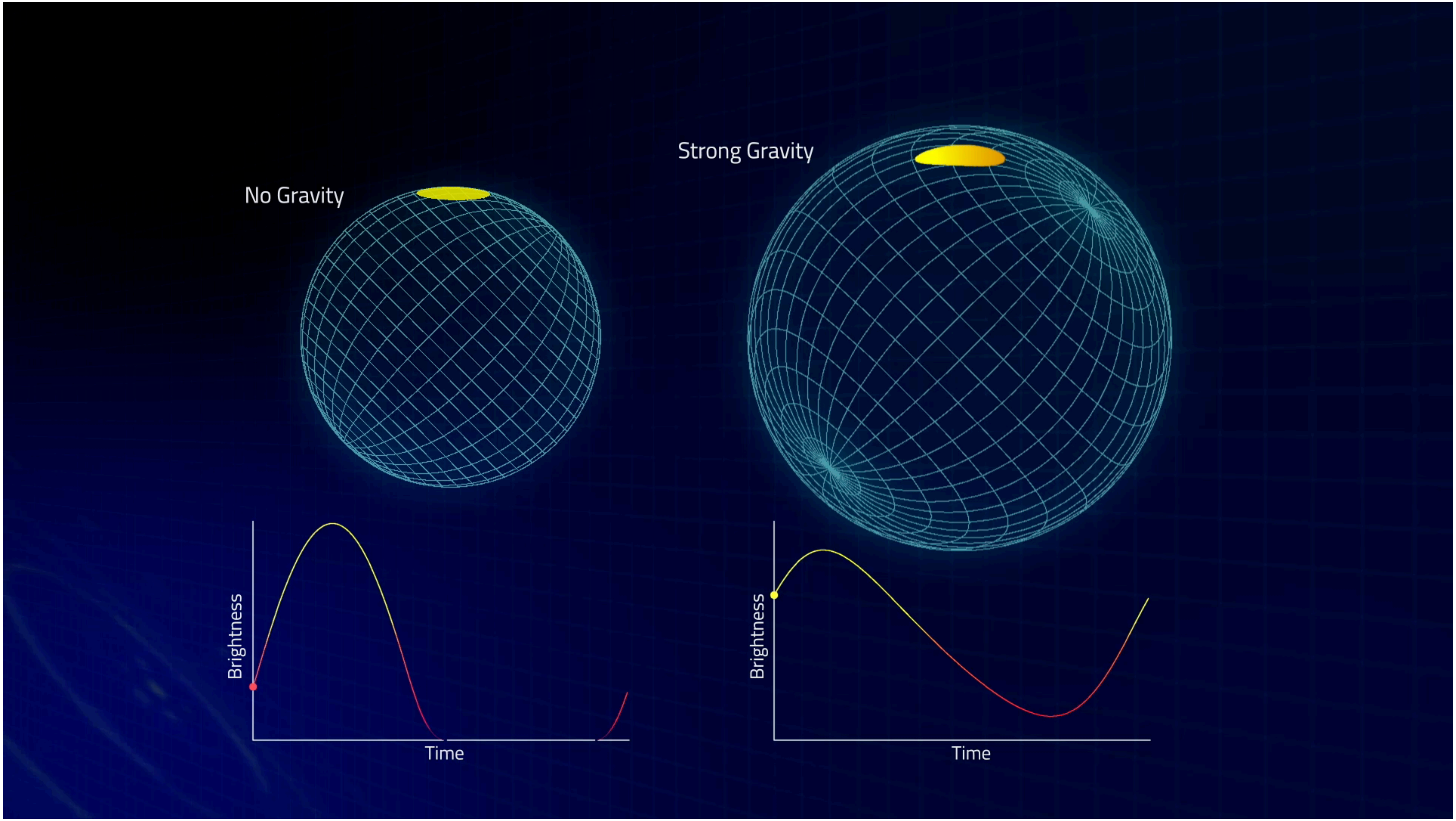


Image courtesy Keith Gendreau

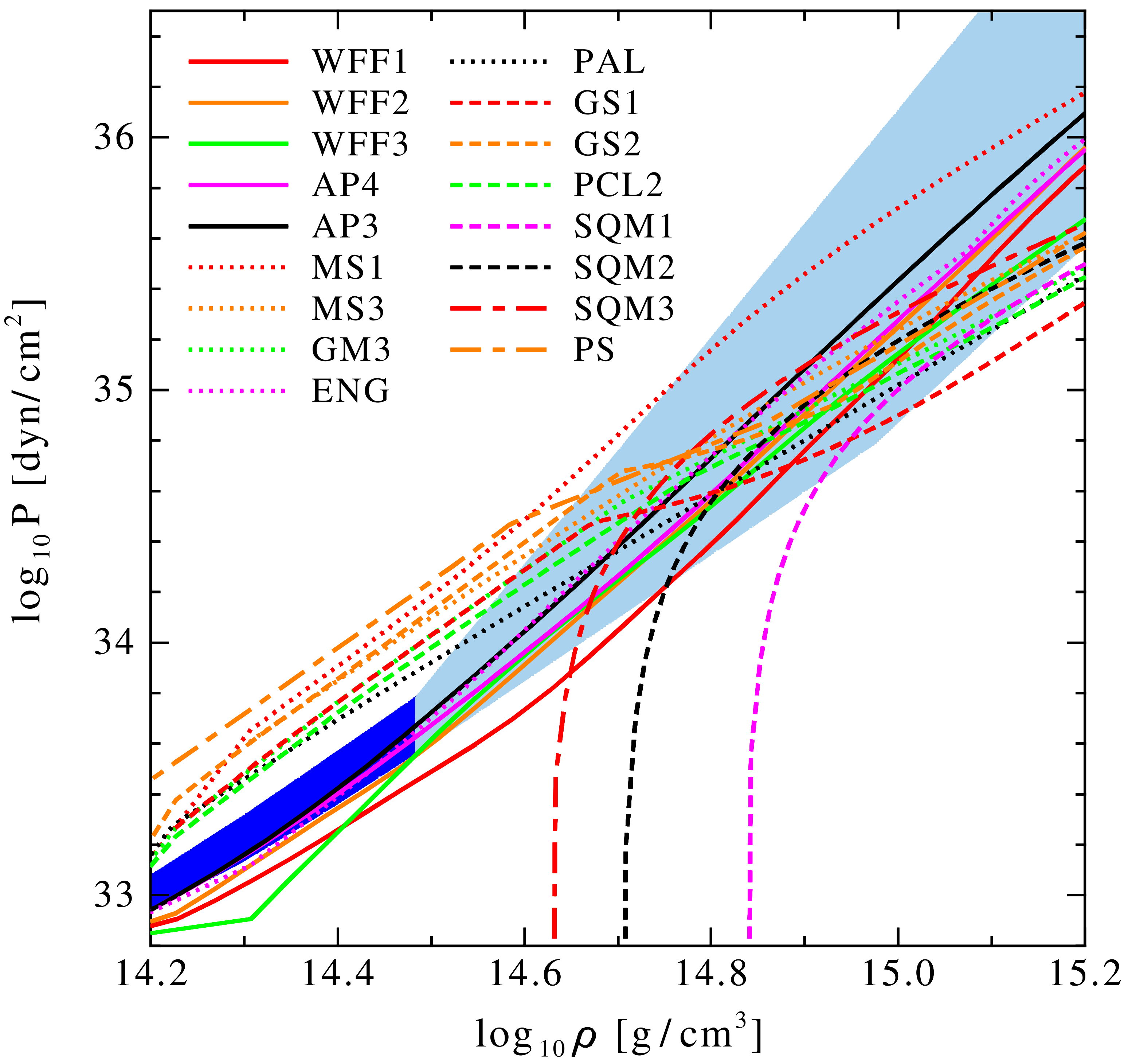
- Look at X-rays from hot spots on neutron star surface. General relativity important!
- 56 channels
- Roughly size of a refrigerator. 400 kg.
- (Also experiment to use pulsars as interplanetary GPS sources)
- See M. Coleman Miller and F. K. Lamb, *Eur. Phys. J. A* **52**, 63 (2016) for details of analysis.
- Data being analyzed, but no articles yet.



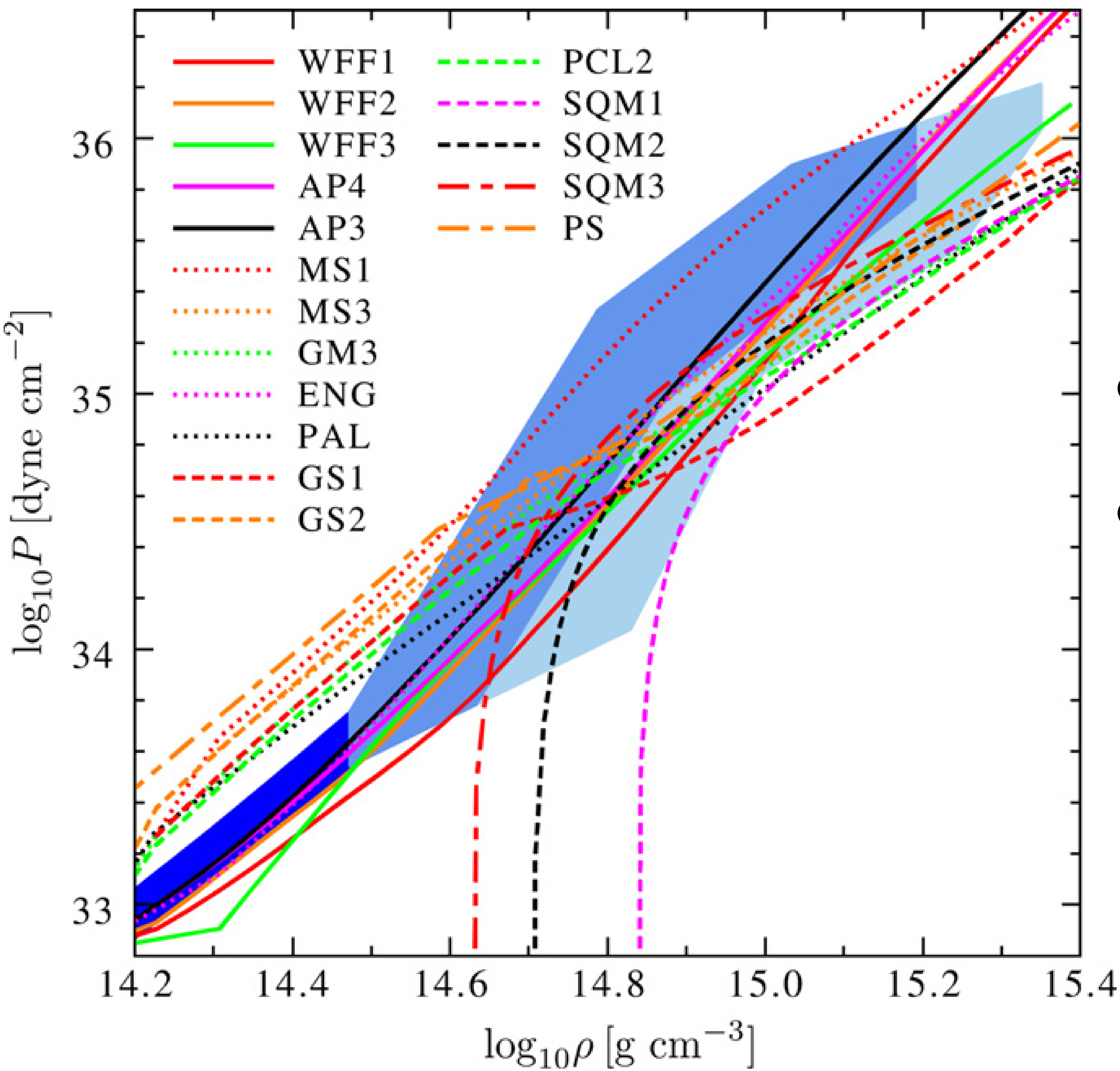
Credit: Morsink/Moir/Arzoumanian/NASA GSFC

X-rays deflected by strong gravitational field

Equations of state



Light blue area: constraint provided by existence of a $1.65 M_{\odot}$ neutron star.
(Hebeler, Lattimer, CJP, Schwenk, Phys. Rev. Lett. **105**, 161102 (2010).)



Light blue.
Can support $1.97 M_\odot$ star.

Dark blue.
Can support $2.4 M_\odot$ star.

Hebeler, Lattimer, CJP, Schwenk, Ap. J 773:11 (2013).

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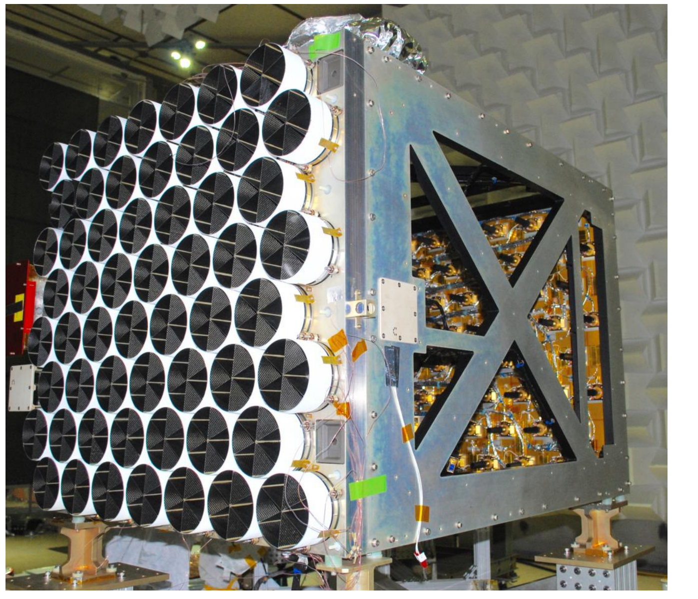

