The effective action in 4-dim CDT

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Outline

- Revision of CDT basics
- The transfer matrix idea and measurement
- Transfer matrix in ‘C’ (de Sitter) phase
- Transfer matrix in ‘A’ (uncorrelated) phase
- Transfer matrix in ‘B’ (collapsed) phase
- Conclusions and prospects

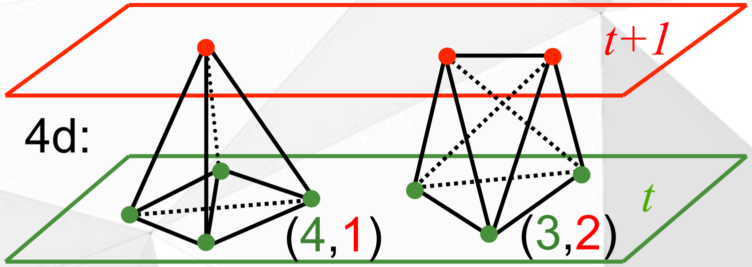
- **Causal Dynamical Triangulations (CDT)** is a non-perturbative approach to Quantum Gravity based on the path integral

$$Z = \int_{Geom} D[g] \exp(iS_{HE}[g])$$

- Regularization of Z is done by summing over all **causal triangulations** T constructed from 4-d simplices

$$Z = \sum_T \frac{1}{C_T} \exp(i\tilde{S}_R[T])$$

- We assume a **global time foliation** S^1 and **fixed spatial topology** $S^3 \Rightarrow$ resulting space-time $S^1 \times S^3$ can be built from two types of **simplices**



- The Einstein-Hilbert action:

$$S_{HE} = \frac{1}{16\pi G}\int dt \int d^3x\sqrt{-g}\left(R-2\Lambda\right)$$

  - G – Newton's constant
  - g – metric determinant
  - R – curvature scalar
  - Λ – cosmological constant

- is regularized by the Regge action where curvature R is determined by the deficit angle „around" 2-d triangles

$$\tilde{S}_R = i\left[-K_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)\right] = iS_R$$

$1/G$ $\qquad$ Λ $\qquad$ $\alpha \;\; (l_t^2 = -\alpha l_s^2)$

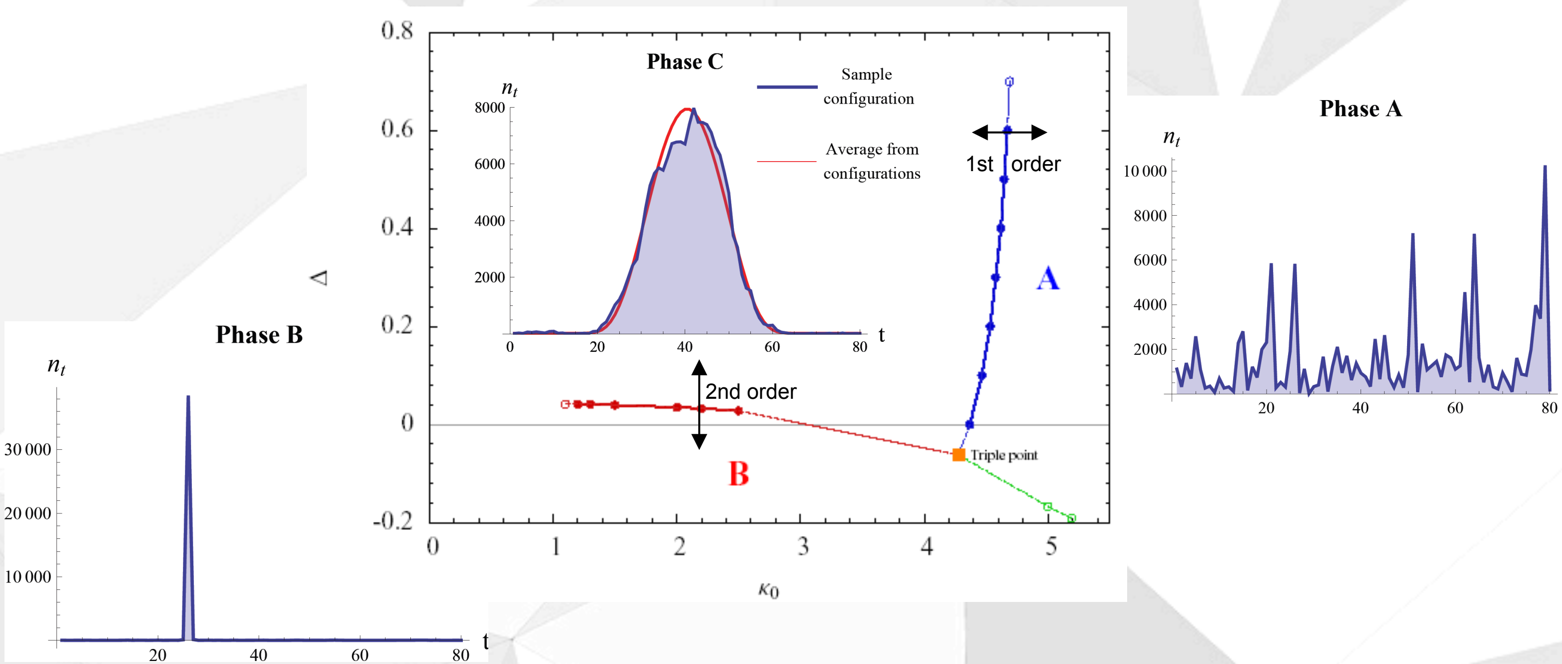
  - N_0 – # of vertices
  - N_4 – # of 4-simplices
  - $N_4^{(4,1)}$ – # of (4,1) & (1,4) simplices

- After Wick rotation: $\alpha \rightarrow -\alpha$ ($|\alpha| > 7/12$) S_R is purely real:

$$Z = \sum_T \frac{1}{C_T}\exp(i\tilde{S}_R[T]) = \sum_T \frac{1}{C_T}\exp(-S_R[T]) \quad \rightarrow \quad \text{random geometry system}$$

$$S_R = -K_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

- Depending on the values of bare couplings K_0 and Δ ($K_4 \approx K_4^{crit}$) ***three phases*** emerge

