Causal Properties of gCDT Models

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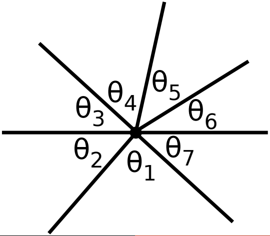
Gravitational Path Integral

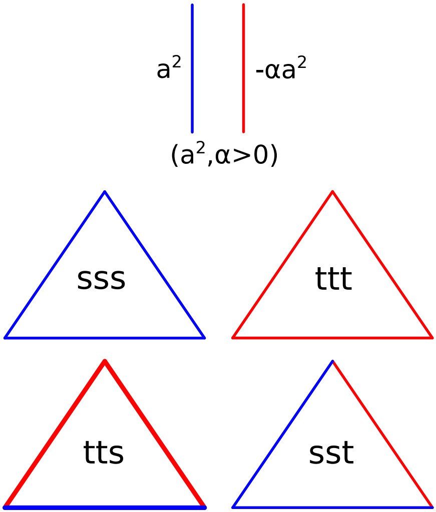
- The full gravitational path integral is $\mathcal{Z} = \int D[g]\, e^{iS_{EH}[g]}$
- It is ill defined and difficult to solve in this form
- By "integrating over" a restricted set of metrics (geometries), it becomes better defined
- A certain class of piecewise flat spacetimes called *triangulations*
- Leads to a simplified nonperturbative "path sum"

$$\mathcal{Z} = \sum_T \frac{1}{C(T)} e^{iS_{Regge}[T]}$$

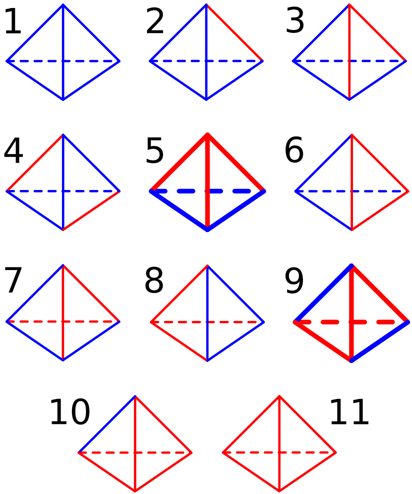
Causal Dynamical Triangulations

- A d dimensional triangulation is an object formed by gluing together d dimensional generalisations of triangles, *simplices*
- Simplices glued together by a set of rules
- Interior of simplices is flat Minkowski space
- This means there is no curvature in the d or d-1 dimensional simplices
- Curvature is introduced at (d-2) dimensional subsimplices (*bones*) where many d dimensional simplices meet
- Related to the "deficit angle" at that subsimplex
  $\varepsilon = 2\pi - \sum_i \theta_i$





- Simplices are built from edges of fixed squared length a^2 (spacelike) or $-\alpha a^2$ (timelike) ($a^2 > 0$, $\alpha > 0$)
- Not all have Lorentzian signature
- 4 possibilities in (1+1) dimensions, only 1 used in CDT
- 11 possibilities in (2+1) dimensions, only 2 used in CDT



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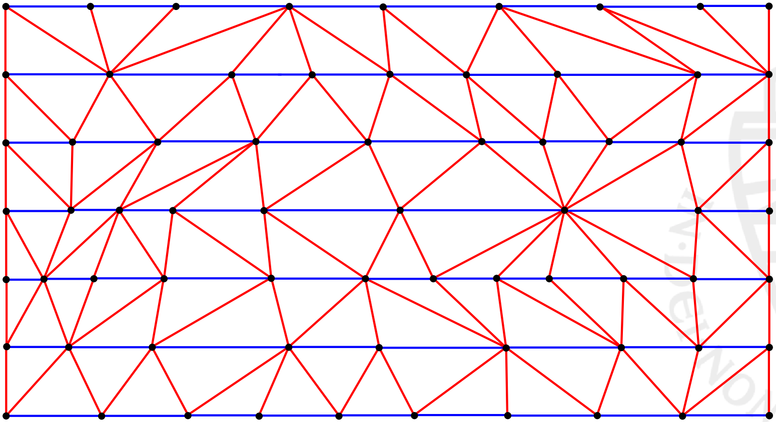
Gluing Rules