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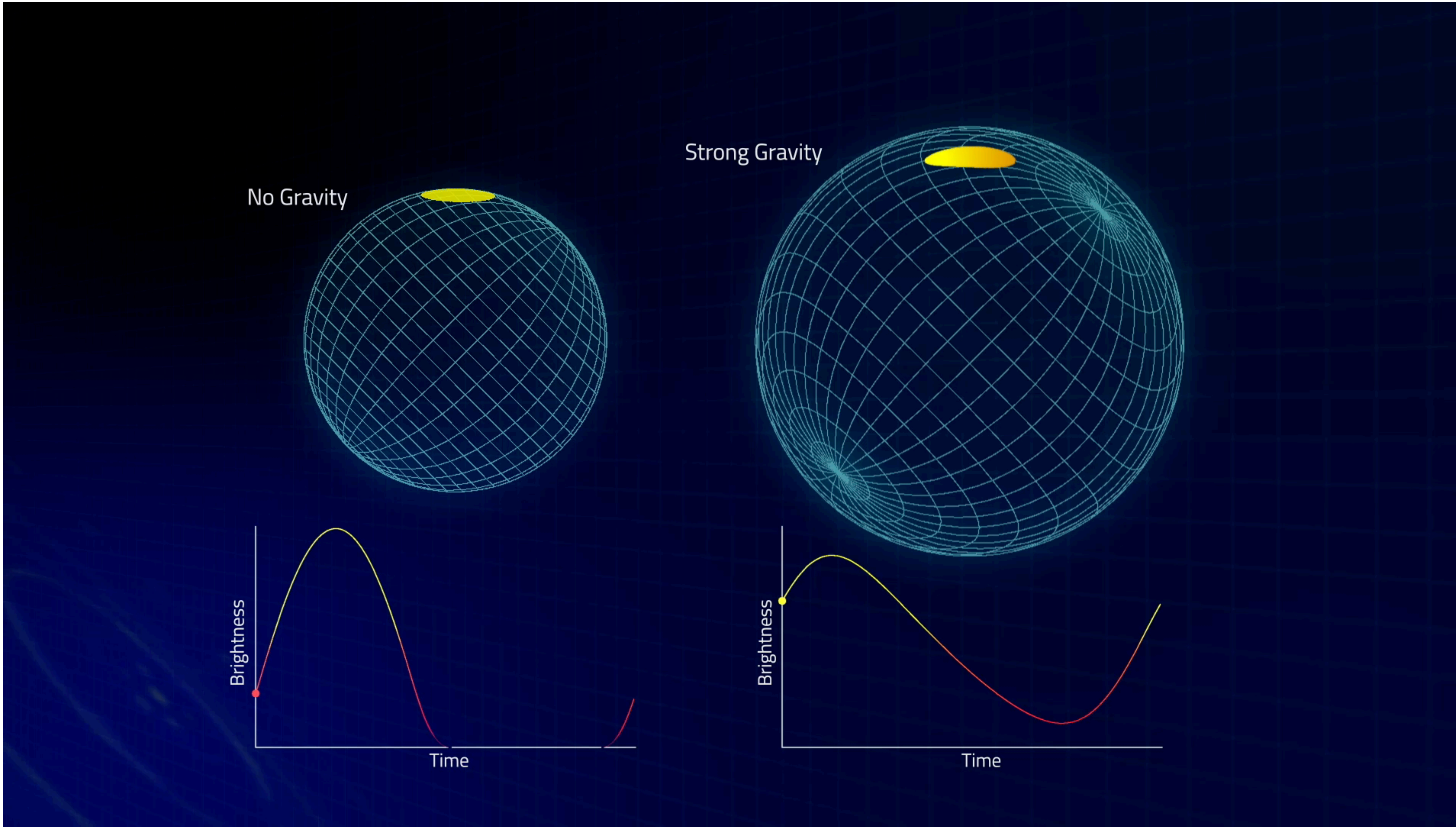

Credit: Morsink/Moir/Arzoumanian/NASA GSFC

X-rays deflected by strong gravitational field

Comments

- Properties of matter at densities less than 1–2 times nuclear density under control
- Simple models with meson condensates or quarks incompatible with measured NS properties
- Hybrid models. Phase transitions are probably artefact.
  Hadron-quark continuity
- Conventional models based on nucleon degrees of freedom
  not in conflict with data but validity questionable
  at high densities
- Masses $> 2M_\odot$ will constrain equation of state further
- Radius measurements useful

Dense Matter and Neutron Stars
Lecture 2.
Superfluidity and superconductivity of nucleons

Chris Pethick
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The Niels Bohr International Academy

59th Cracow School of Theoretical Physics
Zakopane, June 14-22, 2019

Plan

- History. Superfluidity in nuclei. Superfluidity in neutron stars.
- Glitches. Two-component models.
- Affects neutrino emission. (Reddy lectures.)
- Superfluid gaps.
- Neutron superfluid density in crust.

Superconductivity and superfluidity

- Metals. Kammerlingh Onnes (1911).
- Liquid ^{4}He superfluid. Kapitsa, Allen. (1937).
- Fritz London. Superfluidity and Bose–Einstein condensation. "Rigidity of wave function". (1938).
- Mid 1950s. Vortices in liquid helium. (Onsager, Feynman). Flux lines in superconductors. (Abrikosov).
- Weak attraction between two fermions above filled Fermi sphere gives a bound state. (Cooper, 1956)
- Many-body version.
  Bardeen, Cooper and Schrieffer, Phys. Rev. **108**, 1175 (1957).
- Energy gap in spectrum, Δ.
- Effect on total energy $\sim \Delta^2/E_F$ per particle - small compared with E_F.
- Small number of low-lying excitations leads to superfluidity.
- Elementary excitations near Fermi surface are superpositions of an electron and a hole (plus 2 electrons in condensate).

PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

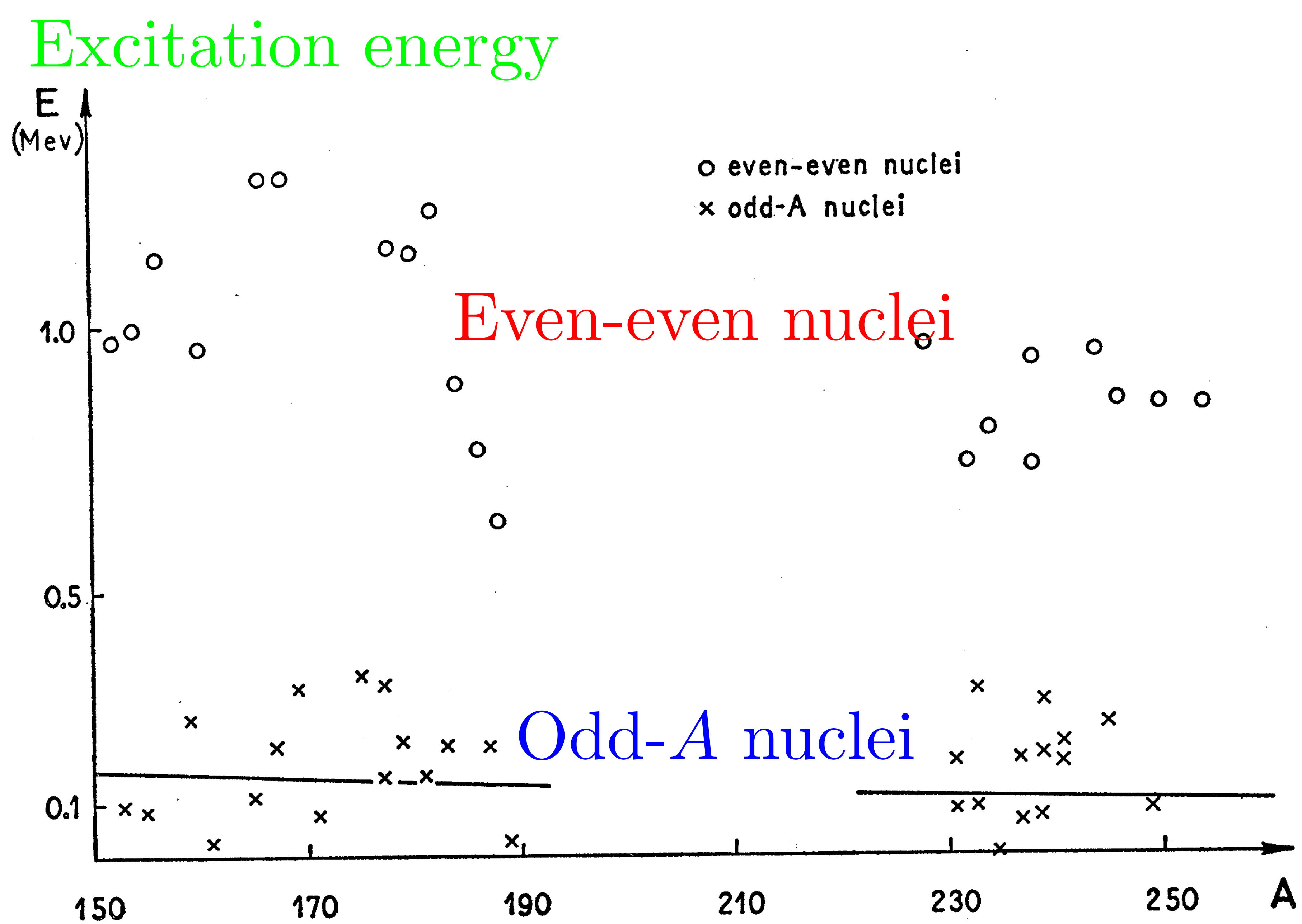
(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150 < A < 190$ and $228 < A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL

Submitted to JETP editor February 13, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 249-263 (July, 1959)

A method is developed for the treatment of superfluidity of nuclei. A formula which agrees satisfactorily with experiment is obtained for the moment of inertia of a nucleus. An expression is found for the change in the energy of "pairing" in the transition from an even-even to an even-odd nucleus, and also for the change in the moment of inertia associated with this transition.

1. INTRODUCTION

SYSTEMS consisting of interacting Fermi particles can be divided into two classes, depending on the type of excited states. When forces of repulsion prevail between the particles, the resultant excitations in the system are the same as in the case of free Fermi particles, but with an effective mass that depends on the forces of interaction between the particles. In the case of attractive forces, "correlated pairs" are formed and lead to an energy gap and to superfluidity.[1-3] One should think that the second type of single particle excitations takes place in nuclei.

In particular, this follows experimentally from the fact that the energy of the first single particle

by this method in a quasiclassical approximation and are in satisfactory agreement with the observed values of the moments of inertia.

The computed value of the moment of inertia in the transition from even-even to even-odd nucleus, and also the gyromagnetic ratio for rotating nuclei, are found to be in agreement with experiments.

These results thus confirm the assumption of the superfluidity of nuclear matter.

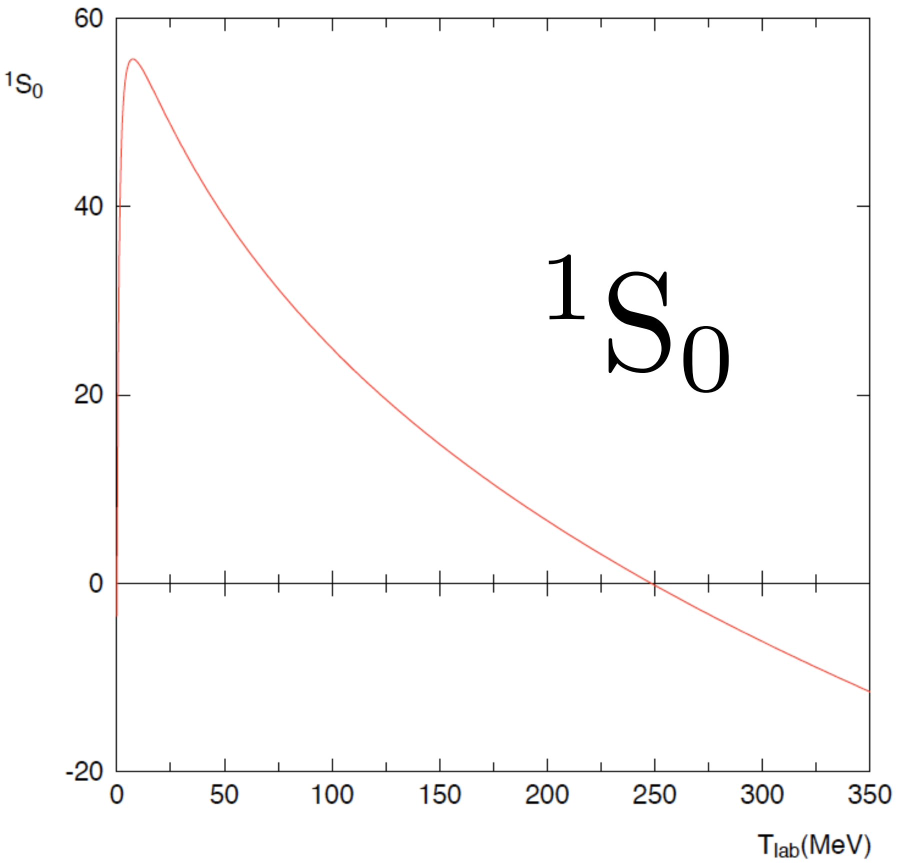
We note that the superfluidity of nuclear matter can lead to interesting macroscopic phenomena if stars with neutron cores exist. Such a star would be in a superfluid state with a transition temperature corresponding to 1 Mev.

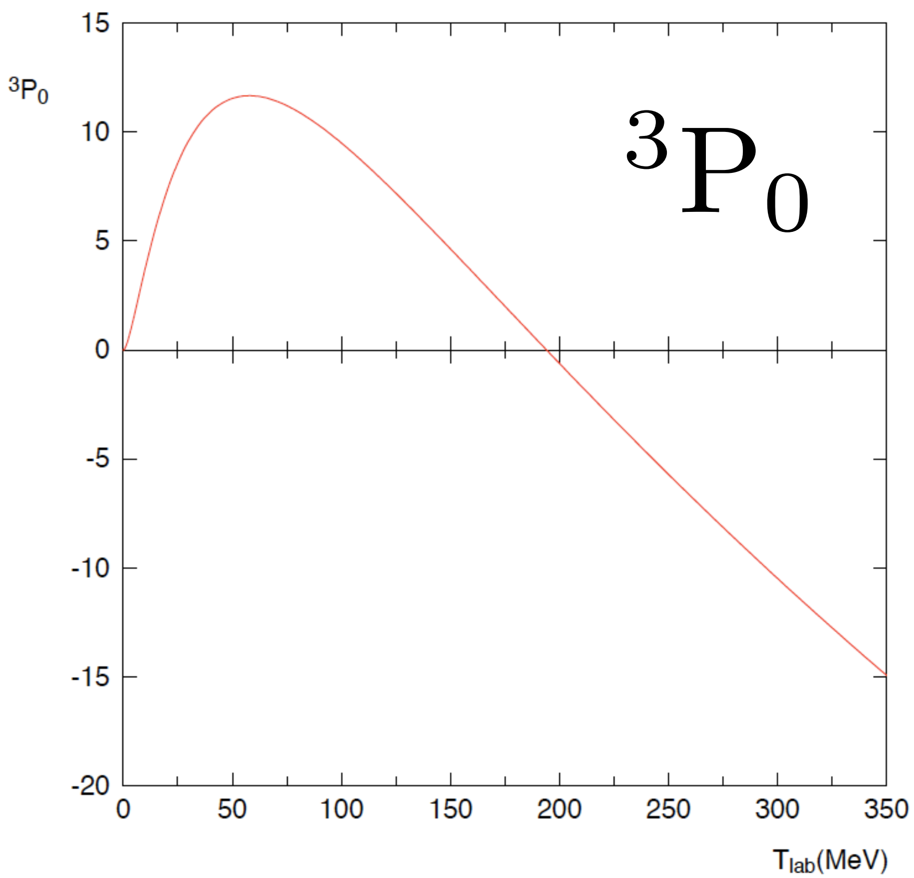
Neutron superfluidity

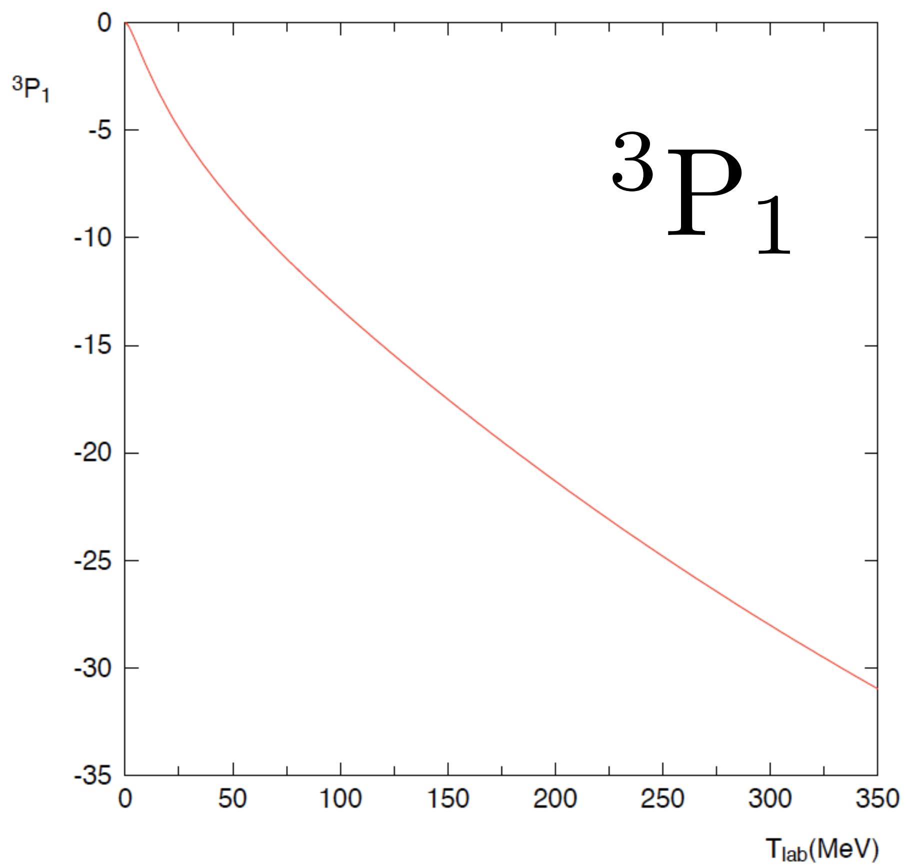
- Phase shifts suggest $^1S_0$ superfluidity (low density) and $^3P_2$-$^3F_2$ (higher density).
- Simplest approach: BCS approximation (mean field).
- Induced interactions (exchange of spin fluctuations) suppress $^1S_0$ gap.
- Inspiration from ultracold atomic gases.
- Reasonable agreement at low densities ($\lesssim n_s/10$).
  (Gor'kov and Melik-Barkhudarov (1961))
- Considerable uncertainties at higher densities.
- Calculations of proton superconductivity more uncertain
  because of the dense neutron medium.