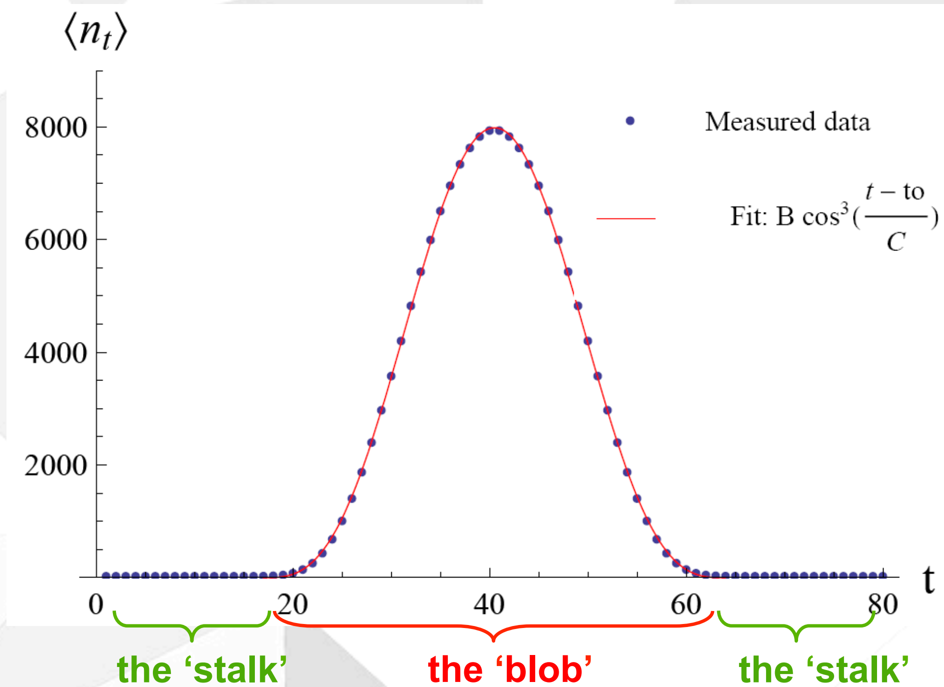


- **In phase ‘C’ the dynamically generated semi-classical background …**

spatial volume at time t: $n_t \equiv N_4^{(4,1)}(t)$

$$\langle n_t \rangle = \frac{3}{4}\tilde{V}_4 \frac{1}{\tilde{A}\tilde{V}_4^{1/4}} \cos^3\left(\frac{t-t_0}{\tilde{A}\tilde{V}_4^{1/4}}\right)$$

$$\sqrt{g_{tt}}\,V_3(t) = \frac{3}{4}V_4 \frac{1}{A V_4^{1/4}} \cos^3\left(\frac{t-t_0}{A V_4^{1/4}}\right)$$



- The solution (in the ‘**blob**’) is consistent with the Euclidean (Wick’s rotation) **de Sitter Universe** (no matter, positive cosmological const.)

- **… and quantum fluctuations are governed by the action:**

$$S = \frac{1}{24\pi G}\int dt\sqrt{g_{tt}}\left(\frac{g^{tt}\dot{V}_3(t)^2}{V_3(t)} + \mu V_3(t)^{1/3} - \lambda V_3(t)\right)$$

  - $\mu = 9\left(\frac{3}{4}\right)^{2/3} A^{-8/3}$
  - $\lambda = 9V_4^{-1/2}A^{-2}$

- This is the (Euclidean) ***minisupespace*** (MS) action obtained from S_{HE} for the maximally symmetric metric: $ds^2 = g_{tt}dt^2 + a^2(t)d\Omega_3{}^2 \Rightarrow V_3(t) \propto a^3(t)$

- **Analysis of the effective propagator (vol-vol correlations)** $\Rightarrow$ effective action in the phase ‘C’ is a discretization of the MS action:

$$S_{dis} = \frac{1}{\Gamma}\sum_t\left(\frac{\left(n_{t+1}-n_t\right)^2}{n_{t+1}+n_t} + \tilde{\mu}n_t^{1/3} - \tilde{\lambda}n_t + \mathrm{O}\left(n_t^{-1/3}\right)\right)$$

Transfer Matrix Motivation:

- The large vol. range in phase 'C' is described by the MS effective action:
  - How good is this description ?
  - Can we measure the effective action directly (not only vol-vol correlations) ?
- How to describe the behavior of the small volume range ?
  - Relatively large quantum fluctuations
  - Small volume discretization effects
  - 'Cleaning' of discretization artifacts might lead to discovery of some non-trivial corrections
- Can we analyze the effective action in other phases ?
- Can we say something more about the dynamics of phase transitions?

$$S_{blob} = \sum_t \frac{1}{\Gamma}\left(\frac{\left(n_{t+1} - n_t\right)^2}{n_{t+1} + n_t} + \tilde{\mu} n_t^{1/3} - \tilde{\lambda} n_t \right) = \sum_t L_{blob}[n_t, n_{t+1}]$$

$$\langle n_t | M_{blob} | n_{t+1} \rangle = \exp\left(-L_{blob}[n_t, n_{t+1}]\right)$$

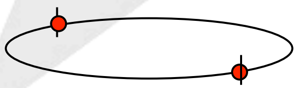
$$Z_{blob} = \sum_{n_1 \ldots n_t \ldots n_T} \exp(-S_{blob}) = Tr\left(M_{blob}{}^T\right)$$

- **Assumptions:**
  - we consider only effective aggregate ‘states’ $|n_t\rangle$
  - the effective interaction is only between neighboring time slices
  - description by the transfer matrix is also viable in other phases
  - time translation symmetry $\Rightarrow M(\cancel{t})$
  - time reflection symmetry $\Rightarrow \langle n_t | M | n_{t+1}\rangle = \langle n_{t+1} | M | n_t \rangle$

$$Z = \sum_{n_1 \ldots n_t \ldots n_T} \prod_{t=1}^{T} \langle n_t | M | n_{t+1} \rangle = Tr\left(M^T\right)$$

- We use measured 2-point probability distribution in systems with $T=2$ time slices…

$$P_2(n_1, n_2) = \frac{1}{Z} \langle n_1 | M | n_2 \rangle^2$$

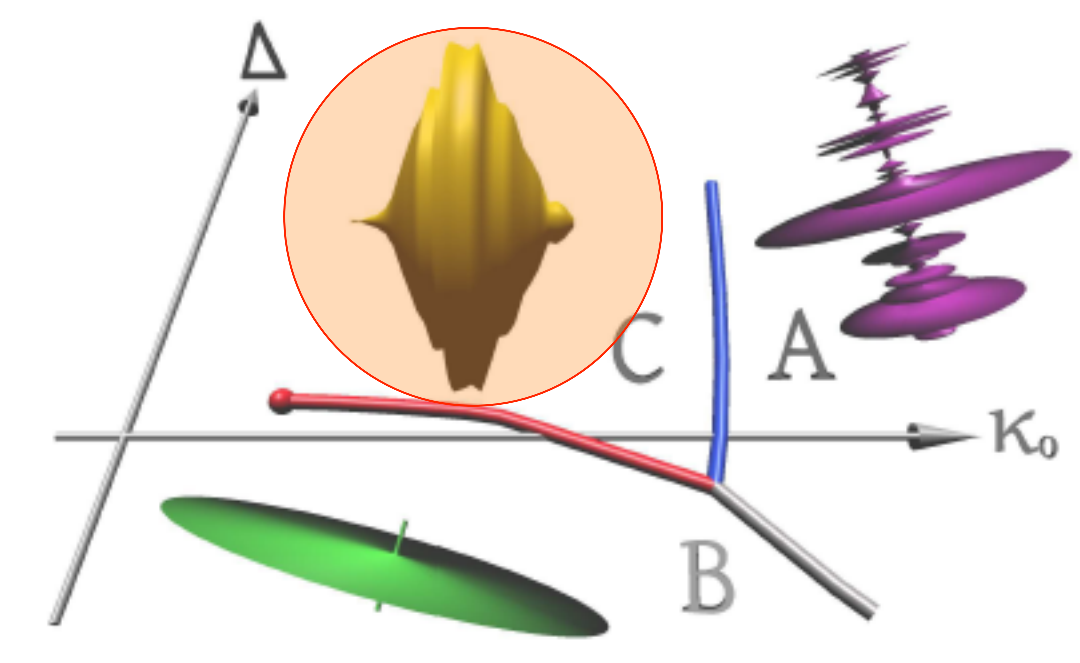


- …to determine transfer matrix elements:

$$\langle n | M | m \rangle = N \sqrt{P_2(n_1 = n, n_2 = m)}$$

- We explicitly symmetrize: $\langle n | M | m \rangle = \langle m | M | n \rangle$

