Classical Stability and Quantum Effects of Warped AdS_3 Black Holes in Topologically Massive Gravity

Hugo Ferreira

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Outline

Topologically Massive Gravity

Einstein gravity in 2+1 dimensions has **no** propagating degrees of freedom!

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Deform theory: **Topologically Massive Gravity**

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with:

$$S_{\text{E-H}} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) \,,$$

$$S_{\text{C-S}} = \frac{\ell}{96\pi G \nu} \int d^3x \sqrt{-g}\, \epsilon^{\lambda\mu\nu}\, \Gamma^{\rho}_{\lambda\sigma} \left(\partial_\mu \Gamma^{\sigma}_{\rho\nu} + \frac{2}{3} \Gamma^{\sigma}_{\mu\tau} \Gamma^{\tau}_{\nu\rho} \right) \,.$$

Deser, Jackiw, Templeton (1982)

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Deser, Jackiw, Templeton (1982)

- **Massive** propagating degree of freedom.
- **Third-order** derivative theory.
- GR solutions $\subset$ TMG solutions.

Warped AdS_3 black hole

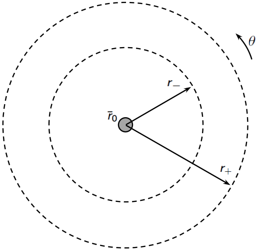
Spacelike stretched black hole:

$$ds^2 = dt^2 + \frac{\ell^2 dr^2}{4R^2(r)N^2(r)} + 2R^2(r)N^\theta(r)dtd\theta + R^2(r)d\theta^2$$

$$R^2(r) = \frac{3(\nu^2-1)}{4}r(r-r_0)$$

$$N^2(r) = \frac{(\nu^2+3)(r-r_+)(r-r_-)}{4R(r)^2}$$

$$N^\theta(r) = \frac{2\nu r - \sqrt{r_+ r_-(\nu^2+3)}}{2R(r)^2}$$

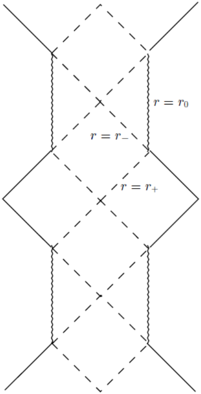


Anninos, Li, Padi, Song, Strominger (2008)

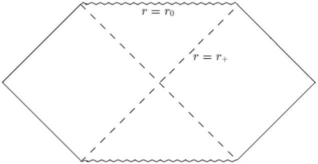
$\nu > 1$ is the **warp factor** of the spacetime.

$\nu \to 1$ recovers the **BTZ black hole**.

Causal structure of warped AdS_3 black holes

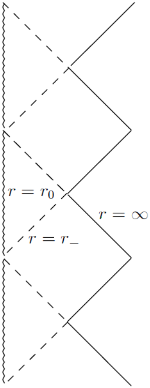


$r_0 < r_- < r_+$



$r_0 = r_- < r_+$

- **Not** asymptotically AdS_3!
- Similar to asymptotically flat black holes!
- Arena to obtain valuable insights for difficult problems with the Kerr black hole!



$r_0 < r_- = r_+$

Jugeau, Moutsopoulos, Ritter (2010)

Outline

Scalar field on the background of a warped AdS_3 black hole

- **Scalar field** Φ on the background of a spacelike stretched black hole:

$$\left(\nabla^2 - m^2\right)\Phi(t, r, \theta) = 0$$

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- **Exact** mode solutions:

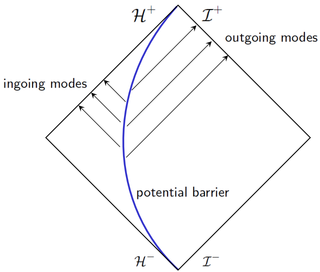
$$\Phi_{\omega k}(t, r, \theta) \sim e^{-i\omega t + ik\theta}\, z^{\alpha}(1-z)^{\beta} F(a, b, c; z)$$