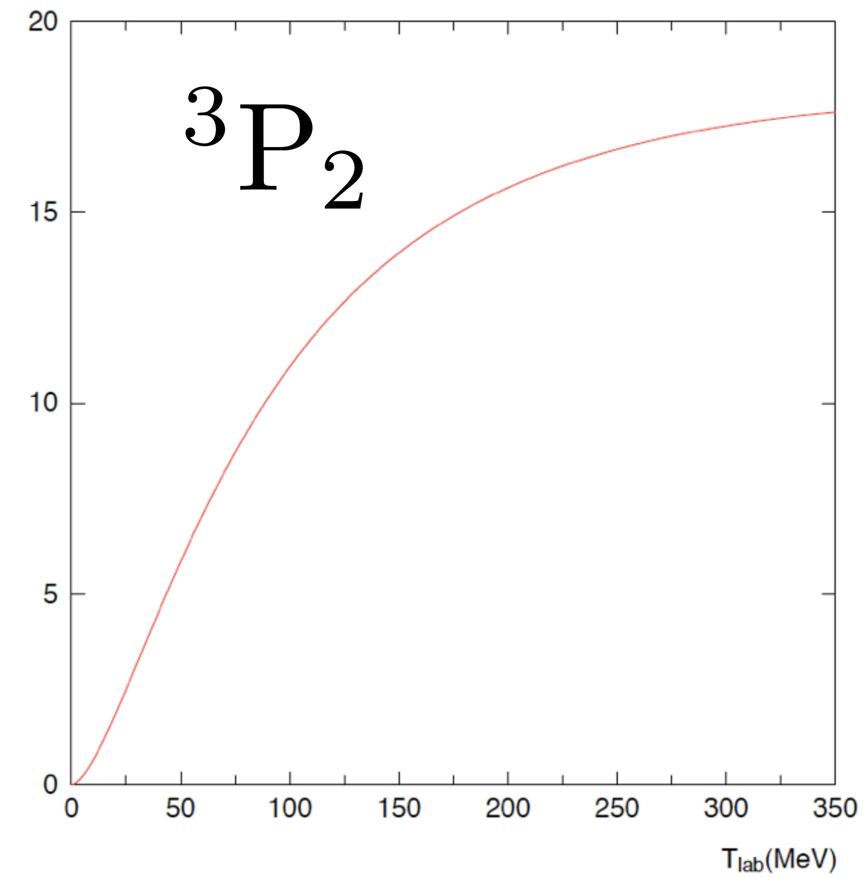
Nucleon-nucleon phase shifts (in degrees)

Positive phase shifts correspond to attraction.
(from nn-online.org, Nijmegen)



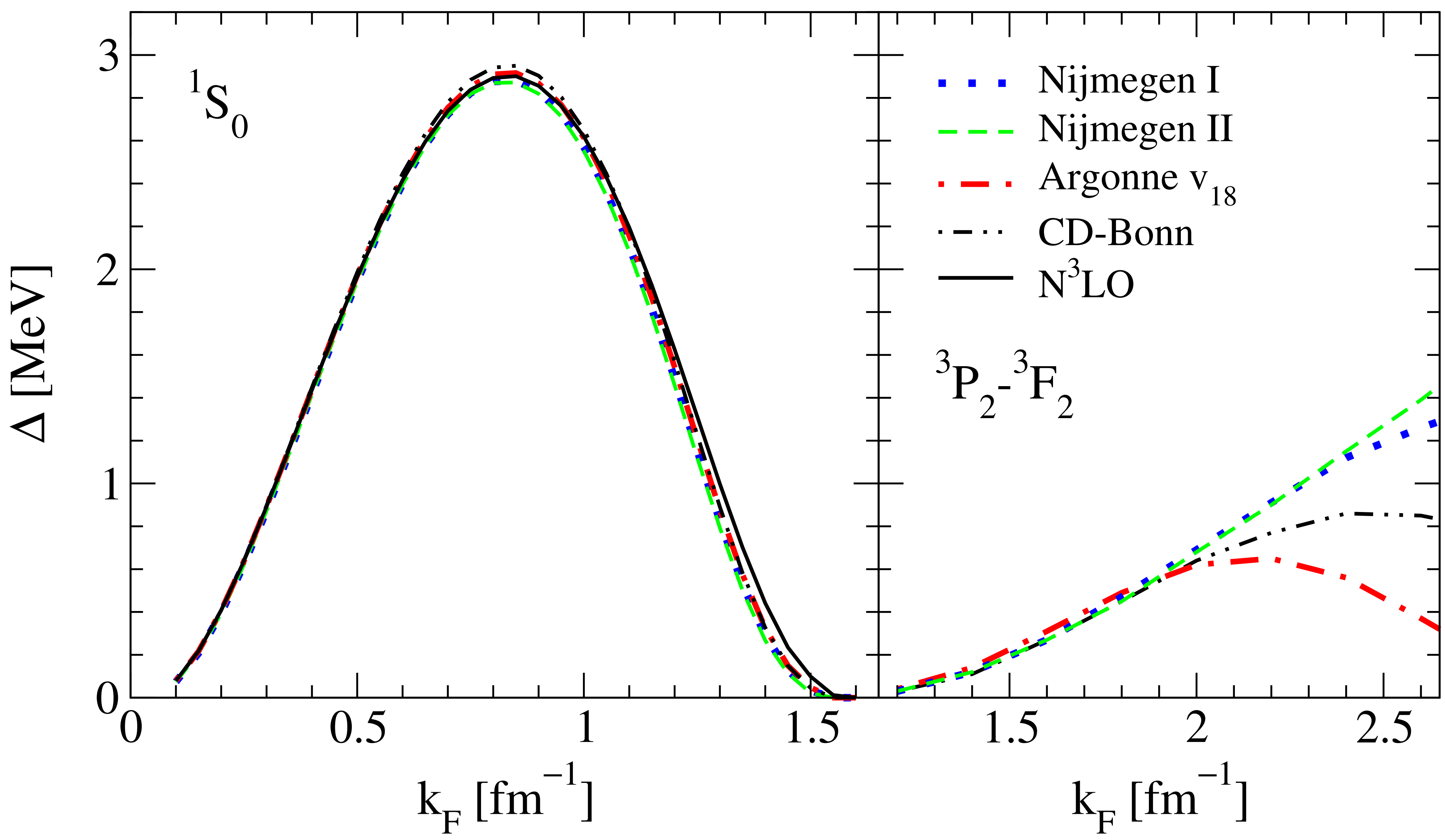






BCS Approximation

Repeated interaction of pairs of nucleons in medium via vacuum interaction



Other nucleons affect two-nucleon interaction

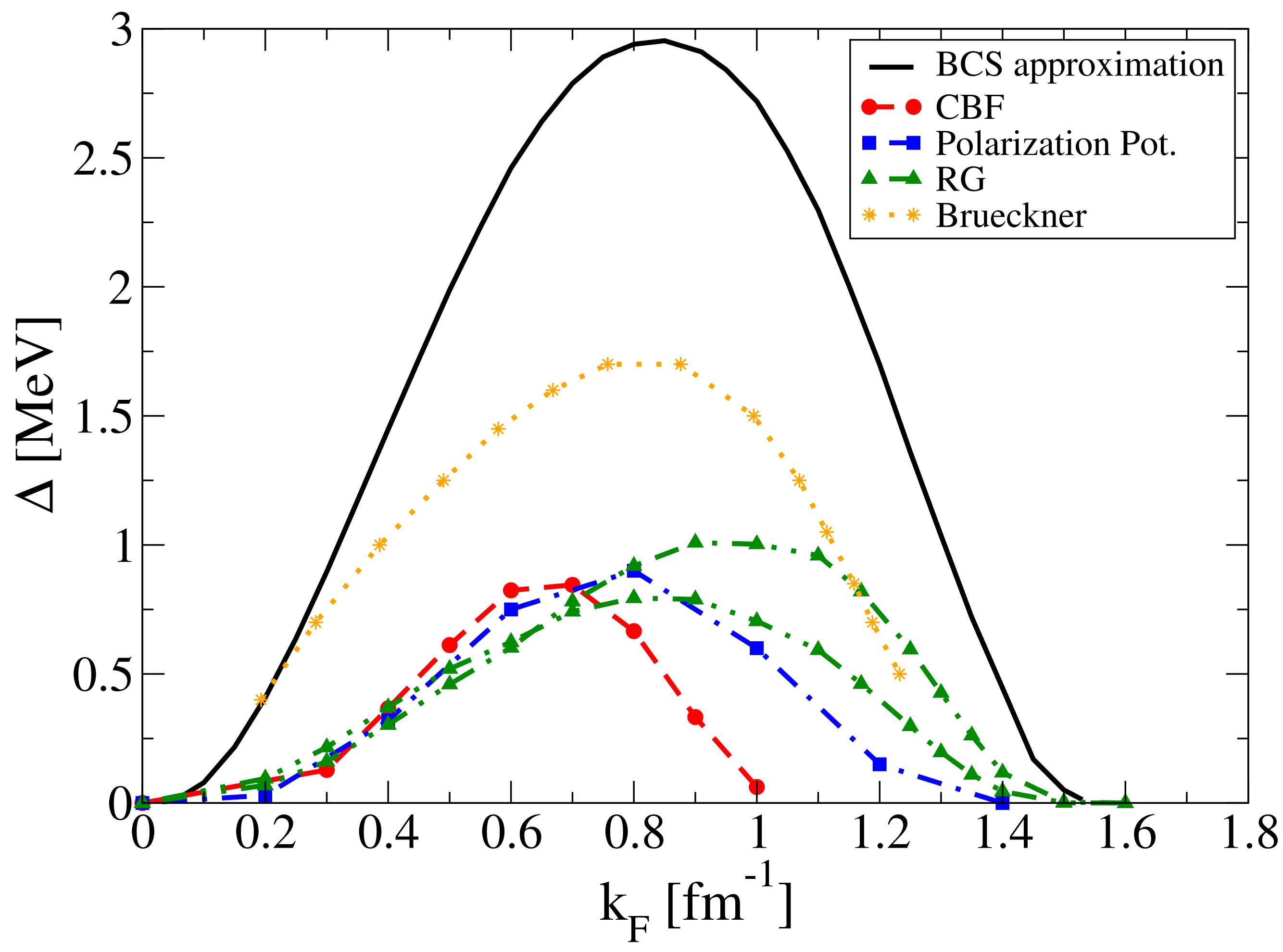


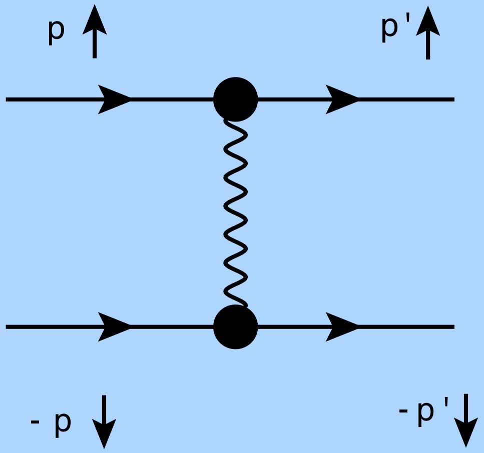
Figure 9: The $^1S_0$ pairing gap Δ at higher densities as a function of Fermi wave number k_F. Results are shown for the BCS approximation (see Fig. 7), for the method of Correlated Basis Functions (CBF) [12], for the polarization potential method, in which induced interactions are calculated in terms of pseudopotentials (Polarization Pot.) [13], for a calculation in which induced interactions in the particle–hole channels are calculated from a renormalization group (RG) approach [42], and for calculations based on Brueckner theory [46].

Nuclear matter density corresponds to $k_{Fn} = 1.68\ \mathrm{fm}^{-1}$

Gezerlis, CJP, and Schwenk, arXiv 1406.6109

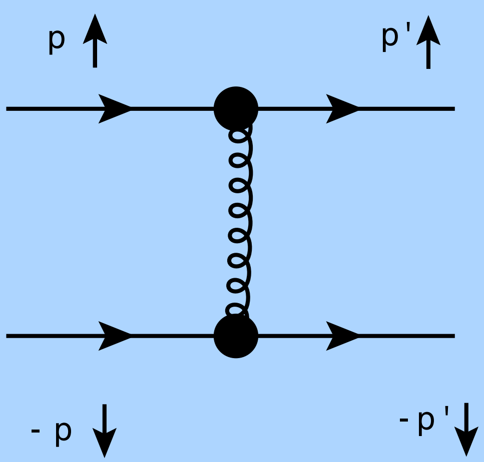
Understanding the effects of the medium

- Exchange of density fluctuations gives attraction.

