Hydrodynamics, nonequilibrium physics and AdS/CFT – the numerical relativity connection

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M. Heller, RJ, P. Witaszczyk, 1103.3452 [PRL 108, 201602 (2012) (physics)
M. Heller, RJ, P. Witaszczyk, 1203.0755 [PRD 85, 126002 (2012)] (technical details)
M. Heller, RJ, P. Witaszczyk, 1302.0697 [PRL 110, 211602 (2013)] (high order hydrodynamics, see talk by P. Witaszczyk)

Outline

Motivation — physics

The AdS/CFT description of a plasma system
- Example: Static uniform plasma

Hydrodynamics versus AdS/CFT

Boost-invariant flow

The metric ansatz and numerical formalism

A short summary of main results

Conclusions

Motivation

- There are strong indications that the quark-gluon plasma produced at RHIC is a strongly coupled system
- This poses numerous problems for the theoretical description
- Static properties:
  - Thermodynamics — entropy/energy density etc.
  - Lattice QCD is an effective tool
  - Directly deals with QCD!
  - Quantitative results
- Real time propeties:
  - Expansion of the plasma in heavy-ion collisions
  - Derivation of hydrodynamic expansion in the later stages of the collision
  - Dynamics far from equilibrium – fast thermalization of the plasma
  - Lattice QCD methods are inherently Euclidean — very difficult to extrapolate to Minkowski signature
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$\mathcal{N} = 4$ SYM (strong coupling)

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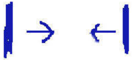

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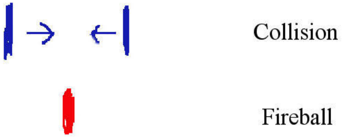

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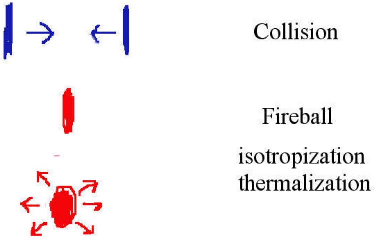

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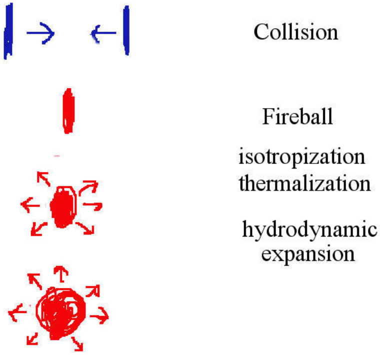

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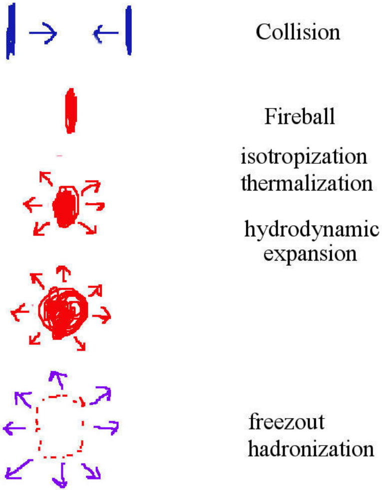

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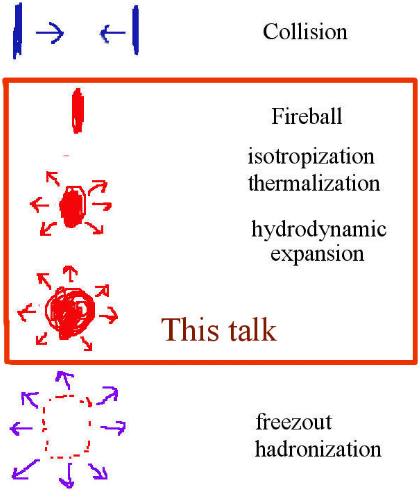

Key question:

Understand the features of (early) thermalization for an evolving (*boost-invariant*) plasma system

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The AdS/CFT description

Aim: How to describe a plasma system in a strongly coupled $\mathcal{N} = 4$ SYM theory?