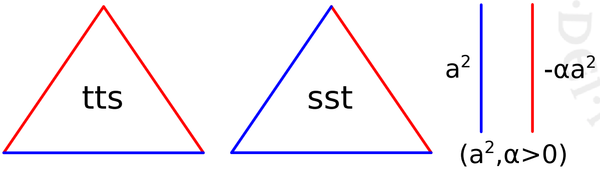
- The CDT gluing rules impose local causal structure on the triangulations
- For example in (1+1) dimensions, conditions about a vertex ensure there is a complete light cone at that vertex
- The CDT gluing rules are yet stronger - they ensure a certain explicit foliation of the triangulation exists
- This has a preferred time label
- Triangulation has product structure $[0,1] \times S^{d-1}$

Example CDT Triangulation in (1+1) Dimensions



CDT to gCDT

- gCDT generalises CDT by allowing more types of simplices
- 2 allowed in (1+1)
- 4 (or 5) allowed in (2+1)
- Local causal structure still given by gluing rules
- Existence of a foliation is no longer explicit, so global causal structure uncertain



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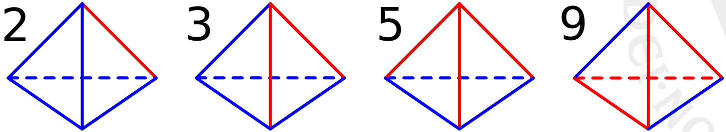

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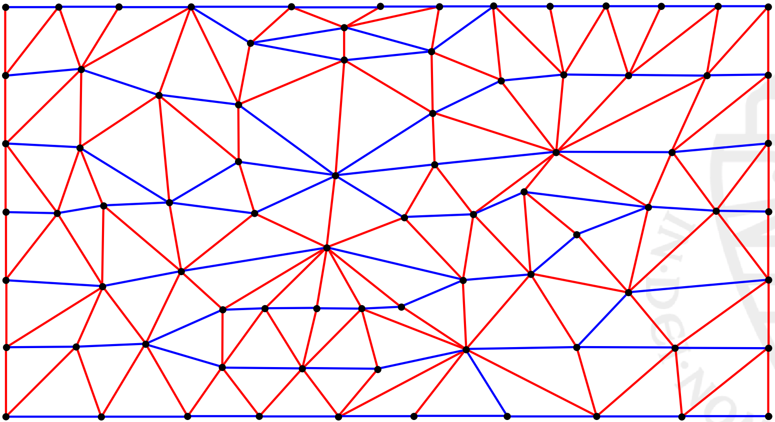

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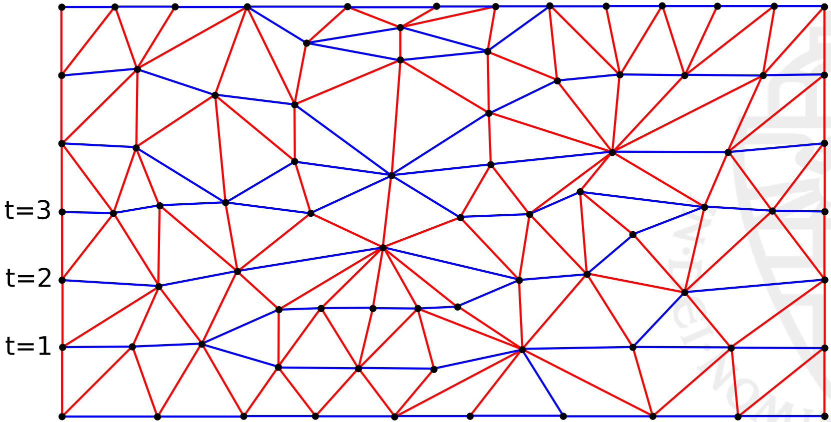

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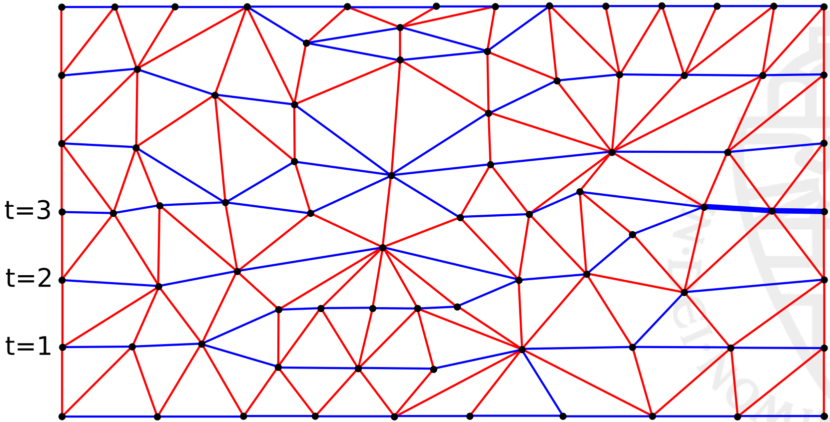