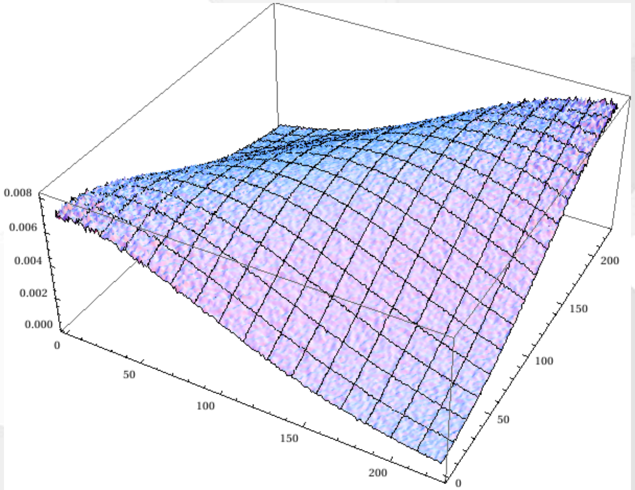
'C' ('de Sitter') phase



- The transfer matrix in phase ‘C’ is measured and fitted with MS action:

$$\langle n | M | m \rangle = N e^{-L_{eff}[n,m]}$$

$$L_{eff}[n,m] = \frac{1}{\Gamma}\left\{ \frac{(n-m)^2}{n+m-2n_o} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right) \right\}$$

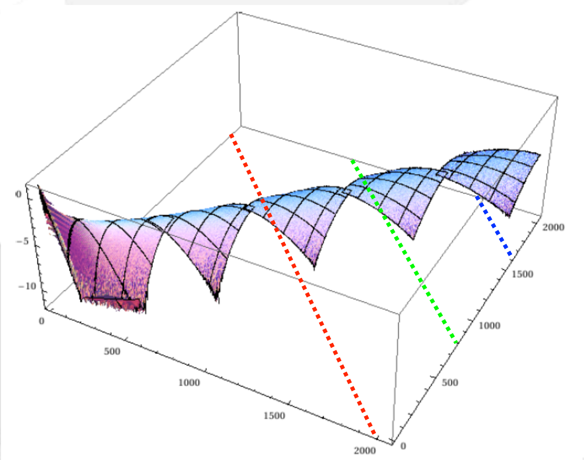


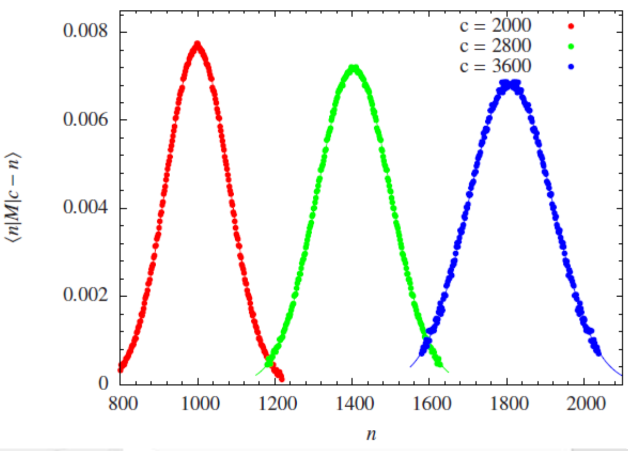
- The kinetic term …

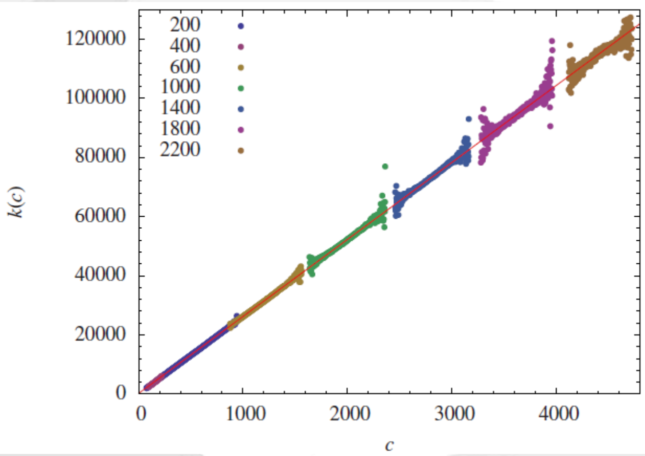
$$\langle n | M | m \rangle = N e^{-\frac{1}{\Gamma}\left\{ \frac{(n-m)^2}{n+m-2n_0} + \mu \left(\frac{n+m}{2}\right)^{1/3} - \lambda \left(\frac{n+m}{2}\right) \right\}}$$

$$k[n+m] = \Gamma\,(n+m-2n_0)$$

  - Gaussian bahaviour for $n+m=s$: $$\langle n | M | s-n \rangle = \tilde{N}[s] \exp\left\{ -\frac{(2n-s)^2}{k[s]} \right\}$$