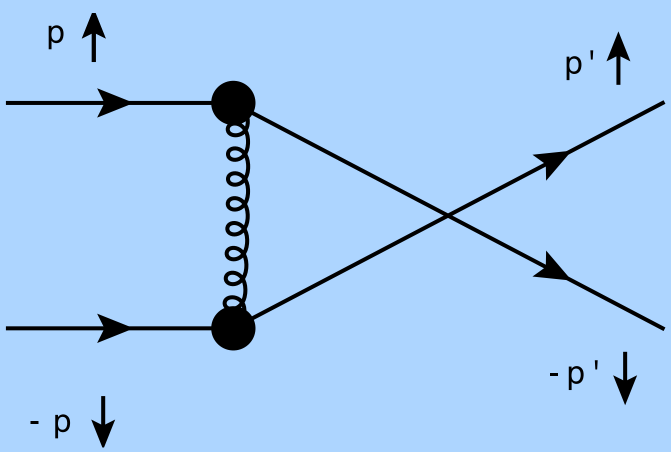


- Exchange of spin fluctuations gives repulsion.

$m_S = 0$



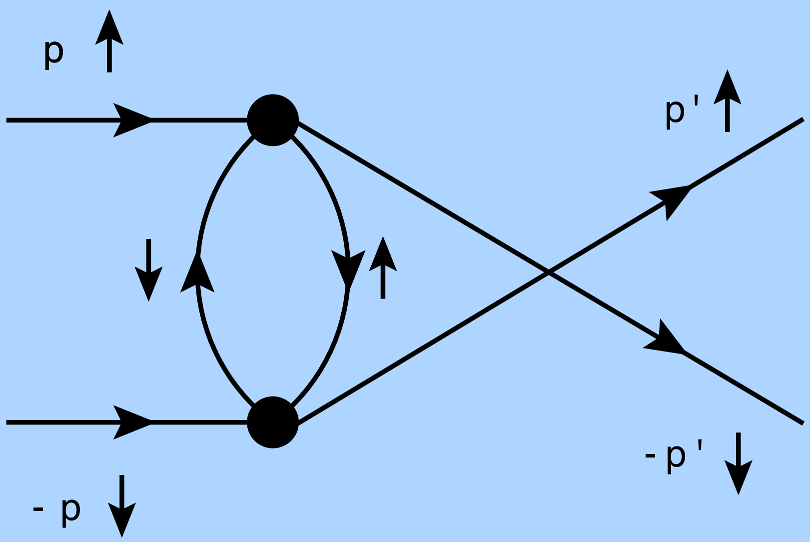
$m_S = \pm 1$



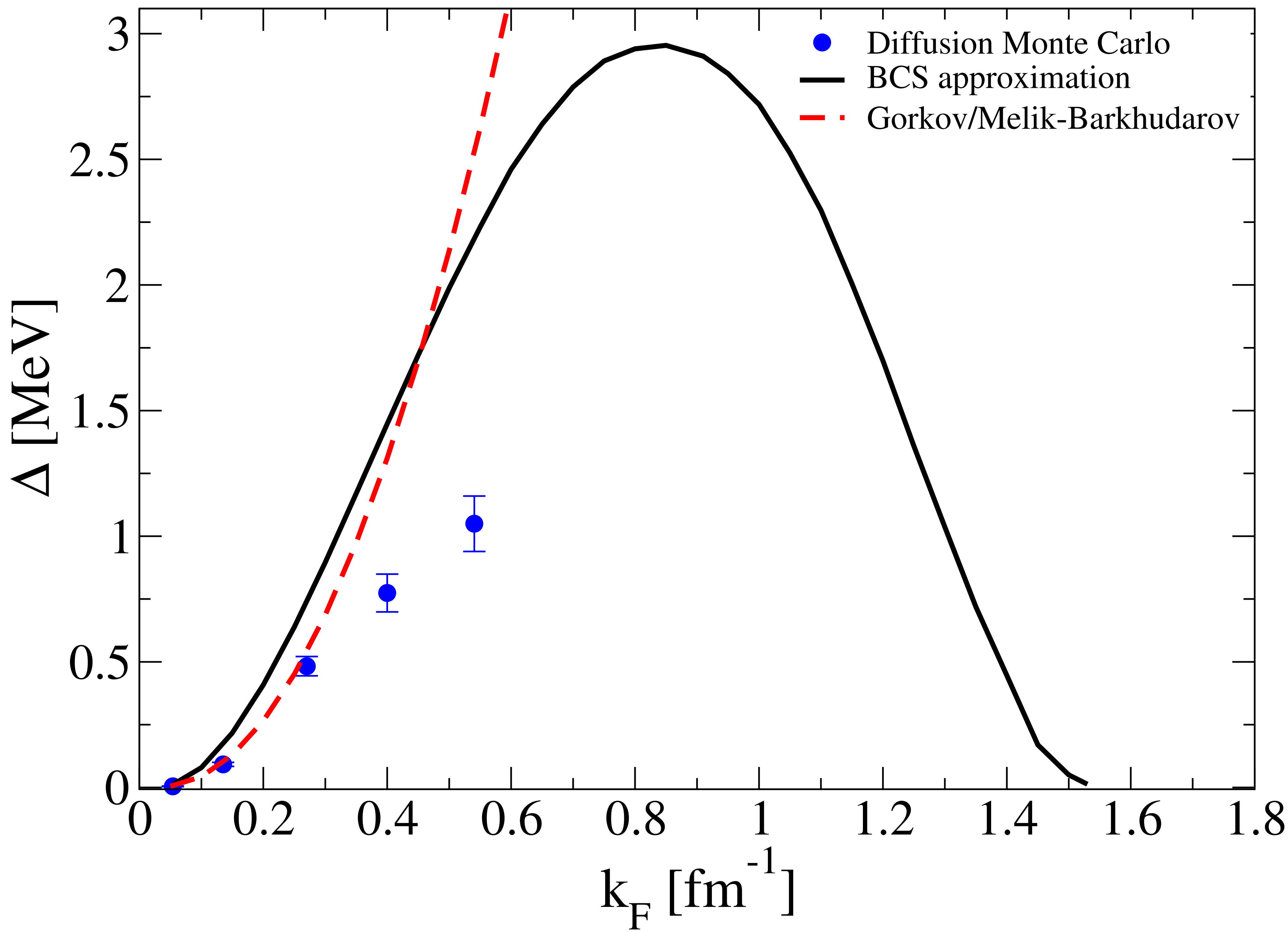
- At low densities, gap lowered, by a factor $1/(4\mathrm{e})^{1/3} \approx 0.45$.
  (Gor'kov and Melik-Barkhudarov,
  Sov. Phys. JETP **13**, 1018 (1961).)

At low densities, spin fluctuations overwhelm density fluctuations.
(3 magnetic substates for spin-1)

Net effect corresponds to the process



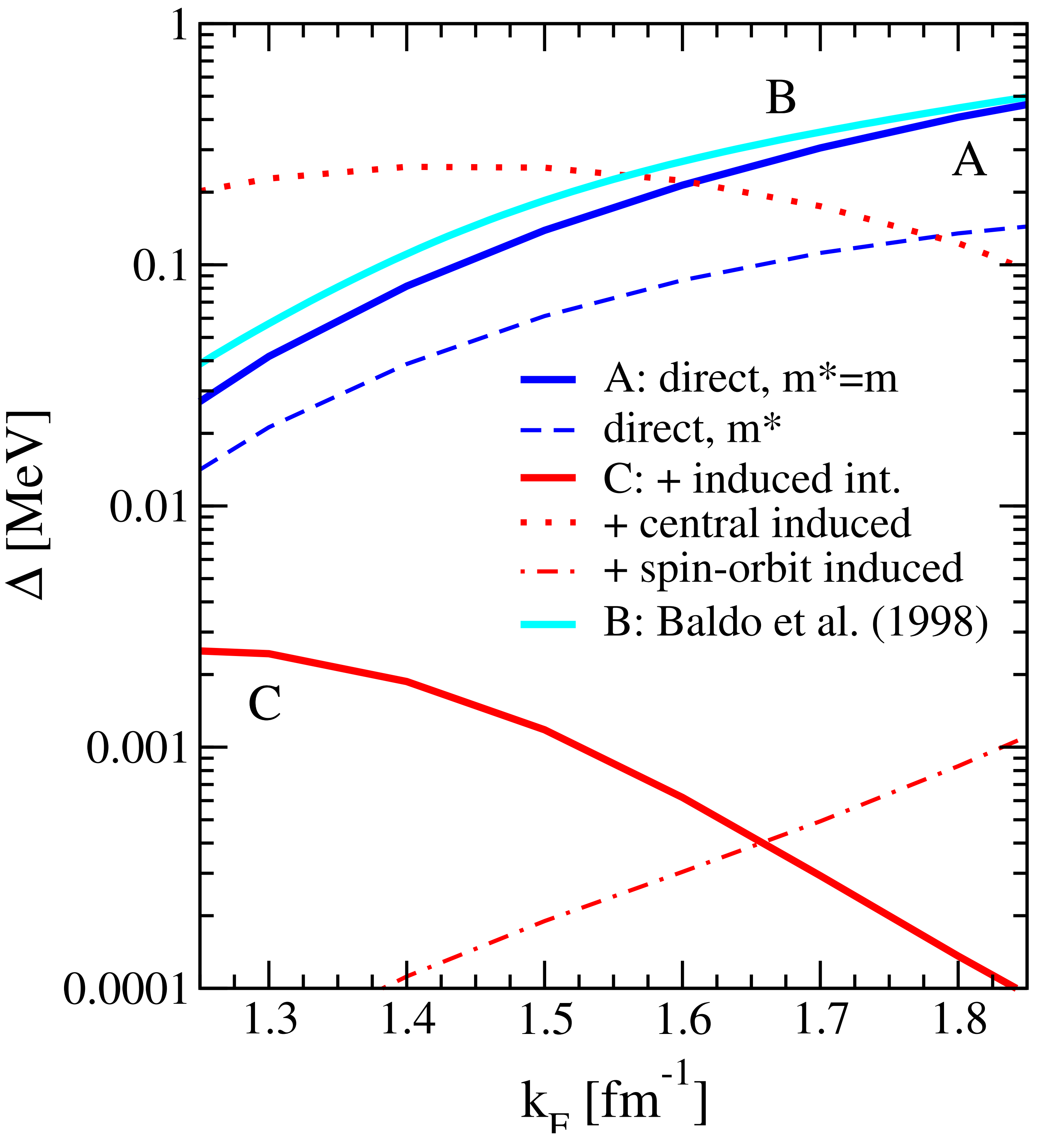
Density fluctuation exchange cancels one of the spin fluctuation channels.



Diffusion Monte Carlo
BCS approximation
Gorkov/Melik-Barkhudarov
Δ [MeV]
k_F [fm^-1]
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

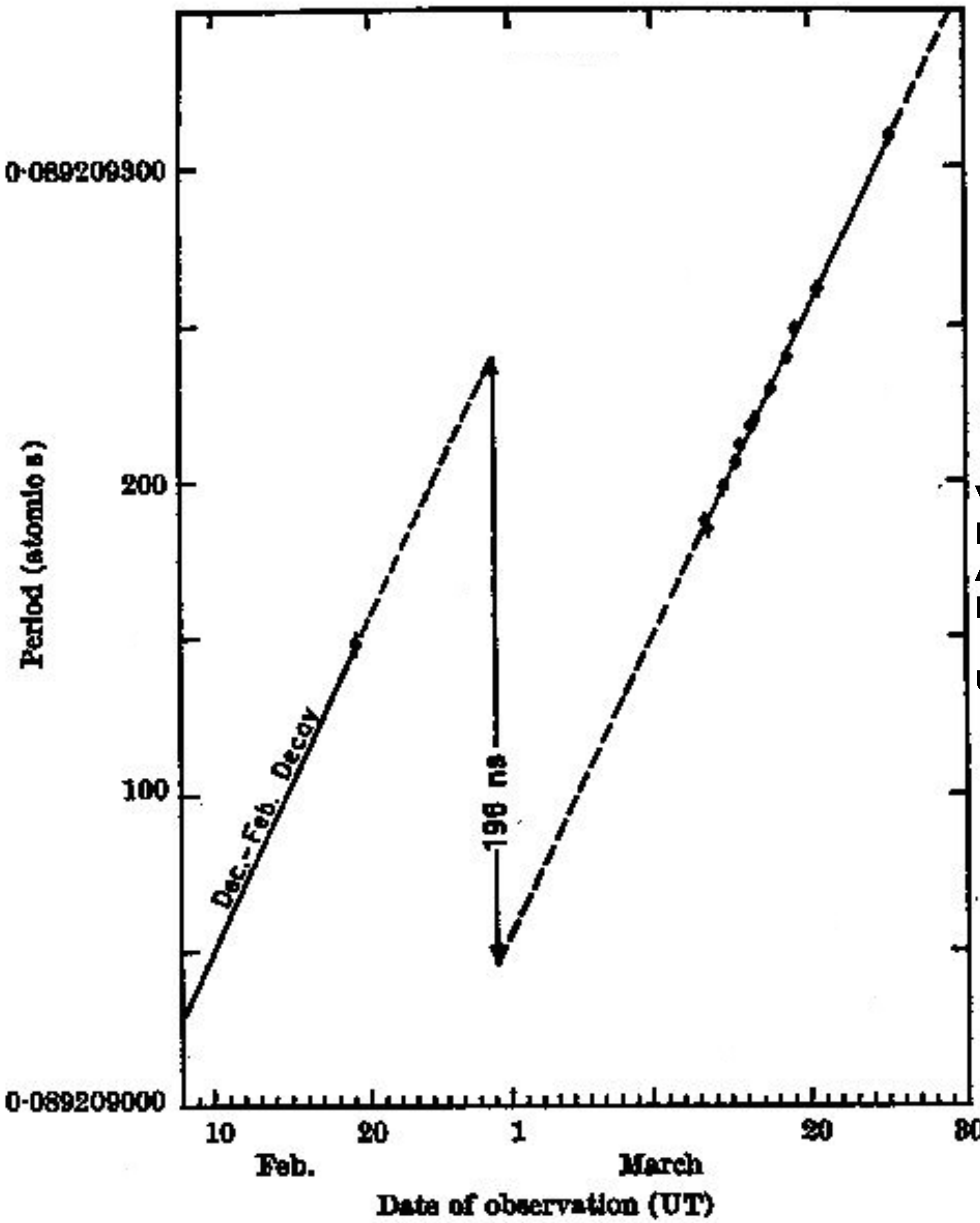
$^3P_2$-$^3F_2$ superfluidity

Inclusion of higher-order processes suppresses gaps.
(A. Schwenk and B. L. Friman, Phys. Rev. Lett. **92**, 082501 (2004).)



Glitch

Sudden speed-up of rotation of a neutron star

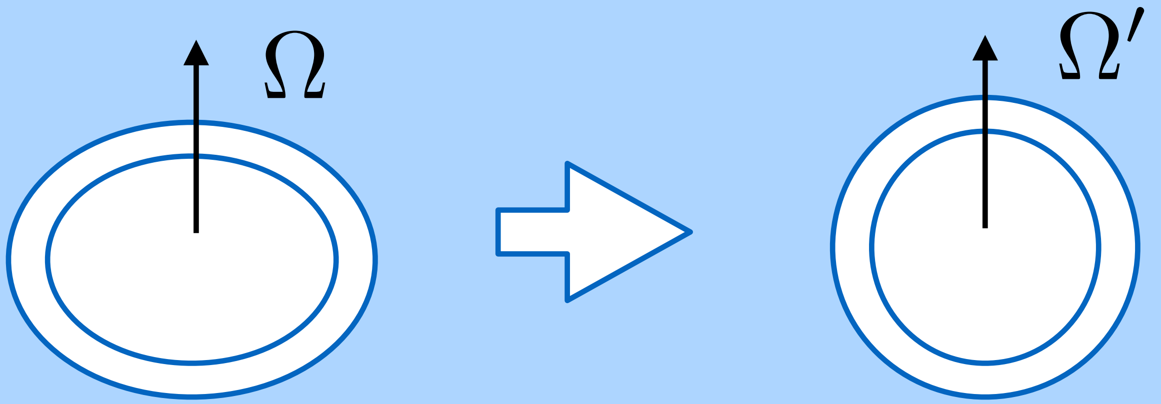


Vela pulsar, Radhakrishnan and Manchester, Nature 222, 228 (1969).
Also Reichley and Downs, Nature 222, 229 (1969).