Method: Describe (possibly time dependent) strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

i) This metric has to satisfy Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

ii) read off $\langle T_{\mu\nu}(x^\rho) \rangle$ from the behaviour of the metric $g_{\mu\nu}(x^\rho, z)$ close to the boundary

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots \qquad \langle T_{\mu\nu}(x^\rho) \rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

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Example: Static uniform plasma

- Start from a constant diagonal energy momentum tensor (with $E = 3p$)

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Solve Einstein's equations with the above boundary condition for $g_{\mu\nu}(x^\rho, z)$...
- The result is a black hole geometry

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2}\, dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

with z_0 expressed in terms of E $\left(E = \frac{3N_c^2}{2\pi^2 z_0^4}\right)$

- There is a horizon at $z = z_0$
- Hawking temperature $T_H = \frac{\sqrt{2}}{\pi z_0} \equiv$ gauge theory temperature
- Bekenstein-Hawking entropy ($\propto$ area of the horizon) $\equiv$ gauge theory entropy

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What is hydrodynamics?

- **Hydrodynamics** isolates long wavelength effective degrees of freedom of a theory
- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- The coefficients of the various tensor structures are the transport coefficients. In a conformal theory these are pure numbers times powers of T.
- Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu} = 0$
- The above form of $T_{\mu\nu}$ for $\mathcal{N} = 4$ SYM at strong coupling is **not** an assumption but can be proven from AdS/CFT Minwalla et.al.

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Linearized hydrodynamics

- Look at small disturbances of the uniform static plasma...
- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i\frac{\eta}{E+p}k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}}k - i\frac{2}{3}\frac{\eta}{E+p}k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

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AdS/CFT, hydrodynamics and nonequilibrium processes

- The uniform static plasma system is described as a static planar black hole
- Small disturbances of the uniform static plasma ≡ small perturbations of the black hole metric (≡ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D,black\ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description...

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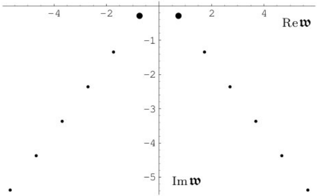

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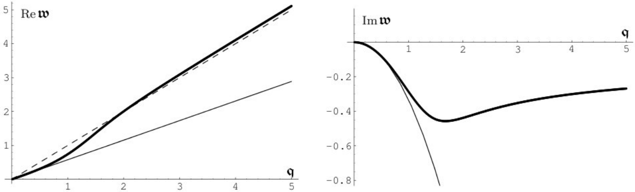

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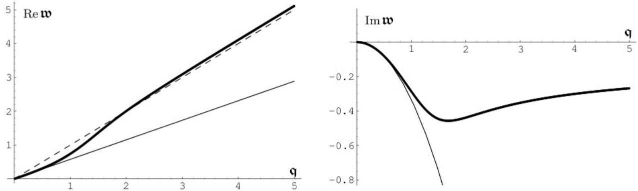

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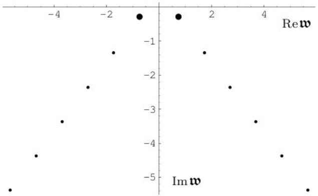

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Einstein's equations in AdS/CFT

- contain all-order viscous hydrodynamic modes (with specific values of all transport coefficients)
- **in addition** contain the dynamics of genuine nonhydrodynamical modes
- incorporate their interactions in a fully nonlinear (and unique) way

Consequence:
Einstein's equations can serve to study nonequilibrium processes in strongly coupled $\mathcal{N} = 4$ SYM and are an effective tool for exploring physics *beyond* hydrodynamics

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How to see *nonlinear* hydrodynamics within AdS/CFT?