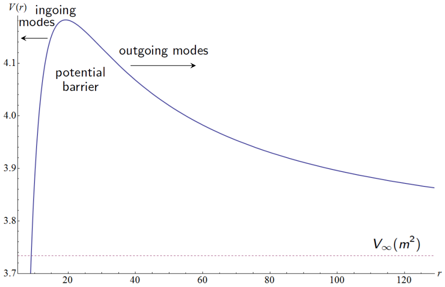
$$z = \frac{r - r_+}{r - r_-}$$

α, β, a, b, c functions of ω and k

$F(a, b, c; z)$ hypergeometric function

Quasinormal modes





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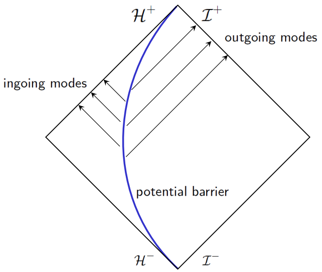
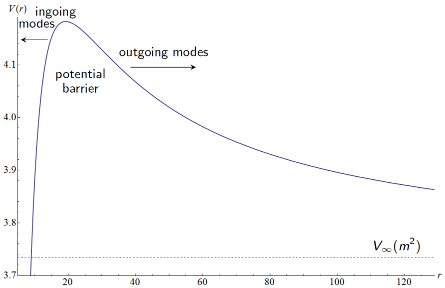
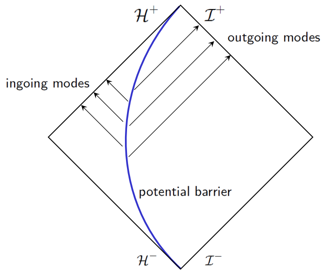


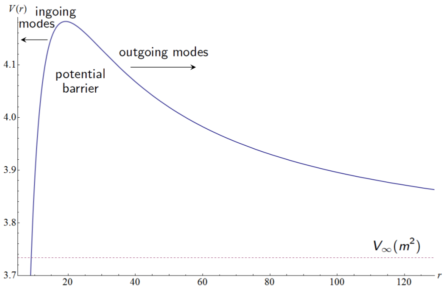


Boundary conditions:

- **Ingoing** modes at the event horizon;
- **Outgoing** modes at infinity.

Quasinormal modes





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$\Longrightarrow$ Discrete set of complex eigenfrequencies $\{\omega_n\}$

$$\Phi_n \sim e^{-i\omega_n t} = e^{-i\mathrm{Re}(\omega_n)t + \mathrm{Im}(\omega_n)t}$$

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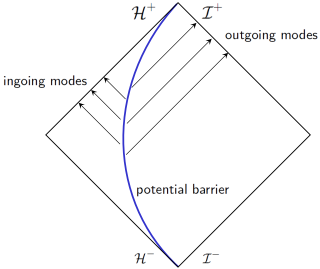

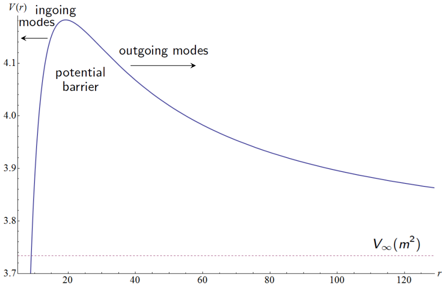

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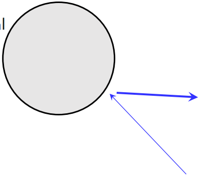
If $\mathrm{Im}(\omega_n) > 0$ for some n: mode is **unstable**!

Superradiant modes



Superradiant modes: amplitude **increases** after reflection by the potential barrier if

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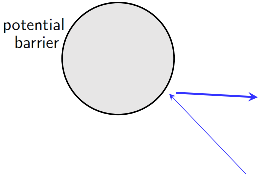

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- boundary conditions imposed on the field;
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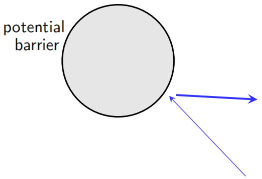

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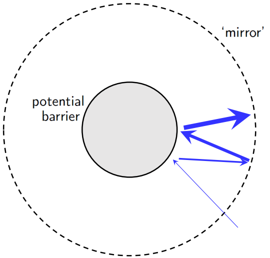
The **existence** of superradiance depends on:

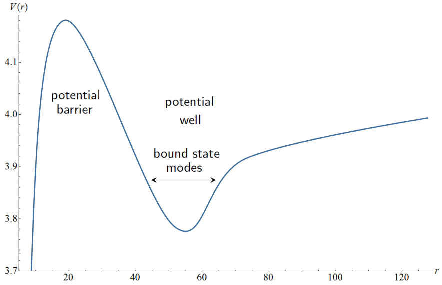
- boundary conditions imposed on the field;
- definition of positive frequency modes.