Up to one part in 10^6

Models of glitches

- Cracking of crust, which becomes more nearly spherical. "Starquake". Moment of inertia reduced, giving spin-up.
  (Ruderman, Nature **223**, 597 (1969).)



$$I\Omega = I'\Omega'$$

- More detailed examination of glitches revealed relaxation processes with long timescale.

- Model. Crust plus superfluid neutrons, with long relaxation time between charged particles and superfluid neutrons.
  (Baym, CJP, Pines, Ruderman, Nature **224**, 872 (1969).)

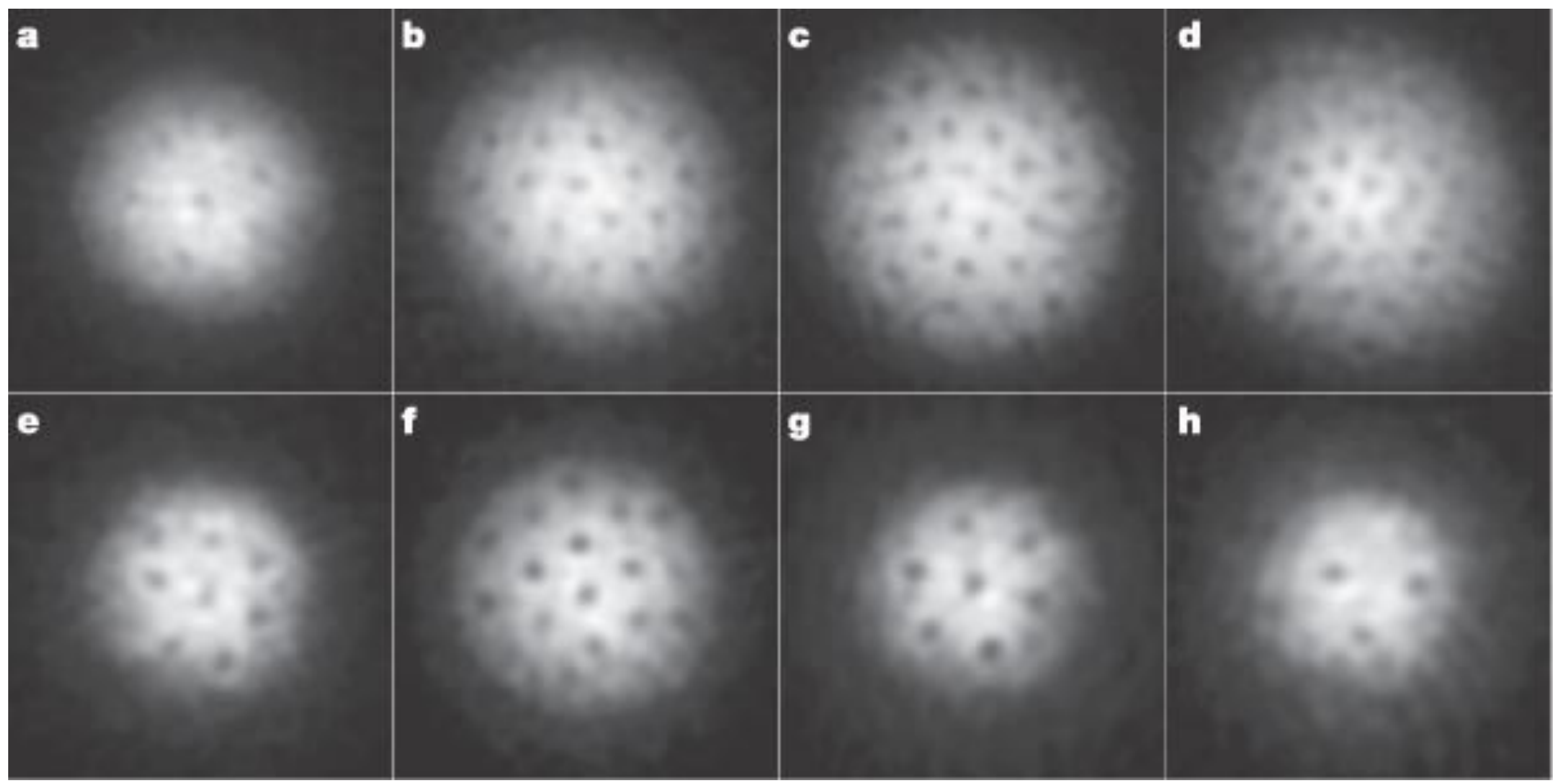
- Model not viable after many more glitches were discovered.

Introduction to superfluidity

- Paired neutrons described by a "condensate wave function".
  $\Psi(\mathbf{r}) = \langle \psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle \propto \mathrm{e}^{\mathrm{i}\boldsymbol{\Phi}(\mathbf{r})}$
- Superfluid velocity, $\mathbf{v}_s = (\hbar/2m_n)\boldsymbol{\nabla}\Phi$.
- Flow irrotational, $\boldsymbol{\nabla} \times \mathbf{v}_s = 0$ if no singularities.
- Superfluids rotate by having quantized vortices, with circulation $\oint \mathbf{v}_s \cdot d\mathbf{l} = \nu h/2m_n$, where ν is an integer. ($\nu = \pm 1$ most stable.)
- Areal density of vortices n_v given by $2\Omega = n_v h/(2m_n)$.

Seeing vortices in rotating ultracold ^{6}Li gas

Zwierlein, Abo-Shaeer, Schirotzek, Schunck and Ketterle,
Nature **435**, 1047 (2005).



Pinned vortices in the crust

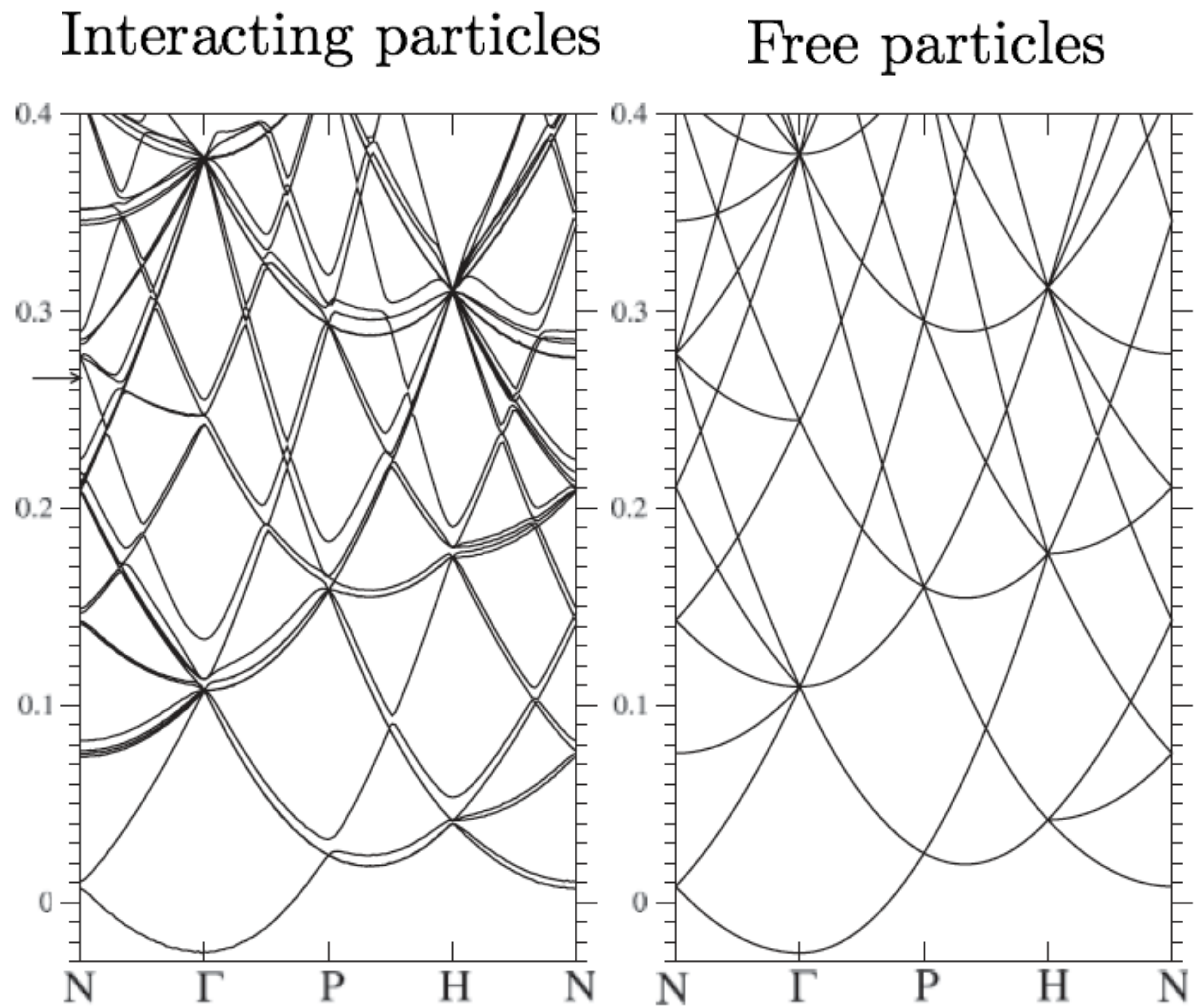
Anderson and Itoh, Nature **256**, 25 (1975).

- Vortices pin to nuclei.
- For superfluid to rotate more slowly, vortex lines must migrate to the exterior of the star.
- Prevented by pinning until force on vortex line due to differential rotation of crust and superfluid can overcome pinning.
- Sudden unpinning event.
- For idea to work, moment of inertia of superfluid much be large enough.

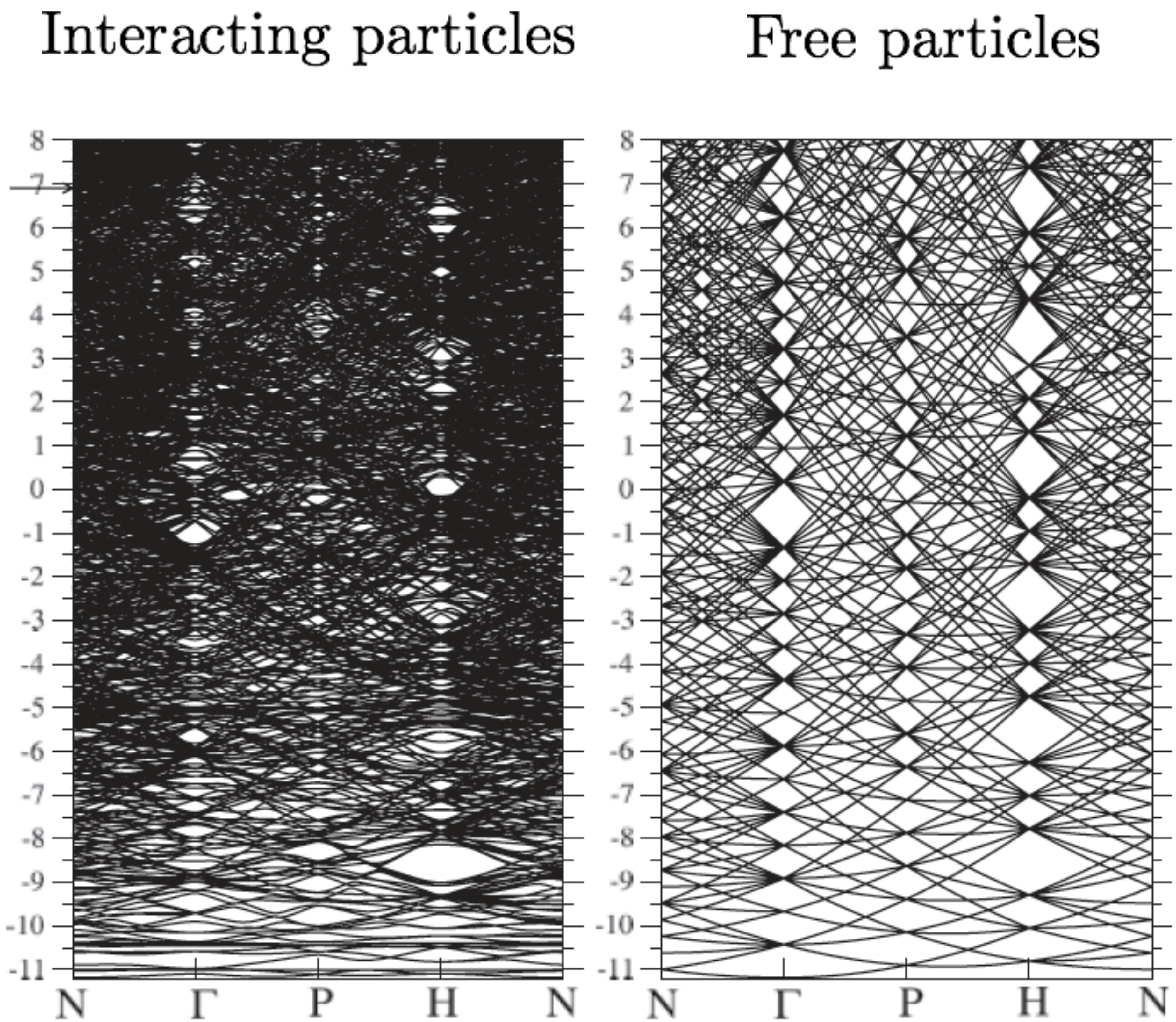
Thanks to Andrew Lyne (Jodrell Bank)