Fluid/gravity duality versus nonequilibrium physics

The approach of [Bhattacharyya,Hubeny,Minwalla,Rangamani]

- Start from a static black hole with fixed temperature T which describes a fluid at rest, $u^\mu = (1,0,0,0)$ with constant energy density
- Perform a boost to obtain a uniform fluid moving with constant velocity u^μ
- The resulting metric (in Eddington-Finkelstein coordinates) is

$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote T and u^μ to (slowly-varying) functions of x^μ

Caveat: The metric is no longer an exact solution of Einstein's equations

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$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left(1 - \frac{T^4}{\pi^4 r^4}\right) u_\mu u_\nu dx^\mu dx^\nu + r^2(\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote T and u^μ to (slowly-varying) functions of x^μ

Caveat: The metric is no longer an exact solution of Einstein's equations

Fluid/gravity duality versus nonequilibrium physics

- Perform an expansion of the Einstein equations in gradients of spacetime fields.
- Find corrections to the metric at first and second order
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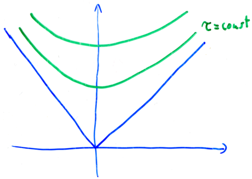
Boost-invariant flow

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau}\varepsilon \quad \text{and} \quad p_T = \varepsilon + \frac{1}{2}\tau \frac{d}{d\tau}\varepsilon \, .$$

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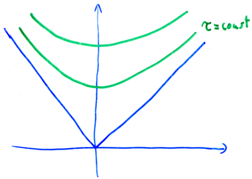

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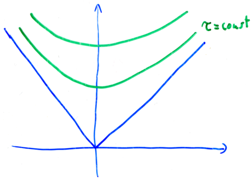

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RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

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  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
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Start from some initial geometry at $\tau = 0$ and solve 5D vacuum Einstein's equations with negative cosmological constant...

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Initial conditions for the evolution of the plasma system

- In a previous work [Beuf, Heller, RJ, Peschanski], we analyzed possible initial conditions in the Fefferman-Graham coordinates

$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- The initial conditions are determined in terms of a *single* function, say $c_0(z)$. $a_0(z) = b_0(z)$ are determined through a constraint equation.
- A typical solution of the constraint equations is

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- There is a *coordinate* singularity at $z = \sqrt{\pi/2}$ where

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$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- The initial conditions are determined in terms of a *single* function, say $c_0(z)$. $a_0(z) = b_0(z)$ are determined through a constraint equation.
- A typical solution of the constraint equations is

$$a_0(z) = b_0(z) = 2\log\cos z^2 \qquad\qquad c_0(z) = 2\log\cosh z^2$$

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The metric ansatz and numerical formalism

- **The key problem is what boundary conditions to impose in the bulk.** For a sample initial profile $c_0(u) = \cosh u$ ($u \equiv z^2$), there is a curvature singularity at $u = \infty$.
- *A-priori* we do not know where is the event horizon!
- We use the ADM freedom of foliation to ensure that all hypersurfaces end on a single spacetime point in the bulk — this ensures that we will control the boundary conditions even though they may be in a strongly curved part of the spacetime

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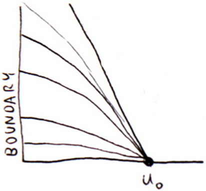

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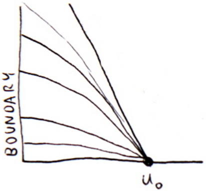

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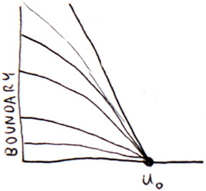

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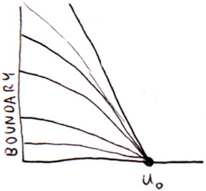

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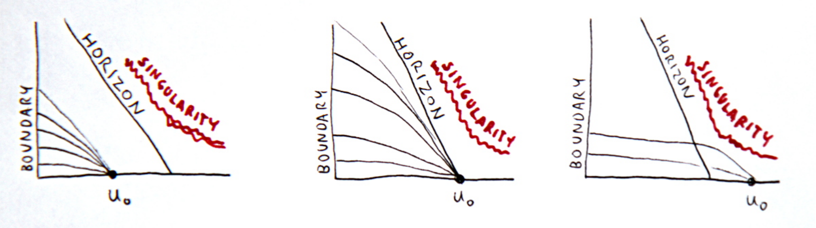