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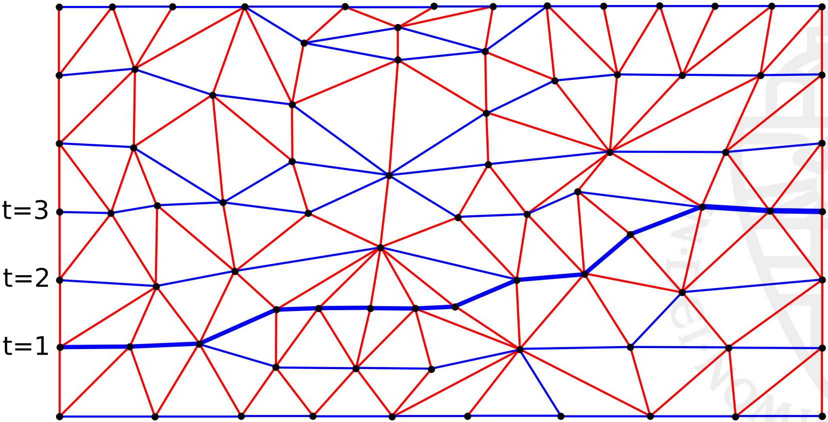

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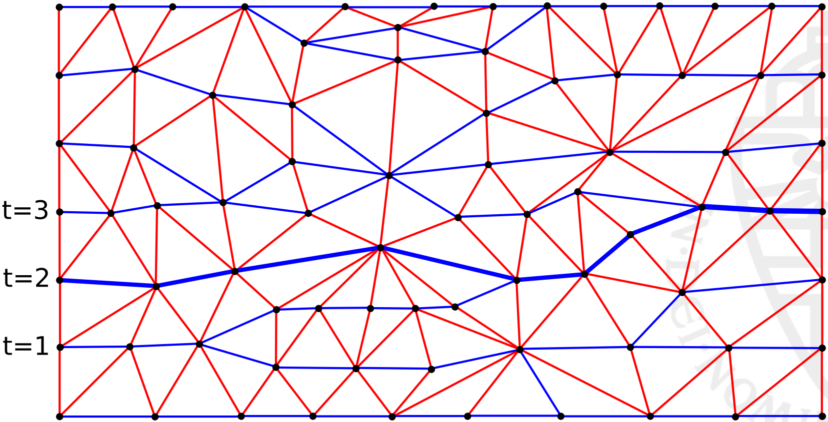

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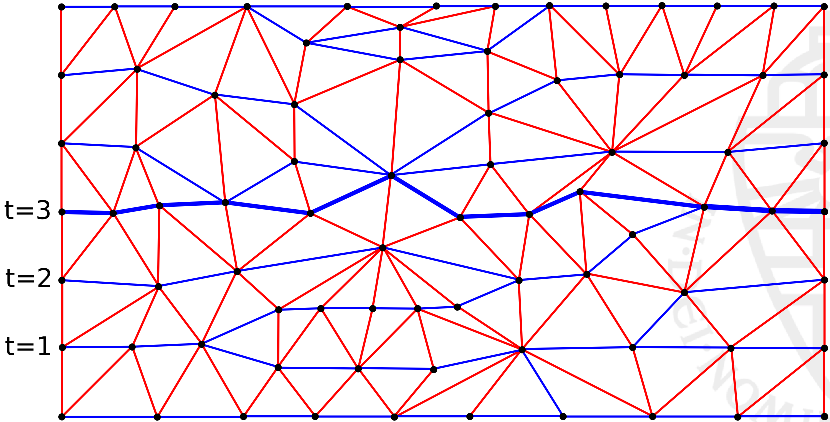


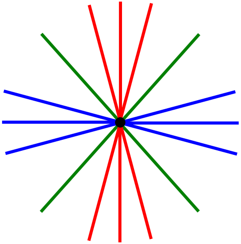
Of What Interest?

- Generalised models may explore the space of metrics more efficiently
- To what extent are the nice results for (higher dimensional) CDT dependent upon the foliation?
- It has recently been shown that in (2+1) dimensional gCDT the results of CDT may be recovered [1]
- Harder to solve $\approx$ more fun!