Two-component model

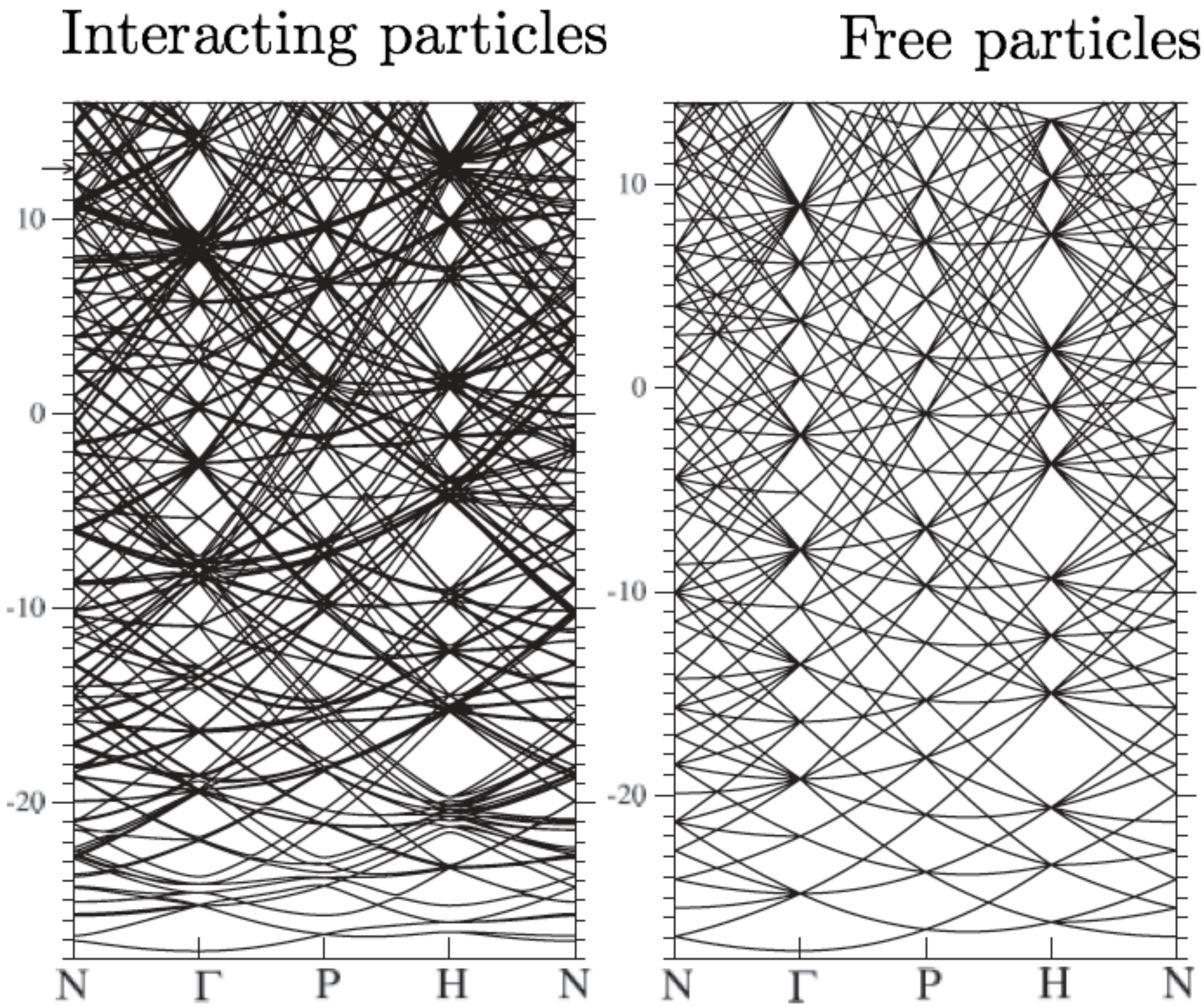
- Is mass of superfluid enough to explain glitches?
- Assume pairing weak. Difficult band structure calculation.
- Many bands (~500)! Chamel, Phys. Rev. C **85,** 035801(2012)
- Reduction of superfluid density by factor of ~10.
- Neutron superfluid density in crust too small to explain glitches (Andersson, Glampedakis, Ho, Espinoza, PRL (2012), Chamel, PRL (2013))
- Need to include both band structure and pairing.



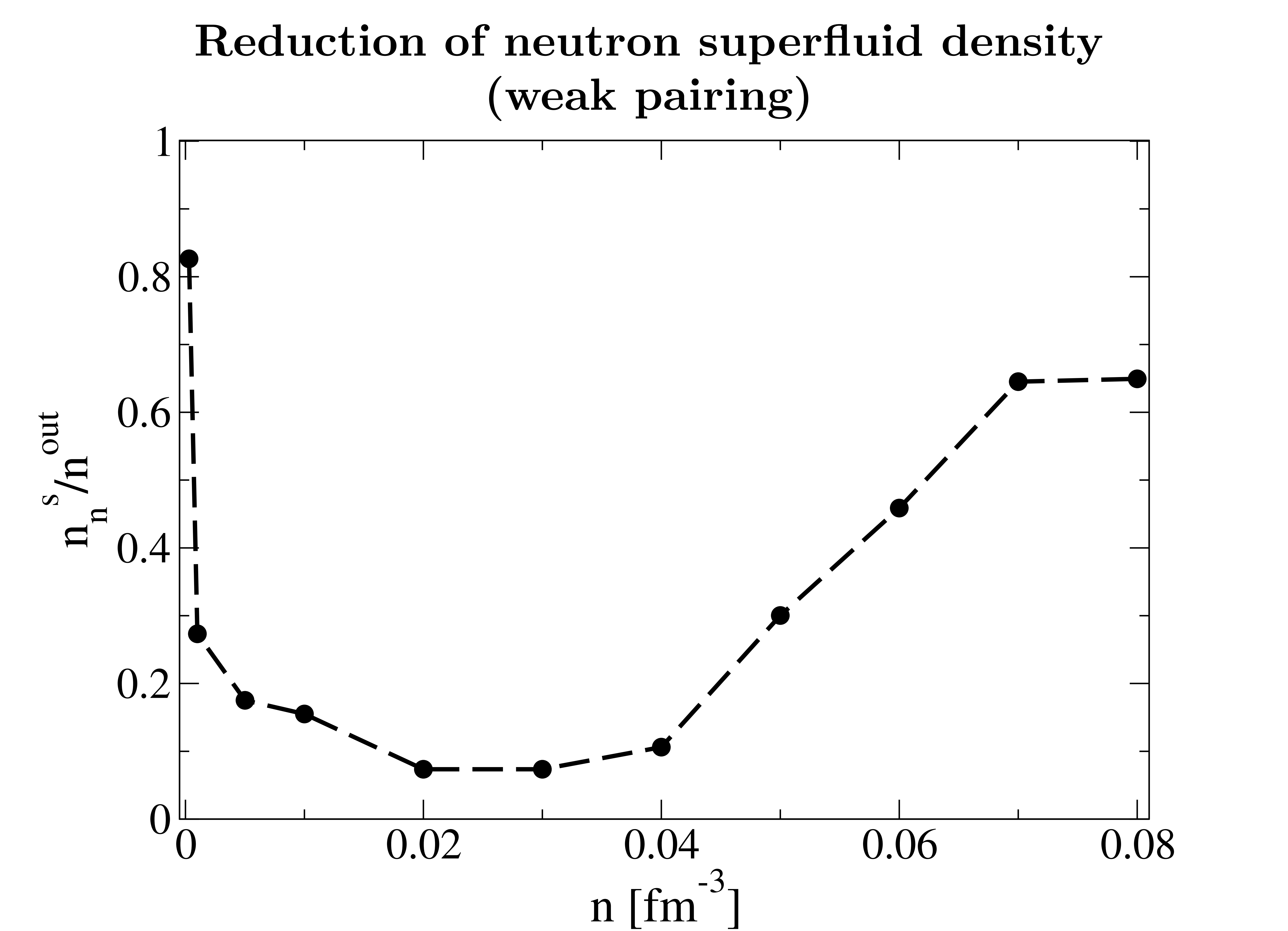
$$n = n_n + n_p = 0.0003 \text{ fm}^{-3}$$



$$n = n_n + n_p = 0.03 \text{ fm}^{-3}$$



$$n = n_n + n_p = 0.08 \text{ fm}^{-3}$$



Reduction of neutron superfluid density
(weak pairing)
n_n^s/n_n^{out}
0
0.2
0.4
0.6
0.8
1
0
0.02
0.04
0.06
0.08
n [fm^{-3}]

Simple considerations

Scattering of BCS quasiparticles by a spin-independent potential, V.

- Quasiparticles at Fermi momentum are half particles and half holes. Interaction of particle component exactly cancels that of hole component.
- NO SCATTERING OF EXCITATION AT THE FERMI MOMENTUM TO ANOTHER STATE WITH THE SAME ENERGY!
- Quasiparticle energy
  $E_k = \pm\sqrt{\xi_k^2 + \Delta^2}$ where $\xi_k = k^2/2m - \mu$
- Matrix element for scattering of quasiparticle from state $|\mathbf{k}+\rangle$ to state $|\mathbf{k}'+\rangle$

$$(u_{\mathbf{k}} u_{\mathbf{k}'} - v_{\mathbf{k}} v_{\mathbf{k}'}) V(\mathbf{k} - \mathbf{k}')$$

where $u_{\mathbf{k}}^2 = (1 + \xi_k/E_k)/2$ and $v_{\mathbf{k}}^2 = (1 - \xi_k/E_k)/2$.

Pairing and band structure

• Cold atoms in a sinusoidal potential.

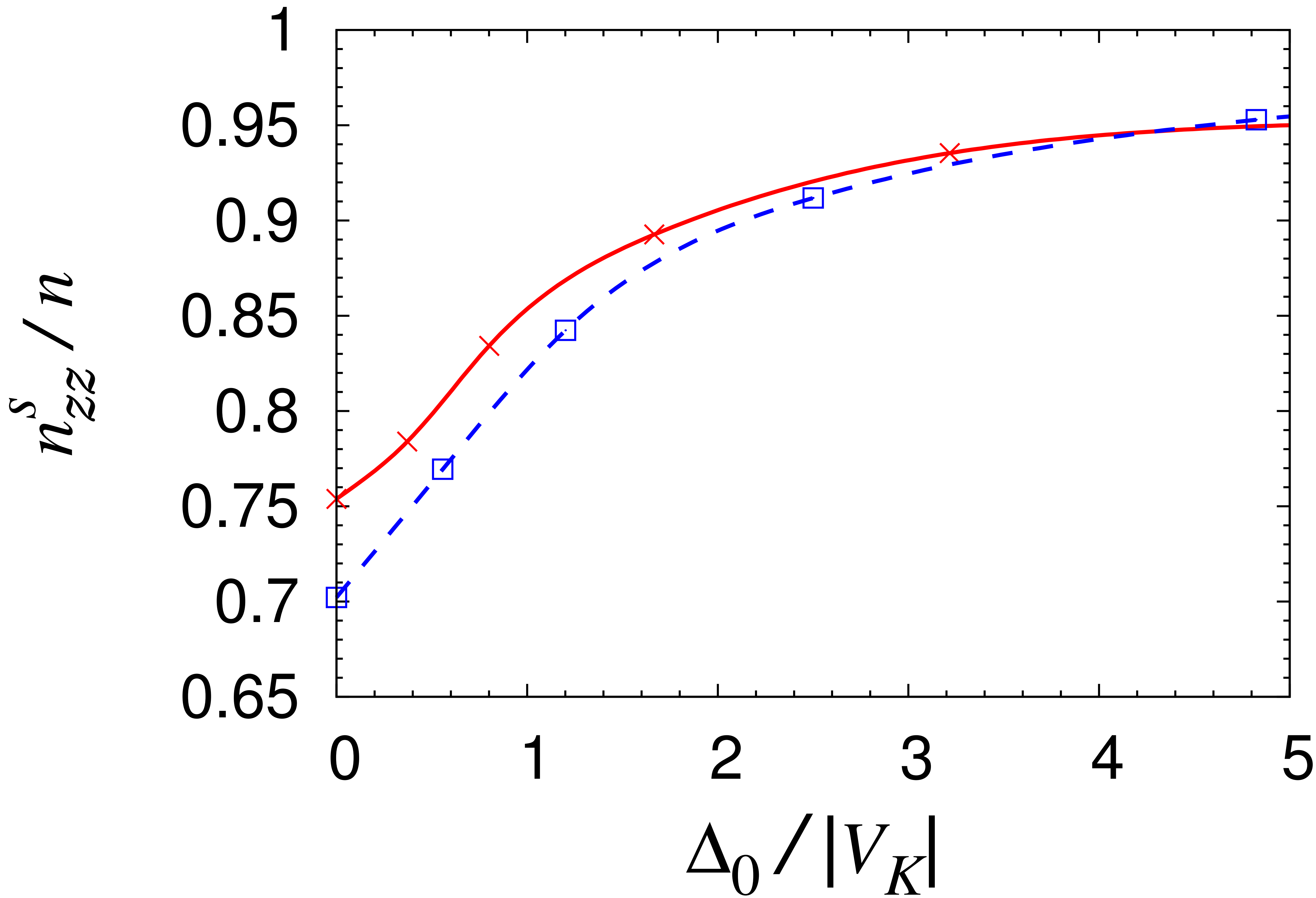
• Include pairing and periodic potential (Hartree-Fock-Bogoliubov).
(G. Watanabe et al., Phys. Rev. A**78**, 063619 (2008))

• Superfluid density little affected if superfluid gap is larger than the strength of the periodic potential. Confirms simple argument.

• In neutron star crust, superfluid gaps are ~ 1-1.5 MeV.

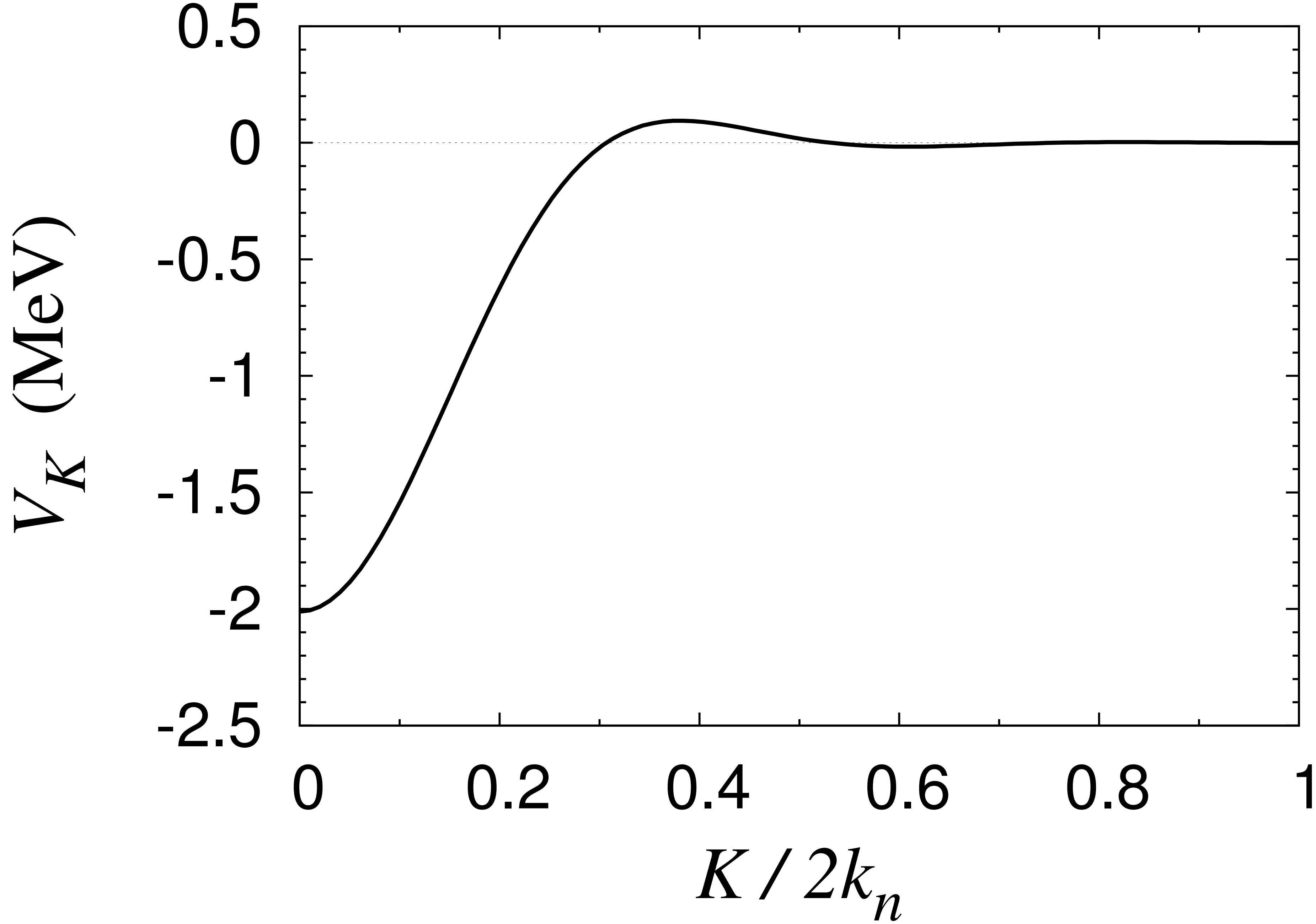
• Strength of periodic potential less than ~ 1 MeV for most reciprocal lattice vectors.