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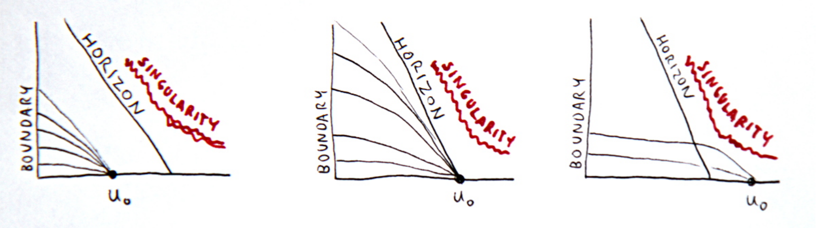

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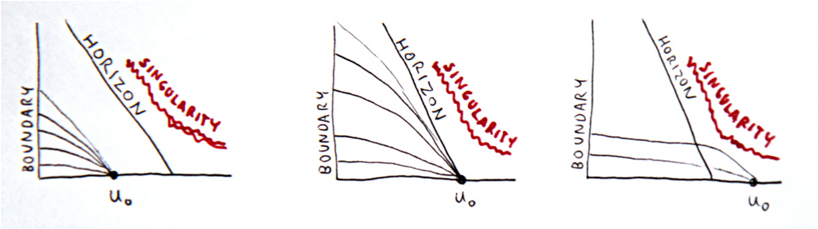

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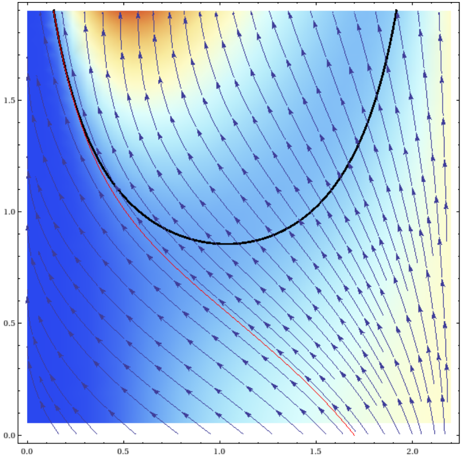

black line – dynamical horizon, arrows – null geodesics, colors represent curvature

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- Consequently, we set the (nondynamical) function $a(u)$ to

$$a(u) = \cos\left(\frac{\pi}{2}\frac{u}{u_0}\right)$$

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- We have to require that the gauge theory metric is ordinary flat Minkowski metric
- In the Fefferman-Graham coordinate system

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2}$$

this amounts to the requirement that $\lim_{z\to 0} g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu}$

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- **Surprisingly,** $b(t,0)$ and $c(t,0)$ can be nontrivial at the boundary — this corresponds to a boundary diffeomorphism!
- The condition of Minkowski boundary metric becomes a relation between extrinsic curvature elements:

$$L(t,0) = b(t,0) + t\frac{b^2(t,0)}{c^2(t,0)}M(t,0)$$

- We implement this in a differential form:

$$\partial_t L = \partial_t \left(b + t\frac{b^2}{c^2}M \right)$$

- The physical gauge theory time τ is related to the ADM coordinate time by

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- We need very accurate spatial derivatives at the boundary in order to reliably extract the physical energy density from the numerical geometry
- For the time evolution we use an adaptive $8^{th}/9^{th}$-order Runge-Kutta method (gnu scientific library)

Numerical checks:

1. We monitor ADM constraints during evolution
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- We have used 29 initial geometries at $\tau = 0$ which encode the initial conditions for the boost-invariant plasma system
- Technically each geometry is determined by a choice of the metric coefficient $c(\tau = 0, u)$.
- We have chosen quite different looking profiles e.g.