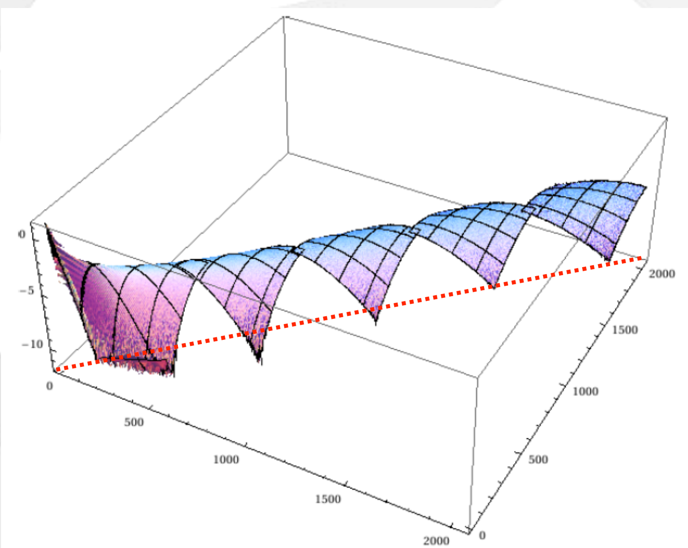


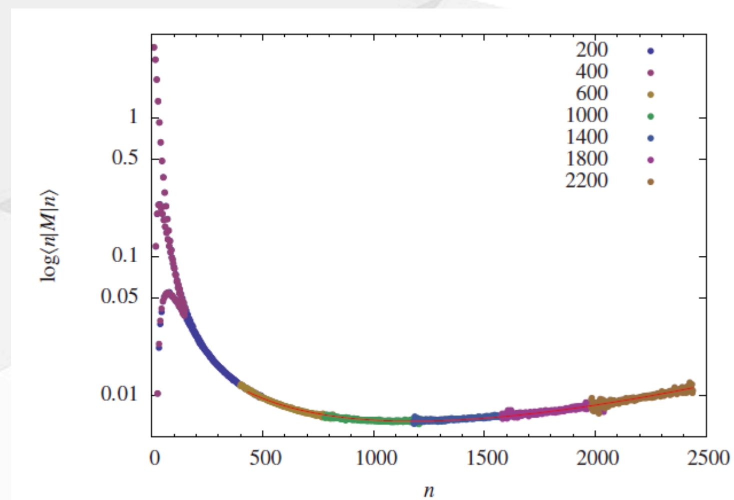
- … and the potential term can be analyzed in detail

$$\langle n|M|m\rangle = N e^{-\frac{1}{\Gamma}\left\{\frac{(n-m)^2}{n+m-2n_0}+\mu\left(\frac{n+m}{2}\right)^{1/3}-\lambda\left(\frac{n+m}{2}\right)\right\}}$$

  - For $n=m$:

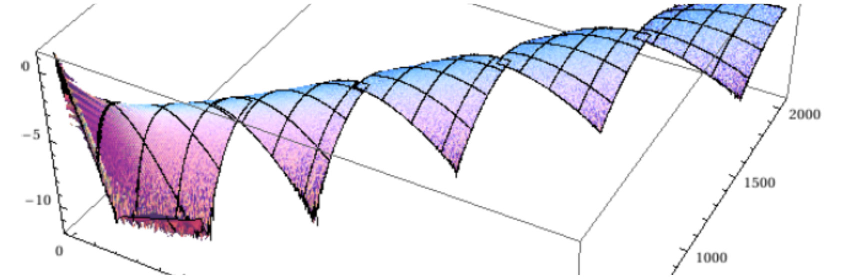
$$\log\langle n|M|n\rangle = -\frac{1}{\Gamma}\left(\mu n^{1/3}-\lambda n\right)+\alpha$$





- The fits agree with the previous method based on the covariance matrix *

$$\langle n|M|m\rangle = Ne^{-\frac{1}{\Gamma}\left\{\frac{(n-m)^2}{n+m-2n_0}+\mu\left(\frac{n+m}{2}\right)^{1/3}-\lambda\left(\frac{n+m}{2}\right)\right\}}$$



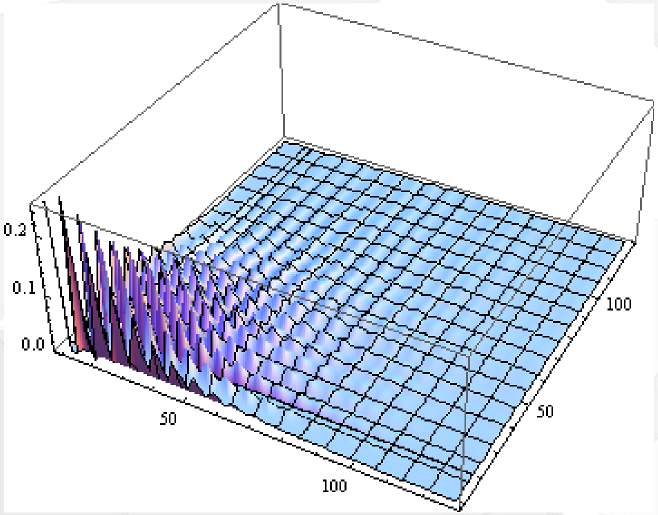
Method	Γ	n_0	μ	λ
Cross-diagonals	26.07 ± 0.02	-3 ± 1	–	–
Diagonal	(26.07)	–	16.5 ± 0.2	0.049 ± 0.001
Full fit	26.17 ± 0.01	7 ± 1	15.0 ± 0.1	0.046 ± 0.001
Previous method*	23 ± 1	–	13.9 ± 0.7	0.027 ± 0.003

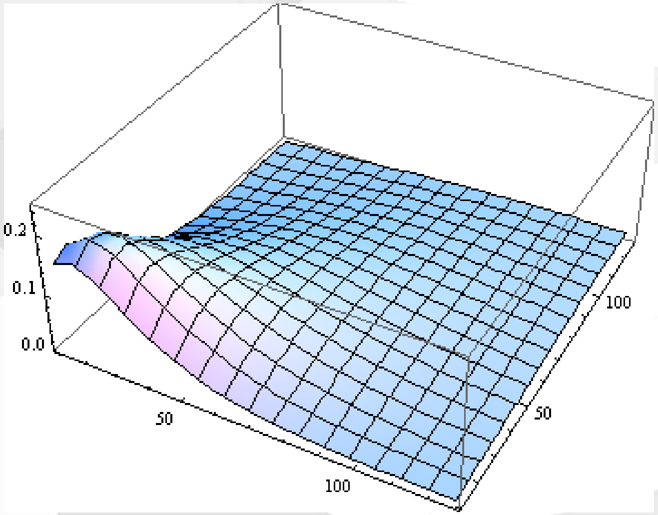
**J. Ambjørn, A. Görlich, J. Jurkiewicz, R. Loll, JGS, T.Trześniewski Nucl. Phys. B 849 (2011) 144*

- The transfer matrix gives access to the effective action in the 'stalk' (small volumes) range:

$$\langle n|M|m\rangle = Ne^{-L_{eff}[n,m]}$$

  - Discretization effects (split into three families of states) makes analytical modeling difficult
  - If we average over these three families M becomes smooth