Suppression of band structure effects by pairing



G. Watanabe and CJP, Phys. Rev. Lett. **119**, 062701 (2017).

Fourier transform of potential of nucleus (Chamel)



Conclusions

- Approximate way of including many reciprocal lattice vectors.
- Conclude that superfluid density could be reduced by tens of per cent but not by an order of magnitude.
- Two-component glitch model still viable!
- Pinning needs to be understood better.

Dense Matter and Neutron Stars
Lecture 3.
Pasta phases

Chris Pethick
(Niels Bohr International Academy, Copenhagen and NORDITA, Stockholm)

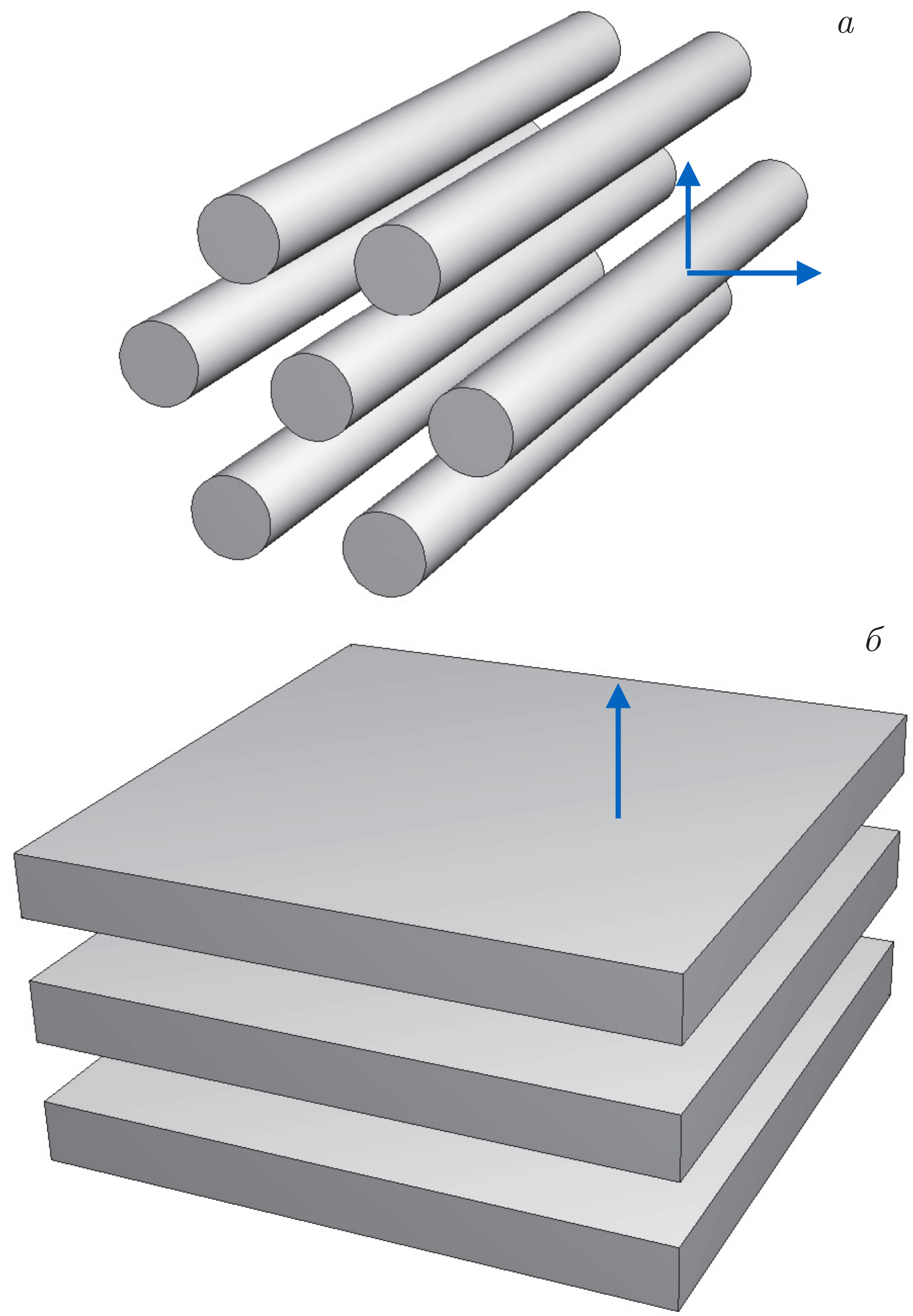
59th Cracow School of Theoretical Physics
Zakopane, June 14-22, 2019

What happens to matter when compressed?

- Electrons become simple
- Matter becomes more rich in neutrons
- Neutron “drip” out of nuclei
- “Solid state” effects (lattice energy) important
- “Inside-out” nuclei
- “Pasta nuclei” (Geoff Ravenhall)

D. G. Ravenhall, CJP, and J. R. Wilson, PRL **50**, 2066 (1983).
M. Hashimoto, H. Seki, and M. Yamada, Prog. Theor. Phys. **71**,320 (1984).

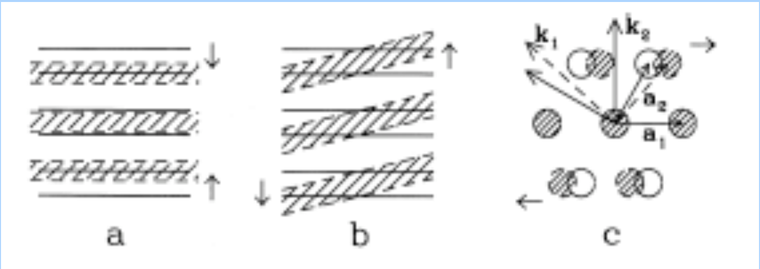
Elastic properties



2-dimensional displacement

1-dimensional displacement

Elastic energy of lasagna



- Elastic energy/unit volume:

$$E_{\mathrm{elast}} = \frac{1}{2} B \left(\frac{\partial u_z}{\partial z} \right)^2$$

($\partial u_z / \partial z$ is the fractional change in the spacing of layers)

- $B = 6E_{\mathrm{Coul}}$, where E_{Coul} is the Coulomb energy per unit volume. (Potekhin and CJP, Phys. Lett. B **427**, 7 (1998).)

- $E_{\mathrm{surf}} = 2E_{\mathrm{Coul}}$ gives

$$B = 6^{2/3}\pi^{1/3}[n_p \sigma e w(1-w)]^{2/3} \approx 2.6\times 10^{33} \left[\frac{n_p}{n_s} \frac{\sigma}{1 \text{ MeV fm}^{-3}} w(1-w) \right]^{2/3} \text{erg cm}^{-3}$$

w - fraction of space filled by nuclear matter, σ - surface tension, and n_p - proton density.

Elastic properties of polycrystals

- Important for stellar oscillations.
  Duncan, (1998), Strohmeyer and Watts, (2006)

- Polycrystal behaves as isotropic system at long wavelengths.

- Estimating elastic constants.

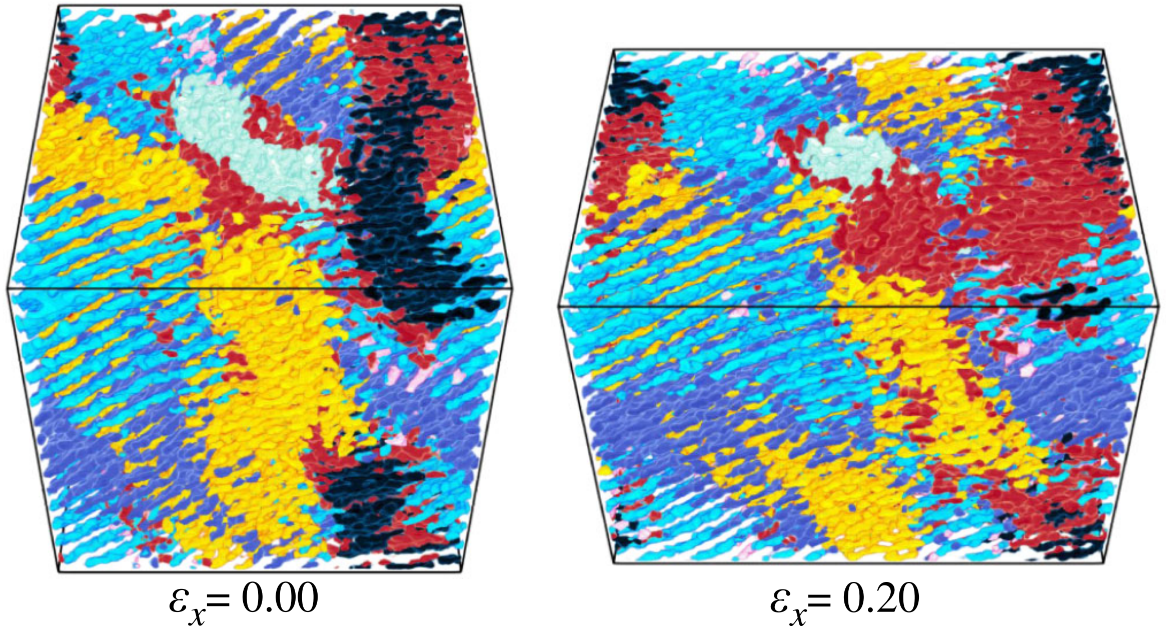
  - Average elastic properties over wave vectors and polarizations.
    Like resistances in series.
    Voigt (1887) Ogata and Ichimaru (1990) (Upper bound)

  - *Harmonic* average of elastic properties over wave vectors and polarisations.
    Like resistances in parallel.
    Reuss (1929) (Lower bound)

  - Self-consistent models give results between limits. Kröner, Eshelby, For astrophysical solids, Kobyakov and CJP, MNRAS **449**, L110 (2015).

Polycrystalline lasagna

- Molecular dynamics simulations of polycrystalline lasagna. Caplan, Schneider, Horowitz, PRL **121**, 132701 (2018).



- Effective shear modulus μ_{eff} of order 10^{30}-10^{31} erg cm^{-3}

- Analytical estimate. Voigt average.

$$\mu_{\text{eff}} = \frac{B}{15},$$

  which is of a similar order of magnitude. (Kobyakov and CJP)

- Power of analytical methods.

Comments

- Reuss average vanishes. (Inverse of elastic constant tensor singular.)
- Need to make self-consistent calculations
- Has been done for bcc solid. 28 % reduction compared with Voigt average
- Complication (or simplification?). Lasagna is modulated!

Pasta phases resemble liquid crystals

- Similar sequences occur in other systems (block copolymers)
- BUT neutrons and protons are superfluid
- Surface and Coulomb energies small compared with bulk energies.
- Low-frequency modes affected by superfluidity.
- Ordinary liquid crystals. No “permeation”. Pattern moves with fluid.
- Not generally the case for pasta phases

Illustrative example: one-component lasagna

- Two-fluid model. Normal component moves with structure. Linearize. Superfluid current proportional to $\boldsymbol{\nabla}\Phi$. Current density

$$\mathbf{j} = n^n \dot{\mathbf{u}}_\parallel + n^s \frac{\hbar}{2m} \boldsymbol{\nabla}_\parallel \Phi + n \frac{\hbar}{2m} \boldsymbol{\nabla}_\perp \Phi$$

- "Potential" energy

$$E_{\rm pot} = \frac{1}{2} B \left(\frac{\partial u_z}{\partial z} \right)^2 + \frac{1}{2} \frac{\partial^2 E}{\partial n^2} (\delta n)^2 + C \delta n \frac{\partial u_z}{\partial z}$$

- B involves surface and Coulomb energies. $B \ll \partial^2 E / \partial n^2$

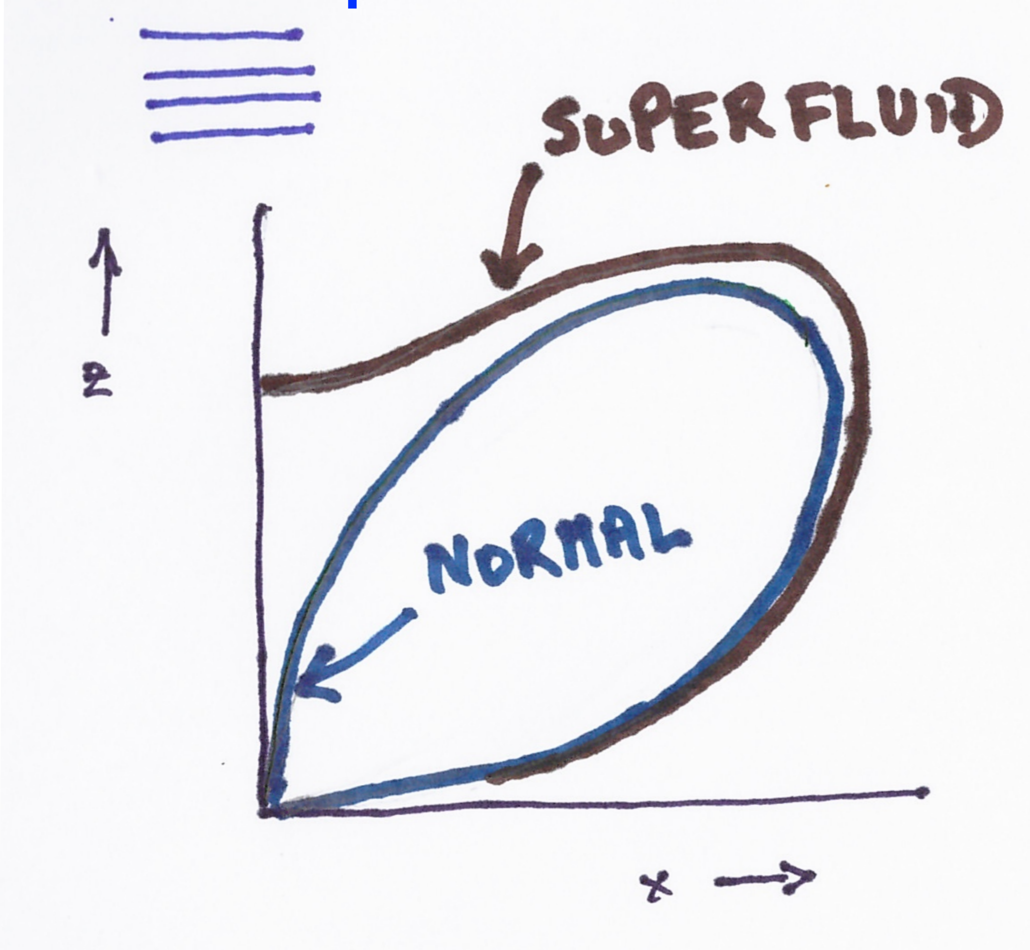
Equations of motion

- Resemble those of a perfect fluid
- Hydrodynamics works because superfluid particles described by a density and a velocity. Here it is a result of "molecular fields", not collisions
- Particle conservation
$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j} = 0$$
- Momentum conservation
$$\begin{aligned} m\frac{\partial \mathbf{j}}{\partial t} &= \text{Pressure force density} + \text{Elastic force density} \\ &= -\boldsymbol{\nabla} p - \frac{\delta E_{\mathrm{pot}}}{\delta \mathbf{u}} \end{aligned}$$
- $\Psi(\mathbf{r}, t) = \langle N-2|\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})|N\rangle \propto \mathrm{e}^{-\mathrm{i}(E_N - E_{N-2})t}$. $E_N - E_{N-2} = 2\mu$. Josephson equation.
$$\frac{\partial \Phi}{\partial t} = -\frac{2\mu}{\hbar}$$
- Take gradient: superfluid acceleration equation
$$m\frac{\partial \mathbf{v}_s}{\partial t} = -\boldsymbol{\nabla}\mu$$

Collective mode velocity

Lasagna

Polar plot of mode velocity



One component

- Liquid crystal. No "permeation".
- Structure moves with local fluid velocity.
- Mode velocity along z-axis zero.