Whatever the choice of boundary conditions and positive frequency, there are **always** superradiant scalar modes on the warped AdS_3 black hole, similarly to the Kerr black hole (but not to the BTZ and Kerr-AdS).

Ferreira (2013)

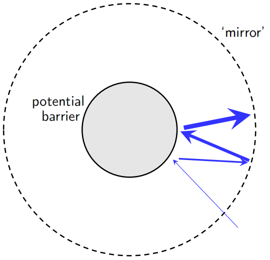
Superradiant and bound state modes

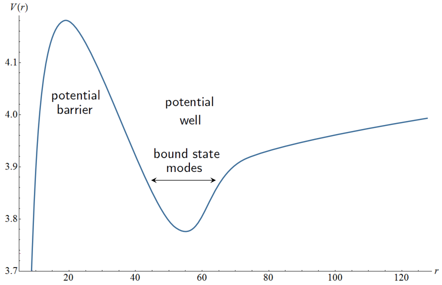




Bound state modes: localised in the potential well (ingoing at event horizon, exponentially decreasing at infinity).

Superradiant and bound state modes





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$\mathrm{Im}(\omega_n) > 0 \implies$ **superradiant bound state mode** $\implies$ instability!

Are there any unstable modes?

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- All modes are **stable**: $\mathrm{Im}(\omega_n) < 0$.
- In particular, **no superradiant instabilities**, in contrast with Kerr!

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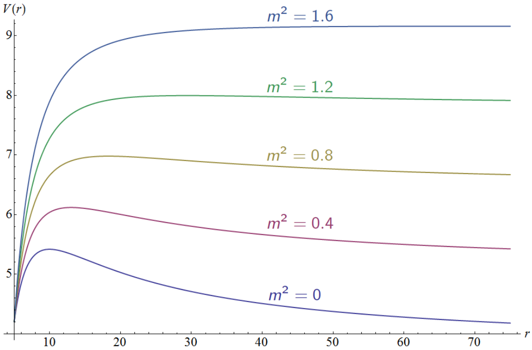
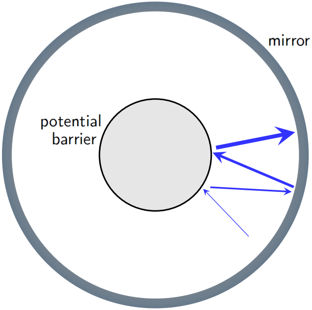


What happens if we add an actual mirror?



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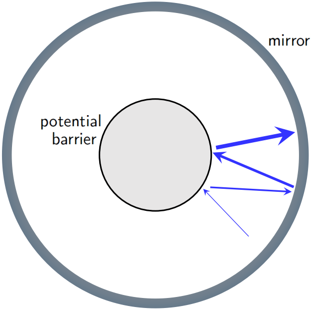
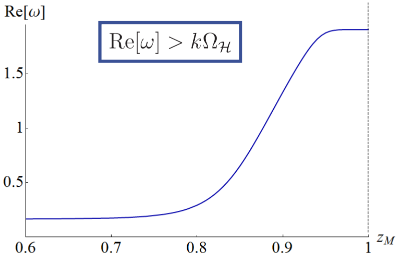


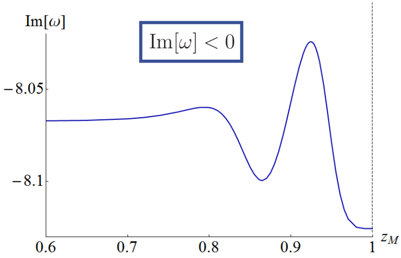
Boundary conditions:

- **Ingoing** modes at the event horizon;
- **Vanishing** modes at the mirror (Dirichlet boundary condition).

What happens if we add an actual mirror?

Frequency vs position of the mirror





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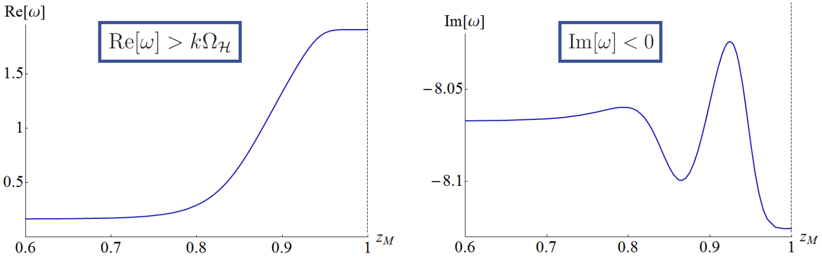

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Bound state modes are still **stable**!