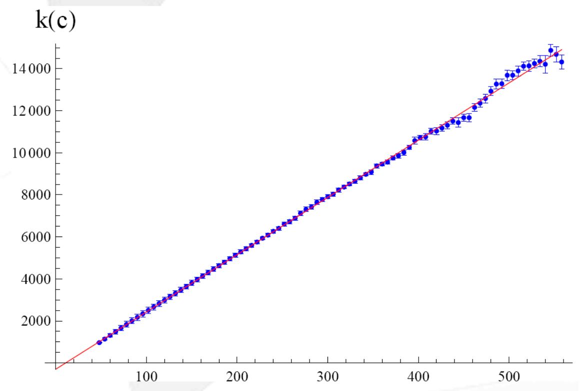




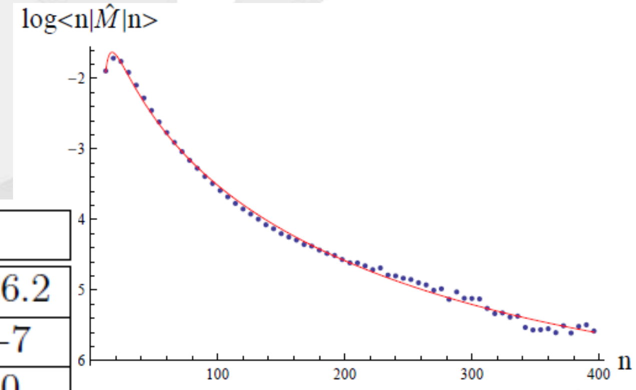
- The effective action for the stalk is basically the same as for the ‘blob’ *

$$S_{eff}^{stalk} = \sum_t \frac{1}{\Gamma}\left\{ \frac{(n_t - n_{t+1})^2}{n_t + n_{t+1} - 2n_0} + \mu\left(\frac{n_t + n_{t+1}}{2}\right)^{1/3} - \lambda\left(\frac{n_t + n_{t+1}}{2}\right)^{1/3} + \delta\left(\frac{n_t + n_{t+1}}{2}\right)^{-\rho} \right\}$$

small volume correction



Parameter	Stalk	Blob
Γ	27.2 ± 0.1	$25.7 - 26.2$
n_0	5 ± 1	$-3 - +7$
μ	34 ± 2	$13 - 30$
λ	0.12 ± 0.02	$0.04 - 0.07$
δ	$(4 \pm 7) \times 10^4$	–
ρ	3 ± 1	–



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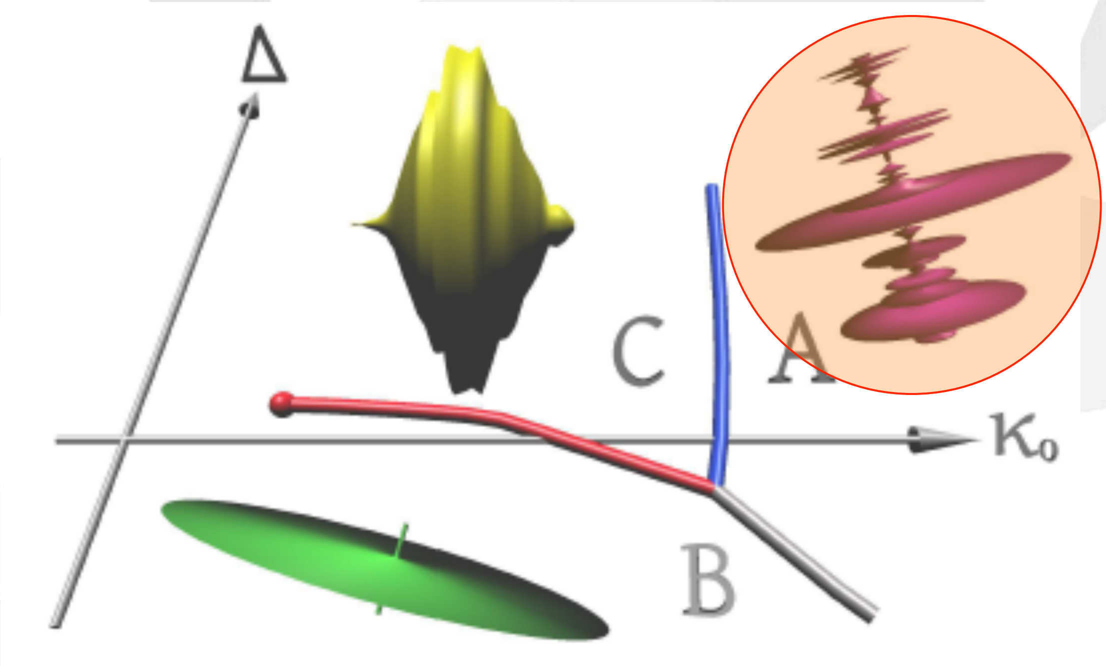
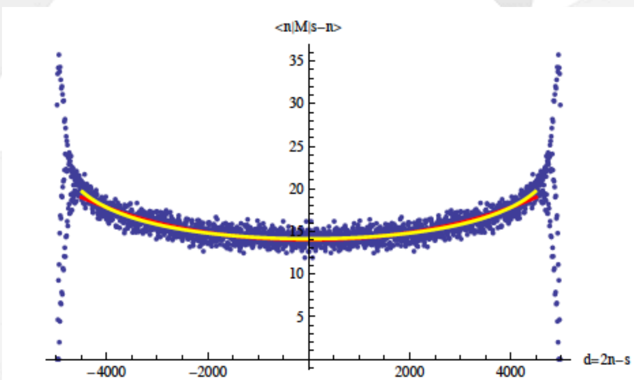
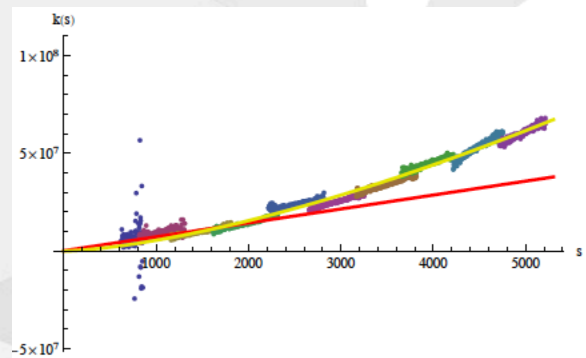
'A' ('uncorrelated') phase



- The kinetic term $\langle n|M|s-n\rangle$ looks anti-gaussian on the first sight …
- Is it possible that we observe a signature change in large Γ regime?

$$L_C[n,m] = \frac{1}{\Gamma}\frac{(n-m)^2}{n+m} + v_C[n+m] \quad ? \quad L_A[n,m] = -\frac{1}{\Gamma}\frac{(n-m)^2}{k[n+m]} + v_A[n+m]$$