[1] S. Jordan, R. Loll, To Appear In Physics Letters B, arxiv 1305.4582

Local gCDT Causal Structure

- Causality conditions ensure a well defined light cone structure around d-2 dimensional simplices ("local causality")
- In (1+1) dimensions, condition imposed around the vertices
- In (2+1) dimensions, condition imposed around the edges



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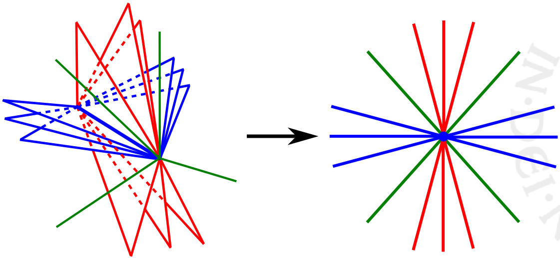


Global gCDT Causal Structure

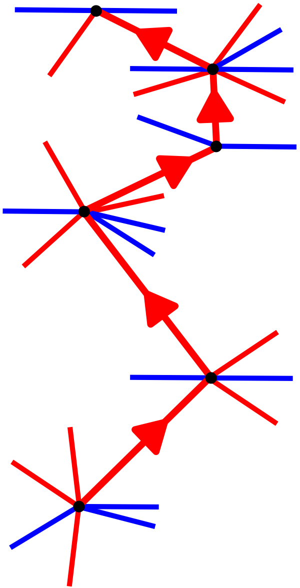
- What global causal structure follows from "local causality"?
- Can a consistent time orientation be defined throughout the triangulation?
- Are there closed timelike curves?

Global gCDT Causal Structure in (1+1) Dimensions

- A proof exists excluding contractible closed timelike curves [2]
- A proof exists that a consistent time orientation can be defined
- Question of non-contractible closed timelike curves is unsettled
- Specific examples of closed timelike curves on a cylindrical topology are known to us
- Necessary/sufficient conditions for such curves?

[2] R. Hoekzema, Master Thesis, 2012

Spirit Of Proof



- Assume two "timewalks" extend from the initial boundary and meet in opposite timelike sectors of a vertex
- Consider the timewalks which extend from the enclosed area of the spacelike boundary
- Use these to shrink the area enclosed by the timewalks and spacelike boundary
- This can be done ad infinitum, but the area is finite - contradiction

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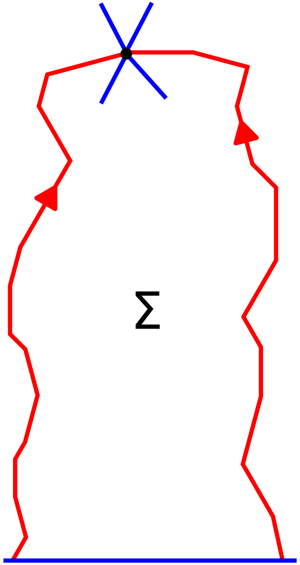

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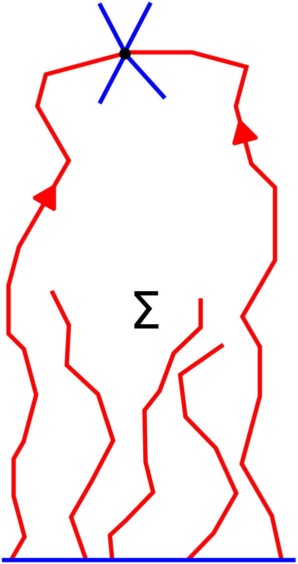

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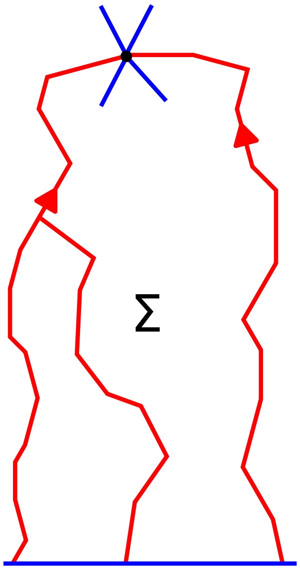

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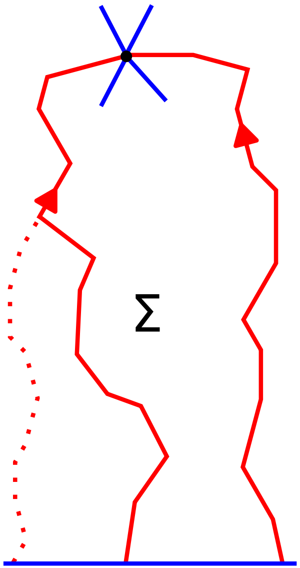