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QFT on warped AdS_3 black holes

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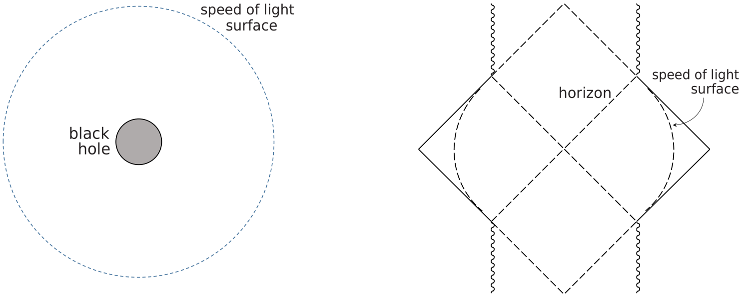
- QFT on rotating black holes is a challenging problem:
  - Superradiant modes require care.
  - The Hartle-Hawking vacuum state is **not** well defined!

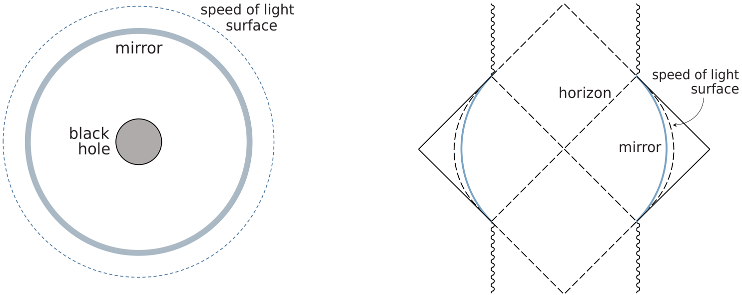
Frolov and Thorne (1989)
Kay and Wald (1991)
Ottewill and Winstanley (2000)
Ottewill and Duffy (2008)

Hartle-Hawking vacuum on a warped AdS_3 black hole



Beyond the speed of light surface, the Hartle-Hawking vacuum would have to rotate with a speed **greater** than the speed of light.

Hartle-Hawking vacuum on a warped AdS_3 black hole



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If a **mirror** is put between the horizon and the speed of light surface, an Hartle-Hawking vacuum is **well defined**.

Stress-energy tensor for a scalar field in the Hartle-Hawking vacuum

- **Aim:** compute the expectation value of the renormalised stress-energy tensor $\langle T_{\mu\nu}(x)\rangle_{\mathsf{ren}}$ for a scalar field in the Hartle-Hawking vacuum

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Work in progress...

Stress-energy tensor in the complex Riemannian section

Consider the **complex Riemannian section**:

- Change to **rotating** coords: $(t, r, \theta) \to (\tilde{t} = t,\ r = r,\ \tilde{\theta} = \theta - \Omega_{\mathcal{H}} t)$
- Analytically continue: $\tilde{t} = -i\tau$;
- Impose periodicity: $\tau \sim \tau + \frac{2\pi}{\kappa_+}$ (Hawking temperature $T_H = \frac{\kappa_+}{2\pi}$).

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Green's functions:

$$G^L(x, x') = \int_0^\infty d\tilde{\omega} \sum_{k=-\infty}^{\infty} G^L_{\tilde{\omega}k}(r, r') \longrightarrow G^{\mathbb{C}}(x, x') = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} G^{\mathbb{C}}_{nk}(r, r')$$

Computation is hoped to be easier in the complex Riemannian section.

Frolov (1982)

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- Warped AdS_3 black holes are classically stable to scalar field perturbations, even if a mirror is added to the spacetime, in contrast with Kerr.
- QFT computations on warped AdS_3 black holes may give valuable insights for the Kerr case.

What's next?

- Is the warped AdS_3 black hole classically stable to other types of perturbations (namely gravitational perturbations)?

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- Is the warped AdS_3 black hole classically stable to other types of perturbations (namely gravitational perturbations)?
- What is the renormalised stress-energy tensor for a field in the Hartle-Hawking vacuum state? What information does it provide for the Kerr case?

THANK YOU FOR YOUR ATTENTION!

More information in:

arXiv:1304.6131 (to appear in Phys. Rev. D)

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