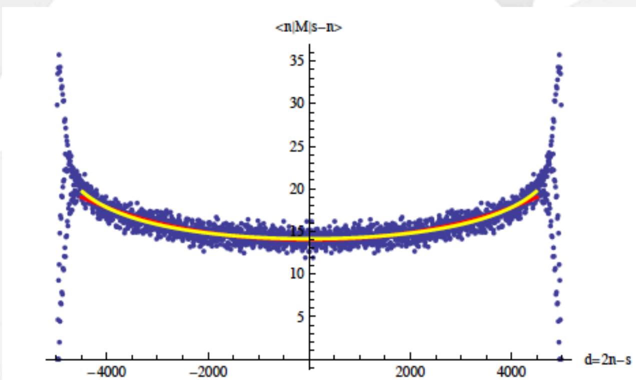


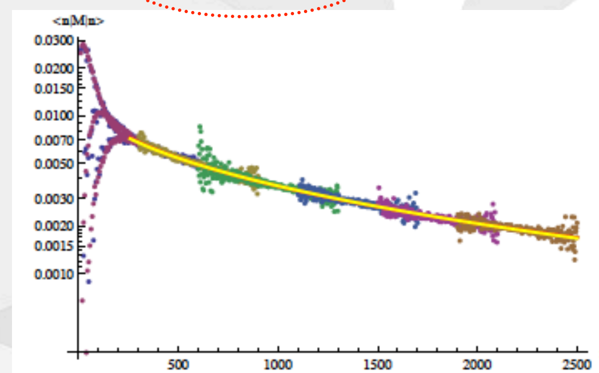
- It looks that $k[n+m]$ is no longer linear, it fits well to:

$$k[n+m] = k_0(n+m)^{2-\alpha}$$

- … but if the kinetic term vanishes as expected and we expand L_A in $s=n+m$ & $d=n\text{-}m$ we recover the effective anti-gaussian behaviour if $\alpha < 1$

$$L_A[n,m] = \mu(n^\alpha + m^\alpha) + \lambda(n+m) = -\frac{\mu\alpha(1-\alpha)}{2^\alpha}\frac{(n-m)^2}{(n+m)^{2-\alpha}} + v_A[n+m] + o(d/s^4)$$

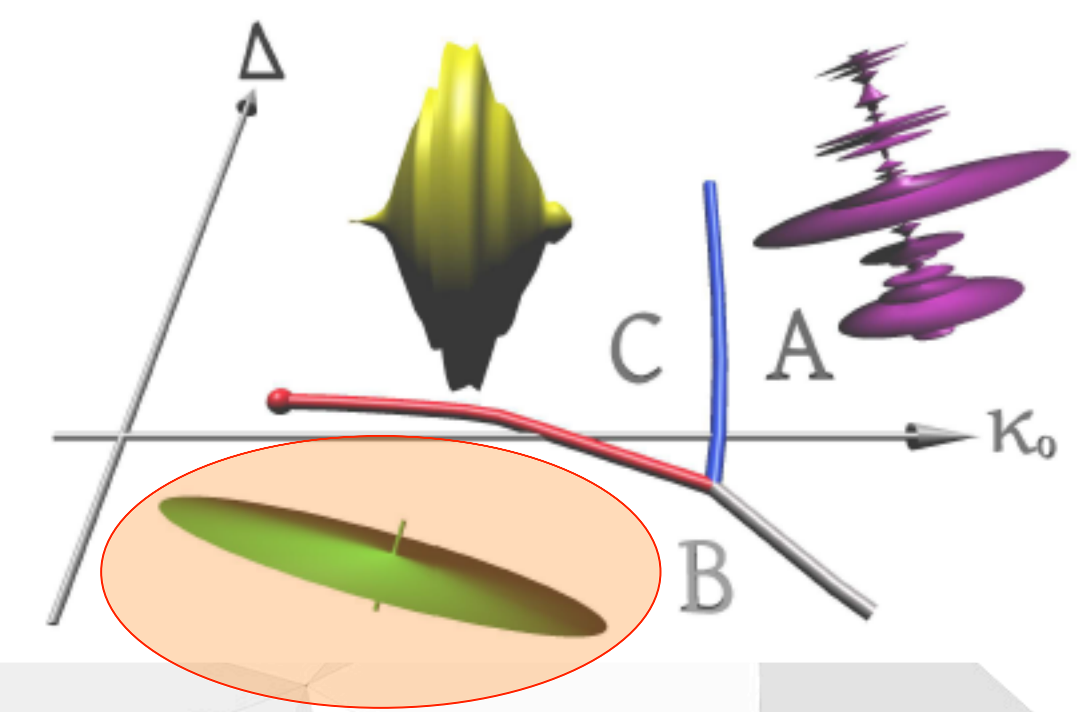




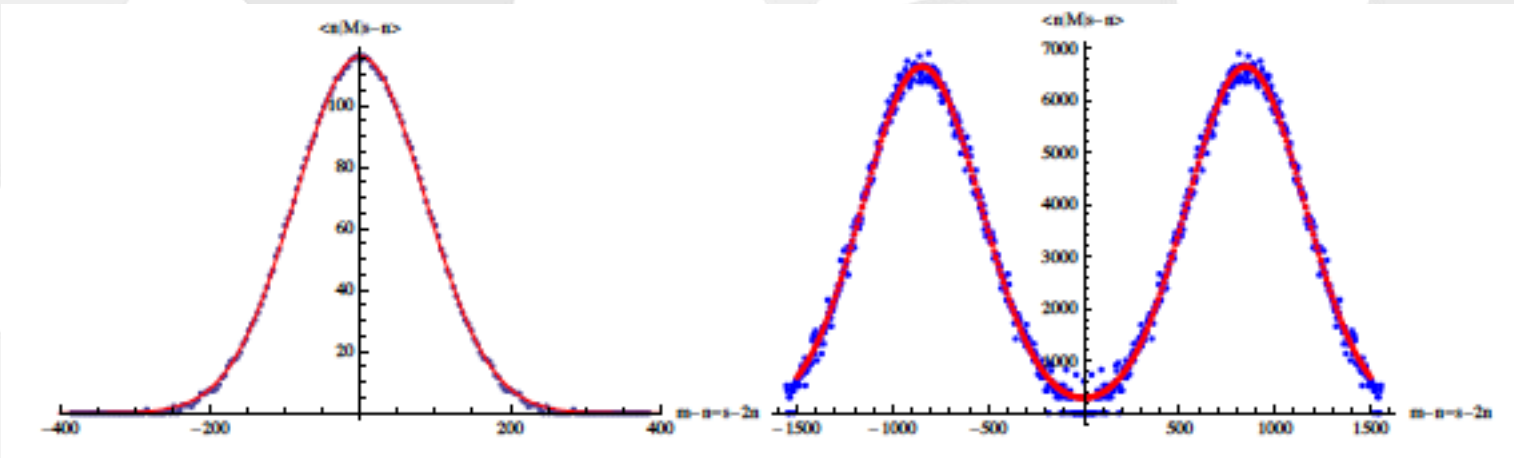
- L_A fits very well to measured potential (diagonal) term $\langle n | M | n \rangle$

$$L_A[n,m] = \mu(n^\alpha + m^\alpha) + \lambda(n+m)$$

'B' ('collapsed') phase



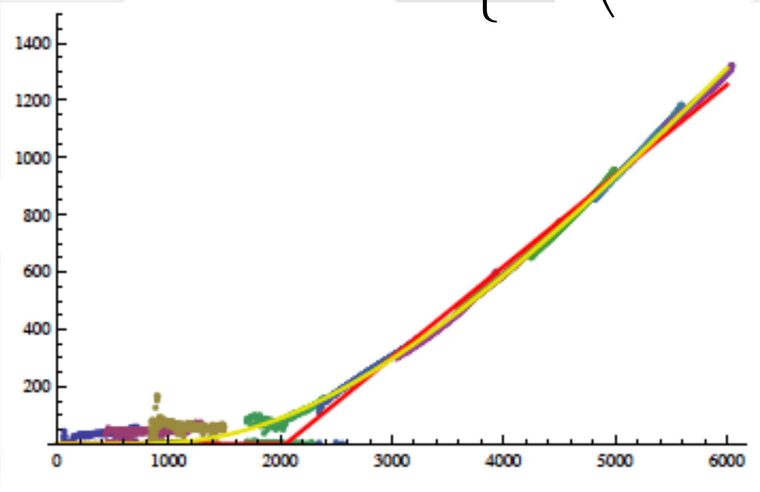
- The kinetic term $\langle n|M|s-n\rangle$ depends strongly on $s=n+m$:

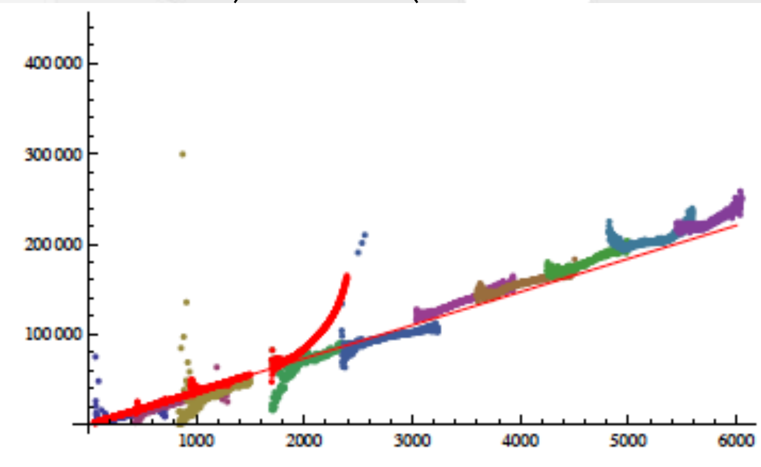


- for s < s_b it fits well to the single gaussian (as in the phase ‘C’)
- for s > s_b the cross-diagonal splits into sum of two gaussians

$$\langle n\,|\,M\,|\,s-n\rangle = N[s]\left\{\exp\left(-\frac{\left((m-n-c[s]\right)^2}{k[s]}\right)+\exp\left(-\frac{\left((m-n+c[s]\right)^2}{k[s]}\right)\right\}$$

$$\langle n|M|s-n\rangle = N[s]\left\{\exp\left(-\frac{\left((m-n-c[s])\right)^2}{k[s]}\right)+\exp\left(-\frac{\left((m-n+c[s])\right)^2}{k[s]}\right)\right\}$$





- c[s] is well fitted by:

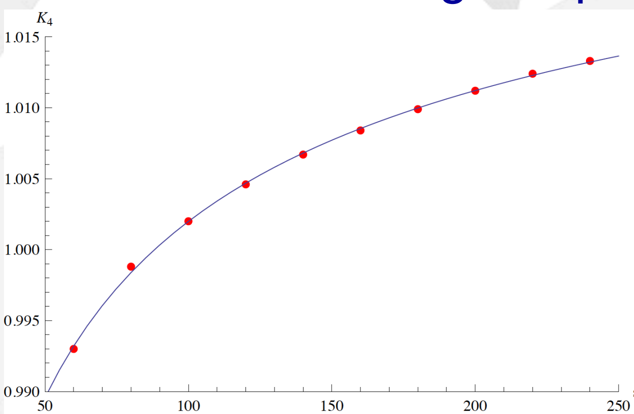
$$c[s] \approx \max\left[0, c_0(s-s_b)\right]$$

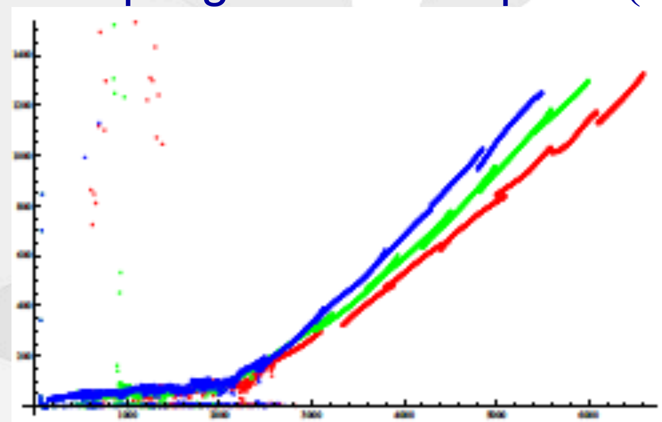
- k[s] is linear (as in the phase ‘C’):

$$k[s] = \Gamma(s-2n_0)$$

$$\langle n | M | m \rangle = N[n+m]\left[\exp\left(-\frac{\left((m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)+\exp\left(-\frac{\left((m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)\right]$$

- This is measured in a given point in the bare coupling constants space (K_0, Δ, K_4)





- But inside the phase ‘B’: K_4^{crit} is volume dependent …

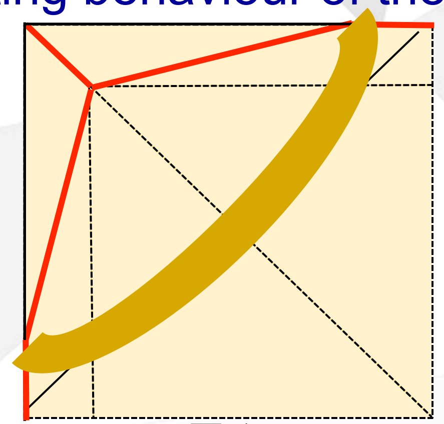
$$K_4^{crit} = K_4^{\infty} - K \cdot s^{-\gamma}$$

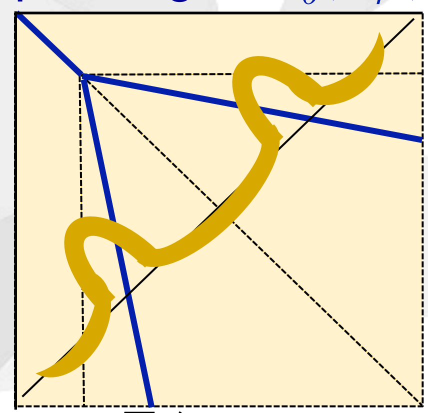
- …and we are interested in large volume behaviour (where K_4 ⇧):

$$S_b = const \quad , \quad c_0 \Uparrow$$

$$\langle n|M|m\rangle = N[n+m]\left[\exp\left(-\frac{\left((m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)+\exp\left(-\frac{\left((m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)\right]$$