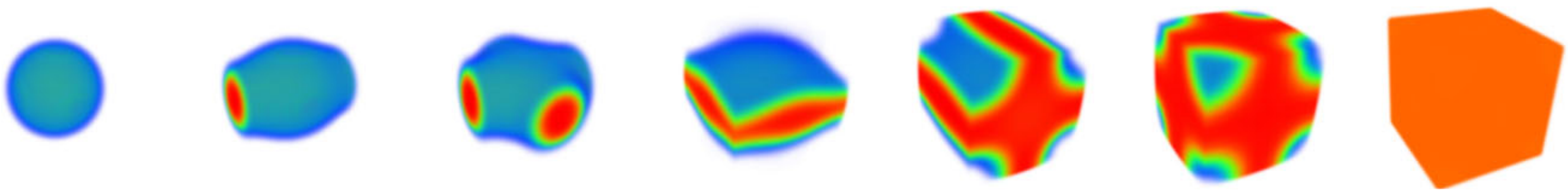
- SF liquid crystal. Permeation occurs.
- Counterflow of normal and superfluid possible.
- Mode velocity along z-axis nonzero.

Also high velocity sound mode

Dmitry Kobyakov and CJP, JETP **127**, 851(2018)
D. Durel and M. Urban, Phys. Rev. C **97**, 065805 (2018)

But pasta is not uniform …

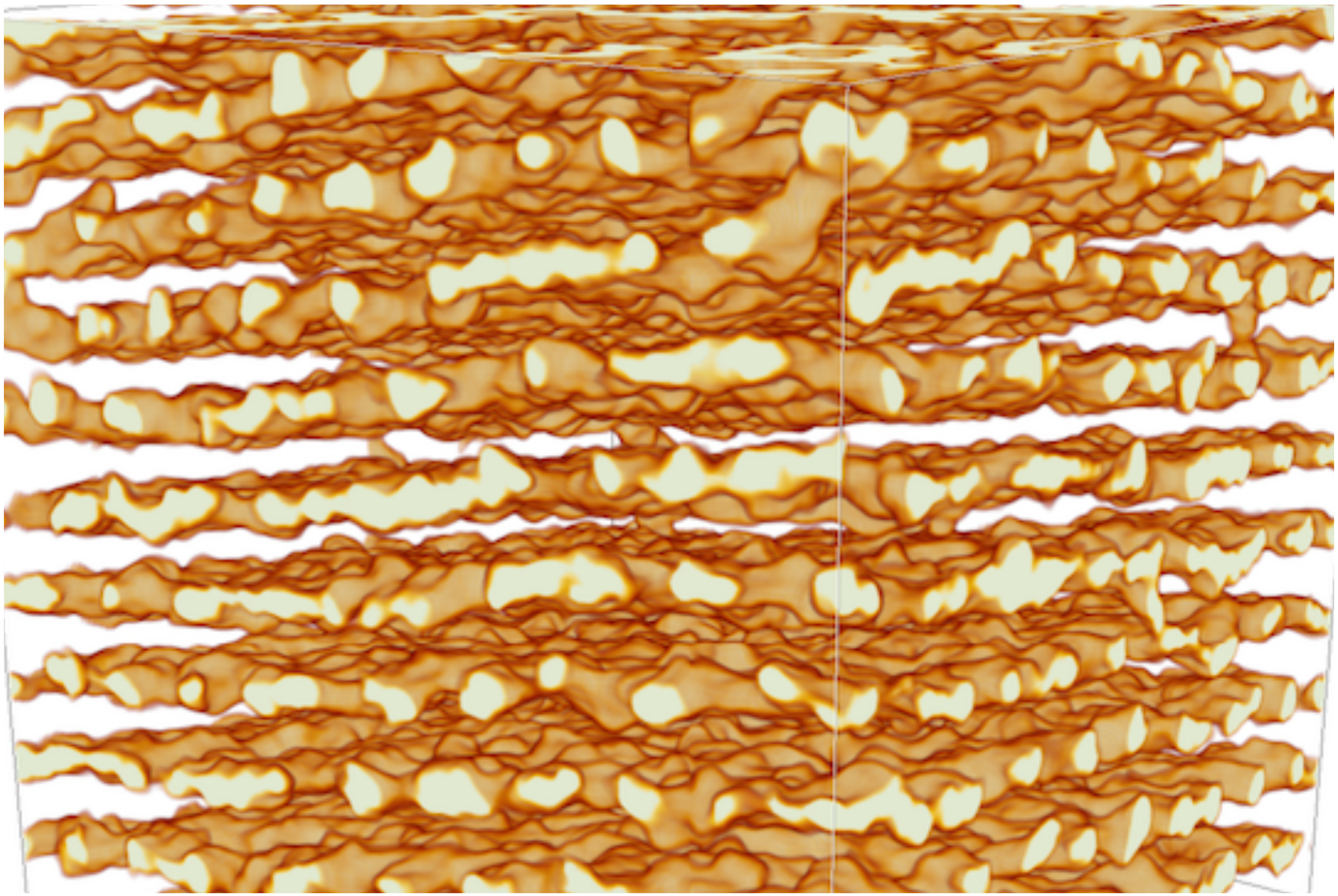
ρ [fm^{-3}]						
0.020	0.032(2)	0.042(2)	0.052(2)	0.090(2)	0.108(2)	0.114(1)



(Red - high density, blue - low density)

H. Pais and J. R. Stone PRL **109**, 151101 (2012)

… and is expected to have defects.



Nuclear waffles
(molecular dynamics simulation)

A. S. Schneider, D. K. Berry, C. M. Briggs, M. E. Caplan, C. J. Horowitz,
Phys. Rev. C **90**, 055805 (2014)

More general approach

- Uniform lasagna (spaghetti) - describe by displacements in 1 (2) directions
- More generally need 3 component vector, ***u***
- More terms in elastic energy
- Two superfluids (neutrons and protons), + normal component (lattice)
- Three more modes: extra phase plus two extra components of **u**

- Three-fluid model. Neutron and proton superfluids, normal fluid associated with lattice.
- Anisotropic.
- Backflow (Entrainment).
- 5 collective modes.
- Surface and Coulomb energies small compared with bulk energies.
- 2 compressional modes (high velocity).
- 3 low-velocity modes, one purely transverse, 2 mixed transverse and longitudinal
  (D. Kobyakov and CJP, in preparation)

Overview of results

- Generally, mode velocity is nonzero in all directions
- Exception. If proton superfluid density vanishes for flow perpendicular to the sheets. In this case the velocity of the mode involving changing the spacing between lasagna sheets vanishes.
- Need better understanding of the proton superfluid density. Tunneling between sheets and defects.

Lessons

- Interesting quantum system combining features of superfluids and solids
- Mode velocity generally nonzero. Thermal fluctuations less important (cf. Landau-Peierls)
- Room for more work! Attenuation of modes. Thermal conduction.

How good electrical conductors are the pasta phases?

- Idea. Pasta poor conductor because of low-lying modes and disorder.
- Could account for decay of magnetic fields and death of isolated X-ray pulsars.
  Pons, Viganò, and Rea, Nature Physics **9,** 431 (2013).

- BUT … Protons superconducting! Pasta is likely a good conductor.
- More work needed. (What happens at boundaries of pasta? Are there weak links? Band structure?)

Where does the crust end?

- Start at higher densities. Lower density until instability sets in.
- Energy density for small deviations from uniformity (no Coulomb or surface effects):

$$E = \frac{1}{2}E_{nn}(\delta n_n)^2 + \frac{1}{2}E_{pp}(\delta n_p)^2 + E_{np}\delta n_n \delta n_p$$

- Stability: Require $E_{nn} > 0, E_{pp} > 0, E_{nn}E_{pp} - E_{np}^2 > 0$
- Instability:

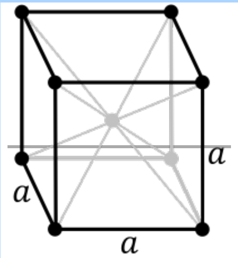
$$E_{pp} - \frac{E_{np}^2}{E_{nn}} < 0$$

  Second term is induced interaction due to change in density of neutrons (phonon exchange)

- Instability occurs at roughly half nuclear matter density.
- Coulomb energy $2\pi e^2(\delta n_p)^2 q^{-2}$. ($\mathbf{q}$ is wave vector)
- Surface (gradient) energy $\propto q^2 \delta n_n^2$, plus terms with δn_p.
- Optimal q when Coulomb and surface effects are equal.

Higher-order terms determine structure

- Terms like $E^{(3)}(\delta n)^3$ and $E^{(4)}(\delta n)^4$
- Minimize energy by choosing lattices with reciprocal lattice vectors of length q_c.
- Example. Cubic term: vectors $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3$ must add to zero (translational invariance)
- For weak instability best lattice is body-centered-cubic.
  (Baym, Bethe, CJP, Nucl. Phys. A **175**, 225 (1971).)
  (For terrestrial solids, Alexander and McTague, PRL **41**, 702 (1978).)



- For stronger instability, obtain pasta sequence.
  Kleinert and Maki, Fortschritte der Physik **29**, 219 (1981).

Radius

7 miles $\approx$ 11 km!

Baade and Zwicky,
Phys. Rev. **46**, 76 (1934).

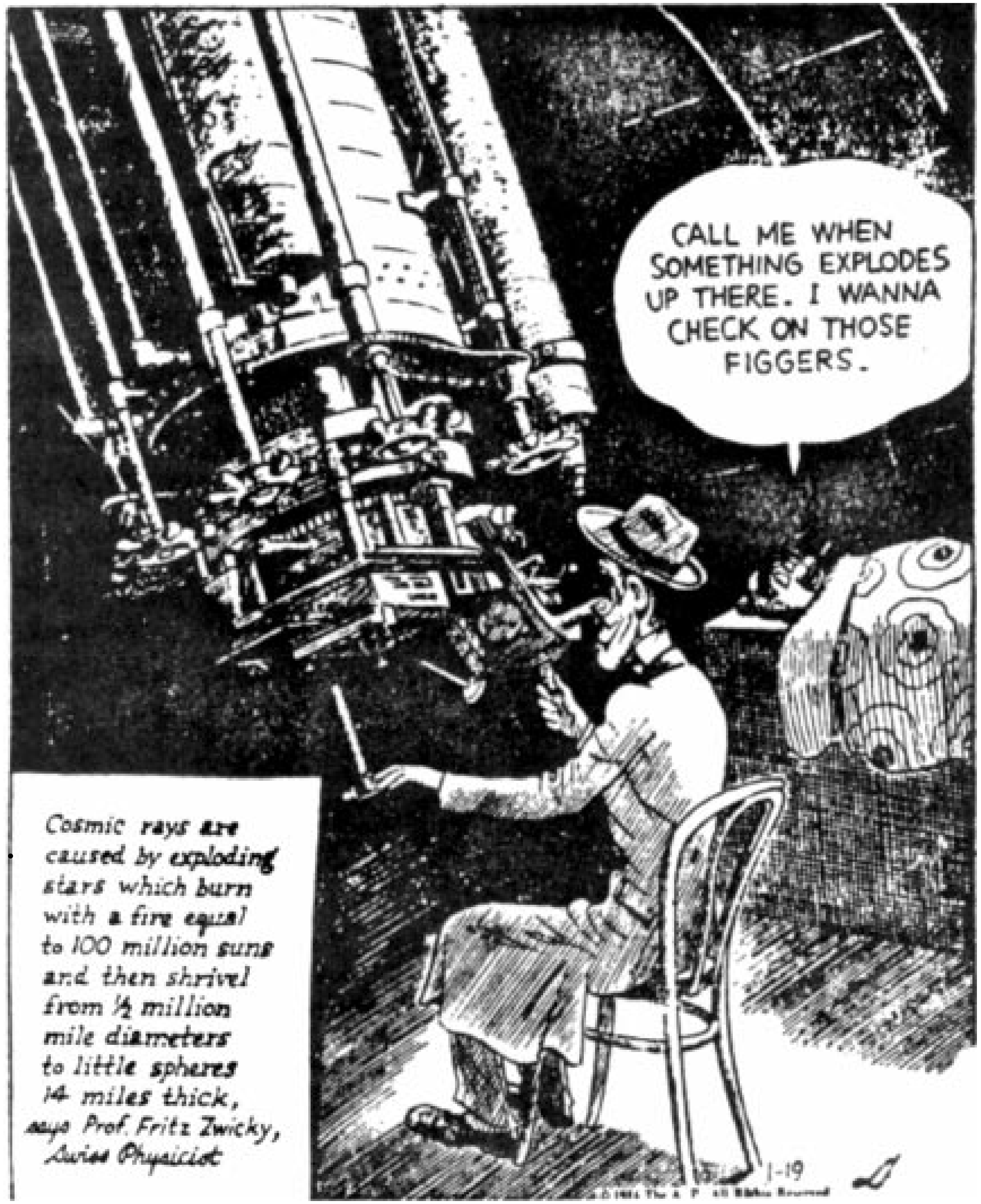


Figure 4. Baade and Zwicky's prediction of neutron stars, reported in this cartoon in the *Los Angeles Times* of 19 January 1934. The lower box reads: "Cosmic rays are caused by exploding stars which burn with a fire equal to 100 million suns and then shrivel from 1/2 million mile diameters to little spheres 14 miles thick, says Prof. Fritz Zwicky, Swiss Physicist." Reproduced with permission of The Associated Press, 1934.

Concluding remarks

- Much progress has been made.
- Strong theoretical, experimental and observational support for nucleon superfluidity.
- More astronomical data coming.
- Nuclear physics. More data and better models for neutron rich matter.
- Neutron stars as a stimulus for gaining a deeper understanding of physics.
- Expect the unexpected.
- Why not join in the fun?