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c_1(u) &= \cosh u \\
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Gauge theory observables

Asymptotics of $g_{\mu\nu}(x^\rho, z)$ at $z \sim 0$ gives the energy-momentum tensor $T_{\mu\nu}(x^\rho)$ of the plasma system (equivalently $\varepsilon(\tau)$)

The area of the apparent horizon defines for us the entropy density (in particular *initial entropy*)

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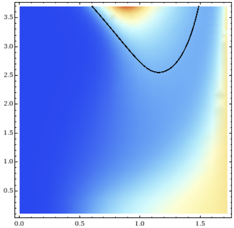

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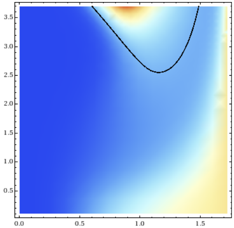

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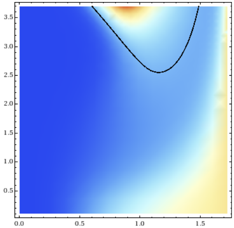

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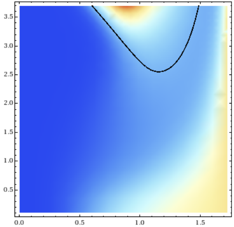

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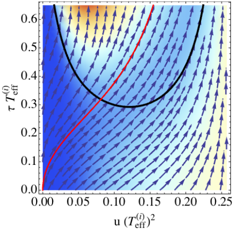

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Thermalization of boost-invariant plasma - lessons from AdS/CFT

1. Using AdS/CFT, we observe a transition to a *viscous* hydrodynamic description for all initial conditions considered ($\equiv$ *effective thermalization*)
2. For all initial conditions considered, *viscous* hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

   (natural values for RHIC: ($\tau_0 = 0.25\, fm$, $T_0 = 500\, MeV$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$)
3. The plasma system is described by *viscous* hydrodynamics even though **it is not in true thermal equilibrium** — there is still a sizable pressure anisotropy

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

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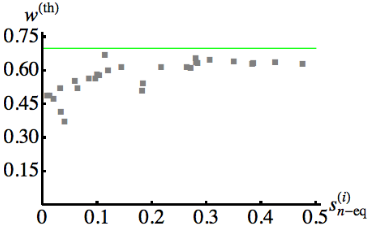

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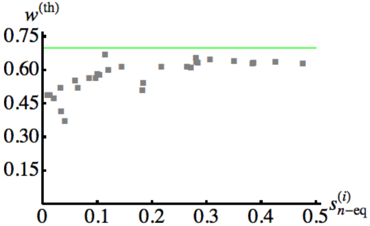

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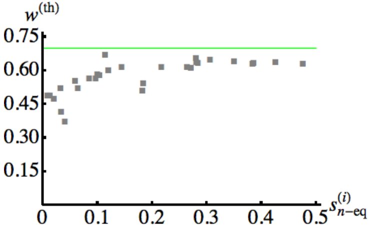

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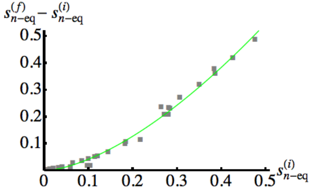

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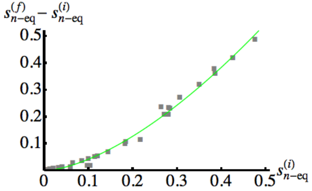

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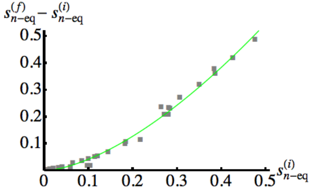

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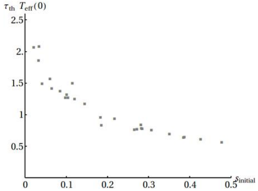

Conclusions

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods *do not* presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- We implemented ADM evolution using spectral methods, freezing the evolution at some interior point by forcing the lapse to vanish there
- Even though genuine nonequilibrium dynamics is very complicated, we observed surprising regularities
- Initial entropy seems to be a key physical characterization of the initial state determining the total entropy production and thermalization time and temperature
- For $w = T_{th} \cdot \tau_{th} > 0.7$ we observe hydrodynamic behaviour but with sizeable pressure anisotropy (described wholly by viscous hydrodynamics)

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