- The limiting behaviour of the system may differ depending on $c_0(K_4^\infty)$

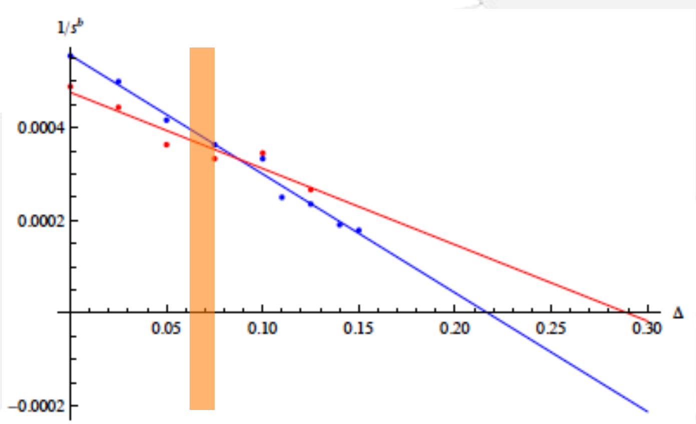


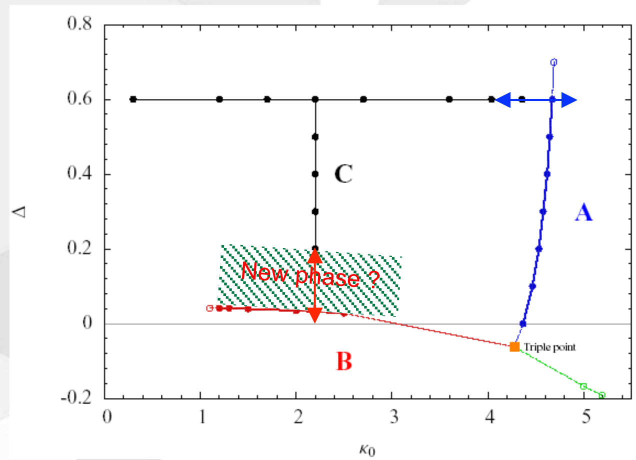


- If $c_0(K_4^\infty) > \sqrt{2}/2$ the effective kinetic term for large vol. looks like “anti-gaussian” and we get generic ‘B’ phase configuration
- If $c_0(K_4^\infty) < \sqrt{2}/2$ “double gaussian” behaviour persists even for large vol. and typical configuration may mimic ‘C’ phase blob structure

$$\langle n | M | m \rangle = N[n+m]\left[\exp\left(-\frac{\left((m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)+\exp\left(-\frac{\left((m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)\right]$$

- After crossing B-C phase transition we should regain MS action ($s_b \to \infty$)

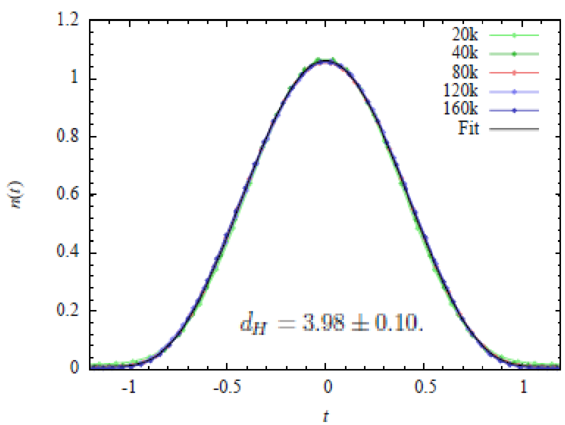


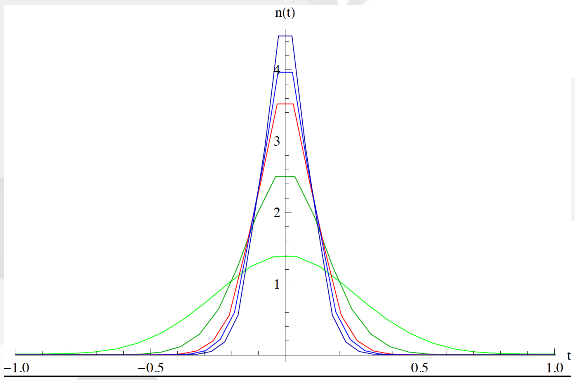


- But it happens for Δ much bigger then suggested by previous phase transitions studies !!!
- It indicates that a newly discovered “bifurcation” phase may exist in CDT

$$\langle n | M | m \rangle = N[n+m]\left[\exp\left(-\frac{\left((m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)+\exp\left(-\frac{\left((m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m-2n_0)}\right)\right]$$

- Generic configurations in the "bifurcation phase" may resemble 'C' phase structure but they should have different scaling properties

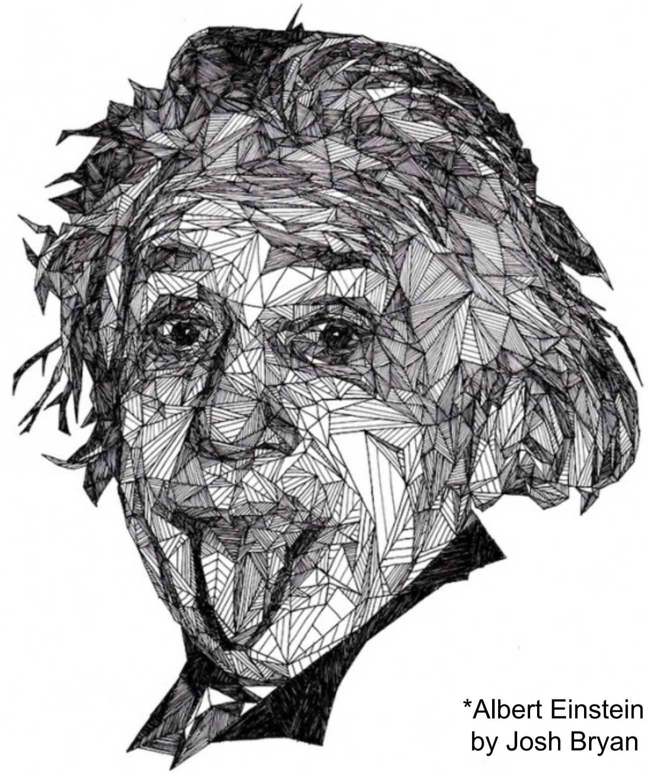




- Typical scaling inside the 'C' phase with Hausdorf dimension 4 …
- … is different that inside the newly discovered "bifurcation phase".

- The transfer matrix approach allows to measure the effective action directly
- The effective action inside phase 'C' is well described by the MS model
- The transfer matrix method gives access to effective action in other phases
- Inside 'A' (uncorrelated) phase the kinetic term vanishes as expected ⇒ possible relation to asymptotic silence ???
- Inside 'B' (collapsed) phase the structure of the transfer matrix is very nontrivial ("double-gaussian" kinetic term)
- "Double-gaussian" structure continues after crossing previously measured B-C phase transition line ⇒ possible existence of new "bifurcation phase" with different scaling properties (will it survive in infinite volume limit ???)
- Open questions:
  - "bifurcation" mechanism in B-C phase transition ⇒ possible signature change ??
  - scaling properties / critical exponents in A-C and B-C phase transitions

Thank You for attention !!!



*Albert Einstein
by Josh Bryan

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