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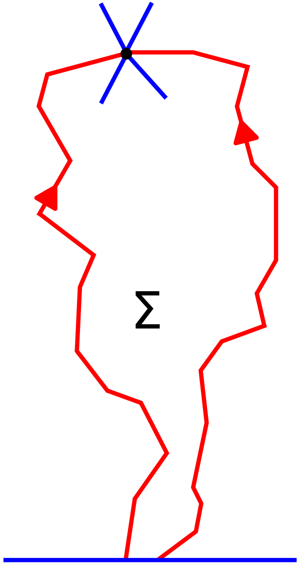

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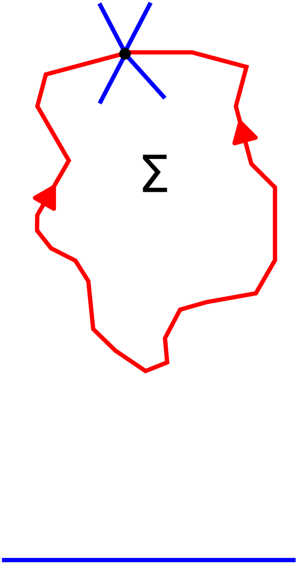

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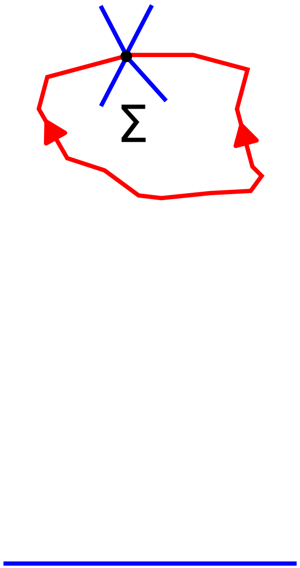

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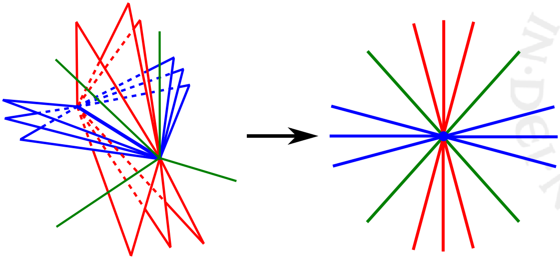
- There is a recent proof of causality at most gCDT vertices
- The question of closed timelike curves and a consistent time orientation are yet to be settled

Vertex Causal Structure in (2+1)

- In (2+1) dimensions the causality conditions are gluing rules for triangles around an edge
- Not clear a priori if this means a *vertex* has desired causal structure, that is, has exactly one complete light cone about it
- A recent proof has been found that edge causality ⇒ correct vertex causal structure [3]



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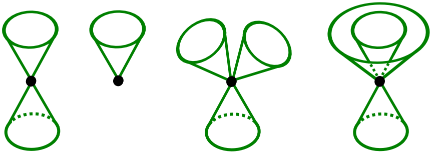
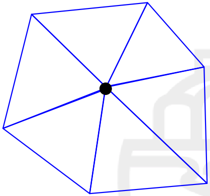
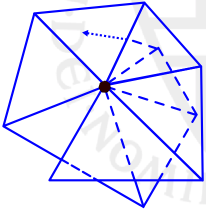


[4] Scott Smith, Master Thesis, 2013

Spirit Of The Proof

- Isolate the local triangulation about the vertex (all tetrahedra at that vertex)
- Identify two dimensional spacelike structures around the vertex
- Two important ones are the *plane* and the *spiral*
- Show that the spacelike objects may be grouped into certain collections at the vertex
- Show that there is a half light cone at both ends of such a collection, and no light cone inside the collection
- Show that there can only be one such collection at a vertex





Future Work

- Investigation into non-contractible closed timelike curves
- An extension of the proofs regarding closed timelike curves and global time orientation in (1+1) to (2+1), using vertex causal stucture

Questions?

My thanks go to Prof. Renate Loll, Samo Jordan, Reneé Hoekzema, The Rest of Renate's Group

My thanks to the organisers for this opportunity, and more generally for the school

Thank You For Listening!

I Invite You To Ask Any Questions You May Have...